Status of Transition Delay Using Compliant Walls

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I. Introduction

UNQUESTIONABLY, the dolphin was the original inspiration for the use of compliant walls for drag reduction and the delay of laminar/turbulent transition. Gray[1] was the first to remark in the scientific literature on the anomalously high swimming speeds apparently achieved by the dolphin, the so-called Gray's paradox. But it was Kramer[2–5] who attempted to exploit the "dolphin's secret" technologically. His pioneering compliant coatings were based closely on the dolphin's epidermis. Nature does not yield up her secrets easily, and undoubtedly Kramer had an imperfect understanding of the structure and function of the dolphin's epidermis. Nevertheless, he observed remarkably large drag reductions in a series of experiments on bodies of revolution covered with his compliant coatings and towed at high speed through seawater. Kramer was convinced that these drag reductions were a result of the transition-delaying properties of his compliant coatings.

In the early part of the 1960s, Kramer's experiments led to a flowering of theoretical studies concerned with hydrodynamic stability and transition in boundary layers over flexible surfaces. Several able theoreticians were attracted to the problem. The work of Benjamin,[6,7] Landahl,[8] Landahl and Kaplan,[9] and Gyorgyfalvy,[10] in particular, have stood the test of time. Their concepts form the foundations of the subject, and their papers remain essential reading. More or less in parallel with the theoretical work, several attempts[11–14] were made to corroborate Kramer's results by means of independent experiments. All such attempts apparently ended in complete failure as far as verification of Kramer's large drag reductions was

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concerned. This state of affairs, combined with some apparently major inconsistencies between Kramer's observations and theory, led to general disaffection in the scientific and engineering community toward compliant walls as a means of achieving transition delay. Consequently, little further work was undertaken on the topic until the 1980s. During the interim period, attention was diverted for the most part to the investigation of the possible use of compliant walls for reducing turbulence levels in fully turbulent boundary layers.

In the 1980s, a reassessment of Kramer's work, among other things, has led to a rebirth of interest in the application of compliant walls for transition delay in water. Kramer's work still remains controversial but, in contradiction to the earlier prevailing view, Carpenter and Garrad[15,16] have demonstrated that in theory, at least, substantial transition delays were possible with Kramer's coatings. Moreover, a fairly careful assessment of the other experimental tests of the Kramer coatings has shown that the facilities and methods used were completely unsuitable for studying transition-related phenomena. In a very real sense, however, this is past history since recent theoretical and experimental work has amply confirmed the transition-delaying potential of compliant walls. Drag reductions, or equivalent transition postponements, of the magnitude reported by Kramer have yet to be confirmed. But, on the basis of present knowledge, there is no reason to suppose that compliant coatings with even better performances will not ultimately be developed. It is the purpose of this review to focus on the recent progress toward the application of compliant walls for transition delay.

There have been several excellent reviews on compliant walls and related topics. Benjamin[17] reviewed the early work. The research in the middle period, which was oriented primarily toward turbulent boundary layers, is covered by Fischer et al.,[18] Bushnell and Hefner,[19] and Dinkelacker.[20] More recent reviews have been carried out by Dowell,[21] Gad-el-Hak,[22,23] and Riley et al.[24] Inevitably, the present review will cover some of the same ground as these recent reviews, particularly the last one cited. Nevertheless, it is hoped that the present paper will bring a narrower focus and a fresh point of view to the topic. Also, much use will be made of current material not yet available in the open literature.

II. Types and Theoretical Models of Compliant Walls

What is meant by a *compliant wall*? The term has been applied, at one time or another, to all the cases illustrated in Fig. 1 except the rigid wall. In the present context, the term will be reserved for the passive flexible surface depicted by case 3. Case 2 could include such thin, possibly flexible layers as slime. Active flexible walls (case 4) have been investigated experimentally by Park et al.[25] and others. Flow-controlled active flexible walls have been studied theoretically by Metcalfe et al.[26]; it will be a challenging task to implement such concepts in practice. These other examples have something in common with case 3, but much of the essential physics is substantially different.

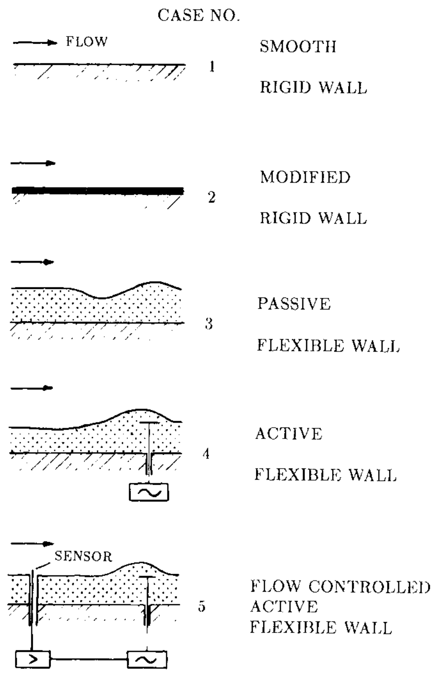


Fig. 1 Types of surface (Ref. 20).

For the study of the interaction of a flowing fluid and a compliant surface, the mechanics of the wall are as important as the fluid mechanics. All compliant walls are not the same, except in a very fundamental sense, in their response to the unsteady forces generated by the flowing fluid. Many of the compliant walls studied experimentally and the corresponding theoretical models are illustrated in Fig. 2. There are two main groups of theoretical models, which will be called *surface-based* and *volume-based* models, respectively. For the former, the equation(s) of motion for the wall can be expressed in terms of variables that do not depend on the coordinate y perpendicular to the compliant surface whereas, for the latter, the equations of motion depend on y, as well as on the coordinate(s) parallel with the surface. Thus, for the wall, the use of a surface-based model reduces the spatial dimensions by one, thereby effecting both a simplification and usually a reduction in computational requirements.

The original Kramer compliant coating is shown in Fig. 2a, and the surface-based plate/spring theoretical model, used in Refs. 15, 16, and 27 to study its hydroelastic and hydrodynamic instabilities, is illustrated in Fig. 2b.

The idea of reducing turbulence or instability growth rates by constraining the wall motion so as to generate a negative Reynolds shear stress was first studied theoretically by Ffowcs Williams.[28] Grosskreutz[29,30] designed and manufactured his compliant walls (Fig. 2c) with the same idea in mind. Figure 2d shows the surface-based theoretical model, based on his concept,

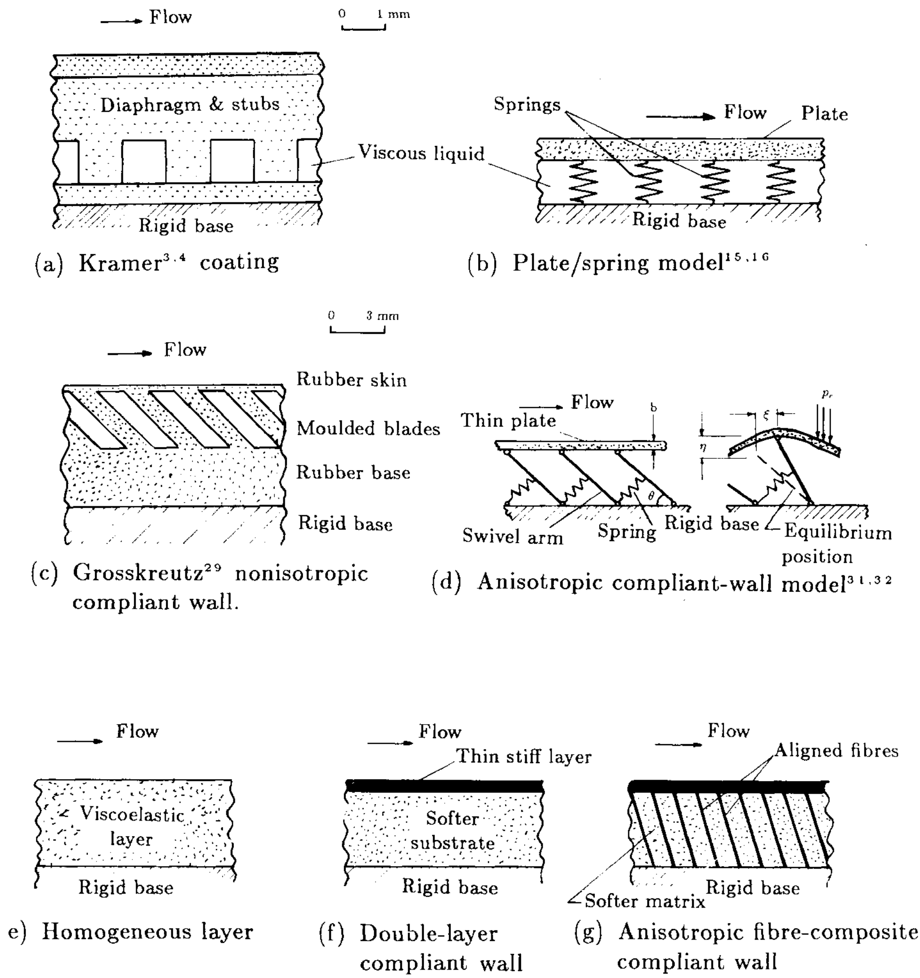


Fig. 2 Types of compliant wall.

developed by Carpenter and Morris.[31–34] It comprises a thin plate supported on inclined, sprung lever arms. Under the action of small-amplitude fluctuating pressure, the ends of the lever arms move in such a way that the horizontal and vertical displacements are simply related, that is,

$$\xi = \eta \tan \theta \tag{1}$$

With the flow direction as shown in Fig. 2d, the product of the streamwise and normal velocity components will always be positive, thereby generating a negative Reynolds shear stress. For such anisotropic compliant walls, the equation of motion can be written in the form[34]

$$\frac{b\rho_m}{\cos^2\theta}\frac{\partial^2\eta}{\partial t^2} + B\frac{\partial^4\eta}{\partial x^4} - Eb\frac{\partial^2\eta}{\partial x^2}\tan^2\theta + \{K - g(\rho - \rho_s)\}\frac{\eta}{\cos^2\theta}$$
$$= -p'_w + \sigma' + \tau'\tan\theta \qquad (2)$$

where ρ_m, ρ, and ρ_s are, respectively, the densities of the plate material, the main-stream fluid, and the substrate material, which may be either fluid or solid; B is the flexural rigidity of the plate, K the spring stiffness, and E the elastic modulus of the plate material; p'_w, σ', and τ' are, respectively, the net fluctuations in dynamic pressure and in the viscous direct and shear stresses acting on the plate. B is related to E and b as follows:

$$B = \frac{Eb^3}{12(1 - \nu_P^2)}$$

where ν_P is the Poisson ratio. The remaining notation is defined in Fig. 2d.

