

DIFFERENTIATION

LIMIT:-

(1) Significant of $\lim_{x \rightarrow a} f(x) = l$

$\lim_{x \rightarrow a} f(x) = l$ indicates that the variable x takes the values $> a$ or $< a$ but $\neq a$.

l is the limiting value of $f(x)$.

(2) ' $\lim_{x \rightarrow a+} f(x)$ ' is called the right hand limit of $f(x)$ at $x = a$.

(3) ' $\lim_{x \rightarrow a-} f(x)$ ' is called the left hand limit of $f(x)$ at $x = a$.

(4) Existence of $\lim_{x \rightarrow a} f(x)$

$\lim_{x \rightarrow a} f(x)$ exists if both $\lim_{x \rightarrow a+} f(x)$ & $\lim_{x \rightarrow a-} f(x)$ exists and $\lim_{x \rightarrow a+} f(x) = \lim_{x \rightarrow a-} f(x)$.

(5) Fundamental Theorems:-

$$(1) \lim_{x \rightarrow a} [f_1(x) \pm f_2(x) \pm f_3(x) \pm \dots] = \lim_{x \rightarrow a} f_1(x) + \lim_{x \rightarrow a} f_2(x) + \dots$$

$$(2) \lim_{x \rightarrow a} [f_1(x) \cdot f_2(x) \dots] = \lim_{x \rightarrow a} f_1(x) \cdot \lim_{x \rightarrow a} f_2(x) \dots$$

$$(3) \lim_{x \rightarrow a} \left[\frac{f(x)}{\phi(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} \phi(x)} \quad [\text{where, } \lim_{x \rightarrow a} \phi(x) \neq 0]$$

$$(4) \lim_{x \rightarrow a} \phi\{f(x)\} = \phi\{\lim_{x \rightarrow a} f(x)\}$$

(6) Formulae

$$(1) \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$(2) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad [x \text{ in radian}]$$

$$(3) \lim_{x \rightarrow 0} \frac{\{1+x\}^n - 1}{x} = n$$

$$(4) \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$(5) \lim_{x \rightarrow 0} \frac{\log_e(1+x)}{x} = 1$$

$$(6) \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

$$(7) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$(8) \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a \quad (a > 0)$$

(7) Some Substitution:-