IB Paper 5: Analysis of circuits

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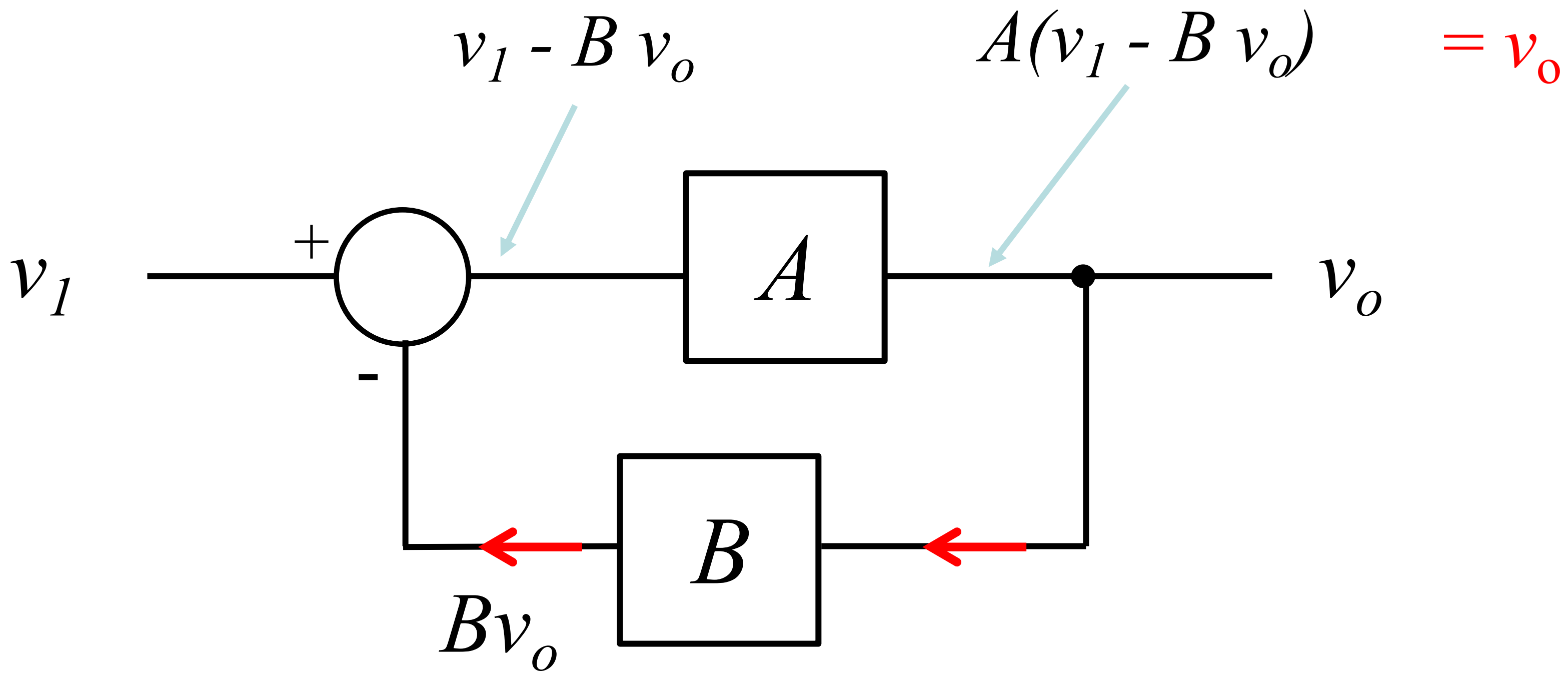
5 *Negative feedback*

