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Chapter 1

Poroelastic natural coatings

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Chapter 2

Volume Average Homogenization theory for Navier-Stokes Equations

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Chapter 3

Drag-model sensitivity of Kelvin-Helmholtz waves in canopy flows

3.1 Introduction

Flows through submerged aquatic plants exhibit large scale vortices at the top of the vegetation, advecting along the flow direction and causing a periodic waving of the plants, referred to as monami [1]. Vortices arise from the nonlinear amplification of a Kelvin-Helmholtz instability mode, related to the presence of an inflection point in the base flow profile; [4] the profile itself is inflectional because the fluid is slowed down by the drag exerted by the canopy, whose modeling has recently been addressed. [28] [31] [42] The correct prediction of the onset and characteristics of the Kelvin-Helmholtz instability is important for assessing the effects of turbulence, in particular to

- understand how the vertical exchange of momentum occurs, 6
- clarify how the transport of CO₂, dissolved nutrients or sediments takes place between the obstructed vegetation flow and the free overflow motion, 7–10 and also
- assess the changes in the morphology of the vegetation in inland or coastal wetlands in response to continuous periodic forcing. [4] [26]

Because of the flexibility of the vegetation, some theoretical studies have focussed on the modeling of the stems of the aquatic plants and their displacement in response to the forcing by the water flow. [28] [26] However, Kelvin-Helmholtz vortices occur whether or not the plants bend and—to ascertain causes and effects to first order—it is acceptable to focus on the flow over and through a submerged array of rigid, cylindrical pillars. This has been the basis of the approach by Ghisalberti and Nepf [18] [16] [17] who have conducted a series

of careful experiments; their results have often been used by fluid dynamicists to put forth and test theoretical hypotheses to predict the frequency and wavelength of the large scale vortical motion, for a variety of conditions. The configuration studied consists of a regular grid of rigid pillars, orthogonal to the surface, of identical height h ; in some of the theoretical models proposed to analyze the stability of this system, the Rayleigh equation is used throughout the water channel, with or without a drag term in correspondence of the canopy. [29] [28] [31] [42] have recently demonstrated that the addition of a drag term through the vegetation reduces the amplification factor of the Kelvin-Helmholtz instability throughout the whole range of wavenumbers and increases mildly the wavelength of the fastest growing mode; further unpublished work by the same authors shows that the addition of a mixing length turbulence model in the stability equations has but a negligible influence on the leading instability mode. Questions remain, however, on the accuracy of the drag model and on its sensitivity. A partial answer to these questions is provided in [42]: there, a different model, applicable within the vegetated layer and based on the equations ruling the behavior of a transversely isotropic porous medium, has been developed and the stability results appear to better match experimental correlations. This conclusion is, however, not consolidated yet, and further studies are needed to assess the influence of the model of the drag force through the vegetation, both in setting up a particular (inflectional) mean flow and on the onset and growth of Kelvin-Helmholtz waves. The present work addresses the points above through an adjoint-based sensitivity analysis along the lines of [7] the direct stability equations are written with account of viscosity, and the adjoint equations are found and solved in the temporal framework. Results in the spatial setting are discussed in Appendix B, where a digression is made on the computation of the group velocity of the instability waves by the use of the adjoint fields. The sensitivity functions to both mild modifications in the base shear layer and in the drag coefficient are computed and discussed. Finally, a different sensitivity analysis is developed on the basis of the recent anisotropic model by [42] and the results qualitatively compared to those obtained with the more conventional isotropic-drag-force model.

3.2 Model of the canopy flow

3.2.1 The mean flow

To obtain the mean flow on top of which small amplitude perturbations are superimposed, the procedure outlined by [16] and recently closely followed by [42] is used. For the sake of conciseness, the procedure which relies on several empirical correlations is not repeated here, aside from a few brief comments. A mildly inclined water channel is considered, with a canopy formed by rigid cylindrical dowels of height h equal to 13.8 cm and diameter $d = 0.64\text{ cm}$. The frontal area of the vegetation per unit volume, i.e., the packing density of the elements, is either $a = 0.04\text{ cm}^{-1}$ or 0.08 cm^{-1} ; the free surface is positioned at a level $H = 46.7\text{ cm}$ from the bottom plate and the flow velocity at the free surface, U_2 , varies

from 4.4 to 13.7 cm/s. The Froude number, $F_r = \frac{U_2}{gH}$ is thus very low and water surface fluctuations can be ignored [9]. To a good approximation the mean flow can be taken as steady and parallel, with the streamwise velocity varying from the value U_1 at the bottom wall (not accounting for the thin bottom boundary layer) to the value U_2 at the top, near the free surface (3.1). The slope of the bottom surface is very small; it is denoted as S and, in the experiments by [16] varies from 1.8×10^{-6}

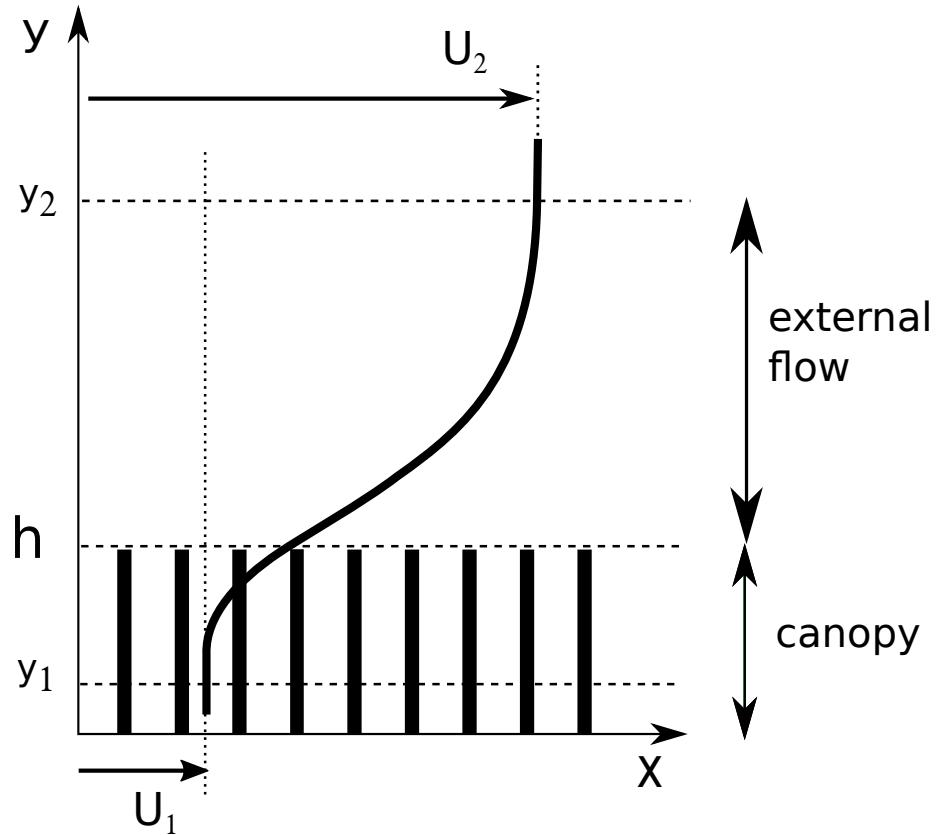


Figure 3.1: Configuration studied with main notations

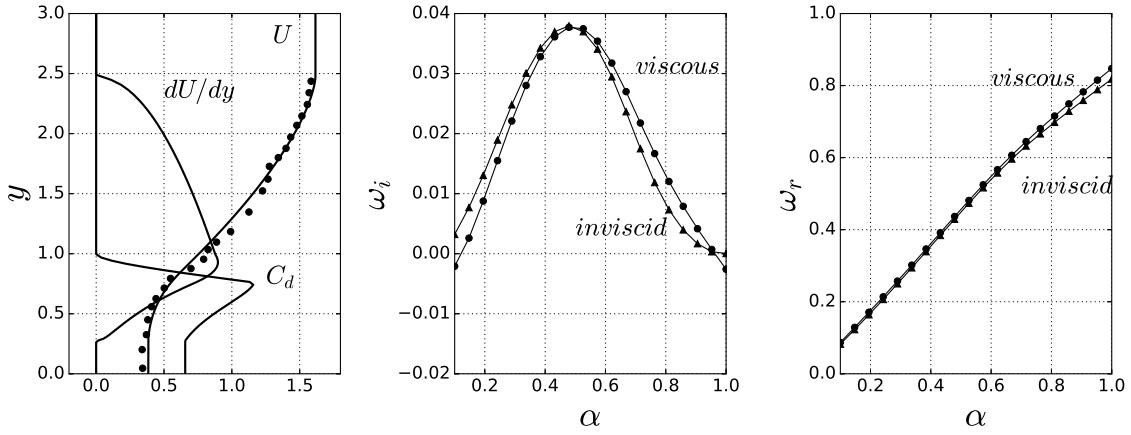


Figure 3.2: Left frame: mean flow U , together with experimental data points [16], its first derivative, and drag coefficient distribution (case G). Center: viscous and inviscid growth rates, ω_i , as a function of the streamwise wavenumber α . Right: corresponding frequencies, ω_r

to 10^{-4} ; such a slope provides the driving force for the motion. The viscous term is small compared to the turbulent diffusion term, so that the mean streamwise momentum equation can be approximated by

$$gS = \frac{\partial \bar{u}' \bar{v}'}{\partial y} + \frac{1}{2} C_D(y) a U(y)^2 \quad (3.1)$$

with g the acceleration of gravity and C_d an isotropic drag function available from the experiments, variable across the canopy and equal to zero when $y \geq h$. The Reynolds stress $\bar{u}' \bar{v}'$ is modelled with the Boussinesq assumption, introducing a turbulent viscosity which depends on a mixing length and on the gradient of the mean velocity U . Referring to [16] for details of the empirical correlations used to close the equations and the solution method, we limit ourselves here to stating that the results obtained for the mean flow are very close to those reported in [42] (cf. their Figure 3) and closely match experimental points for the cases G, H, I, and J considered (we use the same terminology of [18] [16] [17] to indicate the different flow configurations). An example of mean flow is reported in 3.2 (left frame). There, one can observe the computed flow (against discrete measurement points), its first derivative, and the drag coefficient distribution for one representative case (experiment G), used below also to discuss stability and sensitivity results. Other procedures have been employed in the past to calculate the mean flow, with satisfactory results. For example, [31] have considered a constant value of C_d through the canopy, while [42] have coupled, at a fictitious interface, the fluid equations outside the canopy to Darcy's law within the vegetation. Thus, for the purposes of the present paper, the mean flow is assumed as given; it could be, for example, simply a fit through experimental data.

Nonetheless, in Appendix A we provide some considerations on how C_d affects the mean flow in the model used here.

3.2.2 Stability and sensitivity equations

A temporal linear stability analysis is carried out, with the generic perturbation $q'(x, y, t)$ of the form

$$q'(x, y, y, t) = \tilde{q}(y)e^{i(\alpha x - \omega t)} \quad (3.2)$$

with α the real streamwise wavenumber and ω a complex number whose real part, ω_r , is the frequency of the mode and the imaginary part, ω_i , is the growth rate. The dimensionless linear stability equations in primitive variables read

$$\begin{aligned} i\alpha u + Dv &= 0, & D &= d/dy \\ \left[i(\alpha U - \omega) - \frac{D^2 - \alpha^2}{Re} + aC_d U \right] u + U' v + i\alpha p &= 0, & U' &= \frac{dU}{dy} \\ \left[i(\alpha U - \omega) - \frac{D^2 - \alpha^2}{Re} \right] v + Dp &= 0 \end{aligned} \quad (3.3)$$

with the perturbation velocity components which vanish when $y = 0$ and y_∞ . The upper boundary of the computational domain is taken far enough away from the lower boundary to ensure that the results do not vary upon modifications of y_∞ . All the terms in the equations are dimensionless; the mean speed through the shear layer, $U_m = \frac{U_1 + U_2}{2}$, is used to scale the disturbance velocity components, pressure is scaled with ρU_m^2 , distances with h , and time with h/U_m . The Reynolds number in the equations above is thus defined as $Re = \rho U_m / \mu h$, with ρ and μ the fluid's density and dynamic viscosity, respectively. The computations are performed both at the Re values of the experiments and in the inviscid limit ($Re^{-1} \rightarrow 0$), for comparison purposes. In the latter case, the boundary conditions are simply $v = 0$ at $y = 0$ and y_∞ . System 3.3 above and its boundary conditions are, in the following, also written in short notation as $\mathcal{L}q = 0$. The eigenvalues of the system are those complex values of ω which yield non-trivial solutions for u , v , and p . Two numerical collocation codes are written, and successfully compared; one is based on the equations in primitive variables form, the second solves an Orr-Sommerfeld-like equation (with the addition of the drag term) along the lines of [31]. In both cases, a spectral scheme based on N Chebyshev polynomials is used (N is typically equal to 300 to ensure grid-converged results), with an algebraic mapping between the physical and the spectral domains ([20]). Viscous and inviscid stability results for case G are shown in 3.2 (center and right frames); differences are small, in consideration of the fact that the Reynolds number of the viscous case is relatively large ($Re = 3450$). The viscous wavenumber of largest amplification is

found for $\alpha = 0.4790$; the waves are weakly dispersive, particularly at low wavenumbers (an original interpretation of phase and group velocities is proposed in Appendix B). The wavelength of largest growth is smaller than that found by [42] which was 0.73; this is related to the slightly different base flow in the two cases (in the present contribution a smoothing has been applied to the U velocity distribution to render dU/dy continuous across y) and highlights the sensitivity of this stability problem to base flow variations. Following [7] it is assumed that small variations in base flow and drag coefficient entail infinitesimal variations in the system's eigenvalues and eigenfunctions. We stress here the fact that C_d is identically equal to zero outside of the canopy, and this implies that there are no possible variations in C_d for $y \geq 1$. The sensitivity functions to variations in U and C_d are obtained by using the properties of the adjoint system which is defined from the Lagrange identity

$$0 = \delta\langle q^\dagger, \mathcal{L}q \rangle = \langle q^\dagger, \mathcal{L}\delta q \rangle + \langle q^\dagger, \frac{\partial \mathcal{L}}{\partial U} q \delta U \rangle + \langle q^\dagger, \frac{\partial \mathcal{L}}{\partial C_d} q \delta C_d \rangle + \langle q^\dagger, \frac{\partial \mathcal{L}}{\partial \omega} q \rangle \delta \omega \quad (3.4)$$

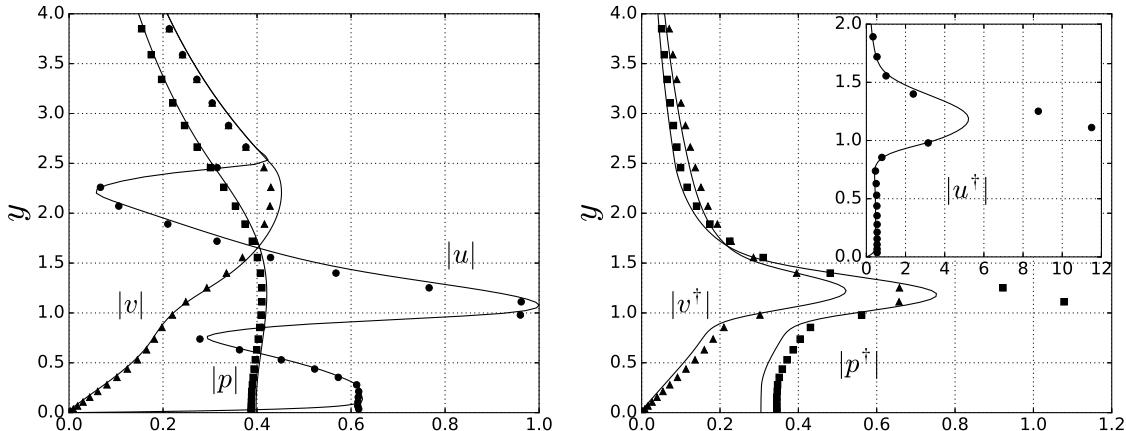


Figure 3.3: Moduli of direct (left frame) and adjoint (right frame) eigenfunctions for the viscous (continuous lines, $Re = 3450$) and the inviscid (symbols) case, in correspondence to the wavenumber of largest amplification.

and considering the effect of independent variations of U and C_d onto q and ω . It is found that

$$\delta\omega = \delta\omega_r + i\delta\omega_i = \int_0^{y_\infty} G_U(y)\delta U(y)dy + \int_0^1 G_{C_D}(y)\delta C_D(y)dy \quad (3.5)$$

with

$$\begin{aligned} G_U &= \alpha \left[\bar{v^\dagger v} + \bar{u^\dagger u} \right] + i(\bar{u^\dagger v})' - iaC_d \bar{u^\dagger u} \\ G_{C_d} &= -iaU \bar{u^\dagger u} \end{aligned} \quad (3.6)$$

the required sensitivity functions; the real parts of G_U and G_{C_d} express sensitivities to variations in the frequency of the mode while the imaginary parts are sensitivities to variations in the growth rate. Direct and adjoint eigenfunctions are normalized so that $N_\omega = 1$, with

$$N_\omega = \int_0^{y_\infty} \left[\bar{v^\dagger v} + \bar{u^\dagger u} \right] dy \quad (3.7)$$

An example of direct and adjoint eigenfunctions is provided in 3.3, both in the viscous case ($Re = 3450$) and in the inviscid limit, for $\alpha = 0.4790$. It is interesting to observe that while the direct eigenfunctions are almost overlapped, the same is not the case for the adjoint eigenfunctions, with the inviscid mode (drawn with symbols) which has a larger amplitude than the viscous one. The shapes of the direct eigenfunctions are very close to those reported in [42]. The adjoint modes reveal that the flow is most sensitive to streamwise forcing, particularly when it occurs slightly above the edge of the canopy. Source terms in the mass conservation and in the vertical momentum equations are much less effective.

