L15

05 November 2025 09:09

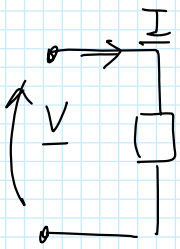
$$\underline{N} = \underline{V}\,\underline{I}^{*} = P + j\,Q$$

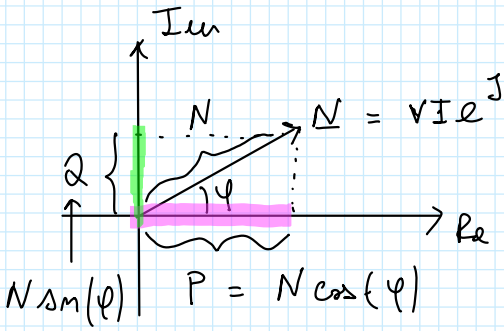
P: potenza attiva [W]

Q: potenza reattiva [VAr]

$$P = \frac{1}{T}\int_{-T/2}^{+T/2} p(t)\,dt$$

ampiezza potenza istantanea reattiva





sfasamento $\underline{V}$ e $\underline{I}$

$$\underline{N} = VI\,e^{j\varphi}$$

$$N = |\underline{N}| = \sqrt{P^2 + Q^2}$$

POTENZA APPARENTE [VA]

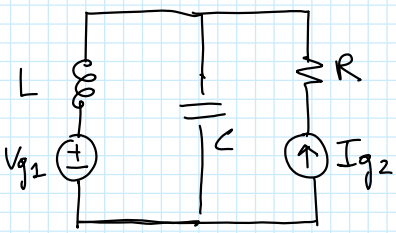
entità delle sollecitazioni subite dal componente

Esercizio

- calcolare fasori correnti in tutti i rami
- verificare conservazione potenza complessa

$$\Rightarrow \sum \underline{N}_E = \sum \underline{N}_A$$

$$\begin{cases} \sum P_E = \sum P_A \\ \sum Q_E = \sum Q_A \end{cases}$$



$R = 1\,\Omega \quad L = 1\,mH$

$C = 500\,\mu F \quad \omega = 10^3\,rad/s$

$v_{g1}(t) = 10\sqrt{2}\,\sin(\omega t + \pi/2)\ V$

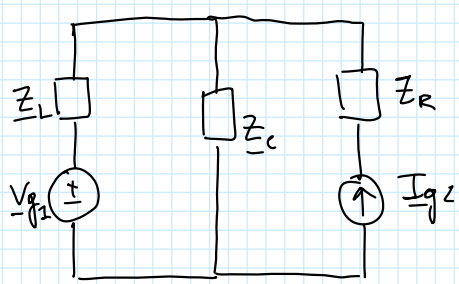
$i_{g2}(t) = 4\sqrt{2}\,\cos(\omega t + \pi/2)\ A$

$$\underline{Z}_L = j\omega L = j$$

(ω: 10^3 rad/s, L: 10^{-3} H)

$$\underline{Z}_C = -\frac{j}{\omega C} = -\frac{j}{0{,}5} = -2j$$

(C: $500\cdot10^{-6}$ F)



$Z_R = R = 1\,\Omega$

$v_{g1}(t) = 10\sqrt{2}\cos\left(\omega t + \pi/2 - \pi/2\right)$

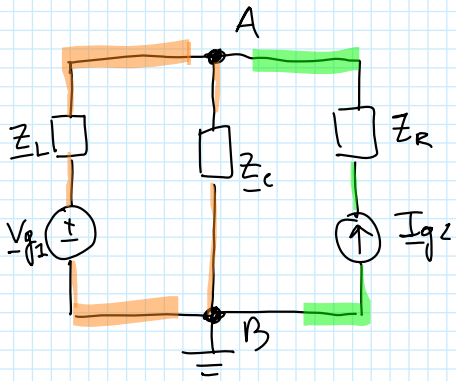
$\sin(x) = \cos(x - \pi/2)$

$\underline{V}_{g1} = S\left[v_{g1}(t)\right] = \frac{10\sqrt{2}}{\sqrt{2}} e^{j0} = 10\cos(0) + j10\sin(0) = 10$

