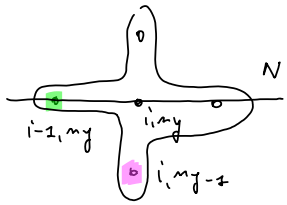
L 14

$$\begin{cases} \left.\frac{\partial \varphi}{\partial n}\right|_N = \frac{\partial \varphi}{\partial y} = \varphi'_N \Rightarrow \left.\frac{\partial \varphi}{\partial y}\right|_{(i,my)} = \varphi'_{(i,my)} = \varphi'_N \\ \left.\frac{\partial^2 \varphi}{\partial x^2}\right|_N + \left.\frac{\partial^2 \varphi}{\partial y^2}\right|_N = t_N \Rightarrow \text{FOR node } (i,my) \quad \left.\frac{\partial^2 \varphi}{\partial x^2}\right|_{(i,my)} + \left.\frac{\partial^2 \varphi}{\partial y^2}\right|_{(i,my)} = t_{i,my} \end{cases}$$

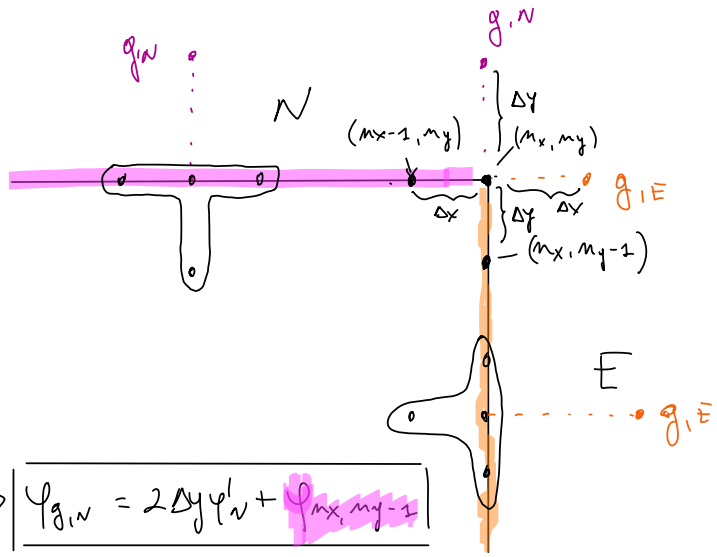


$$\begin{cases} \frac{(\varphi_{g,N} - \varphi_{i,my-1})}{2\Delta y} = \varphi'_N \Rightarrow \varphi_{g,N} = 2\Delta y \varphi'_N + \varphi_{i,my-1} \\ \frac{1}{\Delta x^2}\varphi_{i+1,my} + \frac{1}{\Delta y^2}\varphi_{g,N} - 2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\varphi_{i,my} + \frac{1}{\Delta x^2}\varphi_{i-1,my} + \frac{1}{\Delta y^2}\varphi_{i,my-1} = t_{i,my} \end{cases}$$

NORTH EDGE:

$$\frac{1}{\Delta x^2}\varphi_{i+1,my} - 2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\varphi_{i,my} + \frac{1}{\Delta x^2}\varphi_{i-1,my} + \frac{2}{\Delta y^2}\varphi_{i,my-1} = t_{i,my} - \frac{2\Delta y \varphi'_N}{\Delta y^2}$$

NORTH - EAST CORNER



$$\begin{cases} N: \begin{cases} \partial\varphi/\partial n|_N = \partial\varphi/\partial y|_N = \varphi'_N \\ E: \partial\varphi/\partial n|_E = \partial\varphi/\partial x|_E = \varphi'_E \end{cases} \\ N: (\varphi_{g,N} - \varphi_{mx,my-1})/2\Delta y = \varphi'_N \Rightarrow \boxed{\varphi_{g,N} = 2\Delta y \varphi'_N + \varphi_{mx,my-1}} \\ E: (\varphi_{g,E} - \varphi_{mx-1,my})/2\Delta x = \varphi'_E \Rightarrow \boxed{\varphi_{g,E} = 2\Delta x \varphi'_E + \varphi_{mx-1,my}} \\ \text{Second der in } (mx,my): \frac{1}{\Delta x^2}\varphi_{g,E} + \frac{1}{\Delta y^2}\varphi_{g,N} - 2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\varphi_{mx,my} + \frac{1}{\Delta x^2}\varphi_{mx-1,my} + \frac{1}{\Delta y^2}\varphi_{mx,my-1} = t_{mx,my} \end{cases}$$

$$\Downarrow$$

$$-2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\varphi_{mx,my} + \frac{2}{\Delta x^2}\varphi_{mx-1,my} + \frac{2}{\Delta y^2}\varphi_{mx,my-1} = t_{mx,my} - \frac{2\Delta x \varphi'_E}{\Delta x^2} - \frac{2\Delta y \varphi'_N}{\Delta y^2}$$

obtained algebraic expression for the solution in each node of the grid

⇒ ASSEMBLE a linear system: t, BCs

$$[K]\{\psi\} = \{Rhs\}$$

$$\{\psi\} = \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \vdots \end{Bmatrix}$$

in 1D: $[K]$: $n_x \cdot n_x$

in 2D: $[K]$: $(n_x n_y) \cdot (n_x n_y)$

if $n_x = 5$, $n_y = 4$ ⇒ $[K]$: 20×20