3.3 SENSITIVITY RESULTS FOR THE ISOTROPIC DRAG MODEL

Some representative sensitivity functions are plotted in 3.4; viscous and inviscid results concur in showing that the largest sensitivities to variations of U are found right above the vegetation's edge, where there are peaks in the adjoint eigenfunctions and where d^2U/dy^2 vanishes. The U -sensitivities are negligible within the vegetated layer and for values of y larger than twice the canopy's height. The C_d -sensitivities are non-negligible only in close proximity of the interface. It is interesting to observe that real and imaginary parts of the U -sensitivity functions are shifted in y with respect to one another; this means that, for example, a localized perturbation at a given y position (above the canopy) might have a strong repercussion on the growth rate but not on the frequency of the most unstable Kelvin-Helmholtz mode, or vice versa. Comparing left and right frames of the figure, it is seen that inviscid G_U sensitivity functions display sharper peaks and steeper gradients, and yield larger variations in ω than their viscous counterparts in the proximity of the U inflection point, a clear consequence of the inviscid mechanism ruling the instability. In both the viscous and the inviscid models, the sensitivity to base flow variations is typically one order of magnitude larger than the sensitivity to changes in the drag coefficient.

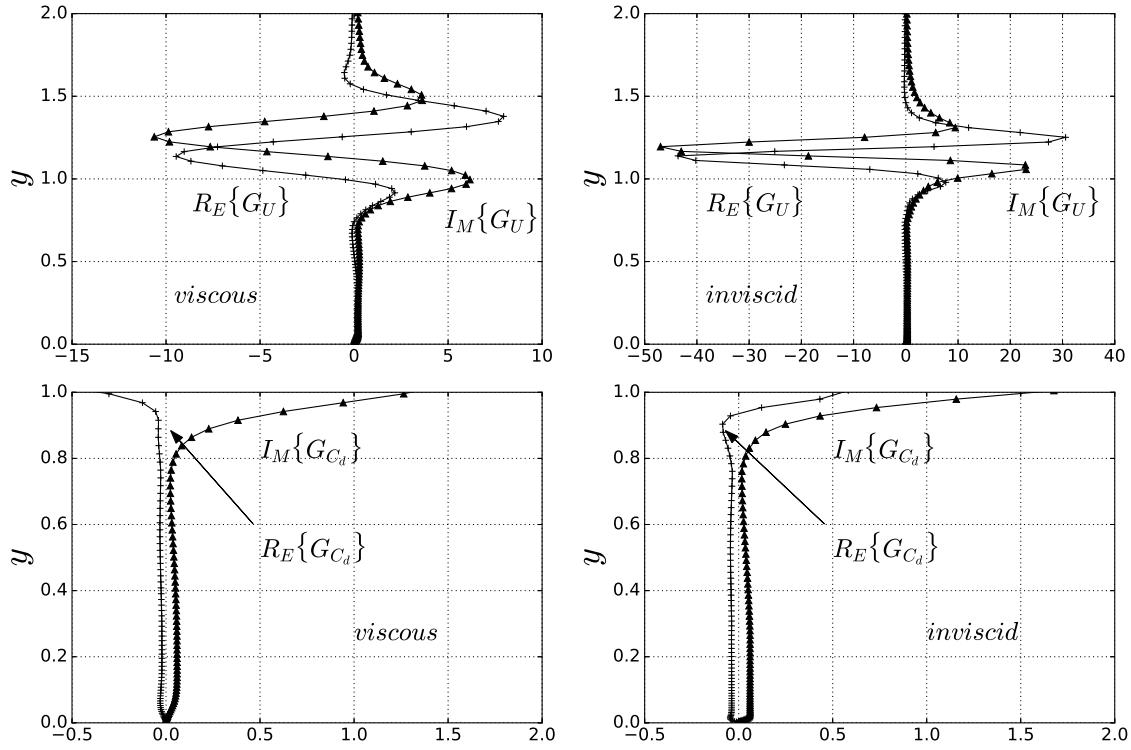


Figure 3.4: Real and imaginary parts of the sensitivities to mean flow variations (top) and to variations in the drag distribution function (bottom), for the parameters of 3.3

The infinite norm of the sensitivities for the four cases studied (G, H, I, and J) is reported in 3.5; the main result found is that $|G_U|_\infty$ grows monotonically with α (and more so in the inviscid case) whereas $|G_{C_d}|_\infty$ does not. It is consistently found that $|G_U|_\infty$ of case H is larger than that of case I, which exceeds the corresponding value of case J, in turn larger than $|G_U|_\infty$ of case G. This is not unexpected in view of the values of the mean shear $\frac{U_2 - U_1}{H}$ which are, going from H to G, equal

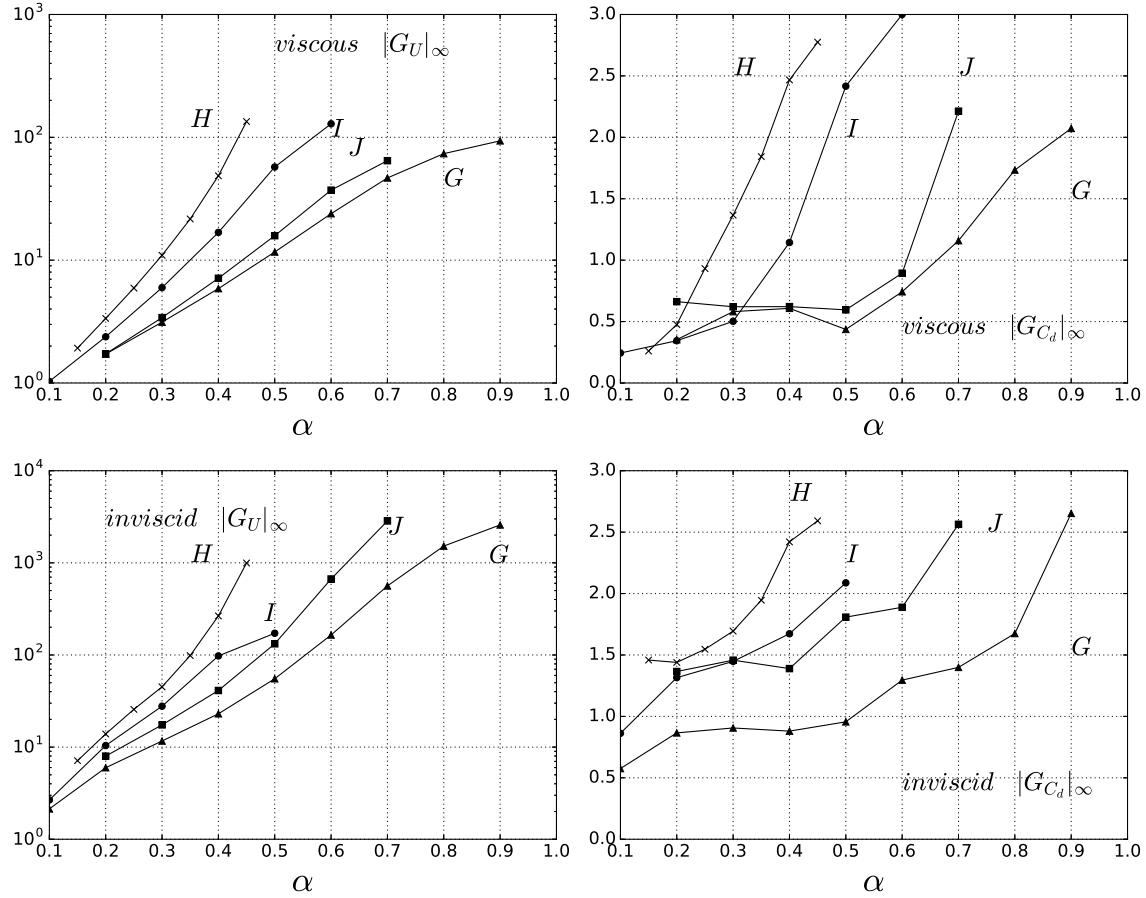


Figure 3.5: Infinite norms of the sensitivity functions for varying α

to 0.236 , 0.158 , 0.084 , and $0.071 s^{-1}$, respectively. The sensitivity of the eigenvalue ω to variations in the mean flow is generally stronger than the corresponding sensitivity to variations in the drag coefficient (aside for the long wave limit, where they are comparable). This might be interpreted positively, considering that the use of a scalar coefficient C_d to represent the drag within the canopy is but a crude approximation. An alternative model to represent the flow throughout a network of rigid, cylindrical dowels has recently been proposed by [42]. The sensitivity results for such a new model are discussed next.

3.4 AN ALTERNATIVE SENSITIVITY MODEL: ACCOUNTING FOR THE CANOPY ANISOTROPICITY

The stability problem in this section is based on the coupling between two regions, one outer region dominated by inertia and ruled by the inviscid equations and an inner one dominated by viscosity and ruled by Darcy's law, with account of the canopy geometry through a tensorial permeability, as described by [42]. Normalizing the disturbance equation which couples pressure and velocity in the inner region with the same scales as previously, we obtain

$$u_i' = -Re \frac{d}{ah^2} \mathcal{K}_{ij} \frac{\partial p'}{\partial x_j}, \quad (x_1, x_2) = (x, y) \quad (3.8)$$

with \mathcal{K}_{ij} the dimensionless permeability. The effective interface between the inertial region and the slow, viscosity-dominated region does not coincide with the edge of the canopy; in fact, the rapid outer flow penetrates through the upper part of the vegetation and an effective matching between outer and inner flows must be enforced some distance δ below the canopy's edge [24]. This distance, a penetration depth, has been successfully computed by [39] for a few cases and is found to increase with the Reynolds number of the flow; for experiment G discussed below it is $\delta = 0.40$ [40]. On account of the results shown in 3.4, with the sensitivities which are negligible for $y \approx 0.60$, we expect that the exact position of the effective interface will not affect the results significantly. Using the fact that the velocity within the orthotropic porous medium is divergence free, the interface condition to be applied at $y_{itf} = 1 - \delta$ is found to be 3.9

$$v|_{itf} + B(\alpha)p|_{itf} = 0 \quad (3.9)$$

with

$$B(\alpha) = Re \frac{d}{ah^2} \sqrt{\mathcal{K}_{11} \mathcal{K}_{22}} \alpha \tanh(\theta), \quad \theta = \alpha \sqrt{\frac{\mathcal{K}_{11}}{\mathcal{K}_{22}}} y_{itf}$$

The second boundary condition that the Rayleigh stability equation must satisfy at y_∞ is simply $v = 0$. Thus, we solve only for the inviscid flow in the outer region, and the permeability of the inner domain enters the equations only through the interface condition 3.9. \mathcal{K}_{ij} is a two- by-two diagonal tensor; \mathcal{K}_{11} is the component of the dimensionless permeability along x and \mathcal{K}_{22} is the y component. For case G considered here, the packing density of the elements is $a = 0.04 \text{ cm}^{-1}$; it is also found that $\mathcal{K}_{11} = 0.0512$ and $\mathcal{K}_{22} = 0.0575$ [40], so that the function $B(\alpha)$ reads $B = 15.727\alpha \tanh(0.566\alpha)$.

3.4.1 The sensitivity equations

The adjoint equations in this case are the same as system 3.3, without the terms containing $1/Re$ and C_d , and the boundary conditions are

$$v^\dagger|_{itf} - B(\alpha)p^\dagger|_{itf} = 0, \quad v^\dagger|_{y_\infty} = 0 \quad (3.10)$$

The variation in the complex frequency is related to variations in the mean flow and in the permeability components through the equation

$$\delta\omega = \int_{y_{itf}}^{y_\infty} G_U(y)\delta U(y)dy + G_{\mathcal{K}_{11}}\delta\mathcal{K}_{11} + G_{\mathcal{K}_{22}}\delta\mathcal{K}_{22}$$

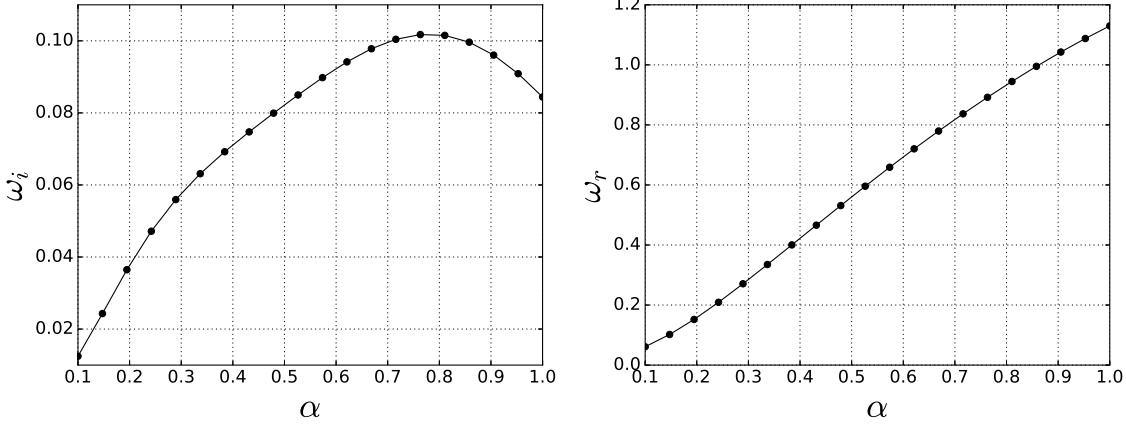


Figure 3.6: Amplification factor (left) and frequency of the most unstable mode as a function of α , for the anisotropic drag model

with

$$\begin{aligned} G_U &= \alpha \left[\overline{v^\dagger v} + \overline{u^\dagger u} \right] + i(\overline{u^\dagger v})' \\ G_{\mathcal{K}_{11}} &= -\frac{i}{2}\alpha Re \frac{d}{ah^2} \left[\overline{p^\dagger p} \right] |_{itf} \sqrt{\frac{\mathcal{K}_{22}}{\mathcal{K}_{11}}} \left\{ \tanh \theta + \frac{\theta}{\cosh^2 \theta} \right\} \\ G_{\mathcal{K}_{22}} &= -\frac{i}{2}\alpha Re \frac{d}{ah^2} \left[\overline{p^\dagger p} \right] |_{itf} \sqrt{\frac{\mathcal{K}_{11}}{\mathcal{K}_{22}}} \left\{ \tanh \theta - \frac{\theta}{\cosh^2 \theta} \right\} \end{aligned} \quad (3.11)$$

the required sensitivities, with the normalization $\int_{y_{itf}}^{y_\infty} [\overline{v^\dagger v} + \overline{u^\dagger u}] = 1$. In writing $\delta\omega$ above, we have made the assumption that the mean flow U does not vary at the two extreme points of the integration domain. The stability results (for the same parameters as in 3.2)

are displayed in 3.6. As already observed in [42], both the growth rate and the frequency are slightly larger with this model than with the isotropic resistance model, for all α 's, and the most unstable mode is found at a larger value of α (here $\alpha \approx 0.8$) in better agreement with experimental correlations [42] [29]. Also in this case the waves are found to be only weakly dispersive. Eigenfunctions are plotted in 3.7, together with the real and imaginary parts of the G_U sensitivity function. As in 3.3, the modulus of the u eigenfunction peaks near the edge of the canopy ($y = 1$), whereas the adjoint eigenfunctions have a maximum value slightly above. As a general remark, the shapes of the direct and adjoint modes are quite similar to those found with the isotropic resistance model; as reported at the end of 3.2.2, it is found that the flow is most sensitive to streamwise momentum forcing. Also, real and imaginary parts of G_U have a double-peak structure, like in the isotropic-drag model, but now the largest absolute value of G_U is

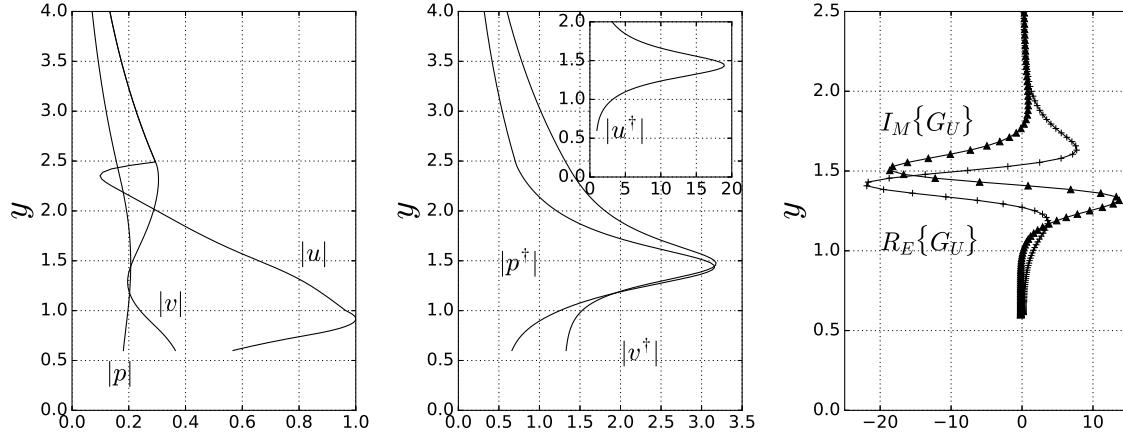


Figure 3.7: Left and center frames: moduli of direct and adjoint eigenfunctions; pressure and “adjoint pressure” are drawn with dashed lines. Right: real and imaginary parts of the sensitivity function G_U ($\alpha = 0.4790$)

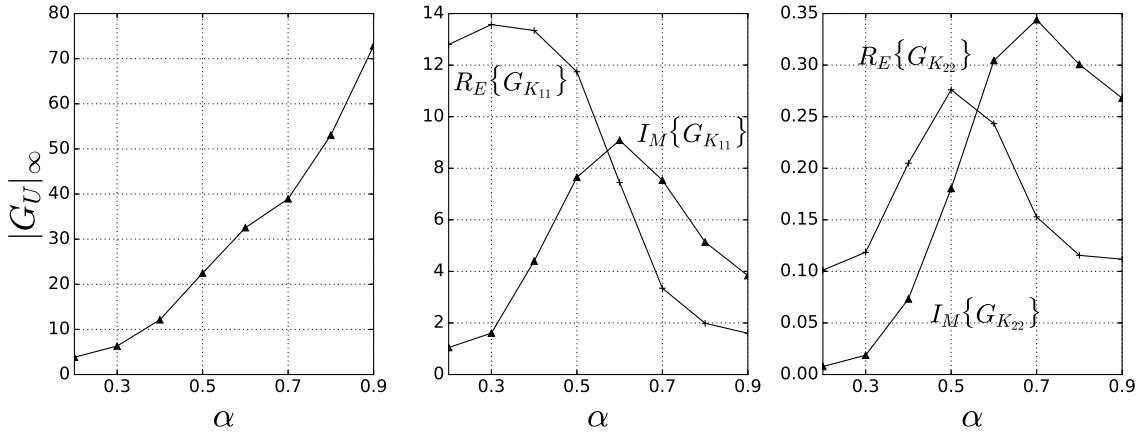


Figure 3.8: Case G. Left: infinite norm of GU for varying α . Center and right frames: real and imaginary parts of the sensitivity coefficients to variations in the permeability components

smaller and shifted towards a larger y than in the previous inviscid case (cf.3.4, top-right frame). This can also be appreciated by the inspection of 3.8 (left); $|GU|_\infty$ still grows monotonically with α , but the sensitivity is smaller than that computed earlier (cf. 3.5) with either the viscous or inviscid model (it is actually closer to the viscous sensitivity, as an effect of the interface condition). Furthermore, it is interesting to observe that both real and imaginary parts of GU vanish for $y = y|_{itf}$ (cf.3.7, right), and this supports the statement made previously that a small shift in the position of the effective interface has but a minor influence on the most unstable mode. The sensitivity coefficients for the two components of the permeability tensors are displayed in 3.8 (center and right frames): the present model is more effective to variations in K_{11} than to K_{22} as far as modifying the complex eigenfrequency. Significantly, different ranges of wavenumbers behave differently as far as the variation in ω is concerned. The frequency ω_r of long waves (around $\alpha \approx 0.3$) is more easily modified by acting on K_{11} (with an almost negligible effect on the growth rate of the wave); conversely, the growth rate of modes with large values of α is affected efficiently by variations in the first component of the permeability tensor.

