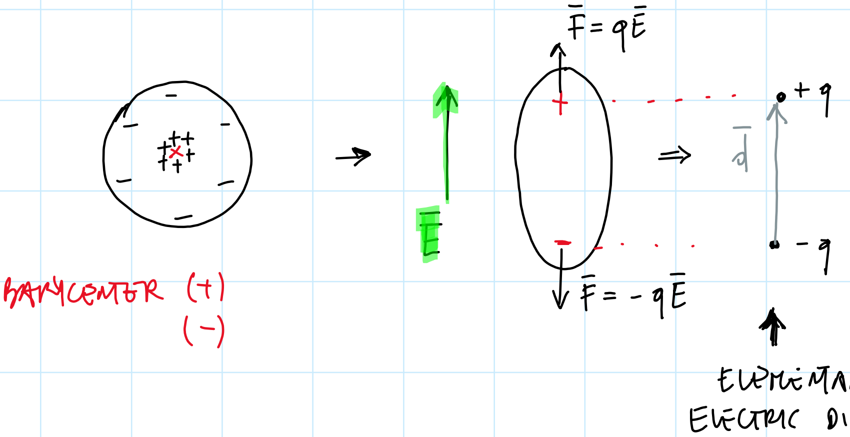
ELECTRIC POLARIZATION

Hp: DIELECTRICS $\sigma \sim 10^{-7 \div -8}$ S/m

NON-POLAR MATTER → INDUCED POLARIZATION

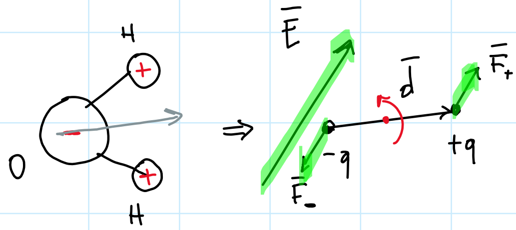


ELECTRIC DIPOLE MOMENT

$$\bar{p} = q\bar{d} \quad [Cm]$$

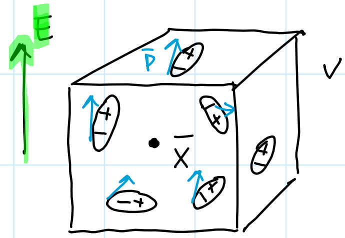
POLAR MATTER → POLARIZATION BY ORIENTATION

Ex. H_2O



MACROSCOPIC DESCRIPTION

consider a point $\bar{x}$



ELECTRIC POLARIZATION FIELD

$$\bar{P}(\bar{x}) = \lim_{\Delta V \to 0} \frac{\Delta \bar{p}}{\Delta V} \quad \left[\frac{Cm}{m^3}\right] = \left[C/m^2\right]$$

↑ local degree of POLARIZATION of matter

$$\sum_i \bar{p}_i \in V = \Delta \bar{p}$$

↑ RESULTANT of electric dipoles in V

it can be proven that: $\boxed{\underbrace{\nabla \cdot \bar{P}}_{\frac{1}{m}\frac{C}{m^2}} = -\rho_p}$ ⇒ POLARIZATION charge density

↙ VOLUME charge density "FREE"

$m \; m^2$

in free - space : $\nabla \cdot \bar{E} = \rho / \varepsilon_0$ ← VOLUME charge density "FREE"

in matter : $\nabla \cdot \bar{E} = \dfrac{\rho + \rho_P}{\varepsilon_0}$ ← $-\nabla \cdot \bar{P}$

$$\nabla \cdot (\varepsilon_0 \bar{E}) = \rho - \nabla \cdot \bar{P}$$

$$\nabla \cdot \big(\underbrace{\varepsilon_0 \bar{E} + \bar{P}}_{\bar{D}}\big) = \rho \implies \boxed{\nabla \cdot \bar{D} = \rho}$$

$\bar{D}$ ELECTRIC DISPLACEMENT FIELD $[C/m^2]$

$$\boxed{\nabla \cdot \bar{E} = \rho}$$

- in free - space no polariz. $\bar{P} = 0$

$$\bar{D} = \varepsilon_0 \bar{E} + \cancel{\bar{P}} = \varepsilon_0 \bar{E}$$

ε_0: $8{,}854 \cdot 10^{-12}$ F/m

- in matter

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} , \qquad \bar{P} = \bar{P}(\bar{E}) \implies \bar{D} = \bar{D}(\bar{E})$$

in general :

$$\bar{P} = \bar{\bar{\alpha}} \, \bar{E} \rightarrow \bar{\bar{\alpha}} = \begin{bmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{bmatrix}$$

$\bar{\bar{\alpha}}$: POLARIZABILITY TENSOR

Pol induced along z by x- component of $\bar{E}$

Polarization along x, y, z induced by field components along x, y, z

Hp: LINEAR, NON - ISOTROPIC MATERIAL

$$\bar{\bar{\alpha}} = \begin{bmatrix} 10 & & \\ & 1 & \\ & & 1 \end{bmatrix} \qquad \bar{P} = \begin{bmatrix} 10 & & \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\alpha = \begin{bmatrix} & 1 & \\ & & 1 \end{bmatrix} \qquad \begin{bmatrix} & 1 & \\ & & 1 \end{bmatrix}\begin{bmatrix} E_y \\ E_z \end{bmatrix}$$