$e^{jx} = \cos(x) + j\sin(x)$

$\underline{I}_{g2} = S\left[I_{g2}(t)\right] = \frac{4\sqrt{2}}{\sqrt{2}} e^{j\pi/2} = 4\cos(\pi/2) + j4\sin(\pi/2) = j4$

$$\begin{cases} \underline{e}_B = 0 \\ (\underline{Y}_L + \underline{Y}_C)\underline{e}_A - \underline{Y}_L \underline{e}_B - \underline{Y}_C \underline{e}_B = \\ \qquad \underline{Y}_L \underline{V}_{g1} + \underline{I}_{g2} \end{cases}$$



conduttanze ↙

calcolo ammettenze

$\underline{Y}_L = \frac{1}{\underline{Z}_L} = \frac{1}{\underline{Z}_L} \cdot \frac{\underline{Z}_L^*}{\underline{Z}_L^*} = \frac{-j\omega L}{\omega^2 L^2} = -\frac{j}{\omega L} = -j$

($j\omega L$ ↗, $-j\omega L$ ↘)

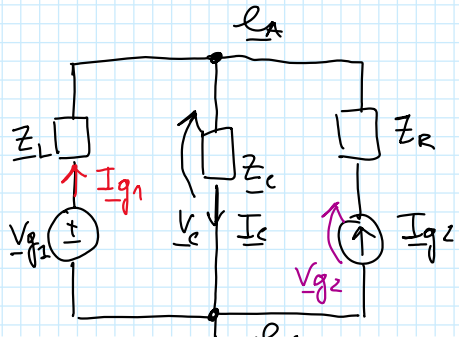
$\underline{Y}_C = \frac{1}{\underline{Z}_C} = \dots = j0.5$

$\underline{e}_A = \frac{\underline{Y}_L \underline{V}_{g1} + \underline{I}_{g2}}{\underline{Y}_L + \underline{Y}_C} = 12$

$e_A = |\underline{e}_A| = 12\ V$

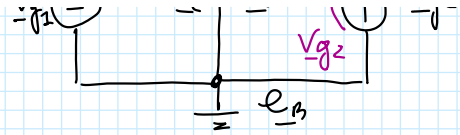
$e_B = |\underline{e}_B| = 0\ V$

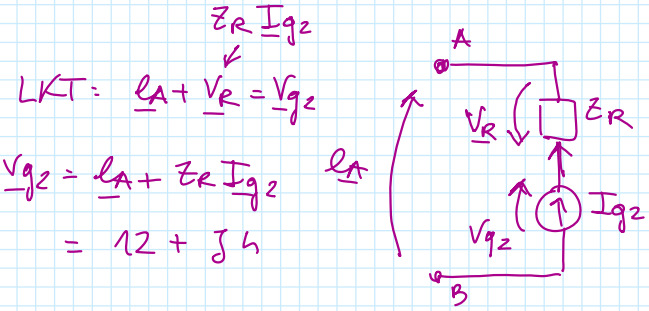
$\underline{I}_C = \underline{V}_C / \underline{Z}_C = \underline{V}_C \underline{Y}_C = \underline{e}_A \underline{Y}_C = j6$ (↙ $\underline{e}_A - \underline{e}_B$)



$$\underline{I}_{g1} = \underline{I}_C - \underline{I}_{g2} = J2$$

(LKC$_A$)





Verifica potenze

$$\underline{N}_{E,g1} = \underline{V}_{g1} \underline{I}_{g1}^* = 10\,(-J2) = -J20$$

($\underline{V}_{g1} = 10$, $\underline{I}_{g1} = J2$)

$$P_{E,g1} = \mathrm{Re}(\underline{N}_{E,g1}) = 0 \qquad Q_{E,g1} = \mathrm{Im}(\underline{N}_{E,g1}) = -20\ \mathrm{VA_r}$$

$$\underline{N}_{E,g2} = \underline{V}_{g2} \underline{I}_{g2}^* = (12 + J4)\cdot(-J4) = 16 - J48$$

$$P_{E,g2} = \mathrm{Re}(\underline{N}_{E,g2}) = 16\ \mathrm{W} \qquad Q_{E,g2} = \mathrm{Im}(\underline{N}_{E,g2}) = -48\ \mathrm{VA_r}$$

$$\underline{N}_{A,L} = \underline{Z}_L \underline{I}_L^2 = \underline{Z}_L \underline{I}_{g1}^2 = J4 \rightarrow P_{A,L} = 0 \;;\; Q_{A,L} = 4\ \mathrm{VA_r}$$

