

$$(7) \cos 3A = 4 \cos^3 A - 3 \cos A$$

$$(8) \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

Sub-multiple Angles

$$(1) \sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5}-1}{4}$$

$$(2) \sin 36^\circ = \cos 54^\circ = \frac{1}{4} \sqrt{10 - 2\sqrt{5}}$$

$$(3) \sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5}+1}{4}$$

$$(4) \sin 72^\circ = \cos 18^\circ = \frac{1}{4} \sqrt{10 + 2\sqrt{5}}$$

Trigonometric Equations

$$(1) (i) \text{ If } \sin \theta = 0, \text{ then } \theta = n\pi$$

$$(ii) \text{ If } \sin \theta = \sin \alpha, \text{ then } \theta = n\pi + (-1)^n \alpha$$

$$(2) (i) \cos \theta = 0, \text{ then } \theta = (2n + 1) \frac{\pi}{2}$$

$$(ii) \cos \theta = \cos \alpha, \text{ then } \theta = 2n\pi \pm \alpha$$

$$(3) (i) \tan \theta = 0, \text{ then } \theta = n\pi$$

$$(ii) \tan \theta = \tan \alpha, \text{ then } \theta = n\pi + \alpha$$

Inverse Circular Function

$$\text{Principle Value} \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$(1) \sin(\sin^{-1} x) = x; \cos(\cos^{-1} x) = x; \tan(\tan^{-1} x) = x$$

$$(2) \sin^{-1}(-x) = -\sin^{-1} x; \cos^{-1}(-x) = \pi - \cos^{-1} x; \tan^{-1}(-x) = -\tan^{-1} x;$$

$$\cot^{-1}(-x) = \pi - \cot^{-1} x; \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x; \sec^{-1}(-x) = \pi - \sec^{-1} x.$$

$$(3) \sin^{-1} x = \operatorname{cosec}^{-1} \frac{1}{x}; \cos^{-1} x = \sec^{-1} \frac{1}{x}; \tan^{-1} x = \cot^{-1} \frac{1}{x} \text{ when } x > 0 \text{ \&}$$

$$\tan^{-1} x = \cot^{-1} \frac{1}{x} - \pi \text{ when } x < 0.$$

$$(4) (i) \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \quad (-1 \leq x \leq 1)$$

$$(ii) \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \quad (-\infty < x < \infty)$$

$$(iii) \sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2} \quad (x \leq -1, \text{ or, } x \geq 1)$$

$$(5) \tan^{-1} x \pm \tan^{-1} y = \tan^{-1} \left(\frac{x \pm y}{1 \mp xy} \right) \text{ [At principal value]}$$

$$(6) \sin^{-1} x \pm \sin^{-1} y = \sin^{-1} (x\sqrt{1-y^2} \pm y\sqrt{1-x^2}) \text{ [At principal value]}$$

$$(7) \cos^{-1} x \pm \cos^{-1} y = \cos^{-1} (xy \mp \sqrt{(1-x^2)(1-y^2)}) \text{ [At principal value]}$$

$$(8) 2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right) = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$