L17

18 November 2025 09:17

Experiments in Matlab with numerical integration

$$\int_a^b f(x)\,dx = \int_{-1}^{1} f(m\tau+q)\,\overbrace{\frac{b-a}{2}}^{m}\,d\tau$$

$$\simeq \sum_{k=1}^{n} w_k \underbrace{\frac{b-a}{2}}_{m}\, f\left(\underbrace{\frac{b-a}{2}}_{m}\tau_k + \underbrace{\frac{a+b}{2}}_{q}\right)$$

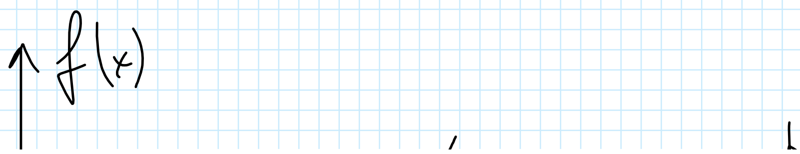
$n = 1 \to m = 2n - 1 = 1$

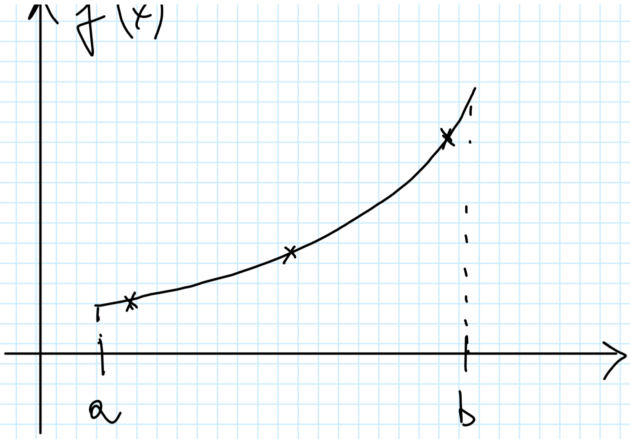
$$> \boxed{\begin{aligned} x_1 &= \pm 1/\sqrt{3} \\ x_2 &= \mp 1/\sqrt{3} \\ w_1 &= w_2 = 1 \end{aligned}}$$

QUADRATURE RULE for $n = 2$

n = 3

$$\begin{cases} w_1 = 5/9 \\ w_2 = 8/9 \\ w_3 = 5/9 \end{cases} \qquad \begin{cases} x_1 = -\sqrt{3/5} \\ x_2 = 0 \\ x_3 = \sqrt{3/5} \end{cases}$$





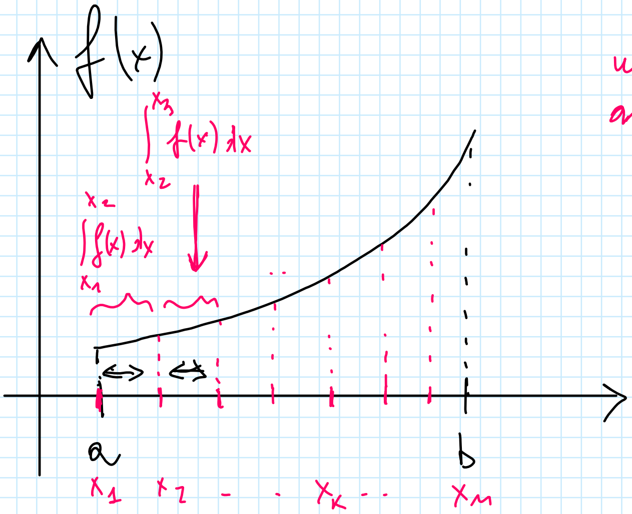
$$\int_a^b f(x)\,dx$$

ADAPTIVE INTEGRATION:

subdivide interval $[a,b]$ into multiple sub-intervals

⇓

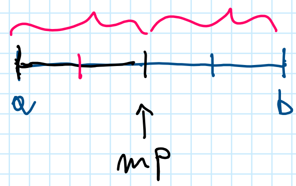
use Gauss integration on every sub-interval and sum the partial integrals



PSEUDO CODE

integrand — # of gauss points — tolerance for integration

$[int] = \text{INT\_ADT}\,(f, a, b, m, tol)$



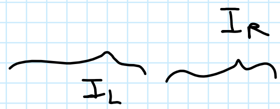
$$I_1 = \int_a^b f(x)\,dx$$

$$I_2 = \int_a^{mp} f(x)\,dx + \int_{mp}^{b} f(x)\,dx$$

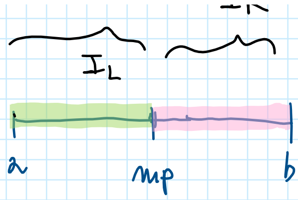
$error = |I_1 - I_2|$ ← if just one interval is enough, I_1 very close to I_2

if error < tolerance ← 10^{-6}

$int = I_2$ ← routine is over



$int = I_2$ ← routine is over



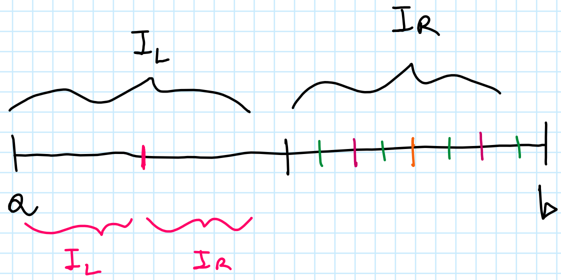
else

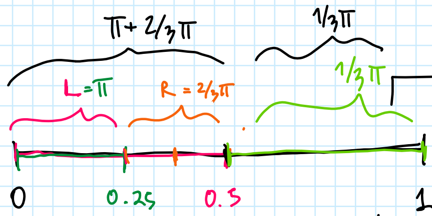
$\rightarrow \rightarrow I_L = INT\_ADT(f, a, mp, n, tol)$

$\rightarrow \rightarrow \rightarrow I_R = INT\_ADT(f, mp, b, n, tol)$

$int = I_L + I_R$

end





$I_1^0 = \int_0^1 \dots$

$I_2^0 = \int_0^{0.5} \dots + \int_{0.5}^1$

error > tol

$I_L^1 = \dots \dots =$

$I_1^1 = \int_0^{0.5} \dots$ $\quad I_2^1 = \int_0^{0.25} \dots + \int_{0.25}^{0.5}$

error > tol ?

$I_1^2 = \int_0^{0.25} \dots$ $\quad I_2^2 = \int_0^{0.125} \dots + \int_{0.125}^{0.25} \dots$

error < tol ⇒ OK

$int^2 = I_2 = \pi$