ALGEBRA

Logarithm

$If\ a^x = M, then\ x = \log_a M$

Rules of Logarithm:-

(1) $\log_a 1 = 0$

(2) $\log_a a = 1$

(3) $a^{\log_a M} = M$

(4) $\log_a MN = \log_a M + \log_a N$

(5) $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

(6) $\log_a M^n = n \log_a M$

(7) $\log_a M = \log_b M \times \log_a b$

(8) $\log_b a \times \log_a b = 1$

(9) $\log_b a = \frac{1}{\log_a b}$

(10) $\log_b M = \frac{\log_a M}{\log_a b}$

(11) $\log_a M = \frac{\log M}{\log a}$

(12) $\log e = 1$

Note: $If\ base\ of\ logarithm\ is\ not\ mentioned, then\ it\ is\ taken\ 10.$

Complex Number

$z = x + iy$

$where,$

$i = \sqrt{-1}\ \&\ x, y \in R.$

$x\ is\ called\ the\ real\ part\ \&\ iy\ is\ called\ the\ imaginary\ part.$

Properties of Complex Number

(1) $|z| = |x + iy| = \sqrt{x^2 + y^2}$

(2) $Amp\ z\ (or\ Arg\ z) = \tan^{-1}\left(\frac{y}{x}\right) = \theta$

$If -\pi < \theta \leq \pi, then\ \theta\ is\ called\ the\ Principal\ value\ of\ the\ argument.$

(3) $If\ z = x + iy\ \&\ in\ complex\ plane,$

$(x, y)\ is\ in\ 1st\ quadrant, \quad then\ 0 < P.V.of\ \theta < \frac{\pi}{2}$

$(x, y)\ is\ in\ 2nd\ quadrant, \quad then\ \frac{\pi}{2} < P.V.of\ \theta < \pi$

(x, y) *is in* 3*rd quadrant*, $\quad$ *then* $-\pi < P.V. of\ \theta < -\frac{\pi}{2}$

(x, y) *is in* 4*th quadrant*, $\quad$ *then* $-\frac{\pi}{2} < P.V. of\ \theta < 0$

(4) **Modulus-Amplitude Form**

$z = r(\cos\theta + i\sin\theta)$

where,

$r = |z|$

$\theta = Arg. z$

(5) $|z_1 + z_2| \leq |z_1| + |z_2|$

(6) $|z_1 z_2| = |z_1||z_2|$

(7) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

(8) (i) $Arg\ (z_1 z_2) = Arg\ z_1 + Arg\ z_2 + m$

(ii) $Arg\ \left(\frac{z_1}{z_2}\right) = Arg\ z_1 - Arg\ z_2 + m$

where, $m = 0$ *or* $\pm 2\pi$

(9) *The three cube roots of* 1 *are* $1, w, w^2$

where, $w = \dfrac{-1 \pm \sqrt{3}i}{2}$

(10) *If* w *is a complex cube root of* 1, *then* $w^3 = 1$ & $1 + w + w^2 = 0$.

(11) **De-Moivre's Theorem**

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(12) **Euler's Identity**

$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$

AP

(1) *If the* 1*st term* $= a$ & *the common difference* $= d$ *in an A.P.*, *then*

(a) $n - th\ term,\ t_n = a + (n-1)d$

(b) *Sum of* 1*st n terms*,

$S_n = \frac{n}{2}(a + l)$

$= \frac{n}{2}\{2a + (n-1)d\}$

(2) $1 + 2 + 3 + \cdots \ldots \ldots .. + n = \frac{n(n+1)}{2}$

(3) $1^2 + 2^2 + 3^2 + \cdots \ldots \ldots .. + n^2 = \frac{n(n+1)(2n+1)}{6}$

(4) $1^3 + 2^3 + 3^3 + \cdots \ldots \ldots .. + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

(5) *If* x *be the AM of two numbers* a & b, *then* $x = \frac{(a+b)}{2}$

GP

(1) $If\ the\ 1st\ term\ =a, common\ ratio\ =r\ in\ a\ GP, then$

(a) $n-th\ term\ =t_n\ =ar^{n-1}$

(b) $Sum\ of\ 1st\ n\ terms,$

$$S_n\ =\frac{a(1-r^n)}{1-r}\qquad when, -1<r<1$$

$$=\frac{a(r^n-1)}{r-1}\qquad when,\ r>1\ or\ r<-1$$

(2) $If\ x\ be\ the\ GM\ of\ two\ numbers\ a\ \&\ b, then\ x\ =\pm\sqrt{ab}$

Infinite GP

$$S_n\ =\frac{a}{1-r}\quad when, -1<r<1$$

$For\ r>1\ or\ r<-1,\qquad the\ series\ doesn't\ exist.$

HP

(1) $If\ a, b, c\ \ are\ in\ HP$

$then\ \frac{1}{a},\frac{1}{b},\frac{1}{c}\ \ are\ in\ AP$

(2) $If\ x\ be\ the\ HM\ of\ two\ numbers\ a\ \&\ b, then\ x\ =\frac{2ab}{a+b}.$

Infinite Series

$If\ n\ is\ a-ve\ integer\ or\ fraction\ \&\ |x|<1, then$

(1) $(1+x)^n\ =1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\cdots\ldots\ldots\infty$

(2) $(1-x)^n\ =1-nx+\frac{n(n-1)}{2!}x^2-\frac{n(n-1)(n-2)}{3!}x^3+\cdots\ldots\ldots\infty$

(3) $(1+x)^{-n}\ =1-nx+\frac{n(n+1)}{2!}x^2-\frac{n(n+1)(n+2)}{3!}x^3+\cdots\ldots\ldots\infty$

(4) $(1-x)^{-n}\ =1+nx+\frac{n(n+1)}{2!}x^2+\frac{n(n+1)(n+2)}{3!}x^3+\cdots\ldots\ldots\infty$

(5) $e^x\ =1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\ldots\ldots+\frac{x^r}{r!}+\cdots\ldots\ldots\infty$

(6) $e^{-x}\ =1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots\ldots\ldots+(-1)^r\frac{x^r}{r!}+\cdots\ldots\ldots\infty$

(7) $e\ =1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots\ldots\ldots\infty$

(8) $e^{-1}\ =1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots\ldots\ldots\infty$

(9) $a^x\ =1+\frac{(\log_e a)}{1!}x+\frac{(\log_e a)^2}{2!}x^2+\frac{(\log_e a)^3}{3!}x^3+\cdots\ldots\ldots\infty$

(10) $\log_e(1+x)\ =x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots\ldots\ldots\infty\ ;\qquad when-1<x\le 1$

(11) $\log_e(1-x)\ =-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}+\cdots\ldots\ldots\infty\ ;\qquad when-1\le x<1$

(12) $\log_e 2=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\ldots\ldots\infty$

Quadratic Equation

General Expression:-

$$ax^2 + bx + c = 0 \qquad [a \neq 0]$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(1) $If\ the\ two\ roots\ of\ a\ quadratic\ equation\ be\ \alpha\ \&\ \beta, then$

(a) $\alpha + \beta = -\frac{b}{a}$

(b) $\alpha\beta = \frac{c}{a}$

(2) **General structure of a quadratic equation:-**

$$x^2 - (Sum\ of\ the\ roots)x + (Product\ of\ the\ roots) = 0$$

(3) $Discriminant,\ D = b^2 - 4ac$

Case I:- (When D>0 & a square no.)

$The\ roots\ will\ be\ real, rational\ \&\ unequal$

Case II:- (When D>0 but not a square no.)

$The\ roots\ will\ be\ real, irrational\ \&\ unequal$

Case III:- (When D=0)

$The\ roots\ will\ be\ real\ \&\ equal$

Case IV:- (When D<0)

$The\ roots\ will\ be\ imaginary$

(4) $If\ a, b, c\ are\ real\ \&\ a\ root\ of\ equation\ (i)\ be\ \alpha + i\beta, then\ the\ other\ root\ will\ be\ \alpha - i\beta\ and\ vice-versa\ (where\ \alpha, \beta\ real).$

(5) $If\ a, b, c\ are\ rational\ \&\ a\ root\ of\ equation\ (i)\ be\ p + \sqrt{q}, then\ the\ other\ root\ will\ be\ p - \sqrt{q}\ \&\ vice-versa\ (where\ p, q\ real).$

(6) $The\ two\ roots\ will\ be\ common\ of\ two\ quadratic\ equation\ ax^2 + bx + c = 0\ \&\ a_1x^2 + b_1x + c_1 = 0$

$iff\ \dfrac{a}{a_1} = \dfrac{b}{b_1} = \dfrac{c}{c_1}.$

(7) $The\ two\ quadratic\ equations\ ax^2 + bx + c = 0\ \&\ a_1x^2 + b_1x + c_1 = 0\ will\ have\ exactly\ one\ common\ root\ iff$

$(a_1c - ac_1)^2 = (ab_1 - a_1b)(bc_1 - b_1c)\ \&\ \dfrac{b}{a} \neq \dfrac{b_1}{a_1}\ or\ \dfrac{c}{a} \neq \dfrac{c_1}{a_1}.$

Permutation

(1) $n! = n(n-1)(n-2)(n-3) \ldots\ldots\ldots 3.2.1$

(2) $0! = 1$

(3) ${}^n_rP = \frac{n!}{(n-r)!} = No.\ of\ permutations\ of\ n\ dissimilar\ things\ taken\ r\ at\ a\ time$

(4) $\frac{n!}{p!q!r!} = The\ permutation\ of\ n\ things\ in\ which\ p\ of\ them\ to\ be\ a, q\ of\ them\ to\ be\ b, r\ of\ them\ to\ be\ c, and\ the\ rest\ to\ be\ unlike.$

(5) $n^r = The\ no.\ of\ permutations\ of\ n\ different\ things\ taken\ r\ at\ a\ time, each\ thing\ may\ be\ repeated\ once, twice, \ldots..upto\ r\ times\ in\ any\ arrangement.$

(6) **Circular Permutation:-**

If no distinction is made between the clockwise & anti − clockwise arrangement then

the no. of permutation $= \frac{1}{2}(n-1)!$.

And if the distinction is made then the no. of total permutation $= (n-1)!$.

Combination

(1) *The no. of combinatios of n dissimilar things taken r at a time* $= {}^{n}_{r}C = \frac{n!}{r!(n-r)!}$

(2) ${}^{n}_{r}C = {}^{n}_{n-r}C$

(3) *If* ${}^{n}_{p}C = {}^{n}_{q}C$, *then* $p + q = n.$ $[p \neq q]$

(4) ${}^{n}_{r}C + {}^{n}_{r-1}C = {}^{n+1}_{r}C$

(5) $\frac{{}^{n}_{r}C}{{}^{n}_{r-1}C} = \frac{n-r+1}{r}$

(6) *The total no. of combinations of n dissimilar things taken one, two, etc. all at a time* =

${}^{n}_{1}C + {}^{n}_{2}C + {}^{n}_{3}C + \cdots \ldots + {}^{n}_{n}C = 2^n - 1$

(7) *The no. of ways in which it is possible to make a selection by taking some or all out of* $p + q + r + \cdots \ldots$ *things, whereof p are alike of one kind, q alike of second kind, r alike of third kind; and so on*

$= (p+1)(q+1)(r+1) \ldots \ldots - 1$

(8) *If* $n = +ve$ *even integer, then* ${}^{n}_{r}C$ *is greatest when* $r = \frac{n}{2}$.

If $n = +ve$ *odd integer then* ${}^{n}_{r}C$ *is greatest when* $r = \frac{n-1}{2}$ *or* $\frac{n+1}{2}$.

(9) *The no. of ways in which* $(m+n)$ *things can be divided into two groups, containing m & n things*

respectively $= \frac{(m+n)!}{m!\,n!}$.

(10) *The no. of ways in which* $(m+n+p)$ *things can be divided into three groups, containing m, n, p*

things respectively $= \frac{(m+n+p)!}{m!\,n!\,p!}$.

Binomial Theorem

(1) *If n is a + ve integer, then*

$(a+x)^n = a^n + {}^{n}_{1}C a^{n-1}x^1 + {}^{n}_{2}C a^{n-2}x^2 + \cdots \ldots \ldots + {}^{n}_{r}C a^{n-r}x^r + \cdots \ldots \ldots + x^n$

(2) *If n is a − ve integer or fraction, then*

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \ldots \ldots \infty$ $\quad [|x| < 1]$

(3) *General term* $= (r+1) - th$ *term* $= t_{r+1} = {}^{n}_{r}C a^{n-r}x^r$

(4) (i) *If n is even, then there exists a middle term & the middle term will be* $\left(\frac{n}{2} + 1\right) - th$ *term.*

(ii) *If n is odd, then there exists two middle terms & the middle terms will be*

$\left(\frac{n-1}{2} + 1\right) - th$ *term &* $\left(\frac{n+1}{2} + 1\right) - th$ *term.*

(5) (i) *In the above expansion, the m − th term will be greatest if* $\frac{(n+1)x}{a+x} = a \ + ve$ *integer* (m)

(ii) $In\ the\ above\ expansion, the\ (m+1)-th\ term\ will\ be\ greatest\ if\ \frac{(n+1)x}{a+x} = a\ \ +ve\ integer\ \ (m)$

$+\ a\ proper\ fraction.$

Determinant

General Form:-

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

Symbolic Form:-

$$\begin{vmatrix} + & - \\ - & + \end{vmatrix} \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Addition of Determinants:-

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} l_1 & m_1 & n_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+l_1 & b_1+m_1 & c_1+n_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} l_1 & b_1 & c_1 \\ l_2 & b_2 & c_2 \\ l_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+l_1 & b_1 & c_1 \\ a_2+l_2 & b_2 & c_2 \\ a_3+l_3 & b_3 & c_3 \end{vmatrix}$$

Properties of Determinants:-

(1) $If\ the\ rows\ \&\ columns\ are\ interchanged\ then\ the\ value\ of\ the\ determinant\ remains\ same.$

(2) $If\ two\ associated\ rows\ (or\ columns)\ are\ interchanged\ then\ the\ sign\ will\ be\ changed.$

(3) $If\ there\ are\ two\ rows\ (or\ columns)\ are\ identical, then\ the\ value\ will\ be\ zero.$

(4) $If\ each\ element\ of\ a\ row\ (or\ a\ column)\ of\ a\ determinant\ is\ multiplied\ by\ a\ constant\ k\ then$ $the\ value\ of\ new\ determinant\ is\ k\ times\ the\ value\ of\ original\ determinant.$

(5) $If\ all\ the\ elements\ in\ a\ row\ (or\ column)\ are\ zero, then\ the\ value\ will\ be\ zero.$

Multiplication of Determinants:-

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} = \begin{vmatrix} a_1p_1 + b_1q_1 & a_1p_2 + b_1q_2 \\ a_2p_1 + b_2q_1 & a_2p_2 + b_2q_2 \end{vmatrix} \qquad (R \times R)$$

To find the area of a triangle:-

$$Area\ of\ \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\ sq.units$$

Cramer's Rule:-

$Let, a_1x + b_1y + c_1z = k_1$

$a_2x + b_2y + c_2z = k_2$

$a_3x + b_3y + c_3z = k_3$

$Using\ cramer's\ rule,\qquad x = \frac{D_1}{D}, y = \frac{D_2}{D}, z = \frac{D_3}{D}$

$where,\quad D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$; $D_1 = \begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}$; $D_2 = \begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}$; $D_3 = \begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}$

***Important Conditions*:−**

Case I:- (When D ≠ 0)

The system of linear equations will be consistent, independent & it has unique solution.

Case II:- (When $\mathbf{D = D_1 = D_2 = D_3 = 0}$)

The system of linear equations will be consistent, dependent & it has infinite no. of solutions.

Case III:- (When D = 0 & at least one of $\mathbf{D_1, D_2, D_3}$ is $\neq 0$)

The system of linear equations will be inconsistent & it has no solution.

Matrix

Basic Form:-

$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$; $Order = R \times C$

(1) *A matrix A is said to be square matrix iff* $R = C$.

