Retiredge ETF Portfolio Optimization Implementation

I want to begin by acknowledging the sources that helped make this document and implementation possible.

Resources:

- [Sample Code] Efficient Frontier Portfolio Optimisation in Python (https://towardsdatascience.com/efficient-frontier-portfolio-optimisation-in-python-e7844051e7f (https://towardsdatascience.com/efficient-frontier-portfolio-optimisation-in-python-e7844051e7f))
- [Sample Code] ML time series forecasting the right way (https://towardsdatascience.com/ml-time-series-forecasting-the-right-way-cbf3678845ff (https://towardsdatascience.com/ml-time-series-forecasting-the-right-way-cbf3678845ff))
- [Sample Data] Quandl ETF Data Bundle Product Overview (https://www.quandl.com/databases/ETFG/documentation (https://www.quandl.com/databases/ETFG/documentation))
- [Documentation] Exponential Smoothing (https://thequackdaddy.github.io/statsmodels.github.io/0.9.0/generated/statsmodels.tsa.holtwinters.Ex
(https://thequackdaddy.github.io/statsmodels.github.io/0.9.0/generated/statsmodels.tsa.holtwinters.Ex
- [Theory] Beating the ETF: Portfolio Optimisation using Python (…and some linear algebra) (https://towardsdatascience.com/beating-the-etf-portfolio-optimisation-using-python-and-some-linear-algebra-e48d0e0e44f (https://towardsdatascience.com/beating-the-etf-portfolio-optimisation-using-python-and-some-linear-algebra-e48d0e0e44f))
- [Theory] PyPortfolioOpt Repo (https://github.com/robertmartin8/PyPortfolioOpt (https://github.com/robertmartin8/PyPortfolioOpt))

Efficient Frontier and Modern Portfolio Theory

Modern Portfolio Theory (MPT) is an investment and finance theory developed by Harry Markowitz in 1952. In this code demo, Retiredge focuses on two of the underlying concepts: risk-return trade-off and diversification.

The goal for this implementation is to predict the best decision our user could make for their portfolio. This means we are not solely considering maximum possible returns and betting all on that one particular ETF. For every financial decision, there is an associated risk with a pattern that can be analyzed and predicted (for example thorugh volatility). Typically, higher risk is associated with a greater chance of a higher return (and lower risk - greater chance for lower return). Thus, for a risk-averse investor, a low risk portfolio is more preferable than a high risk one with the same expected returns.

Efficient Frontier, which is an idea based on the MPT, aligns with this general investing attitude and emphasises that increasing risk is only worth a compounded chance at greater returns. The Efficient Frontier is essentially a line - tracing what the algorithm believes to be the 'smartest' randomly generated portfolios. This will be touched upon further down this document.

Sample ETF Net Asset Value (nav) data retrieved from Quandl will be used for analysis. Quandl is a platform which offers real time and historical data in a variety of financial fields. It also provides a python library for when importing directly through their database.

Diversifiction is another method of reducing portfolio risk, aside from analysing data of individual funds. By investing in multiple stocks, the chances of an overall loss is lower. A diverse, but not randomly chosen portfolio can help build the safety-net for non-aggressive investors and typically less experienced investors such as our target market of college students.

Main Process and Functionalities

1. Importing Data and Lbraries
2. Preprocessing
   A. Initial Visualizations
   B. ML Data Generation
3. Generating Random Portfolio
   A. Feature Visualization
   B. Maximum Sharpe Ratio
   C. Minimum Volatility
4. Calculating Efficient Frontier
5. Display optimized portfolio
   A. Visualizations

Importing Data and Libraries

The data imported below is the free sample Fund Flows data from Quandl's ETF Bundle. It is a small selection of the bundle itself, which has daily-updated information on 2,500+ US ETFs. This notebook analysis simply aims to show a snippet of what could be done with this information.

In [3]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

# Use the two commands below if data should be directly imported from quandl
# ! pip install quandl # For installing onto this notebook only
# import quandl

import scipy.optimize as sco

plt.style.use('fivethirtyeight')
np.random.seed(777)

%matplotlib inline
%config InlineBackend.figure_format = 'retina'
```

In [4]:
```
# Below is code for importing data from quandl directly

# quandl.ApiConfig.api_key = '------------------'
# stocks = ['ETFG/ANLT']
# data = quandl.get_table('WIKI/PRICES', ticker = stocks,
#                          qopts = { 'columns': ['date', 'ticker', 'adj_close'] },
#                          date = { 'gte': '2016-1-1', 'lte': '2017-12-31' }, paginat
# data.head()

