

Timeseries-toolbox.R

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```
# Fable package -----
```

```
#https://cran.r-project.org/web/packages/fable/vignettes/fable.html
```

```
library(fable)
```

```
## Warning: package 'fable' was built under R version 4.1.3
```

```
## Loading required package: fabletools
```

```
## Warning: package 'fabletools' was built under R version 4.1.3
```

```
library(tsibble)
```

```
## Warning: package 'tsibble' was built under R version 4.1.3
```

```
##
## Attaching package: 'tsibble'
```

```
## The following objects are masked from 'package:base':
##
##   intersect, setdiff, union
```

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##   filter, lag
```

```
## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union
```

```
#tsibble:::tourism data set
```

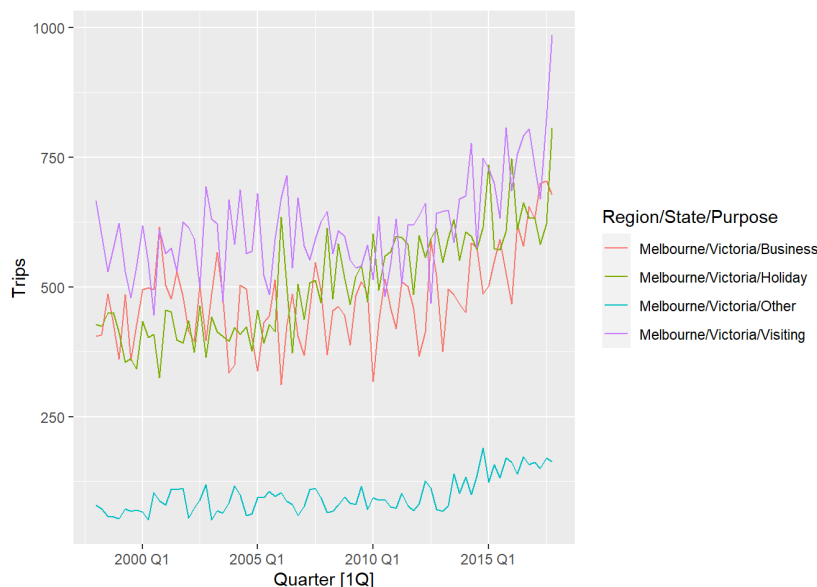
```
tourism_melb <- tourism %>%
  filter(Region == "Melbourne")
```

```
tourism_melb %>%
  group_by(Purpose) %>% head
```

```
## # A tsibble: 6 x 5 [1Q]
## # Key:      Region, State, Purpose [1]
## # Groups:   Purpose [1]
##   Quarter Region   State   Purpose Trips
##   <qtr> <chr>    <chr>    <chr>    <dbl>
## 1 1998 Q1 Melbourne Victoria Business 405.
## 2 1998 Q2 Melbourne Victoria Business 408.
## 3 1998 Q3 Melbourne Victoria Business 486.
## 4 1998 Q4 Melbourne Victoria Business 429.
## 5 1999 Q1 Melbourne Victoria Business 361.
## 6 1999 Q2 Melbourne Victoria Business 486.
```

```
#The variable that we'd like to estimate is the number of overnight trips (in thousands)
#represented by the Trips variable
```

```
tourism_melb %>%
  autoplot(Trips)
```



```
### about ets https://math.unm.edu/~lil/Stat581/8-ets.pdf
```

```
#fit an ets and an arima model
```

```
fit <- tourism_melb %>%
  model(
    ets = ETS(Trips ~ trend("A")),
    arima = ARIMA(Trips)
  )
fit
```

```
## # A mable: 4 x 5
## # Key:   Region, State, Purpose [4]
## # Region State Purpose ets arima
## # <chr> <chr> <chr> <model> <model>
## # 1 Melbourne Victoria Business <ETS(A,A,A)> <ARIMA(0,1,2)(1,0,1)[4] w/ drift>
## # 2 Melbourne Victoria Holiday <ETS(M,A,A)> <ARIMA(0,1,1) w/ drift>
## # 3 Melbourne Victoria Other <ETS(A,A,N)> <ARIMA(0,1,1) w/ drift>
## # 4 Melbourne Victoria Visiting <ETS(M,A,A)> <ARIMA(0,1,1)(1,0,2)[4]>
```

```
fit %>% class
```

```
## [1] "mdl_df" "tbl_df" "tbl" "data.frame"
```

```
#A mable contains a row for each time series (uniquely identified by the key variables),
#and a column for each model specification
```

```
fit %>%
  select(Region, State, Purpose, arima) %>%
  coef()
```

```
## # A tibble: 13 x 9
##   Region State Purpose .model term estimate std.error statistic p.value
##   <chr> <chr> <chr> <chr> <chr> <dbl> <dbl> <dbl> <dbl>
## # 1 Melbour~ Victor~ Business arima ma1 -0.555 0.130 -4.28 5.29e- 5
## # 2 Melbour~ Victor~ Business arima ma2 -0.233 0.129 -1.81 7.47e- 2
## # 3 Melbour~ Victor~ Business arima sar1 0.946 0.0634 14.9 1.08e-24
## # 4 Melbour~ Victor~ Business arima sma1 -0.772 0.145 -5.34 8.81e- 7
## # 5 Melbour~ Victor~ Business arima const~ 0.192 0.213 0.903 3.69e- 1
## # 6 Melbour~ Victor~ Holiday arima ma1 -0.931 0.0851 -10.9 1.77e-17
## # 7 Melbour~ Victor~ Holiday arima const~ 3.65 0.571 6.39 1.06e- 8
## # 8 Melbour~ Victor~ Other arima ma1 -0.750 0.0708 -10.6 8.19e-17
## # 9 Melbour~ Victor~ Other arima const~ 1.24 0.640 1.93 5.70e- 2
## # 10 Melbour~ Victor~ Visiting arima ma1 -0.838 0.0652 -12.8 5.03e-21
## # 11 Melbour~ Victor~ Visiting arima sar1 0.659 0.193 3.41 1.03e- 3
## # 12 Melbour~ Victor~ Visiting arima sma1 -0.402 0.206 -1.95 5.47e- 2
## # 13 Melbour~ Victor~ Visiting arima sma2 0.322 0.143 2.26 2.68e- 2
```

```
fit %>% glance()
```

```
## # A tibble: 8 x 14
##   Region State Purpose .model sigma2 log_lik AIC AICc BIC MSE AMSE
##   <chr>   <chr>   <chr>   <chr>   <dbl>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 Melbour~ Victor~ Busine~ ets    3.53e+3 -498. 1014. 1016. 1035. 3180. 3520.
## 2 Melbour~ Victor~ Busine~ arima  3.67e+3 -435. 882. 883. 896. NA NA
## 3 Melbour~ Victor~ Holiday ets    1.10e-2 -487. 992. 994. 1013. 2548. 2574.
## 4 Melbour~ Victor~ Holiday arima  3.07e+3 -429. 864. 865. 872. NA NA
## 5 Melbour~ Victor~ Other ets    4.97e+2 -422. 853. 854. 865. 472. 512.
## 6 Melbour~ Victor~ Other arima  4.89e+2 -356. 718. 719. 725. NA NA
## 7 Melbour~ Victor~ Visiti~ ets    1.09e-2 -503. 1024. 1026. 1045. 3714. 3860.
## 8 Melbour~ Victor~ Visiti~ arima  4.24e+3 -442. 893. 894. 905. NA NA
## # ... with 3 more variables: MAE <dbl>, ar_roots <list>, ma_roots <list>
```

#If you're working with a single model (or want to look at one model in particular), the report()

```
fit %>%
  filter(Purpose == "Holiday") %>%
  select(ets) %>%
  report()
```

```
## Series: Trips
## Model: ETS(M,A,A)
## Smoothing parameters:
##   alpha = 0.03084501
##   beta  = 0.03084499
##   gamma = 0.0001000967
##
## Initial states:
##   l[0]    b[0]    s[0]    s[-1]    s[-2]    s[-3]
## 424.0777 -2.535481 -26.7441 4.256618 -10.10668 32.59417
##
##   sigma^2: 0.011
##
##           AIC      AICc      BIC
## 991.7305 994.3020 1013.1688
```