(2) *If all the elements in A are zero, then A is said to be a Null (or zero) matrix, denoted by O.*

E.g. $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(3) *If a matrix have only diagonal terms & the other terms are zero, then it is called diagonal matrix.*

E.g. $A = \begin{bmatrix} 17 & 0 \\ 0 & 7 \end{bmatrix}$

(4) *If all the terms in a diagonal matrix are only* 1, *then it is called unit or identity matrix.*

E.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(5) *If* $A^T = A$, *then A is a symmetric matrix.*

(6) *If* $A^T = -A$, *then A is a skew − symmetric matrix.*

(7) *If* $|A| = 0$, *then A is a singular matrix.*

(8) *If* $A^T A = I$, *then A is an orthogonal matrix.*

(9) *If* $A^2 = A$, *then A is an idempotent matrix.*

(10) *If* $A^2 = I$, *then A is an involutory matrix.*

(11) *If* $A^k = O$, *then A is a nilpotent matrix.*

(12) *If the rows & columns of a matrix are interchanged, then it is called transpose matrix* A^T.

(13) *If a diagonal matrix have all the terms same, then it is called scalar matrix.*

(14) **Equal Matrices:-**

If $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ & $B = \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix}$; *then* $A = B$ *iff* $a_1 = c_1, a_2 = c_2, b_1 = d_1, b_2 = d_2$.

(15) **Addition & Subtraction of matrices:-**

$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}, B = \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix}$, *then* $A \pm B = \begin{bmatrix} a_1 \pm c_1 & b_1 \pm d_1 \\ a_2 \pm c_2 & b_2 \pm d_2 \end{bmatrix}$.

(16) **Multiplication by a scalar k:-**

$k\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = \begin{bmatrix} ka_1 & kb_1 \\ ka_2 & kb_2 \end{bmatrix}.$

(17) **Multiplicability of two matrices:-**

$Let\ there\ are\ two\ matrices\ [A]_{R_1 \times C_1}$ & $[B]_{R_2 \times C_2}.$

$If\ C_1 = R_2, then\ A \times B\ exists\ otherwise\ not\ and\ the\ order\ of\ the\ resulting\ matrix\ will\ be\ R_1 \times C_2.$

(18) **Multiplication of two matrices:-**

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \times \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1c_1 + b_1c_2 & a_1d_1 + b_1d_2 \\ a_2c_1 + b_2c_2 & a_2d_1 + b_2d_2 \end{bmatrix}$$

(19) **Adjoint matrix:-**

$$If\ A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad then\ Adj.A = \begin{bmatrix} +\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} & -\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} & +\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \\ -\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} & +\begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} & -\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \\ +\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} & -\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} & +\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \end{bmatrix}$$

(20) **Inverse matrix:-** $A^{-1} = \frac{Adj.\ A}{|A|}$ $\quad [|A| \neq 0]$

(21) $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$where, A = square\ matrix$

$\frac{1}{2}(A + A^T) = symmetric\ matrix$

$\frac{1}{2}(A - A^T) = skew -\ symmetric\ matrix$

(22) **Some useful relations:-**

(i) $(A^T)^T = A$

(ii) $(A \pm B)^T = A^T \pm B^T$

(iii) $(AB)^T = B^T A^T$

(iv) $A^{-1}A = I$

(v) $AA^{-1} = I$

(vi) $(A^{-1})^{-1} = A$

(vii) $(A^{-1})^T = (A^T)^{-1}$

(viii) $(AB)^{-1} = B^{-1}A^{-1}$

(ix) $AB \neq BA\ in\ general$

(x) $(AB)C = A(BC)$

(xi) $A(B + C) = AB + AC$

(xii) $A.O = O.A = O$

(xiii) $AI = IA = A$

(xiv) $If\ A \neq O$ & $B \neq O,\ then\ AB = O\ (it\ may\ be)$

(xv) $If\ CA = CB,\ then\ it\ is\ not\ mandatory\ \ A = B.$

Martin's Rule:-

$Let, a_1x + b_1y + c_1z = k_1$

$a_2x + b_2y + c_2z = k_2$

$a_3x + b_3y + c_3z = k_3$

$Let, A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$

Then the system of equations can be written as $AX = B$, *i.e.*,

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$$

Important Conditions:−

Case I:- **(When $|A| \neq 0$)**

Then the solution will be $X = A^{-1}B$.

Case II:- **(When $|A| = 0$ & $(adj. A)B = O$)**

Then the system may have infinite solution (consistent) or no solution (inconsistent).

Case II:- **(When $|A| = 0$ & $(adj. A)B \neq O$)**

Then solution doesn't exist(inconsistent).

Probability

(1) *In a random experiment, if* S *be a sample space &* E *be an event, then*

(i) $P(E) \geq 0$

(ii) $P(\emptyset) = 0$

(iii) $P(S) = 1$.

(2) $0 \leq P(E) \leq 1$.

(3) $P(E) + P(\bar{E}) = 1$

(4) $P(E - F) = P(E) - P(E \cap F)$

(5) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

(6) $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)$

(7) *If* E_1 & E_2 *be two events such that* $E_1 \subseteq E_2$, *then* $P(E_1) \leq P(E_2)$

(8) *If* E & F *are mutually exclusive events, then*

(i) $P(E \cap F) = 0$

(ii) $P(E \cup F) = P(E) + P(F)$

(9) *If* E & F *are two mutually exclusive exhaustive events, then* $P(E) + P(F) = 1$.

(10) **Independent Event:-**

$$P(E \cap F) = P(E).P(F)$$

(11) **Conditional Probability:-**

If E & F *be two events associated with the same random experiment, then*

$$P(E \cap F) = P(E).P\left(\frac{F}{E}\right) \qquad [where\ P(E) \neq 0]$$

PROBABILITY

Theory of Probability

(1) **Conditional Probability**

Let A & B be the two events associated with the same random experiment . Then the probability of occurence of A under the condition B has already occurred and $P(B) \neq 0$, *is called conditional probability, denoted by* $P\left(\frac{A}{B}\right)$.

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}, \quad where\ P(B) \neq 0$$

(2) *Let A and B be the two events of a sample space S and let E be an event such that* $P(E) \neq 0$.

Then, $$P\left[\frac{A \cup B}{E}\right] = P\left(\frac{A}{E}\right) + P\left(\frac{B}{E}\right) - P\left[\frac{A \cap B}{E}\right]$$

(3) *For any events A & B of a sample space S, prove that*

$$P\left(\frac{\overline{A}}{B}\right) = 1 - P\left(\frac{A}{B}\right).$$

(4) **Multiplication Theorem on Probability**

Let A and B be the two events associated with a sample space S. Then, the simultaneous occurence of two events A and B is denoted by $(A \cap B)$ *is given by*

$$P(AB) = P(A \cap B) = P(A).P\left(\frac{B}{A}\right) = P(B).P\left(\frac{A}{B}\right); \quad where\ P(A) \neq 0\ and\ P(B) \neq 0$$

(5) *For any three events A, B, C of the same sample space, we have*

$$P(A \cap B \cap C) = P(A).P\left(\frac{B}{A}\right).P\left[\frac{C}{A \cap B}\right].$$

(6) **Independent Events**

Two events A and B are said to be independent if

$$P\left(\frac{A}{B}\right) = P(A); \quad where\ P(B) \neq 0 \qquad P\left(\frac{B}{A}\right) = P(B); \quad where\ P(A) \neq 0$$

So, for independent events, $P(A \cap B) = P(A) \times P(B)$

(7) *Two events A and B are said to be mutually exclusive if* $A \cap B = \emptyset$ *and in this case* $P(\emptyset) = 0$

Baye's Theorem and its Application

(1) **Theorem of Total Probability**

Let $E_1, E_2, \ldots, E_n$ *be mutually exclusive and exhaustive events associated with a random experiment and let E be an event that occurs with some* E_i. *Then,*

$$P(E) = \sum_{i=1}^{n} P(E/E_i).P(E_i)$$

(2) **Baye's Theorem**

Let $E_1, E_2, \ldots, E_n$ *be mutually exclusive and exhaustive events associated with a random*

$experiment\ and\ let\ E\ be\ an\ event\ that\ occurs\ with\ some\ E_i. Then,$

$$P\left(\frac{E_i}{E}\right) = \frac{P\left(\frac{E}{E_i}\right).P(E_i)}{\sum_{i=1}^{n} P(E/E_i).P(E_i)}\ ;\ \ i = 1,2,3,\dots,n$$

Probability Distribution

(1) $If\ a\ random\ variable\ X\ takes\ the\ values\ x_1, x_2, \dots, x_n\ with\ respective\ probabilities$

$p_1, p_2, \dots, p_n\ then\ the\ probability\ distribution\ of\ X\ is\ given\ by$

X	x_1	x_2	x_3	$\dots$	$\dots$	x_n
$P(X)$	p_1	p_2	p_3	$\dots$	$\dots$	p_n

$The\ above\ probability\ distribution\ of\ X\ is\ defined\ only\ when$

$(i)\ Each\ \ p_i \geq 0\ ;\quad (ii)\ \sum_{i=1}^{n} p_i = 1.$

(2) $The\ mean\ of\ X, denoted\ by\ \mu, is\ defined\ as$

$$\mu = E(X) = \sum_{i=1}^{n} p_i\, x_i$$

(3) $The\ variance, denoted\ by\ \sigma^2,\ is\ defined\ as$

$$\sigma^2 = \left(\sum x_i{}^2\, p_i - \mu^2\right)$$

(4) $The\ S.D. is\ given\ by\quad \sigma = \sqrt{Variance}.$

Binomial Distribution

(1) **Bernoulli's Theorem**

$Let\ there\ be\ n\ independent\ trials\ in\ an\ experiment\ and\ let\ the\ random\ variable\ X\ denote\ the$ $number\ of\ successes\ in\ these\ trials.\ Let\ the\ probability\ of\ getting\ a\ success\ in\ a\ single\ trial$ $be\ p\ and\ that\ of\ getting\ a\ failure\ be\ q\ so\ that\ p + q = 1.\ Then,$

$$P(X = r) = {}^{n}_{r}C.p^r.q^{n-r}$$

$The\ probability\ distribution\ of\ X\ may\ be\ expressed\ as$

X	0	1	$\dots$	$\dots$	$\dots$	r
$P(X)$	q^n	npq^{n-1}	$\dots$	$\dots$	$\dots$	${}^{n}_{r}C.p^r.q^{n-r}$

$This\ distribution\ is\ called\ a\ binomial\ distribution\ .$

(2) **Condition for the Applicability of a Binomial Distribution**

- $The\ experiment\ is\ performed\ for\ a\ finite\ and\ fixed\ number\ of\ trials.$
- $Each\ trial\ must\ give\ either\ a\ success\ or\ a\ failure.$
- $The\ probability\ of\ a\ success\ in\ each\ trial\ is\ the\ same.$

(3) $The\ mean\ of\ binomial\ distribution\ is\ given\ by\qquad \mu = np.$

(4) $The\ variance\ of\ binomial\ distribution\ is\ given\ by\qquad \sigma^2 = npq.$

(5) $The\ S.D.\ of\ binomial\ distribution\ is\ given\ by \quad \sigma = \sqrt{npq}.$

(6) $The\ recurrence\ relation\ for\ a\ Binomial\ Distribution$

$$P(r+1) = \frac{(n-r)}{(r+1)} \cdot \frac{p}{q} \cdot P(r)$$

TRIGONOMETRY

Associated Angles

How to find $\sin 960°, \cos 960°, \tan 960°$ ***etc.***?

(i) *Find out the quadrant.*

(ii) *If n is even multiple of $\frac{\pi}{2}$, then the t – ratio will be same as given otherwise the t – ratio will be its compliment.*

N. B.: *The rotation must be ACW always.*

sin	All
tan	cos

Compound Angles

(1) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(2) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(3) $\sin(A + B)\sin(A - B) = (\sin A)^2 - (\sin B)^2 = (\cos B)^2 - (\cos A)^2$

(4) $\cos(A + B)\ \cos(A - B) = (\cos A)^2 - (\sin B)^2 = (\cos B)^2 - (\sin A)^2$

(5) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(6) $\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

(7) $\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

Transformation of Sum and Products

(1) $\sin C + \sin D = 2 sin\frac{C+D}{2} cos\frac{C-D}{2}$

(2) $\sin C - \sin D = 2 cos\frac{C+D}{2} sin\frac{C-D}{2}$

(3) $\cos C + \cos D = 2 cos\frac{C+D}{2} cos\frac{C-D}{2}$

(4) $\cos C - \cos D = 2 sin\frac{C+D}{2} sin\frac{D-C}{2}$

(5) $2\sin A \cos B = \sin(A + B) + \sin(A - B)$

(6) $2\cos A \sin B = \sin(A + B) - \sin(A - B)$

(7) $2\cos A \cos B = \cos(A + B) + \cos(A - B)$

(8) $2\sin A \sin B = \cos(A - B) - \cos(A + B)$

Multiple Angles

(1) $\sin 2A = 2\sin A \cos A = \frac{2\tan A}{1+\tan^2 A}$

(2) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(3) $\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$

(4) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(5) $\tan A = \frac{1-\cos 2A}{\sin 2A} = \frac{\sin 2A}{1+\cos 2A}$

(6) $\sin 3A = 3\sin A - 4\sin^3 A$

(7) $\cos 3A = 4\cos^3 A - 3\cos A$

(8) $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Sub-multiple Angles

(1) $\sin 18° = \cos 72° = \frac{\sqrt{5}-1}{4}$

(2) $\sin 36° = \cos 54° = \frac{1}{4}\sqrt{10 - 2\sqrt{5}}$

(3) $\sin 54° = \cos 36° = \frac{\sqrt{5}+1}{4}$

(4) $\sin 72° = \cos 18° = \frac{1}{4}\sqrt{10 + 2\sqrt{5}}$

Trigonometric Equations

(1) (i) $If \sin\theta = 0,\ then\ \theta = n\pi$

(ii) $If \sin\theta = \sin\alpha,\ then\ \theta = n\pi + (-1)^n\alpha$

(2) (i) $\cos\theta = 0,\ then\ \theta = (2n+1)\frac{\pi}{2}$

(ii) $\cos\theta = \cos\alpha,\ then\ \theta = 2n\pi \pm \alpha$

(3) (i) $\tan\theta = 0,\ then\ \theta = n\pi$

(ii) $\tan\theta = \tan\alpha,\ then\ \theta = n\pi + \alpha$

Inverse Circular Function

Principle Value→ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(1) $\sin(\sin^{-1} x) = x$; $\cos(\cos^{-1} x) = x$; $\tan(\tan^{-1} x) = x$

(2) $\sin^{-1}(-x) = -\sin^{-1} x$; $\cos^{-1}(-x) = \pi - \cos^{-1} x$; $\tan^{-1}(-x) = -\tan^{-1} x$;

$\cot^{-1}(-x) = \pi - \cot^{-1} x$; $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$; $\sec^{-1}(-x) = \pi - \sec^{-1} x$.

(3) $\sin^{-1} x = \text{cosec}^{-1}\frac{1}{x}$; $\cos^{-1} x = \sec^{-1}\frac{1}{x}$; $\tan^{-1} x = \cot^{-1}\frac{1}{x}\ when\ x > 0$ &

$\tan^{-1} x = \cot^{-1}\frac{1}{x} - \pi\ when\ x < 0$.

(4) (i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\ (-1 \le x \le 1)$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\ (-\infty < x < \infty)$

(iii) $\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}\ (x \le -1,\ or,\ x \ge 1)$

(5) $\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right)\ [At\ principal\ value]$

(6) $\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)\ [At\ principal\ value]$

(7) $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{(1-x^2)(1-y^2)}\right)\ [At\ principal\ value]$

(8) $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

RELATION AND FUNCTION

RELATION

Cartesian Product:- $A \times B = \{(x, y): x \in A \text{ and } y \in B\}$.

Relation:-
$A\ relation\ R\ is\ a\ set\ of\ ordered\ pairs. A\ relation\ R\ from\ a\ set\ A\ to\ a\ set\ B\ is\ a\ subset\ of$

$A \times B$.

$Mathematically, we\ can\ write\ aRb\ \forall a, b \in R$.

Inverse Relation:-
$If\ A\ \&\ B\ be\ two\ sets\ and\ R\ be\ the\ relation\ from\ A\ to\ B,$

$then\ the\ inverse\ relation\ R^{-1}\ of\ R\ is\ a\ relation\ from\ B\ to\ A\ defined\ by$

$R^{-1} = \{(b, a): (a, b) \in R, a \in A, y \in B\}$.

Classification of Relations:-

(1) Reflexive Relation:-

$Let\ \rho\ be\ a\ relation\ on\ A.\ \rho\ is\ said\ to\ be\ reflexive\ if\ (a, a) \in \rho \forall a \in A$.

