LORENTZ FORCE

- Point charge in free-space q

VECTOR PRODUCT

$|\bar{\alpha} \times \bar{\beta}| = |\bar{\alpha}||\bar{\beta}| \sin\theta$

$$\bar{F} = q\left(\bar{E} + \bar{v} \times \bar{B}\right)$$

$\bar{F}$: electromag force on q

$\bar{v}$: velocity of particle

① $v = |\bar{v}| = 0$

$[J] = [N][m]$

$\Rightarrow [N] = [J]/[m]$

$$\bar{F} = q\bar{E} \Rightarrow \bar{E} = \bar{F}/q = \left[\frac{\text{FORCE}}{\text{CHARGE}}\right] = \left[\frac{N}{C}\right] = [N/C] = \left[\frac{J}{C\,m}\right]$$

$\bar{E}$: ELECTRIC FIELD

$[V/m]$

$[J/C] = [V]$

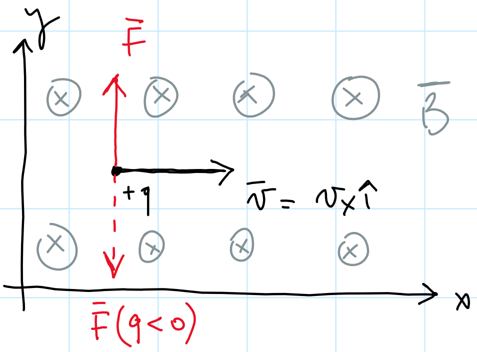
② $v \neq 0$, $E = 0$

B: MAGNETIC FIELD // MAGNETIC FLUX DENSITY

$$\bar{F} = q\,\bar{v} \times \bar{B} \Rightarrow B = \frac{F}{q\,v \sin(\theta)} = \left[\frac{\text{FORCE}}{\text{CHARGE SPEED}}\right]$$

- Direction of $\bar{F}$ (or $\bar{B}$) from RIGHT-HAND RULE

Example.



For a distribution of charged particles

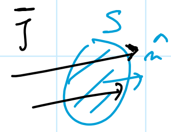
$$\Rightarrow \quad \bar{f} = \rho\,(\bar{E} + \bar{J} \times \bar{B})$$

FORCE DENSITY $[N/m^3]$

ELECTRIC CHARGE DENSITY

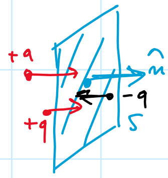
CURRENT DENSITY

$$i = \int_S \bar{J} \cdot d\bar{S}$$

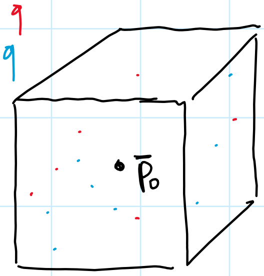


$$\bar{J}(\bar{P}_0) \cdot \hat{n} = \lim_{\substack{\Delta S \to 0 \\ \Delta t \to 0}} \frac{\Delta q}{\Delta S\, \Delta t} \quad \left[\frac{C}{m^2 s}\right] = \left[\frac{A}{m^2}\right]$$

MACRO



• +q
• −q



$$\rho(\bar{P}_0) = \lim_{\Delta V \to 0} \frac{\sum q_{i \in \Delta V}}{\Delta V} = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V} \quad [C/m^3]$$

CONTINUOUS FUNCTION

VOLUME of V

MACROSCOPIC LIMIT

• ΔV → SMALL
→ LARGE ENOUGH that a STATISTICALLY SIGNIFICANT # of PARTICLES are within ΔV

MAXWELL'S EQUATIONS in FREE SPACE [Hp: SCD]