#the augment() function may be more convenient, which provides the original data along with both fitted values and their residuals.

```
fit %>%
  augment()
```

```
## # A tsibble: 640 x 9 [1Q]
## # Key:           Region, State, Purpose, .model [8]
##   Region State Purpose .model Quarter Trips .fitted .resid .innov
##   <chr>   <chr>   <chr>   <chr>   <qtr> <dbl>   <dbl>   <dbl>   <dbl>
## 1 Melbourne Victoria Business ets    1998 Q1 405.   396.    9.54    9.54
## 2 Melbourne Victoria Business ets    1998 Q2 408.   483.   -75.1   -75.1
## 3 Melbourne Victoria Business ets    1998 Q3 486.   487.    -1.13   -1.13
## 4 Melbourne Victoria Business ets    1998 Q4 429.   454.   -25.2   -25.2
## 5 Melbourne Victoria Business ets    1999 Q1 361.   391.   -30.3   -30.3
## 6 Melbourne Victoria Business ets    1999 Q2 486.   466.   19.9    19.9
## 7 Melbourne Victoria Business ets    1999 Q3 359.   492.  -133.   -133.
## 8 Melbourne Victoria Business ets    1999 Q4 426.   424.    1.49    1.49
## 9 Melbourne Victoria Business ets    2000 Q1 495.   364.   130.    130.
## 10 Melbourne Victoria Business ets    2000 Q2 499.   477.   22.0    22.0
## # ... with 630 more rows
```

#arrange arcoding to MASE

```
fit %>%
  accuracy() %>%
  arrange(MASE)
```

```
## # A tibble: 8 x 13
##   Region State Purpose .model .type ME RMSE MAE MPE MAPE MASE RMSSE
##   <chr>   <chr>   <chr>   <chr>   <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 Melbou~ Vict~ Holiday ets    Trai~ 4.67 50.5 37.2 0.217 7.29 0.675 0.697
## 2 Melbou~ Vict~ Busine~ ets    Trai~ 3.31 56.4 42.9 -0.753 9.31 0.691 0.740
## 3 Melbou~ Vict~ Busine~ arima Trai~ 2.54 58.2 46.0 -1.17 10.1 0.741 0.765
## 4 Melbou~ Vict~ Holiday arima Trai~ -4.64 54.3 41.4 -2.44 8.46 0.752 0.751
## 5 Melbou~ Vict~ Other arima Trai~ -0.344 21.7 17.0 -6.16 19.5 0.763 0.772
## 6 Melbou~ Vict~ Other ets    Trai~ -0.142 21.7 17.0 -5.97 19.6 0.767 0.773
## 7 Melbou~ Vict~ Visiti~ ets    Trai~ 8.17 60.9 51.4 0.433 8.28 0.819 0.782
## 8 Melbou~ Vict~ Visiti~ arima Trai~ 6.89 63.1 51.7 0.106 8.44 0.825 0.809
## # ... with 1 more variable: ACF1 <dbl>
```

```
#Forecasts from these models can be produced directly as our specified
#models do not require any additional data.
```

```
fc <- fit %>%
  forecast(h = "5 years")
fc
```

```
## # A tibble: 160 x 7 [1Q]
## # Key:   Region, State, Purpose, .model [8]
##   Region State Purpose .model Quarter      Trips .mean
##   <chr>   <chr>   <chr>   <chr>   <qtr>     <dbl> <dbl>
## 1 Melbourne Victoria Business ets    2018 Q1 N(619, 3533) 619.
## 2 Melbourne Victoria Business ets    2018 Q2 N(709, 3766) 709.
## 3 Melbourne Victoria Business ets    2018 Q3 N(738, 4042) 738.
## 4 Melbourne Victoria Business ets    2018 Q4 N(713, 4364) 713.
## 5 Melbourne Victoria Business ets    2019 Q1 N(664, 4735) 664.
## 6 Melbourne Victoria Business ets    2019 Q2 N(755, 5159) 755.
## 7 Melbourne Victoria Business ets    2019 Q3 N(784, 5640) 784.
## 8 Melbourne Victoria Business ets    2019 Q4 N(759, 6181) 759.
## 9 Melbourne Victoria Business ets    2020 Q1 N(710, 6786) 710.
## 10 Melbourne Victoria Business ets    2020 Q2 N(800, 7458) 800.
## # ... with 150 more rows
```

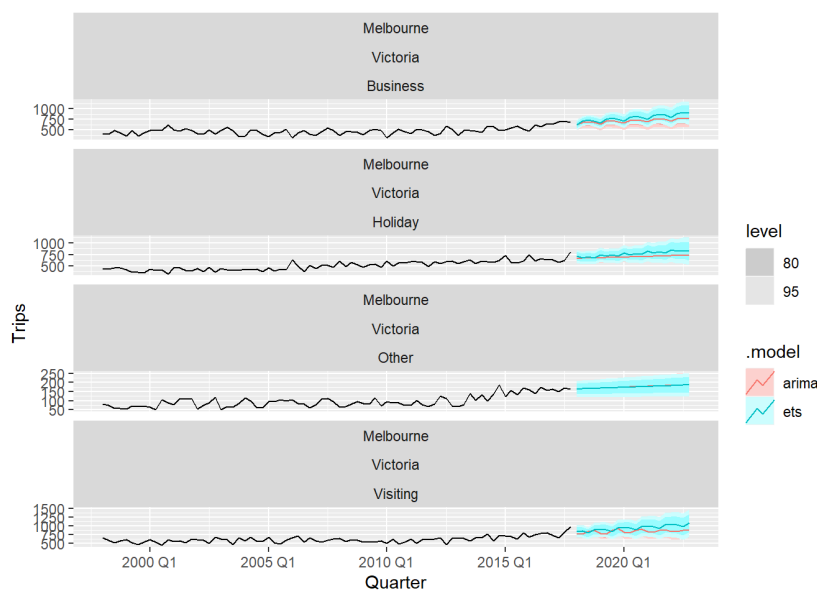
```
#The resulting forecasts are contained in a "fable" (forecast table),
```

```
#Confidence intervals can be extracted from the distribution using the hilo() function.
```

```
fc %>%
  hilo(level = c(80, 95))
```

```
## # A tibble: 160 x 9 [1Q]
## # Key:   Region, State, Purpose, .model [8]
##   Region State Purpose .model Quarter      Trips .mean      `80%`
##   <chr>   <chr>   <chr>   <chr>   <qtr>     <dbl> <dbl>     <hilo>
## 1 Melbo~ Vict~ Busine~ ets    2018 Q1 N(619, 3533) 619. [542.3864, 694.7363]80
## 2 Melbo~ Vict~ Busine~ ets    2018 Q2 N(709, 3766) 709. [630.4443, 787.7426]80
## 3 Melbo~ Vict~ Busine~ ets    2018 Q3 N(738, 4042) 738. [656.9891, 819.9425]80
## 4 Melbo~ Vict~ Busine~ ets    2018 Q4 N(713, 4364) 713. [628.7879, 798.1010]80
## 5 Melbo~ Vict~ Busine~ ets    2019 Q1 N(664, 4735) 664. [575.8480, 752.2196]80
## 6 Melbo~ Vict~ Busine~ ets    2019 Q2 N(755, 5159) 755. [662.5141, 846.6176]80
## 7 Melbo~ Vict~ Busine~ ets    2019 Q3 N(784, 5640) 784. [687.6922, 880.1843]80
## 8 Melbo~ Vict~ Busine~ ets    2019 Q4 N(759, 6181) 759. [658.1603, 859.6735]80
## 9 Melbo~ Vict~ Busine~ ets    2020 Q1 N(710, 6786) 710. [603.9336, 815.0789]80
## 10 Melbo~ Vict~ Busine~ ets    2020 Q2 N(800, 7458) 800. [689.3620, 910.7146]80
## # ... with 150 more rows, and 1 more variable: 95% <hilo>
```

```
fc %>%
  autoplot(tourism_melb)
```



```
# TS studio -----
#https://cran.r-project.org/web/packages/TSstudio/vignettes/Plotting_Time_Series.html
```

```
pacman::p_load(TSstudio)
```

