PROBABILITY

Theory of Probability

(1) **Conditional Probability**

$Let\ A\ \&\ B\ be\ the\ two\ events\ associated\ with\ the\ same\ random\ experiment\ .Then\ the\ probability$

$of\ occurence\ of\ A\ under\ the\ condition\ B\ has\ already\ occurred\ and\ P(B) \neq 0, is\ called$

$conditional\ probability, denoted\ by\ P\left(\frac{A}{B}\right).$

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)},\ \ where\ P(B) \neq 0$

(2) $Let\ A\ and\ B\ be\ the\ two\ events\ of\ a\ sample\ space\ S\ and\ let\ E\ be\ an\ event\ such\ that\ P(E) \neq 0.$

$Then, \qquad P\left[\frac{A \cup B}{E}\right] = P\left(\frac{A}{E}\right) + P\left(\frac{B}{E}\right) - P\left[\frac{A \cap B}{E}\right]$

(3) $For\ any\ events\ A\ \&\ B\ of\ a\ sample\ space\ S, prove\ that$

$P\left(\frac{\overline{A}}{B}\right) = 1 - P\left(\frac{A}{B}\right).$

(4) **Multiplication Theorem on Probability**

$Let\ A\ and\ B\ be\ the\ two\ events\ associated\ with\ a\ sample\ space\ S.\ Then, the\ simultaneous$
$occurence\ of\ two\ events\ A\ and\ B\ is\ denoted\ by\ (A \cap B)\ is\ given\ by$

$P(AB) = P(A \cap B) = P(A).P\left(\frac{B}{A}\right) = P(B).P\left(\frac{A}{B}\right);\ \ where\ P(A) \neq 0\ and\ P(B) \neq 0$

(5) $For\ any\ three\ events\ A, B, C\ of\ the\ same\ sample\ space, we\ have$

$P(A \cap B \cap C) = P(A).P\left(\frac{B}{A}\right).P\left[\frac{C}{A \cap B}\right].$

(6) **Independent Events**

$Two\ events\ A\ and\ B\ are\ said\ to\ be\ independent\ if$

$P\left(\frac{A}{B}\right) = P(A);\ \ where\ P(B) \neq 0 \qquad P\left(\frac{B}{A}\right) = P(B);\ \ where\ P(A) \neq 0$

$So, for\ independent\ events, \qquad P(A \cap B) = P(A) \times P(B)$

(7) $Two\ events\ A\ and\ B\ are\ said\ to\ be\ mutually\ exclusive\ if\ A \cap B = \emptyset\ and\ in\ this\ case\ P(\emptyset) = 0$

Baye's Theorem and its Application

(1) **Theorem of Total Probability**

$Let\ E_1, E_2, \dots, E_n\ \ be\ mutually\ exclusive\ and\ exhaustive\ events\ associated\ with\ a\ random$
$experiment\ and\ let\ E\ be\ an\ event\ that\ occurs\ with\ some\ E_i. Then,$

$$P(E) = \sum_{i=1}^{n} P(E/E_i).P(E_i)$$

(2) **Baye's Theorem**

$Let\ E_1, E_2, \dots, E_n\ \ be\ mutually\ exclusive\ and\ exhaustive\ events\ associated\ with\ a\ random$