Equation (2) is the general form of the equation of motion corresponding to many of the commonly used surface-based theoretical models for compliant walls. For example, when θ is set equal to zero, it reduces to the equation of motion for the isotropic case illustrated in Fig. 2b. If $Eb\tan^2\theta$ is replaced by T, the tension per unit width, and θ is set equal to zero in the remaining terms, together with B and K where appropriate, we obtain the equation for the simple tensioned-membrane surface studied experimentally by Babenko[35] and theoretically by Landahl and Kaplan[8,9] and Domaradski and Metcalfe.[36] In many cases, e.g., the original Kramer coatings, there is a viscous fluid substrate. The dynamic effects of this can be fairly readily modeled theoretically (see Refs. 15, 16, and 27). Viscoelastic materials are commonly used for compliant walls, and viscoelastic effects can also be readily incorporated by the use of complex elastic moduli,[15] which take the form $E(1 - i\tilde{\eta})$: $\tilde{\eta}$ is the viscoelastic loss factor. Viscoelasticity and the effects of a viscous substrate are the main sources of dissipation in compliant walls. The earlier theoretical studies merely incorporated a damping term in the equation of motion for the wall in order to model these effects qualitatively.

Conceptually, the simplest compliant wall is the homogeneous viscoelastic layer illustrated in Fig. 2e. This case has been extensively studied experimentally by Hansen et al., e.g., see Ref. 37, and more recently by Gad-el-Hak et al.[38,39] Volume-based theoretical models based on the Navier equations, suitably extended to incorporate viscoelastic effects, have been developed and studied by Duncan et al.,[40] Fraser and Carpenter,[41] and Yeo.[42,43] Very soft silicone rubber tends to be delicate, so that Gaster[44] and Daniel et al.[45] covered their compliant panels with a thin protective layer of latex rubber. (See Fig. 2f.) It was subsequently discovered that this also improved the performance of the compliant wall. Gaster suggested that the main dynamic effect of the protective covering was to restrain the horizontal motion of the wall. Accordingly, a simple model of such walls is obtained by treating them as homogeneous layers that are free to move only in the vertical direction at the fluid/solid interface. Calculations[45]

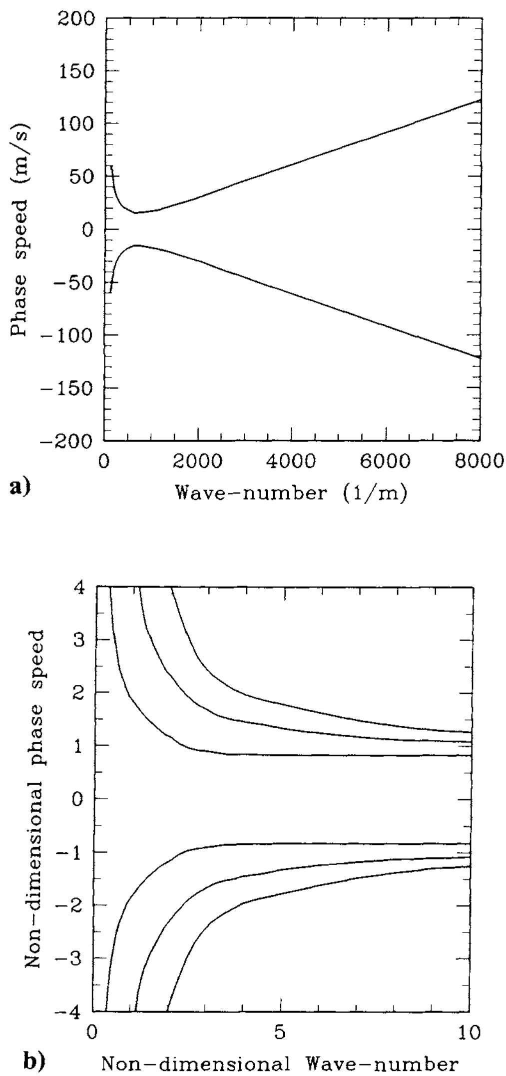


Fig. 3 Dispersion curves for free waves over compliant walls. (Fig. 3b is based on Fig. 2 of Ref. 40.)

indicate that such a model has improved stabilizing characteristics compared to the plain homogeneous layer. More precise volume-based theoretical models of two-layer compliant walls, based on the use of modified Navier equations in both layers, have been developed and studied by Fraser and Carpenter,[41] Willis,[46] Yeo,[42,43] and Duncan.[47] Yeo[42,48] has also developed and studied approximate volume-based models of single- and double-layer fiber-composite anisotropic compliant walls of the type shown in Fig. 2g. More involved computational volume-based models of various compliant walls using finite-element methods were investigated by Buckingham et al.[49,50] Possibly the most complex compliant wall studied experimentally was that of Chung and Merrill.[51]

Some idea of the fundamental wave properties of compliant walls is given by the free-wave dispersion curves shown in Figs. 3a and 3b. In the case of the isotropic plate/spring model of Refs. 15, 16, etc., there are two wave modes, one propagating to the left and one to the right. A significant feature is the existence of a minimum speed. The wave characteristics of the homogeneous elastic layer shown in Fig. 3b differ in two important respects from those in Fig. 3a. First, and most obvious, is the existence of two infinite families of wave modes. Second, it will be noted that the minimum speed occurs at infinite wave number. The wave characteristics of two-layer walls[47] resemble a combination of both Figs. 3a and 3b in that there are two infinite families of waves, but a locally minimum speed occurs at a finite wave number. Both the left-running and right-running waves correspond to different types of hydroelastic instability that occur at sufficiently high flow speeds.

III. Formulation for Flow/Wall Interaction

The boundary conditions at the wall/fluid interface are a crucial aspect in the formulation of the problem of the unsteady interaction between a flowing fluid and a compliant wall. For the full nonlinear problem, the boundary conditions are straightforward; it is required that the velocity and stress components be continuous between fluid and solid. The full nonlinear problem is probably beyond the capabilities of current computational methods. Accordingly, it is necessary to linearize the boundary conditions, and this is when problems may arise. Confining attention to the two-dimensional case, let $\{u',v'\}$ be the perturbations in the horizontal and normal velocity components. Then the linearized conditions on the velocity are given by[6,8,34]

$$u' + DU_w\eta = \dot{\xi} \quad \text{and} \quad v' = \dot{\eta} \quad \text{at} \quad y = 0 \tag{3}$$

Note the presence of the term involving DU_w, the undisturbed velocity gradient at the wall; this is the contribution representing the displacement of the mean flow. Some early theoretical studies erroneously omitted this term.

For surface-based theoretical models, the equation of wall motion, e.g., Eq. (2), can be used as a third boundary condition, thereby automatically insuring that the continuity conditions on stress are satisfied. For linear theory, the amplitude of the disturbance must be small but is otherwise arbitrary. Thus, one of the three boundary conditions is redundant. The number of boundary conditions can be reduced by introducing a surface response coefficient. For example, Benjamin[6] used surface compliance, defined as the normal displacement divided by the pressure, whereas Landahl[8] worked with admittance, defined as velocity divided by normal stress. The forms for the response coefficients were derived from the original three boundary conditions and their values for the fluid and wall required to be equal at the interface, thereby replacing the second and third boundary conditions. This approach has great advantages for deriving general results or formulating approximate theories. For purely numerical studies, however, it confers no advantage and becomes difficult to formulate for more complex situations. Consequently, normalization conditions, e.g., setting the perpendicular wall velocity or displacement equal to unity, have been more commonly used in recent studies.[34,42,43,46,48]

For volume-based wall models, both velocity and stress continuity must be explicitly imposed at the flow/wall interface and at all interfaces located within the wall. The linearized stress boundary conditions are given by

$$-p'_w + \sigma'_f + \rho g \eta = \sigma'_s + \rho_s g \eta \quad \text{and} \quad \tau'_f = \tau'_s \quad \text{at} \quad y = 0 \tag{4}$$

where subscripts f and s are used to denote fluid and solid properties in cases where confusion may arise. The presence of body-force terms in Eq. (4) will be noted. These are analogous to the $DU_w\eta$ term in Eq. (3) in that they represent the displacement of the undisturbed media. They are also analogous to the body-force terms incorporated in the spring-stiffness term on the left-hand side of Eq. (2). A rigorous derivation of the stress-continuity conditions makes it clear that these body-force terms should be included. Nevertheless, they have been omitted by all previous authors but Willis.[46] (Evrensel and Kalnins[52] include the body force in the fluid only.) For multilayer walls, continuity conditions like (3) and (4) should also be applied at the interfaces within the wall. Again, the body-force terms should, in general, be included. It should be remarked that the omission of these body-force terms may not have led to large errors in practice since the materials used for compliant walls usually have densities close to that of water.

In almost all cases studied to date, the linearized equations of motion for the fluid take the form of either the Orr-Sommerfeld or potential-flow equation. Generally, these equations must be solved subject to the boundary conditions discussed earlier. For some approximate asymptotic theories,[53–55] the flow equation(s) can be solved in such a way as to give suitable expressions for the fluid forcing terms in Eq. (2), say, thereby reducing the problem to that of solving the wall equation(s). Similarly, Gaster and Willis[46,56] have developed an accurate numerical method, whereby the Orr-Sommerfeld equation is solved in advance in the form of a series expansion, the coefficients of which are stored in the computer memory. Again, this allows the problem to be reduced largely to solving the wall equation(s), with consequent great savings in computing times.