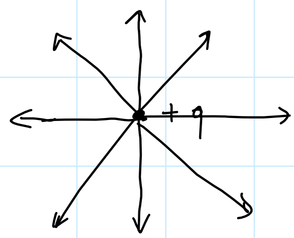
GAUSS LAWS for $\bar{E}$ & $\bar{B}$

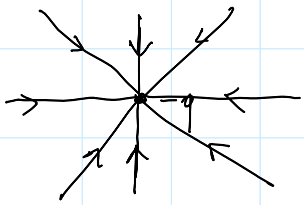
LOCAL FORM

$$\nabla \cdot \bar{E} = \rho / \varepsilon_0$$

PERMITTIVITY of FREE SPACE

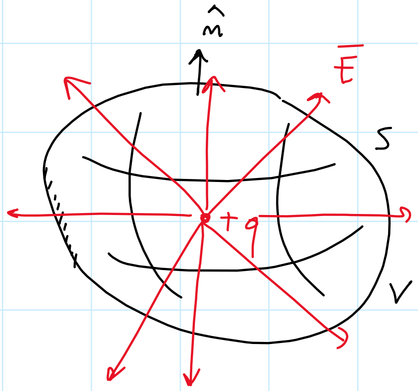
$8{,}854 \cdot 10^{-12}$ F/m





" Charges are SOURCES / SINKS for $\bar{E}$ "

Consider closed surface $S \to V$.



$$\int_V \nabla\cdot\bar{E}\, dV = \int_V \rho/\varepsilon_0\, dV \;\to\; \int_V \rho\, dv = Q$$

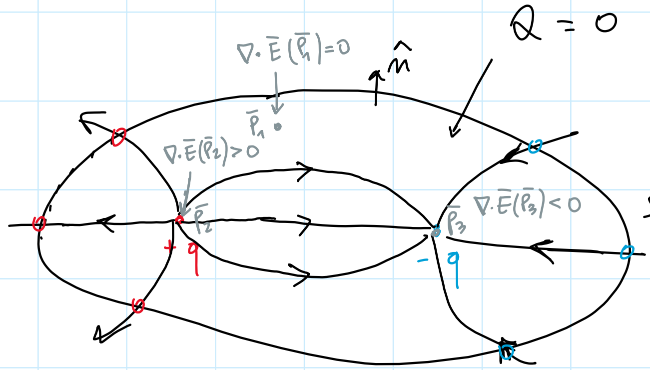
DIV. theorem $\downarrow$

$$\Rightarrow \oint_S \bar{E}\cdot d\bar{S} = Q/\varepsilon_0$$

"the flux of $\bar{E}$ through any closed surf. S is proportional to the net charge Q inside $V(S)$"

Example:

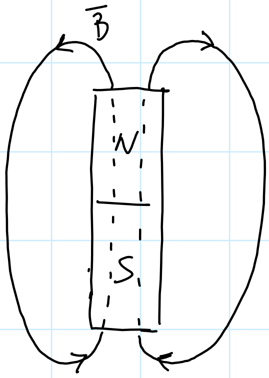
$$\oint_S \bar{E}\cdot d\bar{S} = 0$$



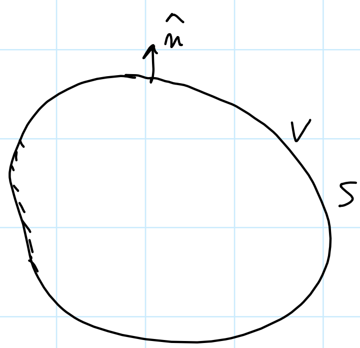
$$\nabla\cdot\bar{B} = 0$$

$\uparrow$

"there are no sources/sinks for $\bar{B}$" $\Rightarrow$ Field Lines CLOSED



Consider closed $S \to V$



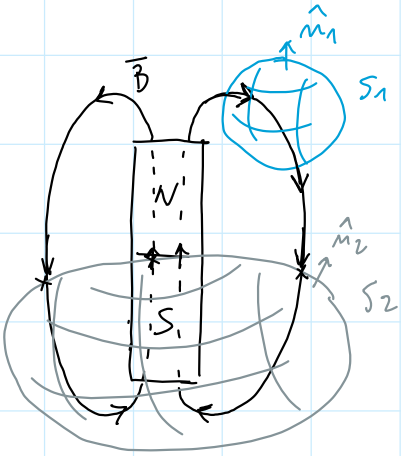
$$\int_V \nabla\cdot\bar{B}\, dV = \int_V 0\, dV$$