3.5 CONCLUDING REMARKS

We have considered two different models of the flow through a vegetated layer experiencing Kelvin-Helmholtz destabilization. One model is based on the use of a single drag coefficient to express the force exerted by the vegetation on the fluid, the second considers the canopy as an orthotropic porous medium and is based on Darcy's equation with a tensorial permeability [39]. Both models have advantages and drawbacks. The main

advantage of the first model is that the drag coefficient can be taken to vary across the canopy; whether this positive consideration, based on macroscopic experimental measurements [18] [16] [17], carries over to the stability problem remains to be established. The second model, applicable to dense porous media, considers two independent parameters to express the disturbance flow perpendicular and parallel to the rigid dowels forming the canopy. Such parameters and components of the transversely isotropic permeability tensor K_{ij} arise from the solution of a local Oseen problem [39]. The drawback of the second model is the fact that an interface (whether real or effective) appears, and adequate matching conditions must be enforced there. Despite much work since the seminal contribution by [6], a consensus on the “best” interface conditions between a pure fluid region and a porous medium has not yet emerged. The models have been put to test through a classical sensitivity analysis [7]. Beyond displaying stability results which correspond better to those to be expected from available experimental correlations [29] [42], the anisotropic model is less sensitive to variations in the base flow (with potentially larger variations in frequency and growth rate of the instability mode for the case of shorter waves). As far as a direct comparison between G_{C_d} and $G_{K_{ii}}$ is concerned, this can hardly be made since the variables represent different objects; in particular, the pressure drop through the canopy depends directly on C_d and inversely on the permeability. The present results indicate that the anisotropic model depends significantly on the value of the apparent [39] permeability component K_{11} , whose evaluation must thus be conducted carefully. This model is also of interest for further developments, in particular for the study of instabilities developing over waving canopies. Darcy’s law in this latter case would need to be modified, as described in [25] and [41].

Acknowledgment

The authors would like to thank the IDEX Foundation of the University of Toulouse for the financial support granted to the last author under the project “Attractivity Chairs.” The computations have been conducted at the CALMIP center, Grant No. P1540. The referees are gratefully acknowledged for their comments leading, in particular, to the correct interpretation of the sensitivity of the drag coefficient and to the material in Appendix A.

APPENDIX A: EFFECT OF C_d ON THE MEAN FLOW

In 3.2 of the paper it is described how the eigenvalue ω varies as an effect of independent variations of U and C_d . However, since C_d is not zero within the canopy and it is used to compute the mean flow profile U , we should in principle have expressed δU as $\delta U = \frac{dU}{dC_d} \delta C_d$ and considered a single sensitivity function $G^*_{C_d} = G_{C_d} + \frac{dU}{dC_d} G_U$, instead of the two sensitivities given in 3.6. This would have certainly been the appropriate line

of action if the mean flow equation were issued from exact equations, in which case we should have considered also the adjoint of the base flow equation in our variational problem. However, the mean flow model by [16] contains empirical approximations and parameters, and alternative models [31], [42]—including very different ones—have been used successfully in the past to predict the mean field; we have thus made the choice, in both 3.3 and 3.4, of considering the mean flow as given, and to take independent variations of U and C_d in the stability analysis to assess the effect of modifications in either variable. If we were to find how much the base flow depends on the drag coefficient in this particular problem, we would need to determine the function $U(C_d)$ and take its derivative. Since both U and C_d are functions of the space coordinate y , the implicit dependence can be found, and we have plotted it for one case on the left frame of 3.9. Clearly, the function $U = f(C_d)$ is not single-valued and therefore the derivative can be calculated only over two separate U (or, equivalently, y) intervals. We have carried out the derivation numerically over each interval, within the range $0.3 \leq y \leq 1$, and the result is reported on the right frame of 3.9. The filled triangle and circle symbols indicate the two y intervals within the canopy. We first observe that both the location where C_d is maximum and the shape of the function $U = f(C_d)$ are strongly correlated to the drag law $C_d(y)$, modeled by [16]

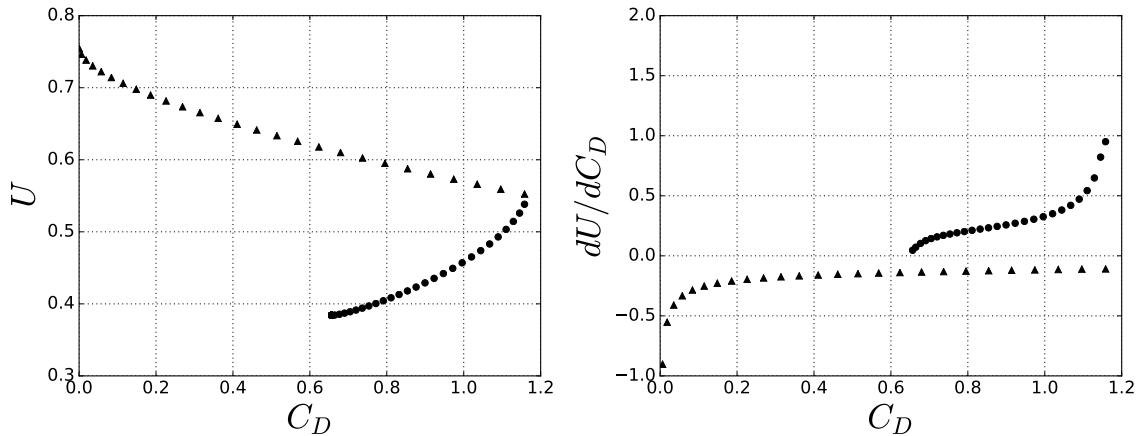


Figure 3.9: Case G. Left: mean velocity profile, U , versus the drag coefficient, C_d . Right: first derivative, dU/dC_d . The triangles denote the region $y \in [0.76, 1]$, the filled circles denote the region $y \in [0.3, 0.76]$.

through their measurement data (cf. their Figure 7 and Equation (18)). We also notice that the derivative dU/dC_d is reasonably small except locally at the point where the derivative of the function is not continuous, where it is of order 1. The discontinuity there is however artificial since the function $C_d(y)$ given in Equation (18) of [16], where C_d is divided into a parabolic and a linear part, can be easily modified to yield a continuous first derivative at $y = 0.76$ if required, still maintaining a mean flow very close to the measured

one.

APPENDIX B: A DIGRESSION ON SPATIAL STABILITY THEORY AND GROUP VELOCITY

Stability problems such as the first one considered in this paper can be approached with the spatial theory framework, with the wavenumber α complex, its imaginary part being a growth rate, and the circular frequency ω a real constant parameter. Let us generalize the sensitivity analysis by considering, as a first step, α and ω as complex numbers which can vary. Equation 3.4 contains one additional term and reads:

$$0 = \delta\langle q^\dagger, \mathcal{L}q \rangle = \langle q^\dagger, \mathcal{L}\delta q \rangle + \langle q^\dagger, \frac{\partial \mathcal{L}}{\partial U} q\delta U \rangle + \langle q^\dagger, \frac{\partial \mathcal{L}}{\partial C_d} q\delta C_d \rangle + \langle q^\dagger, \frac{\partial \mathcal{L}}{\partial \omega} q \rangle \delta \omega + \langle q^\dagger, \frac{\partial \mathcal{L}}{\partial \alpha} q \rangle \delta \alpha \quad (3.12)$$

To obtain the sensitivities in the spatial problem (for which $\delta\omega = 0$) we now have to solve an adjoint system similar to 3.3, where ω^\dagger is replaced by ω and α by α^\dagger . The variation of the wavenumber $\delta\alpha = 0$ is thus given by:

$$\delta\alpha = \delta\alpha_r + i\delta\alpha_i = \int_0^{y_\infty} G_U(y)\delta U(y)dy + \int_0^1 G_{C_d}(y)\delta C_d(y)dy$$

the functions G_U and G_{C_d} maintain the same form as in the temporal theory 3.6, with the direct and adjoint eigenfunctions which are now normalized by imposing that $N_\alpha = -1$, with

$$N_\alpha = \int_0^{y_\infty} \left[\left(U - \frac{2i\alpha}{Re} \right) (\bar{v}^\dagger v + \bar{u}^\dagger u) + \bar{p}^\dagger u + \bar{u}^\dagger p \right] d y$$

Let us now consider a problem in which U and C_d are not allowed to vary, but α and ω are. With reference to Equation 3.12, with any choice of normalization of direct and adjoint modes, it is found that $N_\omega\delta\omega = N_\alpha\delta\alpha$. Thus, once the adjoint problem is solved, it is possible to accurately compute the group velocity c_g of any stability problem using the value of N_ω and N_α , i.e.,

$$c_g := \frac{d\omega_r}{d\alpha_r} \approx \frac{\text{real}(N_\alpha)}{\text{real}(N_\omega)} \quad (3.13)$$

Note that c_g above is different from the “complex group velocity” $C_g := \frac{d\omega}{d\alpha} \approx \frac{N_\alpha}{N_\omega}$, and it is also $c_g \neq \text{real}(C_g)$. Relation 3.13 can be employed in either a spatial or temporal stability analysis and some representative results (for case G) are provided in Table I with the phase velocity $c_r := \omega_r/\alpha_r$ and the group velocity determined from Equation 3.13. The temporal or spatial amplification factors, ω_i or $-\alpha_i$, respectively, are also given for all

cases using Gaster's transformation: $\omega_i = -\alpha_i c_g$. Two types of errors on the calculation of the group velocity (noted *err*) are given in the table; the top four values, relative to the temporal theory, are defined as

$$err = \frac{|c_g|_{3.13} - c_g|_{FD}|}{c_g|_{3.13}}$$

with $c_g|_{FD}$ arising from a first-order finite difference approximation of the group velocity. The bottom four values are defined by the formula

$$err = \frac{|c_g|_{temporal} - c_g|_{spatial}|}{c_g|_{temporal}}$$

The relative difference on c_g between temporal and spatial theory is rather low. It has to be kept in mind, however, that a stability analysis in the spatial framework yields a nonlinear eigenvalue problem, with a consequent larger numerical system than in the temporal framework; therefore, by inverting matrices of the same size, the accuracy is expected to be slightly lower. The accuracy of the growth rate approximated through Gaster's relationship is also found to be acceptable.

Theory	<i>Re</i>	α_r	ω_r	$-\alpha_i$	ω_i	c_r	c_g	<i>err</i> (%)
Temporal	500	0.5	0.4778	<i>0.0248</i>	0.0254	0.9556	1.0245	0.54
	3450	0.5	0.4601	<i>0.0413</i>	0.0404	0.9202	0.9797	0.06
	10^5	0.5	0.4514	<i>0.0436</i>	0.0421	0.9028	0.9661	0.63
	10^9	0.5	0.4508	<i>0.0451</i>	0.0425	0.9016	0.9427	2.90
Spatial	500	0.4993	0.4778	0.0248	0.0250	0.9569	1.0100	1.41
	3450	0.4990	0.4601	0.0427	0.0404	0.9220	0.9471	3.30
	10^5	0.4996	0.4514	0.0449	0.0416	0.9109	0.9371	3.46
	10^9	0.4993	0.4508	0.0450	0.0411	0.9028	0.9143	3.01

Table 3.1: Temporal versus spatial stability, Case G. The model employed here is based on a modified Orr-Sommerfeld equation—rather than a system based on primitive variables as done in the bulk of the paper—which is why the temporal results have slightly larger growth rates ω_i than those displayed in Fig. 3.2; this is related to the need of computing numerically d^2U/dy^2 and dC_d/dy in the Orr-Sommerfeld-like equation. In italics, the growth rates obtained from Gaster's transformation are reported; the parameters imposed in each simulation are indicated with bold characters. The solutions for $Re = 10^9$ coincide with those found using the inviscid equations.

The amplitude of the sensitivity functions, $|G_U(y)|$ and $|G_{C_d}(y)|$, in the spatial and temporal stability frameworks is of same order of magnitude (not shown here) since they are related through temporal spatial the complex group velocity C_g . It is found that

$|G_U^{temporal}| \approx |C_g| |G_U^{spatial}|$ with $|C_g| \approx c_g \approx 1$ in the present case. Obtaining and comparing results in the temporal and spatial stability frameworks, such as in Table I, is a good means to validate the sensitivity functions and to verify the accuracy of the computations of the adjoint stability equations.

APPENDIX C: correction to compare continuous and discrete adjoint eigenfunctions

The discretization operation transform the operator \mathcal{L} into a matrix \mathbf{A} and of course do the same things to the unknown functions that becomes vectors.

continuous	discrete
\mathcal{L}	\mathbf{A}
q	\hat{q}

This has a serious and most often hidden repercussion in th approach to solve the adjoint equations.

As above stated the derivation of the adjoint equation start with the enforcing of the Lagrange identity:

$$\langle q; \mathcal{L}q \rangle = \langle \mathcal{L}^\dagger q^\dagger; q \rangle \quad (3.14)$$

where the scalar product $\langle ; \rangle$ is defined in our case as:

$$\langle a; b \rangle = \int_0^{y_\infty} \bar{a} \cdot b dy \approx \sum_{i=1}^N \sum_{j=1}^N \hat{a}_i^T w_{i,j} \hat{b}_j = \hat{a}_i^T \mathbf{M} \hat{b}_j = \langle a; b \rangle_{\mathbf{M}} \quad (3.15)$$

Is it clear from equation 3.15 that the scalar product takes two different forms in the continuous and in the discrete case. In fact in the discrete case is mandatory to introduce the quadrature rule weights $w_{i,j}$ of the chosen discretization. \mathbf{M} is the matrix representation of the weights and is symmetric and positive defined.

In order to compute and solve the adjoint equation one could proceed as follow:

- The direct problem is defined in the continuous space as $\mathcal{L}q = 0$
- Chose a discretization (FEM, FD, Chebychev polynomials...) and transform the above problem in a discrete one $\mathbf{A}\hat{q}$
- Solve it to obtain the discrete version of the eigenfunctions \hat{q}

For the adjoint problem on should at first compute the adjoint operator, this can be done using the Lagrangian identity at a continuous level:

$$\begin{aligned} \langle q; \mathcal{L}q \rangle &= \langle \mathcal{L}^\dagger q^\dagger; q \rangle \\ \Rightarrow \int_0^{y_\infty} \overline{q^\dagger} \mathcal{L}q dy &= \int_0^{y_\infty} \overline{\mathcal{L}^\dagger q^\dagger} q dy \end{aligned} \quad (3.16)$$

From the last equation starting from the left part is it possible after some manipulation to retrieve the form on the right part and so find the formulation of the adjoint operator.

It is important to pinpoint that in the above equation the scalar product $\langle a; b \rangle$ is enforced at a continuous level.

And now to solve the adjoint system the procedure 3.5 can be used changing the direct system with the adjoint one. The above way of computing the adjoint and solve the system is called **continuous approach**.

To summarize this approach one can straight forward solve the direct problem computationally, mathematically find the adjoint operator using the continuous scalar product and the Lagrange identity and then discretize the adjoint problem and solve it computationally. This is why the **continuous approach** is sometimes known as derive than discretize. And the stability and accuracy problems derive directly from the fact that we discretize the problem two times (the direct first and than the adjoint).

On the contrary in the **discrete approach** the scalar product 3.15 is enforced at the discrete level in order to use the already discretized direct equation to retrive the adjoint system at a discrete level, to limit the computational errors.

$$\begin{aligned} \langle q^\dagger; \mathcal{L}q \rangle &= \langle \mathcal{L}^\dagger q^\dagger; q \rangle \\ \Rightarrow \overline{\hat{q}^\dagger}^T \mathbf{M} \mathbf{A} \hat{q} &= (\overline{\mathbf{A}^\dagger \hat{q}^\dagger})^T \mathbf{M} \hat{q} \\ \Rightarrow \mathbf{M} \mathbf{A} &= \overline{\mathbf{A}^\dagger}^T \mathbf{M} \\ \Rightarrow \mathbf{A}^\dagger &= \mathbf{M}^{-1} \overline{\mathbf{A}}^T \mathbf{M} \end{aligned} \quad (3.17)$$

Chapter 4

Effect of geometrical parameters and inertia on the apparent permeability tensor in fibrous porous media

4.1 Introduction

The flow through porous media is a problem of importance for several natural and technological applications. Since Darcy's original formulation [12], which relates the flow rate through a porous bed to the pressure drop across the bed's sides, many corrections have been made to account, for example, for viscous effects [10] or for the consequences of inertia [15]. All of the cited works are of empirical nature, but homogenisation has been able to recover all of these formulations rigorously starting from the Navier-Stokes equations [36]. This latter approach is sometimes defined VANS, for Volume-Averaged Navier-Stokes.