When volume-based theoretical models are being used, it is necessary to solve the wall equations explicitly, as well as those for the fluid. To date, only compliant walls of infinite length and a breadth have been studied. This permits general solutions of the wall equations to be derived in the form of a superposition of fundamental wavelike solutions. The neatest and most satisfactory way of solving numerically the problem for multilayer walls is due to Yeo,[42,43] whose formulation involves the use of an overall propagation matrix to link the stress and velocity components at the flow/wall interface to their values at the bottom of the lowest layer.

IV. Types and Mechanisms of Instability

Unsteady fluid flow over a compliant wall involves the interaction of two wave-bearing media. Consequently, there are many different types of instability. The various neutral curves given in Fig. 4 give some idea of the complexity of the situation. Actually, this is by no means an unusually complicated case. Only two types of instabilities are present. (For the present, ignore the discrete data points.)

The proliferation of instability modes is perhaps the most significant difference between the transition process over a rigid flat plate and that over a compliant wall. The Tollmien-Schlichting wave (one of the two instabilities in Fig. 4) is what leads to transition over the rigid flat surface. Accordingly, it is clear that this is fundamentally an instability of the

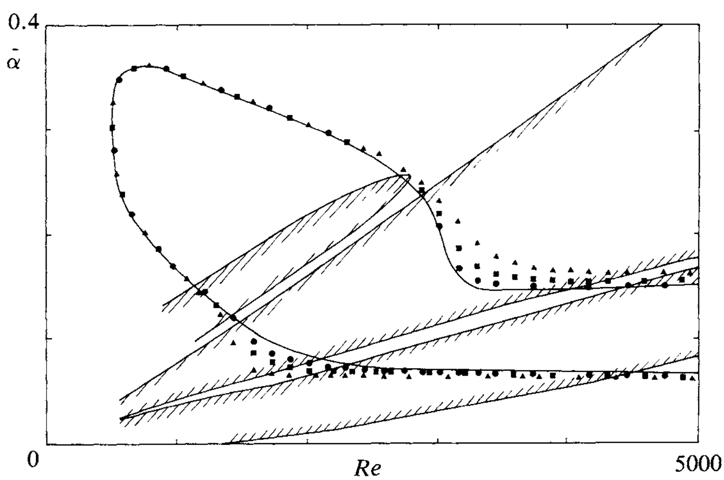


Fig. 4 Neutral stability boundaries for a boundary layer over compliant walls of the type illustrated in Fig. 2b; water flow at 18 m/s. Wall properties: No fluid substrate. $\rho \simeq \rho_m \simeq 1000$ kg/m^3, $E = 0.5$ MN/m^2, $K = 0.115$ GN/m^3, $\nu_P = 0.5$, and $b = 2$ mm. Plain lines, Tollmien-Schlichting waves ($\tilde{\eta} = 0.0$); shaded lines, traveling-wave flutter ($\tilde{\eta} = 0.0$); ●, $\tilde{\eta} = 0.02$; ■, $\tilde{\eta} = 0.05$; ▲, $\tilde{\eta} = 0.1$ (T-S waves). (From Ref. 15.)

boundary-layer flow. It is modified by wall compliance but, for the most part, its basic character remains unchanged. The wave-bearing characteristics of the compliant wall are exemplified by the free-wave modes shown in Fig. 3. Both the left-running and right-running free-wave modes can become unstable if the flow speed is high enough. These instabilities are fundamentally wall-based; consequently, they were termed *flow-induced surface instabilities* by Carpenter and Garrad.[15] They are closely analogous to the instabilities studied in hydro- and aeroelasticity.[57] There are two main types, namely, *divergence*—an essentially static instability—and *traveling-wave flutter*—the other instability present in Fig. 4.

The classification of instabilities, according to whether they are flow-based (Tollmien-Schlichting) or flow-induced surface instabilities, has its merits. It also has a major defect in that, under certain circumstances, the Tollmien-Schlichting and traveling-wave flutter instabilities can coalesce to form a powerful new instability.[15,46,58] Plainly, this new instability has some of the attributes of both the flow-based and flow-induced surface instabilities. It is difficult to find a completely satisfactory name for this new instability. The term *transitional mode*, due to Sen and Arora,[59] will be used in what follows.

The instabilities can also be classified according to whether they are *convective* or *absolute*. The idea of a convective instability, although not the the terminology, originated with Gaster,[60,61] who recognized that the Tollmien-Schlichting waves are better modeled by disturbances that grow with distance downstream from some sort of source than by the temporally growing disturbances used in the classic theory of Tollmien,[62] Schlichting,[63] and Lin.[64] In order to appreciate the essential character of a convective instability, consider a packet of Tollmien-Schlichting waves excited by an impulsive source. Suppose that a probe for measuring velocity fluctuations were placed somewhere downstream of the source. As the wave packet traveled downstream past the probe, the amplitude of the measured signal would initially rise and then fall (see Fig. 5a). It was noted by Gaster[65] that not all instabilities need behave this way. In cases where the group velocity is zero, the probe signal begins to rise at some time after the initiation of the disturbance but does not subsequently decay (see Fig. 5b). Such instabilities are inherently temporally growing and are nowadays termed *absolute*. Both divergence and the transitional mode are absolute, whereas traveling-wave flutter, like Tollmien-Schlichting waves, is convective. Absolute instabilities, if they occur, would lead to profound changes in the transition process. It is pointless to consider reducing their growth rate or postponing their appearance to a higher Reynolds number. Nothing short of complete suppression will work.

A third classification for the instabilities is due to Benjamin[6,7] and Landahl.[8] In this case, the instabilities are classified according to how they respond to irreversible energy transfer to and from the compliant wall. Benjamin's discovery that damping in the wall destabilizes Tollmien-Schlichting waves led to the development of this concept. In simple terms, class A waves are stabilized/destabilized by irreversible energy transfer to/from the compliant wall, whereas class B responds in exactly the opposite way. Tollmien-Schlichting waves belong to class A, whereas

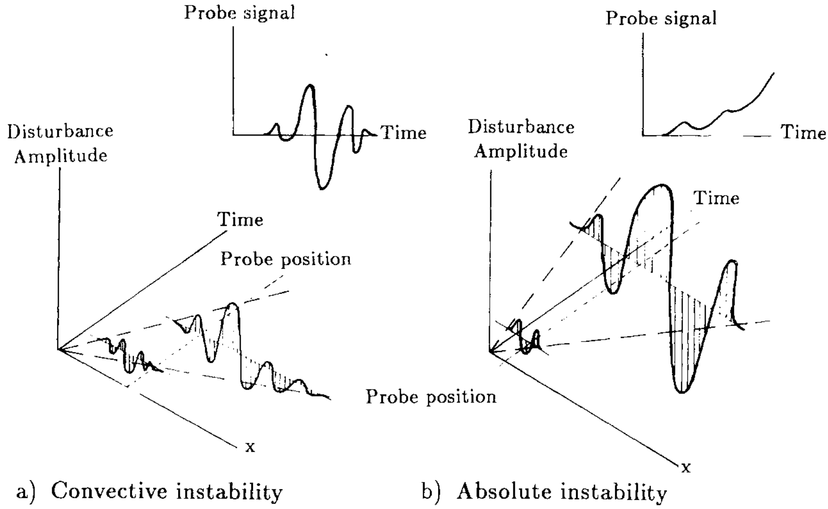
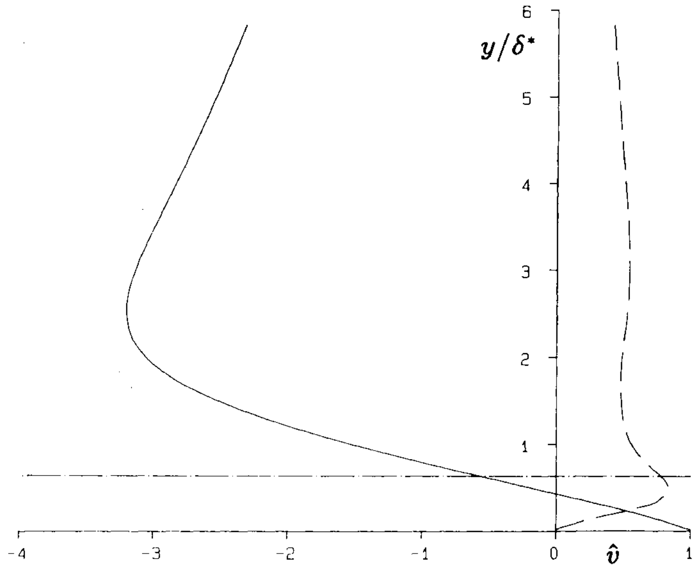


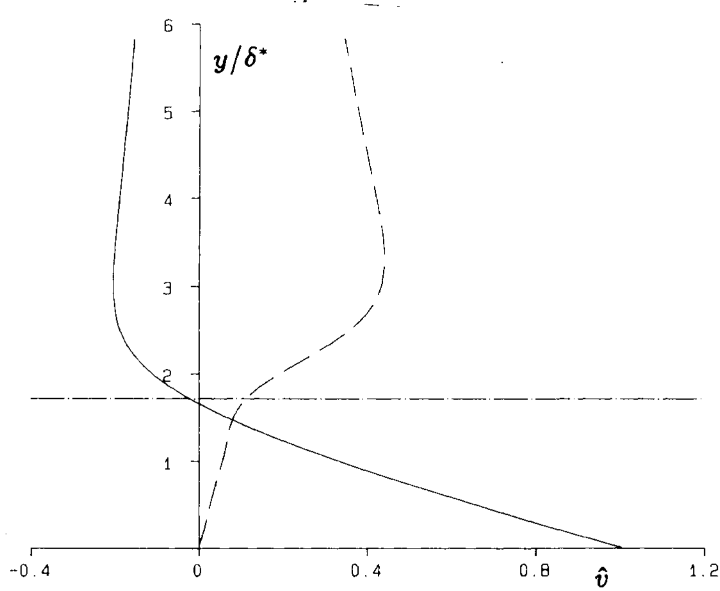
Fig. 5 Schematic illustration comparing convective and absolute instabilities.

traveling-wave flutter is a class B instability. A simple physical explanation for their respective responses to irreversible energy transfer lies with the fact that one is fundamentally an instability in the flow whereas the other is fundamentally an instability in the wall. This is plainly seen from a comparison of the distributions of $\hat{v}$ (the complex amplitude of the normal disturbance velocity) across the boundary layer plotted in Figs. 6a and 6b. In the case of the Tollmien-Schlichting wave, the maximum amplitude is reached well into the flowfield whereas, for traveling-wave flutter, it occurs at the wall itself. Thus, it can be plausibly argued that any mechanism that feeds energy into the wall will tend to decrease the amplitude of the Tollmien-Schlichting wave but increase the amplitude of the traveling-wave flutter. There is also a third class of disturbance, namely class C, which is characterized by indifference to irreversible energy transfer. Both divergence and the transitional mode are class C as well as absolute.

