Дифференциальные уравнения

In [1]:

```
from sympy import *
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
%matplotlib inline
```

Пример 1

In [2]:

```
x = symbols('x')
y = Function('y')
```

In [3]:

```
eq = diff(y(x),x) - (exp(sqrt(x)-2)/sqrt(x))
dsolve(eq,y(x)).simplify()
```

Out[3]:

$$y(x) = C_1 + 2e^{\sqrt{x}-2}$$

Пример 2

In [4]:

```
eq = (x+1)*diff(y(x),x) + x*y(x)
dsolve(eq, y(x))
```

Out[4]:

$$y(x) = C_1 (x+1) e^{-x}$$

Пример 3

In [5]:

```
eq = x*diff(y(x),x) - y(x) - sqrt(y(x)**2-x**2)
dsolve(eq, y(x))
```

Out[5]:

$$y(x) = x \cosh(C_1 - \log(x))$$

Пример 4

In [6]:

```
z = symbols('z')
fz = (1+z**2)/z**3
I1 = integrate(fz)
I1
```

Out[6]:

$$\log(z) - \frac{1}{2z^2}$$

In [7]:

```
I2 = -integrate(1/x)
I2
```

Out[7]:

$$-\log(x)$$

Пример 5

In [8]:

```
u = symbols('u')
v = Function('v')
eq = diff(v(u), u) - v(u)/u - 1/2
dsolve(eq, v(u))
```

Out[8]:

$$v(u) = u\left(C_1 + 0.5\log(u)\right)$$

Пример 6

In [9]:

```
eq = x*diff(y(x),x) - y(x) - x**3
dsolve(eq, y(x))
```

Out[9]:

$$y(x) = x\left(C_1 + \frac{x^2}{2}\right)$$

Пример 7

In [10]:

```
eq = diff(y(x),x)+4*x*y(x)
dsolve(eq,y(x))
```

Out[10]:

$$y(x) = C_1 e^{-2x^2}$$

In [11]:

```
integrate(6*x*exp(x**2),x)
```

Out[11]:

$$3e^{x^2}$$

Пример 8

In [12]:

```
y = symbols('y')
x = Function('x')
eq = diff(x(y),y) - 2*x(y)/y - y**2
dsolve(eq, x(y))
```

Out[12]:

$$x(y) = y^2 (C_1 + y)$$

Пример 9

In [13]:

```
x = symbols('x')
y = Function('y')
eq = diff(y(x),x) - y(x)/x - (x**4)*(y(x)**2)
dsolve(eq,y(x))
```

Out[13]:

$$y(x) = \frac{6x}{C_1 - x^6}$$

Пример 10

In [14]:

```
z = Function('z')
eq = diff(z(x),x)/4 + x*z(x) - x
dsolve(eq, z(x))
```

Out[14]:

$$z(x) = C_1 e^{-2x^2} + 1$$

Пример 11

In [15]:

```
u = Function('u')
eq = diff(u(x),x) + 4*u(x)/x + u(x)**2
des = dsolve(eq, u(x))
des.simplify()
```

Out[15]:

$$u(x) = \frac{3}{C_1 x^4 - x}$$

Пример 12

In [16]:

```
eq = (2*x-3)*diff(y(x),x) + 3*x**2+2*y(x)
dsolve(eq,y(x))
```

Out[16]:

$$y(x) = \frac{C_1 - x^3}{2x - 3}$$

Пример 13

In [17]:

```
x,y = symbols('x y')
Q = x**2 - y**2
I2 = integrate(Q, (y,0,y))
I2
```

Out[17]:

$$x^2 y - \frac{y^3}{3}$$

Пример 14

In [18]:

```
x = symbols('x')
y = Function('y')
eq = (x**2-1)*diff(y(x), x) + 2*x*y(x)**2
dsolve(eq, y(x))
```

Out[18]:

$$y(x) = -\frac{1}{C_1 - \log\left(x^2 - 1\right)}$$

Пример 15

In [19]:

```
eq = x*diff(y(x),x) - y(x) + y(x)**2*(log(x)+2)*log(x)
des = dsolve(eq, y(x))
des
```

Out[19]:

$$y(x) = \frac{x}{C_1 + x\log(x)^2}$$

Пример 16

In [20]:

```
eq = (1+x**2)*diff(y(x),x) + y(x)
des = dsolve(eq, y(x))
des
```

Out[20]:

$$y(x) = C_1 e^{-\operatorname{atan}(x)}$$

Пример 17

In [21]:

```
def Lin_homogen_2(a,yl):
    x = symbols('x')
    u = Function('u')
    z = Function('z')

    y1d = diff(y1,x)

    eq = y1*diff(u(x),x)+(2*y1d+a*y1)*u(x)
    u0 = dsolve(eq,u(x))

    eq = diff(z(x),x)-u0.rhs
    z0 = dsolve(eq,z(x))

    y = y1*z0.rhs
    return y.simplify()
```

Пример 18

In [22]:

```
a = -(2*x+1)/x
y1 = exp(x)
Lin_homogen_2(a,y1)
```

Out[22]:

$$\left(\frac{C_1 x^2}{2} + C_2\right) e^x$$

Пример 19

In [23]:

```
a = -2/x
y1 = x
Lin_homogen_2(a,y1)
```

Out[23]:

$$x\left(C_1 x + C_2\right)$$

Пример 20

In [24]:

```
integrate(log(x),x)
```

Out[24]:

$$x \log(x) - x$$

Пример 21

In [25]:

```
eq = diff(y(x),x,5) - x*cos(2*x)
des = dsolve(eq, y(x))
des
```

Out[25]:

$$y(x) = C_1 + C_2x^2 + C_3x^3 + C_4x^4 + x\left(C_5 + \frac{\sin(2x)}{32}\right) + \frac{5\cos(2x)}{64}$$

Пример 22

In [26]:

```
eq = x*diff(y(x),x,2) - 3*diff(y(x),x)
des = dsolve(eq, y(x))
des
```

Out[26]:

$$y(x) = C_1 + C_2x^4$$

Пример 23

In [27]:

```
z = Function('z')
eq = 2*x*z(x)*diff(z(x),x)-z(x)**2+1
des = dsolve(eq, z(x))
des
```

Out[27]:

```
[Eq(z(x), -sqrt(C1*x + 1)), Eq(z(x), sqrt(C1*x + 1))]
```

In [28]:

```
C1 = symbols('C1')
eq = diff(y(x),x)-sqrt(C1*x+1)
des = dsolve(eq, y(x))
des
```

Out[28]:

$$y(x) = C_2 + \frac{2(C_1x + 1)^{\frac{3}{2}}}{3C_1}$$

Пример 24

In [29]:

```
u = Function('u')
eq = diff(u(x),x) + 4*u(x)/x + u(x)**2
des = dsolve(eq, u(x))
des
```