The theory requires the knowledge of a number of terms, most notably, in the case of an isotropic porous bed, a permeability coefficient and a Forchheimer coefficient. Initial efforts in defining these terms were based on a combination of physical reasoning and measurements, leading to expressions known as the Kozeny-Carman [22, 11] and the Ergun [14] correlations. The first provides the permeability for the laminar flow of a single-phase fluid through a packed bed of sand grains, as function of the porosity and the diameter of the grains, while the second extends Darcy's law to let the pressure drop depend on two terms, one proportional to the velocity and the second to its square, thus accounting for inertia. These approaches do not consider microstructural or geometrical features of the porous bed, which can render the permeability a tensorial quantity, and are often restricted to simple unidirectional flows. In the present work we are concerned with a transversely isotropic material composed by parallel fibers of circular cross-section, with one axis of symmetry,

(O, x_3) ; in such materials the permeability is a diagonal tensor with the component in the direction parallel to the fibers greater than those along the transverse axes. For such an arrangement we will investigate the effects of both the direction of the forcing pressure gradient and inertia. When the latter effect is present, embodied by a Reynolds number Re_d , based on mean velocity through the medium and fibers' diameter, exceeding an order one threshold, the permeability is no more simply defined upon geometrical properties. This new permeability, which arises from a well-defined closure problem, is then called *apparent permeability*.

The influence of the geometry of the solid inclusions has been addressed previously by Yazdchi et al. [37] for arrays of cylinders in both square and hexagonal (or staggered) patterns, with the cylinders' section which can vary in shape. The results, in the two-dimensional and low Reynolds number limits, demonstrate the dependence of the permeability component along the flow direction to both the porosity and the direction of the macroscopic pressure gradient. The direction of the pressure gradient is found to have a weak effect for beds of medium-high porosity ($\varepsilon > 0.7$) and a stronger dependence appears upon the geometry of the solid inclusions.

The influence of the Reynolds number on the permeability and on the Forchheimer correction has been presented in a number of papers. One of the contributions most relevant here is due to Edwards et al. [13]. These authors show that, for arrays of fibers, the apparent permeability decreases with the increase of the Reynolds number, and the rate of this decrease depends on the geometry of the array; also, the Reynolds number is found to have a stronger influence on the apparent permeability when the medium is highly porous. The results of the work by Edwards et al. [13] agree with those by Zampogna and Bottaro [38] and with our own work (as shown later), all for the case of cylindrical fibers, although some issues remain on the persistence of steady solutions in the simulations by Edwards et al. [13] in cases for which a limit cycle should have set in. A fully three-dimensional porous medium, more complex than those discussed so far, has been considered by Soulaine and Quintard [32], confirming the decreasing trend of the apparent permeability with the Reynolds number.

Another contribution which deserves mention is that by Lasseux et al. [23]; they have computed the permeability tensor for various Reynolds numbers, in a two-dimensional geometry with cylinders of square cross-section. Forcing the flow along the main symmetric directions of the fiber, Lasseux et al. [23] have identified different regimes:

- a creeping flow regime for $0 < Re_d < 10^{-3}$, without Forchheimer terms;
- a weak inertia regime for $10^{-3} < Re_d < 1$, with the Forchheimer correction quadratic in Re_d ;
- a strong inertia regime for $1 < Re_d < 10$, where the Forchheimer correction is linear with the Reynolds number;

- a turbulent regime, for $Re_d > 10$, with the Forchheimer correction again quadratic with the Reynolds number.

The boundaries between the different regimes are specific to the geometrical arrangements and to the porosities being considered; a step forward in rendering (some of) these boundaries rigorous and independent of the arrangement of the pores, through the definition of a Reynolds number which accounts for a "topological" coefficient, has been recently made by Pauthenet et al. [27]. For the purposes of the present paper, we must retain that Lasseux et al. [23] have parametrized the Forchheimer correction with the Reynolds number, and have found that the inertial correction is orders of magnitude smaller than the Darcy's term, at least before the turbulent regime sets in. Moreover, Lasseux et al. [23] have studied how a Forchheimer tensor, \mathbf{F} , depends upon the direction of the macroscopic forcing term with respect to the orientation of the square cross-section of the fibers, for Re_d up to 30. It is concluded that a deviation angle, γ , exists between the direction of the pressure gradient and that of the mean flow, because of the fibers' geometry. Finally, the inertial correction is strongly influenced by the orientation of the driving pressure gradient, and the tensor \mathbf{F} is not symmetric (in fact the off-diagonal components are found to be inversely proportional to the diagonal terms, and symmetric with respect to rotations about the diagonal axis of the square, i.e. the direction at 45° in the $x_1 - x_2$ plane).

The effect of variations in the forcing angle, with restrictions to angles in the $x_1 - x_2$ plane, is also examined by Soulaine and Quintard [32] with conclusions in qualitative agreement with those of both the contribution just cited and our results described further below. In all cases, the off-diagonal components of the apparent permeability tensor are small and the diagonal components display but a small variation upon rotation of the driving pressure gradient.

As already anticipated, this work investigates how the direction of the macroscopic pressure gradient, the porosity and the Reynolds number can modify the Darcy and Forchheimer closures arising from a VANS model of a fibrous porous medium. We will consider a three-dimensional unit cell for the microscopic model (such a unit cell is sometimes denoted REV, for Representative Elementary Volume), with a generic forcing whose direction is defined by two Euler angles. Given the formidable space of parameters, some representative results are first shown and discussed. Response surfaces in the space of parameters are then identified by the use of a metamodel based on kriging interpolation. For the sake of space, only the first diagonal component of the apparent permeability tensor is discussed in detail in the paper; however, all components have been computed. They represent an extremely useful data base which we are now in the process of using in macroscopic simulations of flows through bundles of fibers of varying orientation and density.

4.2 The Volume-Averaged Navier-Stokes (VANS) method

4.2.1 A brief description of the method

The system under investigation consists of an incompressible Newtonian fluid which flows through a rigid porous medium. In the following, the subscript β is used to indicate the fluid phase while σ is adopted for the solid phase. The governing equations valid at the microscale are

$$\frac{\partial \mathbf{v}_\beta}{\partial t} + \mathbf{v}_\beta \cdot \nabla \mathbf{v}_\beta = -\frac{1}{\rho_\beta} \nabla p_\beta + \nu_\beta \nabla^2 \mathbf{v}_\beta + \mathbf{f}, \quad (4.1)$$

$$\nabla \cdot \mathbf{v}_\beta = 0, \quad (4.2)$$

where \mathbf{v}_β , p_β , ρ_β and ν_β stand, respectively, for the velocity, the pressure, the density and the kinematic viscosity of the fluid. The right-hand side term, \mathbf{f} , is a force (per unit mass) which drives the fluid motion and can be interpreted as the macroscopic pressure gradient acting on the system.

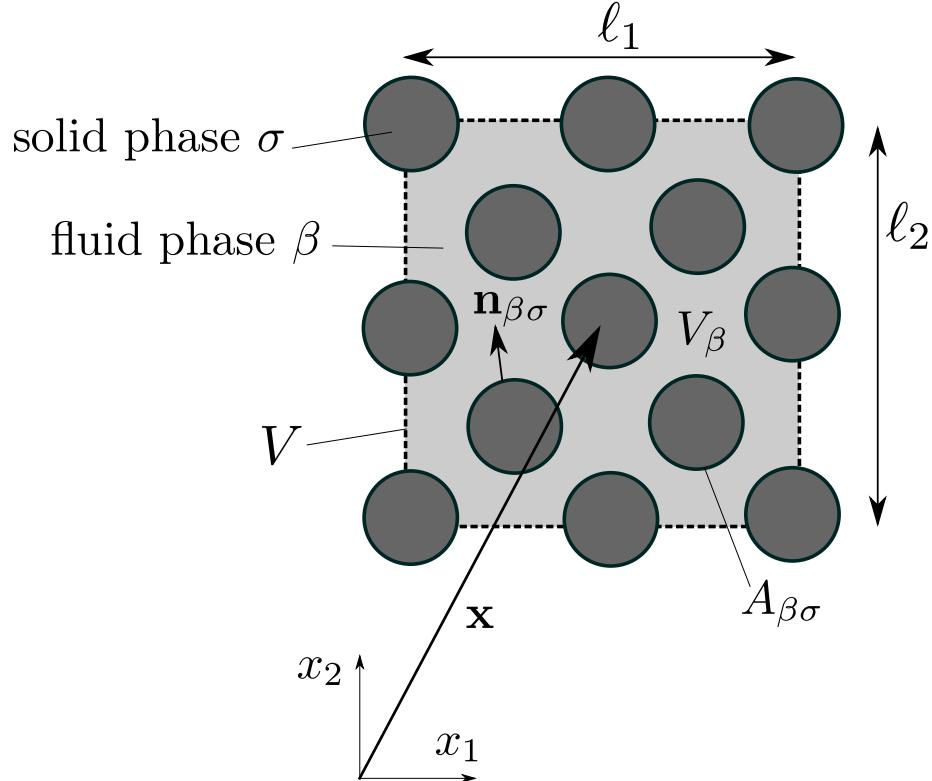


Figure 4.1: Illustration of the REV concept.

The concept of Reference Elementary Volume (REV) of the porous medium is classically

introduced in the framework of the VANS approach. An example of REV is depicted on figure 4.1, together with relevant notations (volume shape and size, indication of the fact that the normal unit vector is directed from the fluid to the solid phase, centroid \mathbf{x} of the REV). The REV represents the domain over which the microscopic problem is solved; its size is defined so as to contain all the microscopic features of the flow. As a rule of thumb, the REV is the smallest fluid domain over which periodic boundary conditions can be applied.

In the computational domain, any flow variable ϕ can be decomposed into an intrinsic average part $\langle \phi \rangle^\beta$ plus a perturbation $\tilde{\phi}$, as:

$$\phi = \langle \phi \rangle^\beta + \tilde{\phi}.$$

The intrinsic average is defined with an integration carried out only on the fluid phase [36]:

$$\langle \psi_\beta \rangle^\beta = \frac{1}{V_\beta} \int_{V_\beta} \psi_\beta(\mathbf{x}) dV_\beta. \quad (4.3)$$

Applying such an operator to equations (4.1) and (4.2), and following Whitaker [35] we have:

$$\begin{aligned} \frac{\partial \langle \mathbf{v}_\beta \rangle^\beta}{\partial t} + \langle \mathbf{v}_\beta \rangle^\beta \cdot \nabla \langle \mathbf{v}_\beta \rangle^\beta &= -\frac{1}{\rho_\beta} \nabla \langle p_\beta \rangle^\beta + \nu_\beta \nabla^2 \langle \mathbf{v}_\beta \rangle^\beta + \mathbf{f} + \\ \frac{1}{V_\beta} \int_{A_{\beta\sigma}} \left(-\frac{\tilde{p}_\beta}{\rho_\beta} \mathbf{I} + \nu_\beta \nabla \tilde{\mathbf{v}}_\beta \right) \cdot \mathbf{n}_{\beta\sigma} dA, \end{aligned} \quad (4.4)$$

$$\nabla \cdot \langle \mathbf{v}_\beta \rangle^\beta = 0, \quad (4.5)$$

upon neglecting in equation (4.4) the sub-REV scale dispersion term (linked to the $\langle \tilde{\mathbf{v}}_\beta \tilde{\mathbf{v}}_\beta \rangle^\beta$ term) which is often small in porous media flows [8].

The surface integral term in equation (4.4) represents the drag (per unit mass) due to surface forces at the fluid-solid interface of the medium. It is called the Darcy-Forchheimer microscale force, \mathbf{F}^m . The equations are however often to be solved at the macroscale, so that a macroscale force model, \mathbf{F}^M , must be used to replace \mathbf{F}^m in the governing equation. Such a model is often based on a permeability tensor, \mathbf{K} , and a Forchheimer tensor, \mathbf{F} , and reads:

$$\mathbf{F}^M = -\nu_\beta \mathbf{K}^{-1} (\mathbf{I} + \mathbf{F}) \langle \mathbf{v}_\beta \rangle^\beta, \quad (4.6)$$

so that the system is closed by imposing

$$\mathbf{F}^m = \mathbf{F}^M. \quad (4.7)$$

The drag force \mathbf{F}^m computed by direct numerical simulations (DNS) with account of all individual pores will be later compared to the model based on the permeability and Forchheimer tensors (whose equations are given below). This is just a useful exercise to demonstrate consistency of the approach and accuracy of the numerical simulations; it does

nothing else since, as briefly described below, to derive the Forchheimer tensor the microscopic velocity field must be known anyhow. Nonetheless, knowledge of the behaviour of these tensors (or, equivalently, of the related *apparent* permeability) might prove both useful and instructive, in particular should one wish to extend the range of applicability of the model to cases for which the microscopic solution is not available.

The core of the VANS approach consists in the identification of the permeability and Forchheimer tensors. This problem, referred to as the closure problem, is discussed at length by Whitaker [34, 35]. He derives two partial differential equation systems, the first valid in the zero Reynolds number limit (system (4.8) below), while the second applies when inertial terms are not negligible (system (4.10)).

In the first system of equations a three component vector \mathbf{d} and a 3×3 tensor \mathbf{D} are introduced. This system can be divided into three separate independent problems which resemble a forced Stokes problem where each component of \mathbf{d} and the corresponding row of \mathbf{D} play, respectively, the role of a pressure and a velocity field. Together with the periodic boundary conditions, the problem reads:

$$\begin{cases} 0 = -\nabla \mathbf{d} + \nabla^2 \mathbf{D} + \mathbf{I}, \\ \nabla \cdot \mathbf{D} = 0, \\ \mathbf{D} = 0 \quad \text{on} \quad A_{\beta\sigma}, \\ \mathbf{d}(\mathbf{x} + \ell_i) = \mathbf{d}(\mathbf{x}), \quad \mathbf{D}(\mathbf{x} + \ell_i) = \mathbf{D}(\mathbf{x}) \quad i = 1, 2, 3. \end{cases} \quad (4.8)$$

The permeability tensor is found by applying the intrinsic average on the \mathbf{D} tensor, i.e. $\mathbf{K} = \varepsilon \langle \mathbf{D} \rangle^\beta$ and, in the Stokes regime, it is

$$\mathbf{F}^M = -\nu_\beta \mathbf{K}^{-1} \langle \mathbf{v}_\beta \rangle^\beta. \quad (4.9)$$

The second closure problem differs from the first only for the presence of a linearised convective term in which the microscopic velocity obtained from the DNS, \mathbf{v}_β , is used as an input. This of course implies knowledge of the microscopic velocity field. A Oseen-like approximation which relaxes this constraint has been proposed by Zampogna and Bottaro [38].

The new unknowns are a vector and a tensor called, respectively, \mathbf{m} and \mathbf{M} , with the same meanings of \mathbf{d} and \mathbf{D} . The system reads:

$$\begin{cases} \frac{1}{\nu_\beta} \mathbf{v}_\beta \cdot \nabla \mathbf{M} = -\nabla \mathbf{m} + \nabla^2 \mathbf{M} + \mathbf{I}, \\ \nabla \cdot \mathbf{M} = 0, \\ \mathbf{M} = 0 \quad \text{on} \quad A_{\beta\sigma}, \\ \mathbf{m}(\mathbf{x} + \ell_i) = \mathbf{m}(\mathbf{x}), \quad \mathbf{M}(\mathbf{x} + \ell_i) = \mathbf{M}(\mathbf{x}) \quad i = 1, 2, 3. \end{cases} \quad (4.10)$$

The average of the tensor \mathbf{M} multiplied by the porosity is the *apparent permeability*, $\mathbf{H} = \varepsilon \langle \mathbf{M} \rangle^\beta$. When inertia is important equation (4.6) can be written as

$$\mathbf{F}^M = -\nu_\beta \mathbf{H}^{-1} \langle \mathbf{v}_\beta \rangle^\beta, \quad (4.11)$$

as shown by Whitaker [35].

Two remarks are in order at this point. First, the equations in the closure problem (4.10) are time-independent because the microscopic velocity \mathbf{v}_β is a solution of a stationary DNS. Thus, the Reynolds number should be sufficiently small for unsteady effects not to be present. Should the wake behind a solid inclusion display regular or irregular temporal oscillations, the equations of system (4.10) may be used, as an approximation, by replacing the instantaneous velocity in the REV with its time-averaged distribution. This case is however not of present concern. Secondly, the closure problems reflect the structure of the solution of the two system (4.8) and (4.10). In particular, the solution of (4.8) depends only on the geometry of the porous medium so that the permeability tensor \mathbf{K} is symmetric. This is not the case for \mathbf{H} , because of the effect of the microscopic velocity amplitude and direction. Clearly, the solution of system (4.8) tends to that of (4.10) when $Re_d \rightarrow 0$.

4.3 Validation and setup

In this section the numerical methodology, the parameters, the setup and the validation for some reference cases are given.

4.3.1 Computational domain

The geometry used for the base REV is shown in figure 4.2: a cylindrical inclusion is present at the centre of the REV and four quarters of cylinders are situated at the corners. The lateral length of the cubic envelop is ℓ , which is used as length scale for the microscopic problem; the diameter d of the cylinders is adapted as a function of the desired porosity ε , ratio between the fluid volume over the total REV volume (ℓ^3).

The forcing term \mathbf{f} of the DNS is a vector whose direction is defined by two Euler angles, with rotations of the form: $\theta \mathbf{e}_3 + \phi \mathbf{e}_2^T$ (cf. figure 4.2). Its amplitude is set a priori and is connected to the Reynolds number, Re_d , defined with the mean velocity over the REV and the fiber diameter, d . Re_d is a result of the calculations, once the mean velocity is evaluated.

