(5) $The\ condition\ for\ three\ given\ points\ A(x_1, y_1, z_1), B(x_2, y_2, z_2)\ and\ C(x_3, y_3, z_3)\ to$ $be\ collinear\ is\ that$

$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} = \frac{z_3 - z_1}{z_2 - z_1}$$

(6) $Three\ points\ A, B\ and\ C\ with\ p.v.'s\ \vec{a}, \vec{b}\ and\ \vec{c}\ respectively\ are\ collinear\ iff$ $there\ exist\ scalars\ d_1, d_2, d_3\ not\ all\ zero\ such\ that$

$d_1\vec{a} + d_2\vec{b} + d_3\vec{c} = \vec{0}\ \ and\ \ d_1 + d_2 + d_3 = 0$

(7) $If\ \theta\ is\ the\ angle\ between\ the\ lines\ \vec{r} = \overrightarrow{r_1} + \lambda\overrightarrow{m_1}\ and\ \vec{r} = \overrightarrow{r_2} + \mu\overrightarrow{m_2}\ \ then$

$$cos\ \theta = \frac{|\overrightarrow{m_1} . \overrightarrow{m_2}|}{|\overrightarrow{m_1}||\overrightarrow{m_2}|}$$

(8) $If\ \theta\ is\ the\ angle\ between\ the\ lines\ whose\ cartesian\ equations\ are$

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}\ \ and\ \ \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}\ \ then$$

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\left(\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\right)\left(\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}\right)}$$

(9) $The\ shortest\ distance\ between\ two\ skew\ (non - coplanar)\ lines\ \vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}\ and$ $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}\ \ is\ given\ by$

$$d = \left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$$

$If\ the\ lines\ intersect\ each\ other\ then\ the\ shortest\ distance\ between\ them\ is\ zero.$

(10) $The\ distance\ between\ two\ parallel\ lines\ \vec{r} = \overrightarrow{a_1} + \lambda\vec{b}\ and\ \vec{r} = \overrightarrow{a_2} + \mu\vec{b}\ \ is\ given\ by$

$$D = \left|\frac{\vec{b} \times (\overrightarrow{a_2} - \overrightarrow{a_1})}{\vec{b}}\right|$$

(11) $The\ shortest\ distance\ between\ two\ skew\ lines$

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}\ \ and\ \ \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}\ \ is\ given\ by$$

$$SD = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{[(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2]}}$$

$The\ two\ lines\ mentioned\ above\ will\ intersect\ if\ SD = 0, i.e.,$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$