$\downarrow$ DIV TH

$$\oint_S \bar{B}\cdot d\bar{S} = 0$$

$$\oint_S \bar{B} \cdot d\bar{S} = 0$$

"the flux of $\bar{B}$ through ANY closed surface S is ZERO"



$$\oint_{S_1} \bar{B} \cdot d\bar{S} = 0$$

$$\oint_{S_2} \bar{B} \cdot d\bar{S} = 0$$

$\bar{B}$ is a SOLENOIDAL FIELD $\overset{SCD}{\longleftrightarrow}$ DIV-FREE FIELD

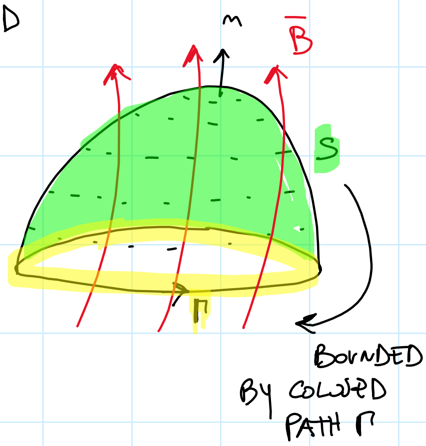
↓

Flux of $\bar{B}$ through any OPEN SURFACE S is a LINKED FLUX

$$\underbrace{\int_S \bar{B}\, d\bar{S}}_{} \Rightarrow \text{DEPEND only on } \Gamma$$

$$\Phi_{B,S} = \Phi_{B,\Gamma}$$

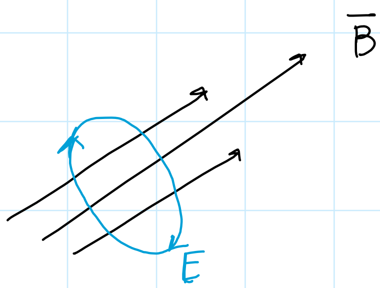
↑ LINKED



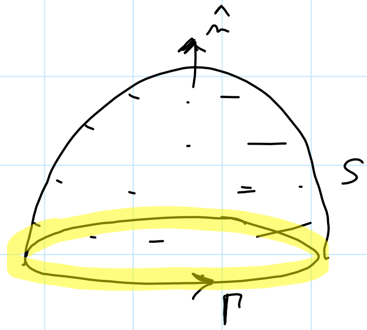
FARADAY - NEUMANN - LENZ

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

"time variations of $\bar{B}$ produce a CIRCULATING electric field"



Consider an OPEN surface S
bounded by closed curve Γ



$$\int_S \nabla \times \bar{E} \cdot d\bar{S} = \int_S -\frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}$$

STOKES theorem

↓ if S does not change

$$\oint_\Gamma \underbrace{\bar{E}}_{[V/m]} \cdot \underbrace{d\bar{l}}_{[m]} = - \frac{d}{dt} \int_S \bar{B} \cdot d\bar{S}$$

" time variations of $\Phi_{B,\Gamma}$ induce an EMF in Γ "

CIRCULATION of $\bar{E}$ along Γ

↓

ELECTROMOTIVE FORCE (EMF)

$\mathcal{E}$ [V]

$$- \frac{d\Phi_{B,S}}{dt} = - \frac{d\Phi_{B,\Gamma}}{dt}$$

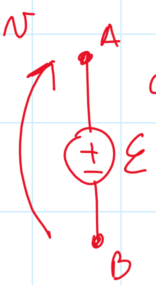
LINKED FLUX

$\int_A^B \bar{E} \cdot d\bar{l} \rightarrow$ VOLTAGE [V]

$\oint_\Gamma \bar{E} \cdot d\bar{l} \rightarrow$ EMF [V]

↓

WORK per unit charge carried out by non conservative electric fields



$\bar{E}$ → $\bar{E}_\rho$ ⇒ CHARGES → CONSERVATIVE FIELD

ELECTROSTATIC FIELD

$$\oint_\Gamma \bar{E}_\rho \cdot d\bar{l} = 0$$

$\bar{E}$ → $\bar{E}_{-\frac{d\Phi_{B,\Gamma}}{dt}}$ ⇒ TIME VAR. of LINKED FLUXES ⇒ NOT CONSERVATIVE