In many ways, the Benjamin-Landahl energy classification and the distinction between convective and absolute instabilities are the two most profound concepts for achieving a deep understanding of the behavior of instabilities over compliant walls, both in a practical and a fundamental sense. The former provides the means for predicting how the instabilities will respond to changes in wall properties and also helps to identify the underlying instability mechanisms. The latter helps us see that certain instabilities may affect the flow much more profoundly than others and may require quite different approaches. It is a rather curious fact that both these concepts were discovered more or less simultaneously in fluid mechanics and plasma dynamics.[66,67] They were then later reintroduced into fluid mechanics from plasma dynamics. For example, Cairns[68] took the concept of "negative-energy waves" (i.e., class A) from Briggs,[66] not



a) Tollmien-Schlichting wave



b) Traveling-wave flutter

Fig. 6 Distribution of the normal disturbance velocity across the boundary layer over a compliant wall. Wall and flow properties are as in Fig. 4. In both cases, $\hat{v}$ is set equal to 1 at the wall, $\alpha\delta^* = 0.11$, and $Re_{\delta^*} = 2000$; complex phase speed $= 0.342 + 0.007i$ in a) and $0.828 + 0.003i$ in b), respectively —c_r, — —c_i, and —·— location of critical point. (From Ref. 54.)

Benjamin or Landahl. Similarly, Brazier-Smith and Scott[69] and Huerre and Monkewitz[70] learned of the concepts of convective and absolute instabilities from Briggs[66] and Bers[71] rather than from Gaster.[65] The terms absolute and convective were, in fact, borrowed from plasma dynamics.

With so many different types of instability, it is natural to ask whether all the possible instabilities have, in fact, been identified. This question can be answered, in principle, by the methods of Sen and Arora,[59] who formulate a sort of inverse problem, whereby they investigate the effect of all possible wall motions on the boundary-layer stability. An alternative approach is to use globally convergent methods to find all the eigenvalues in a given case. This has been done by Carpenter and Morris[32,34] using the methods of Ref. 72. No other instabilities have come to light, but an exhaustive search over all Reynolds numbers and wave numbers or frequencies has not been carried out. Both approaches can be used only with surface-based models of compliant walls. It is worth pointing out that an anomalous spatially growing mode was discovered in Ref. 34. It was established that this mode is not actually unstable, but this was not immediately apparent. The eigenvalue and eigenfunction are not unlike those corresponding to the Tollmien-Schlichting mode, except that the growth rates are typically one or two orders of magnitude higher. It is quite possible that this solution has been mistaken for a genuine instability by previous authors.

V. Convective Instabilities

The Benjamin-Landahl energy classification reveals the importance of irreversible energy transfer for the convective instabilities over a compliant wall. Many mechanisms for irreversible energy transfer are possible, namely:

- Energy transfer within the flowfield:
  - Energy production by Reynolds shear stress
  - Viscous dissipation
- Energy transfer to/from the wall:
  - Irreversible work done by the wall pressure fluctuations
  - Irreversible work done by the wall viscous stress fluctuations
  - Damping in the wall

The effect of the energy-transfer mechanisms one instability growth is also evident from a study of the energy equation for the disturbances, such as that undertaken by Domaradski and Metcalfe.[36] The integral version of this equation, obtained by integrating their equation (24) across the boundary layer, is given by

$$\omega_i \rho \int_0^\infty (\overline{u'^2} + \overline{v'^2})\, \mathrm{d}y = \rho \int_0^\infty (-\overline{u'v'}) DU\, \mathrm{d}y$$

$$- \int_0^\infty (\overline{\sigma'_{ij}\, \partial u'_i / \partial x_j})\, \mathrm{d}y + \overline{p'_w v'_w} - \overline{\sigma'_{22} v'_w} - \overline{\sigma'_{12} \xi} + DU_w \overline{\sigma'_{12} \eta} \tag{5}$$

where ω_i represents the temporal growth rate, σ'_{ij} the components of the fluctuating viscous stress tensor and an overbar an average over a cycle and wavelength. The terms on the right-hand side of Eq. (5) represent, from left to right, the rate of energy production by the Reynolds stress, the viscous dissipation rate (the repeated subscripts convention holds for this term), the rates of work done to the wall by the fluctuations in pressure and normal and shear viscous stresses and, finally, an extra energy-removal term [due to the form of the boundary conditions (3)], which represents the interaction of the displaced undisturbed flow and the fluctuating shear stress. The work done by the fluctuating normal viscous stress is probably always negligible. The contribution of the last energy-removal term of Eq. (5) is surprisingly large, as can be seen from the results of Domaradski and Metcalf.[36] The effect of dissipation in the wall does not appear explicitly in Eq. (5), although it is well known to have a destabilizing effect on the Tollmien-Schlichting waves. It causes a phase lag between the v'_w and p'_w, thereby reducing, or even reversing, the work done by the pressure. The kinetic energy in the flow is a very large fraction of the total energy associated with the instability for the Tollmien-Schlichting waves but a very small one for the traveling-wave flutter, the energy of which is mostly in the wall. This important difference explains why the two types of instabilities have the opposite response to energy transfer to the wall. The energy equation for the wall would be a more appropriate guide for the traveling-wave flutter.

The energy-transfer mechanisms within the flowfield are, of course, present for instabilities over rigid surfaces. Energy production by the Reynolds shear stress is the primary mechanism for the destabilization of the Tollmien-Schlichting instability. In the absence of viscosity, the normal and streamwise disturbance-velocity components are in antiphase, thereby rendering the Reynolds stress zero. As originally postulated by Prandtl,[73] the effects of viscosity, especially in the region between the critical point (where the phase speed c_r of the disturbance equals the undisturbed velocity U in the boundary layer) and the wall, bring about the essential phase shift between the disturbance-velocity components, thereby creating the Reynolds shear stress.

How is this instability mechanism affected by wall compliance? Distributions of the normalized rates of energy production by Reynolds shear stress and viscous dissipation across the boundary layer are plotted in Fig. 7 for the rigid wall, an isotropic compliant wall like Fig. 2b, and two anisotropic compliant walls like Fig. 2d. Disregarding the anisotropic walls for now, a comparison of the rigid and isotropic compliant walls show that, near the wall, there is more energy production in the latter case but, across the bulk of the boundary layer, the energy production is much reduced for compliant walls. Carpenter and Morris[34] showed that wall compliance has both a local influence and a longer-range influence on the disturbance velocities. The former is confined to the viscous wall layer and is detrimental for stabilization because it gives rise to increased Reynolds shear stress. But this effect is more than outweighed by the favorable long-range influence that leads to much reduced Reynolds shear stress over the bulk of the

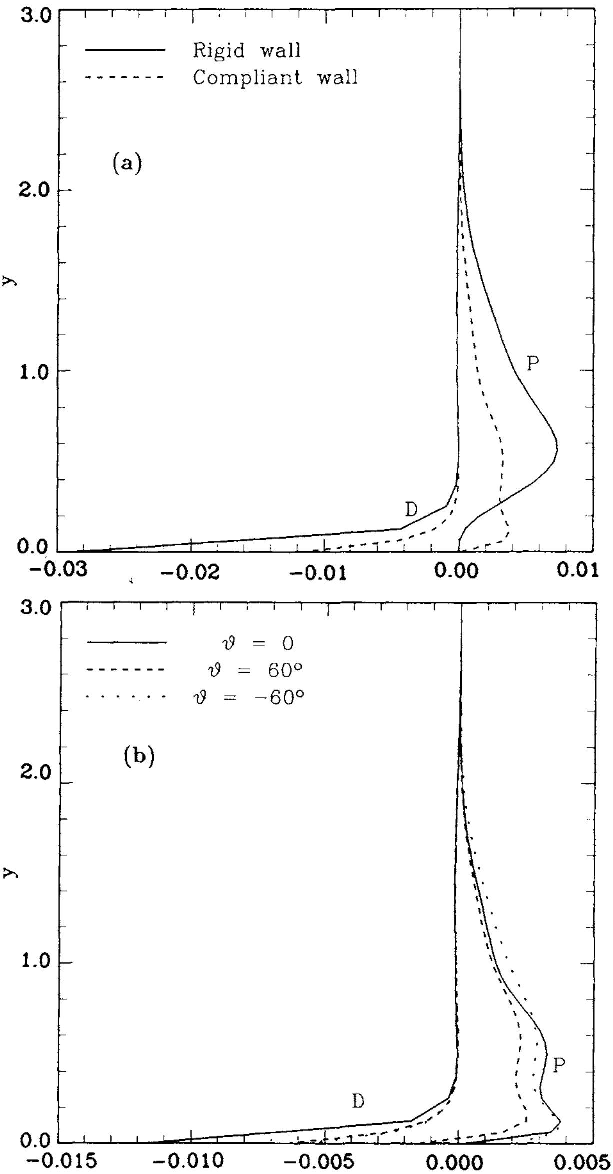


Fig. 7 Comparisons of distributions across the boundary layer of the rates of production of energy by Reynolds stress (P) and viscous dissipation (D). Both quantities have been normalized by dividing by the integral across the boundary layer of disturbance kinetic energy. In all cases, $Re_{\delta^*} = 2240$ and α has been chosen for maximum growth rate; water flow at 20 m/s. Compliant walls are of the types illustrated in Figs. 2b and 2d with properties: $\rho = \rho_m = 1000$ kg/m^3, $\nu_P = 0.5$; a) $\theta = 0$, $E = 1.385$ MN/m^2, $K = 0.354$ GN/m^3, and $b = 0.75$ mm; b) $\theta = \pm 60$ deg, $E = 0.509$ MN/m^2, $K = 0.059$ GN/m^3, and $b = 0.111$ mm. The properties are optimized as explained in Sec. VII (Data supplied by R. D. Joslin.)

boundary layer. Energy balances for instabilities over compliant walls were also studied by Willis[46] and Domaradski and Metcalfe.[36]

Irreversible work done by the fluctuating wall pressure is the primary mechanism for the destabilization of traveling-wave flutter. Benjamin[7,74] showed that, to a good approximation, the wall pressure is given by

$$p'_w = i\alpha \int_0^\infty (U - c_r)v' \, \mathrm{d}y \tag{6}$$

where α is the wave number. Thus, if no phase shift occurs in v' across the boundary layer, then p'_w and v'_w will be in antiphase for nondissipative walls, and there will be no irreversible energy transfer. As shown originally by Miles[75–77] for water waves and Benjamin[7,74] for boundary layers over compliant walls, the essential phase shift occurs at the critical point in the limit as $Re \to \infty$. In Fig. 8, numerical solutions for the complex amplitude of $(U - c_r)v'/U_\infty^2$ are plotted for traveling-wave flutter at a finite Reynolds number and compared to the asymptotic theory of Ref. 54, which is based on an extension of the concepts of Miles and Benjamin. At infinite *Re*, the imaginary part in Fig. 8b would be equivalently zero below the critical point.