# Importing Locally
df = pd.read_csv('ETFG_FUND.csv')
df
```

Out[4]:

	date	ticker	shares_outstanding	nav	flow_daily	as_of_date
0	2018-01-03	EEM	837900000.0	48.12	0.0	2018-01-03
1	2018-01-04	EEM	837900000.0	48.45	0.0	2018-01-04
2	2018-01-05	EEM	845100000.0	48.80	348840000.0	2018-01-05
3	2018-01-08	EEM	855000000.0	49.03	483120000.0	2018-01-08
4	2018-01-09	EEM	855000000.0	48.96	0.0	2018-01-09
...	...	...	...	...	...	...
142	2018-01-29	TLT	57000000.0	122.75	98704000.0	2018-01-29
143	2018-01-30	TLT	57400000.0	121.97	49100000.0	2018-01-30
144	2018-01-31	TLT	54900000.0	122.77	-304925000.0	2018-01-31
145	2018-01-02	TLT	58400000.0	126.94	291157000.0	2017-12-29
146	2018-01-16	TLT	56500000.0	124.61	12437000.0	2018-01-12

147 rows × 6 columns

By looking at the info() of data, it seems like the "date" column is already in datetime format.

In [5]:
```
# Use the two commands below to retrieve more information on the data
#df.info()
df.head() # To retrieve the first 5 items
```

Out[5]:

	date	ticker	shares_outstanding	nav	flow_daily	as_of_date
0	2018-01-03	EEM	837900000.0	48.12	0.0	2018-01-03
1	2018-01-04	EEM	837900000.0	48.45	0.0	2018-01-04
2	2018-01-05	EEM	845100000.0	48.80	348840000.0	2018-01-05
3	2018-01-08	EEM	855000000.0	49.03	483120000.0	2018-01-08
4	2018-01-09	EEM	855000000.0	48.96	0.0	2018-01-09

Preprocessing

In [6]:
```
# select only nav - Net Asset Value
df_mod = df[['date', 'ticker', 'nav']]
df_mod = df_mod.set_index('date')
```

Seven different ETFs are being compared in this analysis. They are:

- iShares MSCI Emerging Markets (EEM)
- SPDR S&P 500 (SPY)
- iShares MSCI EAFE (EFA)
- iShares Barclays 20+ Year Treasury Bond (TLT)
- SPDR Barclays Capital High Yield Bond (JNK)
- SPDR Gold Shares (GLD)
- iShares Russell 2000 (IWM)

In [7]:
```
# Ordering the data by the Funds
table = df_mod.pivot(columns='ticker')
# By specifying col[1] in below list comprehension
# You can select the stock names under multi-level column
table.columns = [col[1] for col in table.columns]
table.head()
```

Out[7]:

	EEM	EFA	GLD	IWM	JNK	SPY	TLT
date							
2018-01-02	47.07	70.49	124.5286	152.44	36.6357	266.552	126.94
2018-01-03	48.12	70.75	124.7978	154.14	36.7982	270.489	126.05
2018-01-04	48.45	71.80	124.7585	154.46	36.9077	271.635	126.04
2018-01-05	48.80	72.21	125.0087	154.89	36.9601	273.541	125.51
2018-01-08	49.03	72.14	125.2703	155.07	36.9548	273.999	125.46

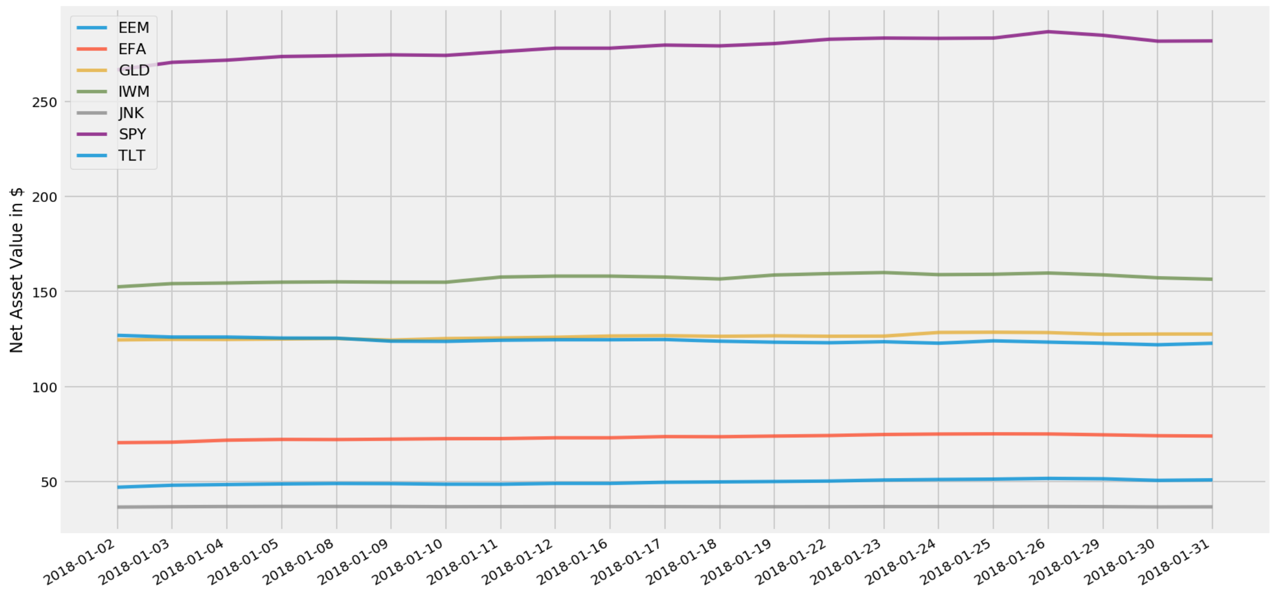
Initial Visualizations

In [8]:

```
# Plotting the sample data
plt.figure(figsize=(14, 7))
for c in table.columns.values:
    plt.plot(table.index, table[c], lw=2.5, alpha=0.8,label=c)

locs, labels = plt.xticks()
plt.setp(labels, rotation=30, horizontalalignment='right')
plt.legend(loc='upper left', fontsize=11)
plt.ylabel('Net Asset Value in $')
```

Out[8]:

```
Text(0, 0.5, 'Net Asset Value in $')
```

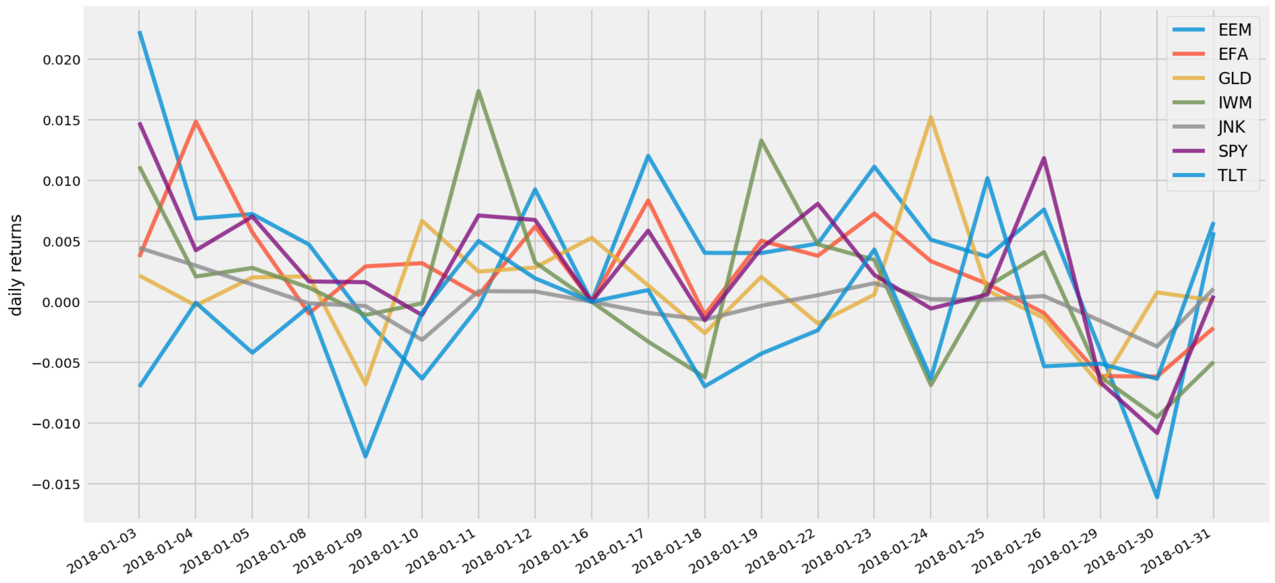


It looks like SPY is consistently at a higher value than the other ETFs.

The below cell plotts daily changes (percent change compared to the day before) in nav, showing the funds' volatility.

In [9]:
```
returns = table.pct_change()