(2) Symmetric Relation:-

$A\ relation\ \rho\ on\ a\ set\ A\ is\ said\ to\ be\ symmetric\ if\ (a, b) \in \rho \Longrightarrow (b, a) \in \rho,$

$where, a, b \in A$.

$A\ relation\ \rho\ is\ said\ to\ be\ an\ anti - symmetric\ relation\ if\ (a, b) \in \rho\ \&\ (b, a) \in \rho$

$\Longrightarrow a = b\ when\ a, b \in A$.

(3) Transitive Relation:-

$A\ relation\ \rho\ on\ a\ set\ A\ is\ said\ to\ be\ transitive\ if\ (a. b) \in \rho\ \&\ (b, c) \in \rho$

$\Longrightarrow (a. c) \in \rho\ \forall a, b, c \in A$.

(4) Equivalent Relation:-
$A\ relation\ \rho\ is\ said\ to\ be\ an\ equivalent\ relation\ if\ \rho\ is\ reflexive, symmetric\ and\ transitive.$

FUNCTION

Real Function:-

$Let\ R\ be\ the\ set\ of\ all\ real\ numbers\ and\ let\ X\ and\ Y\ be\ any\ two\ non - empty\ subsets\ of\ R.$

$Then, a\ rule\ f\ which\ associates\ to\ each\ x \in X, a\ unique\ real\ number\ f(x) \in Y,$

$is\ called\ a\ real\ function\ from\ X\ to\ Y\ and\ we\ write, f: X \to Y.$

Constant Function:- $f(x) = c\ \forall x \in R$.

Identity Function:- $f(x) = x\ \forall x \in R$.

Modulus Function:- $f(x) = |x| = x, when\ x \geq 0$

$= -x, when\ x < 0.$

Reciprocal Function:- $f(x) = \frac{1}{x}.$

Signum Function:- $f(x) = \frac{|x|}{x} = 1, when\ x > 0$

$= 0, when\ x = 0$

$= -1, when\ x < 0.$

Square Root Function:- $f(x) = \sqrt{x}.$

Step/Box/Greatest Integer Function:- $f(x) = [x].$ $e.g. [2.01] = 2, [2.9] = 2.$

Exponential Function:- $f(x) = e^x.$

Logarithmic Function:- $f(x) = \log x.$

Polynomial Function:- $f(x) = a_0x^n + a_1x^{n-1} + \cdots \ldots \ldots \ldots + a_{n-1}x^1 + a_n.$

Rational Function:- $f(x) = \frac{p(x)}{q(x)}, where\ p(x)\ \&\ q(x)\ are\ polynomials\ and\ q(x) \neq 0.$

Trigonometric Function:- $f(x) = \sin x, \cos x, \tan x\ \ etc.$

Periodic Function:- $A\ function\ f(x)\ is\ said\ to\ be\ periodic\ with\ period\ T,$

$$if\ f(x+T) = f(x)\ \forall x.$$

Inverse Function:- $If\ f(y) = x, then\ y = f^{-1}(x).$

Even Function:- $A\ function\ f(x)\ is\ said\ to\ be\ even\ if\ f(-x) = f(x)\ \forall x.$

Odd Function:- $A\ function\ f(x)\ is\ said\ to\ be\ odd\ if\ f(-x) = -f(x)\ \forall x.$

DIFFERENTIATION

LIMIT:-

(1) Significant of $\lim_{x\to a} f(x) = l$

$\lim_{x\to a} f(x) = l$ *indicates that the variable x takes the values* $> a$ *or* $< a$ *but* $\neq a$.

l is the limiting value of $f(x)$.

(2) '$\lim_{x\to a+} f(x)$' *is called the right hand limit of* $f(x)$ *at* $x = a$.

(3) '$\lim_{x\to a-} f(x)$' *is called the left hand limit of* $f(x)$ *at* $x = a$.

(4) Existence of $\lim_{x\to a} f(x)$

$\lim_{x\to a} f(x)$ *exists if both* $\lim_{x\to a+} f(x)$ & $\lim_{x\to a-} f(x)$ *exists and*
$\lim_{x\to a+} f(x) = \lim_{x\to a-} f(x)$.

(5) <u>Fundamental Theorems:-</u>

(1) $\lim_{x\to a}[f1(x) \pm f2(x) \pm f3(x) \pm \cdots \ldots] = \lim_{x\to a} f1(x) + \lim_{x\to a} f2(x) + \ldots \ldots$

(2) $\lim_{x\to a}[f1(x). f2(x) \ldots ..] = \lim_{x\to a} f1(x). \lim_{x\to a} f2(x) \ldots ..$

(3) $\lim_{x\to a}\left[\frac{f(x)}{\emptyset(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} \emptyset(x)}$ $[where,\ \lim_{x\to a} \emptyset(x) \neq 0]$

(4) $\lim_{x\to a} \emptyset\{f(x)\} = \emptyset\{\lim_{x\to a} f(x)\}$

<u>(6) Formulae</u>

(1) $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$

(2) $\lim_{x\to 0} \frac{\sin x}{x} = 1$ $[x\ in\ radian]$

(3) $\lim_{x\to 0} \frac{\{1+x\}^n - 1}{x} = n$

(4) $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

(5) $\lim_{x\to 0} \frac{\log_e(1+x)}{x} = 1$

(6) $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$

(7) $\lim_{x\to \infty}\left(1 + \frac{1}{x}\right)^x = e$

(8) $\lim_{x\to 0} \frac{a^x - 1}{x} = \log_e a$ $(a > 0)$

(7) Some Substitution:-

(1) $If\ \lim_{x\to a} f(x)$, $then\ put\ x-a=h.\quad Then\ \lim_{x\to a} f(x) = \lim_{h\to 0} f(a+h)$

(2) $If\ \lim_{x\to \infty} f(x),\ \ then\ put\ \frac{1}{x}=z.\quad Then\ \lim_{x\to\infty} f(x) = \lim_{z\to 0} f\left(\frac{1}{z}\right)$

(8) L'Hospital's Rule

$If\ a\ function\ is\ in\ the\ form$

$\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, 0\times\infty, 0^0, \infty^0, 1^\infty\ at\ x=a\ then\ f(x) is\ indeterminate\ at\ x=a.$

Case I $\left(\frac{\mathbf{0}}{\mathbf{0}}\right)$

$If\ \lim_{x\to a} f(x) = 0\ \&\ \lim_{x\to a} g(x) = 0$

$then,\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}.$

Case II $\left(\frac{\infty}{\infty}\right)$

$If\ \lim_{x\to a} f(x) = \infty\ \&\ \lim_{x\to a} g(x) = \infty$

$then,\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}.$

CONTINUITY & DIFFERENTIABILITY

Continuity:-

f(x) is said to be continuous at x=a if

(i) f(a) is defined that means f(x) approaches to a definite finite value at x=a
(ii) $\lim_{x\to a} f(x) exists$
(iii) $\lim_{x\to a} f(x) = f(a)$

In brief, f(x) is said to be continuous at x=a if

$$\lim_{x\to a+} f(x) = \lim_{x\to a-} f(x) = f(a)$$

If one of the above condition fails, then f(x) is discontinuous at x=a.

If f(x) is continuous at every point in the interval $a \le x \le b$ then f(x) is continuous in the interval $a \le x \le b$.

Differentiability:-

Let the domain of definition of a function f(x) is D & $(a, b) \in D$.

f(x) is said to be differentiable at x=c ($where\ a < c < b$) iff

$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c}\ exists\ \&\ approaches\ to\ a\ finite\ value.$$

$Differentiability => Continuity$

$Discontinuity => Not\ differentiability$

DERIVATIVES

Definition:- Definition is the process of decreasing of a function.

Mathematically, $\frac{dy}{dx}[or, f'(x)] = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

Derivative from 1st principle

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivative at a point x=a

$$\left(\frac{dy}{dx}\right)_{x=a} = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$\lim_{h \to 0+} \frac{f(a+h)-f(a)}{h}$ is called the Right Hand Derivative of f(x) at x=a and expressed as Rf'(a) or, f'(a+).

$\lim_{h \to 0-} \frac{f(a-h)-f(a)}{-h}$ is called the Left Hand Derivative of f(x) at x=a and expressed as Lf'(a) or, f'(a-).

f'(a) exists if Rf'(a)=Lf'(a).

Formulae:-

(1) $\frac{d}{dx}(x^n) = nx^{n-1} \quad [n = ratonal\ no.]$

(2) $\frac{d}{dx}(e^x) = e^x$

(3) $\frac{d}{dx}(\log x) = \frac{1}{x}$

(4) $\frac{d}{dx}(a^x) = a^x \log_e a$

(5) $\frac{d}{dx}(\sin x) = \cos x$

(6) $\frac{d}{dx}(\cos x) = -\sin x$

(7) $\frac{d}{dx}(\tan x) = \sec^2 x$

(8) $\frac{d}{dx}(\cot x) = -cosec^2 x$

(9) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(10) $\frac{d}{dx}(cosec\ x) = -cosec\ x \cot x$

(11) $\frac{d}{dx}(c) = 0$

(12) $\frac{d}{dx}[cf(x)] = cf'(x)$

(13) $\frac{d}{dx}(e^{mx}) = me^{mx}$

(14) $\frac{d}{dx}(a^{mx}) = ma^{mx}\log_e a$

(15) $\frac{d}{dx}(\sin mx) = m\cos mx$

(16) $\frac{d}{dx}(\cos mx) = -m\sin mx$

(17) $\frac{d}{dx}(\tan mx) = m\sec^2 mx$

(18) $\frac{d}{dx}(\cot mx) = -mcosec^2 mx$

(19) $\frac{d}{dx}(\sec mx) = m\sec mx\tan mx$

(20) $\frac{d}{dx}(cosec\ mx) = -mcosec\ mx\cot mx$

(21) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

(22) $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$

(23) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

(24) $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$

(25) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

(26) $\frac{d}{dx}(\mathrm{cosec}^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$

(27) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

(28) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

<u>Chain Rule:-</u>

$\frac{dy}{dx} = \frac{dy}{dw}.\frac{dw}{dz}.\frac{dz}{dx}$

INTEGRATION

Definition:- Integration is regarded as the inverse of differentiation. It increases a function.

Mathematically, $\frac{d}{dx}$ [F(x)+c] = f(x)

& $\int f(x)dx$ = F(x)+c

where, c= integration constant.

Formulae:-

(1) $\int A\, f(x)dx = A\int f(x)dx$

(2) $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad [n \neq -1]$

(3) $\int \frac{1}{x}dx = \log x + c$

(4) $\int e^x\, dx = e^x + c$

(5) $\int a^x\, dx = \frac{a^x}{\log_e a} + c \qquad [a > 0\ \&\ a \neq -1]$

(6) $\int \sin x\ dx = -\cos x + c$

(7) $\int \cos x\ dx = \sin x + c$

(8) $\int \sec^2 x\ dx = \tan x + c$

(9) $\int \text{cosec}^2\, x\ dx = -\cot x + c$

(10) $\int \sec x \tan x\ dx = \sec x + c$

(11) $\int cosec\ x \cot x\ dx = -cosec\ x + c$

(12) $\int \tan x\ dx = \log|\sec x| + c$

(13) $\int \cot x\ dx = \log|\sin x| + c$

(14) $\int \sec x\ dx = \log|\sec x + \tan x| + c = \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

(15) $\int cosec\ x\ dx = \log|cosec\ x - \cot x| + c = \log|\tan\frac{x}{2}| + c$

(16) $\int e^{mx}\, dx = \frac{e^{mx}}{m} + c$

(17) $\int a^{mx}\, dx = \frac{a^{mx}}{m\log_e a} + c$

(18) $\int \sin mx\ dx = -\frac{\cos mx}{m} + c$

(19) $\int \cos mx\ dx = \frac{\sin mx}{m} + c$

(20) $\int \sec^2 mx\ dx = \frac{\tan mx}{m} + c$

(21) $\int \text{cosec}^2\, mx\ dx = -\frac{\cot mx}{m} + c$

(22) $\int \sec mx \tan mx \; dx = \frac{\sec mx}{m} + c$

(23) $\int cosec\; mx \cot mx \; dx = -\frac{\text{cosec}\, mx}{m} + c$

(24) $\int \tan mx \; dx = \frac{1}{m} \log|\sec mx| + c$

(25) $\int \cot mx \; dx = \frac{1}{m} \log|\sin mx| + c$

(26) $\int \sec mx \; dx = \frac{1}{m} \log|\sec mx + \tan mx| + c = \frac{1}{m} \log\left|\tan\left(\frac{\pi}{4} + \frac{mx}{2}\right)\right| + c$

(27) $\int cosec\; mx \; dx = \frac{1}{m} \log|cosec\; mx - \cot mx| + c = \frac{1}{m} \log\left|\tan\frac{mx}{2}\right| + c$

(28) $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\frac{x}{a} + c \quad [a \neq 0]$

(29) $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c \quad [a \neq 0]$

(30) $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c \quad [a \neq 0]$

(31) $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a} + c = -\cos^{-1}\frac{x}{a} + c$

(32) $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1}\frac{|x|}{a} + c = -\frac{1}{a} \text{cosec}^{-1}\frac{|x|}{a} + c$

Integration by parts:-

$$\int uv\, dx = u \int v\, dx - \int \left[\frac{du}{dx} \int v\, dx\right] dx$$

where, u is the 1st function of x & v is the 2nd function of x

How to choose 1st & 2nd function:-

Use this rule--------ILATE

where,

I= inverse function

L= logarithmic function

A= algebraic function

T= trigonometric function

E= exponential function

(33) $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx) + c = \frac{e^{ax}}{\sqrt{a^2+b^2}} \sin\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

(34) $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx) + c = \frac{e^{ax}}{\sqrt{a^2+b^2}} \cos\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

(35) $\int \sqrt{x^2 \pm a^2}\; dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \log\left|x + \sqrt{x^2 \pm a^2}\right| + c$

(36) $\int \sqrt{a^2 - x^2}\; dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + c$

Definite Integral:-

(1)Definite integral as the limit of a sum

$\int_a^b f(x)dx = \lim_{h\to 0} h\sum_{r=0}^{n-1} f(a+rh) = \lim_{h\to 0} h\sum_{r=1}^{n} f(a+rh)$ [where, nh=b-a]

(2)Fundamental theorem of definite integral

If f(x) is integrable in a $\leq x \leq b$ & f(x) = $\emptyset'(x)$ then

$\int_a^b f(x)dx = \emptyset(b) - \emptyset(a)$

Remember: There is no integration constant in definite integrals.

Formulae on Definite Integrals:-

(1) $\int_a^b f(x)dx = \int_a^b f(z)dz$

(2) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(3) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, a < c < b

(4) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(5) $\int_0^{na} f(x)dx = n\int_0^a f(x)dx$ if f(a+x) = f(x)

(6) $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, $if\ f(2a-x) = f(x)$ = 0, $if\ f(2a-x) = -f(x)$

(7) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, $if\ f(-x) = f(x)\ i.e.\ f(x)\ is\ an\ even\ function$

= 0, $if\ f(-x) = -f(x)\ i.e.\ f(x)\ is\ an\ odd\ function.$

(8) $\int_a^b f(x)dx = \int_a^b f(a+b-x)$

APPLIED CALCULUS

Tangent and Normal

(1) $The\ equation\ of\ a\ tangent\ to\ a\ curve\ y = f(x)\ at\ a\ point\ P(x_1, y_1)\ is\ given\ by$

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} (x - x_1)$$

(2) $The\ equation\ of\ a\ normal\ to\ a\ curve\ y = f(x)\ at\ a\ point\ P(x_1, y_1)\ is\ given\ by$

$$y - y_1 = -\left(\frac{dx}{dy}\right)_{(x_1,y_1)} (x - x_1)$$

(3) $The\ angle\ of\ intersection\ of\ two\ curves\ y = f(x)\ and\ y = g(x)\ is\ given\ by$

$$\tan\alpha = \frac{f'(x) - g'(x)}{1 + f'(x)g'(x)}$$

Monotonic Functions

Increasing Function:- $A\ function\ f(x)\ is\ said\ to\ be\ an\ increasing\ function\ in\ (a, b)\ if$

$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)\ \forall\ x_1, x_2 \in (a, b).$

Decreasing Function:- $A\ function\ f(x)\ is\ said\ to\ be\ an\ decreasing\ function\ in\ (a, b)\ if$

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)\ \forall\ x_1, x_2 \in (a, b).$

Monotonic Function:-
$A\ function\ is\ said\ to\ be\ monotonic\ in\ an\ interval\ if\ it\ is\ either\ increasing\ or\ decreasing\ in\ that\ interval.$

Theorem 1:- $If\ f(x)\ be\ a\ function\ continuous\ on\ [a, b]\ and\ differentiable\ on\ (a, b)\ then$

$f'(x) > 0\ \forall\ x \in (a, b) \Rightarrow f(x)\ is\ increasing\ in\ (a, b)$.