Thus, irreversible energy transfer occurs whenever there is a critical point, i.e., when $0 < c_r < U_\infty$. It can be seen from the free-wave modes in Fig. 3 that, for sufficiently low flow speeds, c_r exceeds U_∞ for all wave numbers. When the minimum c_r falls to U_∞, instability sets in. It can be shown[7,16] that, to a good approximation, p'_w is zero when $c_r = U_\infty$. It therefore follows that the critical onset speed of traveling-wave flutter is obtained by setting U_∞ equal to the minimum value of free-wave speed (see Fig. 3). In this way, simple formulas for the critical speed and wave number can be derived[16,53] for surface-based models. For example, in the case of Fig. 2b,

$$\alpha_c = \left(\frac{K}{B}\right)^{\frac{1}{4}}, \qquad U_c = \left[\frac{2\sqrt{(BK)} + T}{b\rho_m}\right]^{\frac{1}{2}} \tag{7}$$

For nondissipative walls, Eq. (7) holds for all Reynolds numbers but, for dissipative walls,[55] it is only valid in the limit as $Re \to \infty$. It is possible to extend the approach to volume-based models,[40,41,47,55] although a difficulty occurs in the case of the homogeneous layer. In this case, the minimum free-wave speed corresponds to infinite wave number (see Fig. 3b). Not only does this give a critical speed that is independent of depth, in contrast to the experimental results of Gad-el-Hak,[39] but an infinite critical wave number (zero wavelength) is plainly impossible on physical grounds. When the wavelength becomes sufficiently small, the linearized boundary conditions will become invalid, so it is possible in this case that nonlinear effects determine the critical speed and wave number.

Irreversible work done by the wall pressure would have a stabilizing effect on the Tollmien-Schlichting waves according to Eq. (5) and the Benjamin-Landahl energy classification scheme. It is shown in Refs. 54 and

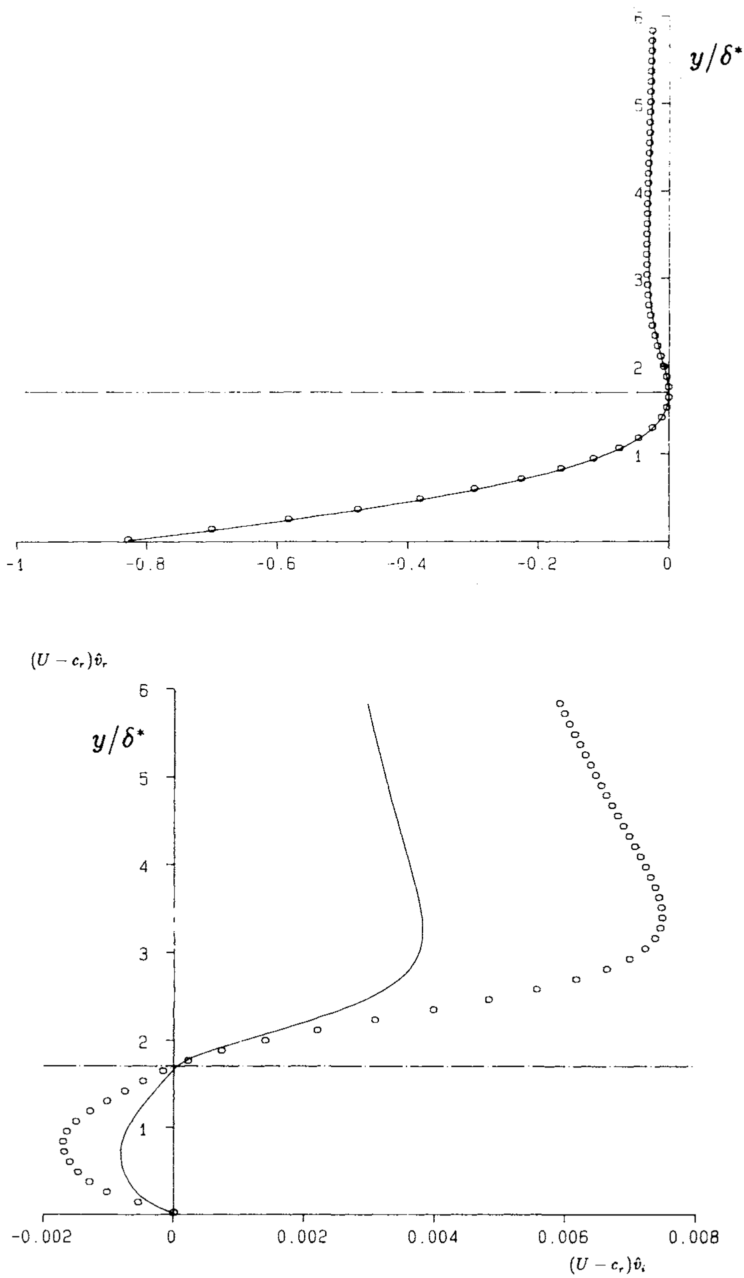


Fig. 8 Distribution of real and imaginary part of $(U - c_r)\hat{v}$ across the boundary layer for traveling-wave flutter. Wall and flow properties are as Fig. 4. Data points correspond to accurate numerical solutions of the Orr-Sommerfeld equation, and solid lines to the asymptotic theory of Carpenter and Gajjar. (From Ref. 54.)

34 to have a significant stabilizing effect for compliant walls, somewhat less than, but comparable in magnitude to, the destabilizing effect on the traveling-wave flutter. Note that the essential phase shift in v' is really the same for both energy production by Reynolds stress and energy transfer by pressure work. The crucial difference is that, because the wall pressure depends on an integral over the entire boundary layer, the energy transfer due to pressure work remains finite in the limit as $Re \to \infty$ unlike Reynolds stress, which only remains nonzero in a region that shrinks to zero as $Re \to \infty$.

In order to have irreversible energy transfer due to the work done by the fluctuating shear stress, it is vital that the compliant surface moves horizontally. Thus, this mechanism of energy transfer is confined to anisotropic walls, like Fig. 2d or 2g, to the single layer (Fig. 2e) or, to a much lesser extent, to the multilayer (Fig. 2f) walls. For isotropic walls, it can be shown[34] that the more the compliance in the horizontal direction, the greater the energy transfer from the wall, implying destabilization/stabilization of the Tollmien-Schlichting/traveling-wave-flutter instabilities. This confirms Gaster's[44,45] hypothesis with regard to favorable effect of a thin protective outer layer on boundary-layer stability over two-layer walls.

The importance of irreversible energy loss due to damping in the wall was recognized in the early theoretical work of Benjamin[6,7] and Landahl.[8] As noted in Sec. II, it usually arises from the viscous or viscoelastic properties of the fluid or solid wall components. It has a destabilizing effect on the Tollmien-Schlichting waves, as shown by the increased size of the unstable region in Fig. 4 with a rise in the viscoelastic loss factor. Even the

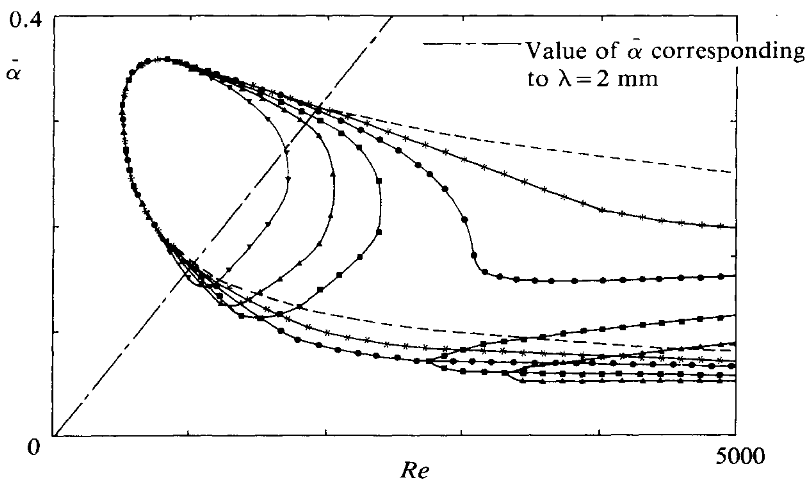


Fig. 9 Effect of a change in wall compliance on the neutral stability boundaries. Properties as in Fig. 4 except as specified. ——, rigid wall; *, $E = 1.0$ MN/m², $K = 0.230$ GN/m³; ●, $E = 0.5$ MN/m², $K = 0.115$ GN/m³; ■, $E = 0.3$ MN/m², $K = 0.069$ GN/m³; ▲, $E = 0.2$ MN/m², $K = 0.046$ GN/m³; ▼, $E = 0.1$ MN/m², $K = 0.23$ GN/m³. (From Ref. 15.)

