Cartesian Form

(i) *If* Φ *is the angle between the line* $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ *and the plane* $a_2x + b_2y + c_2z + d_2 = 0$ *then*

$$\sin\Phi = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\left(\sqrt{a_1^2 + b_1^2 + c_1^2}\right)\left(\sqrt{a_2^2 + b_2^2 + c_2^2}\right)}$$

(ii) *The line* $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ *is perpendicular to the plane* $a_2x + b_2y + c_2z + d_2 = 0$ *only when* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(iii) *The line* $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ *is perpendicular to the plane* $a_2x + b_2y + c_2z + d_2 = 0$ *only when* $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

(13) **Equation of a Plane passing through a given point and parallel to two given Lines**

Vector Form

The vector equation of a plane passing through a point having p.v. $\vec{a}$ *and parallel to the vectors* $\vec{b}$ *and* $\vec{c}$ *is given by*

$$(\vec{r} - \vec{a}).(\vec{b} \times \vec{c}) = 0.$$

Cartesian Form

The cartesian equation of a plane passing through a point $A(x_1, y_1, z_1)$ *and parallel to two non – parallel lines having d.r.'s* b_1, b_2, b_3 *and* c_1, c_2, c_3 *is given by*

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0.$$

(14) **Condition for the Coplanarity of two Lines**

Vector Form

(i) *Two lines* $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ *and* $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ *are coplanar only when*

$$(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0.$$

(ii) *If two lines* $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ *and* $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ *are coplanar, then the equation of the plane containing both of these lines is given by*

$$[(\vec{r} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0] \text{ or } [(\vec{r} - \overrightarrow{a_2}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0].$$

Cartesian Form

(i) *Two lines* $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ *and* $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ *are coplanar only when*

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

(ii) *The equation of the common plane is*

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$