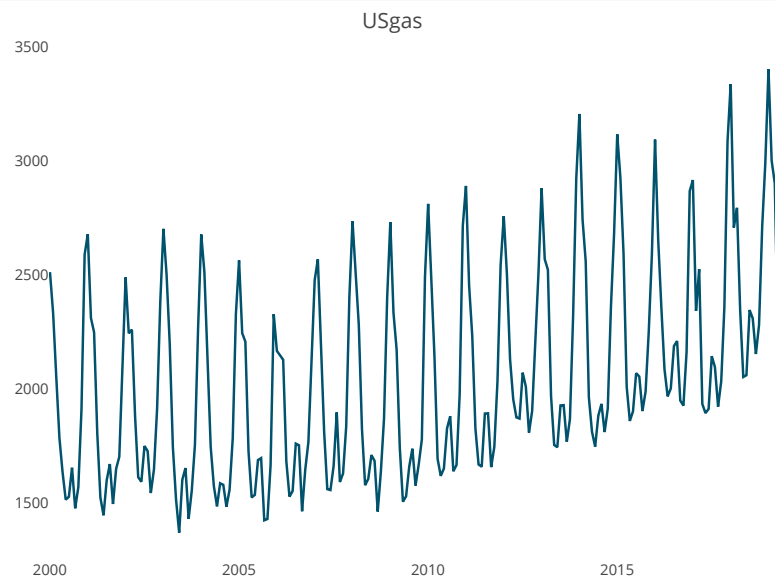
```
data(USgas)
USgas %>% class
```

```
## [1] "ts"
```

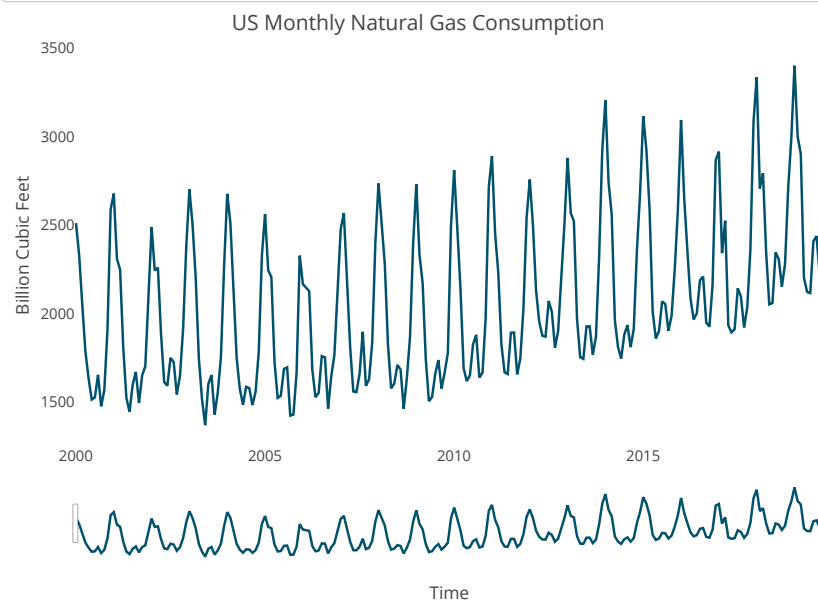
```
ts_info(USgas)
```

```
## The USgas series is a ts object with 1 variable and 238 observations
## Frequency: 12
## Start time: 2000 1
## End time: 2019 10
```

```
ts_plot(USgas)
```



```
ts_plot(USgas,
  title = "US Monthly Natural Gas Consumption",
  Xtitle = "Time",
  Ytitle = "Billion Cubic Feet",
  slider = TRUE)
```



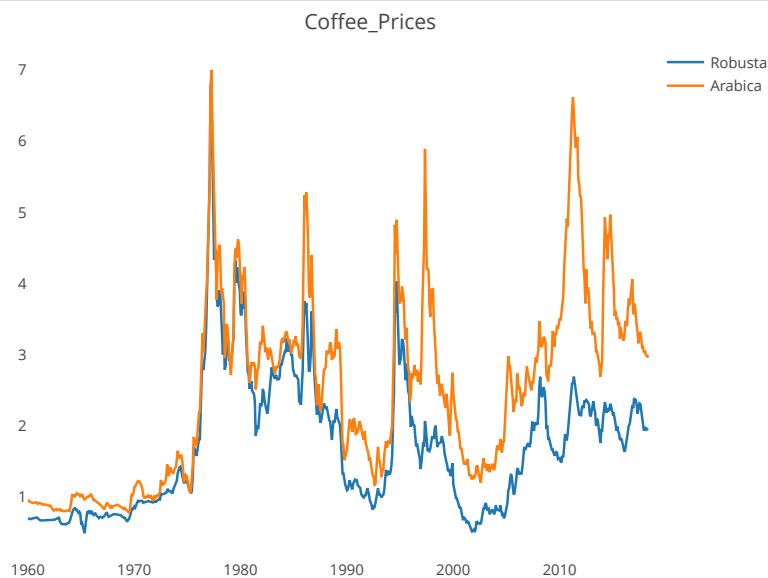
```
## multiple ts plot
data("Coffee_Prices")
Coffee_Prices %>% head
```

```
##           Robusta Arabica
## [1,] 0.6968643 0.9409
## [2,] 0.6887074 0.9469
## [3,] 0.6887074 0.9281
## [4,] 0.6845187 0.9303
## [5,] 0.6906915 0.9200
## [6,] 0.6968643 0.9123
```

```
ts_info(Coffee_Prices)
```

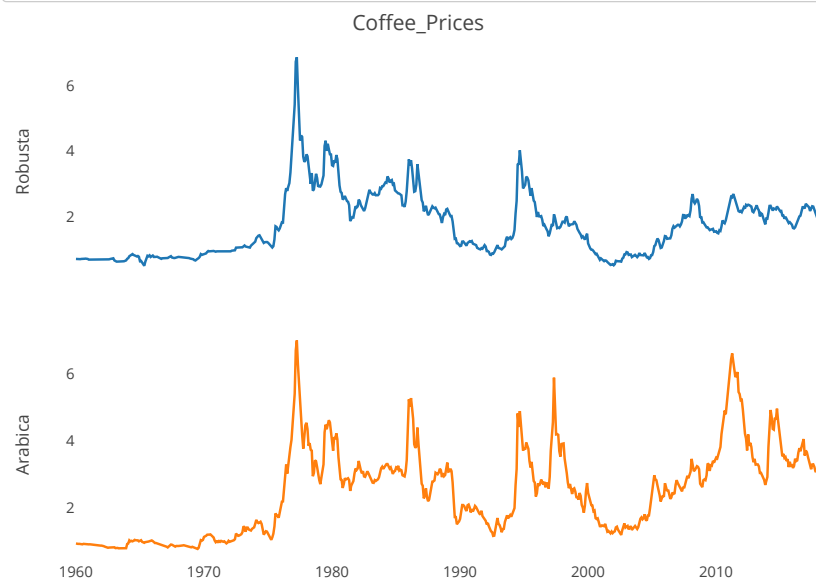
```
## The Coffee_Prices series is a mts object with 2 variables and 701 observations
## Frequency: 12
## Start time: 1960 1
## End time: 2018 5
```

```
ts_plot(Coffee_Prices)
```



```
## Plot on different plots
```

```
ts_plot(Coffee_Prices,
        type = "multiple")
```