4.3.2 Numerical setup

The simulations have been carried out with the open-source code OpenFOAM [33], based on a finite volume discretization with a colocated arrangement for the unknowns. The standard solver icoFoam (incompressible Navier-Stokes) has been modified in order to include a constant pressure gradient acting as a forcing term \mathbf{f} in equation (4.1). The coupling between the velocity and the pressure equations is based on the pressure implicit split operator referred to as the PISO algorithm. The time derivative term is discretized using the second order backward Euler scheme and all the spatial terms use a second-order central difference stencil based on Gauss finite volume approach. The velocity system is solved

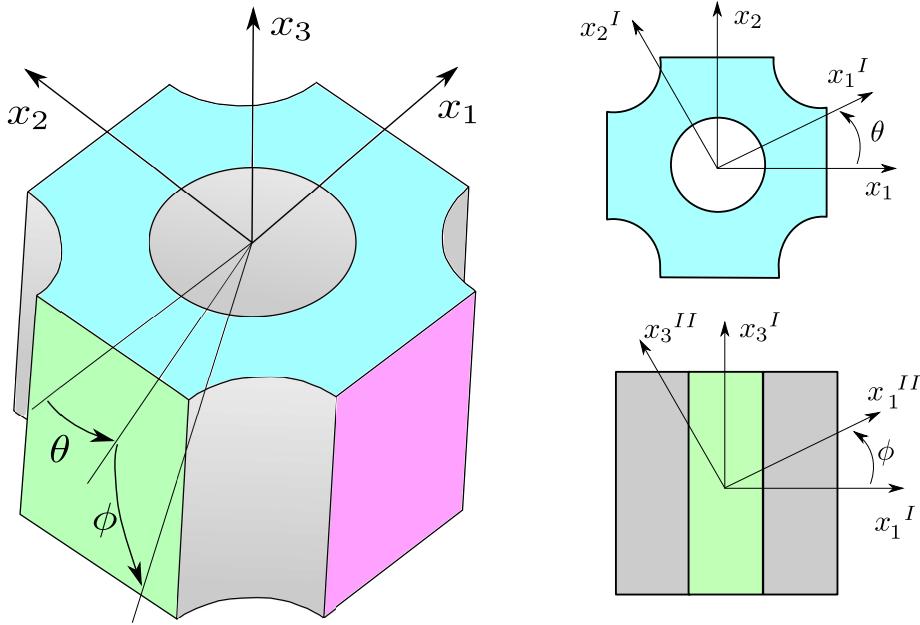


Figure 4.2: REV for the fiber geometry investigated.

with a preconditioned bi-conjugate gradient (PBiCG) iterative solver with the tolerance on the velocity residuals set to 10^{-8} , associated to a diagonal incomplete lower upper preconditioner (DILU). The pressure equation is solved with a geometric-algebraic multigrid (GAMG) algorithm associated to a Gauss-Seidel smoother and the tolerance on the pressure residuals is here equal to 10^{-6} . Cyclic boundary conditions are applied to all fields on all fluid boundaries along the three directions, and the no-slip condition is imposed on the surface of the solid inclusions. The time step Δt is automatically determined to ensure that the maximum Courant number, Co , respects the condition: $Co = \|v_\beta\| \Delta t / \Delta x < 1/2$, in which $\|v_\beta\|$ is the local velocity magnitude in the REV and Δx is the local grid spacing. Co is basically the ratio between the fluid speed and the velocity to propagate information through the mesh and the condition $Co < 1/2$ is found to be sufficient to have a stable solver.

4.3.3 Mesh convergence analysis

The mesh has been computed using the internal OpenFOAM mesher named *snappyHexMesh*. The final grid is mainly composed by hexahedral cells with a refined regular grid in the boundary layer regions next to the solid surfaces. Three different mesh sizes, with 0.65×10^6 , 10^6 and 1.5×10^6 elements, have been tested in order to demonstrate spatial convergence. This has been assessed using the Grid Convergence Index (*GCI*) introduced by Roache [30].

Details of the coarsest mesh used are shown in figure 4.3. On the right frame a close up of the grid in the neighbourhood of the fiber's boundary is displayed: twenty points are used in the structured portion of the mesh along the wall-normal direction.

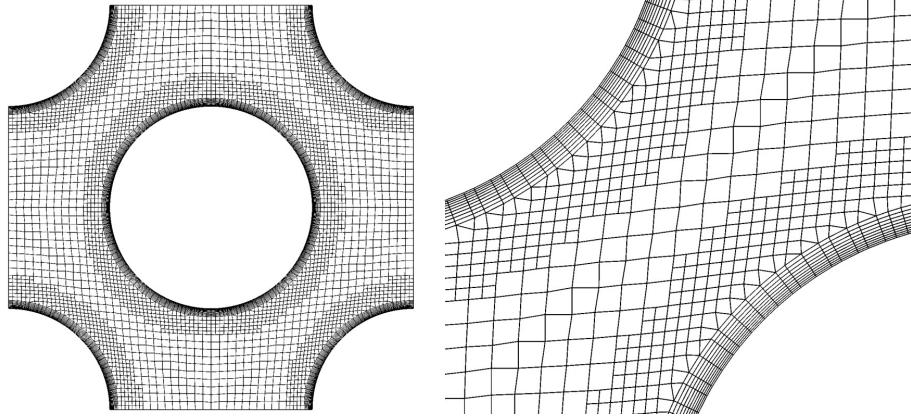


Figure 4.3: Mesh used for the computation; top view (left) and zoom in the boundary layer region (right). $\varepsilon = 0.6$.

The GCI method is based upon a grid refinement error estimator derived from the theory of generalized Richardson extrapolation. It measures the ratio between the computed value of a quantity over the asymptotic numerical value, thus indicating how far the solution is from the asymptotic ("exact") value. The procedure is simple and provides a method to estimate the order of the spatial convergence, based on two or three different grid sizes. First of all, the grids must be generated with the same algorithm and they must have the same final quality. In each simulation a physical scalar quantity representative of the physical phenomenon must be sampled. The method follows the following four steps:

1. Estimate the order of convergence of the procedure, defined as $p = \ln \left(\frac{f_3 - f_2}{f_2 - f_1} \right) / \ln r$, where r is the grid refinement ratio between each grid (it is computed as the ratio between the number of elements of two consecutive grids; the approach imposes that r should remain constant between any couple of consecutive grids and be larger than 1.1), and f_i represents the quantity of interest in each grid (1=coarse, 2=medium and 3=fine).
2. Compute the relative error between grid i and j : $|\epsilon|_{ij} = \frac{f_j - f_i}{f_i}$, for $(i, j) \in \{(1, 2), (2, 3)\}$.
3. Compute $GCI_{ij} = \frac{F_s |\epsilon|_{ij}}{r^p - 1}$, with F_s a safety factor equal to 1.25 if the grids are three, and equal to 3 if the grids are only two [30].

mesh index	mesh identifier	average velocity REV	metric	value
3	fine	1.11	GCI_{23}	0.366%
2	medium	1.07	GCI_{12}	1.11%
1	coarse	1.09	AC	1.006

Table 4.1: Convergence analysis. Left: average velocity within the REV, normalized with $\frac{K_{11}}{\nu_\beta} \|\mathbf{f}\|$. Right: grid convergence metrics. The REV has $\varepsilon = 0.6$, the motion is along x_1 , i.e. $\theta = \phi = 0$ and $Re_d \rightarrow 0$.

4. Check whether each grid level yields a solution that is in the asymptotic range of convergence; this means that the quotient $AC = \frac{GCI_{23}}{GCI_{12}} \frac{1}{r^p}$ should be as close as possible to one.

In our case the quantity of interest chosen is the intrinsic average velocity inside the porous medium, and the results are summarized in table 4.1. From the table it can be seen that the intrinsic velocity difference is very small from one grid to the next and the coarse grid provides results close to the expected asymptotic value. This is taken as a sufficiently convincing argument to carry out all the computations in the following with a grid density equal to that of grid 1.

4.3.4 Validation on two different configurations

The results published in the literature by Zampogna and Bottaro [38] and Yazdchi et al. [37] are now used to validate both the methodology and our choices of the computational parameters. In the cited papers, three-dimensional computations of the permeability components in different cells geometries are presented.

Figure 4.4 displays the comparison for a cell with a square arrangements of the fibers; here the permeability is evaluated along the two principal directions, x_1 and x_3 . A good agreement is found with the published results. Figure 4.5 shows a similar comparison for a staggered arrangement of the inclusions in the unit cell. In this case the section of the cell is rectangular. The agreement for the only permeability component available in the literature is again satisfactory.

Finally, to check the correct implementation of the closure model (4.7) it is important to verify the equality (4.7) between the amplitude F^M of the macroscopic force and its microscopic counterpart obtained through an integration of the DNS fields over the solid boundaries of the inclusions in the REV. Figure 4.6 shows a plot of the relative error between these two forces, i.e. $\frac{\|F^M - F^m\|}{\|F^m\|}$, as function of the Reynolds number. We

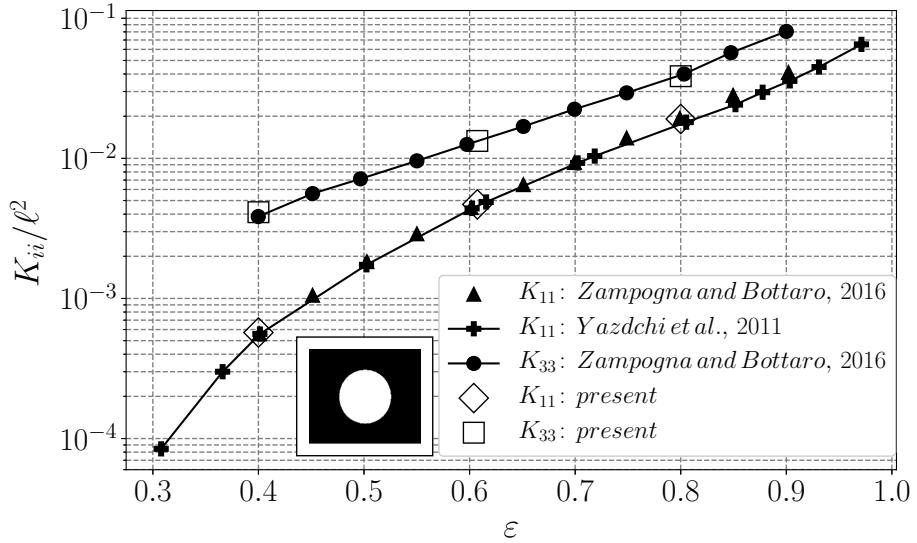


Figure 4.4: Permeability versus porosity for a square arrangement of cylinders. The scaling of the permeability is ℓ^2 and is explicitly indicated in the vertical axis.

consider the successful comparison displayed in figure 4.6 as the conclusive demonstration of the validity of the approach described here. We have nonetheless carried out the same verification displayed in figure 4.6 for each one of the simulations described in the following, to our satisfaction.

4.3.5 Tests with larger REV's

Since the Reference Elementary Volume (REV) is the unit cell within the porous medium over which average quantities of the VANS are computed, it is important to choose its dimensions appropriately in the inertial regime for, if the REV is too small, it might be easy to miss crucial features of the wakes. For example, to predict the critical Reynolds number, Re_c , of the first Hopf bifurcation, a REV containing at least three solid inclusions in the direction of the mean pressure gradient is necessary in the simulations by Agnaou et al. [3]. Among the results reported, it is found that, for a fixed REV size, the error committed in the evaluation of the critical Reynolds number increases with the porosity. This same error is considerably reduced when the mean pressure gradient angle is $\theta = 45^\circ$. Thus, the choice of the number of inclusions in a REV is a task not to be overlooked, and the final choice must account for the porosity, the direction of the pressure gradient and the microscopic Reynolds number.

Here, the influence of the numbers of inclusions present in a REV is assessed by focussing only on the velocity components after averaging over the REV. The unit cubic cell of side

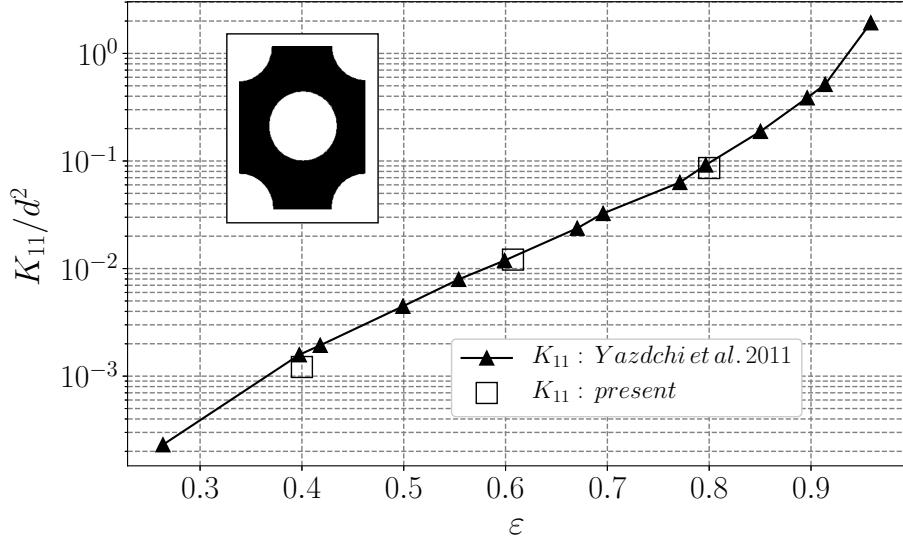


Figure 4.5: Permeability versus porosity for a staggered arrangement of cylinders. The permeability component is here scaled with d^2 (and not ℓ^2), with d the diameter of the inclusions.

ℓ is used as reference: starting from this, two additional REV's are built, as shown in figure 4.7. The first one is doubled in both the x_1 and x_2 directions and the case tested numerically is characterised by $\theta = 0$, $\phi = 0$ (i.e. the forcing pressure gradient is directed along x_1), porosity $\varepsilon = 0.6$ and $Re_d = 50$. The second REV configuration is a composition of 3 reference REVs on top of one another along x_3 , with the parameters set to $\theta = 45^\circ$, $\phi = 45^\circ$, $\varepsilon = 0.6$ and $Re_d = 100$.

For both these test cases, no appreciable differences, neither in the mean velocity nor in the forces on the fibers, have been observed, with relative errors on the mean velocity with respect to the reference case which remain below 2%. We take this as sufficient evidence to use, in the following, only the reference cubic REV of side equal to ℓ , with the understanding that only configurations with Re_d up to around 100 can be considered.

4.4 Microscopic solutions

In this section, some local microscopic fields computed with direct numerical simulations are shown, together with components of the intermediate tensor \mathbf{M} coming from the numerical solution of the closure equations (4.10).

In figure 4.8 (top row) the local x_1 velocity component is drawn for the two-dimensional flow when $\varepsilon = 0.6$, for three Reynolds numbers, to cover the transition from the Stokes

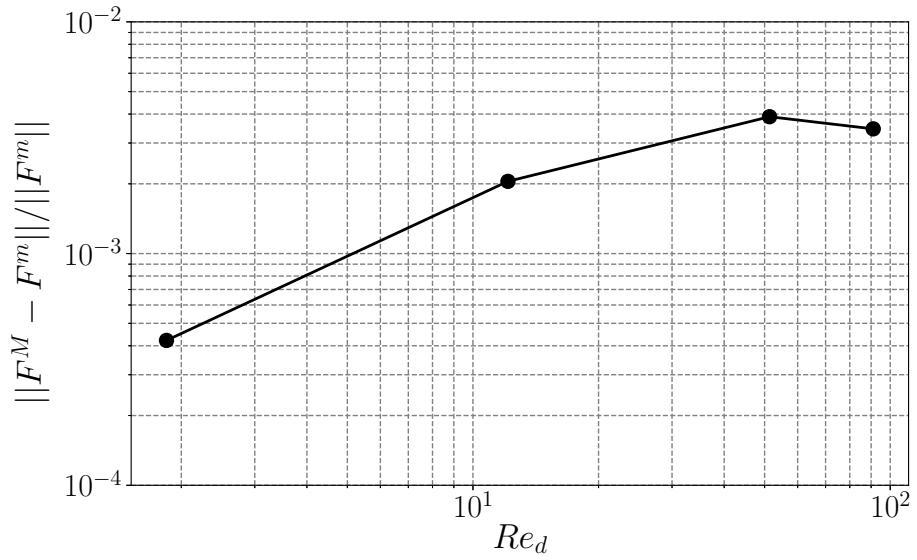


Figure 4.6: Relative error between the microscopically computed forces along the x_1 direction and those arising from the Darcy-Forcheimber model; $\varepsilon = 0.8$ for the REV in the staggered arrangement of Yazdchi et al. [37].

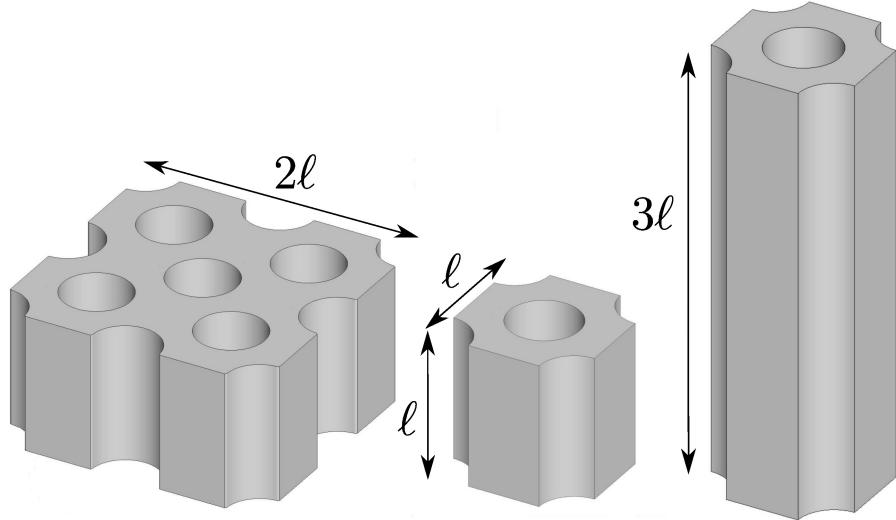


Figure 4.7: REV configurations. Left: $2 \times 2 \times 1$ arrangement; centre: $1 \times 1 \times 1$ arrangement (reference); right $1 \times 1 \times 3$ arrangement.

to the inertial regime. In all plots, the velocities are rendered non-dimensional by the

corresponding value of $\frac{K_{11}}{\nu_\beta} \|\mathbf{f}\|$. When inertia is absent, the flow has a central symmetry; by increasing the Reynolds number, only the symmetry with respect to the x_1 axis is maintained (x_1 is the direction of the forcing pressure gradient), with the wake's length which increases with Re_d . When Re_d is of order 100 the wake spreads to the downstream boundary of the REV, re-entering, because of periodicity, at the upstream side. This Re_d represents the upper limit of validity for the cubic unit cell of side ℓ ; larger values of Re_d could only be investigated with longer/larger/thicker REV's.