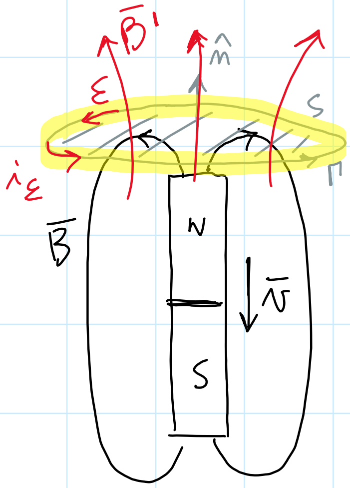
$$\oint_\Gamma \bar{E}_{-\frac{d\Phi}{dt}} \cdot d\bar{l} = \mathcal{E}$$

↓

Produce SAME effects on CHARGES

Example: permanent magnet with closed conductor Γ

$\uparrow \bar{B}'$ $\uparrow \hat{n}$ $\nearrow$



$v = 0 \qquad \Phi_{B,\Gamma} > 0 \Rightarrow \dfrac{d\Phi_{B,\Gamma}}{dt} = 0$

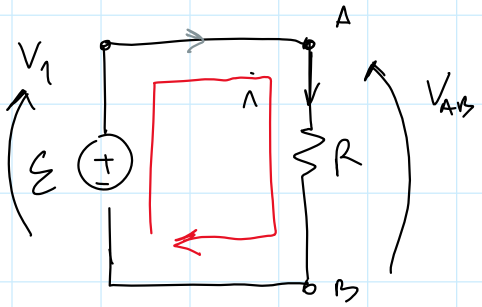
$v > 0 \qquad \Phi_{B,\Gamma} > 0 \Rightarrow \dfrac{d\Phi_{B,\Gamma}}{dt} < 0$

$$\downarrow$$

$$\oint_\Gamma \bar{E} \cdot d\bar{l} = \mathcal{E} \Rightarrow i_\mathcal{E} \Rightarrow \bar{B}'$$

$$\downarrow$$

$$\int_S \bar{B}' \cdot dS = \Phi_{B',\Gamma} > 0$$



$i = V_{AB}/R$

LKT : $V_1 - V_{AB} = 0$

$\oint_\Gamma \bar{E}\, dS = 0$

$\Uparrow$

if $\bar{E}$ is conserv.

AMPÈRE - MAXWELL LAW

$$\nabla \times \left(1/\mu_0 \bar{B} \right) = \bar{J} + \frac{\partial \varepsilon_0 \bar{E}}{\partial t} = \bar{J}_t$$

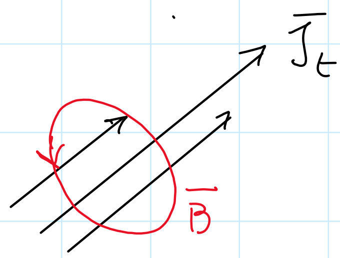
PERMEABILITY of free-space

$\mu_0 = 4\pi \cdot 10^{-7}\ H/m$

CONDUCTION CURRENT DENSITY

DISPLACEMENT CURRENT DENSITY

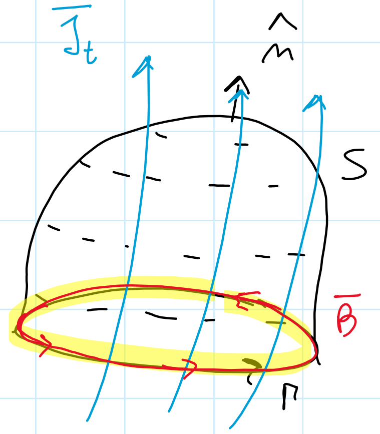
TOTAL CURRENT DENSITY



$\mu_0 = 4\pi \cdot 10^{-7}\ H/m$

" the $\bar{J}_t$ produces a circulating $\bar{B}$ "

consider an open SURFACE S bounded by a closed curve Γ



$$\int_S \nabla \times \left(\frac{1}{\mu_0}\bar{B}\right) \cdot d\bar{S} = \int_S \left(\bar{J} + \frac{\partial \varepsilon_0 \bar{E}}{\partial t}\right) \cdot d\bar{S}$$