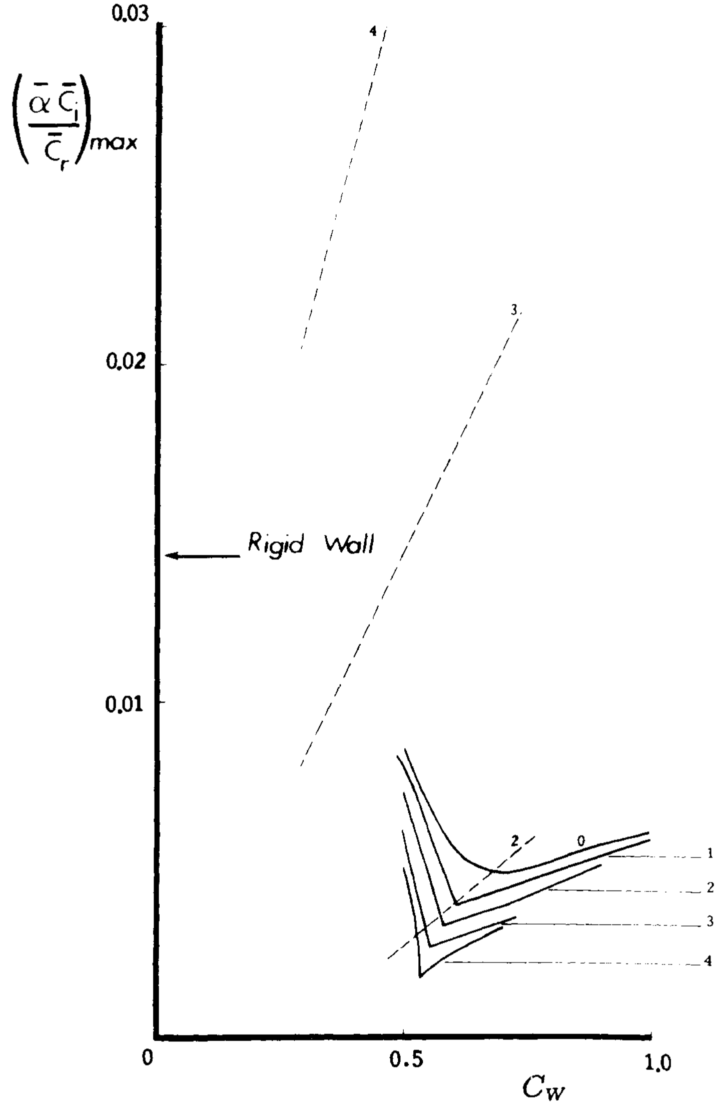


Fig. 10 Effect of plate mass on instability growth. ——, Tollmien-Schlichting waves; — —, traveling-wave flutter; 1, wall properties optimized as explained in Sec. VII; 0, 2, 3, 4, denotes 0, 2, 3, 4 times the optimum plate mass. (From Ref. 78.)

lightest damping completely eliminates the traveling-wave flutter appearing in Fig. 4, showing, as expected, that damping has the opposite, and a more powerful, effect on this instability. In fact, up to a point, damping can be used to control this instability.[15,42,43,78] It is possible[15] that Kramer[2-4] used damping in this way in his original compliant coatings.

Owing to reduced energy production by the Reynolds stress and increased energy transfer to the wall, an increase in compliance has a stabilizing effect on the Tollmien-Schlichting waves. This is shown by the shrinking region of instability in Fig. 9 as the elastic modulus of the plate and spring component of the model in Fig. 2b is made smaller. Indeed, if the wall compliance could be increased sufficiently, complete stabilization of the Tollmien-Schlichting waves could be achieved. What limits the performance of a compliant wall is the appearance of the other instability modes. For instance, as seen from Eq. (7), reducing B, K, or T leads to a drop in the critical speed for traveling-wave flutter. The instabilities are also affected by the oscillating mass of the wall. Again, it can be seen from Eq. (7) that, if the other properties are fixed, a decrease in plate mass leads to a rise in critical speed for traveling-wave flutter. This is because the free-wave speed is pushed up above U_∞. The Tollmien-Schlichting waves, on the other hand, are stabilized by a rise in plate mass.[15,46,78] A satisfactory explanation has never been given for this but, perhaps, it is because a heavier wall can absorb more energy. The effect of plate mass on the convective instabilities is illustrated by Fig. 10 taken from Ref. 78. (Each value of C_W corresponds to a particular set of wall properties, as explained in Sec. X.)

VI. Absolute Instabilities

The mechanism for divergence is simple. In certain respects, it is analogous to the buckling of a strut. It occurs when the hydrodynamic force generated by a disturbance exceeds the restorative mechanical forces in the wall. Despite this simplicity, problems arise with a theoretical treatment. In fact, almost all theoretical studies of divergence, save the recent study of Evrensel and Kalnins,[79] have been confined to the case of unsteady potential flow. There also tends to be a lack of appreciation of the practical importance of the instability. Many authors simply disregard it altogether. Yet a proper understanding of divergence is essential for the design of compliant walls capable of substantial transition delay.

When divergence is treated as a convective instability,[7,8,40,47,79] damping appears to play a vital destabilizing role. Accordingly, many authors have assumed it to be a class A instability. The experimental observations of Gad-el-Hak et al.[38,39] seem to confirm this view. They found that the traveling-wave-flutter instability, which occurred on low-damping walls, was replaced by divergence when the damping was increased. But there is an alternative interpretation.[16] Fraser and Carpenter[41] showed that, for homogeneous elastic layers, the critical speed for traveling-wave flutter is lower than that for divergence. Increased damping tends to stabilize the (class B) traveling wave flutter, thereby allowing divergence to become the dominant instability.

It has been shown[16] that, in fact, divergence is an absolute instability. Accordingly, it may be more appropriate to treat divergence as a standing-wave instability over a finite compliant panel, as is common with a hydroelastic approach.[16,27] Modeled in this way, it appears to be indifferent to damping, except that the growth rate is reduced slightly. This is plainly class C behavior. Moreover, it is what would be expected on physical grounds, given the instability mechanism. The absolute nature of the instability is well-illustrated in Fig. 11 which shows a numerical simulation of divergence over a finite panel of the type shown in Fig. 2b. The methods used were based on those of Ref. 80. The initial disturbance is a bump in the center of the panel. A study of a series of figures like Fig. 11, with flow speed increasing from zero, seems to suggest that divergence comes about

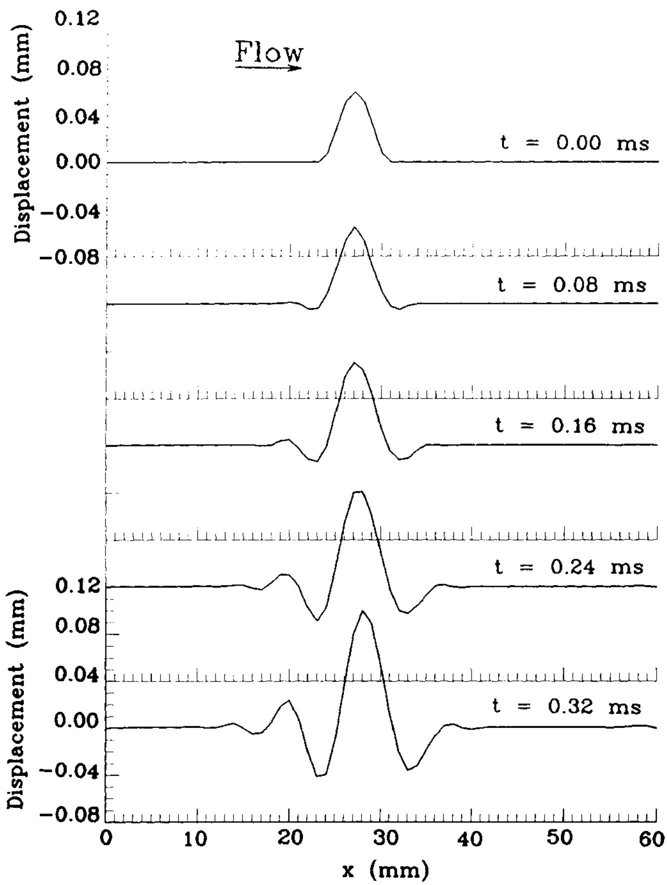


Fig. 11 Numerical simulation of the divergence instability over a compliant panel of finite length. The plots show wall displacements at successive times after an initial disturbance in the form of a bump; water flow at 22.5 m/s. Wall properties are as in Fig. 4 except $E = 0.4\ \mathrm{MN/m^2}$. (Data supplied by A. D. Lucey.[80])

as a coalescence of right-running and left-running waves. With a rise in flow speed above the critical value, divergence in the form of very slowly traveling waves, similar to those observed by Gad-el-Hak et al.,[38] can be simulated. Other studies with the numerical simulations reveal that the only effect of damping is to reduce the instability growth rate slightly.

For the interaction of potential flow with surface-based models of infinite length, simple formulas can be derived[27] for the critical speed and wave number, e.g., for Fig. 2b, they take the forms

$$U_d = 2\left(\frac{BK}{27\rho^4}\right)^{\frac{1}{8}}, \qquad \alpha_d = \left(\frac{K}{3B}\right)^{\frac{1}{4}} \tag{8}$$

Similar predictions do exist for the volume-based models,[40,41,47] but, until recently, it has not been possible to avoid either having an infinite critical wave number, as with the traveling-wave flutter, or the introduction of semiempirical phase-shift parameters. Recently, Evrensel and Kalnins[79] have presented a method that appears to overcome these problems. Nevertheless, as with traveling-wave flutter, it is possible that nonlinear effects are important for determining the critical wave number and speed in these cases.

Divergence is the most commonly observed instability on compliant walls. It occurs on dolphins[81] and humans.[82] It was observed and documented[11] on the original Kramer coatings; Eq. (8) apparently agrees well with these observations.[27] Nevertheless, apart from the work of Gad-el-Hak et al.[38,39] and MacMichael et al.,[83] and even in those cases when the flow was turbulent, there is a dearth of quantitative experimental data. References 24 and 16 should be consulted for more detailed reviews of the work on divergence.

