

debt_fedrate.r

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```
library(tidyverse)
```

```
## Warning: package 'tidyverse' was built under R version 4.1.2
```

```
## -- Attaching packages ----- tidyverse 1.3.1 --
```

```
## v ggplot2 3.3.6    v purrr   0.3.4
## v tibble  3.1.2    v dplyr   1.0.7
## v tidyr   1.1.3    v stringr 1.4.0
## v readr   1.4.0    v forcats 0.5.1
```

```
## Warning: package 'ggplot2' was built under R version 4.1.3
```

```
## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()     masks stats::lag()
```

```
library(ggplot2)
library(readxl)
```

```
## Warning: package 'readxl' was built under R version 4.1.2
```

```
library(ggpubr)
```

```
## Warning: package 'ggpubr' was built under R version 4.1.3
```

```
library(gridExtra)
```

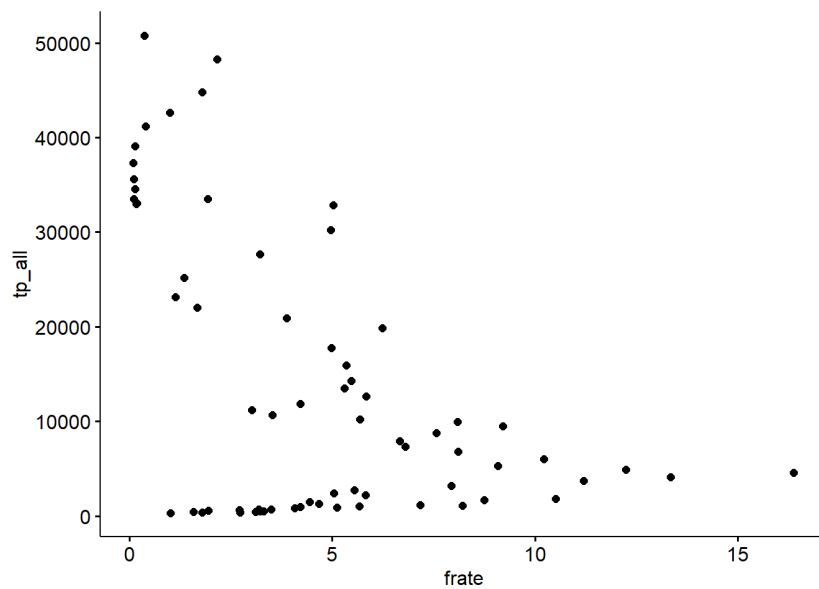
```
## Warning: package 'gridExtra' was built under R version 4.1.2
```

```
##
## Attaching package: 'gridExtra'
```

```
## The following object is masked from 'package:dplyr':
##
##      combine
```

```
df=read_excel("fed_rate_debt.xlsx",sheet="nominal")
```

```
ggscatter(data=df,x="frate",y="tp_all")
```



```
# min max normalization in order to better visualize the kind of relationship between different variables

normalize <- function(x, na.rm = TRUE) {
  return((x- min(x)) / (max(x)-min(x)))
}

#tibble with normalized variables

dfnorm=dfnorm %>% select_if(is.numeric) %>% transmute_all(normalize)

## Scatter plots between the federal rate and the various secotr's debt

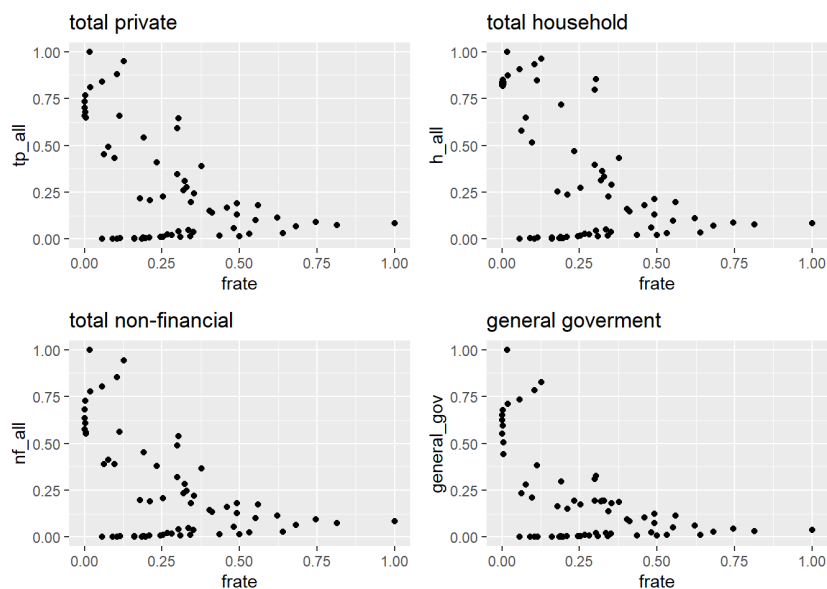
ptp=ggplot(data=dfnorm) +
  geom_point(aes(x=frate,y=tp_all))+
  ggtitle("total private")

ph=ggplot(data=dfnorm) +
  geom_point(aes(x=frate,y=h_all))+
  ggtitle("total household")

pnf=ggplot(data=dfnorm) +
  geom_point(aes(x=frate,y=nf_all))+
  ggtitle("total non-financial")

pg=ggplot(data=dfnorm) +
  geom_point(aes(x=frate,y=general_gov))+
  ggtitle("general government")

grid.arrange(ptp,ph,pnf,pg)
```



```

# from the above plots, the relationship between the federal rate and the level of debt of the various household
#sectors, seems to be negative and non-linear (convex)

# The level of debt is not a good indicator of sustainability, for this reason scatterplots between the various
#sectors shares of debt to gdp and the federal rate are produced.

dfper=read_excel("fed_rate_debt.xlsx",sheet="percent")

ptp=ggplot(data=dfper) +
  geom_point(aes(x=frate,y=tp_all))+
  ggtitle("total private")

