

$$vii. P = x \rightarrow y, Q = \neg(y \rightarrow x)$$

x	y	P	Q	P → Q
0	0	1	0	0
0	1	1	1	1
1	0	0	0	1
1	1	1	0	0

conclusion: neither

$$viii. P = (x \rightarrow y) \wedge (y \rightarrow z), Q = x \rightarrow z$$

x	y	z	P	Q	P → Q
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	0	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	0	1	0	1	1
1	1	0	0	0	1
1	1	1	1	1	1

conclusion: Tautology

0 0   0	0 0   0	0 0   0	0 0   0	0 0   0
0 1   0	0 1   0	0 1   0	0 1   0	0 1   1
1 0   0	1 0   0	1 0   1	1 0   1	1 0   0
1 1   0	1 1   1	1 1   0	1 1   1	1 1   0

∴ there are  $2^4$  possible logical connectives.

Logical connectives operate on 2 binary inputs.

each binary digit can have two values. ∴ 2 inputs can have 4 possible combinations.

⇒ Each logical connective can have 4 possible binary outputs, one corresponding to each possible input combination.

4 possible binary outputs ⇒ 16 possible combinations of outputs.

∴ There are 16 possible binary logical connectives.

x	Q	P	x ⊕ Q	P ⊕ Q
0	0	0	0	0
0	1	0	1	1
1	0	1	1	1
1	1	1	0	0

P	Q	P ⊕ Q
0	0	0
0	1	1
1	0	1
1	1	0

$$\Rightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$$

$$\neg(\neg(P \wedge \neg Q) \wedge \neg(\neg P \wedge Q))$$

$$\neg(\neg P \vee Q) \vee \neg(P \vee \neg Q)$$

$$\neg(P \rightarrow Q) \vee \neg(Q \rightarrow P)$$

$$(P \rightarrow Q) \rightarrow \neg(Q \rightarrow P)$$

$$c. \frac{NAND}{\neg} = \neg(x \wedge x) = A \text{ NAND } A$$

$$\neg = \neg(\neg(x \wedge x)) = (x \text{ NAND } x) \text{ NAND } (x \text{ NAND } x)$$

$$\vee = \neg(\neg x \wedge \neg y) = (x \wedge x) \wedge (y \wedge y)$$

$$\rightarrow = \neg x \vee y = \neg(x \wedge \neg y)$$

$$= x \wedge (y \wedge y)$$

$$d. \bar{\vee} = \neg(x \vee y)$$

$$\neg = x \bar{\vee} x$$

$$\vee = \neg(\neg(x \vee y)) = (x \bar{\vee} y) \bar{\vee} (x \bar{\vee} y)$$

$$\wedge = \neg(\neg x \vee \neg y) = (x \bar{\vee} x) \bar{\vee} (y \bar{\vee} y)$$

$$\rightarrow = \neg x \vee y$$

$$= ((x \bar{\vee} x) \bar{\vee} x) \bar{\vee} ((x \bar{\vee} x) \bar{\vee} y)$$

$$3. a. i. \forall x \in \mathbb{Z} \exists y \in \mathbb{Z} (x^2 < y + 1)$$