

(1) If  $\lim_{x \rightarrow a} f(x)$ , then put  $x - a = h$ . Then  $\lim_{x \rightarrow a} f(x) = \lim_{h \rightarrow 0} f(a + h)$

(2) If  $\lim_{x \rightarrow \infty} f(x)$ , then put  $\frac{1}{x} = z$ . Then  $\lim_{x \rightarrow \infty} f(x) = \lim_{z \rightarrow 0} f\left(\frac{1}{z}\right)$

### **(8) L'Hospital's Rule**

If a function is in the form

$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 0^0, \infty^0, 1^\infty$  at  $x = a$  then  $f(x)$  is indeterminate at  $x = a$ .

#### **Case I** $\left(\frac{0}{0}\right)$

If  $\lim_{x \rightarrow a} f(x) = 0$  &  $\lim_{x \rightarrow a} g(x) = 0$

then,  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ .

#### **Case II** $\left(\frac{\infty}{\infty}\right)$

If  $\lim_{x \rightarrow a} f(x) = \infty$  &  $\lim_{x \rightarrow a} g(x) = \infty$

then,  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ .

## **CONTINUITY & DIFFERENTIABILITY**

### **Continuity:-**

$f(x)$  is said to be continuous at  $x=a$  if

- (i)  $f(a)$  is defined that means  $f(x)$  approaches to a definite finite value at  $x=a$
- (ii)  $\lim_{x \rightarrow a} f(x)$  exists
- (iii)  $\lim_{x \rightarrow a} f(x) = f(a)$

In brief,  $f(x)$  is said to be continuous at  $x=a$  if

$$\lim_{x \rightarrow a+} f(x) = \lim_{x \rightarrow a-} f(x) = f(a)$$

If one of the above condition fails, then  $f(x)$  is discontinuous at  $x=a$ .

If  $f(x)$  is continuous at every point in the interval  $a \leq x \leq b$  then  $f(x)$  is continuous in the interval  $a \leq x \leq b$ .

### **Differentiability:-**

Let the domain of definition of a function  $f(x)$  is  $D$  &  $(a, b) \in D$ .

$f(x)$  is said to be differentiable at  $x=c$  (where  $a < c < b$ ) iff

$\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$  exists & approaches to a finite value.

*Differentiability  $\Rightarrow$  Continuity*