5.1 We can treat negative feedback generally:



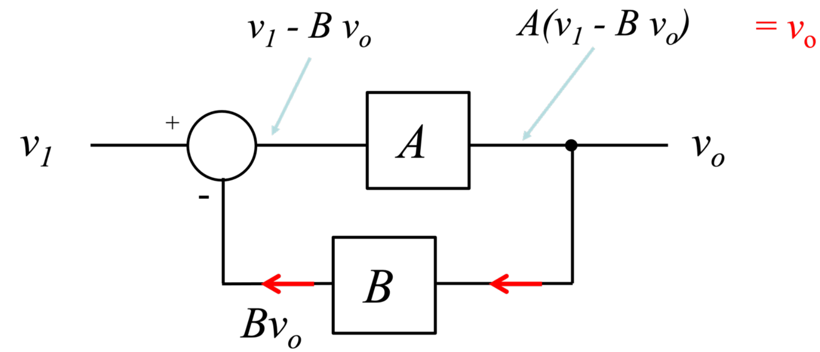
A is the forward gain and B is the fraction of the output fed-back.

Reasons for using negative feedback:

(i) Stabilisation of gain - the gain depends only on resistor values
(ii) Increased input resistance
(iii) Reduced output resistance
(iv) Increased bandwidth

In simple circuits the feedback network may comprise only two resistors, but more complex networks are often used, employing resistors, capacitors, occasionally inductors and in special cases active devices.

Analysis



$$v_o = A(v_1 - Bv_o)$$

$$\Rightarrow v_1(1 + AB) = Av_1$$

$$Gain = \frac{v_0}{v_1} = \frac{A}{(1 + AB)}$$

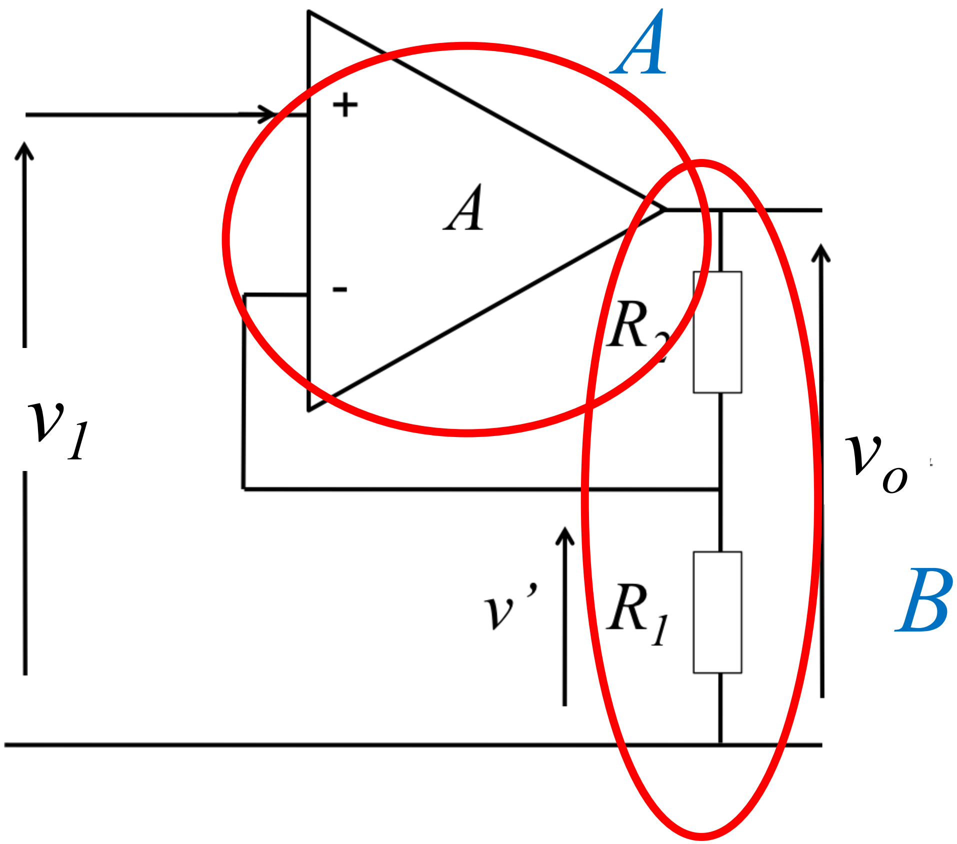
For A large, as is often the case with OpAmps

$$Gain = \frac{v_0}{v_1} \sim \frac{1}{B}$$

In other words, the gain is determined by the feedback network and not the loosely specified parameters of the amplifier itself

The term AB is often called the *loop gain*.