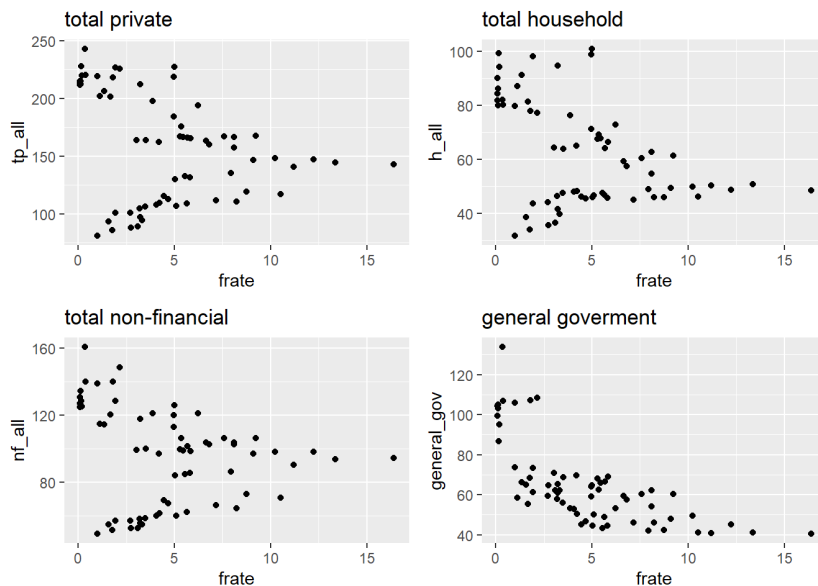
ph=ggplot(data=dfper) +
  geom_point(aes(x=frate,y=h_all))+
  ggtitle("total household")

pnf=ggplot(data=dfper) +
  geom_point(aes(x=frate,y=nf_all))+
  ggtitle("total non-financial")

pg=ggplot(data=dfper) +
  geom_point(aes(x=frate,y=general_gov))+
  ggtitle("general government")

grid.arrange(ptp,ph,pnf,pg)

```



```
# The above plots show a highly non linear relationship, still negative and non-linear. However there is an
# indication of a possible positive relationship (except for the general government's debt). Thus it is very likely
# that there is a confounding variable that distorts the association between the federal rate and the percentage of
# debt to gdp.
```

```
## We will produce a time series plot the various sector debt percentage to gdp ( normalized) and the federal rate
# in order to visualize their intertemporal relationship/
```

```
dfnorm=dfper%% select_if(is.numeric) %>% transmute_all(normalize)
t=c(1954:2020)
```

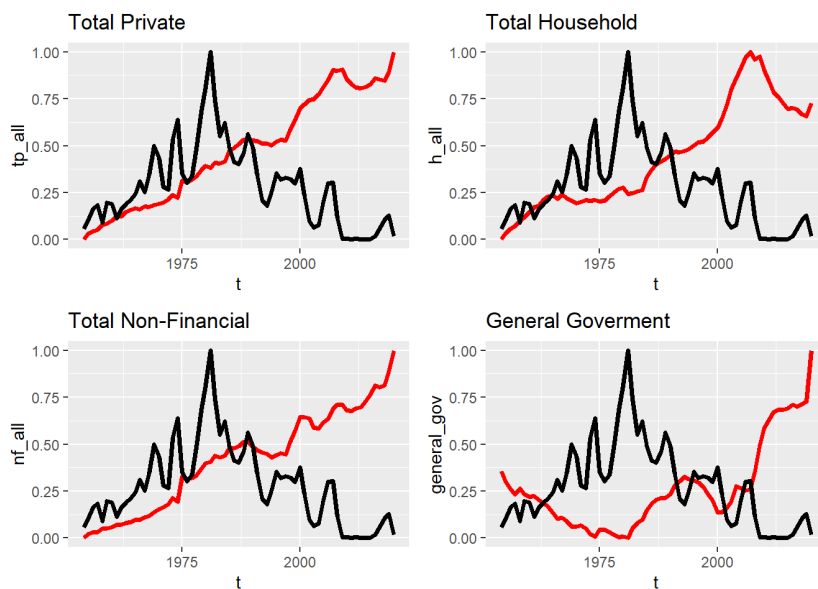
```
ptp <- dfnorm %>%
  ggplot()+
  geom_line(aes(x=t,y=tp_all),color="red",size=1.3)+
  geom_line(aes(x=t,y=frate),color="black",size=1.3)+
  ggtitle("Total Private")
```

```
ph <- dfnorm %>%
  ggplot()+
  geom_line(aes(x=t,y=h_all),color="red",size=1.3)+
  geom_line(aes(x=t,y=frate),color="black",size=1.3)+
  ggtitle("Total Household")
```

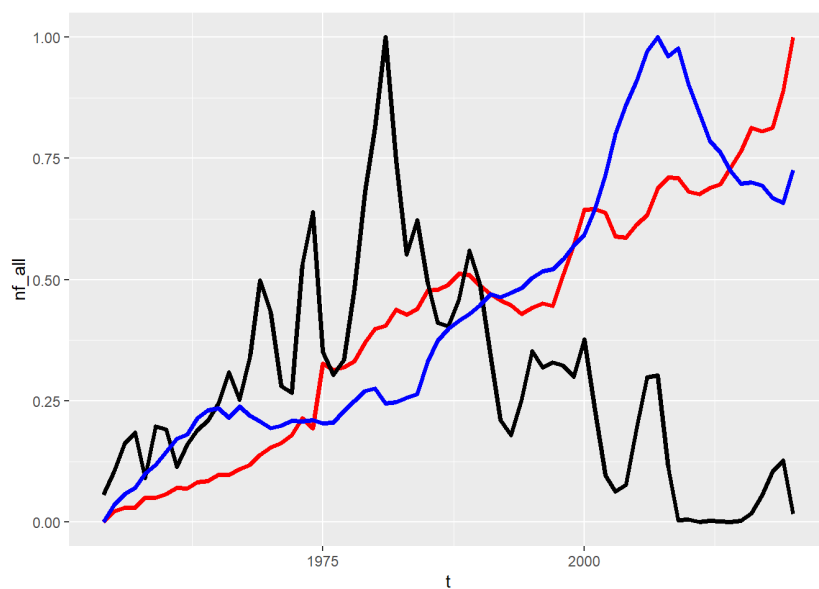
```
pnf <- dfnorm %>%
  ggplot()+
  geom_line(aes(x=t,y=nf_all),color="red",size=1.3)+
  geom_line(aes(x=t,y=frate),color="black",size=1.3)+
  ggtitle("Total Non-Financial")
```

```
pgov <- dfnorm %>%
  ggplot()+
  geom_line(aes(x=t,y=general_gov),color="red",size=1.3)+
  geom_line(aes(x=t,y=frate),color="black",size=1.3)+
  ggtitle("General Government")
```

```
grid.arrange(ptp,ph,pnf,pgov)
```



```
dfnorm %>%
  ggplot()+
  geom_line(aes(x=t,y=nf_all),color="red",size=1.3)+
  geom_line(aes(x=t,y=frate),color="black",size=1.3)+
  geom_line(aes(x=t,y=h_all),color="blue",size=1.3)
```



*#The federal rate reached its maximum value at 1981. Before that year, there was a slight increase in both household
#and non-financial corporations debt to gdp ratio, which accelerated in the period 1981-2020, a period where the
#federal rate shows a decreasing trend*