Theorem 2:- $If\ f(x)\ be\ a\ function\ continuous\ on\ [a, b]\ and\ differentiable\ on\ (a, b)\ then$

$f'(x) < 0\ \forall\ x \in (a, b) \Rightarrow f(x)\ is\ decreasing\ in\ (a, b)$.

Maxima and Minima

Working Rule (2nd Derivative Test)

(i) Find f'(x).

(ii) Solve f'(x)=0. Let its roots be a,b,c etc. Then, these are the candidates for maxima & minima. Let x=c be one of its points.

(iii) Find f''(c).

Now, if f''(c)<0, then x=c is a point of local maxima

if f''(c)>0, then x=c is a point of local minima

if f''(c)=0, then use the 1st derivative test.

Working Rule (1st Derivative Test)

(i) Find f'(x).

(ii) Solve f'(x)=0. Let its roots be a,b,c etc. Then, these are the candidates for maxima & minima. Let x=c be one of its points.

(iii) Determine the sign of f'(x) for values of x slightly < c and that for values of x slightly > c.

Area of Bounded Regions

Theorem 1:-

Let f(x) be continuous & finite in [a,b]. Then,

The area bounded by the curve y=f(x), the X-axis & the ordinates x=a & x=b, is equal to $\int_a^b ydx$.

Mathematically,

$$\int_a^b y\,dx = \int_a^b f(x)dx$$

Theorem 2:-

Let f(y) be continuous & finite in [a,b]. Then,

The area bounded by the curve x=f(y), the Y-axis and the absicca y=c, y=d is equal to $\int_c^d x\,dy$.

Mathematically,

$$\int_c^d x\,dy = \int_c^d f(y)dy$$

Rolle's and Lagrange's Theorem

Rolle's Theorem:-

Let f(x) be a real valued function, defined in the closed interval [a,b] such that

(i) f(x) is continuous in [a,b]
(ii) f(x) is differentiable in [a,b]
(iii) f(a)=f(b)

Then, there exists a real number c in (a,b) such that f'(c)=0.

Lagrange's MVT:-

Let f(x) be a real function such that

(i) f(x) is continuous in [a,b]
(ii) f(x) is differentiable in (a,b)

Then, there exists a real number c ∈ (a,b) such that $f'(c) = \frac{f(b)- f(a)}{b-a}$

Dynamics

(1) The displacement x, velocity v & acceleration f at t times for a particle moving in a straight line.

Then, $v = \frac{dx}{dt}$ & $f = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$

(2) Formulae of motion along a straight line of a particle with uniform acceleration

(i) $v = u + ft$

(ii) $s = ut + \frac{1}{2}ft^2$

(iii) $v^2 = u^2 + 2fs$

(iv) $s_t = u + \frac{1}{2}f(2t-1)$

where, u=initial velocity of the particle v=final velocity of the particle

s=distance covered by the particle upto t times

s_t =distance covered by the particle at t-th sec

(3) Average velocity of a particle at any instant= $\frac{1}{2}$ (Initial velocity + Final velocity).

$$\therefore V = \frac{1}{2}(u + v)$$

DIFFERENTIAL EQUATION

Serial no.	Form	Substitution	Full Differential
1.	$dx \pm dy$	$x \pm y = v$	$d(x \pm y)$
2.	$x\,dx + y\,dy$	$x^2 + y^2 = v$	$\frac{1}{2}d(x^2 + y^2)$
3.	$x\,dy + y\,dx$	$xy = v$	$d(xy)$
4.	$x\,dy - y\,dx$	$y = vx$	$x^2 d(\frac{y}{x})$
5.	$ydx - xdy$	$x = vy$	$y^2 d(\frac{x}{y})$
6.	$\frac{xdy - ydx}{xy}$	$\log\left\lvert\frac{y}{x}\right\rvert = v$	$d(\log\left\lvert\frac{y}{x}\right\rvert)$
7.	$\frac{xdy - ydx}{x^2 + y^2}$	$\tan^{-1}\left(\frac{y}{x}\right) = v$	$d(\tan^{-1}\frac{y}{x})$
8.	$\frac{ydx - xdy}{x^2 + y^2}$	$\tan^{-1}\left(\frac{x}{y}\right) = v$	$d(\tan^{-1}\frac{x}{y})$
9.	$\frac{xdy - ydx}{\sqrt{1 - x^2y^2}}$	$\sin^{-1}(xy) = v$	$d(\sin^{-1} xy)$
10.	$\frac{xdx + ydy}{x^2 + y^2}$	$x^2 + y^2 = v$	$\frac{1}{2}d(\log\lvert x^2 + y^2\rvert)$

Order of Differential Equation:-

Highest order derivative in a differential equation.

Degree of Differential Equation:-

Power of highest order derivative in a differential equation.

Techniques of solving a Differential Equation

Type 1:- $\frac{dy}{dx} = f(x)$

$dy = f(x)dx$

Type 2:- $\frac{dy}{dx} = P(x)Q(y)$

$$\frac{dy}{Q(y)} = P(x)dx$$

Type 3:- $\boldsymbol{\frac{dy}{dx} = P(x+y)}$

Let, $x + y = v\ \ so\ that\ \left(1 + \frac{dy}{dx}\right) = \frac{dv}{dx}$

Type 4:- $\boldsymbol{\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}}$ $\quad [\boldsymbol{Homogeneous\ function}]$

Let, $y = vx\ \ so\ that\ \frac{dy}{dx} = v + x\frac{dv}{dx}$

Type 5:- $\boldsymbol{\frac{dy}{dx} + P(x)y = Q(x)}$

$(i) Find\ I.F. = e^{\int P(x)dx}$

$(ii) The\ solution\ is\ \ y \times (I.F.) = \int [Q(x) \times (I.F.)]dx + C$

Type 6:- $\boldsymbol{\frac{dx}{dy} + P(y)x = Q(y)}$

$(i) Find\ I.F. = e^{\int P(y)dy}$

$(ii) The\ solution\ is\ \ x \times (I.F.) = \int [Q(y) \times (I.F.)]dy + C$

CO-ORDINATE GEOMETRY

Cartesian & Polar Co-ordinates

(1) **Basic Form:-**

Cartesian co-ordinate → (x, y)

Polar co-ordinate → (r, θ)

(2) $x = r\cos\theta,\ y = r\sin\theta$

& $r = \sqrt{x^2 + y^2},\ \theta = \tan^{-1}\frac{y}{x}$

(3) *Distance between* $P(x_1, y_1)$ & $Q(x_2, y_2)$ $is\ PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(4) $The\ co-ordinate\ of\ R\ which\ divides\ PQ\ internally\ (or, externally)\ in\ the\ ratio\ m{:}n\ is$

$\left(\frac{mx_2 \pm nx_1}{m \pm n}, \frac{my_2 \pm ny_1}{m \pm n}\right)$

(5) $The\ mid-point\ of\ P(x_1, y_1)\&\ Q(x_2, y_2)\ is\ \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(6) $The\ centroid\ of\ \Delta ABC\ with\ vertices\ A(x_1, y_1), B(x_2, y_2)\ \&\ C(x_3, y_3)\ is\ \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

(7) $The\ area\ of\ \Delta ABC\ is$

$= \frac{1}{2}|y_1(x_2 - x_3) + y_2(x_3 - x_1) + y_3(x_1 - x_2)|\ unit^2$

$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|\ unit^2$

(8) $Three\ points\ A(x_1, y_1), B(x_2, y_2)\ \&\ C(x_3, y_3)\ will\ be\ collinear\ if$

$y_1(x_2 - x_3) + y_2(x_3 - x_1) + y_3(x_1 - x_2) = 0$

Straight Line

(1) $The\ gradient\ (or, slope)\ of\ a\ straight\ line\ in\ the\ direction\ of\ positive\ X-axis$ $with\ angle\ \theta\ is\ m = \tan\theta.$

(2) $The\ slope\ of\ a\ straight\ line\ joining\ the\ points\ A(x_1, y_1)\ \&\ B(x_2, y_2)\ is\ m = \frac{y_1 - y_2}{x_1 - x_2}$

(3) (i) $y = 0\ \rightarrow equation\ of\ X-axis$

(ii) $y = b\ \rightarrow \parallel X-axis$

(iii) $x = 0\ \rightarrow equation\ of\ Y-axis$

(iv) $x = a\ \rightarrow \parallel Y-axis$

(4) **Forms of straight line**

(i) $General\ form{:}\ ax + by + c = 0$ $\quad [a\ \&\ b\ must\ not\ be\ zero\ simultaneously]$

(ii) $Slope-intercept\ form{:}\ y = mx + c$

(iii) $Point-slope\ form{:}\ y - y_1 = m\ (x - x_1)$

(iv) $Two - point\ form$: $\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$

(v) $Intercept\ form$: $\frac{x}{a} + \frac{y}{b} = 1$

(vi) $Symmetrical\ form$: $\frac{y-y_1}{\sin\theta} = \frac{x-x_1}{\cos\theta} = r$

(vii) $Normal\ form$: $x\cos\alpha + y\sin\alpha = p$, $(p > 0)$

$where, m = slope$

$c = y - intercept$

$a = x - intercept$

$b = y - intercept$

$\theta = inclination\ of\ the\ straight\ line$

$r = distance\ between\ (x, y)\ \&\ (x_1, y_1)$

$\alpha =$ $the\ inclination\ of\ the\ straight\ line\ with\ the\ \perp from\ origin\ in\ the$ $direction\ of\ positive\ X - axis.$

$p = \perp distance\ of\ the\ straight\ line\ from\ origin$

(5) $The\ equation\ of\ the\ straight\ line\ passes\ through\ the\ intersecting\ point\ of$ $a_1x + b_1y + c_1 = 0\ \&\ a_2x + b_2y + c_2 = 0\ is\ a_1x + b_1y + c_1 + k(a_2x + b_2y + c_2) = 0$ $[where, k \neq 0, \infty]$.

(6) $Three\ lines\ will\ be\ concurrent\ if\ one\ of\ these\ three\ lines\ passes\ through\ the$ $intersecting\ point\ of\ other\ two\ lines.$

(7) $The\ angle\ between\ two\ straight\ lines\ y = m_1x + c_1\ \&\ y = m_2x + c_2\ is\ \varphi.$

$$Then, \tan\varphi = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

(8) $If\ two\ straight\ lines\ y = m_1x + c_1\ \&\ y = m_2x + c_2\ are\ \parallel, then\ m_1 = m_2$

(9) $The\ equation\ of\ \parallel line\ of\ the\ straight\ line\ ax + by + c = 0\ is\ ax + by + k = 0$ $[where, k = constant]$.

(10) $If\ two\ straight\ lines\ y = m_1x + c_1\ \&\ y = m_2x + c_2\ are\ \perp, then\ m_1m_2 = -1$

(11) $The\ equation\ of\ \perp on\ the\ straight\ line\ ax + by + c = 0\ is\ bx - ay + k = 0$ $[where, k = constant]$.

(12)
$The\ two\ points\ P(x_1, y_1)\ \&\ Q(x_2, y_2)\ will\ be\ situated\ in\ the\ same\ side\ of\ the\ straight\ line$ $ax + by + c = 0\ if\ the\ expressions\ (ax_1 + by_1 + c)\ \&\ (ax_2 + by_2 + c)\ have\ same\ sign.$ $Otherwise, P\ \&\ Q\ will\ be\ situated\ in\ the\ opposite\ side\ of\ the\ line\ ax + by + c = 0.$

(13) $If\ c\ \&\ (ax_1 + by_1 + c)\ have\ same\ sign$, $then$
$P(x_1, y_1)\ will\ be\ situated\ in\ the\ side\ of\ the\ line\ ax + by + c = 0\ in\ which$
$the\ origin\ (0,0)\ exists.$

$Otherwise, P(x_1, y_1)$ *will be situated in the opposite side of the line* $ax + by + c = 0$ *in which the origin* $(0,0)$ *exists.*

(14) *The* $\perp$ *distance from the external point* $P(x_1, y_1)$ *to the line* $ax + by + c = 0$ *is* $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

(15) *The equation of bisectors of angles between two given straight lines* $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ *are*

$$\frac{a_1x + b_1y + c_1}{\sqrt{{a_1}^2 + {b_1}^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{{a_2}^2 + {b_2}^2}}$$

If c_1 & c_2 *have same sign, then* $(+ve)$ *sign is taken, otherwise* $(-ve)$ *sign is taken.*

(16) *The distance between two* $\parallel$ *lines is* $\frac{|c-k|}{\sqrt{a^2+b^2}}$.

Circle

(1) **Forms of Circle:-**

(i) $x^2 + y^2 = a^2$; *centre* $\rightarrow (0,0)$, *radius* $\rightarrow a$

(ii) *Parametric equation*: $x = a\cos\theta$; $y = a\sin\theta$

(iii) *Centre – radius form*: $(x - \alpha)^2 + (y - \beta)^2 = a^2$; *centre* $\rightarrow (\alpha, \beta)$, *radius* $\rightarrow a$

(iv) *General form*: $x^2 + y^2 + 2gx + 2fy + c = 0$; *centre* $\rightarrow (-g, -f)$, *radius* $\rightarrow \sqrt{g^2 + f^2 - c}$, *x – intercept* $\rightarrow 2\sqrt{g^2 - c}$, *y – intercept* $\rightarrow 2\sqrt{f^2 - c}$.

If $c = 0$, *the circle passes through the origin*;

if $f = 0$, *its centre lies on the X – axis, if* $g = 0$, *its centre lies on the Y – axis.*

(2) *Equation of the circle with the join of two points* (x_1, y_1) & (x_2, y_2) *as diameter is*

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

(3) *Any point on the circle* $x^2 + y^2 = a^2$ *is* $(a\cos\theta, b\sin\theta)$.