The non-dimensional local M_{11} fields for the same parameters are displayed in figure 4.8 (mid row). All values in the figures arise from scaling \mathbf{M} with ℓ^2 . Visually, these local fields are strongly correlated to the local streamwise velocity component in the whole Re_d range. This is not unexpected since the local velocity drives the convective term of system (4.10). The central symmetry of all components of \mathbf{M} in the Stokes regime is coupled to the rotational invariance of the apparent permeability tensor in two-dimensional flows.

The effect of varying the porosity is shown in figure 4.8 (bottom row) where ε is taken equal to 0.4. Even at such a low porosity the stretching of the wake can be noticed, and it increases with Re_d . Interestingly, this effect is milder when the forcing is inclined by an angle ϕ , since the tighter packing of the inclusions causes a strong deviation of the mean flow along the axis of the fiber. In this case, M_{11} and M_{22} behave very similarly to the case $\phi = 90^\circ$.

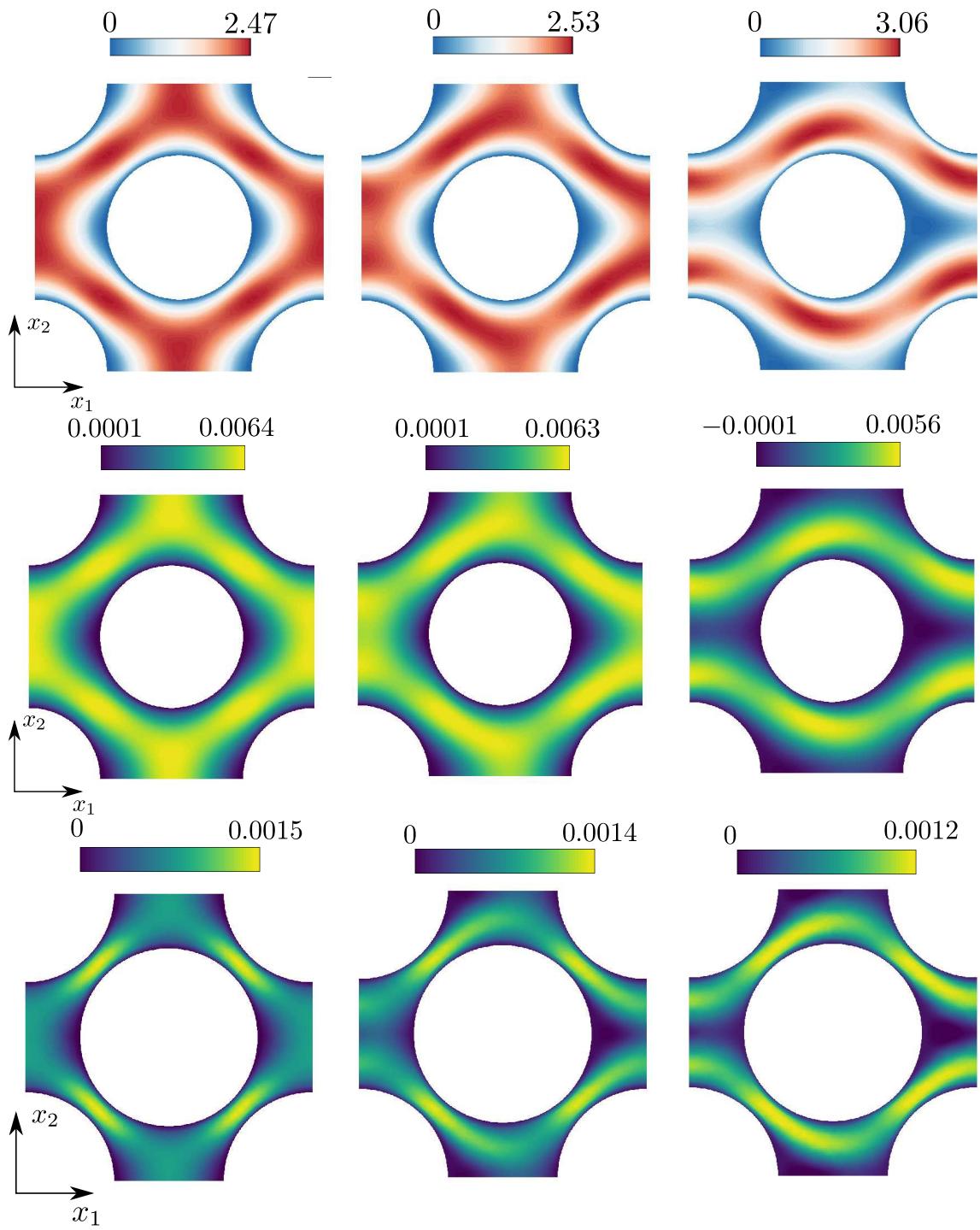


Figure 4.8: Top row: plane view of the dimensionless x_1 component of the local velocity field \mathbf{v}_β for the case $\theta = 0$, $\phi = 0$, $\varepsilon = 0.6$ and for three Reynolds numbers $Re_d = 0, 10, 50$, from left to right. Mid row: microscopic M_{14} fields corresponding to the images in the top row. Bottom row: M_{11} fields for the same Euler angles and Reynolds number as in the top two rows, and smaller porosity ($\varepsilon = 0.4$).

Another interesting point emerges by inspection of figure 4.9 where two off-diagonal components of \mathbf{M} are shown for two porosity values; the first image (left frame) represents a plane flow in the Stokes regime while the second is the plane cut of a three-dimensional solution in the inertial regime. Positive and negative values of the microscopic fields can be seen in both images but, once averaging is applied over the REV, the resulting permeability component is very close to zero (in fact, exactly equal to zero in the Stokes case). This same features occurs for all off-diagonal terms in all cases examined, so that, within the current range of Reynolds numbers, the apparent permeability tensor is, to a good approximation, diagonal¹.

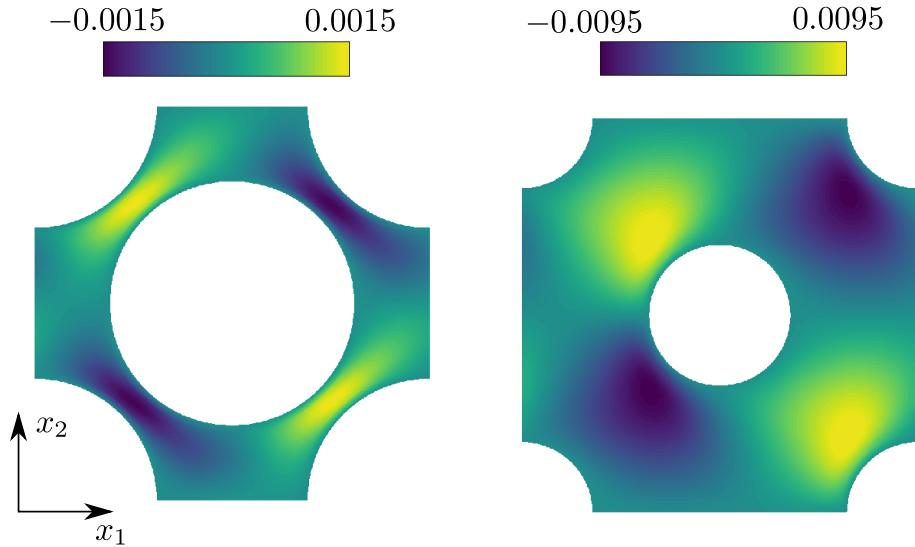


Figure 4.9: right: Non-dimensional M_{21} field for $\theta = 0, \phi = 0, Re_d = 10, \varepsilon = 0.8$, left: Non-dimensional M_{12} field for $\theta = 22.5^\circ, \phi = 45^\circ, Re_d = 50, \varepsilon = 0.4$.

A three-dimensional case is shown in figure 4.10, where all the non-zero terms of the \mathbf{M} tensor are plotted for a porous structure with $\varepsilon = 0.6$. The components shown are $M_{11}, M_{22}, M_{33}, M_{12}$ and M_{21} , while M_{i3} and M_{3j} are not plotted because they are identically zero to machine accuracy. Distinct features are visible in each image; in particular, in the last frame the M_{33} microscopic component displays a low wavelength structure along the cylinder's axis. Increasing the dimensions of the REV along x_3 does not alter such a structure, i.e. the ℓ^3 domain chosen with its periodic boundary conditions does not filter out significant high wave-numbers of the flow. We further note that the tensor \mathbf{M} is not symmetric in this case since each off-diagonal component represents the solution of the

¹In fact, there are always at least two orders of magnitude differences between the diagonal and the off-diagonal components. While the latter should not, in principle, be ignored, we will focus attention here only on the dominant terms of the permeability tensor.

closure problem in a specific direction (first index of the field) and the forcing term acts orthogonally to it (second index of the field). Once averaged over the REV it is found that both H_{12} and H_{21} are very close to zero.

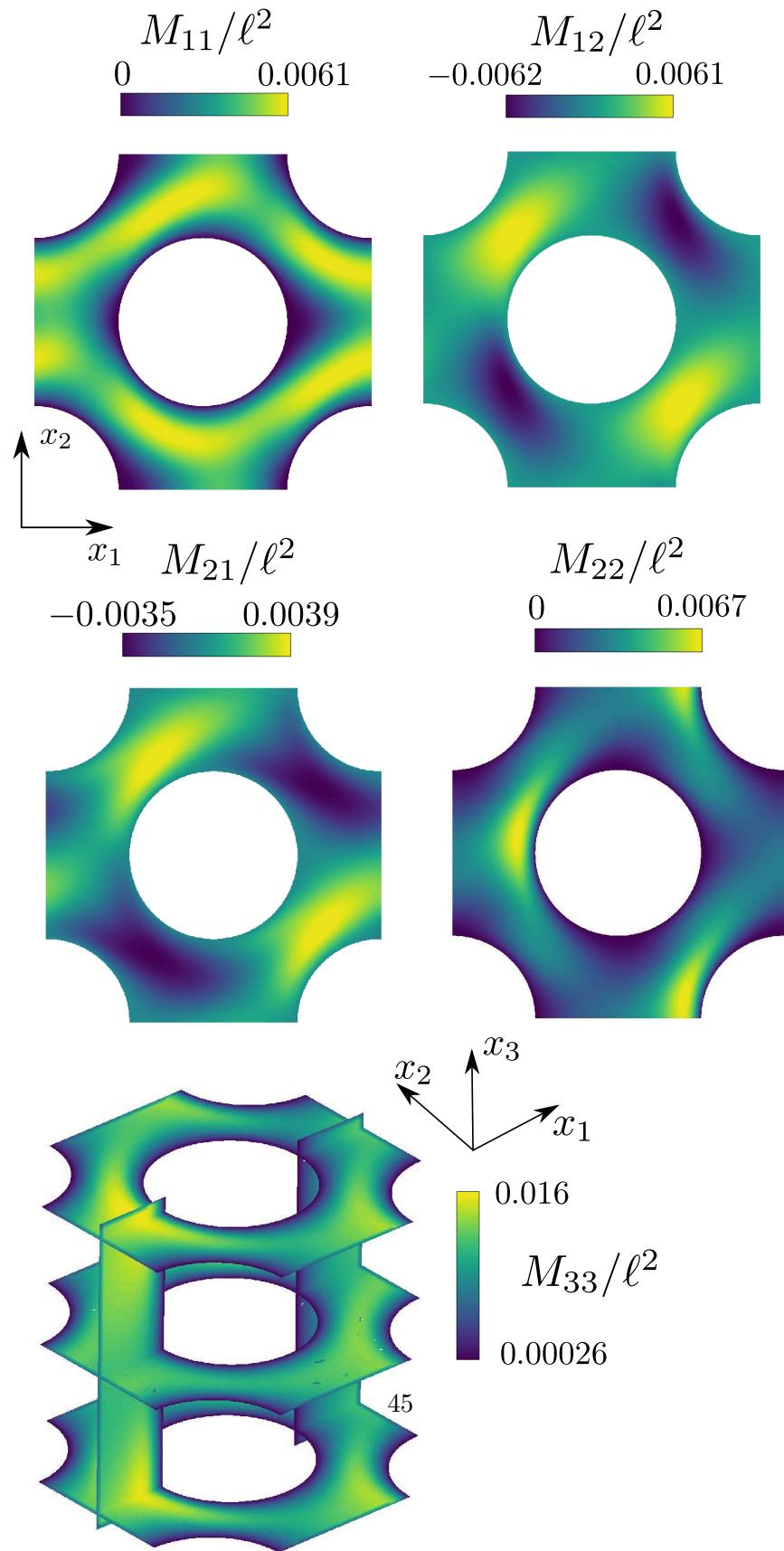


Figure 4.10: Non-dimensional \mathbf{M} components fields for the case $\theta = 22.5^\circ, \phi = 45^\circ, Re_d = 50, \varepsilon = 0.6$.

index	θ	ϕ	field properties
1	0°	0°	2D symmetric
2	22.5°	0°	2D non-symmetric
3	0°	45°	3D symmetric
4	22.5°	45°	3D non-symmetric
5	—	90°	3D symmetric

Table 4.2: Directions of the forcing tested and property of the solutions.

4.5 The apparent permeability tensor

In this section the variations of the diagonal components of the permeability tensor \mathbf{H} are discussed as function of the direction of the mean forcing, the Reynolds number and the porosity. As stated previously, the Reynolds number ranges from 0 to approximately 100 in order to capture phenomena associated with inertia; the cases considered never lead to unsteady signals. The porosity parameter ε is set to either 0.4 (low porosity), 0.6 (medium) or 0.8 (high). The forcing direction is defined by the Euler angles and all the configurations considered in this section are summarized in table 4.2; the choice has been made to explore a reasonably large range of parameters, with both two-dimensional and three-dimensional flows characterized by symmetric and asymmetric patterns.

Let us briefly recall the methodology. First, a DNS is carried out to compute the microscopic flow. Then the closure problem is solved for the tensor \mathbf{M} . Finally, each component of the apparent permeability \mathbf{H} is obtained by averaging (equation (4.3)). The results are collected in figures 4.11, 4.12 and 4.13, showing the variation of the diagonal components of \mathbf{H} .

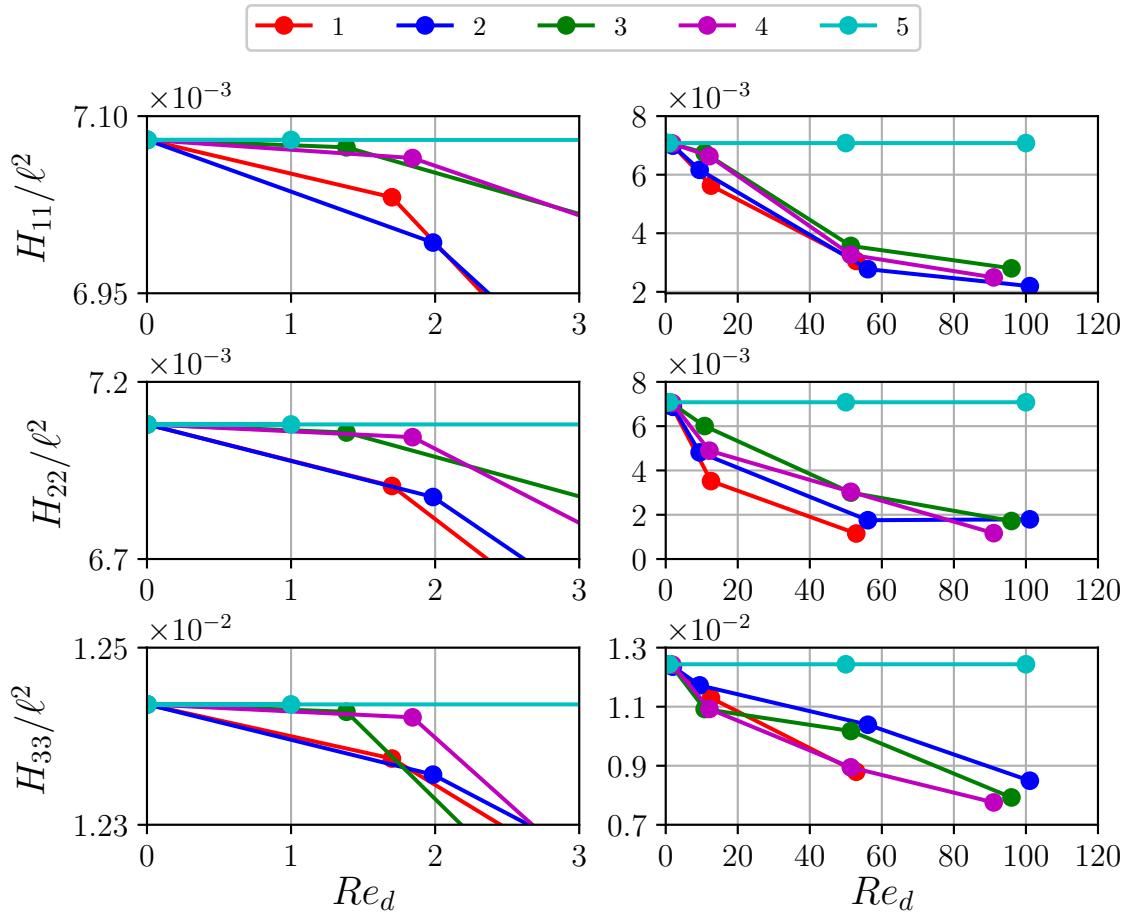


Figure 4.11: Diagonal elements of the apparent permeability \mathbf{H} as function of the Reynolds number for porosity $\varepsilon = 0.8$. The forcing direction is represented through the couple of Euler angles (θ, ϕ) (cf. table 4.2 for the case index). Left column: low- Re_d regime; right column: inertial regime.