Out[29]:

$$u(x) = \frac{3}{x\left(C_1 x^3 - 1\right)}$$

Пример 25

In [30]:

```
y = symbols('y')
z = Function('z')
eq = 2*y*diff(z(y),y) + z(y)
des = dsolve(eq, z(y))
des
```

Out[30]:

$$z(y) = \frac{C_1}{\sqrt{y}}$$

Пример 26

In [31]:

```
x = symbols('x')
y = Function('y')
eq = diff(y(x),x,3)+diff(y(x),x,2)-2*diff(y(x),x)
des = dsolve(eq, y(x))
des
```

Out[31]:

$$y(x) = C_1 + C_2 e^{-2x} + C_3 e^{x}$$

Пример 27

In [32]:

```
lamda=symbols('lamda')
roots(lamda**5-2*lamda**4+9*lamda**3-18*lamda**2)
```

Out[32]:

```
{2: 1, -3*I: 1, 3*I: 1, 0: 2}
```

Пример 28

In [33]:

```
roots(lamda**6+12*lamda**5+61*lamda**4+336*lamda**3 +2016*lamda**2+6400*lamda+7424)
```

Out[33]:

```
{-4: 4, 2 - 5*I: 1, 2 + 5*I: 1}
```

Пример 29

In [34]:

```
x = symbols('x')
y = Function('y')
eq = diff(y(x),x,2)+8*diff(y(x),x)+16*y(x)-4*x**2*exp(3*x)
des = dsolve(eq, y(x))
des
```

Out[34]:

$$y(x) = \frac{4x^2e^{3x}}{49} - \frac{16xe^{3x}}{343} + (C_1 + C_2x)\,e^{-4x} + \frac{24e^{3x}}{2401}$$

Пример 30

In [35]:

```
x,C3,C4 = symbols('x C3 C4')
ych = exp(-x)*(C3*sin(2*x)+C4*cos(2*x))
ych1 = diff(ych,x)
ych1
```

Out[35]:

$$-(C_3\sin(2x) + C_4\cos(2x))\,e^{-x} + (2C_3\cos(2x) - 2C_4\sin(2x))\,e^{-x}$$

Пример 31

In [36]:

```
lamda=symbols('lamda')
roots(lamda**3-5*lamda**2+6*lamda)
```

Out[36]:

```
{3: 1, 2: 1, 0: 1}
```

In [37]:

```
x,C1d,C2d,C3d = symbols('x Cd C2d C3d')
y1 = 1
y2 = exp(2*x)
y3 = exp(3*x)

y1d = diff(y1,x)
y2d = diff(y2,x)
y3d = diff(y3,x)

y1dd = diff(y1,x,2)
y2dd = diff(y2,x,2)
y3dd = diff(y3,x,2)

eq1 = C1d*y1+C2d*y2+C3d*y3
eq2 = C1d*y1d+C2d*y2d+C3d*y3d
eq3 = C1d*y1dd+C2d*y2dd+C3d*y3dd

solve([eq1,eq2,eq3-2**x], [C1d,C2d,C3d])
```

Out[37]:

```
{Cd: 2**x/6, C2d: -2**x*exp(-2*x)/2, C3d: 2**x*exp(-3*x)/3}
```

Пример 32

In [38]:

```
x = symbols('x')
y = Function('y')
eq = diff(y(x),x,4)-3*diff(y(x),x,2)+2*diff(y(x),x)
des = dsolve(eq, y(x))
des
```

Out[38]:

$$y(x) = C_1 + C_4 e^{-2x} + (C_2 + C_3 x)\, e^{x}$$

Пример 33

In [39]:

```
eq = diff(y(x),x,2) + 2*diff(y(x),x) - exp(x)
des = dsolve(eq,y(x))
des
```

Out[39]:

$$y(x) = C_1 + C_2 e^{-2x} + \frac{e^{x}}{3}$$

Пример 34

In [40]:

```
x = symbols('x')
y = Function('y')
eq = x*diff(y(x),x,2)+diff(y(x),x)-sqrt(x)
des = dsolve(eq,y(x))
des
```

Out[40]:

$$y(x) = C_1 + C_2 \log(x) + \frac{4x^{\frac{3}{2}}}{9}$$

Пример 35

In [41]:

```
x = symbols('x')
y = Function('y')
eq = diff(y(x),x,4)-2*diff(y(x),x,3)+diff(y(x),x,2)-2*diff(y(x),x)
des = dsolve(eq,y(x))
des
```

Out[41]:

$$y(x) = C_1 + C_2 e^{2x} + C_3 \sin(x) + C_4 \cos(x)$$

Пример 36

In [42]:

```
t = symbols('t')
x = Function('x')
eq = diff(x(t),t,4) - x(t)
des = dsolve(eq,x(t))
des
```

Out[42]:

$$x(t) = C_1 e^{-t} + C_2 e^{t} + C_3 \sin(t) + C_4 \cos(t)$$

In [43]:

```
diff(des.rhs,t,2)
```

Out[43]:

$$C_1 e^{-t} + C_2 e^{t} - C_3 \sin(t) - C_4 \cos(t)$$

Пример 37

In [44]:

```
C1 = symbols('C1')
eq = diff(x(t),t) - x(t)/(2*t+C1)
des = dsolve(eq,x(t))
des
```

Out[44]:

$$x(t) = C_2\sqrt{C_1 + 2t}$$

Пример 38

In [45]:

```
x = symbols('x')
y1 = Function('y1')
eq = diff(y1(x),x,2)-2*diff(y1(x),x)+5*y1(x)
des = dsolve(eq,y1(x))
des
```

Out[45]:

$$y_1(x) = (C_1 \sin(2x) + C_2 \cos(2x))\, e^x$$

In [46]:

```
C1,C2 = symbols('C1,C2')
y3 = Function('y3')
eq = diff(y3(x), x)-y3(x)-3*exp(x)*(C1*sin(2*x)+C2*cos(2*x))
des = dsolve(eq,y3(x))
des
```

Out[46]:

$$y_3(x) = \left(-\frac{3C_1\cos(2x)}{2} + \frac{3C_2\sin(2x)}{2} + C_3\right) e^x$$

Пример 39

In [47]:

```
A = Matrix([[1,2], [4,3]])
A.eigenvects()
```

Out[47]:

```
[(-1,
  1,
  [Matrix([
   [-1],
   [ 1]])]),
 (5,
  1,
  [Matrix([
   [1/2],
   [  1]])])]
```

In [48]:

```
x,C1,C2 = symbols('t Cl C2')
y1 = C1*exp(-x)+(C2/2)*exp(5*x)
y2 = -C1*exp(-x)+C2*exp(5*x)
print(diff(y1,x)-y1-2*y2, diff(y2,x)-4*y1-3*y2)
```

```
0 0
```

Пример 40

In [49]:

```
A = Matrix([[2,1], [-2,4]])
A.eigenvects()
```

Out[49]:

```
[(3 - I,
  1,
  [Matrix([
   [1/2 + I/2],
   [        1]])]),
 (3 + I,
  1,
  [Matrix([
   [1/2 - I/2],
   [        1]])])]
```