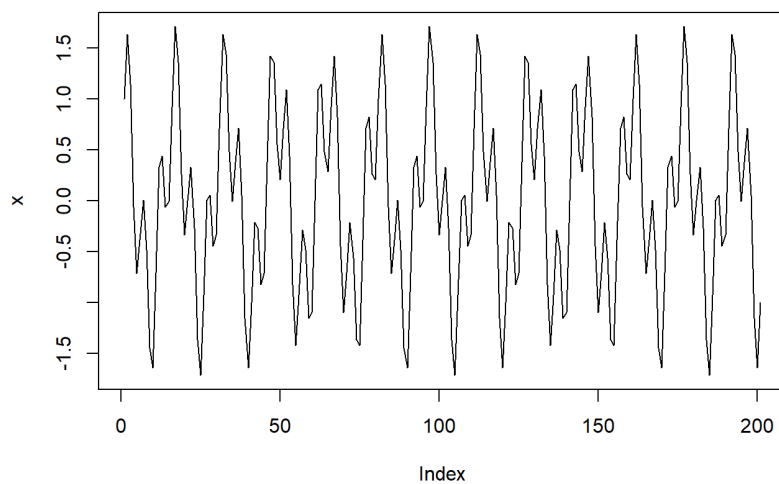
```
# Spectral analysis -----

#https://math.mcmaster.ca/~bolker/eed/2010/Ecology/Spectral.pdf

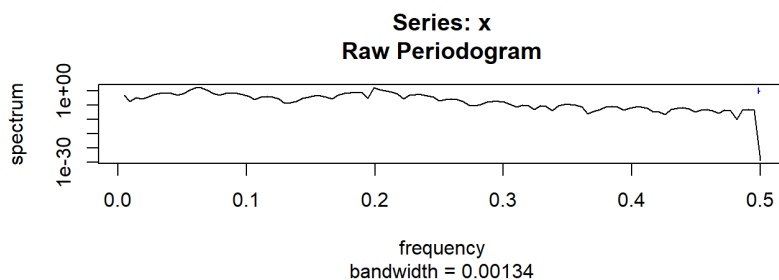
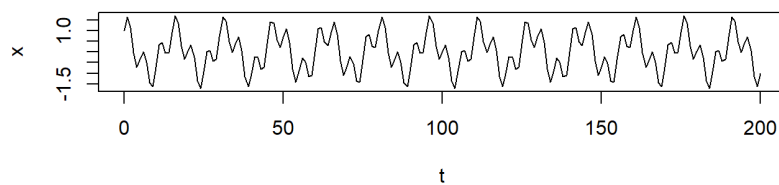
#Simple Example

t <- seq(0,200,by=1)
x <- cos(2*pi*t/16) + 0.75*sin(2*pi*t/5)

plot(x,type="l")
```



```
par(mfrow=c(2,1))
plot(t,x,'l')
spectrum(x)
```



```

par(mfrow=c(1,1))
#Usually, we want to subtract the mean from the time series.

#the spectrum function goes further and automatically removes a linear trend from the series before
#calculating the periodogram

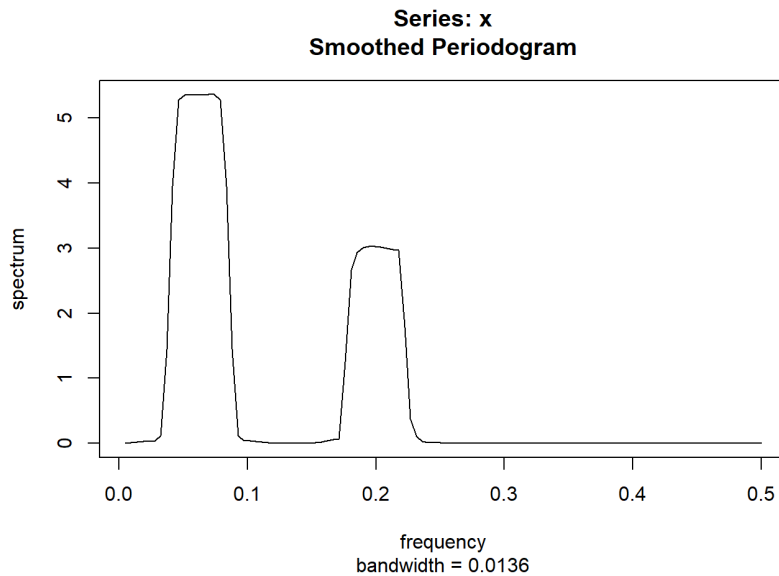
#Smoothing the periodogram

#Spectral analysis in R

#The default frequency axis is in cycles per sampling interval

#It is
#more intuitive to convert the frequency axis to cycles per unit time, we can do this by extracting the
#frequency values that R returns and dividing by the length of the sampling interval. We should also
#multiply the spectral density by 2 so that the area under the periodogram actually equals the variance
#of the time series
spectrum(x,log="no",span=10)

```



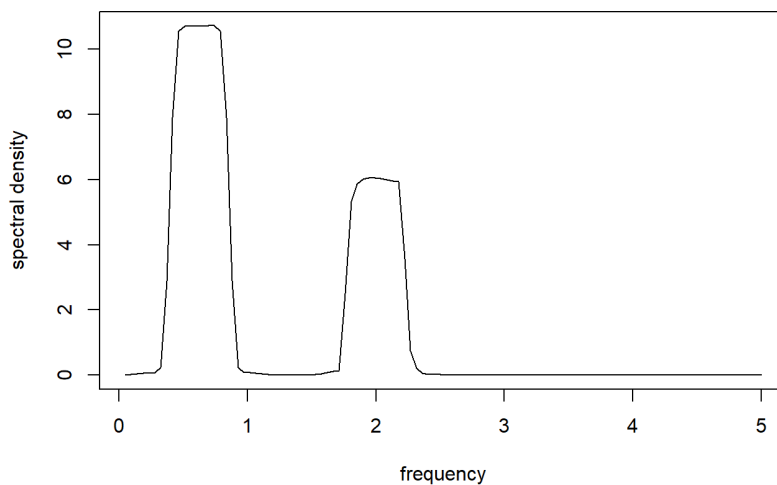
```

del<-0.1 # sampling interval

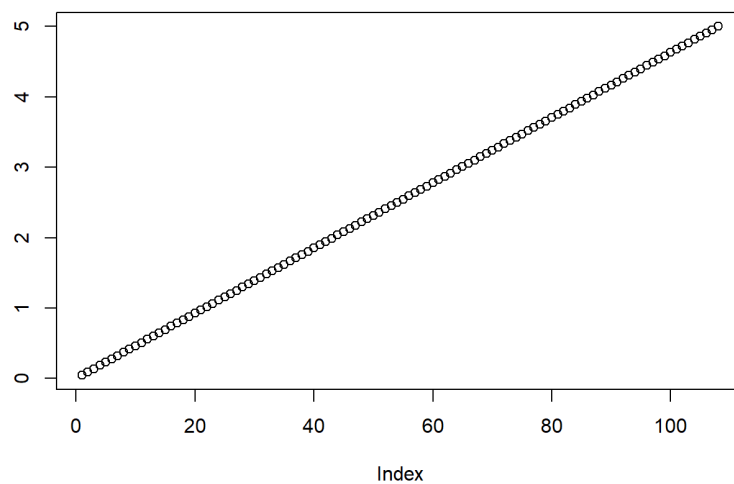
x.spec <- spectrum(x,log="no",span=10,plot=FALSE)

spx <- x.spec$freq/del
spy <- 2*x.spec$spec
plot(spy~spx,xlab="frequency",ylab="spectral density",type="l")