Example: Op-Amp non-inverting amplifier (from IA)



Ideal OpAmp:

Input current = 0

=> R_1 & R_2 have the same current – potential divider

Open-loop gain, A is infinite

$v_1 = v'$

Potential divider:

$$v' = v_o \frac{R_1}{R_1 + R_2}$$

$$B = \frac{R_1}{R_1 + R_2} \Rightarrow Gain = \frac{v_0}{v_1} \sim \frac{1}{B} = 1 + \frac{R_2}{R_1}$$

5.2 Stability of Gain

Without feedback, a change δA in gain results in a fractional change of gain $\delta A/A$. With feedback, the gain $G = \frac{A}{(1+AB)}$

Therefore $$\frac{dG}{dA} = \frac{(1+AB) - AB}{(1+AB)^2} = \frac{1}{(1+AB)^2}$$

The small change δG arising from δA is given by $$\delta G = \frac{dG}{dA}\delta A$$

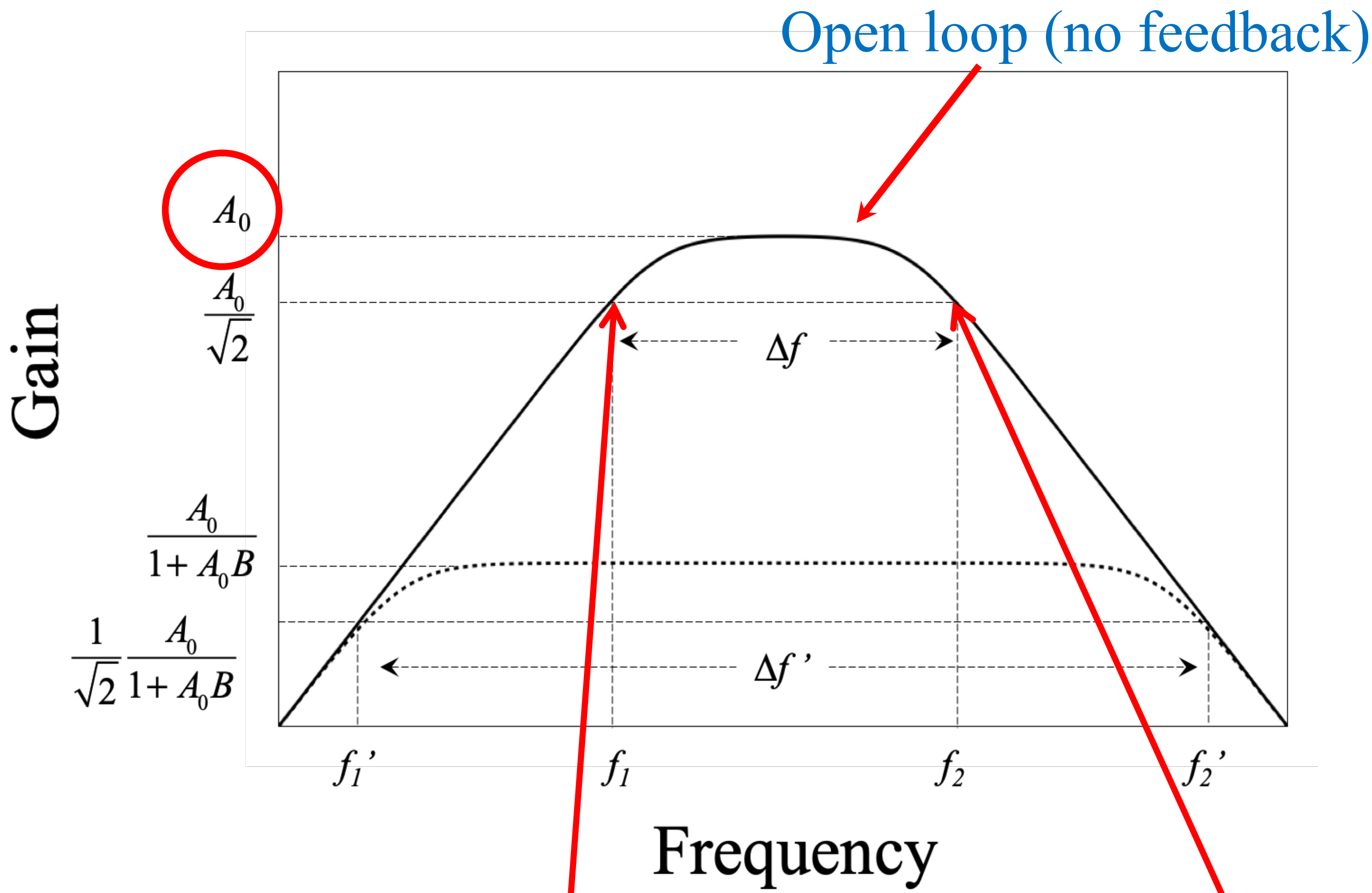
Hence fractional change in gain with feedback, $\delta G/G$ is $$\frac{\delta G}{G} = \frac{1}{G}\frac{dG}{dA}\delta A$$

