

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \pm \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{a^2 + b^2 + c^2}} = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$(13) l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}}; m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}}; n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}.$$

(14) The direction cosines of the join of the two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}.$$

(15) The direction ratios of the line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$x_2 - x_1, y_2 - y_1, z_2 - z_1.$$

(16) If θ be the angle between two lines, then

$$\cos \theta = \frac{\sum a_1 a_2}{\sqrt{\sum a_1^2} \sqrt{\sum a_2^2}}. \quad [\text{where, } a_1 = l_1, m_1, n_1; a_2 = l_2, m_2, n_2].$$

$$\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2; \sin \theta = \sqrt{\sum (m_1 n_2 - m_2 n_1)^2}$$

(17) Two lines with direction cosines l_1, m_1, n_1 & l_2, m_2, n_2 will be parallel if

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}.$$

(18) Two lines with direction cosines l_1, m_1, n_1 & l_2, m_2, n_2 will be perpendicular if

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0.$$

(19) If $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$, then direction ratios of r are a, b, c .

(20) The projection of line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ on a line whose direction cosines are l, m, n is given by

$$l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1).$$

Straight Line in Space

(1) If a line passes through a point with position vector \vec{r}_1 and it is parallel to \vec{m} then its vector equation is $\vec{r} = \vec{r}_1 + \lambda \vec{m}$

(2) If a line passes through a point $A(x_1, y_1, z_1)$ and it has d.r.'s a, b, c then its cartesian equations are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

(3) If a line passes through two points having p.v.'s \vec{r}_1 and \vec{r}_2 , then its vector equation is

$$\vec{r} = \vec{r}_1 + \lambda(\vec{r}_2 - \vec{r}_1)$$

(4) If a line passes through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ then its cartesian equations are

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$