```



```
spx %>% plot
```

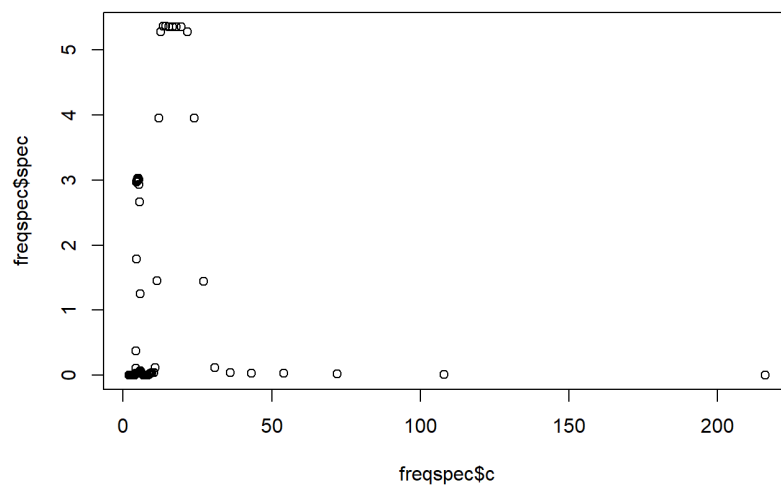



```
attributes(x.spec)
```

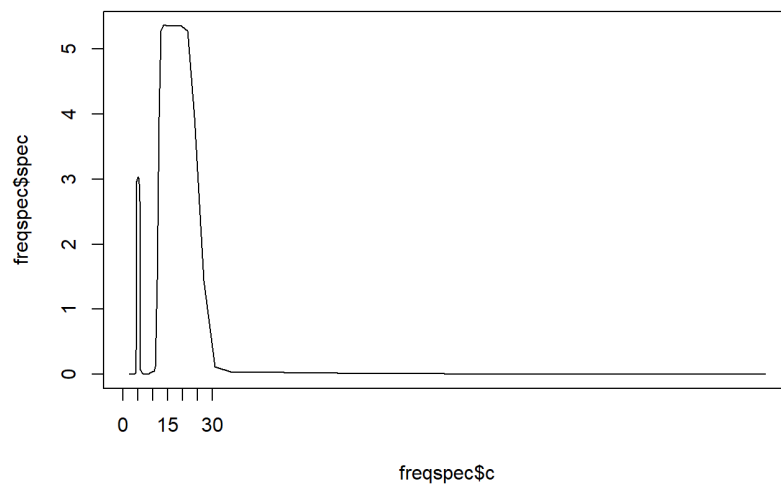
```
## $names
## [1] "freq"      "spec"      "coh"      "phase"    "kernel"   "df"
## [7] "bandwidth" "n.used"    "orig.n"   "series"   "snames"   "method"
## [13] "taper"     "pad"      "detrend"  "demean"
##
## $class
## [1] "spec"
```

```
freqspec=data.frame(freq=x.spec$freq,spec=x.spec$spec)
freqspec=freqspec%% mutate(c=1/freq)

plot(freqspec$spec~freqspec$c)
```



```
plot(freqspec$spec~freqspec$c,type="l",xaxt="n")
axis(1, at = seq(0,30, by = 5))
```



```
## so it seems that we have a cycle every 5 periods, and 1 more every 10-30 periods
```

```
freqspec %>% filter(c>=10 & c<=30) %>% summarize(mean=mean(c))
```

```
##      mean
## 1 16.2386
```

```
#so one more at 16 which is the dominant one
forecast::findfrequency(x)
```

```
## Registered S3 method overwritten by 'quantmod':
##      method      from
##  as.zoo.data.frame zoo
```

```
## [1] 16
```

```
quantile(freqspec$c)
```

```
##      0%      25%      50%      75%     100%
## 2.000000 2.658537 3.963636 7.785714 216.000000
```

```
q=quantile(freqspec$c,probs=c(0.1,0.9))
```

```
freqspec %>% filter(between(c,q[1],q[2])) %>% head #filter for 10% 90% values -->trimming
```

```
##      freq      spec      c
## 1 0.05555556 5.359152 18.00000
## 2 0.06018519 5.356081 16.61538
## 3 0.06481481 5.356592 15.42857
## 4 0.06944444 5.360568 14.40000
## 5 0.07407407 5.360787 13.50000
## 6 0.07870370 5.283165 12.70588
```

```
freqspec %>% filter(between(c,q[1],q[2])) %>% tail
```

```
##      freq      spec      c
## 81 0.4259259 4.408458e-08 2.347826
## 82 0.4305556 3.067506e-08 2.322581
## 83 0.4351852 2.296704e-08 2.297872
## 84 0.4398148 2.164951e-08 2.273684
## 85 0.4444444 2.175318e-08 2.250000
## 86 0.4490741 2.207253e-08 2.226804
```

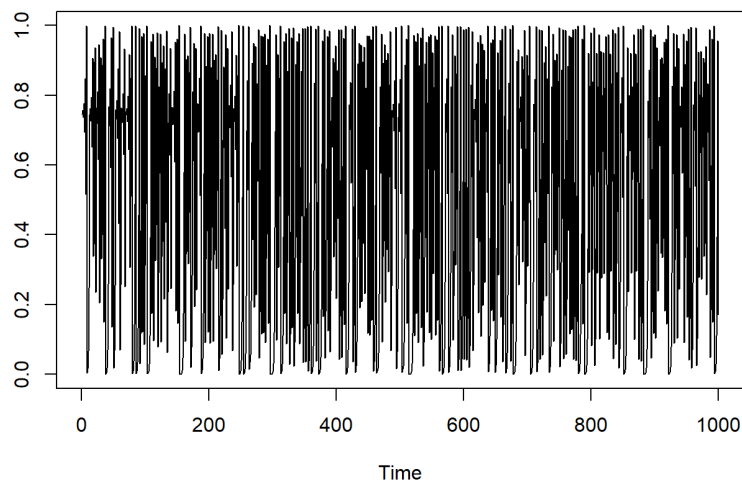
```
# DChaos -----
```

```
pacman::p_load(DChaos)
```

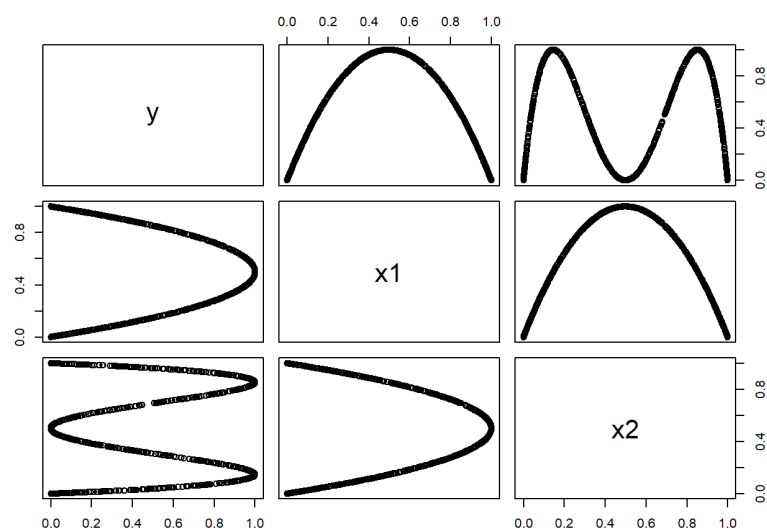
```
set.seed(34)
#Simulates time-series data from the Logistic map with chaos
ts <- DChaos::logistic.sim(n=1000, a=4)
show(head(ts, 5))
```

```
## Time Series:
## Start = 1
## End = 5
## Frequency = 1
## [1] 0.7466701 0.7566155 0.7365940 0.7760930 0.6950905
```

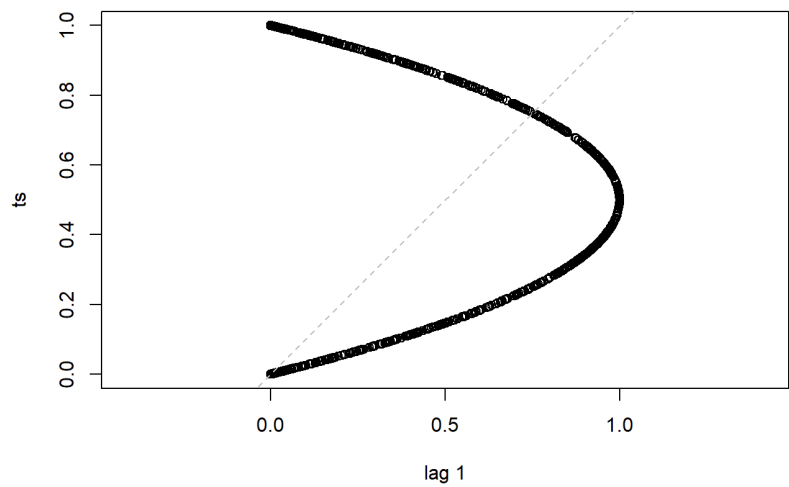
```
ts %>% plot(type="l")
```



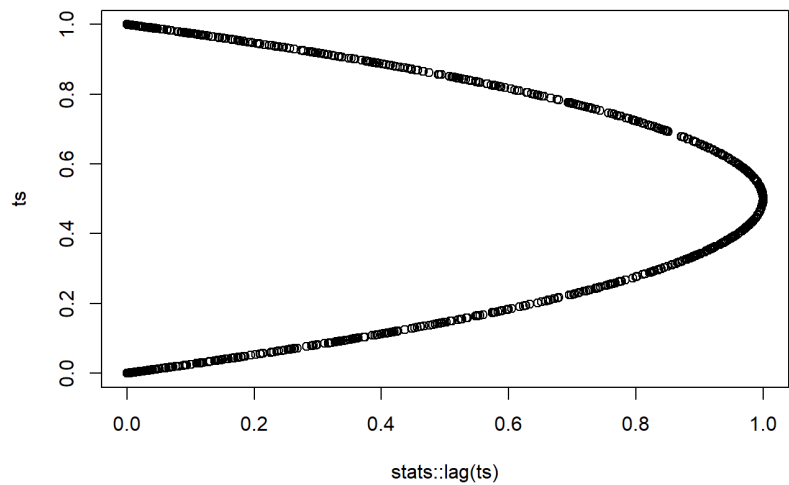
```
#Provides the uniform delayed-coordinate embedding vectors (Backward)
data <- DChaos::embedding(ts, m=3, lag=1, timelapse="FIXED")
data %>% plot
```