$$\Rightarrow \frac{\delta G}{G} = \frac{\delta A}{A}\left(\frac{1}{1+AB}\right)$$

More robust against variations in A

i.e. the fractional change in gain is reduced by a factor $(1 + AB)$.

The Effect on Bandwidth



The frequency-dependent gain, $A(f)$, can be expressed as:

$$A(f) = (\text{low freq. cut off}\,) \times A_o \times (\text{high freq. cut off})$$

where A_o is the gain at mid-band frequencies. The form is similar to what we saw in IA for coupling circuits

Low f Mid-band   High f

$$A(f) = \left(\frac{1}{1+\frac{f_1}{jf}}\right) A_0 \left(\frac{1}{1+\frac{jf}{f_2}}\right)$$

With feedback

$$G(f) = \frac{A(f)}{(1+A(f)B)}$$

Substituting for *A(f)* and rearranging gives

$$G(f) = \frac{A_0}{\left(\frac{1}{1+\frac{f_1}{jf}}\right)\left(\frac{1}{1+\frac{jf}{f_2}}\right) + A_0 B}$$

Consider three cases:

(i) mid-band

(ii) Low frequency

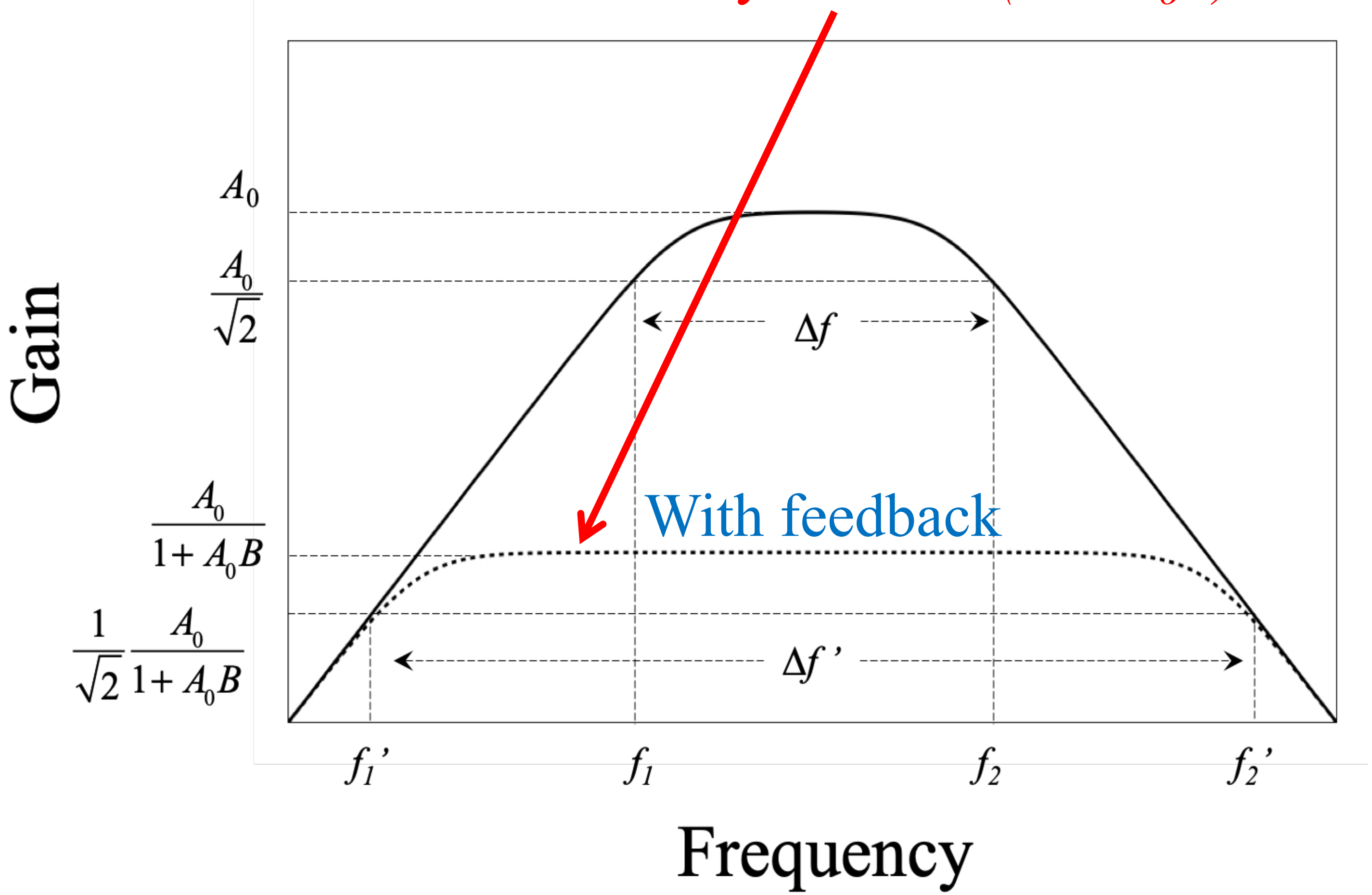
(iii) High frequency

(i) Mid-band, where $f >> f_1$ and $f << f_2$

$$G(f) = \frac{A_0}{\left(\frac{1}{1+\frac{f_1}{jf}}\right)\left(\frac{1}{1+\frac{jf}{f_2}}\right)^{+A_0B}}$$

