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|--------------------------|---------------------------------------|---------------------------------------|---|
| | $\left(-ae, \pm \frac{b^2}{a}\right)$ | $\left(\pm \frac{b^2}{a}, -ae\right)$ | $\left(\alpha - ae, \beta \pm \frac{b^2}{a}\right)$ |
| <u>Parametric</u> | $(a \sec \theta, b \tan \theta)$ | $(b \tan \theta, a \sec \theta)$ | $(\alpha + a \sec \theta, \beta + b \tan \theta)$ |

(1) If $P(x, y)$ be a point on the foci S & S' , then $SP = ex - a; S'P = ex + a; SP - S'P = 2a$.

(2) Any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $(a \sec \theta, b \tan \theta)$.

(3) The rectangular hyperbola is $x^2 - y^2 = a^2$.

Its transverse axis $\rightarrow X - axis$; Conjugate axis $\rightarrow Y - axis$

Eccentricity $\rightarrow \sqrt{2}$; Length of $T - axis$ & $C - axis \rightarrow 2a$.

(4) The two hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ are conjugate to each other.

Eccentricity $\rightarrow b^2 = a^2(e_1^2 - 1)$; $a^2 = b^2(e_2^2 - 1)$.

(5) The position of a point $P(x_1, y_1)$ w.r.t. the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

If $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 > 0$, then inside the hyperbola

$= 0$, then on the hyperbola

< 0 , then outside the hyperbola

(6) The equation of tangent of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P(x_1, y_1)$ is

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1.$$

(7) The equation of normal of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P(x_1, y_1)$ is

$$b^2x_1(y - y_1) + a^2y_1(x - x_1) = 0.$$

Classification of Curves:-

Let us consider a second degree curve in x, y as

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \dots (i) \quad [\text{at least one of } a, h, b \text{ is non - zero constant}]$$

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$$

Identification of Curves:-

The above equation (i) represents

(i) A parabola if $\Delta \neq 0$ & $h^2 = ab$.

(ii) An ellipse if $\Delta \neq 0$ & $h^2 < ab$.

(iii) A hyperbola if $\Delta \neq 0$ & $h^2 > ab$.

(iv) A pair of straight lines if $\Delta = 0$ & $h^2 \geq ab$.

(v) A unique point if $\Delta = 0$ & $h^2 < ab$.