STOKES theorem

$$\underbrace{\oint_\Gamma \frac{1}{\mu_0}\bar{B} \cdot d\bar{l}} = i + i_D = i_T$$

CONDUCTION CURRENT (i)

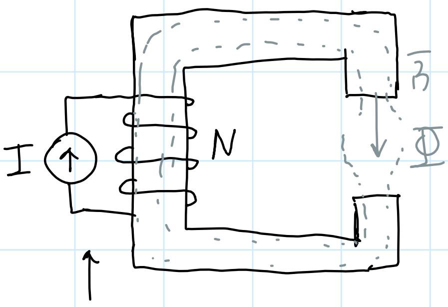
DISPLACEMENT CURRENT (i_D)

MAGNETOMOTIVE FORCE $\mathcal{F}$ [A]

$$\mathcal{F} = \Phi \mathcal{R}$$

mag FLUX (Φ)

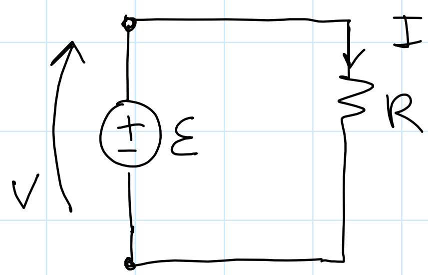
RELUCTANCE ($\mathcal{R}$)



$\mathcal{F} = NI$

$$V = IR$$

$\mathcal{E}$



HOLDS for SOLENOIDAL fields

ANY CLOSED PATH

" the total current density flux linked with Γ produces a

" the total current density flux linked with" Γ produces a $\underbrace{\text{magnetomotive force}}_{\text{MMF}}$ in Γ "

Proof.

$$\underbrace{\nabla\cdot\left(\nabla\times\left(\frac{1}{\mu_0}\bar{B}\right)\right)}_{\nabla\cdot(\nabla\times(\dots))=0} = \nabla\cdot\underbrace{\left(\bar{J} + \frac{\partial \varepsilon_0 \bar{E}}{\partial t}\right)}_{\bar{J}_t} = \nabla\cdot\bar{J}_t$$

$$\boxed{\nabla\cdot\bar{J}_t = 0} \Rightarrow$$

$\bar{J}_t$ is a SOLENOIDAL FIELD (its flux is a LINKED FLUX)

(A.K.A.: charge conservation equation)

CURRENT CONTINUITY EQUATION

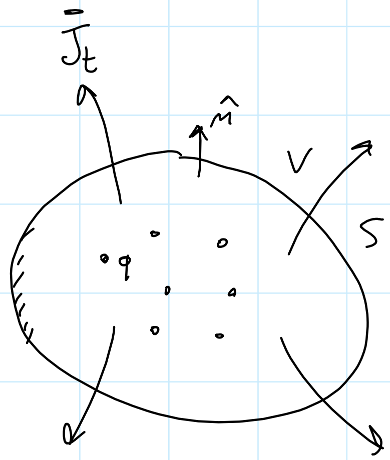
$$\frac{\partial \rho}{\partial t} + \nabla\cdot\bar{J} = 0$$

$$\int_V \nabla\cdot\bar{J}\, dV = -\int_V \frac{\partial \rho}{\partial t}\, dV$$

DIV. Th — if S, V do not change

$$\oint_S \bar{J}\cdot d\bar{S} = -\frac{d}{dt}\int_V \rho\, dV$$

$$\boxed{i = -\frac{dQ}{dt}}$$



" Current leaving a CLOSED SURFACE corresponds to a DECREASE of the NET charge inside V (S) "

$$\oint_S \bar{J} \cdot d\bar{S} = -\int_{S_1} \bar{J} \cdot d\bar{S} + \int_{S_2} \dots$$