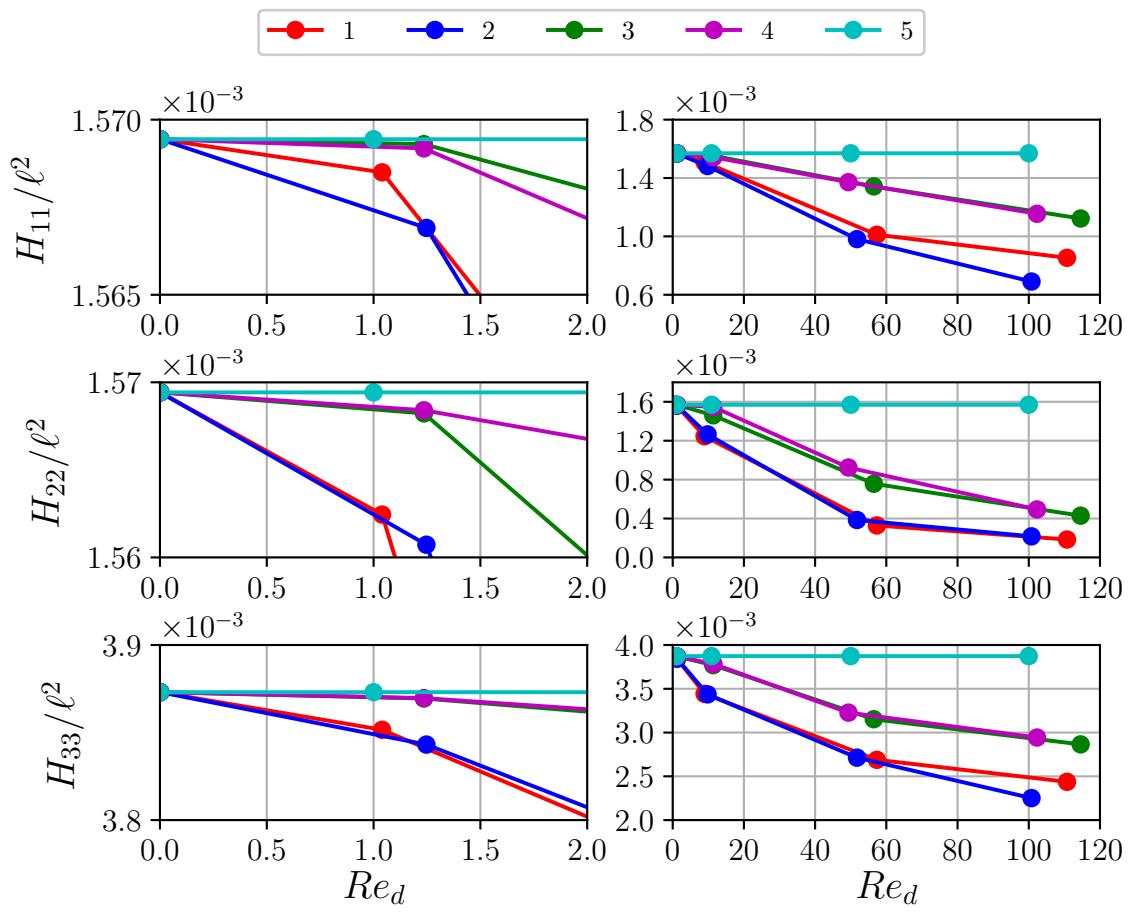


Figure 4.12: Same as figure 4.11 with porosity $\varepsilon = 0.6$.

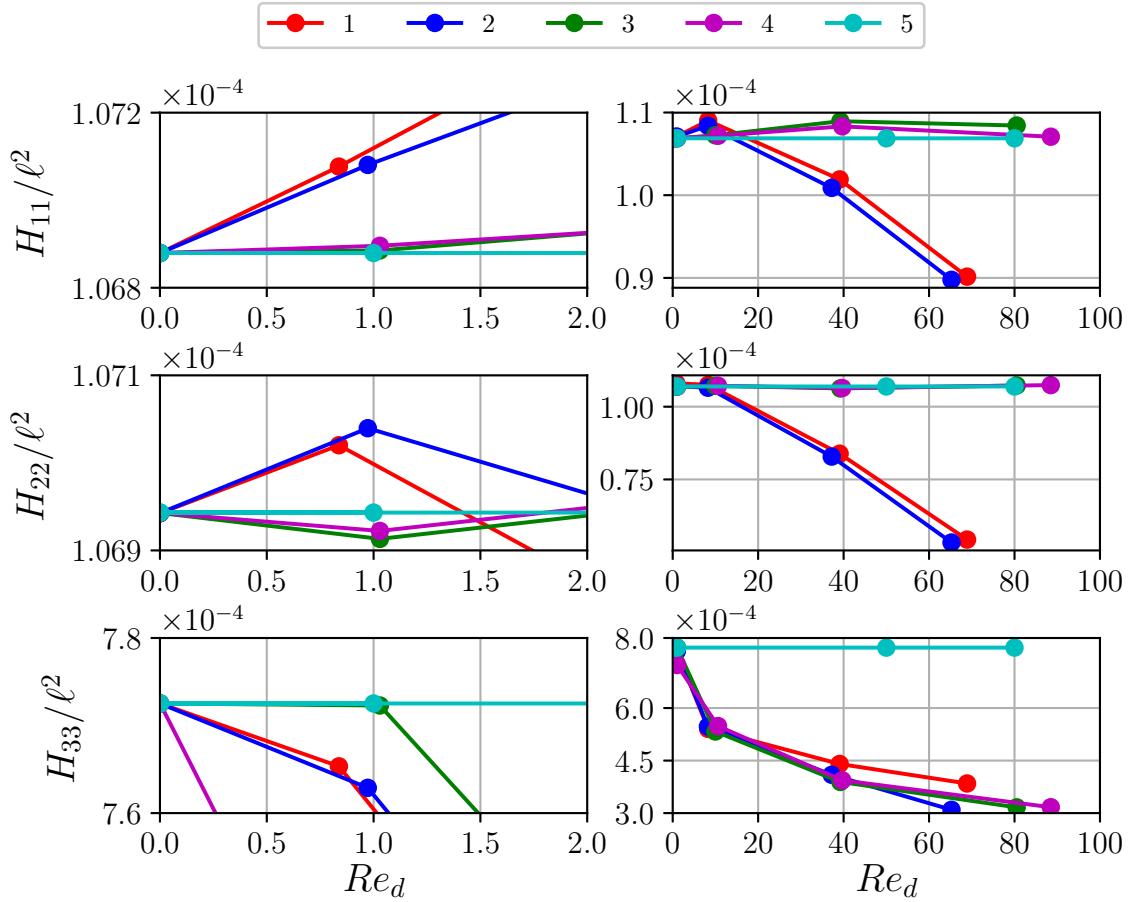


Figure 4.13: Same as figure 4.11 with porosity $\varepsilon = 0.4$.

In the left column of each figure we focus on the low- Re_d regime ($0 < Re_d < 2$), while in the right column the effect of inertia can be assessed. As expected, when Re_d is small the apparent permeability is quasi-Reynolds-number-independent (and can be approximated well by the true permeability). As the Reynolds number increases above a few units, inertial effects grow in importance yielding typically a monotonic decrease of all components of \mathbf{H} , aside from case indexed 5 ($\phi = 90^\circ$) for which the flow remains aligned with the cylinder's axis. In case 5 the microscopic flow solution is invariant with x_3 and does not change with Re_d in the range considered, so that \mathbf{H} is a constant tensor.

When the porosity is large all components show a similar behaviour irrespective of the forcing angle (except, clearly, case 5). Differences start appearing at $\varepsilon = 0.6$; the two cases with $\phi = 0^\circ$ (index 1 and 2) behave similarly, and so do the two cases indexed 3 and 4 (with $\phi = 45^\circ$). This seems to suggest a weaker effect of θ on the permeability components.

For even smaller porosity ($\varepsilon = 0.4$), the blockage which the inclusions cause to the flow produces the unexpected behaviour displayed in figure 4.13. When the flow is purely two-dimensional (cases 1 and 2), variations in the Reynolds number affect \mathbf{H} significantly; when a pressure gradient along x_3 is present the strong packing of the fibers constrain the fluid to flow prevalently along the fibers' axis, and the apparent permeability is almost Re_d -independent. When assessing variations in H_{jj} for this case, attention should also be paid to the fact that the permeability is now at least one order of magnitude smaller than in the previous cases so that variations of the diagonal components shown in figure 4.13 are tiny in absolute terms. This is related to the fact that the inverse of the permeability plays the role of a drag coefficient in the macroscopic expression of the force (cf. equation (4.6)). In other words, materials with higher porosity (larger space between solid inclusions) offer lower resistance to the motion of the fluid.

Applying the intrinsic average operator to the non-diagonal component of the tensor \mathbf{M} results in terms that are negligible with respect to their diagonal counterparts, and these results are true for all the parameters considered. This means that there is a very weak coupling between the principal directions of the fiber. The directional decoupling and the diagonal property of the apparent permeability tensor has also been computationally demonstrated on a completely different REV geometry by Soulaine and Quintard [32]. Conversely, Lasseux et al. [23] have carried out a two-dimensional study with fibers of square cross-section, finding that the off-diagonal terms are non-negligible and only about one order of magnitude smaller than the diagonal components. This result is a consequence of the non-rotationally-invariant geometry considered. The present work and the two articles just cited suggest that the diagonal property of the tensor \mathbf{H} is closely related to the geometry of the porous material, more than to the flow regime.

4.6 A metamodel for \mathbf{H}

The previous sections has shown how the apparent permeability depends on the two Euler angles, the Reynolds number and the porosity. The space of parameters is formidable and the results found so far are not sufficient to treat, for example, cases characterized by multiple inclusions' sizes and orientations in different regions of the domain, or cases involving a poroelastic medium, with temporally and spatially varying porosity, flow direction and local Reynolds number. The complete solution of the closure problem for a single set of parameters takes approximately 4 CPU hours on our two-processor Intel(r) IVYBRIDGE 2.8Ghz, each with 10 cores and 64 GB of RAM, so that a complete parametric study is, to say the least, unpractical. In view of this, the construction of a metamodel capable to provide a full characterisation of the permeability as a function of all parameters is a worthy endeavor. We have tested several surrogate models, before eventually settling on the kriging approach [21] described in the following.

parameter	values			
θ	0°	22.5°	45°	
ϕ	0°	22.5°	45°	67.5°
Re_d	0	10	50	100
ε	0.4	0.6	0.8	

Table 4.3: Sampling parameters.

4.6.1 DACE sampling

The first step to build a metamodel is the collection of relevant samples. The quality of the final metamodel strongly depends on the samples collected and their number and distribution is of primary importance. The apparent permeability tensor, \mathbf{H} , depends on four independent variables; the samples have been generated starting from the set of parameters given in table 4.3.

One of the best options to generate the relevant database would be to use a full factorial design approach in which all the combinations of the four variables from table 4.3 are computed. Because of the large number of computations required, this approach has not been retained. We have resorted to the methodology known as DACE (Design and Analysis of Computer Experiments), a technique to fill in the best possible way the space of the parameters of the problem. The Dakota library [2] has been selected for the purpose and the Monte-Carlo incremental random sampling algorithm [19] has been chosen, in order to make efficient use of the cases already computed. This incremental approach selects in a quasi-random way the new samples to generate, starting from the existing ones. In the end, the set of samples comprises 118 cases.

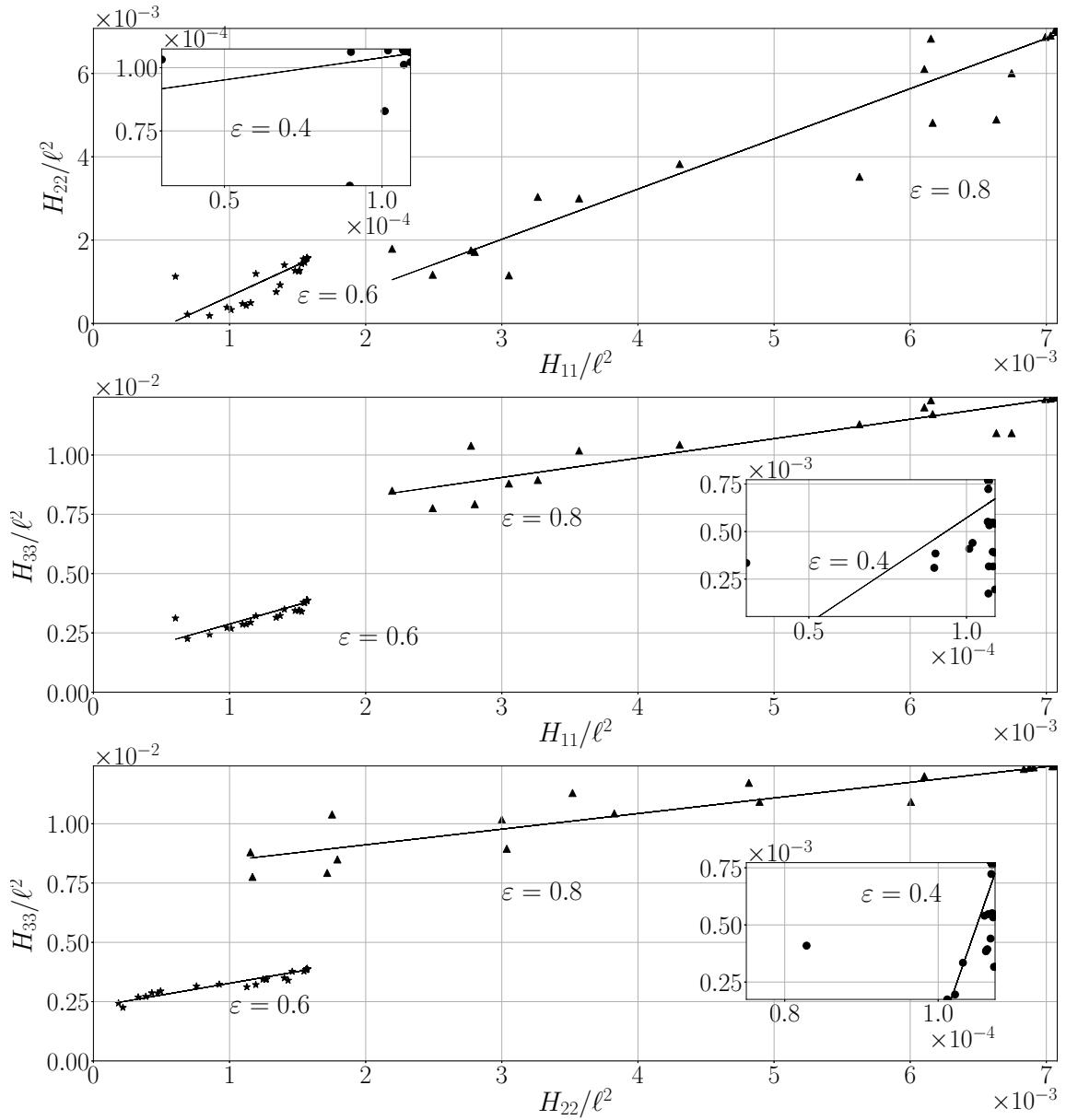


Figure 4.14: Scatter matrix plot for the collected numerical data of the apparent permeability tensor.

In the scatter plot of figure 4.14 the three diagonal components of the permeability tensor are shown as function of one another. The three porosities are separately considered in each of the above plot, and the permeability points are represented with their linear

regression on top. This kind of plot is common in statistical analysis to determine if correlations in the data are present. The permeability components show some correlation with the data points which lie reasonably well on a straight line. This result has a physical implication. Remembering the diagonal dominance of the permeability tensor, we have in the low Re_d limit:

$$\left(\langle u_\beta \rangle^\beta, \langle v_\beta \rangle^\beta, \langle w_\beta \rangle^\beta \right) \sim \left(H_{11} \frac{\partial p}{\partial x_1}, H_{22} \frac{\partial p}{\partial x_2}, H_{33} \frac{\partial p}{\partial x_3} \right). \quad (4.12)$$

It is then possible to compute the angle between the forcing term, ∇p , and the average velocity vector, represented in figure 4.15 for the two-dimensional case, $\phi = 0$. This is achieved by taking the ratio between the first two components of Darcy's equation, calling γ the flow deviation with respect to the mean forcing. We thus have:

$$\tan(\theta + \gamma) = \frac{H_{22}}{H_{11}} \tan \theta. \quad (4.13)$$

If the ratio between the two permeability components is equal to one, the angle γ vanishes. The correlation between H_{11} and H_{22} controls the deviation of the flow in the (x_1, x_2) plane, and the argument can easily be extended to H_{11}/H_{33} and H_{22}/H_{33} for deviation angles in three-dimensions.

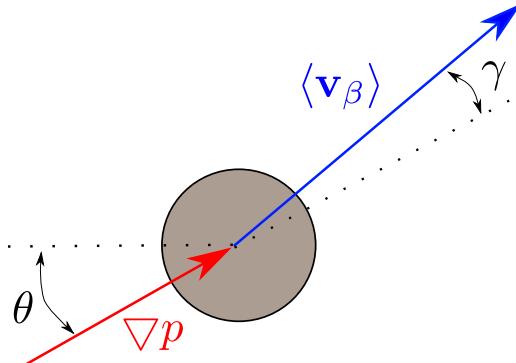


Figure 4.15: Explanatory sketch for the relation between mean pressure gradient and mean velocity field.

