

First Order Conditions Approach with Entropic Latent Variable Integration (ELVIS)

The Data Generating Process

- We have two goods and two time periods, ckt for $t=1,2$ (time), and $k=a,b$ (goods). We consider a case of a Cobb-Douglas Utility function with income $w=1$ almost surely.

In[64]:= **caltrue** = α / **pa1**

Out[64]=
$$\frac{\alpha}{pa1}$$

In[65]:= **cb1true** = $(1 - \alpha)$ / **pb1**

Out[65]=
$$\frac{1 - \alpha}{pb1}$$

In[66]:= **ca2true** = α / **pa2**

Out[66]=
$$\frac{\alpha}{pa2}$$

In[67]:= **cb2true** = $(1 - \alpha)$ / **pb2**

Out[67]=
$$\frac{1 - \alpha}{pb2}$$

- This is measurement error in consumption.

In[68]:= **wa1** = **caltrue** - **ca1**

Out[68]=
$$-ca1 + \frac{\alpha}{pa1}$$

In[69]:= **wa2** = **ca2true** - **ca2**

Out[69]=
$$-ca2 + \frac{\alpha}{pa2}$$

```
In[70]:= wb1 = cb1true - cb1
```

```
Out[70]=
```

$$-cb1 + \frac{1 - \alpha}{pb1}$$

```
In[71]:= wb2 = cb2true - cb2
```

```
Out[71]=
```

$$-cb2 + \frac{1 - \alpha}{pb2}$$

Construction of the Maximum Entropy Distribution conditional on Moments according to ELVIS

- We first create the numerator of the user-provided prior over the problem, this is the etanumerator. The first step is to compute the Euclidean norm of the set of moments.

```
In[72]:= normmomentssquared = (wa1) ^ 2 + (wa2) ^ 2 + (wb1) ^ 2 + (wb2) ^ 2
```

```
Out[72]=
```

$$\left(-cb1 + \frac{1 - \alpha}{pb1}\right)^2 + \left(-cb2 + \frac{1 - \alpha}{pb2}\right)^2 + \left(-ca1 + \frac{\alpha}{pa1}\right)^2 + \left(-ca2 + \frac{\alpha}{pa2}\right)^2$$

- The etanumerator is the product of $\text{Exp}[-\|g\|^2]$ * pdf of α , since pdf of α is a uniform we leave it implicit here as 1 and we will impose the domain constraints later on.

```
In[77]:= etanumerator = Exp[-normmomentssquared]
```

```
Out[77]=
```

$$e^{-\left(-cb1 + \frac{1 - \alpha}{pb1}\right)^2 - \left(-cb2 + \frac{1 - \alpha}{pb2}\right)^2 - \left(-ca1 + \frac{\alpha}{pa1}\right)^2 - \left(-ca2 + \frac{\alpha}{pa2}\right)^2}$$

- This is the vector of moments g

```
In[78]:= g = {wa1, wa2, wb1, wb2}
```

```
Out[78]=
```

$$\left\{-ca1 + \frac{\alpha}{pa1}, -ca2 + \frac{\alpha}{pa2}, -cb1 + \frac{1 - \alpha}{pb1}, -cb2 + \frac{1 - \alpha}{pb2}\right\}$$

- This is the vector of γ the parameter of the ELVIS distribution

```
In[79]:= gamma = {γ1, γ2, γ3, γ4}
```

```
Out[79]=
```

$$\{\gamma1, \gamma2, \gamma3, \gamma4\}$$

- This is the Maximum Entropy Distribution According to ELVIS numerator, the product of $\text{EXP}[\gamma'g]$ etanumerator

```
In[80]:= etastarnumerator = FullSimplify[Exp[gamma.g] * etanumerator]
```

```
Out[80]=
```

$$e^{-\frac{(-1 + cb1 pb1 + \alpha)^2}{pb1^2} - \frac{(-1 + cb2 pb2 + \alpha)^2}{pb2^2} - \left(ca1 - \frac{\alpha}{pa1}\right)^2 - \left(ca2 - \frac{\alpha}{pa2}\right)^2 + \left(-ca1 + \frac{\alpha}{pa1}\right) \gamma1 + \left(-ca2 + \frac{\alpha}{pa2}\right) \gamma2 - cb1 \gamma3 + \frac{\gamma3 - \alpha \gamma3}{pb1} - cb2 \gamma4 + \frac{\gamma4 - \alpha \gamma4}{pb2}}$$

- In order to obtain a pdf of the Maximum Entropy Distribution According to ELVIS, we need to integrate `etastarnumerator` in the domain of α with respect to the uniform distribution. This is the partition function. Typically this cannot be obtained closed form, this is an exception rather than the rule.

```
In[81]:= partitioneta = FullSimplify[Integrate[etastarnumerator, {α, 0, 1}]]
Out[81]=
```

$$\left(e^{-\frac{1}{4 pa^2 pb^2 pb2^2 + 4 pa^2 (pb1^2 pb2^2 + pa2^2 (pb1^2 + pb2^2))}} (2 pa1 pa2 (2 ca1 + \gamma1) (-pb1^2 pb2^2 (2 ca2 + \gamma2) + pa2 (-2 pb2^2 + pb1 pb2^2 (2 cb1 + \gamma3) + pb1^2 (-2 + 2 cb2 pb2 + pb2 \gamma4))) - \right. \\ \left. pa1 pa2 pb1 pb2 \sqrt{\pi} \left(-\text{Erf} \left[(-2 pa2^2 pb1 pb2 + pa1 pa2^2 pb1 pb2 (2 ca1 + \gamma1) - pa1^2 (2 pb1 pb2 - \right. \right. \right. \\ \left. \left. \left. pa2 pb1 pb2 (2 ca2 + \gamma2) + pa2^2 (2 cb2 pb1 + 2 cb1 pb2 + pb2 \gamma3 + pb1 \gamma4) \right) \right] \right) / \\ \left(2 pa1 pa2 \sqrt{pa2^2 pb1^2 pb2^2 + pa1^2 (pa2^2 pb1^2 + (pa2^2 + pb1^2) pb2^2)} \right) \left. + \right. \\ \left. \text{Erf} \left[(pa2 pb1^2 pb2^2 (2 ca1 + \gamma1) + pa1 pb1^2 pb2^2 (2 ca2 + \gamma2) - \right. \right. \\ \left. \left. pa1 pa2 (-2 pb2^2 + pb1 pb2^2 (2 cb1 + \gamma3) + pb1^2 (-2 + 2 cb2 pb2 + pb2 \gamma4))) \right] \right) / \\ \left(2 pb1 pb2 \sqrt{pa2^2 pb1^2 pb2^2 + pa1^2 (pa2^2 pb1^2 + (pa2^2 + pb1^2) pb2^2)} \right) \left. \right) \left. \right) / \\ \left(2 \sqrt{pa2^2 pb1^2 pb2^2 + pa1^2 (pb1^2 pb2^2 + pa2^2 (pb1^2 + pb2^2))} \right)$$

- We capture our knowledge about the domain of symbolic variables in `Assumptionselvis`.

```
In[82]:= Assumptionselvis =
ca1 ≥ 0 && ca2 ≥ 0 && cb1 ≥ 0 && cb2 ≥ 0 && pa1 > 0 && pa2 > 0 && pb1 > 0 && pb2 > 0
Out[82]=
ca1 ≥ 0 && ca2 ≥ 0 && cb1 ≥ 0 && cb2 ≥ 0 && pa1 > 0 && pa2 > 0 && pb1 > 0 && pb2 > 0
```

- We write down the pdf.

```
In[83]:= etadistribution = FullSimplify[etastar numerator / partitioneta, Assumptionselvis]
```

```
Out[83]=
```

$$\begin{aligned}
& - \left(2 \right. \\
& \left. e^{-\frac{(2 pa^2 pb^2 pb^2 \alpha - pa^2 pb^2 pb^2 (2 ca_1 + \gamma_1) + pa^2 (2 pb^2 pb^2 \alpha - pa^2 pb^2 pb^2 (2 ca_2 + \gamma_2) + pa^2 (2 pb^2 (-1 + \alpha) + pb^2 pb^2 (2 cb_1 + \gamma_3) + pb^2 (-2 + 2 cb_2 pb^2 + 2 \alpha + pb^2 \gamma_4))}{4 pa^2 pa^2 pb^4 pb^4 + 4 pa^4 (pa^2 pb^4 pb^2 + pa^2 pb^2 pb^2 (pb^2 + pb^2))}} \right. \\
& \left. \sqrt{pa^2 pb^2 pb^2 + pa^2 (pb^2 pb^2 + pa^2 (pb^2 + pb^2))} \right) / (pa^2 pb^2 pb^2 \\
& pb^2 \sqrt{\pi} \left(\text{Erf} \left[(-2 pa^2 pb^2 pb^2 + pa^2 pb^2 pb^2 (2 ca_1 + \gamma_1) - pa^2 (2 pb^2 pb^2 - \right. \right. \\
& \left. \left. pa^2 pb^2 pb^2 (2 ca_2 + \gamma_2) + pa^2 (2 cb_2 pb^2 + 2 cb_1 pb^2 + pb^2 \gamma_3 + pb^2 \gamma_4)) \right) \right] / \\
& \left(2 pa^2 pb^2 \sqrt{pa^2 pb^2 pb^2 + pa^2 (pa^2 pb^2 + (pa^2 + pb^2) pb^2)} \right) \left. \right] - \\
& \text{Erf} \left[(pa^2 pb^2 pb^2 (2 ca_1 + \gamma_1) + pa^2 pb^2 pb^2 (2 ca_2 + \gamma_2) - \right. \\
& \left. pa^2 pb^2 (-2 pb^2 + pb^2 pb^2 (2 cb_1 + \gamma_3) + pb^2 (-2 + 2 cb_2 pb^2 + pb^2 \gamma_4)) \right) \right] / \\
& \left(2 pb^2 pb^2 \sqrt{pa^2 pb^2 pb^2 + pa^2 (pa^2 pb^2 + (pa^2 + pb^2) pb^2)} \right) \left. \right) \left. \right)
\end{aligned}$$

- The new pdf has to integrate up to 1 in the domain of the latent variable, α . This is because the partition function was used to normalize the numerator.

```
In[85]:= Integrate[etadistribution, {\alpha, 0, 1}]
```

```
Out[85]=
```

```
1
```

Computation of the Maximum Entropy Moments h:

- We write down the integrand or the argument of the expectation that we will use to obtain h, this is the first entry of the vector h with respect to good a and time 1.

In[86]:= **Integrandha1 = FullSimplify[wa1 * etadistribution, Assumptionselvis]**

Out[86]=

$$\left(2 \right. \\ \left. e^{-\frac{(2 pa2^2 pb1^2 pb2^2 \alpha - pa1 pa2^2 pb1^2 pb2^2 (2 ca1 + \gamma 1) + pa1^2 (2 pb1^2 pb2^2 \alpha - pa2 pb1^2 pb2^2 (2 ca2 + \gamma 2) + pa2^2 (2 pb2^2 (-1 + \alpha) + pb1 pb2^2 (2 cb1 + \gamma 3) + pb1^2 (-2 + 2 cb2 pb2 + 2 \alpha + pb2 \gamma 4)))^2}{4 pa1^2 pa2^4 pb1^4 pb2^4 + 4 pa1^4 (pa2^2 pb1^4 pb2^4 + pa2^4 pb1^2 pb2^2 (pb1^2 + pb2^2))}} \right. \\ \left. \sqrt{pa2^2 pb1^2 pb2^2 + pa1^2 (pb1^2 pb2^2 + pa2^2 (pb1^2 + pb2^2))} (ca1 pa1 - \alpha) \right) / \\ (pa1^2 pa2 pb1 pb2 \sqrt{\pi} \\ \left(\text{Erf} \left[(-2 pa2^2 pb1 pb2 + pa1 pa2^2 pb1 pb2 (2 ca1 + \gamma 1) - pa1^2 (2 pb1 pb2 - pa2 pb1 \right. \right. \\ \left. \left. pb2 (2 ca2 + \gamma 2) + pa2^2 (2 cb2 pb1 + 2 cb1 pb2 + pb2 \gamma 3 + pb1 \gamma 4) \right) \right] / \\ \left(2 pa1 pa2 \sqrt{pa2^2 pb1^2 pb2^2 + pa1^2 (pa2^2 pb1^2 + (pa2^2 + pb1^2) pb2^2)} \right) \right] - \\ \text{Erf} \left[(pa2 pb1^2 pb2^2 (2 ca1 + \gamma 1) + pa1 pb1^2 pb2^2 (2 ca2 + \gamma 2) - \right. \\ \left. pa1 pa2 (-2 pb2^2 + pb1 pb2^2 (2 cb1 + \gamma 3) + pb1^2 (-2 + 2 cb2 pb2 + pb2 \gamma 4) \right) \right] / \\ \left. \left(2 pb1 pb2 \sqrt{pa2^2 pb1^2 pb2^2 + pa1^2 (pa2^2 pb1^2 + (pa2^2 + pb1^2) pb2^2)} \right) \right] \right) \right)$$

In[87]:= **ha1 = Integrate[Integrandha1, {\alpha, 0, 1}, Assumptions -> Assumptionselvis]**

Out[87]=

$$\left(2 \right. \\ \left. e^{-\left((-2 pa1 pa2 pb1^2 + 2 cb2 pa1 pa2 pb1^2 pb2 - 2 pa1 pa2 pb2^2 + 2 cb1 pa1 pa2 pb1 pb2^2 - 2 ca2 pa1 pb1^2 pb2^2 - 2 ca1 pa2 pb1^2 pb2^2 - pa2 pb1^2 pb2^2) \right. \right. \\ \left. \left. \sqrt{pa2^2 pb1^2 pb2^2 + pa1^2 (pb1^2 pb2^2 + pa2^2 (pb1^2 + pb2^2))} \right) \right. \\ \left(-\frac{1}{\sqrt{\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2}}} e^{-\frac{\left(\frac{2 ca1}{pa1} + \frac{2 ca2}{pa2} + \frac{2}{pb1^2} - \frac{2 cb1}{pb1} + \frac{2}{pb2^2} - \frac{2 cb2}{pb2} + \frac{\gamma 1}{pa1} + \frac{\gamma 2}{pa2} + \frac{\gamma 3}{pb1} + \frac{\gamma 4}{pb2} \right)^2}{4 \left(-\frac{1}{pa1^2} - \frac{1}{pa2^2} - \frac{1}{pb1^2} - \frac{1}{pb2^2} \right)}} \right. \\ \left. \left(-1 + \right. \right. \\ \left. \left. e^{-\left((2 cb2 pa1^2 pa2^2 pb1 + 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - 2 ca2 pa1^2 pa2 pb1 pb2 + 2 pa2^2 pb1 pb2 - 2 ca1 pa1^2 pb1 pb2 \right. \right. \right. \\ \left. \left. \left. \right) \right) / \left(2 \sqrt{\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2}} \right) \right) - \\ \left. \frac{1}{2 \sqrt{\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2}}} \right. \\ \left. e^{-\frac{(pa2 pb1^2 pb2^2 (2 ca1 + \gamma 1) + pa1 pb1^2 pb2^2 (2 ca2 + \gamma 2) - pa1 pa2 (-2 pb2^2 + pb1 pb2^2 (2 cb1 + \gamma 3) + pb1^2 (-2 + 2 cb2 pb2 + pb2 \gamma 4)))^2}{4 pb1^2 pb2^2 (pa2^2 pb1^2 pb2^2 + pa1^2 (pb1^2 pb2^2 + pa2^2 (pb1^2 + pb2^2)))}} \right. \\ \left. \left(-1 + e^{\frac{(pa2 pb1^2 pb2^2 (2 ca1 + \gamma 1) + pa1 pb1^2 pb2^2 (2 ca2 + \gamma 2) - pa1 pa2 (-2 pb2^2 + pb1 pb2^2 (2 cb1 + \gamma 3) + pb1^2 (-2 + 2 cb2 pb2 + pb2 \gamma 4)))^2}{4 pb1^2 pb2^2 (pa2^2 pb1^2 pb2^2 + pa1^2 (pb1^2 pb2^2 + pa2^2 (pb1^2 + pb2^2)))}} \right) \right) \right) + \\ \left(ca1 \sqrt{\pi} (2 cb2 pa1^2 pa2^2 pb1 + 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - \right. \\ \left. 2 ca2 pa1^2 pa2 pb1 pb2 + 2 pa2^2 pb1 pb2 - 2 ca1 pa1 pa2^2 pb1 pb2 - pa1 pa2^2 \right.$$

[illegible]

$$\begin{aligned}
& \left(cb1 \sqrt{\pi} \left(2 cb2 pa1^2 pa2^2 pb1 + 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - \right. \right. \\
& \quad 2 ca2 pa1^2 pa2 pb1 pb2 + 2 pa2^2 pb1 pb2 - 2 ca1 pa1 pa2^2 pb1 pb2 - pa1 pa2^2 \\
& \quad \left. pb1 pb2 \gamma_1 - pa1^2 pa2 pb1 pb2 \gamma_2 + pa1^2 pa2^2 pb2 \gamma_3 + pa1^2 pa2^2 pb1 \gamma_4 \right) \operatorname{Erf} \left[\right. \\
& \quad \left(\sqrt{ \left(2 cb2 pa1^2 pa2^2 pb1 + 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - 2 ca2 pa1^2 pa2 \right. \right. \\
& \quad \left. \left. pb1 pb2 + 2 pa2^2 pb1 pb2 - 2 ca1 pa1 pa2^2 pb1 pb2 - pa1 pa2^2 pb1 pb2 \right. \right. \\
& \quad \left. \left. \gamma_1 - pa1^2 pa2 pb1 pb2 \gamma_2 + pa1^2 pa2^2 pb2 \gamma_3 + pa1^2 pa2^2 pb1 \gamma_4 \right)^2 } \right] \Bigg/ \\
& \quad \left(2 \sqrt{ pa1^2 pa2^4 pb1^2 pb2^2 + pa1^4 (pa2^2 pb1^2 pb2^2 + pa2^4 (pb1^2 + pb2^2)) } \right) \Bigg] \Bigg/ \\
& \quad \left(2 pb1 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) \sqrt{ \left(2 cb2 pa1^2 pa2^2 pb1 + \right. \right. \\
& \quad 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - 2 ca2 pa1^2 pa2 pb1 pb2 + \\
& \quad 2 pa2^2 pb1 pb2 - 2 ca1 pa1 pa2^2 pb1 pb2 - pa1 pa2^2 pb1 pb2 \gamma_1 - \\
& \quad \left. \left. pa1^2 pa2 pb1 pb2 \gamma_2 + pa1^2 pa2^2 pb2 \gamma_3 + pa1^2 pa2^2 pb1 \gamma_4 \right)^2 } \right) + \\
& \quad \left(\sqrt{\pi} \left(2 cb2 pa1^2 pa2^2 pb1 + 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - 2 ca2 pa1^2 \right. \right. \\
& \quad \left. \left. pa2 pb1 pb2 + 2 pa2^2 pb1 pb2 - 2 ca1 pa1 pa2^2 pb1 pb2 - pa1 pa2^2 pb1 pb2 \gamma_1 - \right. \right. \\
& \quad \left. \left. pa1^2 pa2 pb1 pb2 \gamma_2 + pa1^2 pa2^2 pb2 \gamma_3 + pa1^2 pa2^2 pb1 \gamma_4 \right) \operatorname{Erf} \left[\right. \right. \\
& \quad \left(\sqrt{ \left(2 cb2 pa1^2 pa2^2 pb1 + 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - 2 ca2 pa1^2 pa2 \right. \right. \\
& \quad \left. \left. pb1 pb2 + 2 pa2^2 pb1 pb2 - 2 ca1 pa1 pa2^2 pb1 pb2 - pa1 pa2^2 pb1 pb2 \right. \right. \\
& \quad \left. \left. \gamma_1 - pa1^2 pa2 pb1 pb2 \gamma_2 + pa1^2 pa2^2 pb2 \gamma_3 + pa1^2 pa2^2 pb1 \gamma_4 \right)^2 } \right] \Bigg/ \\
& \quad \left(2 \sqrt{ pa1^2 pa2^4 pb1^2 pb2^2 + pa1^4 (pa2^2 pb1^2 pb2^2 + pa2^4 (pb1^2 + pb2^2)) } \right) \Bigg] \Bigg/ \\
& \quad \left(2 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^2 \sqrt{ \left(2 cb2 pa1^2 pa2^2 pb1 + \right. \right. \\
& \quad 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - 2 ca2 pa1^2 pa2 pb1 pb2 + \\
& \quad 2 pa2^2 pb1 pb2 - 2 ca1 pa1 pa2^2 pb1 pb2 - pa1 pa2^2 pb1 pb2 \gamma_1 - \\
& \quad \left. \left. pa1^2 pa2 pb1 pb2 \gamma_2 + pa1^2 pa2^2 pb2 \gamma_3 + pa1^2 pa2^2 pb1 \gamma_4 \right)^2 } \right) - \\
& \quad \left(cb2 \sqrt{\pi} \left(2 cb2 pa1^2 pa2^2 pb1 + 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - \right. \right. \\
& \quad 2 ca2 pa1^2 pa2 pb1 pb2 + 2 pa2^2 pb1 pb2 - 2 ca1 pa1 pa2^2 pb1 pb2 - pa1 pa2^2 \\
& \quad \left. pb1 pb2 \gamma_1 - pa1^2 pa2 pb1 pb2 \gamma_2 + pa1^2 pa2^2 pb2 \gamma_3 + pa1^2 pa2^2 pb1 \gamma_4 \right) \operatorname{Erf} \left[\right. \\
& \quad \left(\sqrt{ \left(2 cb2 pa1^2 pa2^2 pb1 + 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - 2 ca2 pa1^2 pa2 \right. \right. \\
& \quad \left. \left. pb1 pb2 + 2 pa2^2 pb1 pb2 - 2 ca1 pa1 pa2^2 pb1 pb2 - pa1 pa2^2 pb1 pb2 \right. \right. \\
& \quad \left. \left. \gamma_1 - pa1^2 pa2 pb1 pb2 \gamma_2 + pa1^2 pa2^2 pb2 \gamma_3 + pa1^2 pa2^2 pb1 \gamma_4 \right)^2 } \right] \Bigg/ \\
& \quad \left(2 \sqrt{ pa1^2 pa2^4 pb1^2 pb2^2 + pa1^4 (pa2^2 pb1^2 pb2^2 + pa2^4 (pb1^2 + pb2^2)) } \right) \Bigg] \Bigg/ \\
& \quad \left(2 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2 \sqrt{ \left(2 cb2 pa1^2 pa2^2 pb1 + \right. \right. \\
& \quad 2 cb1 pa1^2 pa2^2 pb2 + 2 pa1^2 pb1 pb2 - 2 ca2 pa1^2 pa2 pb1 pb2 +
\end{aligned}$$

$$\begin{aligned}
& \left(4 \, pb1 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) \sqrt{ \left(2 \, cb2 \, pa1^2 \, pa2^2 \, pb1 + \right. \right. \\
& \quad 2 \, cb1 \, pa1^2 \, pa2^2 \, pb2 + 2 \, pa1^2 \, pb1 \, pb2 - 2 \, ca2 \, pa1^2 \, pa2 \, pb1 \, pb2 + \\
& \quad 2 \, pa2^2 \, pb1 \, pb2 - 2 \, ca1 \, pa1 \, pa2^2 \, pb1 \, pb2 - pa1 \, pa2^2 \, pb1 \, pb2 \, \gamma1 - \\
& \quad \left. \left. pa1^2 \, pa2 \, pb1 \, pb2 \, \gamma2 + pa1^2 \, pa2^2 \, pb2 \, \gamma3 + pa1^2 \, pa2^2 \, pb1 \, \gamma4 \right)^2 \right) - \\
& \left(\sqrt{\pi} \, \gamma4 \left(2 \, cb2 \, pa1^2 \, pa2^2 \, pb1 + 2 \, cb1 \, pa1^2 \, pa2^2 \, pb2 + 2 \, pa1^2 \, pb1 \, pb2 - \right. \right. \\
& \quad 2 \, ca2 \, pa1^2 \, pa2 \, pb1 \, pb2 + 2 \, pa2^2 \, pb1 \, pb2 - 2 \, ca1 \, pa1 \, pa2^2 \, pb1 \, pb2 - pa1 \, pa2^2 \\
& \quad \left. \left. pb1 \, pb2 \, \gamma1 - pa1^2 \, pa2 \, pb1 \, pb2 \, \gamma2 + pa1^2 \, pa2^2 \, pb2 \, \gamma3 + pa1^2 \, pa2^2 \, pb1 \, \gamma4 \right) \operatorname{Erf} \left[\right. \right. \\
& \quad \left(\sqrt{ \left(2 \, cb2 \, pa1^2 \, pa2^2 \, pb1 + 2 \, cb1 \, pa1^2 \, pa2^2 \, pb2 + 2 \, pa1^2 \, pb1 \, pb2 - 2 \, ca2 \, pa1^2 \, pa2 \right. \right. \\
& \quad \left. \left. pb1 \, pb2 + 2 \, pa2^2 \, pb1 \, pb2 - 2 \, ca1 \, pa1 \, pa2^2 \, pb1 \, pb2 - pa1 \, pa2^2 \, pb1 \, pb2 \right. \right. \\
& \quad \left. \left. \gamma1 - pa1^2 \, pa2 \, pb1 \, pb2 \, \gamma2 + pa1^2 \, pa2^2 \, pb2 \, \gamma3 + pa1^2 \, pa2^2 \, pb1 \, \gamma4 \right)^2 \right) \Big/ \\
& \quad \left. \left(2 \, \sqrt{pa1^2 \, pa2^4 \, pb1^2 \, pb2^2 + pa1^4 \, (pa2^2 \, pb1^2 \, pb2^2 + pa2^4 \, (pb1^2 + pb2^2))} \right) \right) \Big/ \\
& \left(4 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2 \sqrt{ \left(2 \, cb2 \, pa1^2 \, pa2^2 \, pb1 + \right. \right. \\
& \quad 2 \, cb1 \, pa1^2 \, pa2^2 \, pb2 + 2 \, pa1^2 \, pb1 \, pb2 - 2 \, ca2 \, pa1^2 \, pa2 \, pb1 \, pb2 + \\
& \quad 2 \, pa2^2 \, pb1 \, pb2 - 2 \, ca1 \, pa1 \, pa2^2 \, pb1 \, pb2 - pa1 \, pa2^2 \, pb1 \, pb2 \, \gamma1 - \\
& \quad \left. \left. pa1^2 \, pa2 \, pb1 \, pb2 \, \gamma2 + pa1^2 \, pa2^2 \, pb2 \, \gamma3 + pa1^2 \, pa2^2 \, pb1 \, \gamma4 \right)^2 \right) - \\
& \left(\sqrt{\pi} \left(-pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) - pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) + pa1 \, pa2 \right. \right. \\
& \quad \left. \left. (-2 \, pb2^2 + pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4)) \right) \operatorname{Erf} \left[\right. \right. \\
& \quad \left(\sqrt{ \left(pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) + pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) - pa1 \, pa2 \, (-2 \, pb2^2 + \right. \right. \\
& \quad \left. \left. pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4)) \right)^2 \right) \Big/ \\
& \quad \left. \left(2 \, \sqrt{pa2^2 \, pb1^4 \, pb2^4 + pa1^2 \, (pb1^4 \, pb2^4 + pa2^2 \, pb1^2 \, pb2^2 \, (pb1^2 + pb2^2))} \right) \right) \Big/ \\
& \left(2 \, pa1 \, pa2 \, pb1^2 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^4 \right. \\
& \quad \left. \sqrt{ \left(\frac{2 \, ca1}{pa1} + \frac{2 \, ca2}{pa2} + \frac{2}{pb1^2} - \frac{2 \, cb1}{pb1} + \frac{2}{pb2^2} - \frac{2 \, cb2}{pb2} + \frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} \right)^2 } \right) + \\
& \left(cb2 \, \sqrt{\pi} \left(-pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) - pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) + pa1 \right. \right. \\
& \quad \left. \left. pa2 \, (-2 \, pb2^2 + pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4)) \right) \operatorname{Erf} \left[\right. \right. \\
& \quad \left(\sqrt{ \left(pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) + pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) - pa1 \, pa2 \, (-2 \, pb2^2 + \right. \right. \\
& \quad \left. \left. pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4)) \right)^2 \right) \Big/ \\
& \quad \left. \left(2 \, \sqrt{pa2^2 \, pb1^4 \, pb2^4 + pa1^2 \, (pb1^4 \, pb2^4 + pa2^2 \, pb1^2 \, pb2^2 \, (pb1^2 + pb2^2))} \right) \right) \Big/ \\
& \left(2 \, pa1 \, pa2 \, pb1^2 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^3 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\frac{2ca1}{pa1} + \frac{2ca2}{pa2} + \frac{2}{pb1^2} - \frac{2cb1}{pb1} + \frac{2}{pb2^2} - \frac{2cb2}{pb2} + \frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} \right)^2} \Bigg) - \\
& \left(\sqrt{\pi} \left(-pa2 pb1^2 pb2^2 (2ca1 + \gamma1) - pa1 pb1^2 pb2^2 (2ca2 + \gamma2) + pa1 pa2 \right. \right. \\
& \quad \left. \left. (-2pb2^2 + pb1 pb2^2 (2cb1 + \gamma3) + pb1^2 (-2 + 2cb2 pb2 + pb2 \gamma4)) \right) \right) \operatorname{Erf} \Bigg[\\
& \quad \left(\sqrt{\left(pa2 pb1^2 pb2^2 (2ca1 + \gamma1) + pa1 pb1^2 pb2^2 (2ca2 + \gamma2) - pa1 pa2 (-2pb2^2 + \right. \right. \\
& \quad \left. \left. pb1 pb2^2 (2cb1 + \gamma3) + pb1^2 (-2 + 2cb2 pb2 + pb2 \gamma4)) \right)^2} \right) \Bigg] / \\
& \quad \left(2 \sqrt{pa2^2 pb1^4 pb2^4 + pa1^2 (pb1^4 pb2^4 + pa2^2 pb1^2 pb2^2 (pb1^2 + pb2^2))} \right) \Bigg] \Bigg) / \\
& \quad \left(2 pa1 pa2 pb1^4 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^2 \right. \\
& \quad \sqrt{\left(\frac{2ca1}{pa1} + \frac{2ca2}{pa2} + \frac{2}{pb1^2} - \frac{2cb1}{pb1} + \frac{2}{pb2^2} - \frac{2cb2}{pb2} + \frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} \right)^2} \Bigg) + \\
& \left(cb1 \sqrt{\pi} \left(-pa2 pb1^2 pb2^2 (2ca1 + \gamma1) - pa1 pb1^2 pb2^2 (2ca2 + \gamma2) + pa1 \right. \right. \\
& \quad \left. \left. pa2 (-2pb2^2 + pb1 pb2^2 (2cb1 + \gamma3) + pb1^2 (-2 + 2cb2 pb2 + pb2 \gamma4)) \right) \right) \operatorname{Erf} \Bigg[\\
& \quad \left(\sqrt{\left(pa2 pb1^2 pb2^2 (2ca1 + \gamma1) + pa1 pb1^2 pb2^2 (2ca2 + \gamma2) - pa1 pa2 (-2pb2^2 + \right. \right. \\
& \quad \left. \left. pb1 pb2^2 (2cb1 + \gamma3) + pb1^2 (-2 + 2cb2 pb2 + pb2 \gamma4)) \right)^2} \right) \Bigg] / \\
& \quad \left(2 \sqrt{pa2^2 pb1^4 pb2^4 + pa1^2 (pb1^4 pb2^4 + pa2^2 pb1^2 pb2^2 (pb1^2 + pb2^2))} \right) \Bigg] \Bigg) / \\
& \quad \left(2 pa1 pa2 pb1^3 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^2 \right. \\
& \quad \sqrt{\left(\frac{2ca1}{pa1} + \frac{2ca2}{pa2} + \frac{2}{pb1^2} - \frac{2cb1}{pb1} + \frac{2}{pb2^2} - \frac{2cb2}{pb2} + \frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} \right)^2} \Bigg) - \\
& \left(ca2 \sqrt{\pi} \left(-pa2 pb1^2 pb2^2 (2ca1 + \gamma1) - pa1 pb1^2 pb2^2 (2ca2 + \gamma2) + pa1 \right. \right. \\
& \quad \left. \left. pa2 (-2pb2^2 + pb1 pb2^2 (2cb1 + \gamma3) + pb1^2 (-2 + 2cb2 pb2 + pb2 \gamma4)) \right) \right) \operatorname{Erf} \Bigg[\\
& \quad \left(\sqrt{\left(pa2 pb1^2 pb2^2 (2ca1 + \gamma1) + pa1 pb1^2 pb2^2 (2ca2 + \gamma2) - pa1 pa2 (-2pb2^2 + \right. \right. \\
& \quad \left. \left. pb1 pb2^2 (2cb1 + \gamma3) + pb1^2 (-2 + 2cb2 pb2 + pb2 \gamma4)) \right)^2} \right) \Bigg] / \\
& \quad \left(2 \sqrt{pa2^2 pb1^4 pb2^4 + pa1^2 (pb1^4 pb2^4 + pa2^2 pb1^2 pb2^2 (pb1^2 + pb2^2))} \right) \Bigg] \Bigg) / \\
& \quad \left(2 pa1 pa2^2 pb1^2 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^2 \right. \\
& \quad \sqrt{\left(\frac{2ca1}{pa1} + \frac{2ca2}{pa2} + \frac{2}{pb1^2} - \frac{2cb1}{pb1} + \frac{2}{pb2^2} - \frac{2cb2}{pb2} + \frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} \right)^2} \Bigg) - \\
& \left(ca1 \sqrt{\pi} \left(-pa2 pb1^2 pb2^2 (2ca1 + \gamma1) - pa1 pb1^2 pb2^2 (2ca2 + \gamma2) + pa1 \right. \right. \\
& \quad \left. \left. pa2 (-2pb2^2 + pb1 pb2^2 (2cb1 + \gamma3) + pb1^2 (-2 + 2cb2 pb2 + pb2 \gamma4)) \right) \right) \operatorname{Erf} \Bigg[\\
& \quad \left(\sqrt{\left(pa2 pb1^2 pb2^2 (2ca1 + \gamma1) + pa1 pb1^2 pb2^2 (2ca2 + \gamma2) - pa1 pa2 (-2pb2^2 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4)}{(2 \, \sqrt{pa2^2 \, pb1^4 \, pb2^4 + pa1^2 \, (pb1^4 \, pb2^4 + pa2^2 \, pb1^2 \, pb2^2 \, (pb1^2 + pb2^2))})} \right) \right) \Bigg/ \\
& \left(2 \, pa1^2 \, pa2 \, pb1^2 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^2 \right. \\
& \quad \left. \sqrt{\left(\frac{2 \, ca1}{pa1} + \frac{2 \, ca2}{pa2} + \frac{2}{pb1^2} - \frac{2 \, cb1}{pb1} + \frac{2}{pb2^2} - \frac{2 \, cb2}{pb2} + \frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} \right)^2} \right) - \\
& \left(\sqrt{\pi} \, \gamma1 \, (-pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) - pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) + pa1 \, pa2 \right. \\
& \quad \left. (-2 \, pb2^2 + pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4)) \right) \operatorname{Erf} \left[\right. \\
& \quad \left. \left(\sqrt{(pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) + pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) - pa1 \, pa2 \, (-2 \, pb2^2 + \right. \right. \\
& \quad \left. \left. pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4))} \right)^2 \right] \Bigg/ \\
& \left(2 \, \sqrt{pa2^2 \, pb1^4 \, pb2^4 + pa1^2 \, (pb1^4 \, pb2^4 + pa2^2 \, pb1^2 \, pb2^2 \, (pb1^2 + pb2^2))} \right) \Bigg/ \\
& \left(4 \, pa1^2 \, pa2 \, pb1^2 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^2 \right. \\
& \quad \left. \sqrt{\left(\frac{2 \, ca1}{pa1} + \frac{2 \, ca2}{pa2} + \frac{2}{pb1^2} - \frac{2 \, cb1}{pb1} + \frac{2}{pb2^2} - \frac{2 \, cb2}{pb2} + \frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} \right)^2} \right) - \\
& \left(\sqrt{\pi} \, \gamma2 \, (-pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) - pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) + pa1 \, pa2 \right. \\
& \quad \left. (-2 \, pb2^2 + pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4)) \right) \operatorname{Erf} \left[\right. \\
& \quad \left. \left(\sqrt{(pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) + pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) - pa1 \, pa2 \, (-2 \, pb2^2 + \right. \right. \\
& \quad \left. \left. pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4))} \right)^2 \right] \Bigg/ \\
& \left(2 \, \sqrt{pa2^2 \, pb1^4 \, pb2^4 + pa1^2 \, (pb1^4 \, pb2^4 + pa2^2 \, pb1^2 \, pb2^2 \, (pb1^2 + pb2^2))} \right) \Bigg/ \\
& \left(4 \, pa1 \, pa2^2 \, pb1^2 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^2 \right. \\
& \quad \left. \sqrt{\left(\frac{2 \, ca1}{pa1} + \frac{2 \, ca2}{pa2} + \frac{2}{pb1^2} - \frac{2 \, cb1}{pb1} + \frac{2}{pb2^2} - \frac{2 \, cb2}{pb2} + \frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} \right)^2} \right) + \\
& \left(\sqrt{\pi} \, \gamma3 \, (-pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) - pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) + pa1 \, pa2 \right. \\
& \quad \left. (-2 \, pb2^2 + pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4)) \right) \operatorname{Erf} \left[\right. \\
& \quad \left. \left(\sqrt{(pa2 \, pb1^2 \, pb2^2 \, (2 \, ca1 + \gamma1) + pa1 \, pb1^2 \, pb2^2 \, (2 \, ca2 + \gamma2) - pa1 \, pa2 \, (-2 \, pb2^2 + \right. \right. \\
& \quad \left. \left. pb1 \, pb2^2 \, (2 \, cb1 + \gamma3) + pb1^2 \, (-2 + 2 \, cb2 \, pb2 + pb2 \, \gamma4))} \right)^2 \right] \Bigg/ \\
& \left(2 \, \sqrt{pa2^2 \, pb1^4 \, pb2^4 + pa1^2 \, (pb1^4 \, pb2^4 + pa2^2 \, pb1^2 \, pb2^2 \, (pb1^2 + pb2^2))} \right) \Bigg/ \\
& \left(4 \, pa1 \, pa2 \, pb1^3 \left(\frac{1}{pa1^2} + \frac{1}{pa2^2} + \frac{1}{pb1^2} + \frac{1}{pb2^2} \right) pb2^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\frac{2 \text{ ca1}}{\text{pa1}} + \frac{2 \text{ ca2}}{\text{pa2}} + \frac{2}{\text{pb1}^2} - \frac{2 \text{ cb1}}{\text{pb1}} + \frac{2}{\text{pb2}^2} - \frac{2 \text{ cb2}}{\text{pb2}} + \frac{\gamma_1}{\text{pa1}} + \frac{\gamma_2}{\text{pa2}} - \frac{\gamma_3}{\text{pb1}} - \frac{\gamma_4}{\text{pb2}} \right)^2} \Bigg) + \\
& \left(\sqrt{\pi} \gamma_4 \left(-\text{pa2 pb1}^2 \text{pb2}^2 (2 \text{ ca1} + \gamma_1) - \text{pa1 pb1}^2 \text{pb2}^2 (2 \text{ ca2} + \gamma_2) + \text{pa1 pa2} \right. \right. \\
& \quad \left. \left. (-2 \text{ pb2}^2 + \text{pb1 pb2}^2 (2 \text{ cb1} + \gamma_3) + \text{pb1}^2 (-2 + 2 \text{ cb2 pb2} + \text{pb2} \gamma_4)) \right) \right) \text{Erf} \Bigg[\\
& \quad \left(\sqrt{\left(\text{pa2 pb1}^2 \text{pb2}^2 (2 \text{ ca1} + \gamma_1) + \text{pa1 pb1}^2 \text{pb2}^2 (2 \text{ ca2} + \gamma_2) - \text{pa1 pa2} (-2 \text{ pb2}^2 + \right. \right. \\
& \quad \left. \left. \text{pb1 pb2}^2 (2 \text{ cb1} + \gamma_3) + \text{pb1}^2 (-2 + 2 \text{ cb2 pb2} + \text{pb2} \gamma_4)) \right)^2} \right) \Bigg] / \\
& \quad \left(2 \sqrt{\text{pa2}^2 \text{pb1}^4 \text{pb2}^4 + \text{pa1}^2 (\text{pb1}^4 \text{pb2}^4 + \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 (\text{pb1}^2 + \text{pb2}^2))} \right) \Bigg] \Bigg) / \\
& \quad \left(4 \text{ pa1 pa2 pb1}^2 \left(\frac{1}{\text{pa1}^2} + \frac{1}{\text{pa2}^2} + \frac{1}{\text{pb1}^2} + \frac{1}{\text{pb2}^2} \right) \text{pb2}^3 \right. \\
& \quad \left. \sqrt{\left(\frac{2 \text{ ca1}}{\text{pa1}} + \frac{2 \text{ ca2}}{\text{pa2}} + \frac{2}{\text{pb1}^2} - \frac{2 \text{ cb1}}{\text{pb1}} + \frac{2}{\text{pb2}^2} - \frac{2 \text{ cb2}}{\text{pb2}} + \frac{\gamma_1}{\text{pa1}} + \frac{\gamma_2}{\text{pa2}} - \frac{\gamma_3}{\text{pb1}} - \frac{\gamma_4}{\text{pb2}} \right)^2} \right) \Bigg) + \\
& \quad \frac{\left(\frac{2 \text{ ca1}}{\text{pa1}} + \frac{2 \text{ ca2}}{\text{pa2}} + \frac{2}{\text{pb1}^2} - \frac{2 \text{ cb1}}{\text{pb1}} + \frac{2}{\text{pb2}^2} - \frac{2 \text{ cb2}}{\text{pb2}} + \frac{\gamma_1}{\text{pa1}} + \frac{\gamma_2}{\text{pa2}} - \frac{\gamma_3}{\text{pb1}} - \frac{\gamma_4}{\text{pb2}} \right)^2}{4 \left(-\frac{1}{\text{pa1}^2} - \frac{1}{\text{pa2}^2} - \frac{1}{\text{pb1}^2} - \frac{1}{\text{pb2}^2} \right)} \\
& \text{ca1 e} \\
& \text{pa1} \\
& \left(\left(\sqrt{\pi} \left(1 - \frac{\frac{2 \text{ ca1}}{\text{pa1}} + \frac{2 \text{ ca2}}{\text{pa2}} + \frac{2}{\text{pb1}^2} - \frac{2 \text{ cb1}}{\text{pb1}} + \frac{2}{\text{pb2}^2} - \frac{2 \text{ cb2}}{\text{pb2}} + \frac{\gamma_1}{\text{pa1}} + \frac{\gamma_2}{\text{pa2}} - \frac{\gamma_3}{\text{pb1}} - \frac{\gamma_4}{\text{pb2}}}{2 \left(\frac{1}{\text{pa1}^2} + \frac{1}{\text{pa2}^2} + \frac{1}{\text{pb1}^2} + \frac{1}{\text{pb2}^2} \right)} \right) \right. \right. \\
& \quad \text{Erf} \Bigg[\left(\sqrt{\left(2 \text{ cb2 pa1}^2 \text{pa2}^2 \text{pb1} + 2 \text{ cb1 pa1}^2 \text{pa2}^2 \text{pb2} + 2 \text{ pa1}^2 \text{pb1 pb2} - 2 \text{ ca2 pa1}^2 \right. \right. \\
& \quad \left. \left. \text{pa2 pb1 pb2} + 2 \text{ pa2}^2 \text{pb1 pb2} - 2 \text{ ca1 pa1 pa2}^2 \text{pb1 pb2} - \text{pa1 pa2}^2 \text{pb1} \right. \right. \\
& \quad \left. \left. \text{pb2 } \gamma_1 - \text{pa1}^2 \text{pa2 pb1 pb2 } \gamma_2 + \text{pa1}^2 \text{pa2}^2 \text{pb2 } \gamma_3 + \text{pa1}^2 \text{pa2}^2 \text{pb1 } \gamma_4 \right)^2} \right) \Bigg] / \\
& \quad \left(2 \sqrt{\text{pa1}^2 \text{pa2}^4 \text{pb1}^2 \text{pb2}^2 + \text{pa1}^4 (\text{pa2}^2 \text{pb1}^2 \text{pb2}^2 + \text{pa2}^4 (\text{pb1}^2 + \text{pb2}^2))} \right) \Bigg] \Bigg) / \\
& \quad \left(2 \sqrt{\frac{1}{\text{pa1}^2} + \frac{1}{\text{pa2}^2} + \frac{1}{\text{pb1}^2} + \frac{1}{\text{pb2}^2}} \right. \\
& \quad \left. \sqrt{\left(1 - \frac{\frac{2 \text{ ca1}}{\text{pa1}} + \frac{2 \text{ ca2}}{\text{pa2}} + \frac{2}{\text{pb1}^2} - \frac{2 \text{ cb1}}{\text{pb1}} + \frac{2}{\text{pb2}^2} - \frac{2 \text{ cb2}}{\text{pb2}} + \frac{\gamma_1}{\text{pa1}} + \frac{\gamma_2}{\text{pa2}} - \frac{\gamma_3}{\text{pb1}} - \frac{\gamma_4}{\text{pb2}}}{2 \left(\frac{1}{\text{pa1}^2} + \frac{1}{\text{pa2}^2} + \frac{1}{\text{pb1}^2} + \frac{1}{\text{pb2}^2} \right)} \right)^2} \right) - \\
& \quad \left(\text{pa1 pa2 } \sqrt{\text{pa2}^2 \text{pb1}^4 \text{pb2}^4 + \text{pa1}^2 (\text{pb1}^4 \text{pb2}^4 + \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 (\text{pb1}^2 + \text{pb2}^2))} \right) \\
& \quad \sqrt{\pi} \left(-\text{pa2 pb1}^2 \text{pb2}^2 (2 \text{ ca1} + \gamma_1) - \text{pa1 pb1}^2 \text{pb2}^2 (2 \text{ ca2} + \gamma_2) + \right. \\
& \quad \left. \text{pa1 pa2} (-2 \text{ pb2}^2 + \text{pb1 pb2}^2 (2 \text{ cb1} + \gamma_3) + \text{pb1}^2 (-2 + 2 \text{ cb2 pb2} + \text{pb2} \gamma_4)) \right) \\
& \quad \text{Erf} \Bigg[\left(\sqrt{\left(\text{pa2 pb1}^2 \text{pb2}^2 (2 \text{ ca1} + \gamma_1) + \text{pa1 pb1}^2 \text{pb2}^2 (2 \text{ ca2} + \gamma_2) - \text{pa1 pa2} \right. \right. \\
& \quad \left. \left. (-2 \text{ pb2}^2 + \text{pb1 pb2}^2 (2 \text{ cb1} + \gamma_3) + \text{pb1}^2 (-2 + 2 \text{ cb2 pb2} + \text{pb2} \gamma_4)) \right)^2} \right) \Bigg] /
\end{aligned}$$

$$\left(\frac{2 \sqrt{\text{pa}^2 \text{pb}_1^4 \text{pb}_2^4 + \text{pa}_1^2 (\text{pb}_1^4 \text{pb}_2^4 + \text{pa}^2 \text{pb}_1^2 \text{pb}_2^2 (\text{pb}_1^2 + \text{pb}_2^2))}} \right) /$$
$$\left(2 (\text{pa}^2 \text{pb}_1^2 \text{pb}_2^2 + \text{pa}_1^2 (\text{pb}_1^2 \text{pb}_2^2 + \text{pa}^2 (\text{pb}_1^2 + \text{pb}_2^2))) \right)$$
$$\sqrt{(\text{pa}_2 \text{pb}_1^2 \text{pb}_2^2 (2 \text{ca}_1 + \gamma_1) + \text{pa}_1 \text{pb}_1^2 \text{pb}_2^2 (2 \text{ca}_2 + \gamma_2) - \text{pa}_1 \text{pa}_2$$
$$(-2 \text{pb}_2^2 + \text{pb}_1 \text{pb}_2^2 (2 \text{cb}_1 + \gamma_3) + \text{pb}_1^2 (-2 + 2 \text{cb}_2 \text{pb}_2 + \text{pb}_2 \gamma_4))^2)} \Bigg) \Bigg) \Bigg) /$$
$$\left(\text{pa}_1^2 \text{pa}_2 \text{pb}_1 \text{pb}_2 \sqrt{\pi} \left(\text{Erf} \left[(-2 \text{pa}^2 \text{pb}_1 \text{pb}_2 + \text{pa}_1 \text{pa}^2 \text{pb}_1 \text{pb}_2 (2 \text{ca}_1 + \gamma_1) - \right. \right.$$
$$\left. \left. \text{pa}_1^2 (2 \text{pb}_1 \text{pb}_2 - \text{pa}_2 \text{pb}_1 \text{pb}_2 (2 \text{ca}_2 + \gamma_2) + \right. \right.$$
$$\left. \left. \text{pa}_2^2 (2 \text{cb}_2 \text{pb}_1 + 2 \text{cb}_1 \text{pb}_2 + \text{pb}_2 \gamma_3 + \text{pb}_1 \gamma_4) \right) \right] /$$
$$(2 \text{pa}_1 \text{pa}_2 \sqrt{\text{pa}^2 \text{pb}_1^2 \text{pb}_2^2 + \text{pa}_1^2 (\text{pb}_1^2 \text{pb}_2^2 + \text{pa}^2 (\text{pb}_1^2 + \text{pb}_2^2))}) \Bigg) -$$
$$\text{Erf} \left[(\text{pa}_2 \text{pb}_1^2 \text{pb}_2^2 (2 \text{ca}_1 + \gamma_1) + \text{pa}_1 \text{pb}_1^2 \text{pb}_2^2 (2 \text{ca}_2 + \gamma_2) - \right.$$
$$\left. \text{pa}_1 \text{pa}_2 \right.$$
$$\left. (-2 \text{pb}_2^2 + \text{pb}_1 \text{pb}_2^2 (2 \text{cb}_1 + \gamma_3) + \text{pb}_1^2 (-2 + 2 \text{cb}_2 \text{pb}_2 + \text{pb}_2 \gamma_4)) \right] /$$
$$(2 \text{pb}_1 \text{pb}_2 \sqrt{\text{pa}^2 \text{pb}_1^2 \text{pb}_2^2 + \text{pa}_1^2 (\text{pb}_1^2 \text{pb}_2^2 + \text{pa}^2 (\text{pb}_1^2 + \text{pb}_2^2))}) \Bigg) \Bigg) \Bigg)$$

```
In[88]:= halattrueconsumption = Simplify[
  ha1 /. {ca1 → caltrue *  $\epsilon$ , ca2 → ca2true *  $\epsilon$ , cb1 → cb1true *  $\epsilon$ , cb2 → cb2true *  $\epsilon$ },
  Assumptionselvis  $\&\&\alpha \geq 0 \&\&\alpha \leq 1$ ]
```

Out[88]=

$$2 \sqrt{pa^2 pb^2 + pa^2 (pb^2 + pb^2)} \left(\frac{1}{4 \sqrt{\frac{1}{pa^2} + \frac{1}{pa^2} + \frac{1}{pb^2} + \frac{1}{pb^2}}} - \frac{2 \left(-1 + e^{-\frac{\left(\frac{\gamma^4}{pa^2} + \frac{\gamma^2}{pa^2} - \frac{\gamma^3}{pb^2} - \frac{\gamma^4}{pb^2} + \frac{2 \alpha e}{pa^2} + \frac{2 \alpha e}{pa^2} + \frac{2 \cdot 2 (-1 \cdot \alpha) e}{pb^2} + \frac{2 \cdot 2 (-1 \cdot \alpha) e}{pb^2} \right)^2}}{4 \left(\frac{1}{pa^2} + \frac{1}{pa^2} + \frac{1}{pb^2} + \frac{1}{pb^2} \right)}} \right) \right) - \frac{pa^2 pb^2 pb^2 \gamma^2 + pa^2 pb^2 pb^2 (-1 + \alpha) + pa^2 (-pa^2 pb^2 \gamma^3 + 2 pa^2 pb^2 (-1 + \alpha) e + pb^2 (-pa^2 pb^2 \gamma^4 + 2 pa^2 (-1 + \alpha) e + pb^2 (-2 \cdot pa^2 \gamma^2 + 2 \alpha e + pb^2 \gamma^2 + 2 \alpha e))}{4 pa^2 pa^2 pb^2 pb^2 (pa^2 pb^2 pb^2 + pa^2 (pb^2 + pb^2))} \right) \right) \left(\sqrt{\frac{1}{pa^2} + \frac{1}{pa^2} + \frac{1}{pb^2} + \frac{1}{pb^2}} \right) +$$

$$\begin{aligned}
& \left(2 \sqrt{\pi} \left(pa1 pa2^2 pb1^2 pb2^2 \gamma1 + 2 pa2^2 pb1^2 pb2^2 \alpha \epsilon + \right. \right. \\
& \quad pa1^2 \left(pa2 pb1^2 pb2^2 \gamma2 + 2 pb1^2 pb2^2 \alpha \epsilon - pa2^2 \left(pb1 pb2^2 \gamma3 - \right. \right. \\
& \quad \quad \left. \left. 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right) \operatorname{Erf}\left[\frac{1}{2} \right. \\
& \quad \left. \sqrt{\left(\left(pa2 pb1^2 pb2^2 \left(\gamma1 + \frac{2 \alpha \epsilon}{pa1} \right) + pa1 pb1^2 pb2^2 \left(\gamma2 + \frac{2 \alpha \epsilon}{pa2} \right) - pa1 pa2 \left(pb1 pb2^2 \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. \gamma3 - 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right)^2 \right. \right. \\
& \quad \left. \left. \left(pa2^2 pb1^4 pb2^4 + pa1^2 \left(pb1^4 pb2^4 + pa2^2 pb1^2 pb2^2 \left(pb1^2 + pb2^2 \right) \right) \right) \right) \right] \right) \Bigg/ \\
& \left(pb1^2 \left(pa2^2 pb1^2 pb2^2 + pa1^2 \left(pb1^2 pb2^2 + pa2^2 \left(pb1^2 + pb2^2 \right) \right) \right) \right. \\
& \quad \left. \sqrt{\left(\frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} + \frac{2 \alpha \epsilon}{pa1^2} + \frac{2 \alpha \epsilon}{pa2^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{pb1^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{pb2^2} \right)^2} \right) + \\
& \left(2 \sqrt{\pi} \left(pa1 pa2^2 pb1^2 pb2^2 \gamma1 + 2 pa2^2 pb1^2 pb2^2 \alpha \epsilon + \right. \right. \\
& \quad pa1^2 \left(pa2 pb1^2 pb2^2 \gamma2 + 2 pb1^2 pb2^2 \alpha \epsilon - pa2^2 \left(pb1 pb2^2 \gamma3 - \right. \right. \\
& \quad \quad \left. \left. 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right) \operatorname{Erf}\left[\frac{1}{2} \right. \\
& \quad \left. \sqrt{\left(\left(pa2 pb1^2 pb2^2 \left(\gamma1 + \frac{2 \alpha \epsilon}{pa1} \right) + pa1 pb1^2 pb2^2 \left(\gamma2 + \frac{2 \alpha \epsilon}{pa2} \right) - pa1 pa2 \left(pb1 pb2^2 \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. \gamma3 - 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right)^2 \right. \right. \\
& \quad \left. \left. \left(pa2^2 pb1^4 pb2^4 + pa1^2 \left(pb1^4 pb2^4 + pa2^2 pb1^2 pb2^2 \left(pb1^2 + pb2^2 \right) \right) \right) \right) \right] \right) \Bigg/ \\
& \left(pb2^2 \left(pa2^2 pb1^2 pb2^2 + pa1^2 \left(pb1^2 pb2^2 + pa2^2 \left(pb1^2 + pb2^2 \right) \right) \right) \right. \\
& \quad \left. \sqrt{\left(\frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} + \frac{2 \alpha \epsilon}{pa1^2} + \frac{2 \alpha \epsilon}{pa2^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{pb1^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{pb2^2} \right)^2} \right) + \\
& \left(\sqrt{\pi} \gamma1 \left(pa1 pa2^2 pb1^2 pb2^2 \gamma1 + 2 pa2^2 pb1^2 pb2^2 \alpha \epsilon + \right. \right. \\
& \quad pa1^2 \left(pa2 pb1^2 pb2^2 \gamma2 + 2 pb1^2 pb2^2 \alpha \epsilon - pa2^2 \left(pb1 pb2^2 \gamma3 - \right. \right. \\
& \quad \quad \left. \left. 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right) \operatorname{Erf}\left[\frac{1}{2} \right. \\
& \quad \left. \sqrt{\left(\left(pa2 pb1^2 pb2^2 \left(\gamma1 + \frac{2 \alpha \epsilon}{pa1} \right) + pa1 pb1^2 pb2^2 \left(\gamma2 + \frac{2 \alpha \epsilon}{pa2} \right) - pa1 pa2 \left(pb1 pb2^2 \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. \gamma3 - 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right)^2 \right. \right. \\
& \quad \left. \left. \left(pa2^2 pb1^4 pb2^4 + pa1^2 \left(pb1^4 pb2^4 + pa2^2 pb1^2 pb2^2 \left(pb1^2 + pb2^2 \right) \right) \right) \right) \right] \right) \Bigg/
\end{aligned}$$

[illegible]

$$\begin{aligned}
& pa1^2 \left(pa2 pb1^2 pb2^2 \gamma2 + 2 pb1^2 pb2^2 \alpha \epsilon - pa2^2 \left(pb1 pb2^2 \gamma3 - \right. \right. \\
& \quad \left. \left. 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right) \operatorname{Erf}\left[\frac{1}{2} \right. \\
& \quad \left. \sqrt{\left(\left(pa2 pb1^2 pb2^2 \left(\gamma1 + \frac{2 \alpha \epsilon}{pa1} \right) + pa1 pb1^2 pb2^2 \left(\gamma2 + \frac{2 \alpha \epsilon}{pa2} \right) - pa1 pa2 \left(pb1 pb2^2 \right. \right. \right. \right. \\
& \quad \left. \left. \left. \gamma3 - 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right)^2 \right. \right. \\
& \quad \left. \left. \left(pa2^2 pb1^4 pb2^4 + pa1^2 \left(pb1^4 pb2^4 + pa2^2 pb1^2 pb2^2 \left(pb1^2 + pb2^2 \right) \right) \right) \right) \right] \right) / \\
& \quad \left(pa2^2 pb1^2 pb2^3 + pa1^2 \left(pb1^2 pb2^3 + pa2^2 pb2 \left(pb1^2 + pb2^2 \right) \right) \right) \\
& \quad \sqrt{\left(\frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} + \frac{2 \alpha \epsilon}{pa1^2} + \frac{2 \alpha \epsilon}{pa2^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{pb1^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{pb2^2} \right)^2} \Bigg) + \\
& \quad \left(2 \sqrt{\pi} (-1 + \alpha) \epsilon \left(pa1 pa2^2 pb1^2 pb2^2 \gamma1 + 2 pa2^2 pb1^2 pb2^2 \alpha \epsilon + \right. \right. \\
& \quad pa1^2 \left(pa2 pb1^2 pb2^2 \gamma2 + 2 pb1^2 pb2^2 \alpha \epsilon - pa2^2 \left(pb1 pb2^2 \gamma3 - \right. \right. \\
& \quad \left. \left. 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right) \operatorname{Erf}\left[\frac{1}{2} \right. \\
& \quad \left. \sqrt{\left(\left(pa2 pb1^2 pb2^2 \left(\gamma1 + \frac{2 \alpha \epsilon}{pa1} \right) + pa1 pb1^2 pb2^2 \left(\gamma2 + \frac{2 \alpha \epsilon}{pa2} \right) - pa1 pa2 \left(pb1 pb2^2 \right. \right. \right. \right. \\
& \quad \left. \left. \left. \gamma3 - 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right)^2 \right. \right. \\
& \quad \left. \left. \left(pa2^2 pb1^4 pb2^4 + pa1^2 \left(pb1^4 pb2^4 + pa2^2 pb1^2 pb2^2 \left(pb1^2 + pb2^2 \right) \right) \right) \right) \right] \right) / \\
& \quad \left(pb1^2 \left(pa2^2 pb1^2 pb2^2 + pa1^2 \left(pb1^2 pb2^2 + pa2^2 \left(pb1^2 + pb2^2 \right) \right) \right) \right) \\
& \quad \sqrt{\left(\frac{\gamma1}{pa1} + \frac{\gamma2}{pa2} - \frac{\gamma3}{pb1} - \frac{\gamma4}{pb2} + \frac{2 \alpha \epsilon}{pa1^2} + \frac{2 \alpha \epsilon}{pa2^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{pb1^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{pb2^2} \right)^2} \Bigg) + \\
& \quad \left(2 \sqrt{\pi} (-1 + \alpha) \epsilon \left(pa1 pa2^2 pb1^2 pb2^2 \gamma1 + 2 pa2^2 pb1^2 pb2^2 \alpha \epsilon + \right. \right. \\
& \quad pa1^2 \left(pa2 pb1^2 pb2^2 \gamma2 + 2 pb1^2 pb2^2 \alpha \epsilon - pa2^2 \left(pb1 pb2^2 \gamma3 - \right. \right. \\
& \quad \left. \left. 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right) \operatorname{Erf}\left[\frac{1}{2} \right. \\
& \quad \left. \sqrt{\left(\left(pa2 pb1^2 pb2^2 \left(\gamma1 + \frac{2 \alpha \epsilon}{pa1} \right) + pa1 pb1^2 pb2^2 \left(\gamma2 + \frac{2 \alpha \epsilon}{pa2} \right) - pa1 pa2 \left(pb1 pb2^2 \right. \right. \right. \right. \\
& \quad \left. \left. \left. \gamma3 - 2 pb2^2 \left(1 + (-1 + \alpha) \epsilon \right) + pb1^2 \left(-2 + pb2 \gamma4 + 2 \epsilon - 2 \alpha \epsilon \right) \right) \right)^2 \right. \right. \\
& \quad \left. \left. \left(pa2^2 pb1^4 pb2^4 + pa1^2 \left(pb1^4 pb2^4 + pa2^2 pb1^2 pb2^2 \left(pb1^2 + pb2^2 \right) \right) \right) \right) \right] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(\text{pb2}^2 \left(\text{pa2}^2 \text{pb1}^2 \text{pb2}^2 + \text{pa1}^2 \left(\text{pb1}^2 \text{pb2}^2 + \text{pa2}^2 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \right) \\
& \sqrt{\left(\frac{\gamma_1}{\text{pa1}} + \frac{\gamma_2}{\text{pa2}} - \frac{\gamma_3}{\text{pb1}} - \frac{\gamma_4}{\text{pb2}} + \frac{2 \alpha \epsilon}{\text{pa1}^2} + \frac{2 \alpha \epsilon}{\text{pa2}^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{\text{pb1}^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{\text{pb2}^2} \right)^2} + \\
& \left(2 \sqrt{\pi} \alpha \epsilon \left(\text{pa1} \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \gamma_1 + 2 \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \alpha \epsilon + \right. \right. \\
& \quad \text{pa1}^2 \left(\text{pa2} \text{pb1}^2 \text{pb2}^2 \gamma_2 + 2 \text{pb1}^2 \text{pb2}^2 \alpha \epsilon - \text{pa2}^2 \left(\text{pb1} \text{pb2}^2 \gamma_3 - \right. \right. \\
& \quad \quad \left. \left. 2 \text{pb2}^2 (1 + (-1 + \alpha) \epsilon) + \text{pb1}^2 (-2 + \text{pb2} \gamma_4 + 2 \epsilon - 2 \alpha \epsilon) \right) \right) \left. \right) \text{Erf} \left[\frac{1}{2} \right. \\
& \quad \left. \sqrt{\left(\left(\text{pa2} \text{pb1}^2 \text{pb2}^2 \left(\gamma_1 + \frac{2 \alpha \epsilon}{\text{pa1}} \right) + \text{pa1} \text{pb1}^2 \text{pb2}^2 \left(\gamma_2 + \frac{2 \alpha \epsilon}{\text{pa2}} \right) - \text{pa1} \text{pa2} \left(\text{pb1} \text{pb2}^2 \right. \right. \right. \right. \\
& \quad \left. \left. \left. \gamma_3 - 2 \text{pb2}^2 (1 + (-1 + \alpha) \epsilon) + \text{pb1}^2 (-2 + \text{pb2} \gamma_4 + 2 \epsilon - 2 \alpha \epsilon) \right) \right) \right)^2} \right. \\
& \quad \left. \left. \left. \left(\text{pa2}^2 \text{pb1}^4 \text{pb2}^4 + \text{pa1}^2 \left(\text{pb1}^4 \text{pb2}^4 + \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \right) \right] \right) \right. \\
& \left(\text{pa1}^2 \left(\text{pa2}^2 \text{pb1}^2 \text{pb2}^2 + \text{pa1}^2 \left(\text{pb1}^2 \text{pb2}^2 + \text{pa2}^2 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \right) \\
& \sqrt{\left(\frac{\gamma_1}{\text{pa1}} + \frac{\gamma_2}{\text{pa2}} - \frac{\gamma_3}{\text{pb1}} - \frac{\gamma_4}{\text{pb2}} + \frac{2 \alpha \epsilon}{\text{pa1}^2} + \frac{2 \alpha \epsilon}{\text{pa2}^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{\text{pb1}^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{\text{pb2}^2} \right)^2} + \\
& \left(2 \sqrt{\pi} \alpha \epsilon \left(\text{pa1} \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \gamma_1 + 2 \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \alpha \epsilon + \right. \right. \\
& \quad \text{pa1}^2 \left(\text{pa2} \text{pb1}^2 \text{pb2}^2 \gamma_2 + 2 \text{pb1}^2 \text{pb2}^2 \alpha \epsilon - \text{pa2}^2 \left(\text{pb1} \text{pb2}^2 \gamma_3 - \right. \right. \\
& \quad \quad \left. \left. 2 \text{pb2}^2 (1 + (-1 + \alpha) \epsilon) + \text{pb1}^2 (-2 + \text{pb2} \gamma_4 + 2 \epsilon - 2 \alpha \epsilon) \right) \right) \left. \right) \text{Erf} \left[\frac{1}{2} \right. \\
& \quad \left. \sqrt{\left(\left(\text{pa2} \text{pb1}^2 \text{pb2}^2 \left(\gamma_1 + \frac{2 \alpha \epsilon}{\text{pa1}} \right) + \text{pa1} \text{pb1}^2 \text{pb2}^2 \left(\gamma_2 + \frac{2 \alpha \epsilon}{\text{pa2}} \right) - \text{pa1} \text{pa2} \left(\text{pb1} \text{pb2}^2 \right. \right. \right. \right. \\
& \quad \left. \left. \left. \gamma_3 - 2 \text{pb2}^2 (1 + (-1 + \alpha) \epsilon) + \text{pb1}^2 (-2 + \text{pb2} \gamma_4 + 2 \epsilon - 2 \alpha \epsilon) \right) \right) \right)^2} \right. \\
& \quad \left. \left. \left. \left(\text{pa2}^2 \text{pb1}^4 \text{pb2}^4 + \text{pa1}^2 \left(\text{pb1}^4 \text{pb2}^4 + \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \right) \right] \right) \right. \\
& \left(\text{pa2}^2 \left(\text{pa2}^2 \text{pb1}^2 \text{pb2}^2 + \text{pa1}^2 \left(\text{pb1}^2 \text{pb2}^2 + \text{pa2}^2 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \right) \\
& \sqrt{\left(\frac{\gamma_1}{\text{pa1}} + \frac{\gamma_2}{\text{pa2}} - \frac{\gamma_3}{\text{pb1}} - \frac{\gamma_4}{\text{pb2}} + \frac{2 \alpha \epsilon}{\text{pa1}^2} + \frac{2 \alpha \epsilon}{\text{pa2}^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{\text{pb1}^2} + \frac{2 + 2 (-1 + \alpha) \epsilon}{\text{pb2}^2} \right)^2} + \\
& \left(2 \text{pa1}^2 \text{pa2}^2 \text{pb1}^2 \sqrt{\pi} \left(-\text{pa1} \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \gamma_1 - 2 \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 (-1 + \alpha) \epsilon + \right. \right. \\
& \quad \text{pa1}^2 \left(\text{pa2}^2 \text{pb1} \text{pb2}^2 \gamma_3 - 2 \text{pa2}^2 \text{pb2}^2 (-1 + \alpha) \epsilon + \right. \\
& \quad \quad \left. \left. \text{pb1}^2 \left(\text{pa2}^2 \text{pb2} \gamma_4 - 2 \text{pa2}^2 (-1 + \alpha) \epsilon - \text{pb2}^2 (-2 + \text{pa2} \gamma_2 + 2 \alpha \epsilon) \right) \right) \right) \left. \right)
\end{aligned}$$

$$\begin{aligned} & \operatorname{Erf}\left[\frac{1}{2pb_1pb_2}\left(\sqrt{\left((pa_1pa^2pb_1^2pb_2^2\gamma_1+2pa^2pb_1^2pb_2^2(-1+\alpha\epsilon)+\right.\right.}\\ & \quad pa_1^2(-pa^2pb_1pb_2^2\gamma_3+2pa^2pb_2^2(-1+\alpha)\epsilon+pb_1^2(-pa^2pb_2\gamma_4+\\ & \quad \left.2pa^2(-1+\alpha)\epsilon+pb_2^2(-2+pa_2\gamma_2+2\alpha\epsilon))\right))^2/\right. \\ & \quad \left.(pa_1^2pa^2pb_1^2pb_2^2+pa_1^4(pa^2pb_1^2pb_2^2+pa^4(pb_1^2+pb_2^2))))\right]\Bigg)/ \\ & \left((pa^2pb_1^2pb_2^2+pa_1^2(pb_1^2pb_2^2+pa^2(pb_1^2+pb_2^2)))\right) \\ & \quad \sqrt{(pa_1pa^2pb_1^2pb_2^2\gamma_1+2pa^2pb_1^2pb_2^2(-1+\alpha\epsilon)+} \\ & \quad pa_1^2(-pa^2pb_1pb_2^2\gamma_3+2pa^2pb_2^2(-1+\alpha)\epsilon+ \\ & \quad pb_1^2(-pa^2pb_2\gamma_4+2pa^2(-1+\alpha)\epsilon+pb_2^2(-2+pa_2\gamma_2+2\alpha\epsilon)))^2)+ \\ & \left(2pa_1^2pa^2pb_2^2\sqrt{\pi}(-pa_1pa^2pb_1^2pb_2^2\gamma_1-2pa^2pb_1^2pb_2^2(-1+\alpha\epsilon)+\right. \\ & \quad pa_1^2(pa^2pb_1pb_2^2\gamma_3-2pa^2pb_2^2(-1+\alpha)\epsilon+ \\ & \quad pb_1^2(pa^2pb_2\gamma_4-2pa^2(-1+\alpha)\epsilon-pb_2^2(-2+pa_2\gamma_2+2\alpha\epsilon)))) \\ & \operatorname{Erf}\left[\frac{1}{2pb_1pb_2}\left(\sqrt{\left((pa_1pa^2pb_1^2pb_2^2\gamma_1+2pa^2pb_1^2pb_2^2(-1+\alpha\epsilon)+\right.\right.}\\ & \quad pa_1^2(-pa^2pb_1pb_2^2\gamma_3+2pa^2pb_2^2(-1+\alpha)\epsilon+pb_1^2(-pa^2pb_2\gamma_4+ \\ & \quad \left.2pa^2(-1+\alpha)\epsilon+pb_2^2(-2+pa_2\gamma_2+2\alpha\epsilon))\right))^2/\right. \\ & \quad \left.(pa_1^2pa^2pb_1^2pb_2^2+pa_1^4(pa^2pb_1^2pb_2^2+pa^4(pb_1^2+pb_2^2))))\right]\Bigg)/ \\ & \left((pa^2pb_1^2pb_2^2+pa_1^2(pb_1^2pb_2^2+pa^2(pb_1^2+pb_2^2)))\right) \\ & \quad \sqrt{(pa_1pa^2pb_1^2pb_2^2\gamma_1+2pa^2pb_1^2pb_2^2(-1+\alpha\epsilon)+} \\ & \quad pa_1^2(-pa^2pb_1pb_2^2\gamma_3+2pa^2pb_2^2(-1+\alpha)\epsilon+ \\ & \quad pb_1^2(-pa^2pb_2\gamma_4+2pa^2(-1+\alpha)\epsilon+pb_2^2(-2+pa_2\gamma_2+2\alpha\epsilon)))^2)+ \\ & \left(pa_1pa^2pb_1^2pb_2^2\sqrt{\pi}\gamma_1(-pa_1pa^2pb_1^2pb_2^2\gamma_1-2pa^2pb_1^2pb_2^2\right. \\ & \quad (-1+\alpha\epsilon)+pa_1^2(pa^2pb_1pb_2^2\gamma_3-2pa^2pb_2^2(-1+\alpha)\epsilon+ \\ & \quad pb_1^2(pa^2pb_2\gamma_4-2pa^2(-1+\alpha)\epsilon-pb_2^2(-2+pa_2\gamma_2+2\alpha\epsilon)))) \\ & \operatorname{Erf}\left[\frac{1}{2pb_1pb_2}\left(\sqrt{\left((pa_1pa^2pb_1^2pb_2^2\gamma_1+2pa^2pb_1^2pb_2^2(-1+\alpha\epsilon)+\right.\right.}\\ & \quad pa_1^2(-pa^2pb_1pb_2^2\gamma_3+2pa^2pb_2^2(-1+\alpha)\epsilon+pb_1^2(-pa^2pb_2\gamma_4+ \\ & \quad \left.2pa^2(-1+\alpha)\epsilon+pb_2^2(-2+pa_2\gamma_2+2\alpha\epsilon))\right))^2/\right. \\ & \quad \left.(pa_1^2pa^2pb_1^2pb_2^2+pa_1^4(pa^2pb_1^2pb_2^2+pa^4(pb_1^2+pb_2^2))))\right]\Bigg)/ \\ & \left((pa^2pb_1^2pb_2^2+pa_1^2(pb_1^2pb_2^2+pa^2(pb_1^2+pb_2^2)))\right) \\ & \quad \sqrt{(pa_1pa^2pb_1^2pb_2^2\gamma_1+2pa^2pb_1^2pb_2^2(-1+\alpha\epsilon)+} \\ & \quad pa_1^2(-pa^2pb_1pb_2^2\gamma_3+2pa^2pb_2^2(-1+\alpha)\epsilon+ \\ & \quad pb_1^2(-pa^2pb_2\gamma_4+2pa^2(-1+\alpha)\epsilon+pb_2^2(-2+pa_2\gamma_2+2\alpha\epsilon)))^2)+ \end{aligned}$$

$$\begin{aligned}
& \left(pa1^2 pa2 pb1^2 pb2^2 \sqrt{\pi} \gamma_2 (-pa1 pa2^2 pb1^2 pb2^2 \gamma_1 - 2 pa2^2 pb1^2 pb2^2 \right. \\
& \quad (-1 + \alpha \epsilon) + pa1^2 (pa2^2 pb1 pb2^2 \gamma_3 - 2 pa2^2 pb2^2 (-1 + \alpha) \epsilon + \\
& \quad \left. pb1^2 (pa2^2 pb2 \gamma_4 - 2 pa2^2 (-1 + \alpha) \epsilon - pb2^2 (-2 + pa2 \gamma_2 + 2 \alpha \epsilon)) \right) \Big) \\
& \operatorname{Erf} \left[\frac{1}{2 pb1 pb2} \left(\sqrt{\left((pa1 pa2^2 pb1^2 pb2^2 \gamma_1 + 2 pa2^2 pb1^2 pb2^2 (-1 + \alpha) \epsilon + \right. \right.} \right. \\
& \quad \left. \left. pa1^2 (-pa2^2 pb1 pb2^2 \gamma_3 + 2 pa2^2 pb2^2 (-1 + \alpha) \epsilon + pb1^2 (-pa2^2 pb2 \gamma_4 + \right. \right. \\
& \quad \left. \left. 2 pa2^2 (-1 + \alpha) \epsilon + pb2^2 (-2 + pa2 \gamma_2 + 2 \alpha \epsilon)) \right)^2 / \right. \\
& \quad \left. (pa1^2 pa2^4 pb1^2 pb2^2 + pa1^4 (pa2^2 pb1^2 pb2^2 + pa2^4 (pb1^2 + pb2^2))) \right) \Big] \Big) / \\
& \left((pa2^2 pb1^2 pb2^2 + pa1^2 (pb1^2 pb2^2 + pa2^2 (pb1^2 + pb2^2))) \right) \\
& \sqrt{\left(pa1 pa2^2 pb1^2 pb2^2 \gamma_1 + 2 pa2^2 pb1^2 pb2^2 (-1 + \alpha) \epsilon + \right.} \\
& \quad \left. pa1^2 (-pa2^2 pb1 pb2^2 \gamma_3 + 2 pa2^2 pb2^2 (-1 + \alpha) \epsilon + \right. \\
& \quad \left. pb1^2 (-pa2^2 pb2 \gamma_4 + 2 pa2^2 (-1 + \alpha) \epsilon + pb2^2 (-2 + pa2 \gamma_2 + 2 \alpha \epsilon)) \right)^2 \Big) + \\
& \left(pa1^2 pa2^2 pb1 pb2^2 \sqrt{\pi} \gamma_3 (pa1 pa2^2 pb1^2 pb2^2 \gamma_1 + 2 pa2^2 pb1^2 pb2^2 (-1 + \alpha) \epsilon + \right. \\
& \quad \left. pa1^2 (-pa2^2 pb1 pb2^2 \gamma_3 + 2 pa2^2 pb2^2 (-1 + \alpha) \epsilon + \right. \\
& \quad \left. pb1^2 (-pa2^2 pb2 \gamma_4 + 2 pa2^2 (-1 + \alpha) \epsilon + pb2^2 (-2 + pa2 \gamma_2 + 2 \alpha \epsilon)) \right) \Big) \\
& \operatorname{Erf} \left[\frac{1}{2 pb1 pb2} \left(\sqrt{\left((pa1 pa2^2 pb1^2 pb2^2 \gamma_1 + 2 pa2^2 pb1^2 pb2^2 (-1 + \alpha) \epsilon + \right. \right.} \right. \\
& \quad \left. \left. pa1^2 (-pa2^2 pb1 pb2^2 \gamma_3 + 2 pa2^2 pb2^2 (-1 + \alpha) \epsilon + pb1^2 (-pa2^2 pb2 \gamma_4 + \right. \right. \\
& \quad \left. \left. 2 pa2^2 (-1 + \alpha) \epsilon + pb2^2 (-2 + pa2 \gamma_2 + 2 \alpha \epsilon)) \right)^2 / \right. \\
& \quad \left. (pa1^2 pa2^4 pb1^2 pb2^2 + pa1^4 (pa2^2 pb1^2 pb2^2 + pa2^4 (pb1^2 + pb2^2))) \right) \Big] \Big) / \\
& \left((pa2^2 pb1^2 pb2^2 + pa1^2 (pb1^2 pb2^2 + pa2^2 (pb1^2 + pb2^2))) \right) \\
& \sqrt{\left(pa1 pa2^2 pb1^2 pb2^2 \gamma_1 + 2 pa2^2 pb1^2 pb2^2 (-1 + \alpha) \epsilon + \right.} \\
& \quad \left. pa1^2 (-pa2^2 pb1 pb2^2 \gamma_3 + 2 pa2^2 pb2^2 (-1 + \alpha) \epsilon + \right. \\
& \quad \left. pb1^2 (-pa2^2 pb2 \gamma_4 + 2 pa2^2 (-1 + \alpha) \epsilon + pb2^2 (-2 + pa2 \gamma_2 + 2 \alpha \epsilon)) \right)^2 \Big) + \\
& \left(pa1^2 pa2^2 pb1^2 pb2 \sqrt{\pi} \gamma_4 (pa1 pa2^2 pb1^2 pb2^2 \gamma_1 + 2 pa2^2 pb1^2 pb2^2 (-1 + \alpha) \epsilon + \right. \\
& \quad \left. pa1^2 (-pa2^2 pb1 pb2^2 \gamma_3 + 2 pa2^2 pb2^2 (-1 + \alpha) \epsilon + \right. \\
& \quad \left. pb1^2 (-pa2^2 pb2 \gamma_4 + 2 pa2^2 (-1 + \alpha) \epsilon + pb2^2 (-2 + pa2 \gamma_2 + 2 \alpha \epsilon)) \right) \Big) \\
& \operatorname{Erf} \left[\frac{1}{2 pb1 pb2} \left(\sqrt{\left((pa1 pa2^2 pb1^2 pb2^2 \gamma_1 + 2 pa2^2 pb1^2 pb2^2 (-1 + \alpha) \epsilon + \right. \right.} \right. \\
& \quad \left. \left. pa1^2 (-pa2^2 pb1 pb2^2 \gamma_3 + 2 pa2^2 pb2^2 (-1 + \alpha) \epsilon + pb1^2 (-pa2^2 pb2 \gamma_4 + \right. \right. \\
& \quad \left. \left. 2 pa2^2 (-1 + \alpha) \epsilon + pb2^2 (-2 + pa2 \gamma_2 + 2 \alpha \epsilon)) \right)^2 / \right. \\
& \quad \left. (pa1^2 pa2^4 pb1^2 pb2^2 + pa1^4 (pa2^2 pb1^2 pb2^2 + pa2^4 (pb1^2 + pb2^2))) \right) \Big] \Big) / \\
& \left((pa2^2 pb1^2 pb2^2 + pa1^2 (pb1^2 pb2^2 + pa2^2 (pb1^2 + pb2^2))) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(pa_1 pa_2^2 pb_1^2 pb_2^2 \gamma_1 + 2 pa_2^2 pb_1^2 pb_2^2 (-1 + \alpha \epsilon) + \right. \\
& \quad pa_1^2 (-pa_2^2 pb_1 pb_2^2 \gamma_3 + 2 pa_2^2 pb_2^2 (-1 + \alpha) \epsilon + \\
& \quad \left. pb_1^2 (-pa_2^2 pb_2 \gamma_4 + 2 pa_2^2 (-1 + \alpha) \epsilon + pb_2^2 (-2 + pa_2 \gamma_2 + 2 \alpha \epsilon)) \right)^2} - \\
& \left(2 pa_1^2 pa_2^2 pb_1^2 \sqrt{\pi} (-1 + \alpha) \epsilon (pa_1 pa_2^2 pb_1^2 pb_2^2 \gamma_1 + 2 pa_2^2 pb_1^2 pb_2^2 \right. \\
& \quad (-1 + \alpha \epsilon) + pa_1^2 (-pa_2^2 pb_1 pb_2^2 \gamma_3 + 2 pa_2^2 pb_2^2 (-1 + \alpha) \epsilon + \\
& \quad \left. pb_1^2 (-pa_2^2 pb_2 \gamma_4 + 2 pa_2^2 (-1 + \alpha) \epsilon + pb_2^2 (-2 + pa_2 \gamma_2 + 2 \alpha \epsilon)) \right) \Big) \\
& \operatorname{Erf}\left[\frac{1}{2 pb_1 pb_2} \left(\sqrt{\left((pa_1 pa_2^2 pb_1^2 pb_2^2 \gamma_1 + 2 pa_2^2 pb_1^2 pb_2^2 (-1 + \alpha \epsilon) + \right. \right. \right. \\
& \quad pa_1^2 (-pa_2^2 pb_1 pb_2^2 \gamma_3 + 2 pa_2^2 pb_2^2 (-1 + \alpha) \epsilon + pb_1^2 (-pa_2^2 pb_2 \gamma_4 + \\
& \quad \left. \left. 2 pa_2^2 (-1 + \alpha) \epsilon + pb_2^2 (-2 + pa_2 \gamma_2 + 2 \alpha \epsilon)) \right)^2} / \right. \\
& \quad \left. (pa_1^2 pa_2^4 pb_1^2 pb_2^2 + pa_1^4 (pa_2^2 pb_1^2 pb_2^2 + pa_2^4 (pb_1^2 + pb_2^2))) \right) \Big] \Bigg/ \\
& \left((pa_2^2 pb_1^2 pb_2^2 + pa_1^2 (pb_1^2 pb_2^2 + pa_2^2 (pb_1^2 + pb_2^2))) \right) \\
& \sqrt{\left(pa_1 pa_2^2 pb_1^2 pb_2^2 \gamma_1 + 2 pa_2^2 pb_1^2 pb_2^2 (-1 + \alpha \epsilon) + \right. \\
& \quad pa_1^2 (-pa_2^2 pb_1 pb_2^2 \gamma_3 + 2 pa_2^2 pb_2^2 (-1 + \alpha) \epsilon + \\
& \quad \left. pb_1^2 (-pa_2^2 pb_2 \gamma_4 + 2 pa_2^2 (-1 + \alpha) \epsilon + pb_2^2 (-2 + pa_2 \gamma_2 + 2 \alpha \epsilon)) \right)^2} - \\
& \left(2 pa_1^2 pa_2^2 pb_2^2 \sqrt{\pi} (-1 + \alpha) \epsilon (pa_1 pa_2^2 pb_1^2 pb_2^2 \gamma_1 + 2 pa_2^2 pb_1^2 pb_2^2 \right. \\
& \quad (-1 + \alpha \epsilon) + pa_1^2 (-pa_2^2 pb_1 pb_2^2 \gamma_3 + 2 pa_2^2 pb_2^2 (-1 + \alpha) \epsilon + \\
& \quad \left. pb_1^2 (-pa_2^2 pb_2 \gamma_4 + 2 pa_2^2 (-1 + \alpha) \epsilon + pb_2^2 (-2 + pa_2 \gamma_2 + 2 \alpha \epsilon)) \right) \Big) \\
& \operatorname{Erf}\left[\frac{1}{2 pb_1 pb_2} \left(\sqrt{\left((pa_1 pa_2^2 pb_1^2 pb_2^2 \gamma_1 + 2 pa_2^2 pb_1^2 pb_2^2 (-1 + \alpha \epsilon) + \right. \right. \right. \\
& \quad pa_1^2 (-pa_2^2 pb_1 pb_2^2 \gamma_3 + 2 pa_2^2 pb_2^2 (-1 + \alpha) \epsilon + pb_1^2 (-pa_2^2 pb_2 \gamma_4 + \\
& \quad \left. \left. 2 pa_2^2 (-1 + \alpha) \epsilon + pb_2^2 (-2 + pa_2 \gamma_2 + 2 \alpha \epsilon)) \right)^2} / \right. \\
& \quad \left. (pa_1^2 pa_2^4 pb_1^2 pb_2^2 + pa_1^4 (pa_2^2 pb_1^2 pb_2^2 + pa_2^4 (pb_1^2 + pb_2^2))) \right) \Big] \Bigg/ \\
& \left((pa_2^2 pb_1^2 pb_2^2 + pa_1^2 (pb_1^2 pb_2^2 + pa_2^2 (pb_1^2 + pb_2^2))) \right) \\
& \sqrt{\left(pa_1 pa_2^2 pb_1^2 pb_2^2 \gamma_1 + 2 pa_2^2 pb_1^2 pb_2^2 (-1 + \alpha \epsilon) + \right. \\
& \quad pa_1^2 (-pa_2^2 pb_1 pb_2^2 \gamma_3 + 2 pa_2^2 pb_2^2 (-1 + \alpha) \epsilon + \\
& \quad \left. pb_1^2 (-pa_2^2 pb_2 \gamma_4 + 2 pa_2^2 (-1 + \alpha) \epsilon + pb_2^2 (-2 + pa_2 \gamma_2 + 2 \alpha \epsilon)) \right)^2} - \\
& \left(2 pa_1^2 pb_1^2 pb_2^2 \sqrt{\pi} \alpha \epsilon (pa_1 pa_2^2 pb_1^2 pb_2^2 \gamma_1 + 2 pa_2^2 pb_1^2 pb_2^2 (-1 + \alpha \epsilon) + \right. \\
& \quad pa_1^2 (-pa_2^2 pb_1 pb_2^2 \gamma_3 + 2 pa_2^2 pb_2^2 (-1 + \alpha) \epsilon + \\
& \quad \left. pb_1^2 (-pa_2^2 pb_2 \gamma_4 + 2 pa_2^2 (-1 + \alpha) \epsilon + pb_2^2 (-2 + pa_2 \gamma_2 + 2 \alpha \epsilon)) \right) \Big) \\
& \operatorname{Erf}\left[\frac{1}{2 pb_1 pb_2} \left(\sqrt{\left((pa_1 pa_2^2 pb_1^2 pb_2^2 \gamma_1 + 2 pa_2^2 pb_1^2 pb_2^2 (-1 + \alpha \epsilon) + \right. \right. \right. \\
& \quad pa_1^2 (-pa_2^2 pb_1 pb_2^2 \gamma_3 + 2 pa_2^2 pb_2^2 (-1 + \alpha) \epsilon + pb_1^2 (-pa_2^2 pb_2 \gamma_4 + \\
& \quad \left. \left. 2 pa_2^2 (-1 + \alpha) \epsilon + pb_2^2 (-2 + pa_2 \gamma_2 + 2 \alpha \epsilon)) \right)^2} / \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\text{pa1}^2 \text{pa2}^4 \text{pb1}^2 \text{pb2}^2 + \text{pa1}^4 \left(\text{pa2}^2 \text{pb1}^2 \text{pb2}^2 + \text{pa2}^4 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \right) \Bigg) \Bigg/ \\
& \left(\left(\text{pa2}^2 \text{pb1}^2 \text{pb2}^2 + \text{pa1}^2 \left(\text{pb1}^2 \text{pb2}^2 + \text{pa2}^2 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \right. \\
& \quad \sqrt{\left(\text{pa1} \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \gamma_1 + 2 \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 (-1 + \alpha \epsilon) + \right.} \\
& \quad \quad \text{pa1}^2 \left(-\text{pa2}^2 \text{pb1} \text{pb2}^2 \gamma_3 + 2 \text{pa2}^2 \text{pb2}^2 (-1 + \alpha) \epsilon + \right. \\
& \quad \quad \quad \left. \left. \text{pb1}^2 \left(-\text{pa2}^2 \text{pb2} \gamma_4 + 2 \text{pa2}^2 (-1 + \alpha) \epsilon + \text{pb2}^2 (-2 + \text{pa2} \gamma_2 + 2 \alpha \epsilon) \right) \right) \right)^2 \Bigg) - \\
& \left(2 \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \sqrt{\pi} \alpha \epsilon \left(\text{pa1} \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \gamma_1 + 2 \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 (-1 + \alpha \epsilon) + \right. \right. \\
& \quad \left. \left. \text{pa1}^2 \left(-\text{pa2}^2 \text{pb1} \text{pb2}^2 \gamma_3 + 2 \text{pa2}^2 \text{pb2}^2 (-1 + \alpha) \epsilon + \right. \right. \right. \\
& \quad \quad \left. \left. \left. \text{pb1}^2 \left(-\text{pa2}^2 \text{pb2} \gamma_4 + 2 \text{pa2}^2 (-1 + \alpha) \epsilon + \text{pb2}^2 (-2 + \text{pa2} \gamma_2 + 2 \alpha \epsilon) \right) \right) \right) \right) \Bigg) \\
& \text{Erf} \left[\frac{1}{2 \text{pb1} \text{pb2}} \left(\sqrt{\left(\left(\text{pa1} \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \gamma_1 + 2 \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 (-1 + \alpha \epsilon) + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \text{pa1}^2 \left(-\text{pa2}^2 \text{pb1} \text{pb2}^2 \gamma_3 + 2 \text{pa2}^2 \text{pb2}^2 (-1 + \alpha) \epsilon + \text{pb1}^2 \left(-\text{pa2}^2 \text{pb2} \gamma_4 + \right. \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. 2 \text{pa2}^2 (-1 + \alpha) \epsilon + \text{pb2}^2 (-2 + \text{pa2} \gamma_2 + 2 \alpha \epsilon) \right) \right) \right) \right)^2 \Bigg/ \\
& \quad \left(\text{pa1}^2 \text{pa2}^4 \text{pb1}^2 \text{pb2}^2 + \text{pa1}^4 \left(\text{pa2}^2 \text{pb1}^2 \text{pb2}^2 + \text{pa2}^4 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \Bigg) \Bigg] \Bigg/ \\
& \left(\left(\text{pa2}^2 \text{pb1}^2 \text{pb2}^2 + \text{pa1}^2 \left(\text{pb1}^2 \text{pb2}^2 + \text{pa2}^2 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \right. \\
& \quad \sqrt{\left(\text{pa1} \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \gamma_1 + 2 \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 (-1 + \alpha \epsilon) + \right.} \\
& \quad \quad \left. \left. \text{pa1}^2 \left(-\text{pa2}^2 \text{pb1} \text{pb2}^2 \gamma_3 + 2 \text{pa2}^2 \text{pb2}^2 (-1 + \alpha) \epsilon + \text{pb1}^2 \right. \right. \right. \\
& \quad \quad \quad \left. \left. \left. \left(-\text{pa2}^2 \text{pb2} \gamma_4 + 2 \text{pa2}^2 (-1 + \alpha) \epsilon + \text{pb2}^2 (-2 + \text{pa2} \gamma_2 + 2 \alpha \epsilon) \right) \right) \right) \right)^2 \Bigg) + \right. \\
& \quad \left. \left(-\text{pa2}^2 \text{pb2} \gamma_4 + 2 \text{pa2}^2 (-1 + \alpha) \epsilon + \text{pb2}^2 (-2 + \text{pa2} \gamma_2 + 2 \alpha \epsilon) \right) \right)^2 \Bigg) \Bigg] \Bigg/ \\
& \left(\sqrt{\pi} \alpha \epsilon \left(\left(\left(\text{pa1} \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \gamma_1 + 2 \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \alpha \epsilon + \text{pa1}^2 \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(\text{pa2} \text{pb1}^2 \text{pb2}^2 \gamma_2 + 2 \text{pb1}^2 \text{pb2}^2 \alpha \epsilon - \text{pa2}^2 \left(\text{pb1} \text{pb2}^2 \gamma_3 - \right. \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. 2 \text{pb2}^2 (1 + (-1 + \alpha) \epsilon) + \text{pb1}^2 (-2 + \text{pb2} \gamma_4 + 2 \epsilon - 2 \alpha \epsilon) \right) \right) \right) \right) \right. \\
& \quad \text{Erf} \left[\frac{1}{2} \sqrt{\left(\left(\text{pa2} \text{pb1}^2 \text{pb2}^2 \left(\gamma_1 + \frac{2 \alpha \epsilon}{\text{pa1}} \right) + \text{pa1} \text{pb1}^2 \text{pb2}^2 \left(\gamma_2 + \frac{2 \alpha \epsilon}{\text{pa2}} \right) - \right. \right. \right. \\
& \quad \left. \left. \left. \text{pa1} \text{pa2} \left(\text{pb1} \text{pb2}^2 \gamma_3 - 2 \text{pb2}^2 (1 + (-1 + \alpha) \epsilon) + \right. \right. \right. \right. \\
& \quad \quad \left. \left. \left. \text{pb1}^2 (-2 + \text{pb2} \gamma_4 + 2 \epsilon - 2 \alpha \epsilon) \right) \right) \right)^2 \Bigg/ \left(\text{pa2}^2 \text{pb1}^4 \text{pb2}^4 + \right. \\
& \quad \left. \left. \text{pa1}^2 \left(\text{pb1}^4 \text{pb2}^4 + \text{pa2}^2 \text{pb1}^2 \text{pb2}^2 \left(\text{pb1}^2 + \text{pb2}^2 \right) \right) \right) \right) \Bigg] \Bigg/
\end{aligned}$$

$$\text{Erf} \left[\left(-2 \text{pa2}^2 \text{pb1} \text{pb2} + \text{pa2}^2 \text{pb1} \text{pb2} (\text{pa1} \gamma_1 + 2 \alpha \epsilon) - \right. \right. \\
\left. \left. \text{pa1}^2 \left(2 \text{pb1} \text{pb2} - \text{pb1} \text{pb2} (\text{pa2} \gamma_2 + 2 \alpha \epsilon) + \right. \right. \right. \\
\left. \left. \left. \text{pa2}^2 \left(\text{pb1} \gamma_4 - \frac{2 \text{pb1} (-1 + \alpha) \epsilon}{\text{pb2}} + \text{pb2} \left(\gamma_3 - \frac{2 (-1 + \alpha) \epsilon}{\text{pb1}} \right) \right) \right) \right) \right] / \\
\left(2 \text{pa1} \text{pa2} \sqrt{\text{pa2}^2 \text{pb1}^2 \text{pb2}^2 + \text{pa1}^2 (\text{pb1}^2 \text{pb2}^2 + \text{pa2}^2 (\text{pb1}^2 + \text{pb2}^2))} \right) \Bigg] \Bigg]$$

- For this simple example we fix a deterministic price sequence for all the sample, in real samples we can have random prices but this keeps things simple.

```
In[57]:= ha1attrueconsumptionpknot =  
Simplify[ha1attrueconsumption /. {pa1 → 1, pb1 → 2, pa2 → 2, pb2 → 1}]
```

Out[57]=

$$\begin{aligned}
& \left(8 \sqrt{10} \left(1 - e^{-\frac{1}{40} (-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) + \right. \\
& 8 \sqrt{10} \left(-1 + e^{-\frac{1}{40} (2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) - \left(8 \sqrt{\pi} \gamma_1 (-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon) \right. \\
& \left. \left. \operatorname{Erf} \left[\frac{1}{2\sqrt{10}} \left(\sqrt{(-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) \right] \right) \right) / \\
& \left(\sqrt{(-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) - \left(4 \sqrt{\pi} \gamma_2 (-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon) \right. \\
& \left. \left. \operatorname{Erf} \left[\frac{1}{2\sqrt{10}} \left(\sqrt{(-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) \right] \right) \right) / \\
& \left(\sqrt{(-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) - \left(4 \sqrt{\pi} \gamma_3 (-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon) \right. \\
& \left. \left. \operatorname{Erf} \left[\frac{1}{2\sqrt{10}} \left(\sqrt{(-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) \right] \right) \right) / \\
& \left(\sqrt{(-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) - \left(8 \sqrt{\pi} \gamma_4 (-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon) \right. \\
& \left. \left. \operatorname{Erf} \left[\frac{1}{2\sqrt{10}} \left(\sqrt{(-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) \right] \right) \right) / \\
& \left(\sqrt{(-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) + \\
& \left(8 \sqrt{\pi} \gamma_1 (2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon) \operatorname{Erf} \left[\frac{\sqrt{(2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2}}{2\sqrt{10}} \right] \right) / \\
& \left(\sqrt{(2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) + \\
& \left(4 \sqrt{\pi} \gamma_2 (2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon) \operatorname{Erf} \left[\frac{\sqrt{(2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2}}{2\sqrt{10}} \right] \right) / \\
& \left(\sqrt{(2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) + \\
& \left(4 \sqrt{\pi} \gamma_3 (2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon) \operatorname{Erf} \left[\frac{\sqrt{(2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2}}{2\sqrt{10}} \right] \right) / \\
& \left(\sqrt{(2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) + \\
& \left(8 \sqrt{\pi} \gamma_4 (2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon) \operatorname{Erf} \left[\frac{\sqrt{(2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2}}{2\sqrt{10}} \right] \right) / \\
& \left(\sqrt{(2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon)^2} \right) \Bigg) / \\
& \left(40 \sqrt{\pi} \left(-\operatorname{Erf} \left[\frac{-10+2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon}{2\sqrt{10}} \right] + \operatorname{Erf} \left[\frac{2\gamma_1+\gamma_2+\gamma_3+2\gamma_4+10\alpha\epsilon}{2\sqrt{10}} \right] \right) \right)
\end{aligned}$$

■ This is a function that obtains $E[h(\gamma)]$ which then you can use for the optimization stage.


```
In[89]:= Expectationhalgamma[γval1_, γval2_, γval3_, γval4_] := NIntegrate[
  (halattrueconsumptionpknot /. {γ1 → γval1, γ2 → γval2, γ3 → γval3, γ4 → γval4}),
  {α, 0, 1}, {ε, 0.9, 1.1}]
```

- We illustrate the use of the function evaluating it at some $\gamma=\{1,1,1,0.1\}$

```
In[90]:= Expectationhalgamma[1, 1, 1, 0.1]
Out[90]= 0.0264676
```

- Minimize the $(E[h(\gamma)])^2$ with respect to γ . There are some regions of the Euclidean space where the moment is not defined this may be an issue of the prior, but overall it does not affect our conclusion here. But it's worth checking.

```
In[91]:= NMinimize[(Expectationhalgamma[γ1, γ2, γ3, γ4])^2, {γ1, γ2, γ3, γ4}]
```

NIntegrate: The integrand
$$\frac{8 \sqrt{10} \left(1 - e^{-\frac{1}{40} \text{Power}[\llbracket 2 \rrbracket]}\right) + \llbracket 12 \rrbracket + \frac{8 \sqrt{\pi} \gamma_4 (2 \gamma_1 + \gamma_2 + \gamma_3 + 2 \gamma_4 + 10 \alpha \epsilon) \text{Erf}\left[\frac{\sqrt{\text{Plus}[\llbracket 5 \rrbracket]^2}}{2 \sqrt{10}}\right]}{\sqrt{(2 \gamma_1 + \gamma_2 + \gamma_3 + 2 \gamma_4 + 10 \alpha \epsilon)^2}}}{40 \sqrt{\pi} \left(-\text{Erf}\left[\frac{-10 + \text{Times}[\llbracket 2 \rrbracket] + \gamma_2 + \gamma_3 + \text{Times}[\llbracket 2 \rrbracket] + \text{Times}[\llbracket 3 \rrbracket]}{2 \sqrt{10}}\right]\right) + \text{Erf}\left[\frac{2 \gamma_1 + \gamma_2 + \gamma_3 + 2 \gamma_4 + 10 \alpha \epsilon}{2 \sqrt{10}}\right]}}$$
 has

evaluated to non-numerical values for all sampling points in the region with boundaries $\{\{0, 1\}, \{0.9, 1.1\}\}$.

NIntegrate: The integrand
$$\frac{8 \sqrt{10} \left(1 - e^{-\frac{1}{40} \text{Power}[\llbracket 2 \rrbracket]}\right) + \llbracket 12 \rrbracket + \frac{8 \sqrt{\pi} \gamma_4 (2 \gamma_1 + \gamma_2 + \gamma_3 + 2 \gamma_4 + 10 \alpha \epsilon) \text{Erf}\left[\frac{\sqrt{\text{Plus}[\llbracket 5 \rrbracket]^2}}{2 \sqrt{10}}\right]}{\sqrt{(2 \gamma_1 + \gamma_2 + \gamma_3 + 2 \gamma_4 + 10 \alpha \epsilon)^2}}}{40 \sqrt{\pi} \left(-\text{Erf}\left[\frac{-10 + \text{Times}[\llbracket 2 \rrbracket] + \gamma_2 + \gamma_3 + \text{Times}[\llbracket 2 \rrbracket] + \text{Times}[\llbracket 3 \rrbracket]}{2 \sqrt{10}}\right]\right) + \text{Erf}\left[\frac{2 \gamma_1 + \gamma_2 + \gamma_3 + 2 \gamma_4 + 10 \alpha \epsilon}{2 \sqrt{10}}\right]}}$$
 has

evaluated to non-numerical values for all sampling points in the region with boundaries $\{\{0, 1\}, \{0.9, 1.1\}\}$.

NIntegrate: The integrand
$$\frac{8 \sqrt{10} \left(1 - e^{-\frac{1}{40} \text{Power}[\llbracket 2 \rrbracket]}\right) + \llbracket 12 \rrbracket + \frac{8 \sqrt{\pi} \gamma_4 (2 \gamma_1 + \gamma_2 + \gamma_3 + 2 \gamma_4 + 10 \alpha \epsilon) \text{Erf}\left[\frac{\sqrt{\text{Plus}[\llbracket 5 \rrbracket]^2}}{2 \sqrt{10}}\right]}{\sqrt{(2 \gamma_1 + \gamma_2 + \gamma_3 + 2 \gamma_4 + 10 \alpha \epsilon)^2}}}{40 \sqrt{\pi} \left(-\text{Erf}\left[\frac{-10 + \text{Times}[\llbracket 2 \rrbracket] + \gamma_2 + \gamma_3 + \text{Times}[\llbracket 2 \rrbracket] + \text{Times}[\llbracket 3 \rrbracket]}{2 \sqrt{10}}\right]\right) + \text{Erf}\left[\frac{2 \gamma_1 + \gamma_2 + \gamma_3 + 2 \gamma_4 + 10 \alpha \epsilon}{2 \sqrt{10}}\right]}}$$
 has

evaluated to non-numerical values for all sampling points in the region with boundaries $\{\{0, 1\}, \{0.9, 1.1\}\}$.

General: Further output of NIntegrate::numr will be suppressed during this calculation.

```
Out[91]= {3.88207 × 10-10, {γ1 → 0.0092729, γ2 → -0.0769732, γ3 → 0.802497, γ4 → -0.369445}}
```

- The output above shows the minimized value that is numerically zero, this is what we wanted because we wrote down the observed data such that the moments are satisfied.