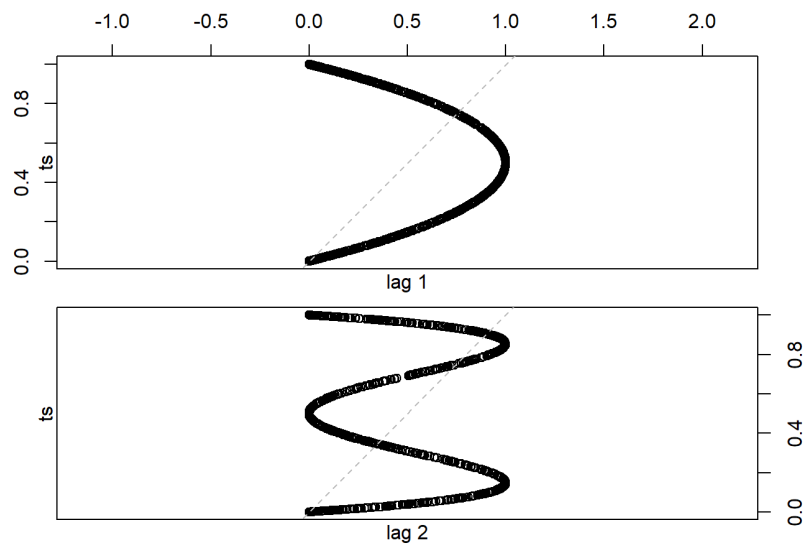
```
#same as
lag.plot(ts)
```



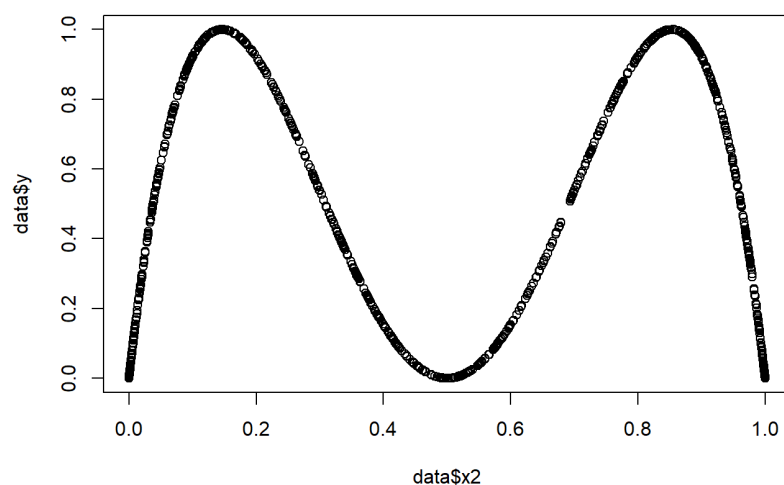
```
plot(ts~stats::lag(ts))
```



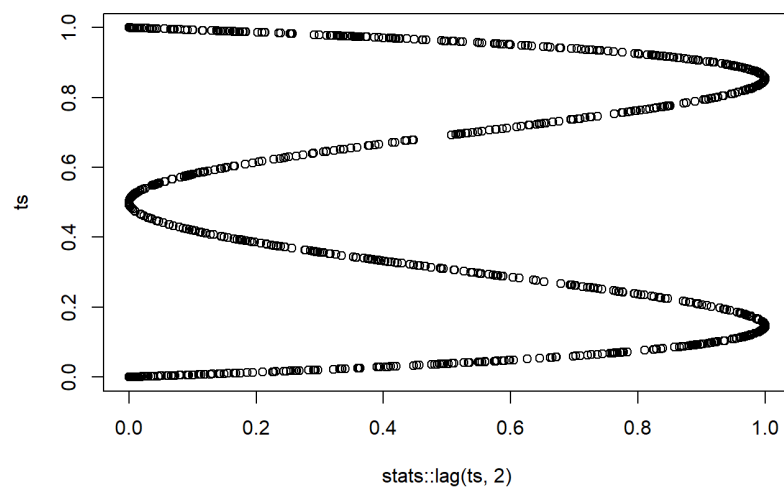
```
lag.plot(ts,2)
```



```
plot(data$x2,data$y)
```



```
#same as  
plot(ts~stats::lag(ts,2))
```



```
stats::lag(ts)==data$x2
```

```
## Warning in `==.default`(stats::lag(ts), data$x2): longer object length is not a  
## multiple of shorter object length
```

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```
## [961] TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE
## [973] TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE
## [985] TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE
## [997] TRUE TRUE FALSE FALSE
```

```
?plot3d
```

```
## No documentation for 'plot3d' in specified packages and libraries:
## you could try '??plot3d'
```

```
library(rgl)
```

```
## Warning: package 'rgl' was built under R version 4.1.2
```

```
#https://en.wikipedia.org/wiki/Logistic_map phase space for logistic a=4
plot3d(
  x=data$x1, y=data$x2, z=data$y,
  type = 's',
  col="red")
#estimates Lyapunov exponent
#https://math.libretexts.org/Bookshelves/Scientific_Computing_Simulations_and_Modeling/Book%3A_Introduction_to_the_Modeling_and_Analysis_of_Complex_Systems_(Sayama)/09%3A_Chaos/9.03%3A_Lyapunov_Exponent

#https://hypertextbook.com/chaos/Lyapunov-1/

#If Lyapunov's exponent is positive, then the behavior of the dynamical system is chaotic.
#if negative the orbit attracts to a stable point or stable periodoc orbit
#if 0 the orbit is a neutral fixed point

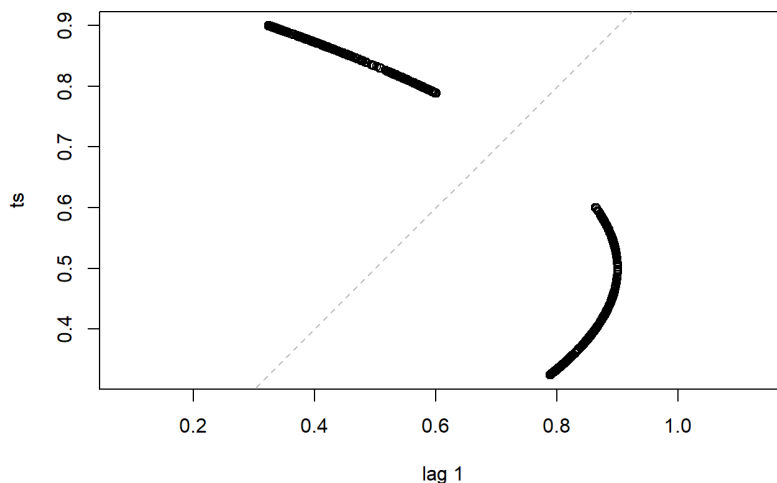
#Lyapunov(ts) too many computations

#jacobian <- DChaos::jacobian.net(data=ts, m=3:3, lag=1:1, timelapse="FIXED", h=2:10)

#Lyapunov.max(jacobian)

ts <- DChaos::logistic.sim(n=1000, a=3.6)
data <- DChaos::embedding(ts, m=3, lag=1, timelapse="FIXED")

lag.plot(ts,type="p")
```



```
?lag.plot
```

```
## starting httpd help server ...
```

```
## done
```

```

plot3d(
  x=data$x1, y=data$x2, z=data$y,
  type = 's',
  col="red")

# ForeCA -----
#https://cran.r-project.org/web/packages/ForeCA/vignettes/Introduction.html

pacman::p_load(ForeCA)

# spectrum control
sc <- list(method = "mvspec")
# entropy control
ec <- list(prior.weight = 1e-2)
data("EuStockMarkets")