$\downarrow$

$$P_x \hat{\imath} = 10\, E_x \hat{\imath}$$
$$P_y \hat{\jmath} = 1\, E_y \hat{\jmath}$$
$$P_z \hat{k} = 1\, E_z \hat{k}$$

ISOTROPIC, LINEAR MATERIALS

$$\bar{P} = \varepsilon_0 \chi_e \bar{E}$$

$\uparrow$ \chi

ELECTRIC SUSCEPTIBILITY [-]

$$\nabla \cdot (\varepsilon_0 \bar{E} + \bar{P}) = \rho$$

$\downarrow$

$$\nabla \cdot (\underbrace{\varepsilon_0 \bar{E} + \varepsilon_0 \chi_e \bar{E}}) = \rho$$

$$\bar{D} = \varepsilon_0 (\underbrace{1+\chi_e}_{\varepsilon_r}) \bar{E}$$

ε_r RELATIVE PERMITTIVITY of the material

$\Downarrow$

$$\nabla \cdot \bar{D} = \rho$$

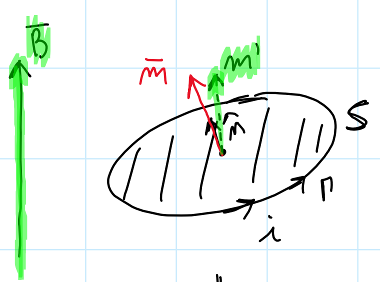
$$\boxed{\nabla \cdot (\varepsilon_0 \varepsilon_r \bar{E}) = \rho}$$

$\uparrow$

$\bar{D}$ same direction of $\bar{E}$ SCALED by $\underbrace{\varepsilon_0 \varepsilon_r}_{\varepsilon}$

ε_r
- ... in AIR ~ 1
- ... in free-space $= 1$
- ... for DIELECTRICS $\sim 2 \div 5$
- for WATER ~ 80

MAGNETIZATION



$\downarrow$

ELEMENTARY MAGNETIC DIPOLE

$$\bar{m} = i \Delta \bar{S} = i \Delta S \hat{n} \quad [A \cdot m^2]$$

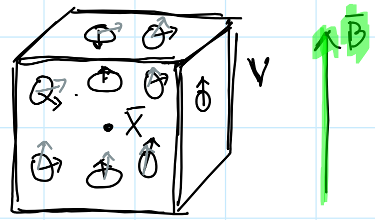
magnetic dipole moment

MACROSCOPIC DESCRIPTION.

MAGNETIZATION VECTOR

$$\bar M(\bar x) = \lim_{\Delta V\to 0} \frac{\Delta \bar m}{\Delta V} \quad \left[\frac{Am^2}{m^3}\right] = \left[A/m\right]$$

Degree of local magnetization



$$\sum \bar m_{i\in V} = \Delta \bar m$$

it can be shown that:

$$\rightarrow \nabla\times\bar M = \bar J_m$$

magnetization current density

$$\left[\frac{1}{m}\,\frac{A}{m}\right]$$

$$\rightarrow \frac{\partial \bar P}{\partial t} = \bar J_p$$

polarization current density

$$\left[\frac{1}{s}\,\frac{C}{m^2}\right] \quad A$$

DISPLACEMENT CURRENT DENSITY

in free space. $\nabla\times\left(1/\mu_0 \bar B\right) = \bar J + \boxed{\frac{\partial \varepsilon_0 \bar E}{\partial t}}$

in matter $\nabla\times\left(1/\mu_0 \bar B\right) = \bar J + \underbrace{\boxed{\bar J_m}}_{\nabla\times\bar M} + \frac{\partial \varepsilon_0 \bar E}{\partial t} + \underbrace{\boxed{\bar J_p}}_{\partial\bar P/\partial t}$

$$\nabla\times\underbrace{\left(1/\mu_0 \bar B - \bar M\right)}_{\bar H\ [A/m]} = \bar J + \frac{\partial}{\partial t}\underbrace{\left(\varepsilon_0\bar E + \bar P\right)}_{\bar D}$$

A-M in matter

$$\boxed{\nabla\times\bar H = \bar J + \frac{\partial \bar D}{\partial t}}$$

$\bar B$	$\bar H$
mag field	(−)
mag. flux density	mag field

$$\rightarrow \bar H = 1/\mu_0 \bar B - \bar M$$

$$\rightarrow \bar{H} = \frac{1}{\mu_0}\bar{B} - \bar{M}$$

$$\Rightarrow \bar{B} = \mu_0(\bar{H} + \bar{M}) \qquad \bar{M} = \bar{M}(\bar{H}) \rightarrow \bar{B} = \bar{B}(\bar{H})$$

For isotropic and linear materials

$$\bar{M} = \chi_m \bar{H}$$

$\Uparrow$

MAGNETIC SUSCEPTIBILITY

[-]

$$\Rightarrow \bar{B} = \mu_0(\bar{H} + \chi_m \bar{H})$$

$$= \mu_0 \underbrace{(1 + \chi_m)}_{\mu_r \; [-]} \bar{H}$$

$\uparrow$

RELATIVE PERMEABILITY

$\Downarrow$

$$\bar{B} = \underbrace{\mu_0}_{4\pi \, 10^{-7} \, H/m} \mu_r \bar{H}$$

μ_r
- .. 1 for free-space
- . ~ 1 for air
- .. $\gg 1$ for ferromagnetic materials — NON-LINEAR; $\sim 10^{3\div4}$ SOFT MATERIALS