(4) *The equation of the concentric circle with the circle* $x^2 + y^2 + 2gx + 2fy + c = 0$ *is*

$$x^2 + y^2 + 2gx + 2fy + c' = 0$$

(5) *Let's consider two circles – – – – – –*

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots.. (ii)$

(i) *The equation of the circle passes through the intersecting points of* (i) & (ii) *is* $x^2 + y^2 + 2g_1x + 2f_1y + c_1 + k(x^2 + y^2 + 2g_2x + 2f_2y + c_2) = 0 \quad [k \neq -1]$

(ii) *The equation of the common chord of* (i) & (ii) *is* $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$

(iii) *The condition that the two circle* (i) & (ii) *will cut orthogonally is* $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

(6) $The\ equation\ of\ any\ circle\ passing\ through\ the\ intersection\ of\ the\ circle$

$x^2 + y^2 + 2gx + 2fy + c = 0\ and\ the\ line\ ax + by + c = 0\ is$

$$x^2 + y^2 + 2gx + 2fy + c + k(ax + by + c) = 0\ \ [where\ k = constant].$$

(7) $The\ position\ of\ a\ point\ P(x_1, y_1)\ w.r.t.the\ circle\ x^2 + y^2 + 2gx + 2fy + c = 0$

$If\ {x_1}^2 + {y_1}^2 + 2gx_1 + 2fy_1 + c > 0$, $then\ outside\ the\ circle$

$= 0$, $then\ on\ the\ circle$

< 0, $then\ inside\ the\ circle$

(8) (i) $The\ equation\ of\ tangent\ to\ the\ circle\ x^2 + y^2 = a^2\ at\ P(x_1, y_1)\ is$

$xx_1 + yy_1 = a^2$ & $its\ length\ is$

$\sqrt{{x_1}^2 + {y_1}^2 - a^2}.$

(ii) $The\ equation\ of\ tangent\ to\ the\ circle\ x^2 + y^2 + 2gx + 2fy + c = 0\ at\ P(x_1, y_1)\ is$

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$ & $its\ length\ is$

$\sqrt{{x_1}^2 + {y_1}^2 + 2gx_1 + 2fy_1 + c}.$

(9) (i) $The\ equation\ of\ normal\ to\ the\ circle\ x^2 + y^2 = a^2\ at\ P(x_1, y_1)\ is$

$xy_1 - x_1y = 0.$

(ii) $The\ equation\ of\ normal\ to\ the\ circle\ x^2 + y^2 + 2gx + 2fy + c = 0\ at\ P(x_1, y_1)\ is$

$(x_1 + g)(y - y_1) = (y_1 + f)(x - x_1).$

(10) (i) $If\ the\ circle\ (x - \alpha)^2 + (y - \beta)^2 = a^2\ touches\ X - axis, then\ its\ equation\ will\ be$

$(x - \alpha)^2 + (y - a)^2 = a^2.$

(ii) $If\ the\ circle\ (x - \alpha)^2 + (y - \beta)^2 = a^2\ touches\ Y - axis, then\ its\ equation\ will\ be$

$(x - a)^2 + (y - \beta)^2 = a^2.$

(iii) $If\ the\ circle\ (x - \alpha)^2 + (y - \beta)^2 = a^2\ touches\ both\ the\ axes, then\ its\ eqn\ will\ be$

$(x - a)^2 + (y - a)^2 = a^2.$

(11) $Two\ circles\ will\ touch\ each\ other\ externally\ if$

$the\ distance\ between\ their\ centres = r_1 + r_2.$

(12) $Two\ circles\ will\ touch\ each\ other\ internally\ if$

$the\ distance\ between\ their\ centres = r_1 - r_2.$

Parabola

$a = distance\ between\ vertex\ to\ focus$ & $a > 0$

Equation	Vertex	Axis	Focus	Length of Latus Rectum	Equation of Directrix	Vertices of Latus Rectum
$y^2 = 4ax$	(0,0)	$+ve\ X - axis$	$(a, 0)$	$4a$	$x + a = 0$	$(a, \pm 2a)$
$y^2 = -4ax$	(0,0)	$-ve\ X - axis$	$(-a, 0)$	$4a$	$x - a = 0$	$(-a, \pm 2a)$

$x^2 = 4ay$	$(0,0)$	$+ve\ Y - axis$	$(0, a)$	$4a$	$y + a = 0$	$(\pm 2a, a)$
$x^2 = -4ay$	$(0,0)$	$-ve\ Y - axis$	$(0, -a)$	$4a$	$y - a = 0$	$(\pm 2a, -a)$
$(y-\beta)^2 = 4a(x-\alpha)$	(α, β)	$\parallel X - axis$	$(a + \alpha, \beta)$	$4a$	$x + a = \alpha$	$(\alpha + a, \beta \pm 2a)$
$(x-\alpha)^2 = 4a(y-\beta)$	(α, β)	$\parallel Y - axis$	$(\alpha, a + \beta)$	$4a$	$y + a = \beta$	$(\alpha \pm 2a, \beta + a)$

(1) $The\ parabola\ x = ay^2 + by + c\ (a \neq 0)\ is\ \parallel to\ X - axis$

(2) $The\ parabola\ y = px^2 + qx + r\ (p \neq 0)\ is\ \parallel to\ Y - axis$

(3) $The\ parametric\ of\ y^2 = 4ax\ is\ (at^2, 2at)$

(4) $The\ position\ of\ a\ point\ P(x_1, y_1)\ w.r.t\ y^2 = 4ax$

$If\ {y_1}^2 - 4ax_1 > 0, then\ outside\ the\ parabola$

$= 0, then\ on\ the\ parabola$

$< 0, then\ inside\ the\ parabola$

(5) $The\ equation\ of\ tangent\ of\ the\ parabola\ y^2 = 4ax\ at\ P(x_1, y_1)\ is$

$yy_1 = 2a(x + x_1).$

(6) $The\ equation\ of\ normal\ of\ the\ parabola\ y^2 = 4ax\ at\ P(x_1, y_1)\ is$

$y_1(x - x_1) + 2a(y - y_1) = 0.$

Ellipse

(1) $If\ P(x, y)\ be\ a\ point\ on\ the\ foci\ S\ \&\ S',\ then\ SP = a - ex; S'P = a + ex; SP + S'P = 2a.$

(2) $The\ equation\ of\ auxiliary\ circle\ of\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a^2 > b^2)\ is\ x^2 + y^2 = a^2.$

(3) $Any\ point\ on\ the\ ellipse\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a^2 > b^2)\ is\ (a\cos\theta, b\sin\theta)\ .$

(4) $The\ position\ of\ a\ point\ P(x_1, y_1)\ w.r.t\ the\ ellipse\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a^2 > b^2)$

$If\ \frac{{x_1}^2}{a^2} + \frac{{y_1}^2}{b^2} - 1 > 0\ , then\ outside\ the\ ellipse$

$= 0\ , then\ on\ the\ ellipse$

$< 0\ , then\ inside\ the\ ellipse$

(5) $The\ equation\ of\ tangent\ of\ the\ ellipse\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ at\ P(x_1, y_1)\ is$

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1.$

(6) $The\ equation\ of\ normal\ of\ the\ ellipse\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ at\ P(x_1, y_1)\ is$

$b^2x_1(y - y_1) = a^2y_1(x - x_1).$

Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a^2 > b^2)$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1\ (a^2 > b^2)$	$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1\ (a^2 > b^2)$	$\frac{(x-\alpha)^2}{b^2} + \frac{(y-\beta)^2}{a^2} = 1\ (a^2 > b^2)$
Centre	$(0,0)$	$(0,0)$	(α, β)	(α, β)

Vertices	$(\pm a, 0)$	$(0, \pm a)$	$(\alpha \pm a, \beta)$	$(\alpha, \beta \pm a)$
Foci	$(\pm ae, 0)$	$(0, \pm ae)$	$(\alpha \pm ae, \beta)$	$(\alpha, \beta \pm ae)$
Eccentricity (e)	$e = \sqrt{1 - \frac{b^2}{a^2}}$	$e = \sqrt{1 - \frac{b^2}{a^2}}$	$e = \sqrt{1 - \frac{b^2}{a^2}}$	$e = \sqrt{1 - \frac{b^2}{a^2}}$
Length of Latus Rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Length of Major axis	$2a$	$2a$	$2a$	$2a$
Length of Minor axis	$2b$	$2b$	$2b$	$2b$
Equation of Major axis	$y = 0$	$x = 0$	$y = \beta$	$x = \alpha$
Equation of Minor axis	$x = 0$	$y = 0$	$x = \alpha$	$y = \beta$
Equation of the Directrix	$x \pm \frac{a}{e} = 0$	$y \pm \frac{a}{e} = 0$	$x \pm \frac{a}{e} = \alpha$	$y \pm \frac{a}{e} = \beta$
Vertices of Latus Rectum	$\left(ae, \pm \frac{b^2}{a}\right)$ $\left(-ae, \pm \frac{b^2}{a}\right)$	$\left(\pm \frac{b^2}{a}, ae\right)$ $\left(\pm \frac{b^2}{a}, -ae\right)$	$\left(\alpha + ae, \beta \pm \frac{b^2}{a}\right)$ $\left(\alpha - ae, \beta \pm \frac{b^2}{a}\right)$	$\left(\alpha \pm \frac{b^2}{a}, \beta + ae\right)$ $\left(\alpha \pm \frac{b^2}{a}, \beta - ae\right)$
Parametric	$(a\cos\theta, b\sin\theta)$ $-\pi < \theta \le \pi$	$(b\cos\theta, a\sin\theta)$ $-\pi < \theta \le \pi$	$(\alpha + a\cos\theta, \beta + b\sin\theta)$ $-\pi < \theta \le \pi$	$(\alpha + b\cos\theta, \beta + a\sin\theta)$ $-\pi < \theta \le \pi$

Hyperbola

Form	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1$
Centre	$(0,0)$	$(0,0)$	(α, β)
Vertices	$(\pm a, 0)$	$(0, \pm a)$	$(\alpha \pm a, \beta)$
Foci	$(\pm ae, 0)$	$(0, \pm ae)$	$(\alpha \pm ae, \beta)$
Eccentricity (e)	$e = \sqrt{1 + \frac{b^2}{a^2}}$	$e = \sqrt{1 + \frac{b^2}{a^2}}$	$e = \sqrt{1 + \frac{b^2}{a^2}}$
Length of Latus Rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Length of Transverse axis	$2a$	$2a$	$2a$
Length of Conjugate axis	$2b$	$2b$	$2b$
Equation of Transverse axis	$y = 0$	$x = 0$	$y = \beta$
Equation of Conjugate axis	$x = 0$	$y = 0$	$x = \alpha$
Equation of Directrix	$x \pm \frac{a}{e} = 0$	$y \pm \frac{a}{e} = 0$	$x \pm \frac{a}{e} = \alpha$
Vertices of Latus Rectum	$\left(ae, \pm \frac{b^2}{a}\right)$	$\left(\pm \frac{b^2}{a}, ae\right)$	$\left(\alpha + ae, \beta \pm \frac{b^2}{a}\right)$

	$\left(-ae, \pm\frac{b^2}{a}\right)$	$\left(\pm\frac{b^2}{a}, -ae\right)$	$\left(\alpha - ae, \beta \pm \frac{b^2}{a}\right)$
Parametric	$(a\sec\theta, b\tan\theta)$	$(b\tan\theta, a\sec\theta)$	$(\alpha + a\sec\theta, \beta + b\tan\theta)$

(1) $If\ P(x, y)\ be\ a\ point\ on\ the\ foci\ S\ \&\ S',\ then\ SP = ex - a; S'P = ex + a; SP - S'P = 2a.$

(2) $Any\ point\ on\ the\ hyperbola\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\ is\ (a\sec\theta, b\tan\theta)$.

(3) $The\ rectangular\ hyperbola\ is\ x^2 - y^2 = a^2.$

$Its\ transverse\ axis\ \rightarrow X - axis; Conjugate\ axis\ \rightarrow Y - axis$

$Eccentricity\ \rightarrow \sqrt{2}\ ; Length\ of\ T - axis\ \&\ C - axis\ \rightarrow 2a.$

(4) $The\ two\ hyperbola\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\ \&\ \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1\ are\ conjugate\ to\ each\ other.$

$Eccentricity\ \rightarrow b^2 = a^2({e_1}^2 - 1)\ ; a^2 = b^2({e_2}^2 - 1).$

(5) $The\ position\ of\ a\ point\ P(x_1, y_1)\ w.r.t.the\ hyperbola\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$If\ \frac{{x_1}^2}{a^2} - \frac{{y_1}^2}{b^2} - 1 > 0$, $then\ inside\ the\ hyperbola$

$= 0$, $then\ on\ the\ hyperbola$

< 0 , $then\ outside\ the\ hyperbola$

(6) $The\ equation\ of\ tangent\ of\ the\ hyperbola\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\ at\ P(x_1, y_1)\ is$

$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1.$

(7) $The\ equation\ of\ normal\ of\ the\ hyperbola\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\ at\ P(x_1, y_1)\ is$

$b^2x_1(y - y_1) + a^2y_1(x - x_1) = 0.$

Classification of Curves:-

$Let\ us\ consider\ a\ second\ degree\ curve\ in\ x, y\ as$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \dots (i)$ $[at\ least\ one\ of\ a, h, b\ is\ non - zero\ constant]$

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$$

Identification of Curves:-

$The\ above\ equation\ (i)\ represents$

$(i)\ A\ parabola\ if\ \Delta \neq 0\ \&\ h^2 = ab.$

$(ii)\ An\ ellipse\ if\ \Delta \neq 0\ \&\ h^2 < ab.$

$(iii)\ A\ hyperbola\ if\ \Delta \neq 0\ \&\ h^2 > ab.$

$(iv)\ A\ pair\ of\ straight\ lines\ if\ \Delta = 0\ \&\ h^2 \geq ab.$

$(v)\ A\ unique\ point\ if\ \Delta = 0\ \&\ h^2 < ab.$

(vi) A $circle$ if $a = b \neq 0, h = 0$ & $(g^2 + f^2 - ac) > 0$

$i.e., a = b > 0, h = 0$ & $(a^2c - ag^2 - af^2) < 0$.

3-D GEOMETRY

Fundamentals of 3-D Geometry

(1) $Distance\ between\ two\ points\ P(x_1, y_1, z_1)\ and\ Q(x_2, y_2, z_2)\ is$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$

(2) $The\ co-ordinate\ of\ the\ point\ R\ which\ divides\ the\ join\ of\ the\ points$

$P(x_1, y_1, z_1)\ and\ Q(x_2, y_2, z_2)\ internally\ in\ the\ ratio\ m\colon n\ are$

$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right).$

(3) $The\ co-ordinate\ of\ the\ point\ R\ which\ divides\ the\ join\ of\ the\ points$

$P(x_1, y_1, z_1)\ and\ Q(x_2, y_2, z_2)\ externally\ in\ the\ ratio\ m\colon n\ are$

$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right).$

(4) $The\ co-ordinates\ of\ the\ mid\ point\ of\ PQ\ are$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$

(5) $If\ A(x_1, y_1, z_1), B(x_2, y_2, z_2)\ and\ C(x_3, y_3, z_3)\ be\ the\ vertices\ of\ \Delta ABC,\ then\ the\ co-ordinates\ of\ the\ centroid\ G\ of\ \Delta ABC\ are$

$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right).$

(6) (i) $Equation\ of\ XY-plane\ is\ z = 0.$

(ii) $Equation\ of\ YZ-plane\ is\ x = 0.$

(iii) $Equation\ of\ ZX-plane\ is\ y = 0.$

(7) (i) $If\ a\ point\ lies\ on\ XY-plane, then\ its\ co-ordinates\ are\ (x, y, 0).$

(ii) $If\ a\ point\ lies\ on\ YZ-plane, then\ its\ co-ordinates\ are\ (0, y, z).$

(iii) $If\ a\ point\ lies\ on\ ZX-plane, then\ its\ co-ordinates\ are\ (x, 0, z).$

(8) (i) $Direction\ cosines\ of\ X-axis\ are\ 1,0,0.$

(ii) $Direction\ cosines\ of\ Y-axis\ are\ 0,1,0.$

(iii) $Direction\ cosines\ of\ Z-axis\ are\ 0,0,1.$

(9) $If\ l, m, n\ be\ the\ direction\ cosines\ of\ any\ line,\ then\ l^2 + m^2 + n^2 = 1.$

(10) $l = \cos\alpha, m = \cos\beta, n = \cos\gamma.$

(11) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$

(12) $If\ a, b, c\ be\ three\ numbers\ proportional\ to\ the\ actual\ direction\ cosines$

$l, m, n\ of\ a\ straight\ line, then$

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c}=\pm\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}}=\pm\frac{1}{\sqrt{a^2+b^2+c^2}}$$

(13) $l=\pm\frac{a}{\sqrt{a^2+b^2+c^2}}; m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}}; n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}}$.

(14) $The\ direction\ cosines\ of\ the\ join\ of\ the\ two\ points\ P(x_1,y_1,z_1)\ and\ Q(x_2,y_2,z_2)\ are$ $\frac{x_2-x_1}{PQ},\frac{y_2-y_1}{PQ},\frac{z_2-z_1}{PQ}$.

(15) $The\ direction\ ratios\ of\ the\ line\ joining\ two\ points\ P(x_1,y_1,z_1)\ and\ Q(x_2,y_2,z_2)\ are$ x_2-x_1,y_2-y_1,z_2-z_1.

(16) $If\ \theta\ be\ the\ angle\ between\ two\ lines,then$

$\cos\theta=\frac{\sum a_1a_2}{\sqrt{\sum a_1{}^2}\sqrt{\sum a_2{}^2}}$. $[where, a_1=l_1,m_1,n_1;\ a_2=l_2,m_2,n_2]$.

$\cos\theta=l_1l_2+m_1m_2+n_1n_2;\ sin\theta=\sqrt{\sum(m_1n_2-m_2n_1)^2}$

(17) $Two\ lines\ with\ direction\ cosines\ l_1,m_1,n_1$ & $l_2,m_2,n_2\ will\ be\ parallel\ if$ $\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{n_1}{n_2}$.

(18) $Two\ lines\ with\ direction\ cosines\ l_1,m_1,n_1$ & $l_2,m_2,n_2\ will\ be\ perpendicular\ if$ $l_1l_2+m_1m_2+n_1n_2=0$.

(19) $If\ \vec{r}=a\hat{\imath}+b\hat{\jmath}+c\hat{k},then\ direction\ ratios\ of\ r\ are\ a,b,c$.

(20) $The\ projection\ of\ line\ segment\ joining\ P(x_1,y_1,z_1)\ and\ Q(x_2,y_2,z_2)\ on\ a\ line\ whose$ $direction\ cosines\ are\ l,m,n\ is\ given\ by$

$l(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1)$.

