# Reliability and Risk Engineering

## Midterm 2


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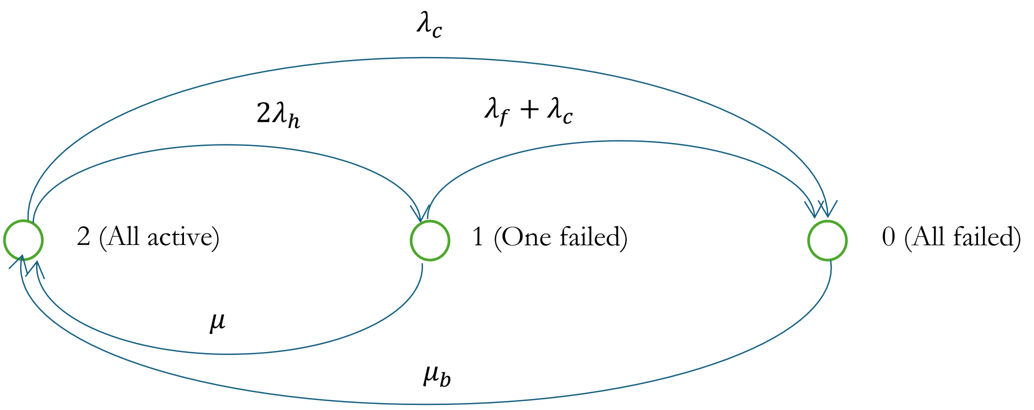
1. A) State Space Diagram.




The transition rate matrix was computed as:

$$\bar{\bar{A}} = \begin{bmatrix} -\mu_b & 0 & \mu_b \\ \lambda_f + \lambda_c & -(\mu + \lambda_f + \lambda_c) & \mu \\ \lambda_c & 2\lambda_h & -(\lambda_c + 2\lambda_h) \end{bmatrix}$$

The fundamental equation is given by $\frac{\partial P}{\partial t} = P * \bar{\bar{A}}$, where $P = [P_1 \quad P_2 \quad P_3]$

In matricial form:

$$[\dot{P}_0 \quad \dot{P}_1 \quad \dot{P}_2] = [P_0 \quad P_1 \quad P_2] * \begin{bmatrix} -\mu_b & 0 & \mu_b \\ \lambda_f + \lambda_c & -(\mu + \lambda_f + \lambda_c) & \mu \\ \lambda_c & 2\lambda_h & -(\lambda_c + 2\lambda_h) \end{bmatrix}$$

At steady state, the rates of change of probabilities are zero. The equation is re-written as:

$$[0 \quad 0 \quad 0] = [\pi_0 \quad \pi_1 \quad \pi_2] * \begin{bmatrix} -\mu_b & 0 & \mu_b \\ \lambda_f + \lambda_c & -(\mu + \lambda_f + \lambda_c) & \mu \\ \lambda_c & 2\lambda_h & -(\lambda_c + 2\lambda_h) \end{bmatrix}$$

To complete the rank of the system of equations, a fourth equation based on the conditions of exhaustiveness and mutual exclusivity is introduced.

$$\pi_0 + \pi_1 + \pi_2 = 0$$

The steady state probabilities are:

$$\pi_0 = 0.0231, \qquad \pi_1 = 0.0182, \qquad \pi_2 = \ 0.9587$$

Steady state (average) availability = $\pi_1 + \pi_2 = 0.0182 + 0.9587 = 0.9769$

This result implies that the system under study is quite available, meaning the system performs well, even with occasional failures, and there is a low likelihood of finding the system in a failed state. This is key for planning maintenance, keeping the power supply steady, and designing backup systems to handle any issues.

B) The indirect Monte-Carlo simulation is performed in the MATLAB file "TwoGeneratorsIndirect.m".

C) The calibration of the number of trials is done in the MATLAB file "Calibration.m". The optimum number of trials for the specified variance was a range between 5,000 and 6,000. It can also be seen that the expected value of the variance decreases for increasing values of simulation trials which confirms what we expect according to the variance equation.

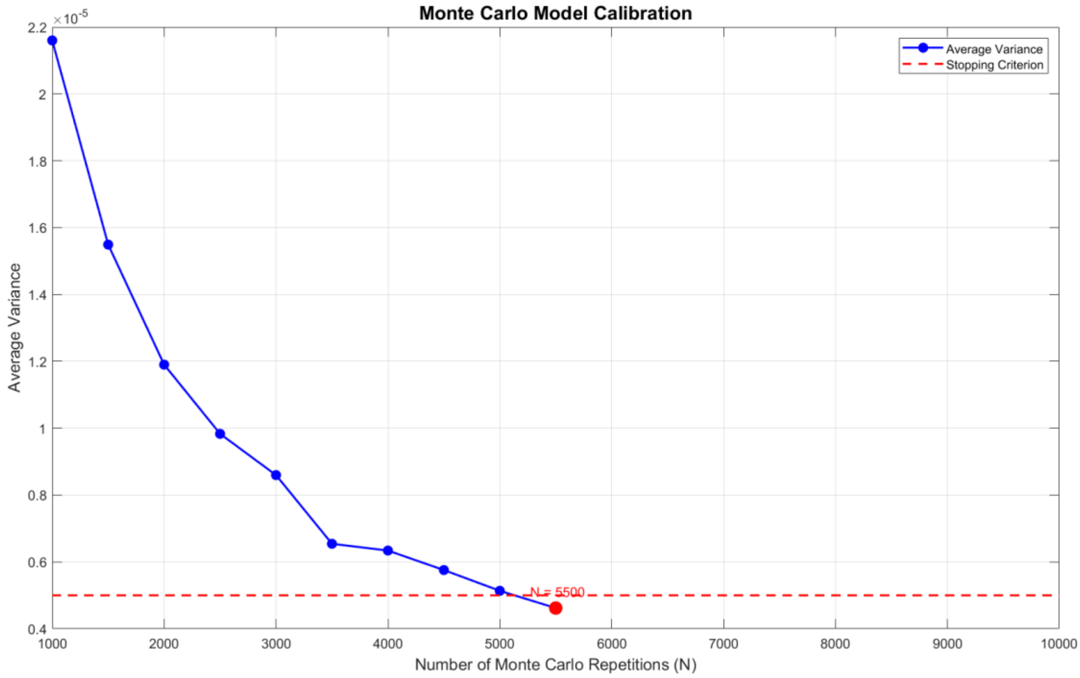



**Figure:** Calibration of number of trials for indirect Monte-Carlo

D) The dimensionality of the transition rate matrix is reduced and the new augmented matrices are:

$$\bar{\bar{A}} = \begin{bmatrix} -(\mu + \lambda_f + \lambda_c) & \mu \\ 2\lambda_h & -(\lambda_c + 2\lambda_h) \end{bmatrix} \qquad w = \ [1 \quad 1] \qquad C = [0 \quad 1]$$

$$R(s) = P_1(s) + P_2(s) = C(sI - A)^{-1}w^T$$

$$R(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} \left( \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -(\mu + \lambda_f + \lambda_c) & \mu \\ 2\lambda_h & -(\lambda_c + 2\lambda_h) \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

After completing the matrix multiplications and simplification, we obtain:

$$R(s) = \frac{2\lambda_h + s + \mu + \lambda_f + \lambda_c}{(s + \mu + \lambda_f + \lambda_c) * (s + \lambda_c + 2\lambda_h) - 2\lambda_h * \mu}$$

The Mean-Time-To-Failure (MTTF) is obtained by setting the Laplace variable (s) to zero:

$$R(0) = \frac{2\lambda_h + \mu + \lambda_f + \lambda_c}{(\mu + \lambda_f + \lambda_c) * (\lambda_c + 2\lambda_h) - 2\lambda_h * \mu} = 2543.256\ hours$$

Thus, the reliability of the system is expected to reach zero at approximately 2500 hours (104.17 days). This number changes upon every run and is displayed on the MATLAB Command Window.

E) A plot of the (average) availability estimate of the system via Monte-Carlo simulation was generated and compared to the solution obtained by solving the steady state probabilities.

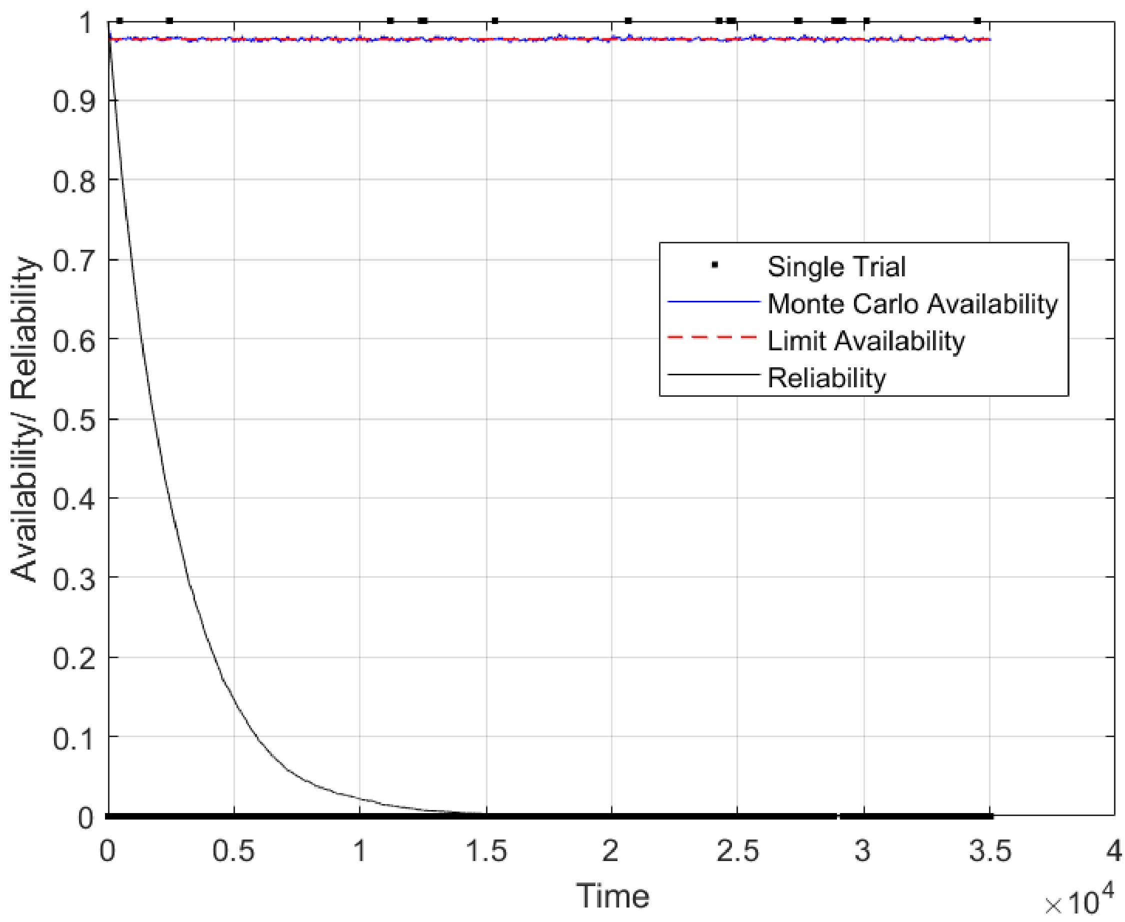



**Figure:** Comparison of analytical and simulation results for indirect Monte-Carlo

The Monte-Carlo estimate is consistent with the analytical solution obtained from the steady state probabilities, thus there is a high confidence in the obtained results. However, there are fluctuations in the availability obtained with Monte-Carlo. These fluctuations are less prominent as the number of Monte-Carlo trials increases, since the variance reduces to zero theoretically with infinite trials.

2. A) The direct Monte-Carlo simulation is performed in the MATLAB file "TwoGeneratorsDirect.m" and the calibration of the number of trials is done in the MATLAB file "Calibration.m". There is a slight difference between the recommended number of trials from the calibration with the indirect and direct functions.

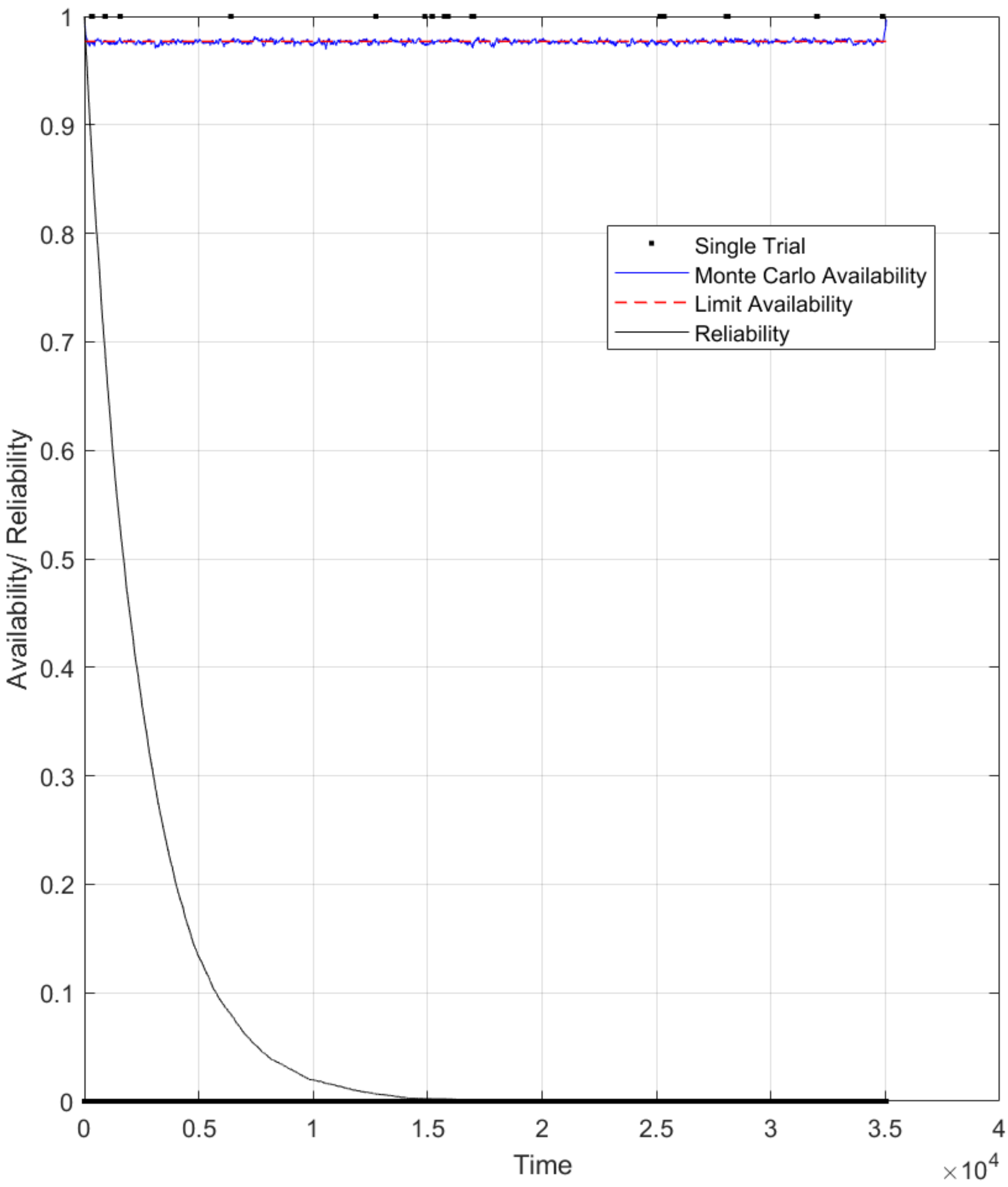



**Figure:** Comparison of analytical and simulation results for indirect Monte-Carlo

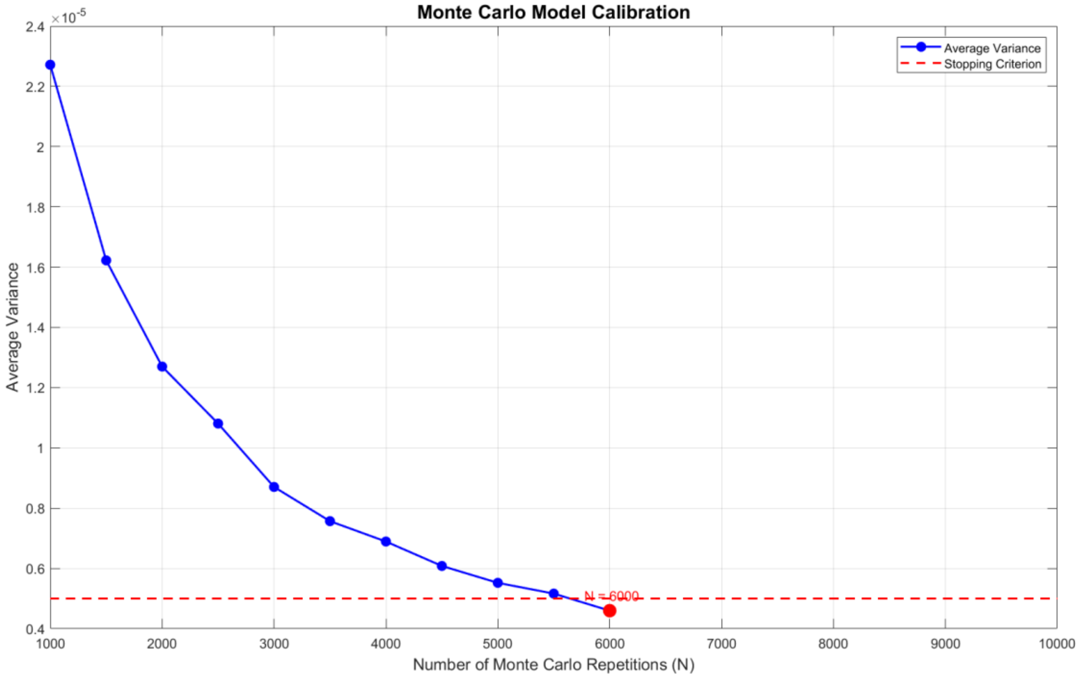



**Figure:** Calibration of number of trials for indirect Monte-Carlo

B)

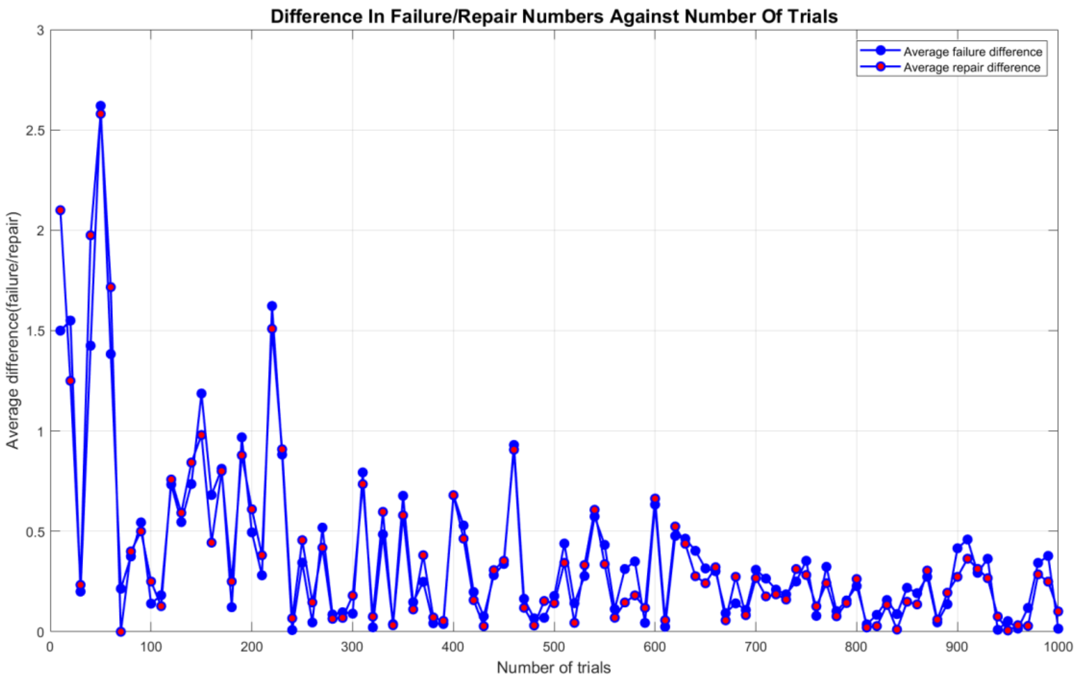



**Figure:** Relationship between number of trials and difference in average failure/repair numbers

In finite simulations, random sampling from the exponential distribution causes one generator to fail and be repaired more frequently, even with identical failure and repair rates. These variations occur due to independent sampling and diminish as the number of trials increases, aligning with the law of large numbers. Since repairs follow failures, the generator with the most frequent failures will also be repaired more often. Increasing the number of trials would reduce this variation and bring the results closer to convergence. Generator 1 failed more in the 10 trials for the first experiment conducted, but this result is not deterministic.