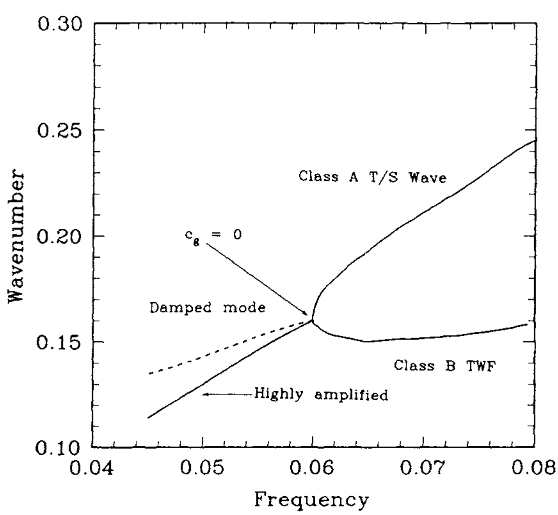


Fig. 12 The appearance of the transitional mode in wave-number–frequency space. $Re_{\delta^*} = 2700$; the flow and wall properties as in Fig. 4 except that there is a fluid substrate of viscosity 50 times that of water. (From Ref. 46.)

The other (probably) absolute instability is the transitional mode that occurs as a coalescence of the Tollmien-Schlichting and traveling-wave-flutter instabilities. This is illustrated in Fig. 12. The fact that, approaching the point of coalescence from the right, the gradient of the curve is infinite shows that the group velocity is probably zero, strongly suggesting an absolute instability.[46] But a more complete analysis has yet to be undertaken. This type of instability was first encountered[15,46,58] in the case of a theoretical model like Fig. 2b with a viscous fluid sublayer. An increase in the substrate viscosity precipitated the coalescence of the class A and class B waves. It is not clear exactly what role viscous damping plays. Presumably, it acts to slow down the traveling-wave flutter so that its phase speed drops to that of the Tollmien-Schlichting wave, thereby creating a necessary, although certainly not sufficient, condition for coalescence. The recent work of Sen and Arora[59] showed that it is a general phenomenon of compliant-wall instability and not merely confined to special cases. It has also been observed with volume-based compliant-wall models.[42,43,48] Much remains to be learned about this powerful instability. It would probably be difficult to distinguish from divergence in an experiment.

VII. Predicted Transition Delays

From a practical standpoint, it is important to be able to determine what sort of compliant wall gives the greatest transition delay and determine, at least approximately, the optimum wall properties.

The proliferation of instability modes and the relatively large number of wall parameters that can be independently varied make these questions difficult to answer without resporting to truly prodigious amounts of computing. Gyorgyfalvy[10] undertook a comprehensive parametric study in the case of the spring-backed-membrane model, although it is doubtful whether he took account of all the possible instabilities. Parametric studies like that of Sen and Arora[59] can also provide guidance on such matters. For the surface-based models, approximate results for the critical speeds, like Eqs. (7) and (8), can be used to derive a subset of optimal wall properties.[34,78,84] It was argued that, for a given class of compliant walls, e.g., the nondissipative spring-backed plate of Fig. 2b without a fluid substrate, the optimal wall properties correspond to those for which both divergence and traveling-wave flutter are marginally stable. Accordingly, U_c and U_d were set equal to U_∞ and, taking fluid and plate densities as equal and the Poisson ratio to be 1/2 (although alternative assumptions could have been made), Eqs. (7) and (8) were used to derive expressions for the various wall parameters that depend on a single nondimensional wall parameter, namely,

$$C_W = 1.02 \times 10^4 \alpha_d \nu / U_\infty$$

(Note that there is no particular physical significance in this choice of wall parameter. It happens to be the most convenient mathematically. The numerical factor was chosen to facilitate the use of results from Ref. 78.)

C_W can now be varied to determine the optimum properties according to a particular criterion, e.g., the largest value of transitional Reynolds number (i.e., corresponding to a specific value of n for the e^n method).

Figure 10 presents maximum growth rates for the two convective instabilites vs C_W for a fixed Reynolds number. The curve labeled 1 corresponds to the case of optimal properties as defined earlier. Those labeled 0, 2, 3, and 4 correspond to walls with similar properties, except that the plate thickness b is 0, 2, etc., times the optimal value. Traveling-wave flutter appears only when b exceeds its optimum value. From curve 1, it can be seen that an optimum value of C_W exists for which the Tollmien-Schlichting growth rate is minimized. This optimum C_W is highly Reynolds number–dependent. It becomes smaller as Re rises. This explains the sigmoid shape of the n vs Re_f curves in Fig. 13. (The ratio of the final instability amplitude at Re_f to the initial amplitude is given by e^n; see Ref. 85.) The location of the local minimum in the n vs Re_f curves indicates the Reynolds number for which the wall properties are optimum. The ratio of the predicted transitional Reynolds number to the rigid-wall value depends on the value chosen for n. For a fairly conservative value of $n = 7$, a four- to fivefold rise in transitional Reynolds number is predicted.

A slight further improvement in performance is possible if wall damping is introduced to control the traveling-wave flutter. This allows heavier plates to be used, with a subsequent improvement in instability growth rate (see Fig. 10), which more than offsets the adverse effect of damping on the Tollmien-Schlichting waves. It is found[78] that the best results are obtained

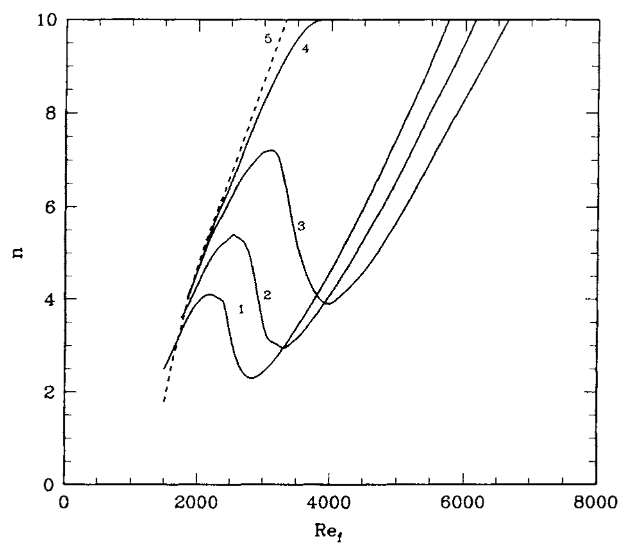


Fig. 13 Accumulated growth parameter plotted against final Re_{δ^*} for various optimal isotropic compliant walls. 1, **$C_W = 0.6$;** 2, **$C_W = 0.5$;** 3, **$C_W = 0.4$;** 4, **$C_W = 0.3$;** 5, **rigid wall. (From Ref. 34.)**

by leaving the springs undamped, with viscoelastic damping in the plate only. Results similar to Fig. 13 were obtained by Yeo[42,43] for single- and multilayer walls. In the case of one multilayer wall, Yeo obtained a 5.5-fold rise in transitional Reynolds number (using $n = 7$). But it should be noted that no account appears to have been taken of divergence, nor were his wall properties optimized. The optimal wall properties obtained in Refs. 78 and 84 vary strongly with U_∞. At 20 m/s, they are fairly similar to those of the original Kramer coatings, as were the properties of Yeo's best case. Plainly, then, these optimal properties can be realized in practice for water. In airflow, very light walls appear to be required,[78] entailing exceedingly delicate structures. Thus, the use of compliant walls to postpone transition in airflow does not appear to hold much promise.

That the optimal wall properties are so dependent on Reynolds number suggests that, in practice, it might be better to use a series of relatively small compliant panels that are tailored for the local Reynolds number. Such multipanel surfaces would also be more resistant to hydroelastic instability. This raises the question of just how small the panels can be without significantly impairing their capabilities for reducing the growth of Tollmien-Schlichting waves. This is one of the more pressing and challenging problems requiring detailed study before compliant walls can be exploited technologically for transition control.

VIII. Measurements of Instability Growth

For a general review of experimental work on compliant walls, Ref. 23 is recommended. Kramer's work and the subsequent attempts at independent corroboration are reviewed in Ref. 15, along with the experimental studies of Babenko[35] and his co-workers. For many years, the latter were the only direct measurements of instabilities on compliant walls, as distinct from drag measurements and other indirect indications. The present section will be devoted to a brief discussion of the recent experimental studies of Gaster et al.[44,45] at British Maritime Technology Ltd., Teddington, England. These represent the first successful attempt to measure the growth of boundary-layer instabilities over a compliant wall and compare the experimental data with the linear theory for Tollmien-Schlichting waves.

The preparatory work and experiments at BMT Ltd. were carried out with great care over about four years. In order to obtain the necessary low level of freestream turbulence, the experiments were conducted in a large towing tank. The model was a 2-m-long flat plate with a 450-mm^2 compliant panel inserted approximately 300 mm from the leading edge. The plate was installed face downward at a depth of 2 m. The instability waves were generated by periodically pumping minute quantities of fluid either through an array of small holes or, in the last series of tests, through a single small hole in the plate surface ahead of the test panel. Thus, both a line and point source were used, creating, respectively, approximately two-dimensional waves and a wedge-shaped wave pattern. The results presented in Fig. 14 correspond to the latter. Only two variables could be independently varied during the test, namely, the frequency of the driven

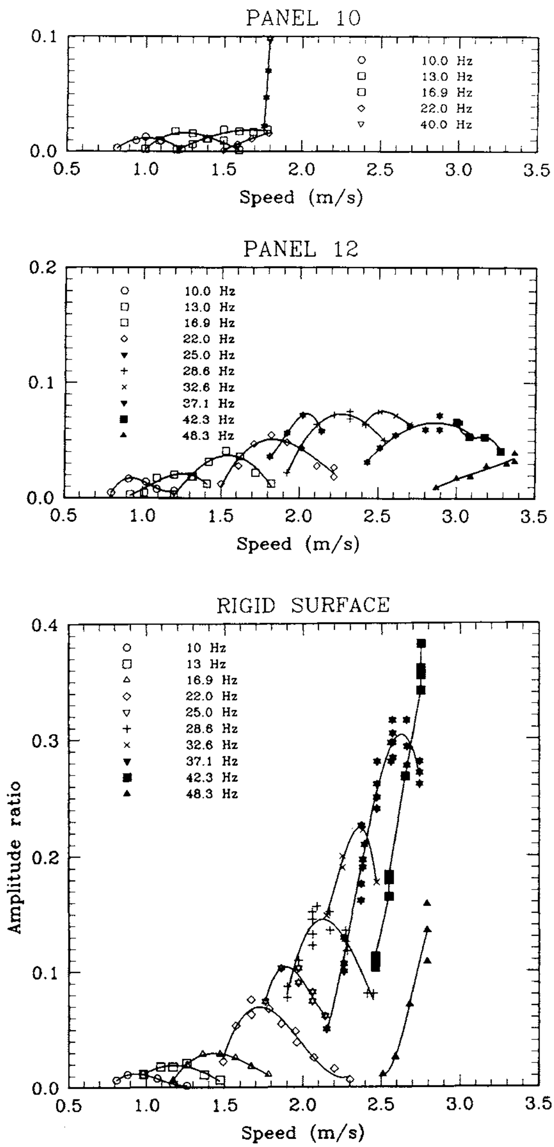


Fig. 14 Experimental data for amplitude ratio a_f/a_i, of disturbance vs flow speed for the rigid wall and two compliant panels. Properties are as in Table 1. (Based on Fig. 4.3 of Ref. 44.)