In [50]:

```
x,C1,C2 = symbols('x C1 C2')
y1 = exp(3*x)*((C1+C2)*cos(x)+(C1-C2)*sin(x))
y2 = exp(3*x)*(2*C1*cos(x)-2*C2*sin(x))
(diff(y1,x)-2*y1-y2).simplify()
```

Out[50]:

0

Пример 41

In [51]:

```
A = Matrix([[0,1], [-1,0]])
A.eigenvects()
```

Out[51]:

```
[(-I,
  1,
  [Matrix([
   [I],
   [1]])]),
 (I,
  1,
  [Matrix([
   [-I],
   [ 1]])])]
```

In [52]:

```
C1d,C2d,x = symbols('C1d,C2d x')
eq1 = C1d*sin(x)-C2d*cos(x)-x
eq2 = C1d*cos(x)+C2d*sin(x)-3
des = solve([eq1,eq2], [C1d,C2d])
des
```

Out[52]:

```
{C1d: x*sin(x)/(sin(x)**2 + cos(x)**2) + 3*cos(x)/(sin(x)**2 + cos(x)**2),
 C2d: -x*cos(x)/(sin(x)**2 + cos(x)**2) + 3*sin(x)/(sin(x)**2 + cos(x)**
2)}
```

Пример 42

In [53]:

```
t = symbols('t')
x = Function('x')
y = Function('y')
eq1 = diff(x(t),t) - x(t) + y(t)
eq2 = diff(y(t),t) - x(t) - 3*y(t)

dsolve((eq1,eq2))
```

Out[53]:

```
[Eq(x(t), -C2*t*exp(2*t) - (C1 - C2)*exp(2*t)),
 Eq(y(t), C1*exp(2*t) + C2*t*exp(2*t))]
```

Пример 43

In [54]:

```
t = symbols('t')
x = Function('x')
y = Function('y')
eq1 = diff(x(t),t)-x(t)-y(t)
eq2 = diff(y(t),t)-y(t)
dsolve((eq1,eq2))
```

Out[54]:

```
[Eq(x(t), C1*exp(t) + C2*t*exp(t)), Eq(y(t), C2*exp(t))]
```

Пример 44

In [55]:

```
t = symbols('t')
x = Function('x')
y = Function('y')
z = Function('z')
eq1 = diff(x(t),t)-2*x(t)-y(t)
eq2 = diff(y(t),t)-x(t)-2*y(t)
eq3 = diff(z(t),t)-x(t)-y(t)-2*z(t)
des = dsolve((eq1,eq2,eq3))
des
```

Out[55]:

```
[Eq(x(t), -C1*exp(t) + C2*exp(3*t)/2),
 Eq(y(t), C1*exp(t) + C2*exp(3*t)/2),
 Eq(z(t), C2*exp(3*t) + C3*exp(2*t))]
```

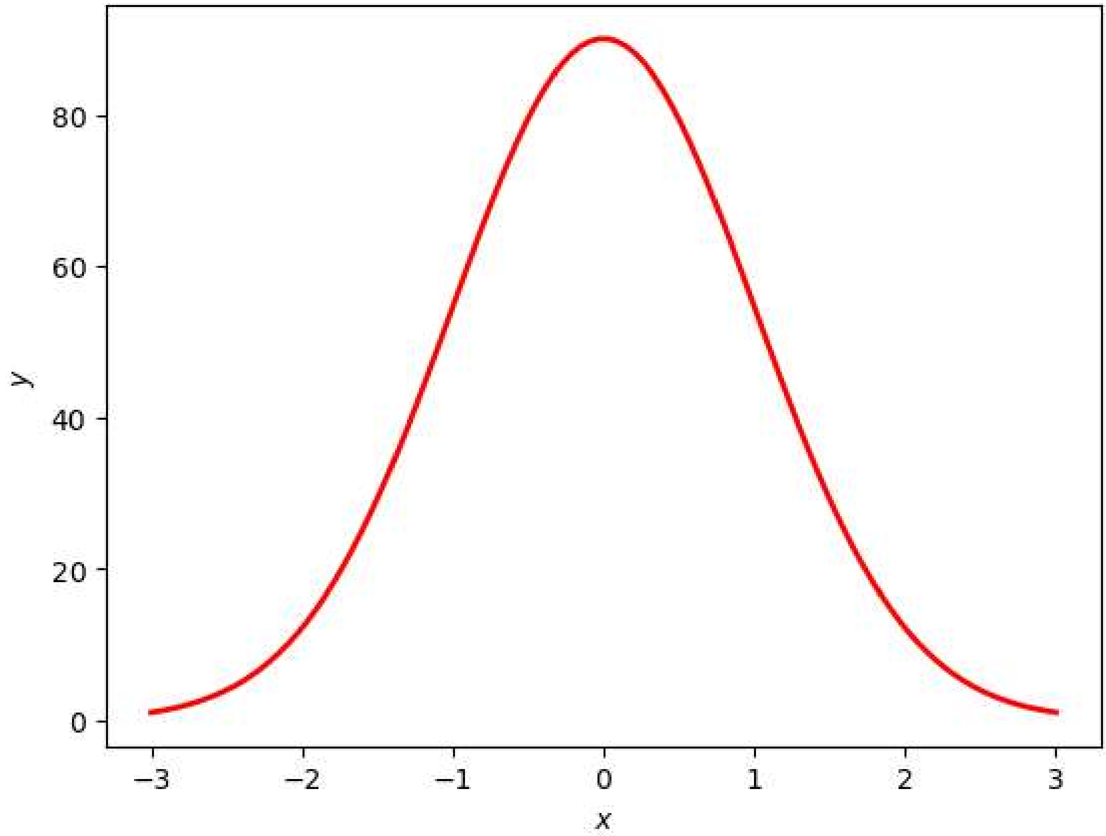
Пример 45

In [56]:

```
def f(y,x):
    return -y*x

x = np.linspace(-3, 3, 100)
y0 = 1
y = odeint(f, y0, x)

plt.plot(x,y,c='r',linewidth=2)
plt.xlabel('$x$')
plt.ylabel('$y$')
plt.show()
```



Пример 46

In [57]:

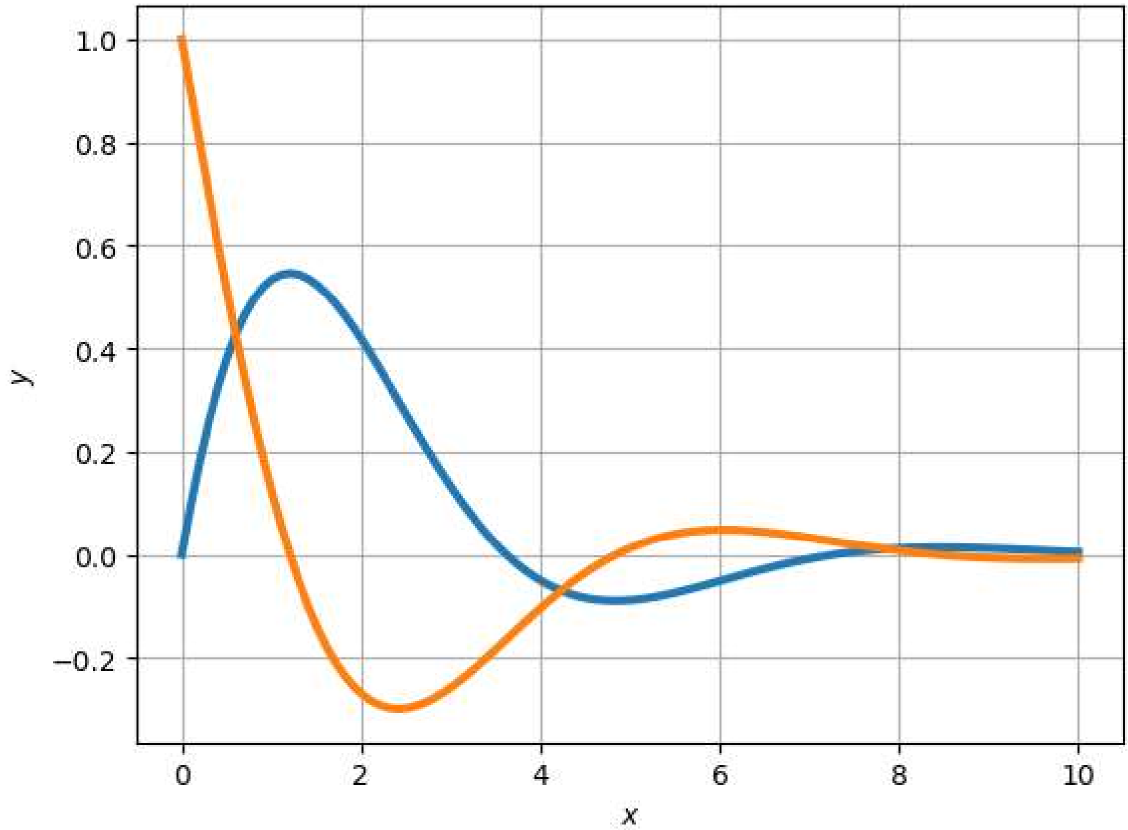
```
def f(y, x):
    y1, y2 = y
    return [y2, -y1-y2]

x = np.linspace(0,10,100)
y0 = [0, 1]
w = odeint(f, y0, x)

y1 = w[:,0]
y2 = w[:,1]
```

In [58]:

```
fig = plt.figure(facecolor='white')
plt.plot(x,y1,x,y2,linewidth=3)
plt.ylabel("$y$")
plt.xlabel("$x$")
plt.grid(True)
plt.show()
```



Пример 47

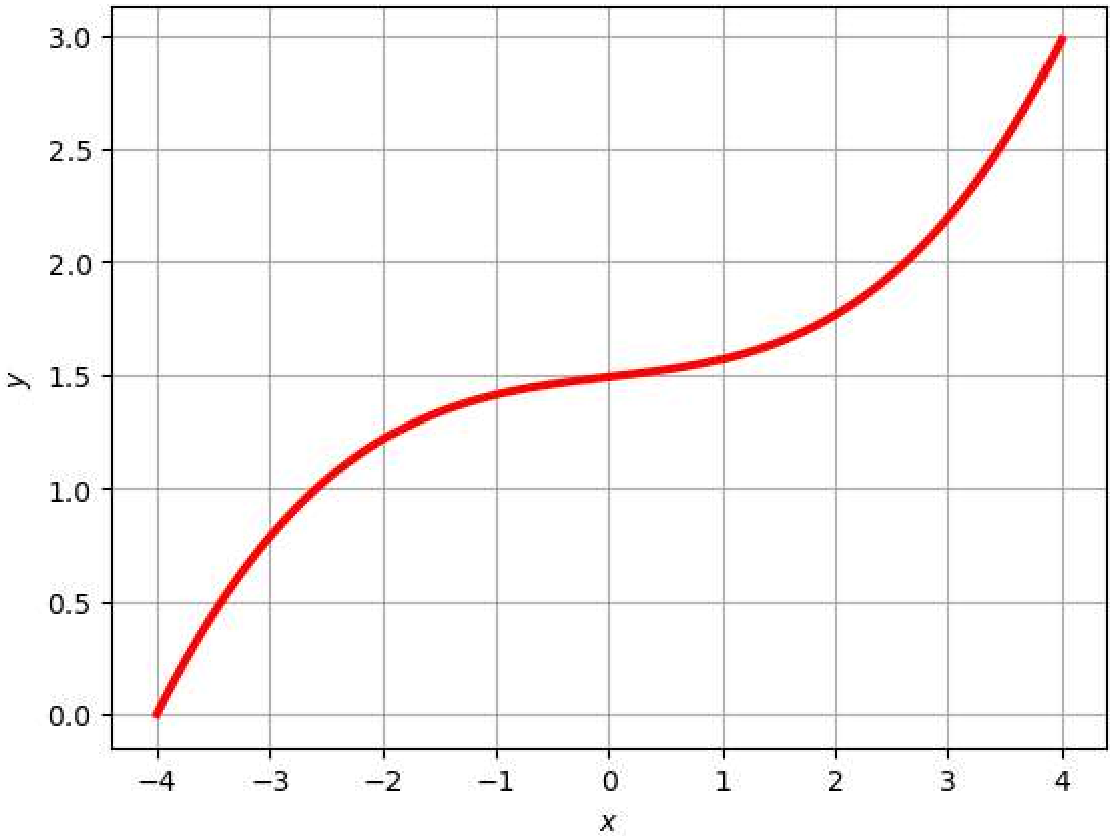
In [59]:

```
def f(y, x):
    y1, y2 = y
    return [y2, 2*x*y2/(x**2+1)]

x = np.linspace(-4, 4, 100)
y0 = [-75, 51]
w = odeint(f, y0, x)

y1 = w[:,0]

fig = plt.figure(facecolor='white')
plt.plot(x,y1,c='r',linewidth=3)
plt.ylabel("$y$")
plt.xlabel("$x$")
plt.grid(True)
plt.show()
```



Пример 48

In [60]:

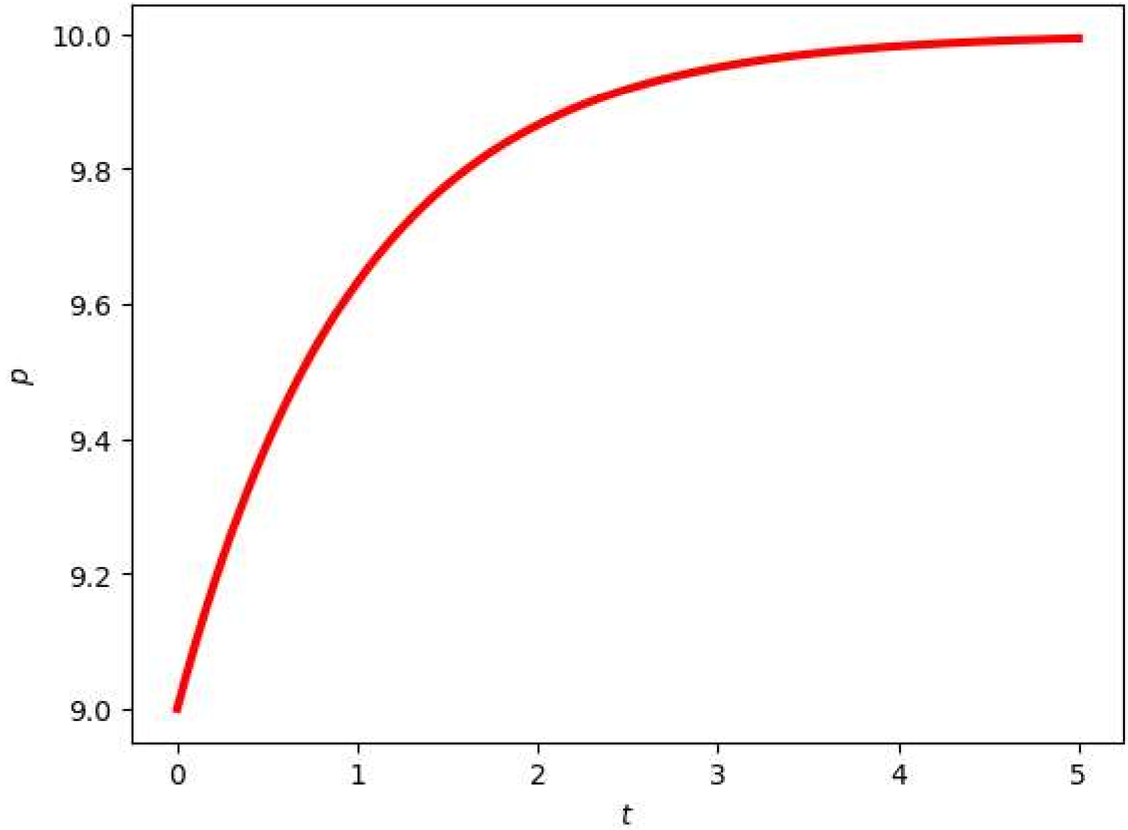
```
t = symbols('t')
p = Function('p')
eq = diff(p(t),t)+p(t)-10
des = dsolve(eq,p(t))
des
```

Out[60]:

$$p(t) = C_1 e^{-t} + 10$$

In [61]:

```
t = np.linspace(0,5,100)
p = 10-np.exp(-t)
plt.plot(t,p,c='r',linewidth=3)
plt.ylabel('$p$')
plt.xlabel("$t$")
plt.show()
```



Пример 49

In [62]:

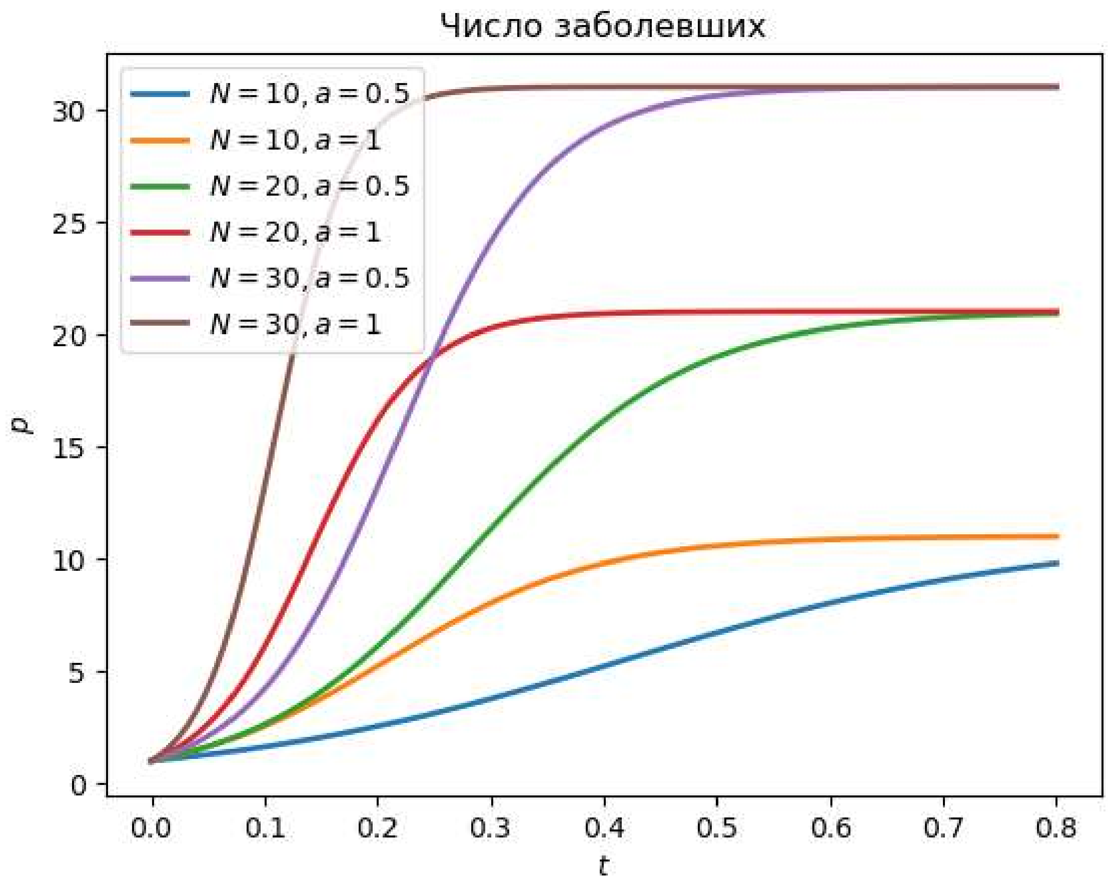
```
u = symbols('u')
N = symbols('N')
integrate(1/((N+1)*u-1),u)
```

Out[62]:

$$\frac{\log(u(N+1)-1)}{N+1}$$

In [63]:

```
t = np.linspace(0,0.8,100)
for param in [[10, 0.5],[10,1],[20,0.5],[20,1],[30,0.5],[30,1]]:
    N = param[0]
    a = param[1]
    X = (N+1)/(N*np.exp(-(N+1)*a*t)+1)
    plt.plot(t, X, lw=2, label="$N=%s, a=%s$" % (N, a))
plt.legend()
plt.ylabel('$p$')
plt.xlabel("$t$")
plt.title("Число заболевших");
```



