L9

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$\frac{1}{n!}$

(1) $f_{k+1} = f_k + \left.\frac{df}{dx}\right|_k \Delta x + \frac{1}{2}\left.\frac{d^2f}{dx^2}\right|_k \Delta x^2 + \frac{1}{6}\left.\frac{d^3f}{dx^3}\right|_k \Delta x^3 + \mathcal{O}(\Delta x^4)$

(2) $f_{k-1} = f_k - \left.\frac{df}{dx}\right|_k \Delta x + \frac{1}{2}\left.\frac{d^2f}{dx^2}\right|_k \Delta x^2 - \frac{1}{6}\left.\frac{d^3f}{dx^3}\right|_k \Delta x^3 + \mathcal{O}(\Delta x^4)$

(1) + (2)

$$f_{k+1} + f_{k-1} = 2f_k + \left.\frac{d^2f}{dx^2}\right|_k \Delta x^2 + \mathcal{O}(\Delta x^4)$$

$$\left.\frac{d^2f}{dx^2}\right|_k = \frac{f_{k+1} - 2f_k + f_{k-1}}{\Delta x^2} + \mathcal{O}(\Delta x^2)$$

Finite Difference formula for second-order derivative

SECOND ORDER ACCURACY