$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

(11) **Angle between two Planes**

Vector Form

The acute angle θ *between the planes* $\vec{r}.\overrightarrow{n_1} = q_1$ *and* $\vec{r}.\overrightarrow{n_2} = q_2$ *is given by*

$$\cos\theta = \frac{|\overrightarrow{n_1}.\overrightarrow{n_2}|}{|\overrightarrow{n_1}||\overrightarrow{n_2}|}.$$

(i) *Two planes* $\vec{r}.\overrightarrow{n_1} = q_1$ *and* $\vec{r}.\overrightarrow{n_2} = q_2$ *are perpendicular to each other* $\Leftrightarrow \overrightarrow{n_1}.\overrightarrow{n_2} = 0$.

(ii) *Two planes* $\vec{r}.\overrightarrow{n_1} = q_1$ *and* $\vec{r}.\overrightarrow{n_2} = q_2$ *are parallel to each other* $\Leftrightarrow \overrightarrow{n_1} = \lambda\overrightarrow{n_2}$ *for some scalar* λ.

(iii) *Any plane parallel to* $\vec{r}.\vec{n} = q$ *and passing through a point with p.v.* $\vec{a}$ *is*

$$(\vec{r} - \vec{a}).\vec{n} = 0.$$

Cartesian Form

The acute angle θ *between the planes* $a_1x + b_1y + c_1z + d_1 = 0$ *and*

$a_2x + b_2y + c_2z + d_2 = 0$ *is given by*

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\left(\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}\right)\left(\sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}\right)}$$

(i) *Two planes* $a_1x + b_1y + c_1z + d_1 = 0$ *and* $a_2x + b_2y + c_2z + d_2 = 0$ *are perpendicular to each other* $\Leftrightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$.

(ii) *Two planes* $a_1x + b_1y + c_1z + d_1 = 0$ *and* $a_2x + b_2y + c_2z + d_2 = 0$ *are parallel to each other* $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(iii) *The equation of a plane passing through the point* (x_1, y_1, z_1) *and parallel to the plane* $ax + by + cz + d = 0$ *is given by*

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$$

(iv) *Any plane parallel to the yz* $-$ *plane is* $x = \lambda$.
Any plane parallel to the xz $-$ *plane is* $y = \lambda$.
Any plane parallel to the xy $-$ *plane is* $z = \lambda$.

(12) **Angle between a line and a plane**

Vector Form

(i) *If* Φ *is the angle between the line* $\vec{r} = \vec{a} + \lambda\vec{b}$ *and the plane* $\vec{r}.\vec{n} = q$ *then*,

$$\sin\Phi = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}.$$

(ii) *The line* $\vec{r} = \vec{a} + \lambda\vec{b}$ *is perpendicular to the plane* $\vec{r}.\vec{n} = q$ *only when* $\vec{b} = t\vec{n}$ *for some scalar* t.

(iii) (a) *The line* $\vec{r} = \vec{a} + \lambda\vec{b}$ *is parallel to the plane* $\vec{r}.\vec{n} = q$ *only when* $\vec{b}.\vec{n} = 0$.

(b) *If the line* $\vec{r} = \vec{a} + \lambda\vec{b}$ *is parallel to the plane* $\vec{r}.\vec{n} = q$ *then the distance between them is* $\frac{|\vec{a}.\vec{n} - q|}{|\vec{n}|}$.