In [64]:

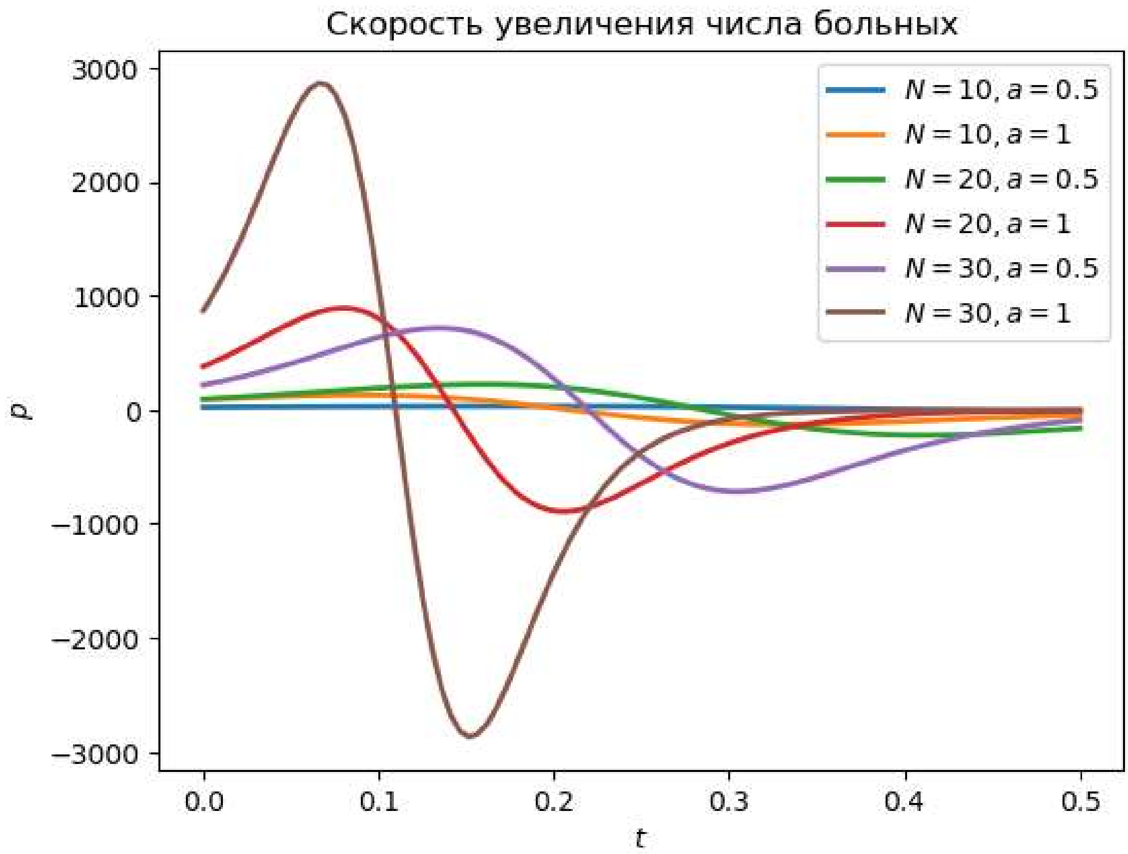
```
t,N,a = symbols('t N a')
X = (N+1)/(N*exp(-(N+1)*a*t)+1)
Xprim = diff(X,t,2)
Xprim.simplify()
```

Out[64]:

$$\frac{Na^2(N+1)^3\left(N-e^{at(N+1)}\right)e^{at(N+1)}}{\left(N+e^{at(N+1)}\right)^3}$$

In [65]:

```
t = np.linspace(0,0.5,100)
for param in [[10, 0.5],[10,1],[20,0.5],[20,1],[30,0.5],[30,1]]:
    N = param[0]
    a = param[1]
    Xprim = a**2*N*(N+1)**3*(N-np.exp((N+1)*a*t))*np.exp((N+1)*a*t) / (N+np.exp((N+1)*a*t
    plt.plot(t, Xprim, lw=2, label = "$N=%s, a=%s$" % (N, a))
plt.legend()
plt.ylabel('$p$')
plt.xlabel("$t$")
plt.title("Скорость увеличения числа больных");
```



Примеры решения задач

Решить задачу Коши $y' = \frac{xy^2 - yx^2}{x^3}$, $y(-1) = 1$

In [66]:

```
x = symbols('x')
y = Function('y')
eq = diff(y(x),x) - (x*y(x)**2-y(x)*x**2)/x**3
des = dsolve(eq, y(x))
des
```

Out[66]:

$$y(x) = \frac{2x}{C_1 x^2 + 1}$$

Решить уравнение $(x + y - 4)y' = 2x + y + 3$

In [67]:

```
u = symbols('u')
z = Function('z')
eq = u*(1+z(u))*diff(z(u),u)-2+z(u)**2
des = dsolve(eq, z(u))
des.simplify()
```

Out[67]:

$$C_1 = \log(u) + \frac{\left(\sqrt{2} + 2\right)\log\left(z(u) - \sqrt{2}\right)}{4} + \frac{\left(2 - \sqrt{2}\right)\log\left(z(u) + \sqrt{2}\right)}{4}$$

Решить уравнение $xy' + y = y^2$.

In [68]:

```
eq = x*diff(y(x),x) + y(x) - y(x)**2
dsolve(eq, y(x))
```

Out[68]:

$$y(x) = -\frac{1}{C_1 x - 1}$$

Найти общее решение уравнения $y'' - 2\left(1 + \operatorname{tg}^2 x\right) y = 0$, если известно одно его частное решение $y_1 = \operatorname{tg} x$.

In [69]:

```
a = 0
y1 = tan(x)
Lin_homogen_2(a,y1)
```

Out[69]:

$$\left(C_1 \int \frac{e^{-x}}{\tan^2(x)}\, dx + C_2\right) \tan(x)$$

Решить уравнение $\left(1 + x^2\right) y'' + 2xy' = x^3$.