Straight Line in Space

(1) $If\ a\ line\ passes\ through\ a\ point\ with\ position\ vector\ \overrightarrow{r_1}\ and\ it\ is\ parallel\ to\ \vec{m}\ then\ its$ $vector\ equation\ is$ $\vec{r}=\overrightarrow{r_1}+\lambda\vec{m}$

(2) $If\ a\ line\ passes\ through\ a\ point\ A(x_1,y_1,z_1)\ and\ it\ has\ d.r.'s\ a,b,c\ then\ its\ cartesian$ $equations\ are$

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

(3) $If\ a\ line\ passes\ through\ two\ points\ having\ p.v.'s\ \overrightarrow{r_1}\ and\ \overrightarrow{r_2},then\ its\ vector\ equation\ is$

$$\vec{r}=\overrightarrow{r_1}+\lambda(\overrightarrow{r_2}-\overrightarrow{r_1})$$

(4) $If\ a\ line\ passes\ through\ two\ points\ A(x_1,y_1,z_1)\ and\ B(x_2,y_2,z_2)\ then\ its\ cartesian$ $equations\ are$

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

(5) $The\ condition\ for\ three\ given\ points\ A(x_1, y_1, z_1), B(x_2, y_2, z_2)\ and\ C(x_3, y_3, z_3)\ to\ be\ collinear\ is\ that$

$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} = \frac{z_3 - z_1}{z_2 - z_1}$$

(6) $Three\ points\ A, B\ and\ C\ with\ p.v.'s\ \vec{a}, \vec{b}\ and\ \vec{c}\ respectively\ are\ collinear\ iff\ there\ exist\ scalars\ d_1, d_2, d_3\ not\ all\ zero\ such\ that$

$d_1\vec{a} + d_2\vec{b} + d_3\vec{c} = \vec{0}\ \ and\ \ d_1 + d_2 + d_3 = 0$

(7) $If\ \theta\ is\ the\ angle\ between\ the\ lines\ \vec{r} = \vec{r_1} + \lambda\vec{m_1}\ and\ \vec{r} = \vec{r_2} + \mu\vec{m_2}\ \ then$

$cos\ \theta\ = \dfrac{|\vec{m_1}\ .\vec{m_2}|}{|\vec{m_1}||\vec{m_2}|}$

(8) $If\ \theta\ is\ the\ angle\ between\ the\ lines\ whose\ cartesian\ equations\ are$

$\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}\ \ and\ \ \dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}\ \ then$

$\cos\theta\ = \dfrac{|a_1a_2 + b_1b_2 + c_1c_2|}{\left(\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}\right)\left(\sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}\right)}$

(9) $The\ shortest\ distance\ between\ two\ skew\ (non - coplanar)\ lines\ \vec{r} = \vec{a_1} + \lambda\vec{b_1}\ and\ \vec{r} = \vec{a_2} + \mu\vec{b_2}\ \ is\ given\ by$

$$d\ = \left|\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

$If\ the\ lines\ intersect\ each\ other\ then\ the\ shortest\ distance\ between\ them\ is\ zero.$

(10) $The\ distance\ between\ two\ parallel\ lines\ \vec{r} = \vec{a_1} + \lambda\vec{b}\ and\ \vec{r} = \vec{a_2} + \mu\vec{b}\ \ is\ given\ by$

$$D\ = \left|\frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{\vec{b}}\right|$$

(11) $The\ shortest\ distance\ between\ two\ skew\ lines$

$\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}\ \ and\ \ \dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}\ \ is\ given\ by$

$SD\ = \dfrac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{[(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2]}}$

$The\ two\ lines\ mentioned\ above\ will\ intersect\ if\ SD\ = 0\ , i.e.,$

$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

The Plane

(1) $The\ general\ equation\ of\ a\ plane\ is\ ax + by + cz + d = 0.\ The\ d.r.'s\ of\ the\ normal\ to\ the\ plane\ are\ a, b, c.$

(2) $The\ equation\ of\ the\ plane\ passing\ through\ the\ point\ P(x_1, y_1, z_1)\ is$

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$

(3) $If\ a\ plane\ makes\ intercepts\ a, b\ and\ c\ with\ the\ X - axis, Y - axis\ and\ Z - axis\ respectively, then\ its\ equation\ is\ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$

(4) $The\ equation\ of\ a\ plane\ passing\ through\ three\ points\ A(x_1, y_1, z_1), B(x_2, y_2, z_2)\ and\ C(x_3, y_3, z_3)\ is\ given\ by$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

(5) (i) $Let\ a\ plane\ be\ at\ a\ distance\ p\ from\ the\ origin\ and\ let\ \hat{n}\ be\ a\ unit\ vector\ perpendicular\ to\ the\ plane.$

$Then, equation\ of\ the\ plane\ is\ \vec{r}.\hat{n} = p.$

(ii) $The\ cartesian\ form\ of\ equation\ of\ this\ plane\ is\ lx + my + nz = p, where\ l, m, n\ are\ the\ d.c.'s\ of\ normal\ to\ the\ plane.$

(iii) $If\ \vec{n}\ is\ a\ vector\ normal\ to\ a\ given\ plane\ then\ \vec{r}.\hat{n} = q\ represents\ a\ plane.$

(iv) $The\ cartesian\ form\ is\ ax + by + cz + d = 0, where\ a, b, c\ are\ the\ d.r.'s\ of\ \vec{n}.$

(6) (i) $The\ vector\ equation\ of\ a\ plane\ passing\ through\ a\ point\ A\ with\ p.v.\ \vec{a}\ and\ perpendicular\ to\ \vec{n}\ is\ (\vec{r} - \vec{a}).\vec{n} = 0.$

(ii) $The\ cartesian\ equation\ of\ a\ plane\ passing\ through\ a\ point\ A(x_1, y_1, z_1)\ and\ perpendicular\ to\ a\ line\ having\ d.r.'s\ a, b, c\ is$

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$$

(7) **Distance of a Plane from a point**

Vector Form

(i) $Let\ p\ be\ the\ length\ of\ perpendicular\ drawn\ from\ a\ point\ P\ with\ p.v.\ \vec{a}\ to\ the\ plane\ \vec{r}.\vec{n} = q. Then,$

$$p = \frac{|\vec{a}.\vec{n} - q|}{|\vec{n}|}.$$

(ii) $Let\ p\ be\ the\ length\ of\ perpendicular\ drawn\ from\ the\ origin\ to\ the\ plane\ \vec{r}.\vec{n} = q. Then,$

$p = \frac{|q|}{|\vec{n}|}.$

Cartesian Form

(i) $Let\ p\ be\ the\ length\ of\ perpendicular\ drawn\ from\ a\ point\ P(x_1, y_1, z_1)\ to\ the$ $plane\ ax + by + cz + d = 0. Then,$

$$p = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

(ii) $Let\ p\ be\ the\ length\ of\ perpendicular\ drawn\ from\ the\ origin\ to\ the\ plane$ $plane\ ax + by + cz + d = 0. Then,$

$$p = \frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$$

(8) **Equation of a Plane parallel to a given Plane**

Vector Form

$Any\ plane\ parallel\ to\ \vec{r}.\vec{n} = q_1\ \ is\ given\ by\ \vec{r}.\vec{n} = q_2, where\ the\ constant\ q_2\ is\ determined$ $by\ a\ given\ condition.$

Cartesian Form

$Any\ plane\ parallel\ to\ ax + by + cz + d = 0\ \ is\ given\ by\ ax + by + cz + k = 0, where$ $the\ constant\ k\ is\ determined\ by\ a\ given\ condition.$

(9) **Plane passing through the intersection of two planes**

Vector Form

$The\ vector\ equation\ of\ a\ plane\ passing\ through\ the\ intersection\ of\ two\ planes$ $\vec{r}.\overrightarrow{n_1} = q_1\ \ and\ \ \vec{r}.\overrightarrow{n_2} = q_2\ \ is\ given\ by$

$$\vec{r}.(\overrightarrow{n_1} + \lambda\overrightarrow{n_2}) = (q_1 + \lambda q_2).$$

Cartesian Form

$The\ equation\ of\ a\ plane\ passing\ through\ the\ intersection\ of\ two\ planes$ $a_1x + b_1y + c_1z + d_1 = 0\ \ and\ \ a_2x + b_2y + c_2z + d_2 = 0\ \ is\ given\ by$

$$a_1x + b_1y + c_1z + d_1 + \lambda(a_2x + b_2y + c_2z + d_2) = 0.$$

(10) **Equation of a Plane passing through Three Non-collinear Points**

Vector Form

$The\ vector\ equation\ of\ a\ plane\ passing\ through\ three\ non-collinear\ points\ having$ $p.v.'s\ \ \vec{a}, \vec{b}, \vec{c}\ \ is\ given\ by$

$$(\vec{r} - \vec{a}).\left[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})\right] = 0.$$

Cartesian Form

$The\ cartesian\ equation\ of\ a\ plane\ passing\ through\ three\ non-collinear\ points$ $A(x_1, y_1, z_1), B(x_2, y_2, z_2)\ and\ C(x_3, y_3, z_3)\ is\ given\ by$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

(11) **Angle between two Planes**

Vector Form

The acute angle θ *between the planes* $\vec{r}.\overrightarrow{n_1} = q_1$ *and* $\vec{r}.\overrightarrow{n_2} = q_2$ *is given by*

$$\cos\theta = \frac{|\overrightarrow{n_1}.\overrightarrow{n_2}|}{|\overrightarrow{n_1}||\overrightarrow{n_2}|}.$$

(i) *Two planes* $\vec{r}.\overrightarrow{n_1} = q_1$ *and* $\vec{r}.\overrightarrow{n_2} = q_2$ *are perpendicular to each other* $\Leftrightarrow \overrightarrow{n_1}.\overrightarrow{n_2} = 0$.

(ii) *Two planes* $\vec{r}.\overrightarrow{n_1} = q_1$ *and* $\vec{r}.\overrightarrow{n_2} = q_2$ *are parallel to each other* $\Leftrightarrow \overrightarrow{n_1} = \lambda\overrightarrow{n_2}$ *for some scalar* λ.

(iii) *Any plane parallel to* $\vec{r}.\vec{n} = q$ *and passing through a point with p.v.* $\vec{a}$ *is*

$$(\vec{r}-\vec{a}).\vec{n} = 0.$$

Cartesian Form

The acute angle θ *between the planes* $a_1x + b_1y + c_1z + d_1 = 0$ *and*

$a_2x + b_2y + c_2z + d_2 = 0$ *is given by*

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\left(\sqrt{a_1^2 + b_1^2 + c_1^2}\right)\left(\sqrt{a_2^2 + b_2^2 + c_2^2}\right)}$$

(i) *Two planes* $a_1x + b_1y + c_1z + d_1 = 0$ *and* $a_2x + b_2y + c_2z + d_2 = 0$ *are perpendicular to each other* $\Leftrightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$.

(ii) *Two planes* $a_1x + b_1y + c_1z + d_1 = 0$ *and* $a_2x + b_2y + c_2z + d_2 = 0$ *are parallel to each other* $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(iii) *The equation of a plane passing through the point* (x_1, y_1, z_1) *and parallel to the plane* $ax + by + cz + d = 0$ *is given by*

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0.$$

(iv) *Any plane parallel to the* $yz-$*plane is* $x = \lambda$.
Any plane parallel to the $xz-$*plane is* $y = \lambda$.
Any plane parallel to the $xy-$*plane is* $z = \lambda$.

(12) **Angle between a line and a plane**

Vector Form

(i) *If* Φ *is the angle between the line* $\vec{r} = \vec{a} + \lambda\vec{b}$ *and the plane* $\vec{r}.\vec{n} = q$ *then*,
$\sin\Phi = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}$.

(ii) *The line* $\vec{r} = \vec{a} + \lambda\vec{b}$ *is perpendicular to the plane* $\vec{r}.\vec{n} = q$ *only when* $\vec{b} = t\vec{n}$ *for some scalar* t.

(iii) (a) *The line* $\vec{r} = \vec{a} + \lambda\vec{b}$ *is parallel to the plane* $\vec{r}.\vec{n} = q$ *only when* $\vec{b}.\vec{n} = 0$.
(b) *If the line* $\vec{r} = \vec{a} + \lambda\vec{b}$ *is parallel to the plane* $\vec{r}.\vec{n} = q$ *then the distance between them is* $\frac{|\vec{a}.\vec{n} - q|}{|\vec{n}|}$.

Cartesian Form

(i) $If\ \Phi\ is\ the\ angle\ between\ the\ line\ \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}\ and\ the\ plane$ $a_2x + b_2y + c_2z + d_2 = 0\ then$

$$\sin\Phi = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\left(\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\right)\left(\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}\right)}$$

(ii) $The\ line\ \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}\ is\ perpendicular\ to\ the\ plane\ a_2x + b_2y + c_2z + d_2 = 0$ $only\ when\ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$

(iii) $The\ line\ \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}\ is\ perpendicular\ to\ the\ plane\ a_2x + b_2y + c_2z + d_2 = 0$ $only\ when\ a_1a_2 + b_1b_2 + c_1c_2 = 0.$

(13) **Equation of a Plane passing through a given point and parallel to two given Lines**

Vector Form

$The\ vector\ equation\ of\ a\ plane\ passing\ through\ a\ point\ having\ p.v.\ \vec{a}\ and\ parallel\ to$

$the\ vectors\ \vec{b}\ and\ \vec{c}\ is\ given\ by$

$$(\vec{r} - \vec{a}).(\vec{b} \times \vec{c}) = 0.$$

Cartesian Form

$The\ cartesian\ equation\ of\ a\ plane\ passing\ through\ a\ point\ A(x_1, y_1, z_1)\ and\ parallel$

$to\ two\ non-parallel\ lines\ having\ d.r.'s\ b_1, b_2, b_3\ and\ c_1, c_2, c_3\ is\ given\ by$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0.$$

(14) **Condition for the Coplanarity of two Lines**

Vector Form

(i) $Two\ lines\ \vec{r} = \vec{a_1} + \lambda\vec{b_1}\ and\ \vec{r} = \vec{a_2} + \lambda\vec{b_2}\ are\ coplanar\ only\ when$

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 0.$$

(ii) $If\ two\ lines\ \vec{r} = \vec{a_1} + \lambda\vec{b_1}\ and\ \vec{r} = \vec{a_2} + \lambda\vec{b_2}\ are\ coplanar,\ then\ the\ equation$ $of\ the\ plane\ containing\ both\ of\ these\ lines\ is\ given\ by$

$$[(\vec{r} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 0]\ or\ [(\vec{r} - \vec{a_2}).(\vec{b_1} \times \vec{b_2}) = 0].$$

Cartesian Form

(i) $Two\ lines\ \frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}\ and\ \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}\ are\ coplanar\ only\ when$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

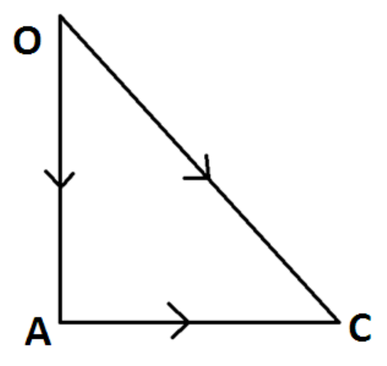
(ii) $The\ equation\ of\ the\ common\ plane\ is$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0\ \ or\ \ \begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

VECTOR

(1) $The\ unit\ vector\ along\ \vec{a} = \frac{\vec{a}}{|\vec{a}|}$.

(2) **Triangle law of addition of vectors**



$If\ \overrightarrow{OA} = \vec{a}, \overrightarrow{AC} = \vec{b}, then\ \vec{a} + \vec{b} = \vec{c}$

$$\Rightarrow \overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}.$$

(3) **Position vectors**

(i) $The\ position\ vector\ of\ a\ point\ P\ w.r.t.\ origin\ O\ is\ \overrightarrow{OP}\ vector.$

(ii) $If\ the\ position\ vector\ of\ P\ \&\ Q\ are\ \vec{a}\ \&\ \vec{b}\ respectively\ w.r.t.\ origin\ O, then$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \vec{b} - \vec{a}.$$

(iii) $If\ a\ point\ R\ divides\ \overline{PQ}\ internally\ in\ the\ ratio\ m{:}n,$

$then\ the\ position\ vector\ of\ R\ will\ be\ \frac{m\vec{b}+n\vec{a}}{m+n}.$

(iv) $If\ a\ point\ R\ divides\ \overline{PQ}\ externally\ in\ the\ ratio\ m{:}n,$

$then\ the\ position\ vector\ of\ R\ will\ be\ \frac{m\vec{b}-n\vec{a}}{m-n}.$

(v) $The\ position\ vector\ of\ the\ mid\ point\ of\ \overline{PQ}\ is\ \frac{\vec{a}+\vec{b}}{2}.$

(4) $If\ \vec{r} = x\vec{a} + y\vec{b}, then$

$x\vec{a}, y\vec{b} = Vector\ components\ of\ \vec{r}\ along\ \vec{a}\ \&\ \vec{b}.$

$x, y = Scalar\ components\ of\ \vec{r}\ along\ \vec{a}\ \&\ \vec{b}.$

(5) **2-Dimensional (2-D)**

$If\ P(x, y)\ be\ a\ point\ in\ 2-D\ plane, then\ the\ position\ vector\ of\ P\ w.r.t.\ origin\ O\ (\vec{r})\ is$

$\overrightarrow{OP} = \vec{r} = x\hat{\imath} + y\hat{\jmath}$

$where, \hat{\imath}$ & $\hat{\jmath}$ $are\ the\ unit\ vectors\ along\ positive\ X - axis$ & $Y - axis\ respectively.$

$|\overrightarrow{OP}| = |\vec{r}| = \sqrt{x^2 + y^2}$.

$x\hat{\imath}$ & $y\hat{\jmath}$ $= Vector\ components\ along\ X - axis$ & $Y - axis$.

x & y $=$ $Scalar\ components\ along\ X - axis$ & $Y - axis$.

(6) **3-Dimensional (3-D)**

$If\ P(x, y, z)\ be\ a\ point\ in\ 3 - D\ plane, then\ the\ position\ vector\ of\ P\ w.r.t. origin\ O\ (\vec{r})\ is$

$\overrightarrow{OP} = \vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

$where, \hat{\imath}, \hat{\jmath}$ & $\hat{k}$ $are\ the\ unit\ vectors\ along\ positive\ X - axis, Y - axis$ & $Z - axis$ $respectively.$

$|\overrightarrow{OP}| = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

$x\hat{\imath}, y\hat{\jmath}$ & $z\hat{k}$ $= Vector\ components\ along\ X - axis, Y - axis$ & $Z - axis$.

x, y & z $=$ $Scalar\ components\ along\ X - axis, Y - axis$ & $Z - axis$.

(7) **Direction Ratios & Direction Cosines of a Vector**

$Consider\ the\ vector\ \vec{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$(i) Then\ direction\ ratios\ of\ \vec{r}\ are\ a, b, c.$

$(ii) The\ direction\ cosines\ of\ \vec{r}\ are\ given\ by$

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}\ ;\ m = \frac{b}{\sqrt{a^2 + b^2 + c^2}};\ n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$(iii) l^2 + m^2 + n^2 = 1.$

$(iv) If\ A(x_1, y_1, z_1)\ and\ B(x_2, y_2, z_2)\ be\ any\ two\ points\ in\ space\ then\ direction$ $ratios\ of\ \overrightarrow{AB}\ are\ (x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\ and\ direction\ cosines\ of\ \overrightarrow{AB}\ are$

$$\frac{(x_2 - x_1)}{r}, \frac{(y_2 - y_1)}{r}, \frac{(z_2 - z_1)}{r}\ \ where\ r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

(8) **Scalar product / Dot product of vectors**

(i) $The\ scalar\ product\ of\ two\ vectors\ \vec{a}$ & $\vec{b}, which\ meets\ in\ a\ point\ is\ given\ by$

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$where\ \theta\ is\ the\ angle\ between\ these\ vectors.$

(ii) $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1.$

(iii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0.$

(iv) $\lambda(a\hat{\imath} + b\hat{\jmath}) = \lambda a\hat{\imath} + \lambda b\hat{\jmath}.$

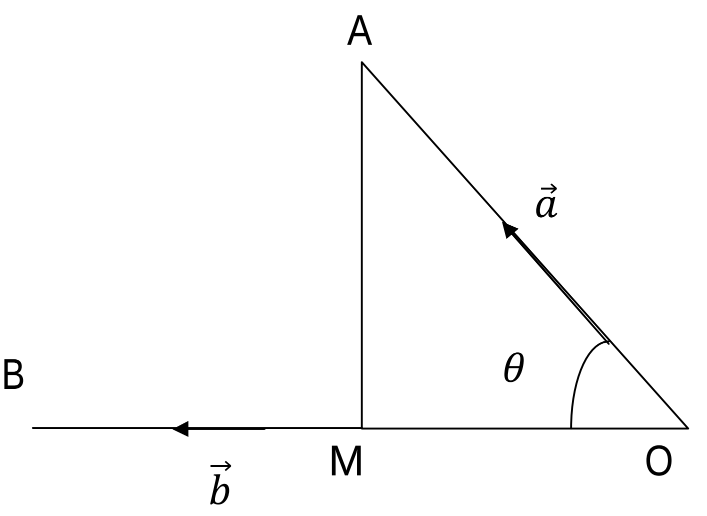
(v) $If\ \vec{a} = \vec{0}\ or\ \vec{a} = \vec{0}, we\ define\ \vec{a}.\vec{b} = 0.$

(vi) $If\ \vec{a}\ and\ \vec{b}\ are\ like\ (collinear)\ vectors, we\ have\ \theta = 0;\ \vec{a}.\vec{b} = ab\cos\theta = ab.$

(vii) $If\ \vec{a}\ and\ \vec{b}\ are\ unlike\ vectors, we\ have\ \theta = \pi;\ \vec{a}.\vec{b} = ab\cos\theta = -ab.$

(viii) $If\ \vec{a}\ and\ \vec{b}\ are\ orthogonal\ vectors, we\ have\ \theta = \frac{\pi}{2};\ \vec{a}.\vec{b} = ab\cos\theta = 0.$

(9) **Projection**



$Let\ \overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}\ and\ \angle BOA = \theta; also\ AM \perp OB.$

$Then, OM\ is\ the\ projection\ of\ \vec{a}\ on\ \vec{b}.$

$$OM = OAcos\theta = |\vec{a}|cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{b}|}.$$

(10) **Condition of Perpendicularity**

$Let\ \vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}\ and\ \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\vec{a}\ will\ be\ perpendicular\ on\ \vec{b}\ if\ \vec{a}.\vec{b} = 0 \Leftrightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0.$

(11) **Vector product / Cross product of vectors**

$Let\ \vec{a}\ and\ \vec{b}\ be\ two\ non-zero, non-parallel\ vectors, and\ let\ \theta\ be\ the\ angle\ between$ $them\ such\ that\ 0 < \theta < \pi.$

$Then, the\ cross\ product\ of\ \vec{a}\ and\ \vec{b}\ is\ defined\ as\quad \vec{a} \times \vec{b} = (|\vec{a}||\vec{b}|sin\theta)\hat{n}$

$where\ \hat{n}\ is\ a\ unit\ vector\ perpendicular\ to\ both\ \vec{a}\ and\ \vec{b}.$

(i) $If\ \vec{a}\ and\ \vec{b}\ are\ parallel\ or\ collinear, i.e., when\ \theta = 0\ or\ \theta = \pi\ then,\ \ \vec{a} \times \vec{b} = \vec{0}.$

(ii) $If\ \vec{a} = \vec{0}\ or\ \vec{b} = \vec{0}, we\ define\ \vec{a} \times \vec{b} = \vec{0}.$

(iii) $For\ any\ vector\ \vec{a}, we\ have\ \vec{a} \times \vec{a} = (|\vec{a}||\vec{a}|sin0)\hat{n} = \vec{0}.$

(iv) $The\ angle\ \theta\ between\ two\ vectors\ is\ defined\ by\quad \theta = \sin^{-1}\left[\frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}\right].$

(v) $A\ unit\ vector\ \hat{n}\ perpendicular\ to\ each\ one\ of\ \vec{a}\ and\ \vec{b}\ is\ given\ by\ \ \hat{n} = \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}.$

(vi) $\vec{a} \times \vec{b}.\ \vec{c} = \vec{a}.\ \ \vec{b} \times \vec{c}$

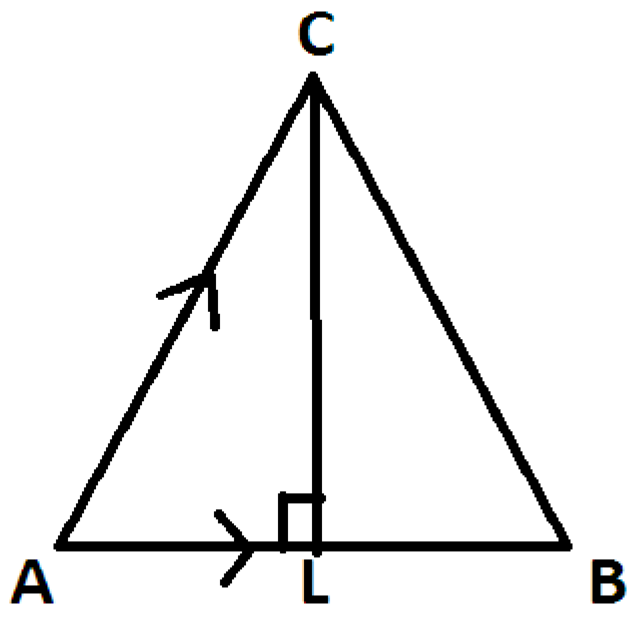
(vii) $\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$

(viii) $(-\vec{b}) \times \vec{a} = (\vec{a} \times \vec{b}) = \vec{b} \times (-\vec{a})$

(12) **Area of Triangle**

$Let\ us\ consider\ \Delta ABC\ in\ which\ \overrightarrow{AB} = \vec{a}\ \&\ \overrightarrow{AC} = \vec{b}$

$Area\ of\ the\ \Delta ABC\ is\ given\ by\ \frac{1}{2}\left|\vec{a}\ \times\ \vec{b}\right|\ and\ it\ is\ called\ vector\ area\ of\ \Delta ABC.$



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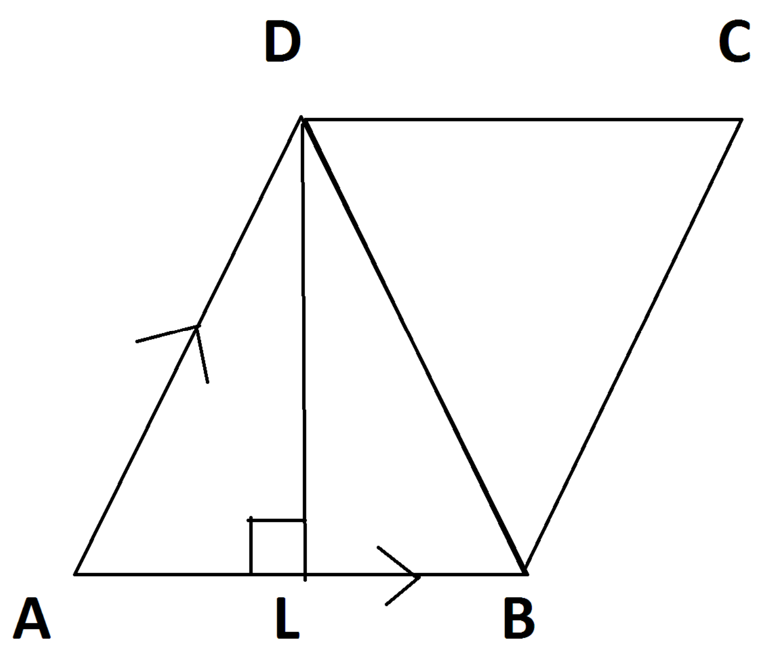
(13) **Area of Parallelogram**

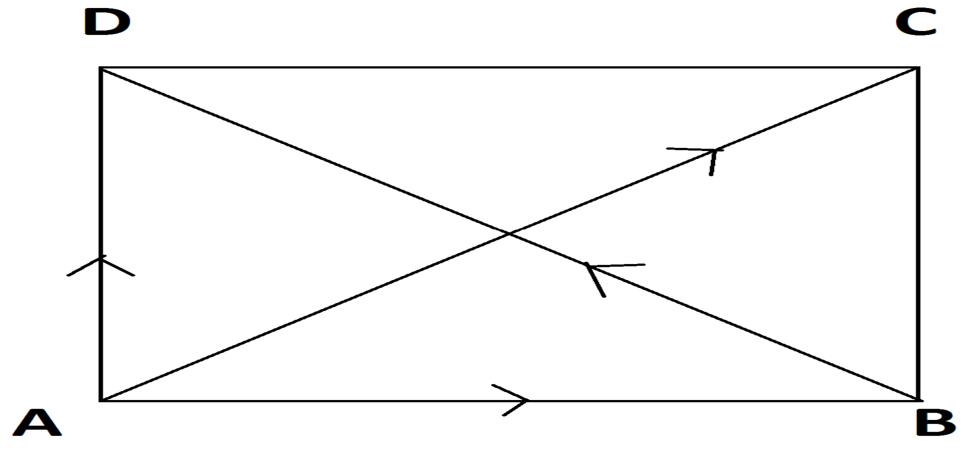
$Let\ us\ consider\ ||gm\ ABCD\ in\ which\ \overrightarrow{AB}\ =\ \vec{a}\ \&\ \overrightarrow{AC}\ =\ \vec{b}$

$Area\ of\ the\ ||gm\ ABCD\ is\ given\ by\ \left|\vec{a}\ \times\ \vec{b}\right|$



(14) **Area of Quadrilateral**

$Area\ of\ the\ quad.ABCD\ is\ given\ by\ \frac{1}{2}\left|\overrightarrow{AC}\ \times\ \overrightarrow{BD}\right|$



(15) **Vector product of an orthonormal vector triad**

$For\ mutually\ perpendicular\ unit\ vectors\ \hat{\imath}, \hat{\jmath}, \hat{k}, we\ have$

(i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \hat{0}$

(ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

(16) **Vector product in terms of componenets**

$Let\ \vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}\ and\ \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}. Then,$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

(17) **Scalar Triple product**

(i) $[\vec{a}\, \vec{b}\, \vec{c}] = (\vec{a} \times \vec{b}). \vec{c}$

$[\vec{a}\, \vec{b}\, \vec{c}]\ represents\ the\ volume\ of\ the\ parallelopiped\ with\ coterminous\ edges\ \vec{a}, \vec{b}, \vec{c}$

$forming\ a\ right-handed\ system.$

(ii) $(\vec{a} \times \vec{b}). \vec{c} = (\vec{b} \times \vec{c}). \vec{a} = (\vec{c} \times \vec{a}). \vec{b}$

(iii) $The\ scalar\ triple\ product\ changes\ in\ sign\ but\ not\ in\ magnitude\ when\ the\ cyclic\ order$

$of\ vectors\ is\ changed, i.e.,\qquad [\vec{c}\, \vec{b}\, \vec{a}] = -[\vec{a}\, \vec{b}\, \vec{c}]$

(iv) $The\ scalar\ triple\ product\ vanishes\ if\ any\ two\ of\ its\ vectors\ are\ equal, i.e.,$

$[\vec{a}\, \vec{a}\, \vec{b}] = 0, [\vec{a}\, \vec{b}\, \vec{a}] = 0\ and\ [\vec{b}\, \vec{a}\, \vec{a}] = 0.$

(v) $The\ scalar\ triple\ product\ vanishes\ if\ any\ two\ of\ its\ vectors\ are\ parallel\ or\ collinear.$

$Let\ \vec{a}||\vec{b}\ or\ \vec{a}\ and\ \vec{b}\ are\ collinear. Then, \vec{a} = m\vec{b}$

$\therefore\ [\vec{a}\, \vec{b}\, \vec{c}] = [m\vec{b}\, \vec{b}\, \vec{c}] = 0$

(vi) **Scalar triple product in terms of components**

$Let\ \vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k};\ \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}\ and\ \vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

$$[\vec{a}\, \vec{b}\, \vec{c}] = \begin{vmatrix} \boldsymbol{a_1} & \boldsymbol{a_2} & \boldsymbol{a_3} \\ \boldsymbol{b_1} & \boldsymbol{b_2} & \boldsymbol{b_3} \\ \boldsymbol{c_1} & \boldsymbol{c_2} & \boldsymbol{c_3} \end{vmatrix}$$