```
which(dfnorm$frate==1)
```

```
## [1] 28
```

```
cbind(t,dfnorm)
```

| ## | t | frate | tp_all | tp_ld | h_all | h_ld | nf_all |
|-------|------|--------------|-------------|-------------|------------|--------------|------------|
| ## 1 | 1954 | 0.0564448702 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| ## 2 | 1955 | 0.1043148778 | 0.03012383 | 0.02619475 | 0.03534233 | 0.03637158 | 0.02177145 |
| ## 3 | 1956 | 0.1620517372 | 0.04563044 | 0.05219200 | 0.05840542 | 0.05950403 | 0.02995464 |
| ## 4 | 1957 | 0.1851070482 | 0.05089428 | 0.06909844 | 0.07115775 | 0.07220810 | 0.02967553 |
| ## 5 | 1958 | 0.0910464772 | 0.07762389 | 0.10556406 | 0.10042978 | 0.10157498 | 0.05029037 |
| ## 6 | 1959 | 0.1976694146 | 0.08524891 | 0.11692890 | 0.11885440 | 0.11972931 | 0.04991585 |
| ## 7 | 1960 | 0.1916358900 | 0.10158717 | 0.14493066 | 0.14479196 | 0.14602392 | 0.05752116 |
| ## 8 | 1961 | 0.1141976761 | 0.12250230 | 0.17471014 | 0.17210842 | 0.17368241 | 0.07091225 |
| ## 9 | 1962 | 0.1608682204 | 0.12533148 | 0.18416009 | 0.18153011 | 0.18359047 | 0.06916798 |
| ## 10 | 1963 | 0.1896105381 | 0.14815413 | 0.21533198 | 0.21434147 | 0.21684617 | 0.08191545 |
| ## 11 | 1964 | 0.2090826106 | 0.15745219 | 0.23069188 | 0.23053867 | 0.23346179 | 0.08535186 |
| ## 12 | 1965 | 0.2447114673 | 0.16830811 | 0.24190714 | 0.23502155 | 0.23774185 | 0.09832253 |
| ## 13 | 1966 | 0.3083755833 | 0.16003119 | 0.23547457 | 0.21600714 | 0.21811867 | 0.09811721 |
| ## 14 | 1967 | 0.2532813782 | 0.17737792 | 0.26486563 | 0.23880928 | 0.24072242 | 0.10913281 |
| ## 15 | 1968 | 0.3419707290 | 0.17498940 | 0.25922815 | 0.21898641 | 0.22016448 | 0.11797471 |
| ## 16 | 1969 | 0.4985812074 | 0.18444418 | 0.26546579 | 0.20795296 | 0.20839579 | 0.13854620 |
| ## 17 | 1970 | 0.4345607122 | 0.18861416 | 0.27850279 | 0.19359544 | 0.19295056 | 0.15351234 |
| ## 18 | 1971 | 0.2809848169 | 0.19737840 | 0.28822641 | 0.19907816 | 0.19832537 | 0.16282674 |
| ## 19 | 1972 | 0.2668225885 | 0.21347721 | 0.30645163 | 0.20975148 | 0.20916922 | 0.17956220 |
| ## 20 | 1973 | 0.5309764117 | 0.23612459 | 0.31817574 | 0.20776885 | 0.20767988 | 0.21365909 |
| ## 21 | 1974 | 0.6395258140 | 0.22297778 | 0.33903457 | 0.20995368 | 0.20935601 | 0.19322388 |
| ## 22 | 1975 | 0.3517445770 | 0.31305566 | 0.31687528 | 0.20463080 | 0.20422396 | 0.32724982 |
| ## 23 | 1976 | 0.3041250753 | 0.30405755 | 0.30387224 | 0.20554318 | 0.20620494 | 0.31362527 |
| ## 24 | 1977 | 0.3346333329 | 0.31848725 | 0.32566667 | 0.22925621 | 0.23121848 | 0.31984210 |
| ## 25 | 1978 | 0.4815590581 | 0.33636639 | 0.33899469 | 0.25063419 | 0.25369897 | 0.33251460 |
| ## 26 | 1979 | 0.6819483855 | 0.37047057 | 0.35917042 | 0.27007266 | 0.27398470 | 0.36993712 |
| ## 27 | 1980 | 0.8136792912 | 0.39250732 | 0.36679132 | 0.27514274 | 0.27904177 | 0.39876879 |
| ## 28 | 1981 | 1.0000000000 | 0.38377921 | 0.34785710 | 0.24461232 | 0.24789553 | 0.40505906 |
| ## 29 | 1982 | 0.7454460792 | 0.40772426 | 0.37182082 | 0.24745608 | 0.25065667 | 0.43804238 |
| ## 30 | 1983 | 0.5523289044 | 0.40437243 | 0.38608507 | 0.25670539 | 0.26003404 | 0.42743526 |
| ## 31 | 1984 | 0.6219581554 | 0.41626051 | 0.41547060 | 0.26419499 | 0.26833595 | 0.44003692 |
| ## 32 | 1985 | 0.4915477615 | 0.47167970 | 0.47714985 | 0.33226024 | 0.33813630 | 0.47819914 |
| ## 33 | 1986 | 0.4117709356 | 0.49017463 | 0.53573292 | 0.37380055 | 0.38124770 | 0.47924638 |
| ## 34 | 1987 | 0.4032346454 | 0.50794991 | 0.56037983 | 0.39889807 | 0.39762901 | 0.48945867 |
| ## 35 | 1988 | 0.4591589782 | 0.53087648 | 0.58124487 | 0.41550245 | 0.41521275 | 0.51241994 |
| ## 36 | 1989 | 0.5599759276 | 0.53559126 | 0.58417116 | 0.42982256 | 0.42644835 | 0.51037060 |
| ## 37 | 1990 | 0.4913544426 | 0.52789082 | 0.58247851 | 0.44776366 | 0.44299148 | 0.48805600 |
| ## 38 | 1991 | 0.3433898385 | 0.52491403 | 0.56384825 | 0.46999228 | 0.46454289 | 0.46993423 |
| ## 39 | 1992 | 0.2106148776 | 0.51348582 | 0.52734857 | 0.46402897 | 0.45856378 | 0.45705227 |
| ## 40 | 1993 | 0.1799529983 | 0.51053681 | 0.52351014 | 0.47291122 | 0.46678918 | 0.44725765 |
| ## 41 | 1994 | 0.2526610767 | 0.50263287 | 0.52703705 | 0.48338247 | 0.47760212 | 0.42928579 |
| ## 42 | 1995 | 0.3525497922 | 0.52019773 | 0.54602148 | 0.50305646 | 0.49625019 | 0.44256023 |
| ## 43 | 1996 | 0.3197947685 | 0.53265178 | 0.55298205 | 0.51818842 | 0.51102113 | 0.45123803 |
| ## 44 | 1997 | 0.3296759639 | 0.52989616 | 0.56564482 | 0.52108180 | 0.51374942 | 0.44544256 |
| ## 45 | 1998 | 0.3228913111 | 0.58365731 | 0.61040147 | 0.54220371 | 0.53609763 | 0.51034599 |
| ## 46 | 1999 | 0.2996627846 | 0.63783398 | 0.65366794 | 0.57006808 | 0.56459691 | 0.57166601 |
| ## 47 | 2000 | 0.3772849568 | 0.69816662 | 0.68307768 | 0.59331150 | 0.58888058 | 0.64478872 |
| ## 48 | 2001 | 0.2325374198 | 0.72155030 | 0.72568095 | 0.64591178 | 0.64329502 | 0.64606332 |
| ## 49 | 2002 | 0.0968410011 | 0.74554490 | 0.76286474 | 0.71584958 | 0.71202737 | 0.63745954 |
| ## 50 | 2003 | 0.0636893315 | 0.74819757 | 0.79766668 | 0.79994053 | 0.79807287 | 0.58909660 |
| ## 51 | 2004 | 0.0774233234 | 0.77221704 | 0.83347260 | 0.86062221 | 0.85903102 | 0.58627606 |
| ## 52 | 2005 | 0.1919623045 | 0.81154328 | 0.87308644 | 0.90917396 | 0.90950444 | 0.61320024 |
| ## 53 | 2006 | 0.2992374830 | 0.85138522 | 0.93502306 | 0.97040862 | 0.97049469 | 0.63299793 |
| ## 54 | 2007 | 0.3024213612 | 0.90189401 | 1.00000000 | 1.00000000 | 1.00000000 | 0.68792280 |
| ## 55 | 2008 | 0.1126336119 | 0.90065517 | 0.99654167 | 0.96069370 | 0.95770745 | 0.71053048 |
| ## 56 | 2009 | 0.0043017658 | 0.90668794 | 0.98434958 | 0.97675770 | 0.96993986 | 0.70931102 |
| ## 57 | 2010 | 0.0053843516 | 0.85521855 | 0.90424548 | 0.90159240 | 0.89731859 | 0.68128920 |
| ## 58 | 2011 | 0.0007984911 | 0.82702215 | 0.85842270 | 0.84346140 | 0.83726640 | 0.67646442 |
| ## 59 | 2012 | 0.0031955490 | 0.81153800 | 0.82781034 | 0.78593216 | 0.77919206 | 0.68971404 |
| ## 60 | 2013 | 0.0011565513 | 0.80628864 | 0.81949477 | 0.76218579 | 0.75553768 | 0.69684042 |
| ## 61 | 2014 | 0.0000000000 | 0.81303056 | 0.81058182 | 0.72517426 | 0.71690757 | 0.72960493 |
| ## 62 | 2015 | 0.0027753870 | 0.82558209 | 0.81084060 | 0.69802106 | 0.68826095 | 0.76467925 |
| ## 63 | 2016 | 0.0186959217 | 0.86031568 | 0.82806207 | 0.70011986 | 0.69099687 | 0.81378147 |
| ## 64 | 2017 | 0.0560125127 | 0.85190964 | 0.84356249 | 0.69455152 | 0.68575474 | 0.80504004 |
| ## 65 | 2018 | 0.1043758195 | 0.84632867 | 0.83601311 | 0.66850328 | 0.66163535 | 0.81311442 |
| ## 66 | 2019 | 0.1272912023 | 0.89441983 | 0.83501430 | 0.65822184 | 0.65241237 | 0.88928807 |
| ## 67 | 2020 | 0.0168215348 | 1.00000000 | 0.96384965 | 0.72667985 | 0.72229208 | 1.00000000 |
| ## | | nf_ld | general_gov | Central_gov | | gdp | |
| ## 1 | | 0.0000000000 | 0.354730315 | 0.40923767 | | 0.0000000000 | |
| ## 2 | | 0.005861529 | 0.300528809 | 0.34983509 | | 0.001648610 | |
| ## 3 | | 0.028185911 | 0.260192819 | 0.30447564 | | 0.002769554 | |
| ## 4 | | 0.045682296 | 0.234569987 | 0.27036808 | | 0.003880914 | |
| ## 5 | | 0.080918020 | 0.262345073 | 0.28578237 | | 0.004188774 | |
| ## 6 | | 0.080438277 | 0.234637763 | 0.25563231 | | 0.006157535 | |
| ## 7 | | 0.102728792 | 0.218568604 | 0.23313805 | | 0.007130916 | |
| ## 8 | | 0.126823363 | 0.224560351 | 0.23169101 | | 0.008021353 | |
| ## 9 | | 0.133029095 | 0.202944968 | 0.20953612 | | 0.010016907 | |
| ## 10 | | 0.152770413 | 0.187797190 | 0.19224188 | | 0.011585539 | |
| ## 11 | | 0.162206442 | 0.166776126 | 0.17021498 | | 0.013815289 | |
| ## 12 | | 0.179093635 | 0.135466858 | 0.13959486 | | 0.016534170 | |
| ## 13 | | 0.191266779 | 0.104290435 | 0.11023244 | | 0.019910510 | |
| ## 14 | | 0.221023501 | 0.106976949 | 0.10856191 | | 0.022095129 | |
| ## 15 | | 0.235970320 | 0.089239432 | 0.09070316 | | 0.025870336 | |