In [70]:

```
x = symbols('x')
z = Function('z')
eq = (1+x**2)*diff(z(x),x)+2*x*z(x)-x**3
dsolve(eq,z(x))
```

Out[70]:

$$z(x) = \frac{C_1 + \frac{x^4}{4}}{x^2 + 1}$$

In [71]:

```
z2 = x**4/(4*(x**2+1))
integrate(z2,x)
```

Out[71]:

$$\frac{x^3}{12} - \frac{x}{4} + \frac{\operatorname{atan}(x)}{4}$$

Решить уравнение $y'' - 3y' + 2y = e^{2x} \sin x$

In [72]:

```
eq = diff (y(x),x,2)-3*diff(y(x),x)+2*y(x)-exp(2*x)*sin(x)
dsolve(eq,y(x))
```

Out[72]:

$$y(x) = \left(C_1 + \left(C_2 - \frac{\sin(x)}{2} - \frac{\cos(x)}{2}\right) e^x\right) e^x$$

Решить систему уравнений $\begin{cases} \frac{dy_1}{dx} = 2y_1 - y_2 \\ \frac{dy_2}{dx} = 2y_2 - y_1 - 5e^x \sin x \end{cases}$

In [73]:

```
x = symbols('x')
y1 = Function('y1')
y2 = Function('y2')
eq1 = diff(y1(x),x)-2*y1(x)+y2(x)
eq2 = diff(y2(x),x)-2*y2(x)+y1(x)
des = dsolve((eq1,eq2))
des
```

Out[73]:

```
[Eq(y1(x), C1*exp(x) - C2*exp(3*x)), Eq(y2(x), C1*exp(x) + C2*exp(3*x))]
```

Функции спроса D и предложения S, выражающие зависимость от цены p и ее производных, имеют вид: $D(t) = 3p'' - p' - 200p + 600, \quad S(t) = 4p'' + p' + 201p - 603$. Найти зависимость равновесной цены от времени.

In [74]:

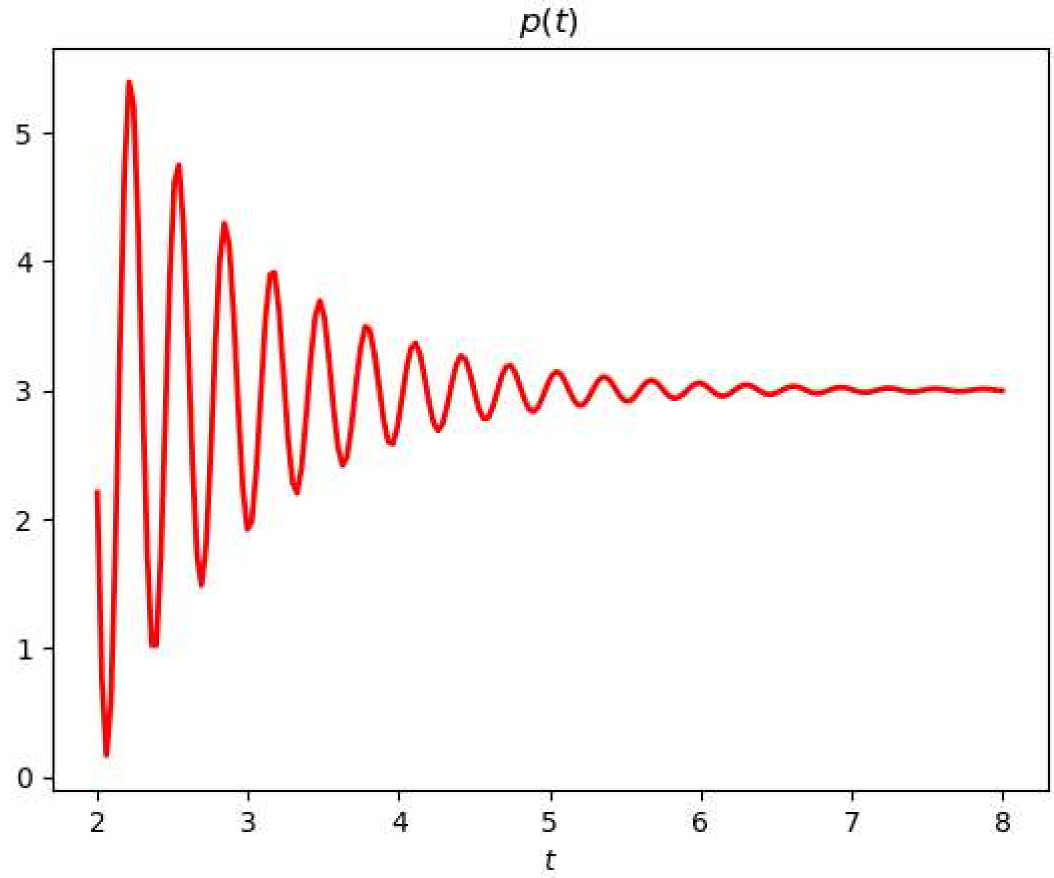
```
t = symbols('t')
p = Function('p')
eq = diff(p(t),t,2)+2*diff(p(t),t)+401*p(t)-1203
des = dsolve(eq,p(t))
des
```

Out[74]:

$$p(t) = (C_1 \sin(20t) + C_2 \cos(20t))\, e^{-t} + 3$$

In [75]:

```
t = np.linspace(2,8,200)
y = 3 + np.exp(-t)*(10*np.sin(20*t)+20*np.cos(20*t))
plt.plot(t, y, c = 'r', lw=2)
plt.xlabel("$t$")
plt.title("$p(t)$")
plt.show()
```



Решить задачу Коши $\left(x^2y - y\right)^2 y' = x^2y - y + x^2 - 1, \quad y(\infty) = 0$

In [76]:

```
x = symbols('x')
y = Function('y')
eq = (x**2*y(x)-y(x))**2*diff(y(x),x)-x**2*y(x)+y(x)-x**2+1
des = dsolve(eq, y(x))
des
```

Out[76]:

$$\frac{y^2(x)}{2} - y(x) - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} + \log(y(x)+1) = C_1$$

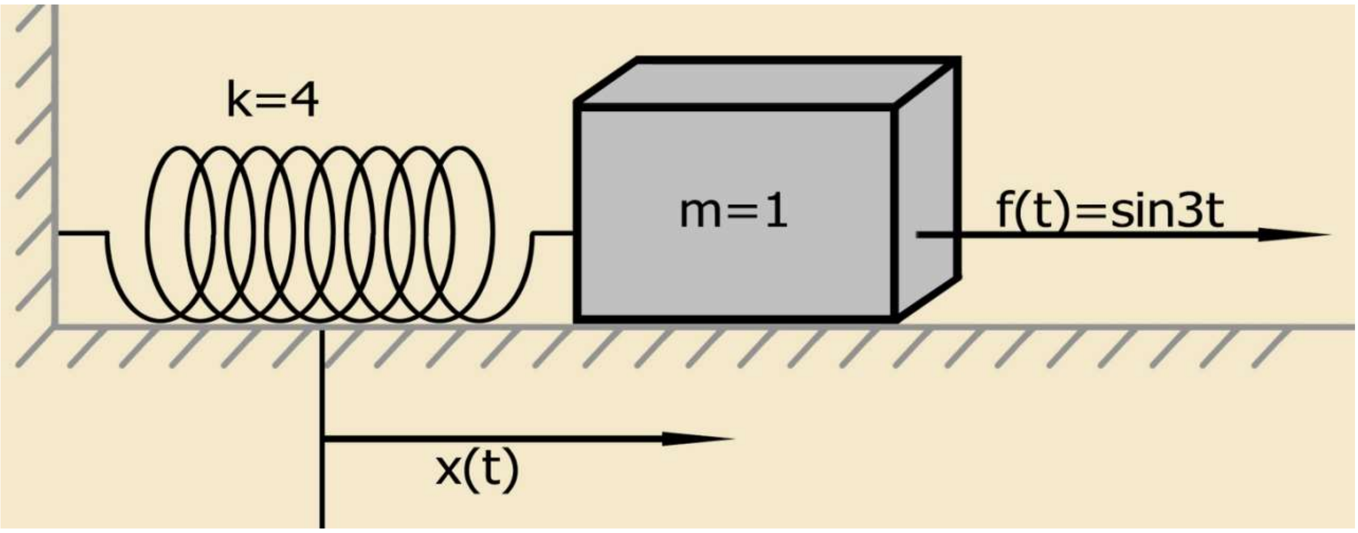
Индивидуальное задание

Необходимо решить дифференциальное уравнение, характеризующее объект находящийся под действием силы и прикрепленный к определенной пружине. Уравнение имеет следующий вид:

$$x'' + 4x = \sin(3t); x(0) = 1.2; x'(0) = 1, (4)$$

где $x(0)$ - приведенное начальное положение массы, $x'(0)$ - приведенная начальная скорость массы.

Упрощённая физическая модель, заданная уравнением при ненулевых начальных условиях



Система, состоящая из материальной точки заданной массы, закрепленной на пружине, удовлетворяет задаче Коши (задаче с начальными условиями). Материальная точка заданной массы первоначально находится в покое в положении ее равновесия.

In [89]:

```
var('s')
var('t', positive=True)
var('X', cls=Function)
```

Out[89]:

X

In [90]:

```
x0 = Rational(6, 5)
x0
```

Out[90]:

$$\frac{6}{5}$$

In [91]:

```
x01 = Rational(1, 1)
x01
```

Out[91]:

$$1$$

In [92]:

```
g = sin(3*t)
g
```

Out[92]:

$$\sin(3t)$$

In [93]:

```
Lg = laplace_transform(g, t, s, noconds=True)
Lg
```

Out[93]:

$$\frac{3}{s^2 + 9}$$

In [94]:

```
d2 = s**2*X(s) - s*x0 - x01
d2
```

Out[94]:

$$s^2 X(s) - \frac{6s}{5} - 1$$

In [95]:

```
d0 = X(s)
d0
```

Out[95]:

$$X(s)$$

In [96]:

```
d = d2 + 4*d0
d
```

Out[96]:

$$s^2 X(s) - \frac{6s}{5} + 4X(s) - 1$$

In [97]:

```
de = Eq(d, Lg)
de
```

Out[97]:

$$s^2 X(s) - \frac{6s}{5} + 4X(s) - 1 = \frac{3}{s^2 + 9}$$

In [98]:

```
rez = solve(de, X(s))[0]
rez
```

Out[98]:

$$\frac{6s^3 + 5s^2 + 54s + 60}{5\left(s^4 + 13s^2 + 36\right)}$$

In [99]:

```
soln = inverse_laplace_transform(rez, s, t)
soln
```

Out[99]:

$$\frac{4\sin(2t)}{5} - \frac{\sin(3t)}{5} + \frac{6\cos(2t)}{5}$$

In [100]:

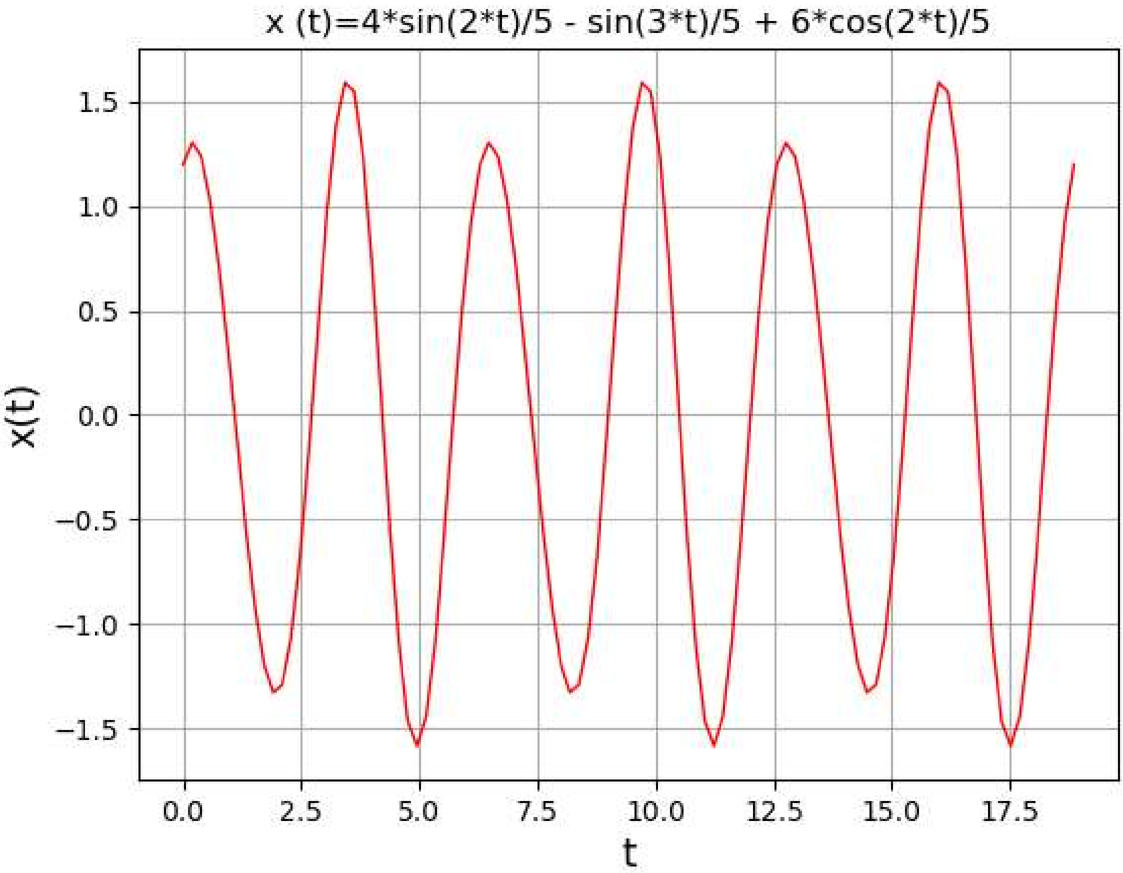
```
f = lambdify(t, soln, 'numpy')
f
```

Out[100]:

```
<function _lambdifygenerated(t)>
```

In [101]:

```
x = np.linspace(0, 6*np.pi, 100)
plt.title('x (t)=%s' % soln)
plt.grid(True)
plt.xlabel('t', fontsize=14)
plt.ylabel('x(t)', fontsize=14)
plt.plot(x, f(x), 'r', linewidth=1)
plt.show()
```



In [ ]: