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**Mathematisch-
Naturwissenschaftliche
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Computergrafik

Masters Thesis

Investigations on Backbone Computation

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Jonas Bollgrün (Matrikelnummer 3353424), 4. August 2019

Abstract

Template

Acknowledgments

If you have someone to Acknowledge ;)

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1. Introduction

introduction

wofür brauche ich backbones

wer hat sich damit beschäftigt

welche paper waren relevant

2. Disambiguation

definiere formel,klausel,literal,variable,prim implikante ,satisfiable/unsatisfiable

This thesis is an investigation on the calculation of backbones for boolean formulas in conjunctive normal form (or CNF in short). A CNF formula F is a conjunction of a set of clauses $C(F)$, meaning that all of these clauses have to be satisfied (or fulfilled) to satisfy the formula. A clause c in turn is a disjunction of a set of literals, meaning at least one of said literals must be fulfilled. A literal l can be defined as the occurence of a boolean variable v which may or not be negated and to fulfill such a literal, it's variable must be assigned $\perp$ for negated literals and $\top$ for those literals without negation. The same variable can occur in multiple clauses of the same formula and must have the same assignment in all occurences. A complete assignment of all variables of F (written as $Var(F)$) that leads to the formula being fulfilled is called a model. A formula for which no model can be found is called unsatisfiable.

The exact terminology can differ depending on the paper and project that you read. A formula can be called a problem and the assignment of a variable can be called it's phase. Clauses can also be called constraints and sometimes sentences. A synonym for a formula, clause or literal being fulfilled is it being satisfied. Models can also be called solutions of formulas

The backbone is a problem specific set of literals that contains all literals that occur in every model of that problem. We can also say that a variable is not part of the backbone, if neither it's positive or it's negative assignment is in the backbone. If we have an unsatisfiable formula, it's backbone can be considered undefinable, which is why this thesis concerns itself only with satisfiable CNF formulas. (TODO ref unsat backbone, aber keine einigkeit drüber)

3. Base Algorithms

The algorithms that I investigated for this thesis can be grouped very broadly into two approaches, which I will describe in the following two sections.

3.1. Probing algorithms

You can define the backbone as all literals that occur with the same assignment in all models of it's problem, which implies that enforcing that variable to either True or False should make the formula unsatisfiable. This definition already leads to a simple algorithm that can calculate the backbone, by checking both assignments of every literal for whether it would make the formula unsatisfiable, see Algorithm 1. This algorithm is referenced in TODO ref BB paper

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Algorithm 1: Iterative algorithm (two tests per variable)

Input: A formula F in CNF
Output: All literals of the backbone of F v_r
$v_r \leftarrow \emptyset$
for $x \in Var(F)$ **do**
    $(outc_1, v) \leftarrow SAT(F \cup \{x\})$
    $(outc_2, v) \leftarrow SAT(F \cup \{\neg x\})$
    if $outc_1 = \bot \wedge outc_2 = \bot$ **then**
        return $\emptyset$
    else if $outc_1 = \bot$ **then**
        $v_r = v_r \cup \{\neg x\}$
        $F = F \cup \{\neg x\}$
    else if $outc_2 = \bot$ **then**
        $v_r = v_r \cup \{x\}$
        $F = F \cup \{x\}$
return v_r

As is commonly written in literature about boolean satisfiability, the two calls to the SAT function return a pair which consists first of whether the given function was satisfiable at all and, secondly, the found model, which in this case is discarded.

There is no good algorithm that can tell whether a boolean formula is satisfiable or not without trying to find a model for said formula. We will see after the next section how we can make use of these models.

3.2. Intersection algorithms

An alternative definition of the backbone is the intersection of all models. If a literal is not part of the backbone, there must exist a model that contains the negation of that literal.

3.3. Combining both approaches

BB

What is this all about?

Cite like this: [**?**]

3.4. Problem Statement

A. Blub