plt.figure(figsize=(14, 7))
for c in returns.columns.values:
    plt.plot(returns.index, returns[c], lw=3, alpha=0.8,label=c)
locs, labels = plt.xticks()
plt.setp(labels, rotation=30, horizontalalignment='right')
plt.legend(loc='upper right', fontsize=12)
plt.ylabel('daily returns')
```

Out[9]:
```
Text(0, 0.5, 'daily returns')
```



It seems EEM (Top Blue line at the left hand side) is slightly more volatile than the other ETFs. And TLT (bottom blue line) tends to perfrom worse than other ETFs. The least volatile is JNK.

Generating Future Values

For the sake of this class, we decided to use the free sample data. Quandl's many more complete and in-depth datasets are available for purchase for $3000 a year, which has been taken into consideration in our slidedeck and pitch.

However, to better illustrate our technology with a realistic dataset, machine learning, specifically time series forcasting will be performed below to increase the size of our data beyond one month. This same technology can be used for additional portfolio prediction research to complement this efficient frontier implementation.

In [10]:
```
# Getting the Model
from statsmodels.tsa.holtwinters import ExponentialSmoothing

def getmodelfit(y):
    model = ExponentialSmoothing(y,
                                 trend = 'multiplicative',
                                 seasonal = 'multiplicative',
                                 seasonal_periods = 6)
    fit = model.fit(smoothing_level = 0.05, smoothing_slope = 0.05, smoothing_season
    return model, fit
```

In [11]:
```
# Creating lag features which helps in utilizing supervized learning
from sklearn.preprocessing import StandardScaler
from statsmodels.tsa.stattools import pacf

def create_lag_features(y):

    scaler = StandardScaler()
    features = pd.DataFrame()

    partial = pd.Series(data=pacf(y, nlags=2)) # you can change value of nlags to be
    lags = list(partial[np.abs(partial) >= 0.2].index)

    df = pd.DataFrame()

    # avoid to insert the time series itself
    lags.remove(0)

    for l in lags:
        df[f"lag_{l}"] = y.shift(l)

    features = pd.DataFrame(scaler.fit_transform(df[df.columns]),
                            columns=df.columns)
    features.index = y.index

    return features
```

In [12]:
```
# Make a prediction using the fit and plot the prediction and original
def makepred(name, orig, fit, n):
    pred = fit.forecast(n)
    plt.plot(pred, label = name)
    plt.plot(orig)
    plt.xticks([table.index[0], table.index[-1]], visible=True, rotation=90)
    plt.legend();
    return pred
```

The following function: recursive_forecast is another implementation of makepred, however, it has more potential for implementing a greater degree of ML supervised learning processes to make future time series predictions. The default number for n_steps can be changed to reflect how long of a prediction one would want to make. Here we are predicting 300 days into the future.

In [13]:
```
# Make a prediction and plot the original and prediction. Adds date values to predic
# returns a Series of the prediction
def recursive_forecast(name, y, fit, lags,
                       n_steps=300, step="D"):

    """
    Parameters
    ----------
    y: pd.Series holding the input time-series to forecast
    model: pre-trained machine learning model
    lags: list of lags used for training the model
    n_steps: number of time periods in the forecasting horizon
    step: forecasting time period

    Returns
    -------
    fcast_values: pd.Series with forecasted values
    """

    # get the dates to forecast
    last_date = '2018-02-01'
    fcast_range = pd.date_range(last_date,
                                periods=n_steps,
                                freq=step)
    fcasted_values = []
    target = y.copy()
    i = 0

    pred = fit.forecast(n_steps)

    #plot for viewing
    plt.plot(pred, label = name)
    plt.plot(y)
    plt.xticks([table.index[0], table.index[-1]], visible=True, rotation="horizontal
    plt.legend();

    for date in fcast_range:

        # build target time series using previously forecast value
        new_point = fcasted_values[-1] if len(fcasted_values) > 0 else 0.0
        target = target.append(pd.Series(index=[date], data=new_point))

        # The Code below can be modified to caculate real features for supervised le
#         # build feature vector using previous forecast values
#         features = create_ts_features(target, lags=lags)

#         # forecast
#         predictions = model.predict(features)
        fcasted_values.append(pred.iloc[i])
        i += 1

    return pd.Series(index=fcast_range, data=fcasted_values)
```

Below is the execution. The red info/warning messages can be ignored, as they do not intefer with the algorithm and visualizations.

In [14]:

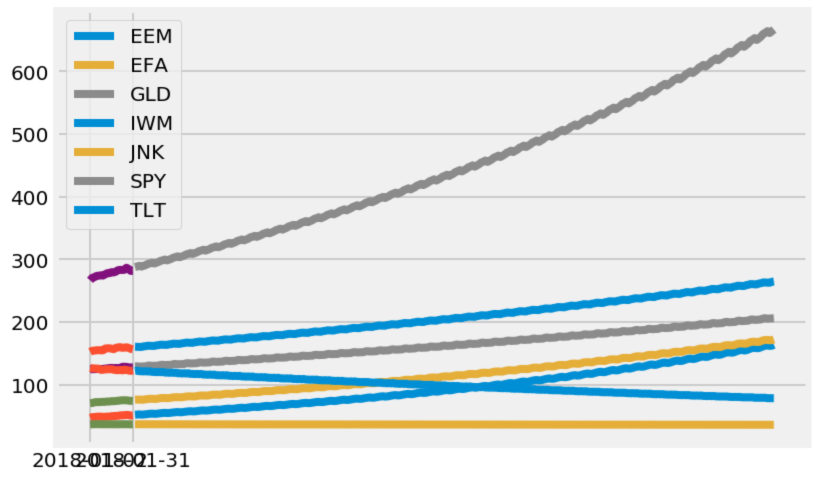
```
# The list of ETFs we are tracking.
etfs = ['EEM', 'EFA', 'GLD', 'IWM', 'JNK', 'SPY','TLT']
```

In [17]:

```python
mfps = {}
#index = pd.date_range('2018-02-01', periods=120, freq='D')

for s in etfs:
    y = table[s]
    lag = create_lag_features(y)
    model, fit = getmodelfit(y)
    #pred = makepred(s, y, fit, 48)
    pred = recursive_forecast(s, y, fit, lag)
    mfps[s] = [model, lag, fit, pred]
```

