

$$\textcircled{7} \quad \left. \begin{array}{l} f \text{ surj.} \\ g \circ f = g' \circ f \end{array} \right\} \Rightarrow g = g'$$

06.10.2022

Lemner I

Paradoxul băiețului

2 p-iesit la table și tene
denel bolon G-unibuc.ro

ex. $A = \{3n - 2 \mid n \in \mathbb{N}\}$, $B = \{1003 - 2m \mid m \in \mathbb{N}\}$
 $x \in A \cap B \Leftrightarrow x \in A \wedge x \in B \Leftrightarrow \exists n, m \in \mathbb{N} \wedge$

$$x = 3n - 2 = 1003 - 2m$$

$$3n + 2m = 1005$$

Obs. $3 \mid 3n \quad 3 \mid 1005 \quad \Rightarrow 3 \mid 2m \Rightarrow 3 \mid m$

$$\Rightarrow m = 3m', m' \in \mathbb{Z}$$

$$3n + 6m' = 1005$$

$$n + 2m' = 335$$

$$\Leftrightarrow \begin{cases} 0 \leq m' \leq 167 \\ n = 335 - 2m' \end{cases}$$

$$A \cap B = \{1003 - 6m' \mid 0 \leq m' \leq 167\} \subset \mathbb{Z}$$

$$ax + by = c, a, b, c \in \mathbb{Z}^*$$

Ecuație de soluții în $\mathbb{Z} \times \mathbb{Z} \Leftrightarrow (a, b) \mid c$

Fie $(x_0, y_0) \in \mathbb{Z} \times \mathbb{Z}$ soluție

$$(a, b) \mid a \wedge (a, b) \mid b$$

$$\Rightarrow (a, b) \mid ax_0 + by_0 = c \Rightarrow (a, b) \mid c$$

$$d = (a, b) \mid \mathbb{Z}$$

$$\text{Not}(a, b) = d$$

$$\Rightarrow a = da', (a', b') = 1$$

$$b = db'$$

$$\Rightarrow d \mid c \Rightarrow c = dc', c' \in \mathbb{Z}$$

$$da'x + db'y = dc' \quad \text{/: } d$$

$$a'x + b'y = c'$$

$$(a', b') = 1$$

$$\Leftrightarrow \exists u, v \in \mathbb{Z} \quad a'u + b'v = 1$$

$$\Rightarrow a'u + b'v = c'$$

$$a'ua' + b'vc'd = c'd = c$$

$$a'ua' + b'vc' = c$$

$$\underline{\text{Ex.}} \quad 281x - 133y = 3$$

$$281 = 133 \cdot 2 + 15$$

$$133 = 15 \cdot 8 + 13$$

$$15 = 13 \cdot 1 + 2$$

$$13 = 2 \cdot 6 + 1$$

$$2 = 1 \cdot 2 + 0$$

$$\cancel{281 = 133 \cdot 2 + 15}$$

$$\cancel{281 = 15 \cdot 8}$$

$$15 = 281 - 133 \cdot 2$$

$$133 = 133 - (281 - 133 \cdot 2) \cdot 8$$

$$2 = 15 - 13 = 15 - (133 - (281 - 133 \cdot 2) \cdot 8)$$

$$1 = 13 - 2 \cdot 6 = 13 - (15 - (133 - (281 - 133 \cdot 2) \cdot 8)) \cdot 2$$

$$= -62 \cdot 281 + 131 \cdot 133$$

$$-186 \cdot 281 + 393 \cdot 133 = 3$$

$$\Rightarrow x = -186, y = 393$$

$$ax + by = c$$

$$a, b, c \in \mathbb{Z}^*$$

$$d = (a, b) \mid \mathbb{Z}$$

$$a = da', (a', b') = 1$$

$$b = d \cdot b'$$

$$c = d \cdot c'$$

$$ax + by = c$$

$$\Leftrightarrow a'x + b'y = c'$$

Die (x_0, y_0) sol. part.

Deswegen (x, y) allg. sol.

$$\Rightarrow e' x_0 + b' y_0 = c' = e' x + b' y$$

$$\Rightarrow e'(x_0 - x) = b'(y - y_0) \Rightarrow e' \mid b'(y - y_0)$$

$$\Rightarrow (e', b') = 1$$

$$\Rightarrow e' \mid (y - y_0) \Rightarrow \exists t \in \mathbb{Z} \text{ s.d. } y - y_0 = e' t$$

$$\Rightarrow x_0 - x = b' t$$

$$\Rightarrow (x, y) = (x_0 - b' t, y_0 + e' t), t \in \mathbb{Z}$$

$$281x - 133y = 3$$

$$x_0 = -186$$

$$y_0 = -393$$

$$\Rightarrow \begin{cases} x = -186 + 133t \\ y = -393 + 281t \end{cases}, t \in \mathbb{Z}$$

(allg. Lsg. o. ec.)

ex. $3n + 2m = 1005, n, m \in \mathbb{Z}$

$$A = \{3n - 2 \mid n \in \mathbb{Z}\}$$

$$B = \{1003 - 2m \mid m \in \mathbb{Z}\}$$

$$x \in A \cap B \Rightarrow x = 3n - 2 = 1003 - 2m$$

$$3 = 2 + 1 + 0$$

$$1 = 3 \cdot 1 - 2 \cdot 1$$

$$3 \cdot 1005 - 2 \cdot 1005 = 1005$$

$$n_0 = 1005, m_0 = -1005$$

$$\begin{cases} n = n_0 + 2t \\ m = m_0 + 3t \end{cases}, t \in \mathbb{Z}$$

$$\begin{aligned} \Rightarrow A \cap B &= \{3(1005 + 2t) - 2 \mid t \in \mathbb{Z}\} \\ &= \{-6t + 3015 - 2 \mid t \in \mathbb{Z}\} \\ &= \{6t + 1115 \mid t \in \mathbb{Z}\} \end{aligned}$$

Ex 6. $M = \left\{ x \in \mathbb{Q} \mid x = \frac{n^2+3}{n^2+n}, n \in \{1, \dots, 50\} \right\}$

$$\frac{n^2+3}{n^2+n} = \frac{m^2+3}{m^2+m}$$

$$n^2 m^2 + n^2 n + 3 n^2 + 3 n = n^2 m^2 + n m^2 + 3 n^2 + 3 n$$

$$n^2 m + 3 n^2 + 3 n = n m^2 + 3 n^2 + 3 n$$

$$n m (n - m) = 3 (n^2 - m^2) + 3 (n - m)$$

$$(n - m) (n m - 3 (n + m) - 3) = 0$$

$$n \neq m$$

$$\Rightarrow n m - 3 (n + m) - 3 = 0$$

$$(m - 3)(n - 3) = -9 - 3 = 0$$

$$(m - 3)(n - 3) = 12$$

$$n - 3 \mid 12$$

$$n - 3 \in \{ \pm 1, \pm 2, 4, 6, 12 \}$$

I $n - 3 = -1 \Rightarrow m - 3 = -12$ ✗

II $n - 3 = 1 \Rightarrow n = 4, m = 15$ (4, 15)

III $n - 3 = -2 \Rightarrow n = 1, m = -8$ (contradictoire)

IV $n - 3 = 2 \Rightarrow n = 5, m - 3 = 6, m = 9$ (5, 9)

V $n - 3 = 3 \Rightarrow n = 6, m - 3 = 4, m = 7$ (6, 7)

$$\Rightarrow |M| = 50 - 3 = 47$$