

$$(1) \quad f_{k+1} = f_k + \frac{df}{dx}\bigg|_k \Delta x + \frac{1}{2} \frac{d^2f}{dx^2}\bigg|_k \Delta x^2 + \frac{1}{6} \frac{d^3f}{dx^3}\bigg|_k \Delta x^3 + O(\Delta x^4)$$

$$(2) \quad f_{k-1} = f_k - \frac{df}{dx}\bigg|_k \Delta x + \frac{1}{2} \frac{d^2f}{dx^2}\bigg|_k \Delta x^2 - \frac{1}{6} \frac{d^3f}{dx^3}\bigg|_k \Delta x^3 + O(\Delta x^4)$$

(1) + (2)

$$f_{k+1} + f_{k-1} = 2f_k + \frac{d^2f}{dx^2}\bigg|_k \Delta x^2 + O(\Delta x^4)$$

$$\frac{d^2f}{dx^2}\bigg|_k = \frac{f_{k+1} - 2f_k + f_{k-1}}{\Delta x^2} + O(\Delta x^2)$$

Finite Difference
formula for second-
order derivative

SECOND ORDER ACCURACY