$$G = \frac{A_0}{1 + A_0B}$$

Gain is reduced by a factor $(1 + A_oB)$

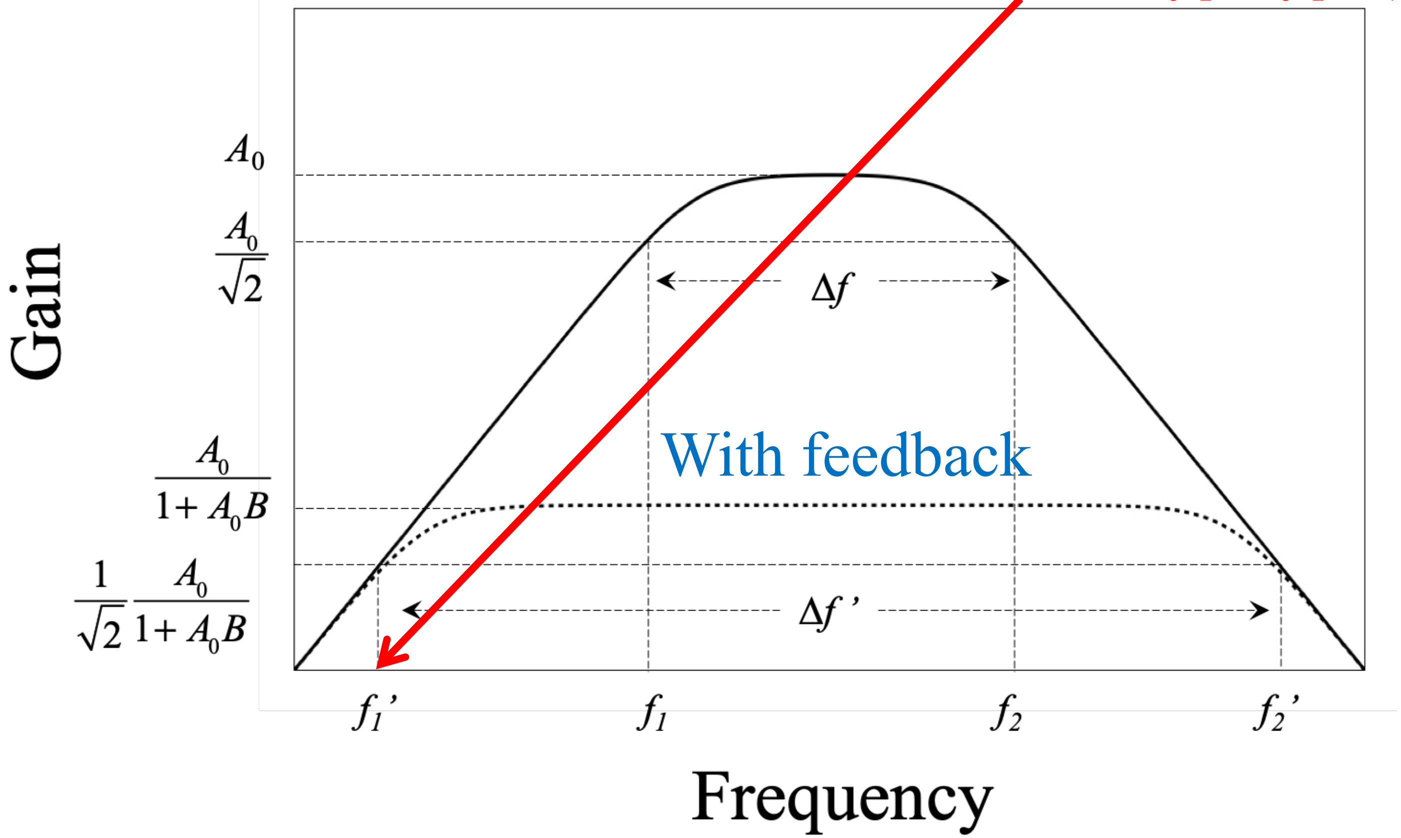


(ii) Low frequency $f << f_2$

$$G(f) = \frac{A_0}{\left(\frac{1}{1+\frac{f_1}{jf}}\right)\left(\frac{1}{1+\frac{jf}{f_2}}\right) + A_0 B}$$

$$G \cong \frac{A_0}{1 + A_0 B}\left(\frac{1}{1 + \frac{f_1}{jf(1 + A_0 B)}}\right)$$

