

$$\text{where, } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}; D_1 = \begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}; D_2 = \begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}; D_3 = \begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}$$

Important Conditions: –

Case I:- (When $D \neq 0$)

The system of linear equations will be consistent, independent & it has unique solution.

Case II:- (When $D = D_1 = D_2 = D_3 = 0$)

The system of linear equations will be consistent, dependent & it has infinite no. of solutions.

Case III:- (When $D = 0$ & at least one of D_1, D_2, D_3 is $\neq 0$)

The system of linear equations will be inconsistent & it has no solution.

Matrix

Basic Form:-

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}; \text{ Order} = R \times C$$

(1) A matrix A is said to be square matrix iff $R = C$.

(2) If all the elements in A are zero, then A is said to be a Null (or zero) matrix, denoted by O.

$$\text{E.g. } A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(3) If a matrix have only diagonal terms & the other terms are zero, then it is called diagonal matrix.

$$\text{E.g. } A = \begin{bmatrix} 17 & 0 \\ 0 & 7 \end{bmatrix}$$

(4) If all the terms in a diagonal matrix are only 1, then it is called unit or identity matrix.

$$\text{E.g. } A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(5) If $A^T = A$, then A is a symmetric matrix.

(6) If $A^T = -A$, then A is a skew – symmetric matrix.

(7) If $|A| = 0$, then A is a singular matrix.

(8) If $A^T A = I$, then A is an orthogonal matrix.

(9) If $A^2 = A$, then A is an idempotent matrix.

(10) If $A^2 = I$, then A is an involutory matrix.

(11) If $A^k = O$, then A is a nilpotent matrix.

(12) If the rows & columns of a matrix are interchanged, then it is called transpose matrix A^T .

(13) If a diagonal matrix have all the terms same, then it is called scalar matrix.

(14) Equal Matrices:-

$$\text{If } A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \& B = \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix}; \text{ then } A = B \text{ iff } a_1 = c_1, a_2 = c_2, b_1 = d_1, b_2 = d_2.$$

(15) Addition & Subtraction of matrices:-

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}, B = \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix}, \text{ then } A \pm B = \begin{bmatrix} a_1 \pm c_1 & b_1 \pm d_1 \\ a_2 \pm c_2 & b_2 \pm d_2 \end{bmatrix}.$$

(16) Multiplication by a scalar k:-