waves and U_∞. The main measurements were made with a hot-film foil gauge, glued to the trailing edge of the panel, and a pressure transducer, connected to the plenum chamber leading to the driver hole(s). In order to determine a_f/a_i, the ratio of the amplitude of the wave at the trailing edge of the panel to its initial amplitude at the driver, it was necessary to evaluate a single constant of proportionality linking the plenum pressure level and the initial amplitude of the wave. This constant was evaluated once and for all by equating the theoretical value of a_f/a_i to the measured value for the rigid wall at one specific frequency and speed. For further details of the experimental procedures, see Refs. 44–46.

Typical experimental results from the last series of tests are presented in Fig. 14. The compliant panels were of the two-layer type (see Fig. 2f), comprising a relatively deep and soft lower layer made of silicone rubber and a thin upper layer made of latex rubber. The properties are given in Table 1. It is plain from Fig. 14 that instability growth over the compliant panels is very much lower than for the rigid wall. Laminar flow could be maintained at speeds up to 2.7 m/s for the rigid wall, as compared to about 3.7 m/s for the stiffest compliant panel (No. 12). The comparison between experiment and theory is not shown here but may be found elsewhere.[24,44–46] There was good agreement between Willis' theoretical curves, corresponding to Figs. 14, and the other tests, thereby establishing for the first time that, in agreement with theory, compliant walls can very substantially reduce the growth of Tollmien-Schlichting waves. The two main shortcomings of the experiments were the lack of any means of measuring the surface displacement and problems with accurately determining the material properties of the wall. Also, limitations in the model and facilities restricted the potential gains in transitional Reynolds number.

It can be seen from Fig. 14 that some sort of very rapid growth or breakdown occurs for Panel 10 when $U_\infty \geqslant 1.75$ m/s. The breakdown speed U_B for the other cases is given in Table 1. This breakdown phenomenon was not predicted as such by the theory but appears to be what limits the performance of the compliant walls. The simple "inviscid" theory[41] gives the critical speed, for traveling-wave flutter over two-layer walls with an infinitely deep lower layer, in the form

$$U_c = 0.0166\sqrt{E}$$

Table 1 Properties of compliant walls used by Gaster et al.[45]

Panel no.	E N/m^2	$\bar{\eta}$	b mm	U_B m/s	$0.017\sqrt{E}$ m/s
Top layer	10^6	0.06	0.17	—	—
10	3000	0.03	7.83	1.75	0.95
11	6000	0.02	7.83	2.65	1.35
12	11,400	0.01	7.83	3.70	1.86

where E is the elastic modulus of the lower layer and it is assumed that the Poisson ratio is 1/2 and $\rho = \rho_s = 1000\ \text{kg/m}^3$. This gives a lower limit for the onset of traveling-wave flutter and, as shown in Table 1, is about half the breakdown speed. The breakdown speed itself is roughly proportional to $\sqrt{E}$ and occurred at the highest excitation frequency. (It also occurred at a slightly higher speed in the natural flow without artificial excitation.) Moreover, from the hot-film signals and power spectra, it appears[44] that the frequency of the instability after breakdown was about three times the excitation frequency. Also, numerical studies[44,46] of the coupled Orr-Sommerfeld/wall equations suggested that the traveling-wave flutter instability was present at the experimental flow speeds in question and that the frequency corresponding to maximum growth was three to four times, and the growth rates several times, those corresponding to the Tollmien-Schlichting waves. This all suggests that traveling-wave flutter was the cause of the breakdown. Set against these factors is the abruptness of the breakdown. This suggests that the powerful transitional instability mode or some nonlinear resonance phenomenon may be responsible for the breakdown. Plainly, additional experimental studies are required to investigate further the breakdown phenomenon and the other instability modes.

IX. More Advanced Topics

Three-Dimensional Instabilities

The effective wall compliance increases with flow speed, whereas the effective flow speed for obliquely propagating waves is less than U_∞. Thus, oblique waves experience decreased effective wall compliance compared to two-dimensional ones. This has a relative destabilizing (stabilizing) effect on the oblique Tollmien-Schlichting waves (traveling-wave flutter), which can dominate the stablizing effect of reduced effective Reynolds number. This explains the results of numerical studies[42,86] showing that three-dimensional instabilities can be more unstable than two-dimensional ones over compliant walls with good transition-delaying properties. It has also been found that the degree of dominance of the three-dimensional instabilities is highly Reynolds-number–dependent. It is clear from this recent work that, for compliant walls, three-dimensional instabilities are highly significant, even in the linear regime of transition, so that the estimates of transitional Reynolds number in Sec. VII may need to be modified accordingly.

Anisotropic Wall Compliance

The studies of Yeo,[42,48] Carpenter et al.,[31–34,54] and Joslin and Morris[87] showed that transition-delaying performance can be substantially enhanced by the use of anisotropic compliant walls like those illustrated in Figs. 2g and 2d. Using fairly conservative criteria, almost a tenfold rise in transitional Reynolds number is theoretically attainable.[34] The effect can be explained[34] by a rise in the irreversible energy transfer to the wall as a result of pressure work and a large reduction in the production of disturbance energy by Reynolds stress. See Fig. 7, and compare the case of $\theta > 0$ with that of $\theta < 0$. Irreversible work by the fluctuating shear stress also occurs, and this reduces the performance somewhat because of its relative destabilizing effect on the Tollmien-Schlichting waves. This adverse effect is weaker for oblique waves, so that anisotropic compliant walls have the added advantage of being considerably less vulnerable than isotropic ones to three-dimensional instabilities.[86]

Nonlinear Instability

Both Benjamin[17] and Landahl and Kaplan[9] briefly discussed nonlinear instability and presented some speculative preliminary results. And, until very recently, there matters have rested. Owing to the presence of additional instability modes, the nonlinear problem for flows over compliant walls is very rich in possibilities. In addition, as noted earlier, in some cases, nonlinear interface conditions between the fluid and wall may need to be investigated to provide accurate estimates of the critical speeds for the hydroelastic instabilities. It is already known[88] that mild nonlinearities arise in the theoretical treatment of compliant walls having finite length. Some steps toward an investigation of nonlinear secondary instabilities have recently been undertaken,[89] and Thomas and Craik[90] have looked at three-wave resonance for free-surface flows over flexible surfaces; the latter could be regarded as a very approximate model for the boundary-layer problem.

Two important tools for the study of nonlinear instability over rigid surfaces are direct numerical simulations and asymptotic techniques, particularly the triple deck. Work on the extension of these techniques to boundary layers over compliant walls has been undertaken recently. Domaradski and Metcalfe[36] have used pseudospectral methods with a surface-based model of simple-membrane type to obtain direct numerical simulations of the development of disturbances in a boundary layer over a compliant wall. They have carried out a careful study to show that they can reproduce the results of linear stability theory. As yet, their methods have not been applied to nonlinear instability, although the results for the linear case are of great interest in their own right. Hall[91] has also developed a code based on pseudospectral methods. In this case though, finite-element methods are used for the volume-based compliant-wall models and the coupled problem solved. No comparisons with linear theory appear to have been made, so that it is difficult to be sure what exactly has been achieved. The triple-deck asymptotic technique has been extended to linear stability of boundary layers over compliant walls by Mackerrell,[92] who used the model illustrated in Fig. 2b, and Rothmayer and Hiemcke,[93] who studied the homogeneous layer (Fig. 2e). Reasonable agreement with numerical solutions to the Orr-Sommerfeld equation was shown in both cases. These recent studies represent a necessary and important first step toward nonlinear applications of the triple-deck method.

X. The Way Ahead

There has been considerable recent work, making a variety of substantial contributions toward the eventual practical application of compliant walls for transition delay. Something like a tenfold increase in transitional Reynolds number compared to the rigid wall is in prospect. Certainly, taken as a whole, this work suggests that it would be a very promising method of transition control in water. Nevertheless, much remains to be done before compliant walls can be used technologically for transition control. Following is a partial list of the tasks remaining.

1) Building on the work of Gaster et al., further experimental studies should be undertaken to explore the various instability modes and breakdown processes and to achieve greater transition delays. It is important that the wall displacement be measured and the material properties accurately determined.

2) Fairly simple formulas for the critical speeds and wave numbers of the hydroelastic instabilities over volume-based compliant-wall models are required, so that the properties of such walls can be optimized.

3) Anisotropic compliant walls appear to be particularly promising, and further work on both the theory and the practical realization of the concept is required.

4) Three-dimensional instabilities appear to be prevalent even in the linear stage of transition. Many aspects of this topic remain to be explored.

5) The stabilizing properties of compliant walls are highly Reynolds-number–dependent. For this and other reasons, it is expected that a multiple-panel surface would have a substantially better performance than a single-panel one. Virtually no work has been done on the former.

6) The effects of external pressure gradient and axisymmetry remain largely unexplored but would be important for practical applications.

7) As noted in Sec. IX, there are a large variety of nonlinear instability topics requiring study. Of particular practical importance is the question of the interactions between the various instability modes.

8) Finally, it is not without possibility that compliant walls may favorably modify the processes responsible for generating the initial disturbances. To date, though, no work has been reported concerning the influence of wall compliance on receptivity.

Acknowledgments

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