```
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:216:
ValueWarning: A date index has been provided, but it has no associated frequen
cy information and so will be ignored when e.g. forecasting.
  warnings.warn('A date index has been provided, but it has no'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:580:
ValueWarning: No supported index is available. Prediction results will be give
n with an integer index beginning at `start`.
  warnings.warn('No supported index is available.'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:216:
ValueWarning: A date index has been provided, but it has no associated frequen
cy information and so will be ignored when e.g. forecasting.
  warnings.warn('A date index has been provided, but it has no'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:580:
ValueWarning: No supported index is available. Prediction results will be give
n with an integer index beginning at `start`.
  warnings.warn('No supported index is available.'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:216:
ValueWarning: A date index has been provided, but it has no associated frequen
cy information and so will be ignored when e.g. forecasting.
  warnings.warn('A date index has been provided, but it has no'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:580:
ValueWarning: No supported index is available. Prediction results will be give
n with an integer index beginning at `start`.
  warnings.warn('No supported index is available.'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:216:
ValueWarning: A date index has been provided, but it has no associated frequen
cy information and so will be ignored when e.g. forecasting.
  warnings.warn('A date index has been provided, but it has no'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:580:
ValueWarning: No supported index is available. Prediction results will be give
n with an integer index beginning at `start`.
  warnings.warn('No supported index is available.'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:216:
ValueWarning: A date index has been provided, but it has no associated frequen
cy information and so will be ignored when e.g. forecasting.
  warnings.warn('A date index has been provided, but it has no'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:580:
ValueWarning: No supported index is available. Prediction results will be give
n with an integer index beginning at `start`.
  warnings.warn('No supported index is available.'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:216:
ValueWarning: A date index has been provided, but it has no associated frequen
cy information and so will be ignored when e.g. forecasting.
  warnings.warn('A date index has been provided, but it has no'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:580:
ValueWarning: No supported index is available. Prediction results will be give
n with an integer index beginning at `start`.
  warnings.warn('No supported index is available.'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:216:
ValueWarning: A date index has been provided, but it has no associated frequen
cy information and so will be ignored when e.g. forecasting.
```

```
  warnings.warn('A date index has been provided, but it has no'
/opt/conda/lib/python3.8/site-packages/statsmodels/tsa/base/tsa_model.py:580:
ValueWarning: No supported index is available. Prediction results will be give
n with an integer index beginning at `start`.
  warnings.warn('No supported index is available.'
```



The predicted trajectory follows the paths our data has taken in January 2018. Realistically, of course, the data would would be much more volatile with a lot of noise. However, the Efficient Frontier method is effective even when the data trends seem monotone, and the following code would not need to be modified to be applied to other highly different real world data.

In [18]:
```python
# mfps['EEM'][3].to_frame().rename({0:})
id = pd.date_range('2018-02-01', periods=300, freq='D')
columns = etfs
df_f = pd.DataFrame(index=id, columns=columns)
for s in etfs:
    df_f[s] = mfps[s][3]
df_f.head()

df_p = pd.concat([table, df_f])
df_p
```

Out[18]:

	EEM	EFA	GLD	IWM	JNK	SPY	TLT
2018-01-02	47.070000	70.490000	124.528600	152.440000	36.635700	266.552000	126.940000
2018-01-03	48.120000	70.750000	124.797800	154.140000	36.798200	270.489000	126.050000
2018-01-04	48.450000	71.800000	124.758500	154.460000	36.907700	271.635000	126.040000
2018-01-05	48.800000	72.210000	125.008700	154.890000	36.960100	273.541000	125.510000
2018-01-08	49.030000	72.140000	125.270300	155.070000	36.954800	273.999000	125.460000
...	...	...	...	...	...	...	...
2018-11-23 00:00:00	162.256400	171.286724	206.072306	263.089381	36.031412	661.645364	79.089855
2018-11-24 00:00:00	162.829005	171.326604	205.341395	262.813101	36.015387	664.345810	78.426219
2018-11-25 00:00:00	161.894321	170.948506	205.614502	262.632118	35.901892	661.271681	78.388672
2018-11-26 00:00:00	162.199485	171.006309	205.798565	264.117150	35.920285	664.169494	78.177747
2018-11-27 00:00:00	163.648830	172.249494	206.000019	264.662421	35.982464	666.932311	78.417635

321 rows × 7 columns

Random Portfolios Generation

There are 7 funds in this portfolio. One decision the user could make is how to allocate their budget to each of ETFs (i.e. diversification). The below functions help with generating random weights to each fund, then calcualte and return the annualized return and volatility for comparision.

These functions were modified adapted from Ricky Kim's sample code (the first linked source).

In [19]:
```python
# Calculate the returns and volatility. The number 365 can change depending on the r
def portfolio_annualised_performance(weights, mean_returns, cov_matrix):
    returns = np.sum(mean_returns *weights ) *365
    std = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)
    return std, returns
```

In [20]:
```
# Generate portfolios with random weight allocation. Returns the results and the wei
# The number generated can be modified
def random_portfolios(num_portfolios, mean_returns, cov_matrix, risk_free_rate):
    results = np.zeros((3,num_portfolios))
    weights_record = []
    for i in range(num_portfolios):
        weights = np.random.random(7) # modify number to be number of etfs being anc
        weights /= np.sum(weights)
        weights_record.append(weights)
        portfolio_std_dev, portfolio_return = portfolio_annualised_performance(weigh
        results[0,i] = portfolio_std_dev
        results[1,i] = portfolio_return
        results[2,i] = (portfolio_return - risk_free_rate) / portfolio_std_dev
    return results, weights_record
```

The below terminology commonly refers to general stock indexes, but can be applied to the net asset value of ETFs as well. Daily change or returns can be found by calling pct_change on the dataframe. The below also sets the mean returns, covariance matrix, number of random portfolios generated, and the risk free rate.

In [52]:
```
returns = df_p.pct_change()
mean_returns = returns.mean()
cov_matrix = returns.cov()
num_portfolios = 25000
risk_free_rate = 0.0178
```

In [46]:
```
mean_returns
```

Out[46]:
```
EEM    0.003915
EFA    0.002804
GLD    0.001581
IWM    0.001731
JNK   -0.000055
SPY    0.002877
TLT   -0.001495
dtype: float64
```

Visualization and Output of Random Portfolios

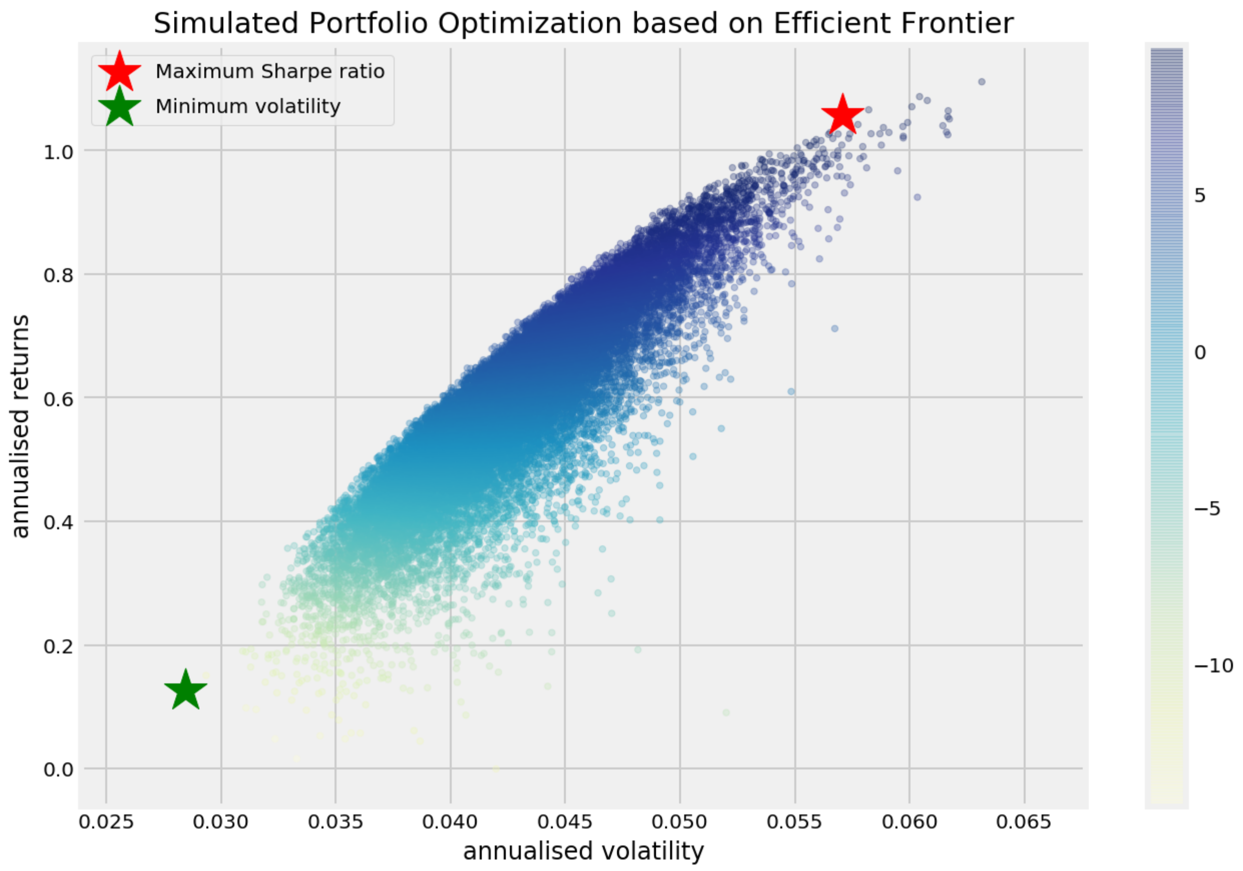
The visualization below plots all randomly generated portfolios, the bluer the color, the higher its Sharpe ratio. The red star is the portfolio with the highest sharpe ratio, while the green star (the lower left star) is the portfolio with the lowest volatility. The specific weight/budget allocation for these two portfolios will also be displayed.

The Sharpe Ratio is a metric which calculates the return on investment compared to its risk, offering additonal insight into portfolio gains.