($\underline{I}_L = \underline{I}_{g1}$)

$$\underline{N}_{A,C} = \underline{Z}_C \underline{I}_C^2 = -J72 \rightarrow P_{A,C} = 0 \;;\; Q_{A,C} = -72\ \mathrm{VA_r}$$

$$N_{A,R} = Z_R I_R^2 = R I_{g2}^2 = 16\ \mathrm{W} = P_{A,R}$$

($I_R = I_{g2}$)

Verifica: $\underbrace{\underline{N}_{E,g1} + \underline{N}_{E,g2}}_{\sum \underline{N}_E} = \underbrace{\underline{N}_{A,L} + \underline{N}_{A,C} + \underline{N}_{A,R}}_{\sum \underline{N}_A}$

alternativa

Re: $\cancel{P_{E,g1}} + P_{E,g2}\ (16\ \mathrm{W}) = \cancel{P_{A,L}} + \cancel{P_{A,C}} + P_{A,R}\ (16\ \mathrm{W})$

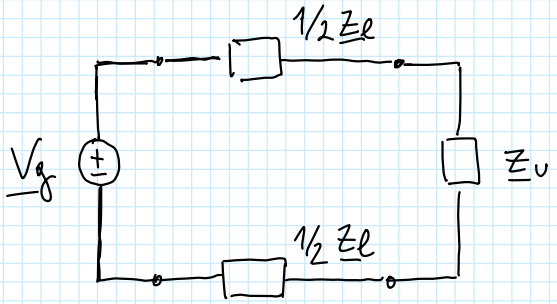
Im: $Q_{E,g1} + Q_{E,g2} = Q_{A,L} + Q_{A,C} + \cancel{Q_{A,R}}$

$-20\ \mathrm{VA_r} \quad -48\ \mathrm{VA_r} \qquad 4\ \mathrm{VA_r} \quad -72\ \mathrm{VA_r}$

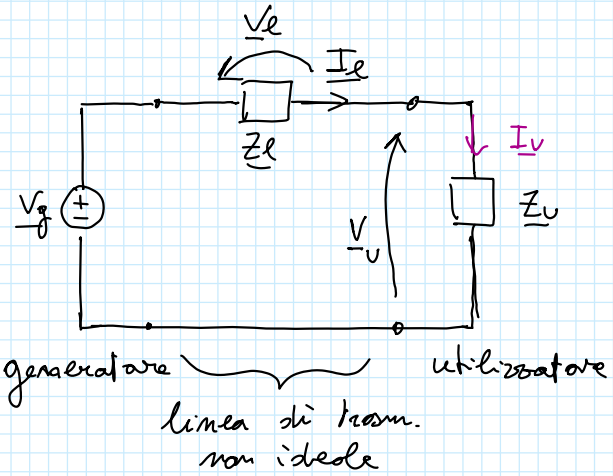
$-68\ \mathrm{VA_r} \qquad -68\ \mathrm{VA_r}$

Rifasamento

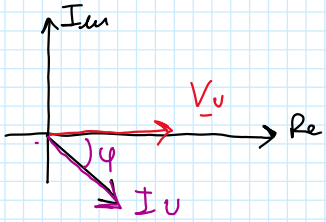
Hp. regime sinusoidale — linea di trasmissione non ideale $\Rightarrow \underline{Z}_l$



$\Rightarrow$



Hp utilizzatore $\mathcal{R}-L$



OBIETTIVO del distributore

- garantire P_u
- garantire $\underline{V}_u$

Problematiche

① "caduta di tensione" nella linea l

LKT. $\underline{V}_u + \underline{V}_l = \underline{V}_g \Rightarrow \underline{V}_u = \underline{V}_g - \underline{V}_l$

