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Masters Thesis

Investigations on Backbone Computation

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Abstract

Template

Acknowledgments

If you have someone to Acknowledge ;)

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1 Introduction

introduction

wofür brauche ich backbones

wer hat sich damit beschäftigt

welche paper waren relevant

2 Disambiguation

definiere formel,klausel,literal,variable,prim implikante ,satisfiable/unsatisfiable

This thesis is an investigation on the calculation of backbones for boolean formulas in conjunctive normal form (or CNF in short). A CNF formula F is a conjunction of a set of clauses $C(F)$, meaning that all of these clauses have to be satisfied (or fulfilled) to satisfy the formula. A clause c in turn is a disjunction of a set of literals, meaning at least one of said literals must be fulfilled. A literal l can be defined as the occurence of a boolean variable v which may or not be negated and to fulfill such a literal, it's variable must be assigned $\perp$ for negated literals and $\top$ for those literals without negation. The same variable can occur in multiple clauses of the same formula and must have the same assignment in all occurences. A complete assignment of all variables of F (written as $Var(F)$) that leads to the formula being fulfilled is called a model. A formula for which no model can be found is called unsatisfiable.

The exact terminology can differ depending on the paper and project that you read. A formula can be called a problem and the assignment of a variable can be called it's phase. Clauses can also be called constraints and sometimes sentences. A synonym for a formula, clause or literal being fulfilled is it being satisfied. Models can also be called solutions of formulas

The backbone is a problem specific set of literals that contains all literals that occur in every model of that problem. We can also say that a variable is not part of the backbone, if neither it's positive or it's negative assignment is in the backbone. If we have an unsatisfiable formula, it's backbone can be considered undefinable, which is why this thesis concerns itself only with satisfiable CNF formulas. (TODO ref unsat backbone, aber keine einigkeit drüber)

TODO implikante

3 Base Algorithms

The algorithms that I investigated for this thesis can be grouped very broadly into two approaches, which I will describe in the following two sections.

3.1 Intersection algorithms

A simple definition of the backbone is that it is the intersection of all models of it's formula. If a literal is not part of the backbone, there must exist a model that contains the negation of that literal. Therefore if we had a way to iterate over every single model of the formula and, starting with the set of both literals for every variable and removing every literal from that set that was missing in one of these models, that set would end up being the backbone of the formula. [MSJL10] as well as [JLMS15] list an algorithm that does exactly this.

Here, found models are prevented from being found again by adding a blocking clause of said model and the algorithm terminates once all models are prohibited and the formula became unsatisfiable through this. This blocking clause is a disjunction *verodertnegiertallesinschnittbekanntermodels* which enforces that at least one variable must be different.

Clearly, calculating every single model of a formula leaves room for optimization. Most models of a common boolean formula differ by small, independent differences that can just as well occur in other models. Therefore the intersection of only a handful of models can suffice to result in the backbone, as long as these models are chosen to be as different as possible. This was achieved in [JLMS15] as is described in algorithm X.

It generates an upper bound ν_r of the backbone by intersecting found models and inhibits this upper bound instead of individual models. This blocking clause is much more powerful, because it enforces not only that a new model is found, but also that this new model will reduce the upper bound estimation of the backbone. Eventually

v_r will be reduced to the backbone. This can be easily recognized, because the blocking clause of the backbone or any of it's subsets makes the formula unsatisfiable.

This algorithm is implemented in the Sat4J library under the designation *IBB*.

3.2 Probing algorithms

Alternatively, you can define the backbone as all literals that occur with the same assignment in all models of it's problem, which implies that enforcing that variable to it's negation should make the formula unsatisfiable. This definition already leads to a simple algorithm that can calculate the backbone, by checking both assignments of every literal for whether it would make the formula unsatisfiable, see Algorithm 1. This algorithm is referenced in [MSJL10]

Algorithm 1: Iterative algorithm (two tests per variable)

Input: A formula F in CNF
Output: All literals of the backbone of F v_r

$v_r \leftarrow \emptyset$
for $x \in Var(F)$ **do**
    $(outc_1, v) \leftarrow SAT(F \cup \{x\})$
    $(outc_2, v) \leftarrow SAT(F \cup \{\neg x\})$
    if $outc_1 = \bot \wedge outc_2 = \bot$ **then**
        return $\emptyset$
    else if $outc_1 = \bot$ **then**
        $v_r = v_r \cup \{\neg x\}$
        $F = F \cup \{\neg x\}$
    else if $outc_2 = \bot$ **then**
        $v_r = v_r \cup \{x\}$
        $F = F \cup \{x\}$
return v_r

As is commonly written in literature about boolean satisfiability, the two calls to the *SAT* function return a pair which consists first of whether the given function was satisfiable at all and, secondly, the found model, which in this case is discarded. There is no good algorithm that can tell whether a boolean formula is satisfiable or not without trying to find a model for said formula.

Literaturverzeichnis

[JLMS15] Mikoláš Janota, Inês Lynce, and Joao Marques-Silva. Algorithms for computing backbones of propositional formulae. *AI Commun.*, 28(2):161–177, April 2015.

[MSJL10] João Marques-Silva, Mikoláš Janota, and Inês Lynce. On computing backbones of propositional theories. *Frontiers in Artificial Intelligence and Applications*, 215:15–20, 01 2010.