```
## 16 0.263377387 0.059332133 0.05953591 0.029475995
## 17 0.309007238 0.062421616 0.05548786 0.032126568
## 18 0.321529803 0.066810583 0.05392545 0.036465312
## 19 0.344021164 0.050067650 0.03888024 0.041921370
## 20 0.369267450 0.019960041 0.01159043 0.049002170
## 21 0.408676394 0.008236997 0.00000000 0.054635096
## 22 0.371077538 0.043079077 0.03904272 0.061181634
## 23 0.342658119 0.044994974 0.05141844 0.070390944
## 24 0.354217025 0.031120605 0.05042714 0.080513866
## 25 0.352139096 0.018137351 0.04035753 0.093417877
## 26 0.366493636 0.002968651 0.02493847 0.106615854
## 27 0.375232923 0.009388466 0.03367671 0.117577462
## 28 0.377178698 0.000000000 0.03076641 0.134243872
## 29 0.421389995 0.049421354 0.07614986 0.140762454
## 30 0.437859693 0.080509193 0.10798296 0.154594335
## 31 0.485814647 0.098135435 0.12779909 0.173830766
## 32 0.519786251 0.147709910 0.16815797 0.188192855
## 33 0.581572047 0.184686079 0.20619191 0.199660841
## 34 0.609802649 0.203954774 0.21916092 0.212796899
## 35 0.628970260 0.213279145 0.22344921 0.230963381
## 36 0.620493048 0.214537844 0.22388272 0.250273683
## 37 0.596059052 0.232786366 0.24664978 0.265597295
## 38 0.531515067 0.279647618 0.28462284 0.274890833
## 39 0.466433964 0.303850896 0.30914688 0.292152982
## 40 0.448316576 0.325100266 0.32654494 0.308272503
## 41 0.441573736 0.312084791 0.31898930 0.328703987
## 42 0.455640752 0.307068203 0.31522394 0.345503842
## 43 0.450697098 0.297704279 0.30832556 0.366158133
## 44 0.472442933 0.272098680 0.27961381 0.390198607
## 45 0.533127535 0.237503462 0.24651875 0.413326408
## 46 0.583012661 0.199305950 0.21137054 0.440413494
## 47 0.610667694 0.136629860 0.15161479 0.469951453
## 48 0.626236536 0.136379994 0.14373713 0.485725445
## 49 0.612782096 0.161948161 0.15970550 0.502271515
## 50 0.572540626 0.194954915 0.18760372 0.527404576
## 51 0.566240172 0.274922968 0.19633934 0.563660093
## 52 0.580872944 0.268012275 0.19783279 0.602837118
## 53 0.626574219 0.254498274 0.18924953 0.639839694
## 54 0.718417625 0.258503319 0.19297595 0.671230402
## 55 0.765377202 0.353226780 0.28301030 0.685318451
## 56 0.725520687 0.493864545 0.40940021 0.671411507
## 57 0.658445274 0.585394764 0.50423054 0.698621316
## 58 0.643642400 0.631833729 0.55757537 0.724869600
## 59 0.656614802 0.669453358 0.60416722 0.756049417
## 60 0.670170269 0.685382508 0.62981080 0.784133772
## 61 0.701603228 0.684940346 0.63838465 0.817850248
## 62 0.738591770 0.689474951 0.64918726 0.849083681
## 63 0.769407113 0.711401297 0.67592371 0.872392587
## 64 0.806952514 0.701312131 0.67180541 0.909782393
## 65 0.822626001 0.712806624 0.69376440 0.959706559
## 66 0.832379534 0.727822240 0.71708490 1.000000000
## 67 1.000000000 1.000000000 1.00000000 0.977178409
```

```
trend=1:67
```

```
## Simple Regressions
```

```
model_total=lm(data=df,log(tp_all)~log(frate))
```

```
model_total %>% summary
```

```
##
## Call:
## lm(formula = log(tp_all) ~ log(frate), data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.391 -1.112  0.454  1.103  2.047
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   9.1479     0.2306  39.661 < 2e-16 ***
## log(frate)   -0.4927     0.1386  -3.555 0.000711 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.473 on 65 degrees of freedom
## Multiple R-squared:  0.1628, Adjusted R-squared:  0.1499
## F-statistic: 12.64 on 1 and 65 DF, p-value: 0.0007114
```

```
model_house=lm(data=df,log(h_all)~log(frate))
```

```
model_house %>% summary
```

```
##
## Call:
## lm(formula = log(h_all) ~ log(frate), data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.4122 -1.0781  0.4082  1.0544  2.1803
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   8.2294      0.2264  36.350 < 2e-16 ***
## log(frate)   -0.5084      0.1360  -3.737 0.000395 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.446 on 65 degrees of freedom
## Multiple R-squared:  0.1769, Adjusted R-squared:  0.1642
## F-statistic: 13.97 on 1 and 65 DF,  p-value: 0.0003947
```

```
model_nf=lm(data=df,log(nf_all)~log(frate))
```

```
model_nf %>% summary
```

```
##
## Call:
## lm(formula = log(nf_all) ~ log(frate), data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.3739 -1.1369  0.5097  1.1010  2.1035
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   8.6351      0.2337  36.950 < 2e-16 ***
## log(frate)   -0.4829      0.1404  -3.439 0.00102 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.493 on 65 degrees of freedom
## Multiple R-squared:  0.1539, Adjusted R-squared:  0.1409
## F-statistic: 11.83 on 1 and 65 DF,  p-value: 0.001024
```

```
model_gov=lm(data=df,log(general_gov)~log(frate))
```

```
model_gov %>% summary
```

```
##
## Call:
## lm(formula = log(general_gov) ~ log(frate), data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.6888 -1.0782  0.3392  1.0374  2.1440
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   8.3536      0.2005  41.663 < 2e-16 ***
## log(frate)   -0.5787      0.1205  -4.803 9.56e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.281 on 65 degrees of freedom
## Multiple R-squared:  0.262, Adjusted R-squared:  0.2506
## F-statistic: 23.07 on 1 and 65 DF,  p-value: 9.558e-06
```

```
model_total$coefficients[2]
```

```
## log(frate)
##  -0.492665
```

```
model_house$coefficients[2]
```



```
## log(frate)
## -0.5084442
```

```
model_nf$coefficients[2]
```

```
## log(frate)
## -0.4829396
```

```
model_gov$coefficients[2]
```

```
## log(frate)
## -0.5787272
```

```
##correlation and cross correlation
```

```
df %>% select_if(is.numeric) %>% cor
```

```
##
## frate      tp_all      tp_ld      h_all      h_ld      nf_all
## frate      1.0000000 -0.5273538 -0.5269532 -0.5397819 -0.5388292 -0.5149127
## tp_all     -0.5273538  1.0000000  0.9990802  0.9915550  0.9913194  0.9966366
## tp_ld      -0.5269532  0.9990802  1.0000000  0.9957803  0.9956245  0.9924736
## h_all      -0.5397819  0.9915550  0.9957803  1.0000000  0.9999937  0.9775923
## h_ld       -0.5388292  0.9913194  0.9956245  0.9999937  1.0000000  0.9772140
## nf_all     -0.5149127  0.9966366  0.9924736  0.9775923  0.9772140  1.0000000
## nf_ld      -0.5072416  0.9970707  0.9940263  0.9798219  0.9794783  0.9992958
## general_gov -0.5463644  0.9690883  0.9603308  0.9376937  0.9368863  0.9804955
## Central_gov -0.5365957  0.9522045  0.9408392  0.9123998  0.9114742  0.9690728
## gdp        -0.4971859  0.9949478  0.9949730  0.9877537  0.9874689  0.9908379
##           nf_ld general_gov Central_gov      gdp
## frate      -0.5072416 -0.5463644 -0.5365957 -0.4971859
## tp_all      0.9970707  0.9690883  0.9522045  0.9949478
## tp_ld       0.9940263  0.9603308  0.9408392  0.9949730
## h_all       0.9798219  0.9376937  0.9123998  0.9877537
## h_ld        0.9794783  0.9368863  0.9114742  0.9874689
## nf_all      0.9992958  0.9804955  0.9690728  0.9908379
## nf_ld       1.0000000  0.9770688  0.9647085  0.9927093
## general_gov 0.9770688  1.0000000  0.9974560  0.9580405
## Central_gov 0.9647085  0.9974560  1.0000000  0.9399979
## gdp         0.9927093  0.9580405  0.9399979  1.0000000
```

```
### - means x causes y, Lead/+ means y causes x
```

```
ccf(x=df$frate, df$tp_all, lag.max = 5,plot=F)
```

```
##
## Autocorrelations of series 'X', by lag
##
##      -5      -4      -3      -2      -1      0      1      2      3      4      5
## -0.410 -0.447 -0.472 -0.486 -0.496 -0.527 -0.530 -0.529 -0.525 -0.516 -0.511
```

```
#in differences
```

```
ccf(y=diff(df$tp_all), x=diff(df$frate), lag.max = 5,plot=F)
```

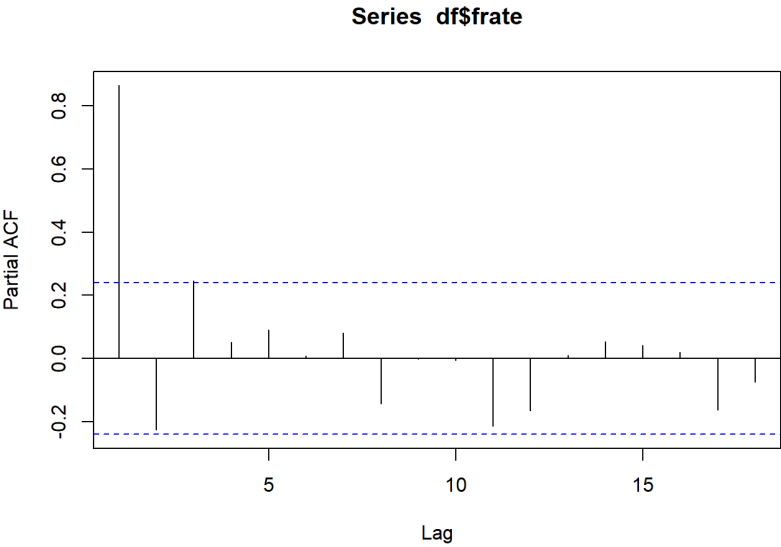
```
##
## Autocorrelations of series 'X', by lag
##
##      -5      -4      -3      -2      -1      0      1      2      3      4      5
## -0.132 -0.159 -0.145 -0.014  0.112  0.069 -0.086 -0.156 -0.173 -0.154 -0.057
```

```
####
```

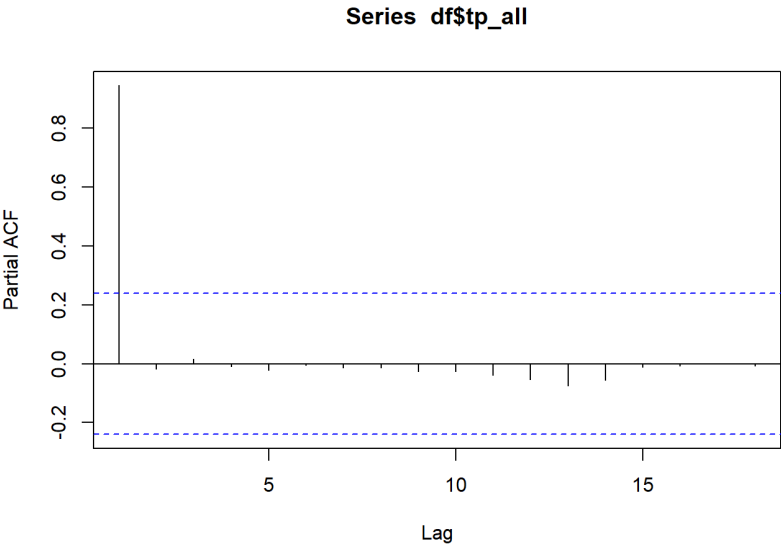
```
ccf(x=df$frate, df$tp_all, lag.max = 5,plot=F)
```

```
##
## Autocorrelations of series 'X', by lag
##
##      -5      -4      -3      -2      -1      0      1      2      3      4      5
## -0.410 -0.447 -0.472 -0.486 -0.496 -0.527 -0.530 -0.529 -0.525 -0.516 -0.511
```

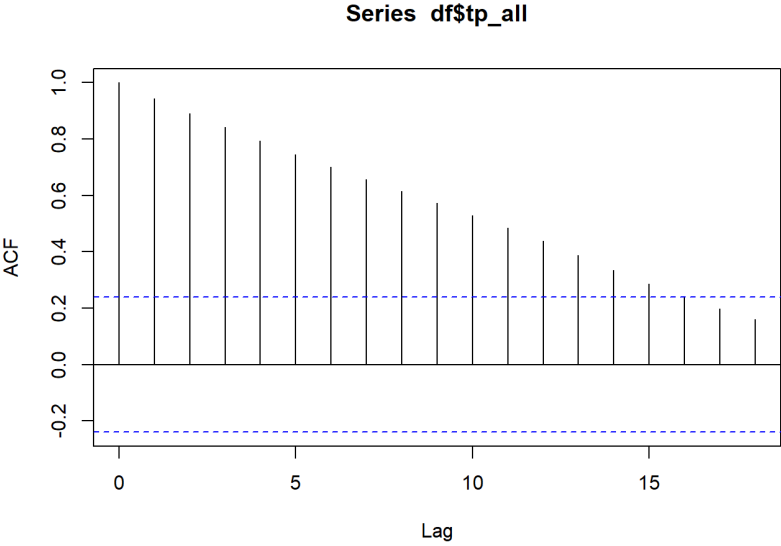
```
pacf(df$frate)
```



```
pacf(df$tp_all)
```



```
acf(df$tp_all)
```



```
# FORECASTING -----
```

```
df$class="Train"
```

```
df$class %>% length
```

```
## [1] 67
```

```
df$class[63:67]="Test"
```

```
df %>% tail(10)
```

```
## # A tibble: 10 x 12
##   year   frate tp_all tp_ld h_all h_ld nf_all nf_ld gener~1 Centr~2   gdp
##   <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 2011  0.102 33524. 23905. 14054. 13633. 19470. 10272. 15519. 12250. 15600.
## 2 2012  0.141 34522. 24382. 13994. 13568. 20528. 10814. 16742. 13457. 16254.
## 3 2013  0.107 35630. 25118. 14224. 13791. 21406. 11327. 17600. 14340. 16843.
## 4 2014  0.0885 37319. 26008. 14371. 13912. 22948. 12096. 18332. 15080. 17551.
## 5 2015  0.134 39083. 26984. 14564. 14080. 24519. 12904. 19094. 15823. 18206.
## 6 2016  0.393 41186. 28049. 14983. 14492. 26204. 13557. 19990. 16705. 18695.
## 7 2017  1.00 42649. 29545. 15536. 15032. 27113. 14513. 20645. 17333. 19480.
## 8 2018  1.79 44757. 30970. 16000. 15506. 28756. 15464. 21976. 18677. 20527.
## 9 2019  2.16 48268. 32223. 16507. 16011. 31761. 16212. 23181. 19903. 21373.
## 10 2020  0.363 50765. 34344. 17130. 16638. 33635. 17706. 27981. 24865. 20894.
## # ... with 1 more variable: Class <chr>, and abbreviated variable names
## #   1: general_gov, 2: Central_gov
```