Using a linear correlation such as that shown in table 4.4 and figure 4.14, it is observed that in the low porosity case ($\varepsilon = 0.4$) the ratio can become very large indicating a strong deviation of the flow from the forcing direction, because of the strong constraint provided by the inclusions. As the porosity increases, the ratio does not differ much from unity, which means that the deviation remains limited. It is simple to see that the deviation

ε	H_{11}/H_{22}	H_{11}/H_{33}	H_{22}/H_{33}
0.4	1.57	11.06	96.03
0.6	1.50	1.62	0.99
0.8	1.20	0.82	0.66

Table 4.4: Permeability components ratio for three values of the porosity. The permeability ratios here are given by the angular coefficients of the linear correlations displayed in figure 4.14.

angle, for example in the (x_1, x_2) plane, satisfies the approximate relation

$$\tan \gamma = \frac{\left(1 - \frac{H_{11}}{H_{22}}\right) \tan \theta}{\frac{H_{11}}{H_{22}} + \tan^2 \theta},$$

so that for $\frac{H_{11}}{H_{22}}$ equal to, say, 1.5, the largest deviation remains always below 12° for any θ . It should however be kept in mind that trends based on these ratios are valid only as long as Darcy's law and linear correlations are acceptable. Cases exists for which such trends are violated; for example, a flow with $\theta = 45^\circ$ and $\phi = 0^\circ$ has deviation angle γ equal to zero, for whatever porosity. In this case H_{11}/H_{22} is equal to one and such a point is an outlier in the regression plots of figure 4.14.

4.6.2 Kriging interpolation method

The kriging approach is a linear interpolation/extrapolation method that aims to build a predictor field based on a set of observations $(\mathbf{x}_i, y(\mathbf{x}_i))$, for $i = 1, \dots, n$.

The predictor $\hat{f}(\mathbf{x})$ is a sum of a trend function $t(\mathbf{x})$ and a Gaussian process error model $e(\mathbf{x})$:

$$\hat{f}(\mathbf{x}) = t(\mathbf{x}) + e(\mathbf{x}). \quad (4.14)$$

The aim of the error model is to make adjustments on the trend function so that, for any point of the sampling the predictor is exactly equal to the sample, i.e. $\hat{f}(\mathbf{x}_i) = y(\mathbf{x}_i)$. This property represents one of the main qualities of this approach. In addition, when the model parameters are conveniently set, the trend function and the covariance model can take into account both smooth and steep variations in the data set.

The trend function defined here is based on a second order least-square regression, with the coefficients found from the solution of the associated linear system. The Gaussian process error model has zero-mean and its covariance between two generic data-points, x_i and x_j , is written as

$$\text{Cov}(y(\mathbf{x}_i), y(\mathbf{x}_j)) = \sigma^2 r(\mathbf{x}_i, \mathbf{x}_j).$$

The coefficient σ is an amplitude parameter and $r(x^i, x^j)$ is a correlation function, based on the Matérn covariance model that reads:

$$r(\mathbf{x}_i, \mathbf{x}_j) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu}|\mathbf{x}_i - \mathbf{x}_j|}{|\boldsymbol{\lambda}|} \right)^\nu K_\nu \left(\frac{\sqrt{2\nu}|\mathbf{x}_i - \mathbf{x}_j|}{|\boldsymbol{\lambda}|} \right), \quad (4.15)$$

where $K_\nu(\cdot)$ is a modified Bessel function and $\Gamma(\cdot)$ is the gamma function. The parameters that can be used to tune the metamodel are the amplitude parameter σ , the exponent ν and the scale vector $\boldsymbol{\lambda}$. The kriging metamodel outputs can show different behaviours for different selections of the above three parameters and their setting is thus crucial. The amplitude parameter σ is chosen to be equal to 1; larger value lead to steeper gradients and undesirable local extrema around the data points. The vector $\boldsymbol{\lambda} = (\lambda_\theta, \lambda_\phi, \lambda_{Re_d}, \lambda_\varepsilon)$ is a scaling parameter for the distance $|\mathbf{x}_i - \mathbf{x}_j|$. In this study, through systematic variations of the parameters it is found that the choice $\boldsymbol{\lambda} = (1.2, 1, 1, 1)$ yields acceptable results; in particular, the weight along θ is mildly larger than in the other directions in order to obtain smoother metamodel surfaces in this direction. The exponent ν controls the covariance function and more especially its gradients. When $\nu = 1/2$ the covariance can be approximated by a negative exponential, $\exp(-\alpha x)$ and when ν goes to infinity it behaves as $\exp(-\alpha x^2)$. In the present study, the best (i.e. smoother) results are obtained for ν equal to 1.9. The above parameters have been chosen in order to avoid unphysical or unrealistic behaviour of the apparent permeability such as, for instance, negative values or steep, spurious local maxima/minima. The method above is implemented in OpenTURNS and full details are provided by Baudin et al. [5].

The metamodel provides a scalar function (for each term of the \mathbf{H} tensor) defined in a four-dimensional space. In each of the following figures two parameters are fixed and the response surface is displayed as function of the remaining two, focussing on the H_{11} component. The other diagonal components of the apparent permeability tensor behave in a similar fashion and will not be shown for brevity. All the results of the metamodel are, however, available from the authors upon request.

In figure 4.16 the angle ϕ is fixed to zero, and the isolines display H_{11} as function of the angle θ and of the Reynolds number, Re_d , for three values of porosity. The white square symbols indicate the samples used to build the metamodel. The maximum value of each surface is always found for Re_d equal to zero and H_{11} typically decreases with Re_d , when the porosity is sufficiently large. As seen previously, for a porosity approximately greater or equal to 0.6 the variation of the apparent permeability with the angle θ is weak in this two-dimensional configuration. For the lowest porosity studied (left frame) the permeability has very small values and the isolines display an irregular behaviour; this is a feature common to all plots relative to the smaller value of ε , signaling that it is probably necessary, in this specific case, to insert additional sample points in building the response surfaces.

In figure 4.17 the parameter θ is set to 0° and the response surface is displayed in the $Re_d - \phi$ plane. As already indicated, the results confirm that an increase of the Reynolds

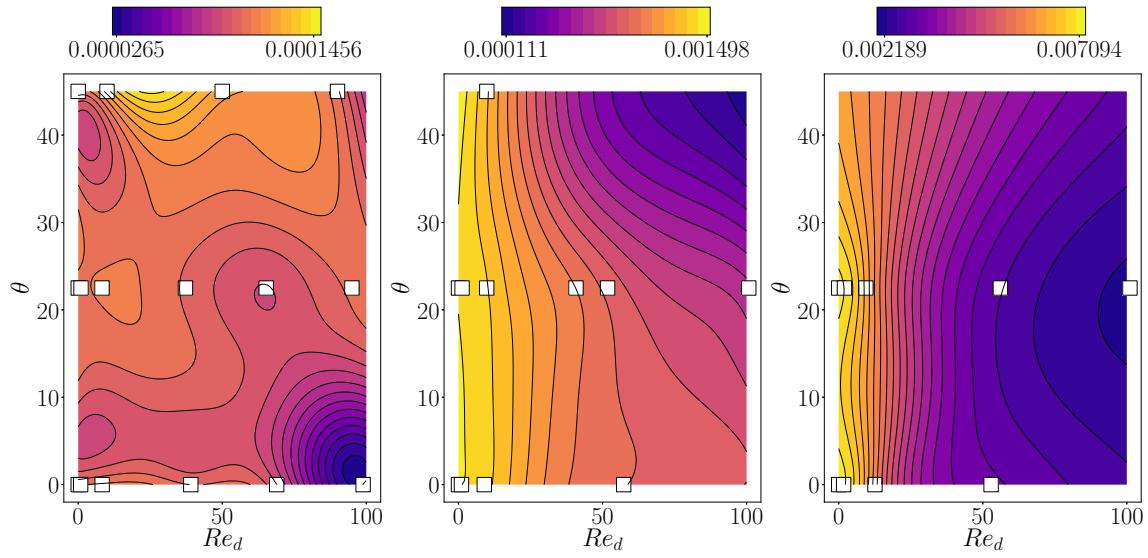


Figure 4.16: Response surfaces of H_{11} with $\phi = 0^\circ$ for porosity $\varepsilon = 0.4, 0.6, 0.8$, from left to right.

number is generally associated to a decrease of the first diagonal component of the apparent permeability tensor. However, the H_{11} variations with respect to ϕ are more pronounced than those found with respect to θ and are due to a real three-dimensionalization of the flow. This conclusion remains to be verified in the lower porosity case (left frame) where the variations are very tiny and more irregular.

In figure 4.18 the Reynolds number is set to the inertial range value of 40 and the response surface is displayed in the $\theta - \phi$ plane. For the two highest porosity values, 0.6 and 0.8, the results confirm that H_{11} has a much stronger dependence on ϕ than on θ , suggesting that the real test of permeability models must include three-dimensional effects. As seen earlier, the behaviour of the permeability when the porosity is low (left frame in the figure) is not intuitive, with a significant effect of the angle ϕ and a minor influence of θ . Again this occurs from the constraint provided to the flow by the inclusions, and from the occurrence of a large deviation γ in these cases.

The response surface is shown in the $Re_d - \varepsilon$ plane of figure 4.19 for three sets of $\theta - \phi$ angles. Here a significant effect of the porosity with respect to the Reynolds number is observable. In fact the surface gradient is almost aligned with the porosity direction, i.e. a quasi-Reynolds independence is demonstrated in this plane, and the apparent permeability can change by one order of magnitude in the range of the analysed porosity.

Some relatively small Reynolds number effects are visible at porosity equal to 0.8, when the wake of the flow has more space to develop in the inertial regime. In the central figure the flow is aligned with the direction of the fibers and, as expected, it shows practically no

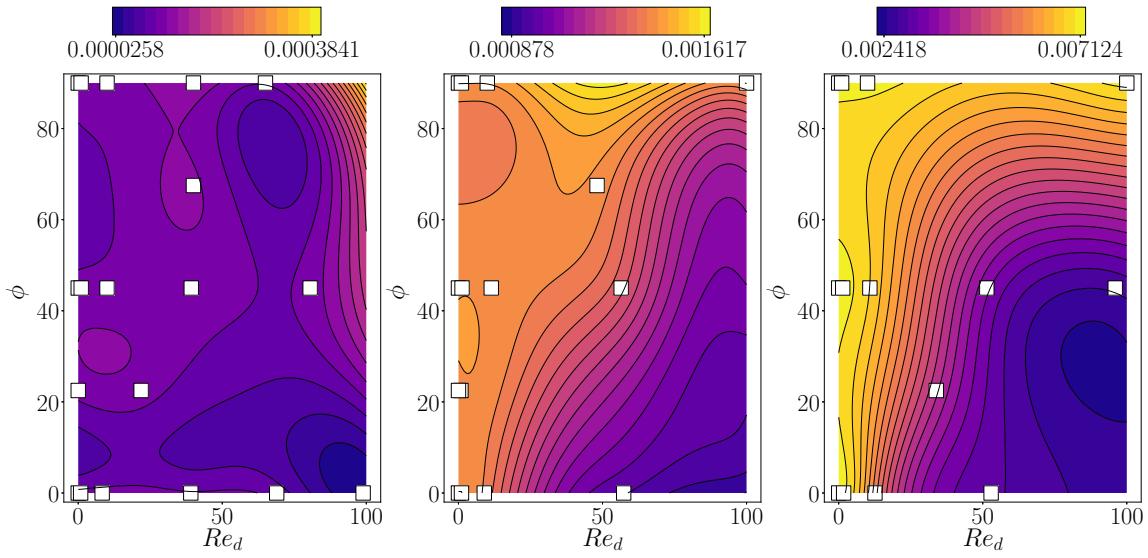


Figure 4.17: Response surfaces of H_{11} with $\theta = 0^\circ$ for porosity $\varepsilon = 0.4, 0.6, 0.8$, from left to right.

dependence with respect to the Reynolds number.

The response surface analysis has confirmed the qualitative trends which had been reached earlier on the basis of a few selected flow cases, yielding at the same time much more detailed information on the behaviour of the apparent permeability with the parameters of the problem. The data base which has been built will be used in future work which will focus, via the VANS approach, on configurations for which neither the porosity nor the local Reynolds number are constant in space or time.

4.7 Concluding remarks

The components of the permeability tensor are essential ingredients for any solution of flow through anisotropic porous media. When the flow through the pores resents of significant acceleration effects, the permeability must be modified (it is then called *apparent*) by the presence of a second tensor, the Forchheimer tensor \mathbf{F} , defined by

$$\mathbf{F} = \mathbf{K}\mathbf{H}^{-1} - \mathbf{I}.$$

The permeability, \mathbf{K} , and the apparent permeability, \mathbf{H} , can be formally deduced by two closure problems which have been briefly recalled in section 4.2. The real obstacle to the solution of the problem for \mathbf{H} is the need to know the microscopic velocity fields through the pores. We have solved for such fields in a unit cell (the REV), varying the forcing amplitude

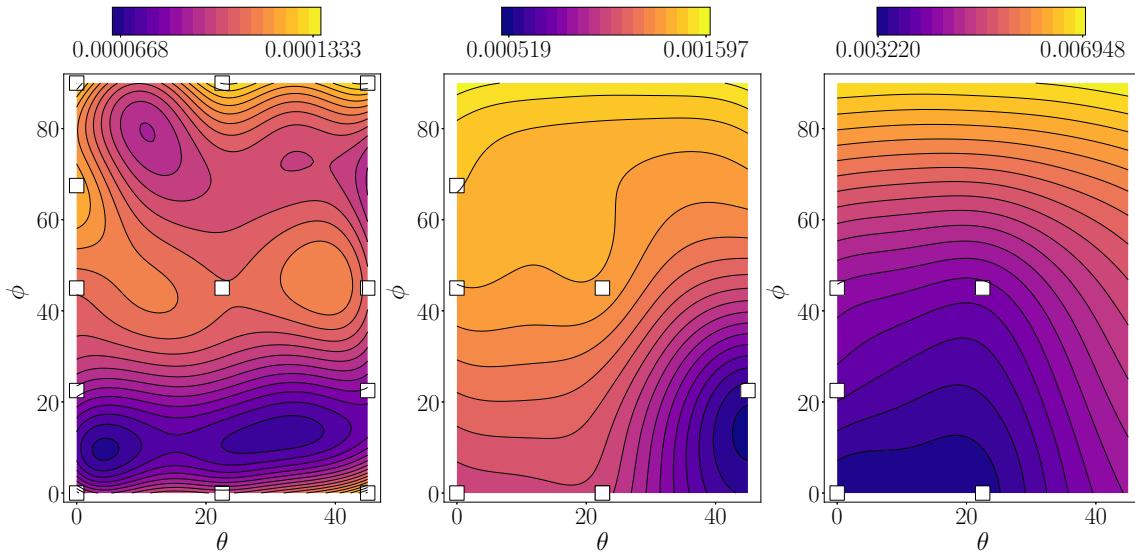


Figure 4.18: Response surfaces of H_{11} with $Re = 40$ for porosity $\varepsilon = 0.4, 0.6, 0.8$, from left to right.

and direction, treating over one hundred different cases of flows through arrangements of parallel fibers. From this, we have thus been able to solve the linear system (4.12) for all the unknown elements of the intermediate tensor \mathbf{M} , from which, through averaging, we have computed the apparent permeability. Such a tensor is indispensable to evaluate accurately the drag force caused by the presence of the fibers, for a macroscopic solution of the flow on the basis of equations [36] when inertial effects are present.

It has been found that the apparent permeability tensor is strongly diagonally dominant for whatever forcing direction and porosity, provided the local Reynolds number remains below a value approximately equal to 100; this results – which is a direct consequence of the transverse isotropy of the material which has been considered here – can be used to compute \mathbf{H} rapidly, approximating it as a diagonal tensor.

Finally, a metamodel has been used to produce results so as to cover the whole space of parameters, and this has allowed the construction of a complete data base. This data base is now being used in simulations of poroelastic media based on the VANS approach.

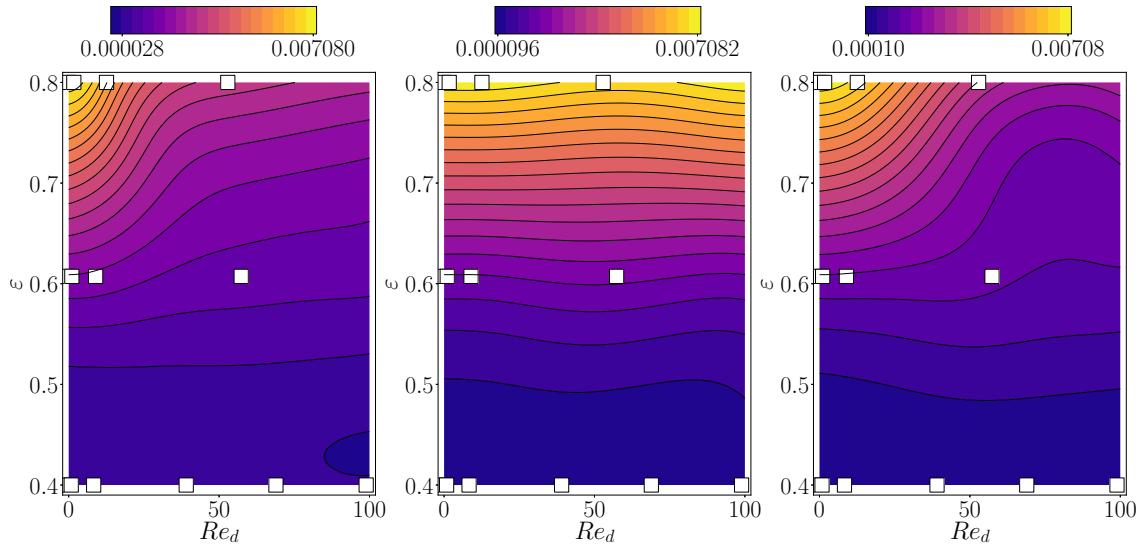


Figure 4.19: Response surface of H_{11} ; in the left frame $\phi = \theta = 0$, in the centre frame $\phi = 90^\circ$, $\theta = 0$ and on the right $\phi = 45^\circ$, $\theta = 22.5^\circ$.

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