# Log-returns in %
ret <- diff(log(EuStockMarkets)) * 100
cor.ret <- cor(ret)
knitr::kable(format(cor.ret, digits = 2),
  caption = "Correlation matrix")

```

Correlation matrix

	DAX	SMI	CAC	FTSE
DAX	1.00	0.70	0.73	0.64
SMI	0.70	1.00	0.62	0.58
CAC	0.73	0.62	1.00	0.65
FTSE	0.64	0.58	0.65	1.00

```

#the meaning of the inverted correlation matrix
#http://www2.tulane.edu/~PsysStat/dunlap/Phys613/RI2.html

knitr::kable(format(solve(cor.ret), digits = 2),
  caption = "Conditional covariance given other variables")

```

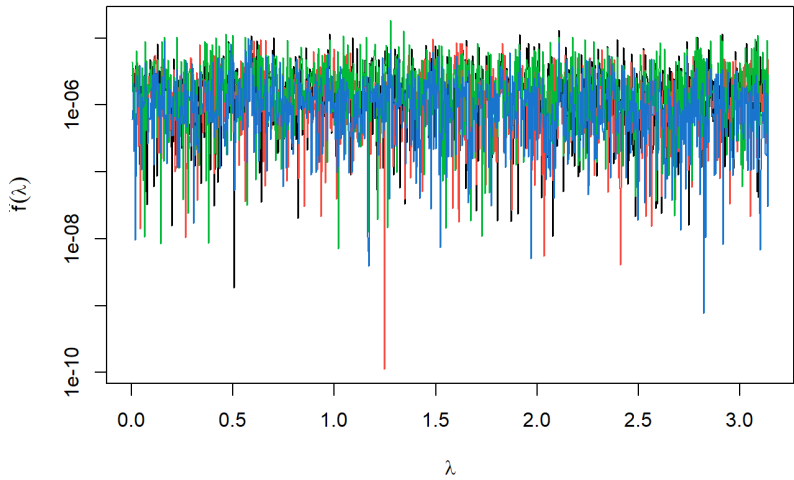
Conditional covariance given other variables

	DAX	SMI	CAC	FTSE
DAX	2.90	-1.03	-1.18	-0.49
SMI	-1.03	2.14	-0.31	-0.40
CAC	-1.18	-0.31	2.51	-0.69
FTSE	-0.49	-0.40	-0.69	1.99

```

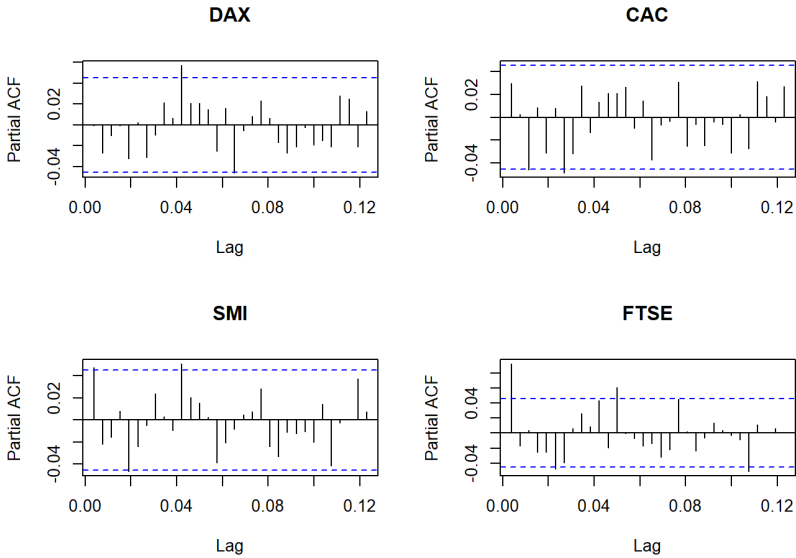
ret.spec <- mvspecpectrum(ret, method = sc$method)
plot(ret.spec)

```

```
?mvspectrum

layout(matrix(seq_len(ncol(ret)), ncol = 2))
for (nn in colnames(ret)) {
  pacf(ret[, nn], main = nn)
}
```



```
#More specifically, we can estimate ForeCA measure of forecastability,  $\Omega$ , for each series:
ret.omega <- Omega(ret, spectrum.control = sc, entropy.control = ec)
ret.omega
```

```
##      DAX      SMI      CAC      FTSE
## 5.353323 5.135365 4.966076 5.253096
## attr(,"unit")
## [1] "%"
```

```
#According to the estimates CAC is the Least forecastable, DAX is the most forecastable stock market.
```

```
#However, we can ask if there are linear combinations of stock markets, i.e.,  
#a portfolio, that are even easier to forecast. That's exactly what ForeCA is doing.
```

```
mod.foreca.ret <- foreca(ret, n.comp = ncol(ret),  
                        spectrum.control = sc,  
                        entropy.control = ec)  
#mod.foreca.ret # this uses the print.foreca method
```

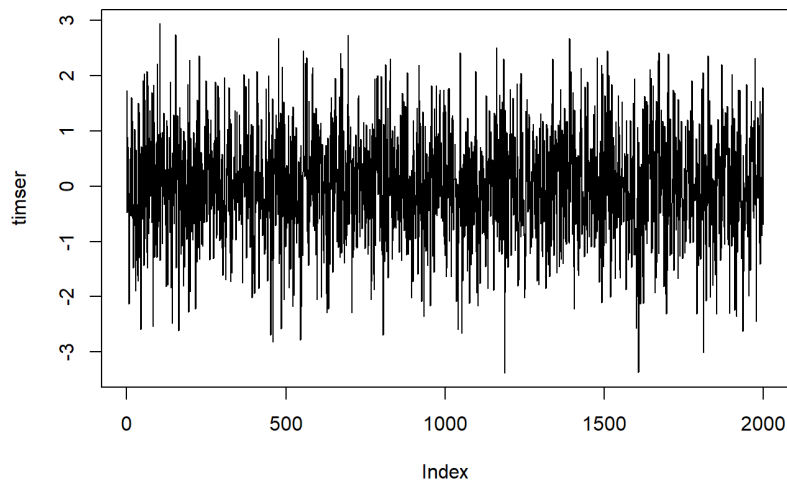
```
# Entropy -----
```

```
pacman::p_load(TSEntropies)  
#https://en.wikipedia.org/wiki/Approximate_entropy
```

```
# A time series containing many repetitive patterns has a relatively small ApEn;  
#a less predictable process has a higher ApEn., where ApEn is approximate entropy  
par(mfrow=c(1,1))  
timser <- rnorm(2000)  
ApEn(timser)
```