```
dat=df
dat %>% tail
```

```
## # A tibble: 6 x 12
##   year   frate tp_all tp_ld h_all h_ld nf_all nf_ld general~1 Centr~2   gdp
##   <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 2015  0.134 39083. 26984. 14564. 14080. 24519. 12904. 19094. 15823. 18206.
## 2 2016  0.393 41186. 28049. 14983. 14492. 26204. 13557. 19990. 16705. 18695.
## 3 2017  1.00 42649. 29545. 15536. 15032. 27113. 14513. 20645. 17333. 19480.
## 4 2018  1.79 44757. 30970. 16000. 15506. 28756. 15464. 21976. 18677. 20527.
## 5 2019  2.16 48268. 32223. 16507. 16011. 31761. 16212. 23181. 19903. 21373.
## 6 2020  0.363 50765. 34344. 17130. 16638. 33635. 17706. 27981. 24865. 20894.
## # ... with 1 more variable: Class <chr>, and abbreviated variable names
## #   1: general_gov, 2: Central_gov
```

```
dat_train = subset(dat, Class == 'Train')
dat_test = subset(dat, Class == 'Test')

nrow(dat_train); nrow(dat_test)
```

```
## [1] 62
```

```
## [1] 5
```

```
### CONVERT TO TS for forecast package
library(forecast)
```

```
## Warning: package 'forecast' was built under R version 4.1.1
```

```
## Registered S3 method overwritten by 'quantmod':
##   method      from
##   as.zoo.data.frame zoo
```

```
##
## Attaching package: 'forecast'
```

```
## The following object is masked from 'package:ggpubr':
##
##   gghistogram
```

```
dat_train %>% tail
```

```
## # A tibble: 6 x 12
##   year   frate tp_all tp_ld h_all h_ld nf_all nf_ld genera~1 Centr~2   gdp
##   <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 2010  0.176 33029. 23789. 14165. 13762. 18864. 10027. 14318. 11083. 15049.
## 2 2011  0.102 33524. 23905. 14054. 13633. 19470. 10272. 15519. 12250. 15600.
## 3 2012  0.141 34522. 24382. 13994. 13568. 20528. 10814. 16742. 13457. 16254.
## 4 2013  0.107 35630. 25118. 14224. 13791. 21406. 11327. 17600. 14340. 16843.
## 5 2014  0.0885 37319. 26008. 14371. 13912. 22948. 12096. 18332. 15080. 17551.
## 6 2015  0.134 39083. 26984. 14564. 14080. 24519. 12904. 19094. 15823. 18206.
## # ... with 1 more variable: Class <chr>, and abbreviated variable names
## #   1: general_gov, 2: Central_gov
```

```
dat_ts <- ts(dat_train[, 3], start = c(1954, 1), end = c(2015, 1), frequency = 1)
```

```
##Mean absolute error percentage
```

```
#lines 2 to 4
mape <- function(actual,pred){
  mape <- mean(abs((actual - pred)/actual))*100
  return (mape)
}
```

```
#NAIVE FORECASTING
```

```
naive_mod <- naive(dat_ts, h = 5)
summary(naive_mod)
```

```
##
## Forecast method: Naive method
##
## Model Information:
## Call: naive(y = dat_ts, h = 5)
##
## Residual sd: 947.7004
##
## Error measures:
##           ME      RMSE      MAE      MPE      MAPE  MASE      ACF1
## Training set 635.5375 947.7004 652.5945 7.531884 7.583594 1 0.8045716
##
## Forecasts:
##      Point Forecast   Lo 80   Hi 80   Lo 95   Hi 95
## 2016      39082.82 37868.29 40297.35 37225.36 40940.28
## 2017      39082.82 37365.22 40800.42 36455.98 41709.66
## 2018      39082.82 36979.20 41186.44 35865.61 42300.03
## 2019      39082.82 36653.77 41511.87 35367.90 42797.74
## 2020      39082.82 36367.06 41798.58 34929.42 43236.22
```

```
#add naive forecast to test
```

```
dat_test$naive=722.773
```

```
mape(dat_test$tp_all, dat_test$naive) #98%
```

```
## [1] 98.40287
```

```
####Simple Exponential Smoothing
se_model <- ses(dat_ts, h = 5)
summary(se_model)
```

```
##
## Forecast method: Simple exponential smoothing
##
## Model Information:
## Simple exponential smoothing
##
## Call:
## ses(y = dat_ts, h = 5)
##
## Smoothing parameters:
##   alpha = 0.9999
##
## Initial states:
##   l = 314.6853
##
##   sigma: 955.6492
##
##      AIC      AICc      BIC
## 1110.786 1111.200 1117.167
##
## Error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE      ACF1
## Training set 625.3522 940.1092 642.1319 7.412839 7.46371 0.9839678 0.805956
##
## Forecasts:
##      Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## 2016      39082.64 37857.93 40307.36 37209.61 40955.68
## 2017      39082.64 37350.72 40814.56 36433.90 41731.39
## 2018      39082.64 36961.52 41203.77 35838.66 42326.62
## 2019      39082.64 36633.40 41531.89 35336.85 42828.44
## 2020      39082.64 36344.32 41820.97 34894.74 43270.55
```

```
df_fc = as.data.frame(se_model) ##save output in dataframe

dat_test$simplexp = df_fc$`Point Forecast`

mape(dat_test$tp_all, dat_test$simplexp) #13.6%
```

```
## [1] 13.63783
```

```
### holt's trend method
```

```
holt_model <- holt(dat_ts, h = 5)
summary(holt_model)
```

```
##
## Forecast method: Holt's method
##
## Model Information:
## Holt's method
##
## Call:
## holt(y = dat_ts, h = 5)
##
## Smoothing parameters:
##   alpha = 0.9999
##   beta  = 0.9999
##
## Initial states:
##   l = 278.0224
##   b = 58.8622
##
##   sigma: 418.6783
##
##      AIC      AICc      BIC
## 1010.348 1011.420 1020.984
##
## Error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE      ACF1
## Training set 27.50987 404.9474 214.069 0.4112807 2.415873 0.3280276 0.2266918
##
## Forecasts:
##      Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
## 2016      40847.12 40310.56 41383.67 40026.52 41667.71
## 2017      42611.42 41411.74 43811.10 40776.66 44446.18
## 2018      44375.72 42368.29 46383.15 41305.63 47445.82
## 2019      46140.03 43201.46 49078.59 41645.88 50634.17
## 2020      47904.33 43925.50 51883.16 41819.24 53989.42
```

