

# A Study of the Silent Flight of the Owl

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Many species of owl, including the barn and barred owl, use both visual and bi-aural location to search for prey around dusk and at night. Their bi-aural location system has a maximum sensitivity between 3-6kHz although the hearing of the owl has an upper limit in excess of 20kHz. Its prey, typically voles and mice, squeak and squeal in the frequency range of 3-6kHz and this range of frequency includes the rustling of leaves made by prey. The hearing of these prey is acute between 2-20kHz. The owl in both gliding and flapping flight generates noise at low frequencies below 2kHz, but is almost totally silent at frequencies above 2kHz. Hence the flight of the owl is almost silent to its prey. When an owl attacks its prey from its perch from a height of 6-10m the prey are unaware of its approach before they are captured by its long talons.

The noise suppression devices developed by owls during their long evolution period of millions of years have been identified. They comprise (i) leading edge feathers in the form of a comb, (ii) trailing edge feathers in the form of a fringe, and (iii) fluffy down on the wings and legs.

The paper discusses the aerodynamic characteristics of each of these devices and suggests tentative explanations as to how these flow characteristics lead to a large reduction in the noise generated by the owl in flight and especially in its critical range of frequencies above 2kHz.

## 1. INTRODUCTION

Many species of owl have perfected their use of their habitat and their means of survival through the capture of their prey, in the form of small animals such as voles and mice. The owl as we observe it today has been in existence for over 12 million years. It is suggested it developed its strategy of a silent predator when the small animals were in glut as a result of an abundance of bushes carrying red berries some 10 million years ago. The strategy used by many species of owls involves capturing their prey by day and night and especially at dusk from a perch at a height of 3-6m by gliding steeply towards the prey in daylight or in flapping flight after dark. Some owls today use different techniques for capturing prey. Here we will discuss the 'noiseless' flight of owls such as the barn owl (*Tyto alba*) and the barred owl (*Strix varia*), both of which use the strategies discussed above.

In the aural response of the prey the owl's flight is completely silent to within 1m of the ground. At this height, even if the approach of the owl is heard, the prey have no time to escape before being struck by the owl's talons. The owl has a highly sensitive bi-aural hearing and location system enabling it to both hear the movements of the prey and its squeaks and squeals together with the rustling of leaves as well as its location. The frequency of these squeaks and squeals and rustling of leaves is in the range 2-10kHz. The maximum hearing response of the bi-aural and location

system of the owl lies between 3-6kHz, which is within the range of frequencies made by its prey. The upper limit of hearing of the owl is about 20kHz with a lower hearing response of about 100Hz. The prey have very acute hearing above 2kHz but are not disturbed by sounds of lower frequency. Thus the prey are unaware of the owl's approach either in flapping or gliding flight. How is this silent flight of the owl achieved?

It was Lt. Comdr. Graham R.N.<sup>1</sup> who observed the owl's wing feathers differed from almost all other birds by :

- (1) The addition of feathers in the form of a leading edge comb.
- (2) A fringe formed by the feathers at the trailing edge.
- (3) A velvety covering on the wing upper surface and a downy lower surface as well as thick down on its legs.

Graham concluded that these attributes resulted in the the silent flight of the owl, since without them, as in almost all other birds, they would be noisy in gliding and flapping flight from their perch at 3-6m from the ground. Without these attributes the owl would be heard by its prey as soon as it left its perch for in this short flight time of about 1s the prey could take evasive action and escape capture.

However Graham could not explain how these three devices could generate silent flight especially in the frequency range above 2kHz and even render the owl's

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flight silent to most humans. The main research into the silent flight of the owl comes from the work of Kroeger et al<sup>2</sup> in the early 1970's. Kroeger built a special chamber in which the flight of the barred owl (*Strix varia*) was observed and its noise at different heights from the ground was measured. The owls were trained to fly from a perch 3m from the ground towards forwards food placed on the ground. The flight of the owl was observed, its speed measured as well as its noise at different heights from the ground.

This research confirmed Graham's conclusions for when the trailing edge fringe and the leading edge comb were removed the modified owl emitted sounds as strong as other birds of similar mass and speed and its flying characteristics were seriously impaired. In fact it could no longer fly along a straight-line trajectory between its perch and food. There can be no doubt that without these devices incorporated into the structure of their feathers the owl would have had to develop a different strategy to capture prey and survive.

The interest shown by the aeronautical community today in the silent flight of the owl stems from the fact that of all the flying vehicles known to man only the owl is capable of almost silent flight. Of course its speed is low, its mass is small and yet, as we will show below, airframe noise (aircraft noise less engine noise) can be determined for a given height and speed with a small scatter for most aircraft, gliders and birds, (excluding the owl with its noise suppression devices intact), from a single formula covering a mass range from 1kg to 500,000kg. This implies that in spite of the different materials used in the construction of flying vehicles the global mechanism for noise generation is almost the same throughout this enormous mass range and size. The evidence from the work of Kroeger et al is that the owl is considerably quieter and thus its 'airframe' noise follows a different law. When this new law is fully understood the challenge is then to find how to exploit it in the reduction of noise in the vastly different range of mass, size and speed encountered in aircraft flying today, as compared to the owl.

This picture of airframe noise, compared to that of the owl, of course does not highlight the special problems of the increase in noise due to certain undercarriages, flaps and slats for high-lift as well as part span flaps. The differences in noise made by these components on various aircraft types is within the scatter band referred to above covering this vast range of size and mass. The suggestion is that at best the aircraft designer can lower airframe noise through careful and dedicated research to an extrapolation of the noise made by birds other than the owl. If the technology of the noise suppression devices on the owl are better known then, if it can be applied to aircraft, a further noise reduction is possible.

In our discussion below on the flight of the silent owl we will assume the owl in its steep approach path and legs extended can be classed as typical of a vehicle in a landing configuration.

## 2. FLOW FIELD

In the experiments of Kroeger et al<sup>2</sup> the average flight Reynolds number of the owl was  $1.5 \times 10^5$  with a flight speed of about 8m/s. The average mass of the owl was 0.6kg and its flight  $C_L = 1$ . The owl's wing is cambered but at this low Reynolds number laminar separation is inevitable near to the position of maximum camber close to the leading edge. The poor flying performance of the owl with the leading edge comb removed is proof that under these conditions the wing upper surface flow is almost stalled. The presence of the leading edge comb stabilizes the wing upper surface flow and removes this laminar separation. The upper surface flow is evidently 'pseudo-turbulent'. This is surprising for a wing at low Reynolds number having a large adverse pressure gradient on its upper surface. That the flow is turbulent on the upper surface is proven by observation of the broadband noise spectrum measured for the owl by Kroeger et al in the frequency range below 2kHz. For the modified owl with its leading edge comb removed the low frequency portion of the spectrum is increased and there is no cut-off above 2kHz. The broadband spectrum continues well beyond 10kHz typical of all other birds, gliders and aircraft. The noise from flapping is in the lower frequencies and from the measurements of Kroeger et al there appears little difference between the noise emitted during either gliding or flapping flight.

It appears that the leading edge comb behaves as a set of closely spaced co-rotating vortex generators. The spacing is very regular and from an estimate of the value of the local friction velocity,  $u_\tau$ , the value of  $z^* = u_\tau \Delta z / \nu = 18$ . In experiments on drag reduction a value of  $z^* = 18$  is near the optimum spacing for riblets. These vortex generators at the owl's leading edge, on the other hand, are not used for drag reduction at low incidence but are used at this low Reynolds number to control separation. We can only assume that  $z^* = 18$  is an optimum spacing for streamwise vortices to prevent separation over the upper surface of the wing and provide an attached flow up to the wing trailing edge. Experiments by Kroeger et al confirmed the presence of streamwise vortices over the whole upper surface of the owl's wing.

Thus Graham's device (1), discussed above, is designed to allow the owl to fly at this high value of  $C_L$  in stable flight with fully attached 'pseudo-turbulent' flow at an extremely low Reynolds number. The effect on noise emission will be discussed more fully below. However it can be seen from our discussion on the upper surface flow in this section that the presence of the streamwise vortices have drastically reduced the

boundary layer thickness everywhere over the wing chord and in particular at the wing trailing edge. Since the noise emitted will be shown below to be proportional to the volume of turbulence crossing the trailing edge the noise emitted by the owl will be lower than a bird of comparable mass and speed. (Of course we cannot find such a bird other than the modified owl with its comb removed. The measurements of Kroeger in this case show a substantial increase in the noise at both low and high frequencies. )

### 3. Noise from an aerofoil and wing

A simplistic view of the noise emitted by the owl in flight is to assume it involves a similar noise generation problem to that of an aircraft on the final approach. Thus we assume that the farfield noise intensity can be derived from the Brooks and Hodgson<sup>3</sup> formula. They showed this agreed with their experimental results of the noise from a wing of large aspect ratio at small angles of attack. This formula is based on the theoretical work of Ffowcs Williams and Hall<sup>4</sup>, Howe<sup>5</sup>, Goldstein<sup>6</sup>, and Kambe<sup>7</sup> for a flat plate at zero incidence.

We find for the farfield noise intensity,

$$I \simeq \frac{\rho_\infty V_\infty^5}{2\pi^3 c_\infty^2} \frac{u_0^5}{V_\infty^5} \frac{S}{h^2} \frac{\ell_0}{\bar{c}} \quad (1)$$

where  $S = b \times \bar{c}$  is the wing area,  $b$  is the wing span,  $\bar{c}$  is the mean chord,  $h$  is the height of the wing from the ground and corresponds to the height of the aircraft in the flyover plane,  $u_0$  is a measure of the fluctuations in velocity near the wing trailing edge, and  $\ell_0$  is a corresponding correlation length scale of the acoustic sources near the trailing edge, which Brooks and Hodgson<sup>3</sup> define as  $\delta_0$ , the boundary layer displacement thickness at the wing trailing edge. The correlation volume is  $\ell_0^3$  and the number of correlation volumes along the span at the trailing edge is  $b/\ell_0$ . Strictly this formula can only apply to a 2D-wing section at small angles of attack. *Equ.(1)* does not contain the important angular dependencies and the important Döppler factors derived by Howe<sup>5</sup> associated with convection effects. *Equ.(1)* is written for the flyover plane. The formula derived by Howe<sup>5</sup> is, (in our notation),

$$I = \frac{\langle p^2 \rangle}{\rho_\infty c_\infty} \simeq \rho_\infty u_0^2 M_V^2 V \frac{b \ell_0}{h^2} \sin \alpha \sin^2(\theta/2) \cos^3 \beta \quad (2)$$

where  $V_\infty$  is the freestream velocity,  $V(x_2)$  is the convection velocity of the turbulence crossing the trailing edge at an angle  $\beta$  to the freestream direction, and  $M_V = V/c_\infty$ .  $\alpha$  is the wing incidence and  $\theta$  is the angle in the flight plane with reference to the downstream direction. When overhead  $\theta = 90$  deg. The

neglect of the angular dependencies in *equ.(1)* must be noted. These are given correctly in *equ.(2)*. The most important of these is  $\cos^3 \beta$ , where  $\beta$  is the sweepback angle of the trailing edge, either the total or the average value, if serrations or some other similar geometric alteration is applied to the trailing edge. An angle of  $\beta = 60$  deg would reduce the noise given by *equ.(1)* by 9dB. Recent work on serrated edges has been performed in wind turbine noise reduction research and a summary of this work is given by Wagner, Bareiss, and Guidati<sup>8</sup>.

Measurements of Brooks and Hodgson<sup>3</sup> and Brooks and Marcolini<sup>9</sup> on wings of large aspect ratio showed that with increase in incidence, the extent of the influence of the tip vortices is confined to an area close to the wing tips. In that region the boundary layer thickness is greatly reduced but recovers its near-2D values a short distance inboard. The outer limit of the boundary defined approximately by the contour of  $u'/V_\infty = 0.01$  is about  $0.05\bar{c}$  above the wing near the wing tip at an incidence of 0 deg. At 14 deg incidence the boundary layer thickness has reduced to  $0.02\bar{c}$  but the height of the vortex is  $0.13\bar{c}$  and of comparable width spanwise. The latter dimension amounted to about 17% of the wing span. The axial speed in the region of the tip vortex where the greatest turbulent intensity existed was increasing linearly with incidence from  $V_\infty$  at zero incidence to  $1.37V_\infty$  at 14 deg incidence. The overall effect of incidence is a small, but noticeable, increase in the noise intensity provided the flow is attached to the wing upper surface. We can expect that *equ.(1)* can apply to the case of the owl in flight but a correction would be needed for an aircraft flying at a much higher aircraft  $C_L$  with part span flaps extended, leading edge devices operable, and undercarriage down.

For the owl on the approach the mean lift coefficient,  $\bar{C}_L$ , is given by

$$\bar{C}_L = 2 \frac{mg}{\rho_\infty V_\infty^2 S} \quad (3)$$

where  $m$  is the mass of the owl. The strength of the wing tip vortices is approximately  $\Gamma_0$  where

$$\Gamma_0 = \frac{V_\infty S}{2b} \bar{C}_L. \quad (4)$$

Thus associated with the port and starboard wings of the owl there is a 'cloud' of time dependent vorticity with its mean value being approximately  $\Gamma_0$  on each wing panel. This time dependent vorticity is shed from the wing forming the wake and is convected downstream. Since the strength of the mean circulation around the tip vortices is proportional to the mean lift coefficient we may assume the root mean square value

of the fluctuating trailing vorticity is also proportional to the lift coefficient on the wing.

We now assume that the unsteady vortical flow over the wing is controlled by the magnitude of the total circulation generated by the aircraft. (This can only be partially true since there is already vorticity in the boundary layers even when the lift is zero. However this vorticity generates no net contribution to the circulation). Since the unsteady flow has a circulation proportional to  $u_v \ell_v$  our hypothesis is that  $u_v \ell_v$  is proportional to  $\Gamma_0$ .  $u_v$  is a measure of the fluctuations of velocity in the vortex and  $\ell_v$  is a general characteristic length scale of the turbulence in the unsteady vortical flow region. We may assume  $u_v$  is greater than  $u_0$ , the corresponding turbulent velocity in the trailing edge boundary layer, since it arises from the growth of the unsteady mixing regions formed from the separating boundary layers at the wing tips. Similarly we may assume  $\ell_v$  is greater than  $\ell_0$ , which is proportional to  $\delta_0$  close to the trailing edge, where  $\delta_0$  has been used above in *equ.*(1). We cannot assume a fluctuating lift coefficient, say,  $< c_L^2 >^{1/2}$ , exists on the wing proportional to the mean  $\bar{C}_L$ . It is known that on a large flat plate without edges immersed in uniform flow the total fluctuating force on the plate is zero. Similarly on an aircraft, except in the presence of gusts, the fluctuating lift is small and usually regarded as negligible. Nevertheless the trailing vortices are turbulent and we make the assumption that the root mean square value of the fluctuations in the trailing vortex circulation is proportional to the mean circulation,  $\Gamma_0$ , as stated above. Let

$$u_v \ell_v = K \Gamma_0 \quad (5)$$

where  $\Gamma_0$  is given by *equ.*(4). *Equ.*(5) is clearly a very approximate result for an aerofoil or wing, and we now apply it to the complete aircraft or bird. Thus we find on replacing  $u_0$  and  $\ell_0$  by  $u_v$  and  $\ell_v$  respectively in *equ.*(1)

$$I = \frac{mgV_\infty^3}{\pi^3 c_\infty^2 \bar{C}_L} \left( \frac{u_v}{V_\infty} \right)^5 \frac{\ell_v}{\bar{c}} \quad (6)$$

and using *equ.*(5) we find

$$I = C \frac{mgV_\infty^3}{c_\infty^2 h^2} \quad (7)$$

where

$$C = \frac{K}{2\pi^3} \left( \frac{u_v}{V_\infty} \right)^4 \quad (8)$$

which reduces to  $C = 1.6 \times 10^{-7}$  when we write approximately,  $K = 0.1$ ,  $u_v/V_\infty = 0.1$  and  $\bar{C}_L \ell_v / \bar{c} = 0.5$ . For a height  $h = 45.72m$  we find  $C/c_\infty^2 h^2 = 6.7 \times 10^{-16}$

and with  $I_{ref} = 10^{-12} \text{ watts/m}^2$ , corresponding to  $p_{ref} = 2 \times 10^{-5} Pa$ ,

$$NdB = 10 \log_{10} \frac{I}{I_{ref}} = 10 \log_{10} 0.0066 m V_\infty^3 \quad (9)$$

compared with the simple correlation found for all 'clean' aircraft, gliders, and birds with a range of masses from  $1kg$  to  $400,000kg$  we find

$$NdB = 10 \log_{10} 0.007 m V_\infty^3. \quad (10)$$

The matching of full-scale flight data with a simple formula is clearly fortuitous especially as it is based on data for the wing alone.

The simple formula given by *equ.*(7) is a very crude extrapolation formula but it does convey certain important conclusions. It suggests that for birds, gliders and aircraft in spite of their wide variety of geometries their farfield sound intensity follows the velocity scaling law  $V_\infty^5$ . But this velocity scaling law arises from sound generated by dipoles and quadrupoles close to a sharp edge, such as the trailing edge of the wing, which is then scattered by the edge, say within an acoustic wave length, whereas such sources would radiate as  $V_\infty^6$  or  $V_\infty^8$  respectively in the absence of a sharp trailing edge. Now there are many sources of noise on an aircraft and bird which appear remote from the wing trailing edge, yet measurements suggest that they each in turn make a contribution to the noise proportional to  $V_\infty^5$ , whereas in isolation such sources would radiate proportional to  $V_\infty^6$  or  $V_\infty^8$ . The only explanation is since at approach the aircraft Mach number is small, the dominant noise sources have a sound wave length greater than the distance of the component to the wing trailing edge. In this case the dominant noise sources are compact. Hence the farfield noise is dominated for both bird and aircraft by sound scattered by the wing trailing edge. Our *equ.*(7) above shows the sound intensity proportional to aircraft mass and  $V_\infty^5$  and inversely proportional to  $h^2$ . This result however does not apply to the owl. It does apply to all other birds, including the owl when its L.E. comb and T.E. fringe are removed.

#### 4. Noise from the trailing edge of the the owl's wing

We now examine the noise properties of the owl's fringe at its trailing edge. This is item(2) on Graham's list of noise reduction devices. *Equ.*(2) above shows that if the sweep angle of the wing trailing edge  $\beta$  is 60 deg the scattered noise can be reduced by about 9dB. Howe<sup>10</sup> has studied the noise generated by a saw-toothed trailing edge and Wagner et al<sup>8</sup> have discussed its application to noise reduction on wind turbines.

The fringe of the owl's trailing edge resembles a serrated or sawtoothed edge and suggests the scattering can be reduced or even eliminated by such a device. If that were the case the velocity scaling law for the owl should be  $V_\infty^6$  in place of  $V_\infty^5$  which, at the owl's speed of  $6m/s$ , would result in a noise reduction of  $18dB$ . Similarly for an aircraft flying at  $M = 0.3$  the reduction would be  $5dB$ .

However even though with Graham's devices (1) and (2) we see the owl can fly quieter than all other birds, it is not 'silent' and would be heard by its prey. The reason is the spectrum of noise from all flying vehicles including the owl is broadband and extends up to at least  $10kHz$ . All sound will be captured by prey above  $2kHz$  due to their acute hearing above that frequency. Kroeger et al's measurements on the modified owl deprived of its L.E.comb and T.E.fringe showed a typical broadband spectrum. Thus unless some modification to the acoustic spectrum could be achieved by devices (1) and (2), and this appears unlikely, but not impossible, the owl flies quietly but not silently.

## 5. Noise from the velvety down on the owl's wing

The final item (3) on Graham's list of noise suppression devices is possibly the the most difficult to explain. We have seen from from section(4) that devices (1) and (2) both reduce noise, the former by reducing the volume of turbulence crossing the trailing edge and the latter by reducing or eliminating trailing edge scattering. Thus although the noise emitted by the owl is greatly reduced it appears to continue to generate noise of high frequency and in particular above  $2kHz$ . If the velvety surface on the owl's wing were to act as a sound absorber it is unlikely to eliminate all sound generated at high frequencies, since most of the sound emission would have little contact with the wing's surface and would radiate away. Thus the speculation is that the owl has a mechanism to eliminate high frequency sound at source. In other words the owl does not generate sound at high frequencies or at least that sound it does generate is of sufficiently low amplitude it fails to be heard by prey. Kroeger et al's measurements show that when the owl is at a distance of less than  $1m$  from the ground it is just audible at frequencies above  $2kHz$  but the prey have no time to escape the owl's talons.

In the case of the flight of a rigid wing or body the intensity of turbulence in the boundary layer is related to the friction due to its motion through the air. The energy to create the turbulence is derived from the motion of the air relative to the aircraft. This turbulence has however a finite life span, when it is replaced by new turbulence and the cycle is repeated. The turbulence that dies has a much smaller length scale than the large scale energetic structures from which it is de-

rived by an evolutionary (cascade) process. This small scale turbulence finally loses its energy by viscous processes and its kinetic energy is transformed into heat. This is measurable even though the resultant temperature rise of the surrounding air is minute and normally ignored. The entire process of energy conversion into heat is known as dissipation. The presence of small scale turbulence within the boundary layers of the air flowing over the wings, tail, body and legs of the bird is a source of noise and since the eddies in the turbulence are small the noise generated by these eddies is of high frequency. It represents the the upper limit of the energy spectrum of the dissipation and of the radiated noise. We conclude that for both aircraft and birds, with the exception of the owl, the noise spectrum extends into the high frequencies and is continuous up to a cut off frequency associated with dissipation. Thus birds, other than the owl, can be heard by prey on the ground scurrying around for nuts and insects. Unless they adopt a strategy to attack them, such as the high speed dive of the hawk, their prey will have time to react to their presence and so scurry away before being caught. The downy, velvety, fluffy surface of the owl has a special property in respect of the airflow past it, for it is a compliant surface and as such damps the turbulence of the boundary layer air. Since the fibers of the down are so small in diameter the down absorbs the energy in the small eddies. The down therefore offers a bypass mechanism for energy dissipation at frequencies somewhat smaller than the conventional dissipation range of frequencies associated with viscous dissipation. The result is the spectrum of the radiated noise down to ground level receives a near cut off in frequencies above  $2kHz$ . Thus the owl's prey are unaware of the owl's approach, since to the prey the owl is silent above  $2kHz$  until it is within  $1m$ , and then it is too late to escape.

## 6. CONCLUSIONS

The current research on the silent flight of the owl is part of our aim towards a greater understanding of the the increased noise at ground level made by aircraft in flight, other than engine noise, especially in the landing phase, when the aircraft are flying at low speeds and high incidence. It would be remarkable indeed if the noise reduction and noise suppression devices, developed by the owl during its long period of evolution, had some application towards a reduction in the landing phase of current civil transports. Nevertheless if we ignore the technical and scientific mechanisms associated with the owl's noise suppression system we may miss one of the golden opportunities to generate the quiet transport aircraft of the future.

The owl has three major devices associated with its feathers which are different from the feathers of all other birds. We have discussed all three devices in this

paper and have offered tentative but plausible reasons for the necessary changes in the flow field past the owl's wing, body and legs which combined result in a major noise reduction and notably a suppression of all noise above 2kHz.

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