```
## [1] 1.92855
```

```
plot(timser,type="l")
```

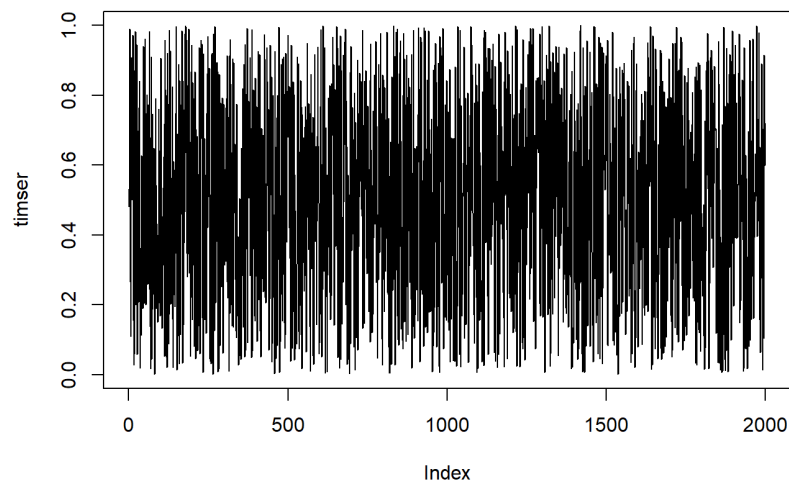


```
timser=runif(2000)
```

```
ApEn(timser)
```

```
## [1] 1.998601
```

```
plot(timser,type="l")
```



```
timser <- rnorm(2000)
ApEn(timser)
```

```
## [1] 1.895816
```

```
ApEn(ts) # the chaotic one
```

```
## [1] 0.2031768
```

```
#### Transfer entropy
pacman::p_load(RTransferEntropy)
#https://en.wikipedia.org/wiki/Transfer_entropy
```

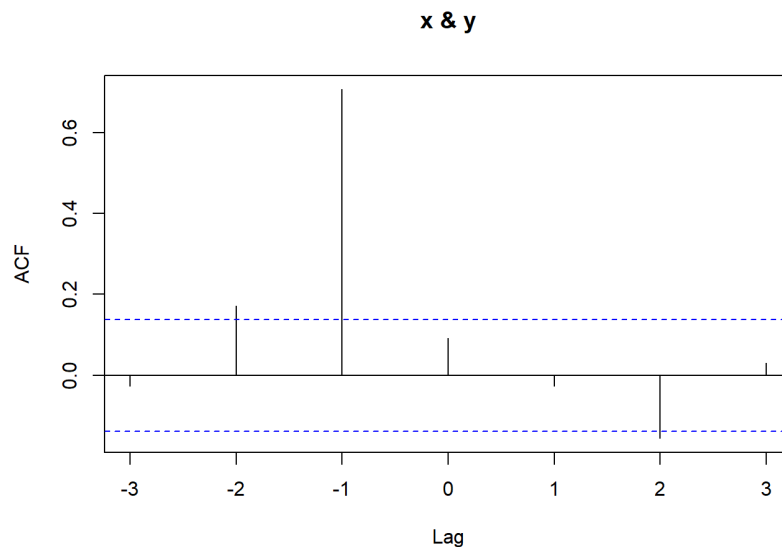
```
#simulated series
```

```
set.seed(12345)
n <- 200
x <- rep(0, n + 1)
y <- rep(0, n + 1)

for (i in 2:(n + 1)) {
  x[i] <- 0.2 * x[i - 1] + rnorm(1, 0, 2)
  y[i] <- x[i - 1] + rnorm(1, 0, 2)
}

x <- x[-1]
y <- y[-1]
```

```
ccf(x=x,y=y,lag=3)
print(ccf(y=y, x=x,lag=3))
```



```
##
## Autocorrelations of series 'X', by lag
##
##   -3   -2   -1    0    1    2    3
## -0.026 0.171 0.706 0.092 -0.026 -0.155 0.031
```

```
ApEn(x)
```

```
## [1] 0.8690917
```

```
ApEn(y)
```

```
## [1] 0.8657891
```

```
set.seed(12345)
shannon_te <- transfer_entropy(x, y)
```

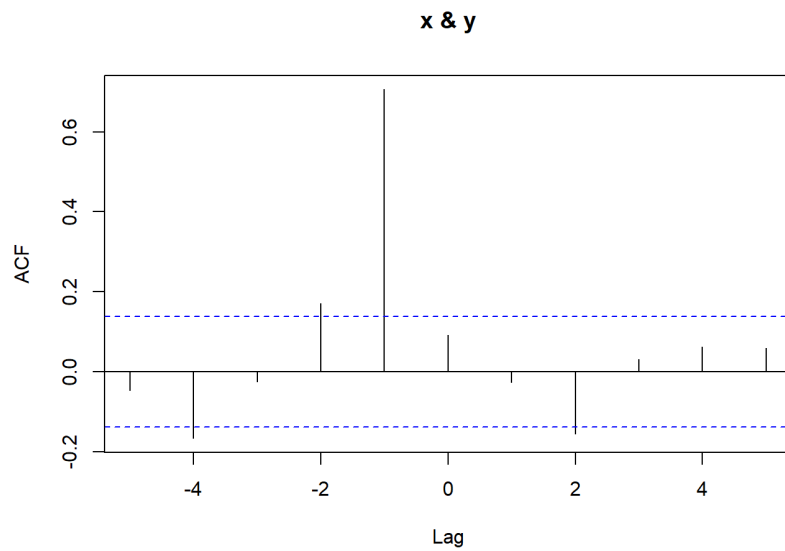
```
## Shannon's entropy on 1 core with 100 shuffles.
## x and y have length 200 (0 NAs removed)
## [calculate] X->Y transfer entropy
## [calculate] Y->X transfer entropy
## [bootstrap] 300 times
## Done - Total time 7.37 seconds
```

```
shannon_te
```

```
## Shannon Transfer Entropy Results:
## -----
## Direction      TE   Eff. TE  Std.Err.  p-value   sig
## -----
##      X->Y    0.1030   0.0865   0.0115   0.0000   ***
##      Y->X    0.0141   0.0000   0.0082   0.5233
## -----
## Bootstrapped TE Quantiles (300 replications):
## -----
## Direction      0%    25%    50%    75%   100%
## -----
##      X->Y  0.0018 0.0114 0.0144 0.0200 0.0614
##      Y->X  0.0040 0.0125 0.0162 0.0258 0.0787
## -----
## Number of Observations: 200
## -----
## p-values: < 0.001 '***', < 0.01 '**', < 0.05 '*', < 0.1 '.'
```

```
## from the above we can see that there is a significant flow of information from x to y
## but not vice versa
```

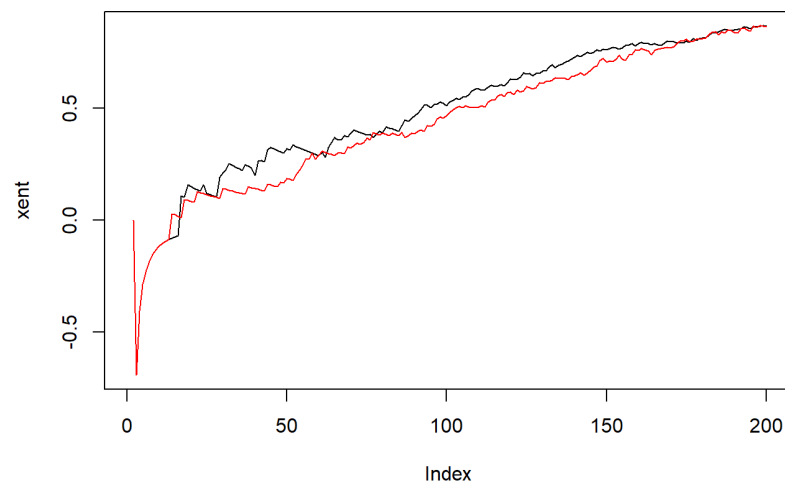
```
##the cross correlation agrees that x causes y and not vice versa (that much)
ccf(x=x,y=y,lag=3)
print(ccf(y=y, x=x,lag=5))
```



```
##
## Autocorrelations of series 'X', by lag
##
##      -5      -4      -3      -2      -1      0      1      2      3      4      5
## -0.047 -0.167 -0.026  0.171  0.706  0.092 -0.026 -0.155  0.031  0.062  0.059
```

```
yent=vector()
xent=vector()
for (i in 2:200) {
  xent[i]=ApEn(x[1:i])
  yent[i]=ApEn(y[1:i])
}
```

```
plot(xent,type="l")
lines(yent,col="red")
```



```
cor(yent,xent,use="complete.obs")
```

```
## [1] 0.9886563
```

```
r1=rnorm(100)
r2=rnorm(100)

transfer_entropy(r1, r2)
```

```
## Shannon's entropy on 1 core with 100 shuffles.  
## x and y have length 100 (0 NAs removed)  
## [calculate] X->Y transfer entropy  
## [calculate] Y->X transfer entropy  
## [bootstrap] 300 times  
## Done - Total time 5.38 seconds
```

```
## Shannon Transfer Entropy Results:  
## -----  
## Direction      TE    Eff. TE  Std.Err.  p-value  sig  
## -----  
##      X->Y    0.0146    0.0000    0.0191    0.7467  
##      Y->X    0.0400    0.0055    0.0164    0.1033  
## -----  
## Bootstrapped TE Quantiles (300 replications):  
## -----  
## Direction      0%      25%      50%      75%      100%  
## -----  
##      X->Y  0.0059  0.0146  0.0204  0.0273  0.1353  
##      Y->X  0.0032  0.0144  0.0237  0.0394  0.1063  
## -----  
## Number of Observations: 100  
## -----  
## p-values: < 0.001 '***', < 0.01 '**', < 0.05 '*', < 0.1 '.'
```