(vii) $For\ any\ three\ vectors\ \vec{a}, \vec{b}\ and\ \vec{c}$

$[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\, \vec{b}\, \vec{c}]$

(viii) $The\ necessary\ and\ sufficient\ condition\ for\ three\ non-zero, non-collinear$

$vectors\ \vec{a}, \vec{b}, \vec{c}\ to\ be\ coplanar\ is\ that\ [\vec{a}\, \vec{b}\, \vec{c}] = 0.$

(ix) $For\ any\ three\ vectors\ \vec{a}, \vec{b}, \vec{c},\ the\ vectors\ (\vec{a} - \vec{b}), (\vec{b} - \vec{c}) and\ (\vec{c} - \vec{a})\ are\ coplanar.$

$Also, (\vec{a} + \vec{b}), (\vec{b} + \vec{c}) and\ (\vec{c} + \vec{a})\ are\ coplanar.$

(x) $For\ any\ three\ vectors\ \vec{a}, \vec{b}, \vec{c},\ [\vec{a}\ \ \vec{a} + \vec{b}\ \ \vec{a} + \vec{b} + \vec{c}] = 0$

STATISTICS

MEAN

(1) $Simple\ mean,\ \bar{x} = \frac{\sum x_i}{n}$

(2) $Weighted\ mean,\ \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

(3) $Short - cut\ method:\ \bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$

(4) $Step - deviation\ method:\ \bar{x} = A + \frac{\sum f_i u_i}{\sum f_i} \times h$

(5) $Mean\ of\ a\ composite\ sample = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2}$

$where,$

$n = Total\ observations$

$x_i = Mid\ values\ of\ the\ class\ intervals$

$f_i = Frequencies\ of\ the\ class\ intervals$

$A = Assumed\ mean$

$d_i = x_i - A$

$$u_i = \frac{x_i - A}{h}$$

$h = Width\ of\ each\ class$

MEDIAN

$$\boldsymbol{Median = l + \frac{\frac{N}{2} - CF}{f_m} \times h}$$

$where,$

$l = Lower\ boundary\ of\ median\ class$

$N = Total\ frequency$

$CF = Cumulative\ frequency\ of\ the\ class\ preceeding\ to\ the\ median\ class$

$f_m = Frequency\ of\ the\ median\ class$

$h = Width\ of\ the\ median\ class$

Quartiles, Deciles & Percentiles

Measure	Discrete series and ungrouped frequency distribution	Grouped frequency distribution
Q_1	$size\ of\ \frac{n+1}{4}\ th\ term$	$\boldsymbol{l + \frac{\frac{N}{4} - CF}{f_m} \times h}$
Q_3	$size\ of\ \frac{3(n+1)}{4}\ th\ term$	$\boldsymbol{l + \frac{\frac{3N}{4} - CF}{f_m} \times h}$
D_1	$size\ of\ \frac{n+1}{10}\ th\ term$	$\boldsymbol{l + \frac{\frac{N}{10} - CF}{f_m} \times h}$
D_7	$size\ of\ \frac{7(n+1)}{10}\ th\ term$	$\boldsymbol{l + \frac{\frac{7N}{10} - CF}{f_m} \times h}$
P_1	$size\ of\ \frac{n+1}{100}\ th\ term$	$\boldsymbol{l + \frac{\frac{N}{100} - CF}{f_m} \times h}$
P_{47}	$size\ of\ \frac{47(n+1)}{100}\ th\ term$	$\boldsymbol{l + \frac{\frac{47N}{100} - CF}{f_m} \times h}$

MODE

$$\boldsymbol{Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h}$$

$where,$

$l =\ Lower\ boundary\ of\ modal\ class$

$f_1 = Frequency\ of\ the\ modal\ class$

$f_0 = Frequency\ of\ the\ class\ preceeding\ the\ modal\ class$

$f_2 =\ Frequency\ of\ the\ class\ succeeding\ the\ modal\ class$

$h = Width\ of\ the\ modal\ class$

Relation between mean, median and mode

$Mode = 3 \times (Median) - 2 \times (Mean).$

S.D.

$\boldsymbol{Variance = (S.D.)^2}$

$\boldsymbol{Co-efficient\ of\ variance = \frac{S.D.}{Mean} \times 100}.$

$$\boldsymbol{S.D. = \sqrt{\frac{\sum f_i u_i^2}{\sum f_i} - \left(\frac{\sum f_i u_i}{\sum f_i}\right)^2} \times h}$$

$where,$

$x_i = Mid\ values\ of\ the\ class\ intervals$

$f_i = Frequencies\ of\ the\ class\ intervals$

$A = Assumed\ mean$

$u_i = \frac{x_i - A}{h}$

$h = Width\ of\ each\ class$

[**N.B.**:- The S.D. can be determined by three methods explained in mean using simple method & deviation method.]

CORRELATION & REGRESSION

Correlation

(1) $Co-variance$

$$Cov(x,y)=\frac{1}{n}\sum_{i=1}^{n}x_iy_i-\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)\left(\frac{1}{n}\sum_{i=1}^{n}y_i\right)$$

(2) $\sigma_x{}^2=\frac{1}{n}\sum x_i{}^2-\left(\frac{1}{n}\sum x_i\right)^2$

(3) $\sigma_y{}^2=\frac{1}{n}\sum y_i{}^2-\left(\frac{1}{n}\sum y_i\right)^2$ $\quad [where,\ \sigma=s.d.]$

(4) $Correlation\ co-efficient,$

$r_{xy}=\dfrac{Cov(x,y)}{\sigma_x\sigma_y}$

(5) $For\ any\ bi-variate\ data,\ -1\le r_{xy}\le 1$

(6) $r_{xx}=1\ \&\ r_{x(-x)}=-1$

(7) $If\ u=ax+b\ \&\ v=cy+d$

$then,\qquad r_{uv}=\dfrac{ac}{|a||c|}r_{xy}$

(8) $Var(x+y)=\sigma_x{}^2+\sigma_y{}^2+2\sigma_x\sigma_y r_{xy}$

(9) $Var(x-y)=\sigma_x{}^2+\sigma_y{}^2-2\sigma_x\sigma_y r_{xy}$

(10) $Var(x+y)=\frac{1}{n}\sum_{i=1}^{n}\{(x_i+y_i)-\overline{x+y}\}^2$

$\qquad =\frac{1}{n}\sum_{i=1}^{n}\{(x_i-\bar{x})+(y_i-\bar{y})\}^2$

(11) $If\ u=\frac{x-\bar{x}}{\sigma_x},v=\frac{y-\bar{y}}{\sigma_y},$

$then,\qquad r_{xy}=Cov(x,y)$

(12) $If\ r_{xy}=\pm 1,\ then\ y\ is\ a\ linear\ function\ of\ x\ \&\ vice-versa.$

(13) $If\ x,y\ are\ independent\ then\ they\ are\ un-correlated.$

(14) $If\ u=a_1x+b_1y+c_1$

$\qquad v=a_2x+b_2y+c_2$

$then,$

$$Cov(u,v)=a_1a_2Var(x)+(a_1b_2+a_2b_1)Cov(x,y)+b_1b_2Var(y)$$

Regression

(1) $Regression\ line\ of\ x\ on\ y\ is$

$(x-\bar{x})=b_{xy}(y-\bar{y})$

$where,\quad b_{xy} = r_{xy}\frac{\sigma_x}{\sigma_y}$

(2) $Regression\ line\ of\ y\ on\ x\ is$

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$where,\quad b_{yx} = r_{xy}\frac{\sigma_y}{\sigma_x}$

$[b_{xy}, b_{yx} \rightarrow Regression\ co - efficient]$

(3) $The\ two\ regression\ lines\ intersect\ at\ (\bar{x}, \bar{y}).$

(4) $b_{xy} = \frac{Cov(x,y)}{{\sigma_y}^2},\ b_{yx} = \frac{Cov(x,y)}{{\sigma_x}^2}$

(5) $b_{xy} = \frac{1}{b_{yx}}\qquad i.e., b_{xy}b_{yx} = 1.$

(6) $The\ angle\ between\ two\ regression\ line\ is\ tan\theta = \left|\frac{1-{r_{xy}}^2}{b_{xy}+b_{yx}}\right|.$

Spearman Rank Correlation Coefficient

(1) $The\ Spearman\ rank\ correlation\ coefficient\ is\ given\ by$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}.$$

$where,\quad d = difference\ between\ the\ ranks$

$\qquad\quad n = total\ observations$

(2) $If\ there\ are\ repeatation\ of\ a\ rank\ for\ m\ items\ then\ a\ correction\ of\ the\ factor$

$\frac{1}{12}(m^3 - m)\ is\ needed.$

$Then\ the\ rank\ will\ be\ r = 1 - \frac{6[\sum d^2 + \frac{1}{12}(m^3 - m)]}{n(n^2-1)}.$

BOOLEAN ALGEBRA

$The\ basic\ function\ of\ Boolean\ Algebra\ is$ 1 & 0, $i.e. ON - OFF\ process.$

$There\ are\ basically\ three\ logic\ gates - AND, OR$ & $NOT.$

Properties of Boolean Algebra:-

(1) ***Commutative Property***:

$$A + B = B + A$$

$$A.B = B.A$$

(2) ***Associative Property***:

$$A + (B + C) = (A + B) + C$$

$$A.(B.C) = (A.B).C$$

(3) ***Distributive Property***:

$$A + BC = (A + B)(A + C)$$

$$A.(B + C) = A.B + A.C$$

(4) ***Absorption Law***:

$$A + AB = A$$

$$A.(A + B) = A$$

(5) ***De Morgan's Theorem***:

$$\overline{AB} = \overline{A} + \overline{B}$$

$$\overline{A + B} = \overline{A}.\overline{B}$$

(6) ***Involution Law***:

$$\overline{\overline{A}} = A$$

(7) $A + 0 = A$

(8) $A + 1 = 1$

(9) $A + A = A$

(10) $A + \overline{A} = 1$

(11) $A.1 = A$

(12) $A.0 = 0$

(13) $A.A = A$

(14) $A.\overline{A} = 0$

COMMERCIAL MATHEMATICS

(1) Average Due Date

(1) $d = \frac{\sum_{i=1}^{n} P_i d_i}{\sum_{i=1}^{n} P_i}$

$where,$

$d = equated\ time$

$P_i = different\ payments$

$d_i = times\ counted\ from\ the\ zero\ date$

(2) $Zero\ date + Equated\ time \rightarrow Average\ due\ date.$

(2) Discount

(1) $True\ discount,\ TD = Interest\ on\ present\ value\ of\ bill = Pni$

(2) $A = P + TD$

(3) $P = \frac{A}{1+ni}$

(4) $Discounted\ value = A(1 - ni)$

(5) $TD = Pni = \frac{Ani}{1+ni}$

(6) $BD = Interest\ on\ amount\ of\ bill = Ani$

(7) $BD = (1 + ni)TD$

(8) $Banker's\ gain,\ BG = BD - TD$

(9) $BG = Interest\ on\ TD = \frac{A(ni)^2}{1+ni}$

(10) $Amount\ of\ bill = A = \frac{BD \times TD}{BD - TD}$

(3) Annuities

(1) $Amount\ of\ an\ annuity,\ M = \frac{A}{r}[(1 + r)^n - 1]$

(2) $Present\ value\ of\ an\ annuity,\ V = \frac{A}{r}[1 - (1 + r)^{-n}]$

(3) $Amount\ of\ an\ annuity\ due,\ M = \frac{A}{r}(1 + r)[(1 + r)^n - 1]$

(4) $Present\ value\ of\ an\ annuity\ due,\ V = \frac{A}{r}(1 + r)[1 - (1 + r)^{-n}]$

(5) $Amount\ of\ deferred\ annuity,\ M = \frac{A}{r}[(1 + r)^n - 1]$

(6) $Present\ value\ of\ deferred\ annuity,\ V = \frac{A}{r(1+r)^m}[1 - (1 + r)^{-n}]$

(7) $Amount\ of\ sinking\ fund,\ M = \frac{A}{r}[(1+r)^n - 1]$

(8) $Present\ value\ of\ perpetuity,\ V = \frac{A}{r}$

(9) $Present\ value\ of\ deferred\ perpetuity,\ V = \frac{A}{r(1+r)^m}$

$where,$ $A = Amount\ of\ each\ instalment$

$V = Present\ value\ of\ annuity$

$M = Future\ amount\ of\ annuity$

$r = Rate\ of\ interest\ p.a.$

$n = Number\ of\ instalment$

$m = Payment\ start\ after\ which\ deferring\ interval$

(4) Application of Derivative in Commerce & Economics

(1) $Total\ cost,\ TC = C(x)$

(2) $Total\ fixed\ cost,\ TFC = [C(x)]_{x=0}$

(3) $Total\ variable\ cost,\ TVC = TC - TFC$

(4) $TC = TFC + TVC$

(5) $Average\ cost,\ AC = \frac{C(x)}{x}$

(6) $Average\ fixed\ cost,\ AFC = \frac{TFC}{x}$

(7) $Average\ variable\ cost,\ AVC = \frac{TVC}{x}$

(8) $AC = AFC + AVC$

(9) $Cost\ function = C(x)$

(10) $Demand\ function,\ x = f(p)$

(11) $Price\ function,\ p = f(x)$

(12) $Revenue\ function,\ R(x) = px$

(13) $Average\ revenue,\ AR = \frac{R(x)}{x} = p$

(14) $Profit\ function,\ P(x) = R(x) - C(x)$

(15) $Average\ profit,\ \frac{P(x)}{x} = \frac{R(x)}{x} - \frac{C(x)}{x}$

$\Rightarrow AP = AR - AC$

(16) $Breakdown\ point,\ R(x) = C(x)\ \ i.e.,\ P(x) = 0$

(17) $Marginal\ cost,\ MC = \frac{dC}{dx}$

(18) $Marginal\ revenue,\ MR = \frac{dR}{dx} = p\left(1 + \frac{x}{p}.\frac{dp}{dx}\right)$

(19) $In\ purely\ competitive\ environment,\ \frac{dp}{dx} = 0 \Rightarrow MR = p = AR$

(20) $In\ a\ monopolistic\ economy,\ \frac{dp}{dx} < 0 \Rightarrow MR < AR$

(5) Index Number

(1) Simple aggregative method:-

$$P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$$

(2) Simple average of price relatives method:-

$$P_{01} = \frac{1}{N}\sum\left(\frac{P_1}{P_0} \times 100\right) = \frac{\sum I}{N}$$

(3) Weighted aggregate method:-

$$P_{01} = \frac{\sum P_1 w}{\sum P_0 w} \times 100$$

(4) Weighted aggregate of price relative method:-

$$P_{01} = \frac{\sum Iw}{\sum w}$$

$where,\quad I = \frac{P_1}{P_0} \times 100 = Price\ relative$

$w = Weight$

$P_0 = Base\ price$

$P_1 = Current\ price$

$N = Number\ of\ items$

(6) Moving Averages

$If\ x_1, x_2, x_3, \ldots \ldots, x_n\ is\ given\ annual\ time\ series,\ then$

(1) 3-yearly moving averages:-

$\frac{x_1 + x_2 + x_3}{3}, \frac{x_2 + x_3 + x_4}{3}, \frac{x_3 + x_4 + x_5}{3}, \ldots \ldots\ which\ are\ placed\ against\ years\ 2,3,4, \ldots \ldots$ $respectively.$

(2) 5- yearly moving averages:-

$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}, \frac{x_2 + x_3 + x_4 + x_5 + x_6}{5}, \ldots \ldots\ which\ are\ placed\ against\ years\ 3,4, \ldots \ldots$ $respectively.$

(3) 4- yearly moving averages:-

$\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{x_2 + x_3 + x_4 + x_5}{4}, \ldots \ldots\ which\ are\ placed\ against\ years\ 2.5, 3.5 \ldots \ldots\ respectively.$

$Further, to\ synchronize\ time\ frame\ for\ moving\ averages\ and\ original\ data, we\ have\ to$ $average\ every\ two\ moving\ averages; average\ of\ first\ \&\ second\ moving\ average\ in\ this\ case$ $would\ be\ placed\ against\ \frac{2.5 + 3.5}{2} = 3rd\ year; average\ of\ second\ \&\ third\ moving\ average$ $would\ be\ placed\ against\ \frac{3.5 + 4.5}{2} = 4th\ year, and\ so\ on.$

$This\ is\ called\ \ \boldsymbol{4 - yearly\ centred\ moving\ average}.$