APPLIED CALCULUS

Tangent and Normal

(1) $The\ equation\ of\ a\ tangent\ to\ a\ curve\ y = f(x)\ at\ a\ point\ P(x_1, y_1)\ is\ given\ by$

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} (x - x_1)$$

(2) $The\ equation\ of\ a\ normal\ to\ a\ curve\ y = f(x)\ at\ a\ point\ P(x_1, y_1)\ is\ given\ by$

$$y - y_1 = -\left(\frac{dx}{dy}\right)_{(x_1,y_1)} (x - x_1)$$

(3) $The\ angle\ of\ intersection\ of\ two\ curves\ y = f(x)\ and\ y = g(x)\ is\ given\ by$

$$\tan\alpha = \frac{f'(x) - g'(x)}{1 + f'(x)g'(x)}$$

Monotonic Functions

Increasing Function:- $A\ function\ f(x)\ is\ said\ to\ be\ an\ increasing\ function\ in\ (a, b)\ if$

$x_1 < x_2 \implies f(x_1) < f(x_2)\ \forall\ x_1, x_2 \in (a, b).$

Decreasing Function:- $A\ function\ f(x)\ is\ said\ to\ be\ an\ decreasing\ function\ in\ (a, b)\ if$

$x_1 < x_2 \implies f(x_1) > f(x_2)\ \forall\ x_1, x_2 \in (a, b).$

Monotonic Function:-
$A\ function\ is\ said\ to\ be\ monotonic\ in\ an\ interval\ if\ it\ is\ either\ increasing\ or\ decreasing\ in\ that\ interval.$

Theorem 1:- $If\ f(x)\ be\ a\ function\ continuous\ on\ [a, b]\ and\ differentiable\ on\ (a, b)\ then$

$f'(x) > 0\ \forall\ x \in (a, b) \implies f(x)\ is\ increasing\ in\ (a, b)$.

Theorem 2:- $If\ f(x)\ be\ a\ function\ continuous\ on\ [a, b]\ and\ differentiable\ on\ (a, b)\ then$

$f'(x) < 0\ \forall\ x \in (a, b) \implies f(x)\ is\ decreasing\ in\ (a, b)$.

Maxima and Minima

Working Rule (2nd Derivative Test)

(i) Find f'(x).

(ii) Solve f'(x)=0. Let its roots be a,b,c etc. Then, these are the candidates for maxima & minima. Let x=c be one of its points.

(iii) Find f''(c).

Now, if f''(c)<0, then x=c is a point of local maxima

if f''(c)>0, then x=c is a point of local minima

if f''(c)=0, then use the 1st derivative test.

Working Rule (1st Derivative Test)

(i) Find f'(x).

(ii) Solve f'(x)=0. Let its roots be a,b,c etc. Then, these are the candidates for maxima & minima. Let x=c be one of its points.

(iii) Determine the sign of f'(x) for values of x slightly < c and that for values of x slightly > c.