```
In [47]:  # Generates random portfolios and gets results and corresponding weights. Then locat
           # portfolio as well as the minimum volatilit portfolio. Then all the portfolios are
           def display_simulated_ef_with_random(mean_returns, cov_matrix, num_portfolios, risk_
               results, weights = random_portfolios(num_portfolios,mean_returns, cov_matrix, ri

               max_sharpe_idx = np.argmax(results[2])
               sdp, rp = results[0,max_sharpe_idx], results[1,max_sharpe_idx]
               max_sharpe_allocation = pd.DataFrame(weights[max_sharpe_idx],index=table.columns
               max_sharpe_allocation.allocation = [round(i*100,2)for i in max_sharpe_allocation
               max_sharpe_allocation = max_sharpe_allocation.T

               min_vol_idx = np.argmin(results[0])
               sdp_min, rp_min = results[0,min_vol_idx], results[1,min_vol_idx]
               min_vol_allocation = pd.DataFrame(weights[min_vol_idx],index=table.columns,colum
               min_vol_allocation.allocation = [round(i*100,2)for i in min_vol_allocation.alloc
               min_vol_allocation = min_vol_allocation.T

               print( "-"*80)
               print( "Maximum Sharpe Ratio Portfolio Allocation\n")
               print( "Annualised Return:", round(rp,2))
               print( "Annualised Volatility:", round(sdp,2))
               print( "\n")
               print( max_sharpe_allocation)
               print( "-"*80)
               print( "Minimum Volatility Portfolio Allocation\n")
               print( "Annualised Return:", round(rp_min,2))
               print( "Annualised Volatility:", round(sdp_min,2))
               print( "\n")
               print( min_vol_allocation)

               plt.figure(figsize=(10, 7))
               plt.scatter(results[0,:],results[1,:],c=results[2,:],cmap='YlGnBu', marker='o',
               plt.colorbar()
               plt.scatter(sdp,rp,marker='*',color='r',s=500, label='Maximum Sharpe ratio')
               plt.scatter(sdp_min,rp_min,marker='*',color='g',s=500, label='Minimum volatility
               plt.title('Simulated Portfolio Optimization based on Efficient Frontier')
               plt.xlabel('annualised volatility')
               plt.ylabel('annualised returns')
               plt.legend(labelspacing=0.8)
```

In [48]:
```
display_simulated_ef_with_random(mean_returns, cov_matrix, num_portfolios, risk_free
```

```
--------------------------------------------------------------------------------
Maximum Sharpe Ratio Portfolio Allocation

Annualised Return: 1.06
Annualised Volatility: 0.06


              EEM    EFA   GLD    IWM   JNK   SPY   TLT
allocation  32.78  11.2  9.98  13.16  0.21  32.1  0.58
--------------------------------------------------------------------------------
Minimum Volatility Portfolio Allocation

Annualised Return: 0.13
Annualised Volatility: 0.03


             EEM   EFA   GLD    IWM    JNK   SPY    TLT
allocation  1.11  5.05  9.69  10.23  55.86  3.04  15.02
```



For minimum risk portfolio, we can see that over half of our budget is allocated to JNK. This can be identified as the least volatile ETF from the daily returns visualization.

However for a higher return, at the cost of a higher risk, our budget is much more evenly allocated, with JNK and TLT having close to 0 weights. This parallels the mean_returns calculated earlier. The least volatile often offer much less in returns than the most volatile.

Efficient Frontier

From the above visualization, an arch line can be identified at the top left edge of the dot cluster. The efficient frontier is precisely this line. It identifies the points (portfolios) that given a target return has the lowest risk. Because points along the line will give you the lowest risk for a given target return. Thus all of these points are the most efficient portfolio choices.

The below functions utlizes Scipy's optimize function to get the maximum Sharpe ratio portfolio. This is a more accurate than the previous method of generating randomized portfolios. The "neg_sharpe_ratio" computes the negative Sharpe ratio, due to Scipy only having a minimzation option. In "max_sharpe_ratio" function, you first define arguments (this should not include the variables you would like to change for optimisation, in this case, "weights"). At first, the construction of constraints was a bit difficult for me to understand, due to the way it is stated.

In [39]:

```
def neg_sharpe_ratio(weights, mean_returns, cov_matrix, risk_free_rate):
    p_var, p_ret = portfolio_annualised_performance(weights, mean_returns, cov_matri
    return -(p_ret - risk_free_rate) / p_var

# Calculates maximum sharpe ration portfolio using Scipy. Defining arguments and con
def max_sharpe_ratio(mean_returns, cov_matrix, risk_free_rate):
    num_assets = len(mean_returns)
    args = (mean_returns, cov_matrix, risk_free_rate)
    constraints = ({'type': 'eq', 'fun': lambda x: np.sum(x) - 1}) # all the weights
    bound = (0.0,1.0)
    bounds = tuple(bound for asset in range(num_assets)) # any weight should be incl
    result = sco.minimize(neg_sharpe_ratio, num_assets*[1./num_assets,], args=args,
                        method='SLSQP', bounds=bounds, constraints=constraints)
    return result
```

Similarly, Scipy can be used for a more accurate calculation of volatility.

In [40]:

```
# Calculating volatility of a portfolio
def portfolio_volatility(weights, mean_returns, cov_matrix):
    return portfolio_annualised_performance(weights, mean_returns, cov_matrix)[0]

# Returning the minimum variance, i.e. least volatile portfolio result.
def min_variance(mean_returns, cov_matrix):
    num_assets = len(mean_returns)
    args = (mean_returns, cov_matrix)
    constraints = ({'type': 'eq', 'fun': lambda x: np.sum(x) - 1}) # same constraint
    bound = (0.0,1.0)
    bounds = tuple(bound for asset in range(num_assets))

    result = sco.minimize(portfolio_volatility, num_assets*[1./num_assets,], args=ar
                        method='SLSQP', bounds=bounds, constraints=constraints)

    return result
```

The code below computes the efficient frontier line in increments of return changes.

In [41]:

```
# Calculates the efficient return: minimum volatility for a target return
def efficient_return(mean_returns, cov_matrix, target):
    num_assets = len(mean_returns)
    args = (mean_returns, cov_matrix)

    def portfolio_return(weights):
        return portfolio_annualised_performance(weights, mean_returns, cov_matrix)[1

    constraints = ({'type': 'eq', 'fun': lambda x: portfolio_return(x) - target},
                   {'type': 'eq', 'fun': lambda x: np.sum(x) - 1})
    bounds = tuple((0,1) for asset in range(num_assets))
    result = sco.minimize(portfolio_volatility, num_assets*[1./num_assets,], args=ar
    return result

# Finds the efficient frontier for a range of return targets
def efficient_frontier(mean_returns, cov_matrix, returns_range):
    efficients = []
    for ret in returns_range:
        efficients.append(efficient_return(mean_returns, cov_matrix, ret))
    return efficients
```

Efficient Frontier Visualization

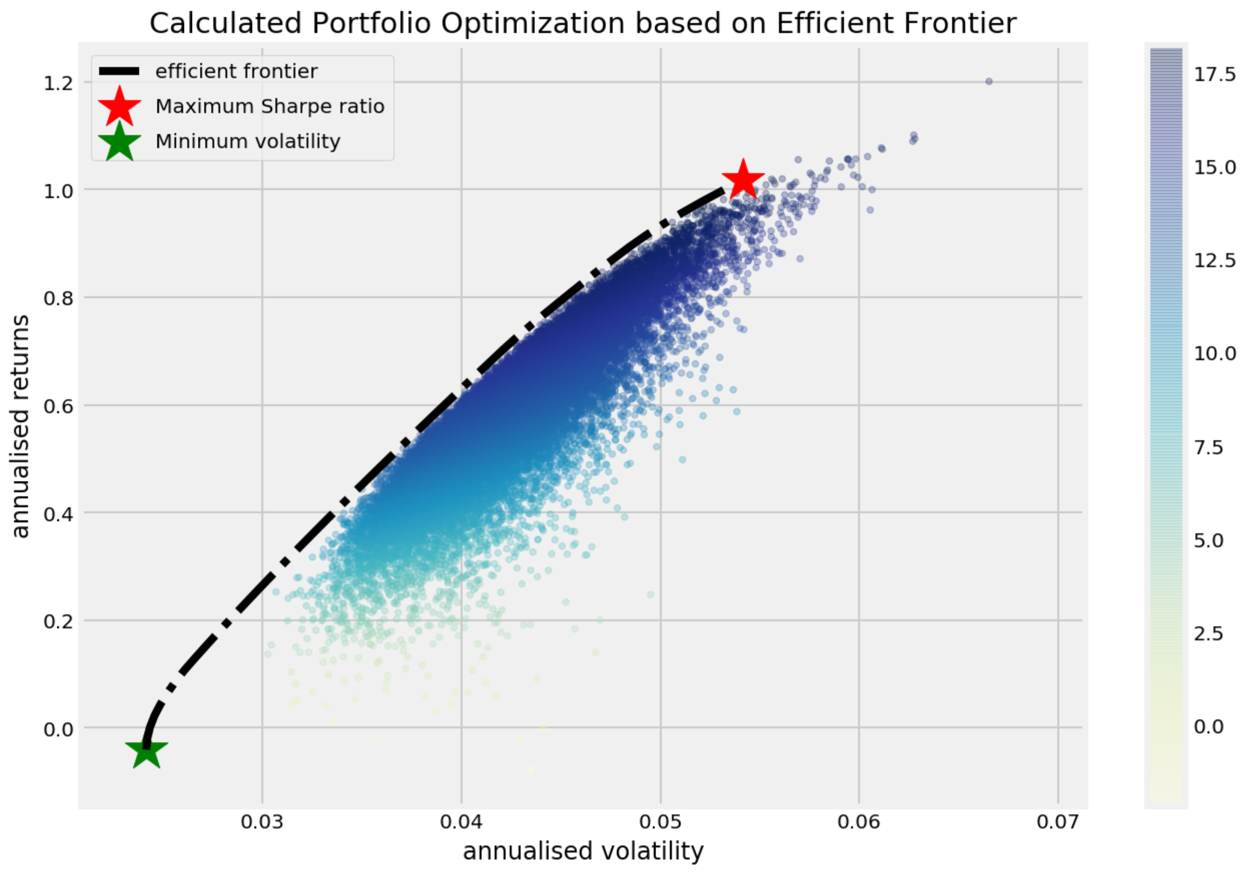
Now we plot the efficient frontier, then display maximum Sharpe ratio and minimum volatility with all the previously randomly generated portfolios. Note this is different to the previously found two optimal portfolio, which were simply selected from the random generations.

In [57]:

```
def display_calculated_ef_with_random(mean_returns, cov_matrix, num_portfolios, risk
    results, _ = random_portfolios(num_portfolios,mean_returns, cov_matrix, risk_fre

    max_sharpe = max_sharpe_ratio(mean_returns, cov_matrix, risk_free_rate)
    sdp, rp = portfolio_annualised_performance(max_sharpe['x'], mean_returns, cov_ma
    max_sharpe_allocation = pd.DataFrame(max_sharpe.x,index=table.columns,columns=['
    max_sharpe_allocation.allocation = [round(i*100,2)for i in max_sharpe_allocatior
    max_sharpe_allocation = max_sharpe_allocation.T
    max_sharpe_allocation

    min_vol = min_variance(mean_returns, cov_matrix)
    sdp_min, rp_min = portfolio_annualised_performance(min_vol['x'], mean_returns, c
    min_vol_allocation = pd.DataFrame(min_vol.x,index=table.columns,columns=['alloca
    min_vol_allocation.allocation = [round(i*100,2)for i in min_vol_allocation.alloc
    min_vol_allocation = min_vol_allocation.T

    print( "-"*80)
    print( "Maximum Sharpe Ratio Portfolio Allocation\n")
    print( "Annualised Return:", round(rp,2))
    print( "Annualised Volatility:", round(sdp,2))
    print( "\n")
    print( max_sharpe_allocation)
    print( "-"*80)
    print( "Minimum Volatility Portfolio Allocation\n")
    print( "Annualised Return:", round(rp_min,2))
    print( "Annualised Volatility:", round(sdp_min,2))
    print( "\n")
    print( min_vol_allocation)

    plt.figure(figsize=(10, 7))
    plt.scatter(results[0,:],results[1,:],c=results[2,:],cmap='YlGnBu', marker='o',
    plt.colorbar()
    plt.scatter(sdp,rp,marker='*',color='r',s=500, label='Maximum Sharpe ratio')
    plt.scatter(sdp_min,rp_min,marker='*',color='g',s=500, label='Minimum volatility

    target = np.linspace(rp_min, 1, 50) # the length of the line can be modified
    efficient_portfolios = efficient_frontier(mean_returns, cov_matrix, target)
    plt.plot([p['fun'] for p in efficient_portfolios], target, linestyle='-.', color
    plt.title('Calculated Portfolio Optimization based on Efficient Frontier')
    plt.xlabel('annualised volatility')
    plt.ylabel('annualised returns')
    plt.legend(labelspacing=0.8)
```

```
In [58]:  display_calculated_ef_with_random(mean_returns, cov_matrix, num_portfolios, risk_fre
```

```
--------------------------------------------------------------------------------
Maximum Sharpe Ratio Portfolio Allocation

Annualised Return: 1.02
Annualised Volatility: 0.05


               EEM  EFA    GLD    IWM  JNK    SPY  TLT
allocation   32.54  0.0  23.47  10.66  0.0  33.33  0.0
--------------------------------------------------------------------------------
Minimum Volatility Portfolio Allocation

Annualised Return: -0.04
Annualised Volatility: 0.02


             EEM  EFA   GLD  IWM    JNK  SPY   TLT
allocation   0.0  0.0  4.91  0.0  85.55  0.0  9.54
```



The calculated Scipy's minimum volatility has allocated most (over 85%) of the budget to JNK despite a below 0 annualized return. Evidently the least volatile also tends to yield the least returns, and a smart invester would not chose to 100% risk averse. For the maximum Sharpe ratio portfolio calculated through Scipy, we can see it yields a higher return than found through randomized generation. There is a less even distribution across the ETFs, with EEM and SPY covering a combined 2/3 of the budget allocation. Refering back to the visualization of our generated data, we can identify those two funds as the fastest growing funds.

Final Portfolio Decisions Output

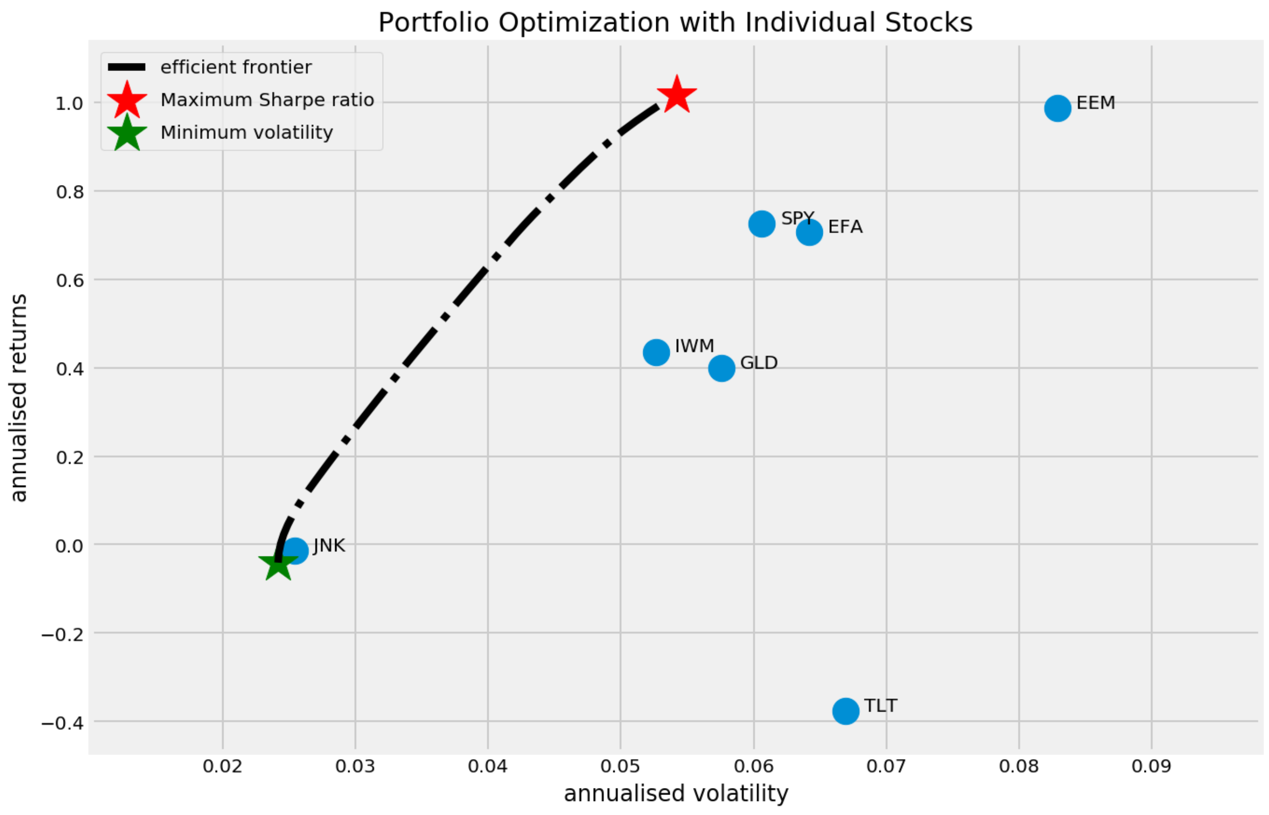
Removing the irrelevant information for our users, the randomly generated portfolios, we can finally display the efficient frontier line with two extreme portfolio options along with the risk and returns of each individual fund. This can help accurately illustrate the effects diversification and risk-minization have on the overall portfolio's perfomance. The user can also utilize this information to better make informed decisions for how they wish to construct their portfolio.

In [62]:

```python
def display_ef_with_selected(mean_returns, cov_matrix, risk_free_rate):
    max_sharpe = max_sharpe_ratio(mean_returns, cov_matrix, risk_free_rate)
    sdp, rp = portfolio_annualised_performance(max_sharpe['x'], mean_returns, cov_ma
    max_sharpe_allocation = pd.DataFrame(max_sharpe.x,index=table.columns,columns=['
    max_sharpe_allocation.allocation = [round(i*100,2)for i in max_sharpe_allocation
    max_sharpe_allocation = max_sharpe_allocation.T
    max_sharpe_allocation

    min_vol = min_variance(mean_returns, cov_matrix)
    sdp_min, rp_min = portfolio_annualised_performance(min_vol['x'], mean_returns, c
    min_vol_allocation = pd.DataFrame(min_vol.x,index=table.columns,columns=['alloca
    min_vol_allocation.allocation = [round(i*100,2)for i in min_vol_allocation.alloc
    min_vol_allocation = min_vol_allocation.T

    an_vol = np.std(returns) * np.sqrt(252)
    an_rt = mean_returns * 252

    print( "-"*80)
    print( "Maximum Sharpe Ratio Portfolio Allocation\n")
    print( "Annualised Return:", round(rp,2))
    print( "Annualised Volatility:", round(sdp,2))
    print( "\n")
    print( max_sharpe_allocation)
    print( "-"*80)
    print( "Minimum Volatility Portfolio Allocation\n")
    print( "Annualised Return:", round(rp_min,2))
    print( "Annualised Volatility:", round(sdp_min,2))
    print( "\n")
    print( min_vol_allocation)
    print( "-"*80)
    print( "Individual Stock Returns and Volatility\n")
    for i, txt in enumerate(table.columns):
        print( txt,":","annuaised return",round(an_rt[i],2),", annualised volatility
    print( "-"*80)

    fig, ax = plt.subplots(figsize=(10, 7))
    ax.scatter(an_vol,an_rt,marker='o',s=200)

    for i, txt in enumerate(table.columns):
        ax.annotate(txt, (an_vol[i],an_rt[i]), xytext=(10,0), textcoords='offset poi
    ax.scatter(sdp,rp,marker='*',color='r',s=500, label='Maximum Sharpe ratio')
    ax.scatter(sdp_min,rp_min,marker='*',color='g',s=500, label='Minimum volatility'

    target = np.linspace(rp_min, 1, 50) # the length of the line can be modified
    efficient_portfolios = efficient_frontier(mean_returns, cov_matrix, target)
    ax.plot([p['fun'] for p in efficient_portfolios], target, linestyle='-.', color=
    ax.set_title('Portfolio Optimization with Individual Stocks')
    ax.set_xlabel('annualised volatility')
    ax.set_ylabel('annualised returns')
    ax.legend(labelspacing=0.8)
```

```
In [63]:  display_ef_with_selected(mean_returns, cov_matrix, risk_free_rate)
```

```
--------------------------------------------------------------------------------
Maximum Sharpe Ratio Portfolio Allocation

Annualised Return: 1.02
Annualised Volatility: 0.05


              EEM  EFA    GLD    IWM  JNK    SPY  TLT
allocation  32.54  0.0  23.47  10.66  0.0  33.33  0.0
--------------------------------------------------------------------------------
Minimum Volatility Portfolio Allocation

Annualised Return: -0.04
Annualised Volatility: 0.02


            EEM  EFA   GLD  IWM    JNK  SPY   TLT
allocation  0.0  0.0  4.91  0.0  85.55  0.0  9.54
--------------------------------------------------------------------------------
Individual Stock Returns and Volatility

EEM : annuaised return 0.99 , annualised volatility: 0.08
EFA : annuaised return 0.71 , annualised volatility: 0.06
GLD : annuaised return 0.4 , annualised volatility: 0.06
IWM : annuaised return 0.44 , annualised volatility: 0.05
JNK : annuaised return -0.01 , annualised volatility: 0.03
SPY : annuaised return 0.73 , annualised volatility: 0.06
TLT : annuaised return -0.38 , annualised volatility: 0.07
--------------------------------------------------------------------------------
```



The Efficient Frontier can be seen tracing from the minimized volatility towards the maximum Sharpe Ratio. In implementation of our application, the sample portfolio as well as the graph outputted here can be displayed on our UI. Ultimately guiding our users to make more informed decisions for retirement investing. Some features through this particular implementation I want to highlight are the degree of flexibility in the technical methodologies. Variables such as, number of generated portfolios, risk rate the user wishes to tolerate, as well as specific functions for outputting the optimized portfolio allocations can all be modified. This is only one sliver of how this generalized function can become personalized for each individual user, providing a friendly and rewarding investing experiencelike no other.