(15) **Vector product of an orthonormal vector triad**

$For\ mutually\ perpendicular\ unit\ vectors\ \hat{\imath}, \hat{\jmath}, \hat{k}, we\ have$

(i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \hat{0}$

(ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

(16) **Vector product in terms of componenets**

$Let\ \vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}\ and\ \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}. Then,$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

(17) **Scalar Triple product**

(i) $[\vec{a}\, \vec{b}\, \vec{c}] = (\vec{a} \times \vec{b}). \vec{c}$

$[\vec{a}\, \vec{b}\, \vec{c}]\ represents\ the\ volume\ of\ the\ parallelopiped\ with\ coterminous\ edges\ \vec{a}, \vec{b}, \vec{c}$

$forming\ a\ right-handed\ system.$

(ii) $(\vec{a} \times \vec{b}). \vec{c} = (\vec{b} \times \vec{c}). \vec{a} = (\vec{c} \times \vec{a}). \vec{b}$

(iii) $The\ scalar\ triple\ product\ changes\ in\ sign\ but\ not\ in\ magnitude\ when\ the\ cyclic\ order$

$of\ vectors\ is\ changed, i.e., \qquad [\vec{c}\, \vec{b}\, \vec{a}] = -[\vec{a}\, \vec{b}\, \vec{c}]$

(iv) $The\ scalar\ triple\ product\ vanishes\ if\ any\ two\ of\ its\ vectors\ are\ equal, i.e.,$

$[\vec{a}\, \vec{a}\, \vec{b}] = 0, [\vec{a}\, \vec{b}\, \vec{a}] = 0\ and\ [\vec{b}\, \vec{a}\, \vec{a}] = 0.$

(v) $The\ scalar\ triple\ product\ vanishes\ if\ any\ two\ of\ its\ vectors\ are\ parallel\ or\ collinear.$

$Let\ \vec{a}||\vec{b}\ or\ \vec{a}\ and\ \vec{b}\ are\ collinear. Then, \vec{a} = m\vec{b}$

$\therefore\ [\vec{a}\, \vec{b}\, \vec{c}] = [m\vec{b}\, \vec{b}\, \vec{c}] = 0$

(vi) **Scalar triple product in terms of components**

$Let\ \vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k};\ \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}\ and\ \vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

$$[\vec{a}\, \vec{b}\, \vec{c}] = \begin{vmatrix} \boldsymbol{a_1} & \boldsymbol{a_2} & \boldsymbol{a_3} \\ \boldsymbol{b_1} & \boldsymbol{b_2} & \boldsymbol{b_3} \\ \boldsymbol{c_1} & \boldsymbol{c_2} & \boldsymbol{c_3} \end{vmatrix}$$

(vii) $For\ any\ three\ vectors\ \vec{a}, \vec{b}\ and\ \vec{c}$

$[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\, \vec{b}\, \vec{c}]$

(viii) $The\ necessary\ and\ sufficient\ condition\ for\ three\ non-zero, non-collinear$

$vectors\ \vec{a}, \vec{b}, \vec{c}\ to\ be\ coplanar\ is\ that\ [\vec{a}\, \vec{b}\, \vec{c}] = 0.$

(ix) $For\ any\ three\ vectors\ \vec{a}, \vec{b}, \vec{c},\ the\ vectors\ (\vec{a} - \vec{b}), (\vec{b} - \vec{c}) and\ (\vec{c} - \vec{a})\ are\ coplanar.$

$Also, (\vec{a} + \vec{b}), (\vec{b} + \vec{c}) and\ (\vec{c} + \vec{a})\ are\ coplanar.$

(x) $For\ any\ three\ vectors\ \vec{a}, \vec{b}, \vec{c},\ [\vec{a}\ \ \vec{a} + \vec{b}\ \ \vec{a} + \vec{b} + \vec{c}] = 0$