$|\underline{V}_u| = |\underline{V}_g - \underline{V}_l|$

$\uparrow$ $\underline{I}_u$

$|\underline{Z}_l \cdot \underline{I}_l|$

$\uparrow$

$\underline{I}_l$

$R_l = \rho \, l/S$ — $[\Omega m]$ resistività; lunghezza; sezione conduttori

② Perdite joule nella linea

$\underline{Z}_l \longrightarrow R_l + jX_l$

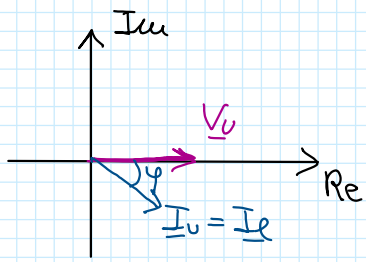
$P_{Jl} = R_l I_l^2$

OBIETTIVO: minimizzare I_l $\uparrow$ modulo fasore corrente linea

$I_u = I_l$

$P_u = V_u I_u \cos(\varphi) \Rightarrow I_l = \dfrac{P_u}{V_u \cos(\varphi)}$

$\uparrow$ SFASAMENTO fra fasori $\underline{V}_u$, $\underline{I}_u$



- I_l minima se $\varphi = 0 \Rightarrow \cos\varphi = 1$

- I_l minima se $\varphi = 0 \Rightarrow \cos\varphi = 1$
- $I_l \uparrow$ se $\cos\varphi \downarrow$

SFASAMENTO fra fasori $\underline{V}_U$, $\underline{I}_U \updownarrow \underline{I}_l$

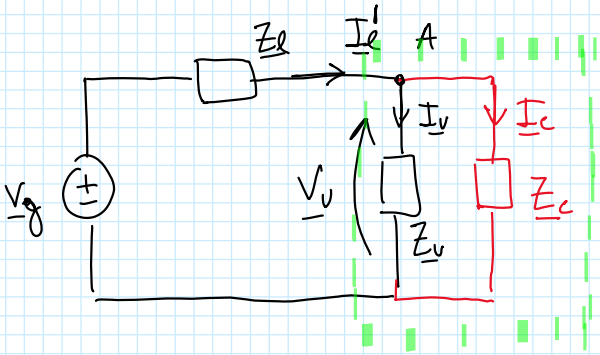
$\underline{I}_U = \underline{I}_l$

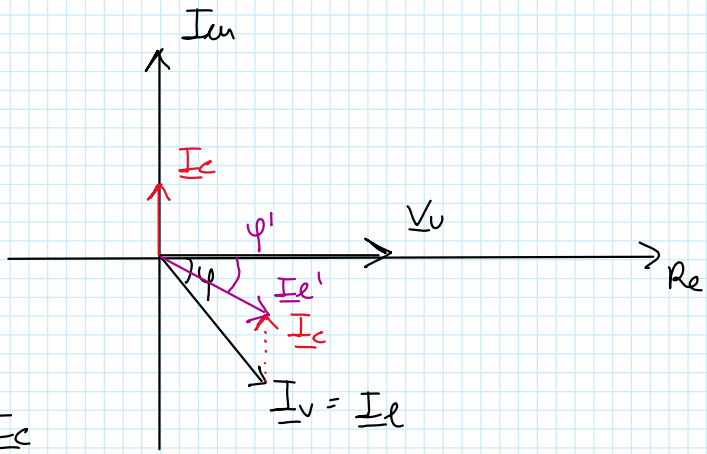
effetto $\cos\varphi < 1 \Rightarrow$ aumentare I_l a pari P_U $\Rightarrow$ costo per il gestore

V_U

Se $\cos\varphi = 1$ $(\varphi = 0)$: $P_U = VI\cos\varphi$ ($\cos\varphi = 1$) $\quad Q_U = VI\sin\varphi$ ($\sin\varphi = 0$)

$\cos\varphi < 1$ $(\varphi \neq 0)$ $\quad P_U = VI\cos\varphi$ ($\cos\varphi < 1$) $\quad Q_U = VI\sin\varphi$





Ω - L $X > 0$ $\quad LKC_A$ $\quad \underline{I}_l' = \underline{I}_U + \underline{I}_C$

C $X < 0$

- grazie a I_C . $\cos(\varphi') > \cos(\varphi)$

$$I_l = \frac{P_U}{V_U \cos\varphi} \qquad I_l' = \frac{P_U}{V_U \cos(\varphi')} < I_l$$

effetto condensatore rifasamento: compensazione reattanza induttiva dell'utilizzatore

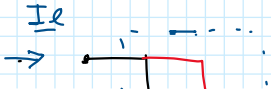
normativa:

$\cos\varphi \geq 0.95$

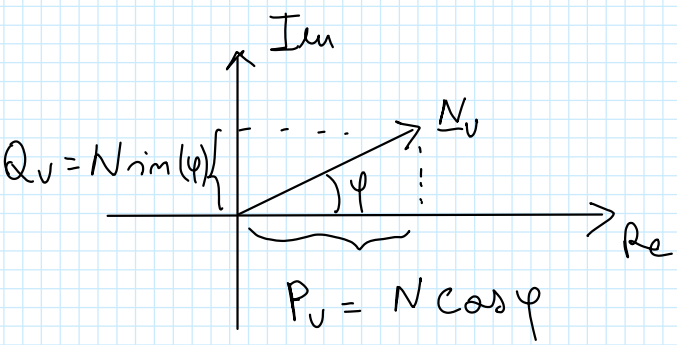
$0.7 \leq \cos\varphi < 0.95$ penali proporzionali alla Q_A

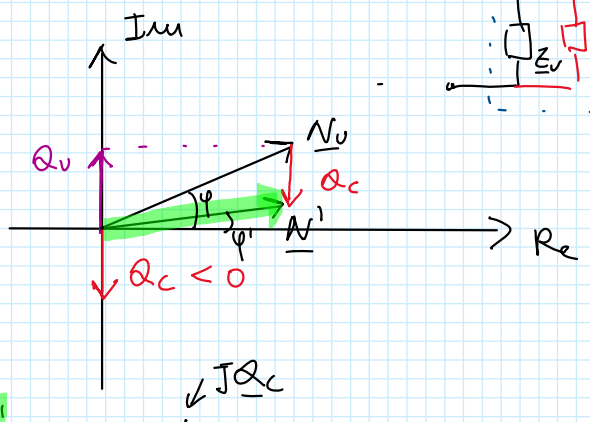
$\cos\varphi < 0.7$ rifasamento obbligatorio

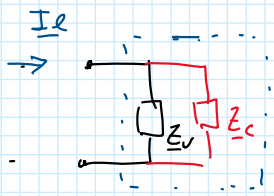
- Quanto deve valere C per $\cos\varphi \rightarrow \cos\varphi'$



- Quanto deve valere C per $\cos\varphi \to \cos\varphi'$







Ω - L: $P_U > 0$ $Q_U > 0$

$\underline{N}' = \underline{N}_U + \underline{N}_C$

$$\tan\varphi = Q_U / P_U$$

$$\tan\varphi' = \frac{Q_U + Q_C}{P_U}$$

$$\tan\varphi' - \tan\varphi = \frac{Q_U + Q_C}{P_U} - \frac{Q_U}{P_U} = \frac{Q_C}{P_U} \Leftarrow -\omega C V_C^2$$

↑ tensione applicata condensatore

$$\tan\varphi' - \tan\varphi = -\frac{\omega C V_C^2}{P_U}$$

$$\boxed{C = \frac{\tan\varphi - \tan\varphi'}{\omega V_C^2} P_U}$$

capacità da porre in $\parallel$ a utilizzatore per $\varphi \to \varphi'$

Proprietà dei circuiti lineari

Sovrapposizione degli effetti

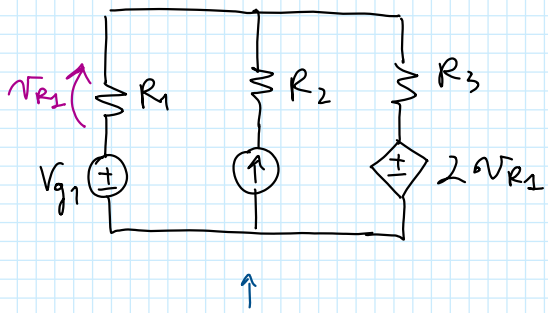
In un circuito lineare qualunque $v(t)$, $i(t)$ è data dalla somma algebrica degli effetti dei generatori indipendenti quando essi agiscono uno alla volta

$g_1, g_2, \dots g_m$

$$v(t) = f(g_1, g_2, \dots, g_m) = \sum_{i=1}^{m} f_i(g_i)$$

$$i(t) = h(g_1, g_2, \dots, g_m) = \sum_{i=1}^{m} h_i(g_i)$$

Esercizio



2 gen indip

2 sotto-circuiti

1. Assegno VDR correnti / tensioni
2. Spengo tutti i generatori indipendenti tranne g_i-esimo
3. Calcolo correnti / tensioni parziali dovute ad effetto di g_i-esimo

   ottengo $f_i(g_i)$

   $h_i(g_i)$
4. Sommo correnti / tensioni parziali