```
df_holt = as.data.frame(holt_model) #save results on df_holt
dat_test$holt = df_holt$`Point Forecast` #add holt forecast to test
mape(dat_test$tp_all, dat_test$holt) #2.3%
```

```
## [1] 2.361173
```

```
###arima
```

```
arima_model <- auto.arima(dat_ts)
summary(arima_model)
```

```
## Series: dat_ts
## ARIMA(2,2,1)
##
## Coefficients:
##      ar1      ar2      ma1
##      0.9709  -0.4352  -0.8432
## s.e.   0.1276   0.1169   0.0830
##
## sigma^2 estimated as 136268: log likelihood=-438.69
## AIC=885.39  AICc=886.11  BIC=893.76
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE      ACF1
## Training set 56.8127 353.9476 205.8245 0.9188444 2.406438 0.3153942 -0.09027856
```

```
fore_arima = forecast::forecast(arima_model, h=5)
df_arima = as.data.frame(fore_arima)
dat_test$arima = df_arima$`Point Forecast`
mape(dat_test$tp_all, dat_test$arima) ## 5%
```

```
## [1] 5.094956
```

```
#Tbats
```

```
model_tbats <- tbats(dat_ts)
summary(model_tbats)
```

```
##              Length Class  Mode
## lambda           1  -none- numeric
## alpha            1  -none- numeric
## beta             1  -none- numeric
## damping.parameter 1  -none- numeric
## gamma.values      0  -none- NULL
## ar.coefficients   0  -none- NULL
## ma.coefficients   0  -none- NULL
## likelihood        1  -none- numeric
## optim.return.code 1  -none- numeric
## variance          1  -none- numeric
## AIC               1  -none- numeric
## parameters        2  -none- list
## seed.states        2  -none- numeric
## fitted.values     62    ts    numeric
## errors            62    ts    numeric
## x                 124  -none- numeric
## seasonal.periods   0  -none- NULL
## y                 62    ts    numeric
## call              2  -none- call
## series            1  -none- character
## method            1  -none- character
```

```
for_tbats <- forecast::forecast(model_tbats, h = 5)
df_tbats = as.data.frame(for_tbats)
dat_test$tbats = df_tbats$`Point Forecast`
mape(dat_test$tp_all, dat_test$tbats) #3.8%
```

```
## [1] 3.812087
```

```
#### percentage deviation of preds
pred_results=dat_test %>% select(-c(frate,tp_ld,h_ld,nf_all,h_all,nf_ld,general_gov,Central_gov,gdp,Class))

pred_results[, -1] %>% transmute_all(.funs=function(x) 1- x/pred_results$tp_all)
```

```
## # A tibble: 5 x 6
##   tp_all naive simplexp      holt      arima      tbats
##   <dbl> <dbl>      <dbl>      <dbl> <dbl>      <dbl>
## 1      0 0.982    0.0511 0.00823 0.0134 0.0141
## 2      0 0.983    0.0836 0.000880 0.0165 0.0127
## 3      0 0.984    0.127  0.00851 0.0366 0.0247
## 4      0 0.985    0.190  0.0441  0.0831 0.0626
## 5      0 0.986    0.230  0.0564  0.105  0.0765
```

```
df1960=df %>% filter(year>1960)
```

```
rep(1:3,each=20)
```

```
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2
## [39] 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
```

```
df1960=df1960 %>% mutate(decade=rep(1:3,each=20))
df1960
```

```
## # A tibble: 60 x 13
##   year  frate tp_all tp_ld h_all  h_ld nf_all nf_ld gener~1 Centr~2  gdp Class
##   <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <chr>
## 1 1961  1.95  562.  453.  243.  238.  319.  215.  343.  272.  559. Train
## 2 1962  2.71  607.  493.  265.  260.  342.  233.  356.  280.  600. Train
## 3 1963  3.18  664.  540.  294.  288.  369.  252.  367.  286.  633. Train
## 4 1964  3.50  723.  591.  323.  317.  399.  274.  381.  293.  680. Train
## 5 1965  4.08  796.  650.  353.  346.  444.  304.  391.  297.  737. Train
## 6 1966  5.11  862.  707.  376.  368.  486.  338.  405.  304.  808. Train
## 7 1967  4.22  935.  773.  411.  402.  524.  371.  430.  320.  854. Train
## 8 1968  5.66 1018.  839.  436.  427.  582.  413.  455.  334.  933. Train
## 9 1969  8.21 1116.  914.  464.  453.  652.  461.  463.  332. 1009. Train
## 10 1970  7.17 1185.  979.  479.  467.  706.  512.  492.  347. 1064. Train
## # ... with 50 more rows, 1 more variable: decade <int>, and abbreviated
## #   variable names 1: general_gov, 2: Central_gov
```

```
df1960 %>% group_by(decade) %>% summarize(sd=sd(frate)/mean(frate))
```

```
## # A tibble: 3 x 2
##   decade  sd
##   <int> <dbl>
## 1      1 0.491
## 2      2 0.438
## 3      3 1.09
```

```
df1960 %>% split(.$decade) %>%
  map_dfr(~sort(.$frate,decreasing=TRUE))
```

```
## # A tibble: 20 x 3
##   `1` `2` `3`
##   <dbl> <dbl> <dbl>
## 1 13.3  16.4  5.02
## 2 11.2  12.2  4.97
## 3 10.5  10.2  3.88
## 4  8.74  9.21  3.22
## 5  8.21  9.09  2.16
## 6  7.94  8.10  1.92
## 7  7.17  8.10  1.79
## 8  5.82  7.57  1.67
## 9  5.66  6.80  1.35
## 10  5.54  6.66  1.13
## 11  5.11  6.24  1.00
## 12  5.05  5.83  0.393
## 13  4.67  5.69  0.363
## 14  4.44  5.46  0.176
## 15  4.22  5.35  0.159
## 16  4.08  5.30  0.141
## 17  3.50  4.97  0.134
## 18  3.18  4.21  0.107
## 19  2.71  3.52  0.102
## 20  1.95  3.02  0.0885
```

```
boxplot(frate~decade,data=df1960)
```