Lower 3dB frequency is reduced to $f_1' = f_1 / (1 + A_o B)$

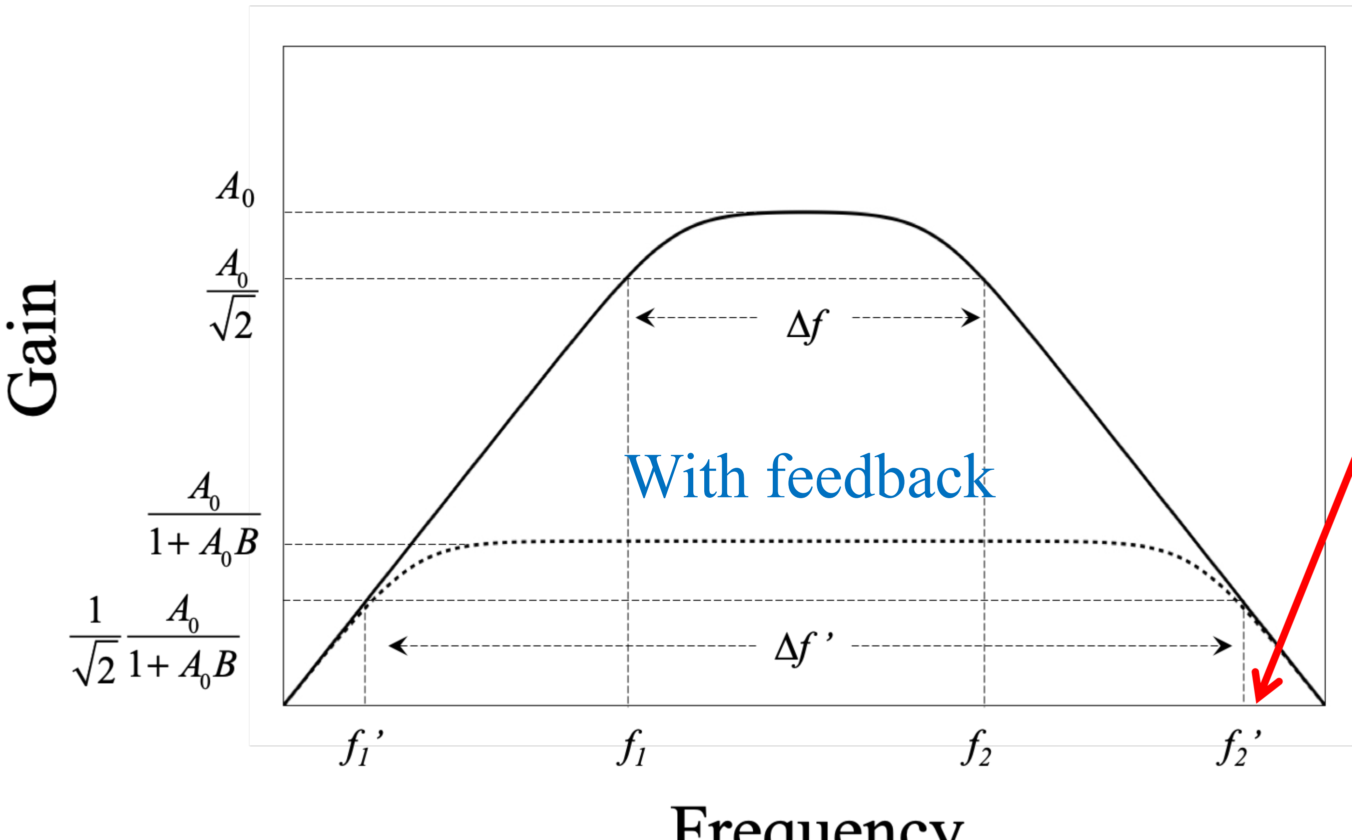


(iii) High frequency $f >> f_1$

$$G(f) = \frac{A_0}{\left(\frac{1}{1+\frac{f_1}{jf}}\right)\left(\frac{1}{1+\frac{jf}{f_2}}\right)^{-1} + A_0 B}$$

$$G \cong \frac{A_0}{1+A_0B}\left(\frac{1}{1+\frac{jf}{f_2(1+A_0B)}}\right)$$

Upper 3dB frequency is increased to $f_{2'} = f_2\,(1 + A_oB)$



Hence the overall bandwidth of the amplifier (f_2 - f_1) has been increased.

In general, feedback is used to sacrifice gain in order to increase bandwidth.

5.3 Gain-Bandwidth Product

From feedback theory, the closed loop gain of an amplifier circuit was $G = A/(1 + AB)$ where A was the open loop gain and B the fraction of the output fed back.

Many operational amplifiers are designed to give stable dc performance, hence the lower frequency cut-off is at $f_1 = 0$.

Op-Amps are also internally compensated to ensure a high frequency drop-off and guarantee stability.

As seen from the analysis above, the consequence of adding negative feedback:

The open loop (dc) gain falls by $(1 + A_o B)$

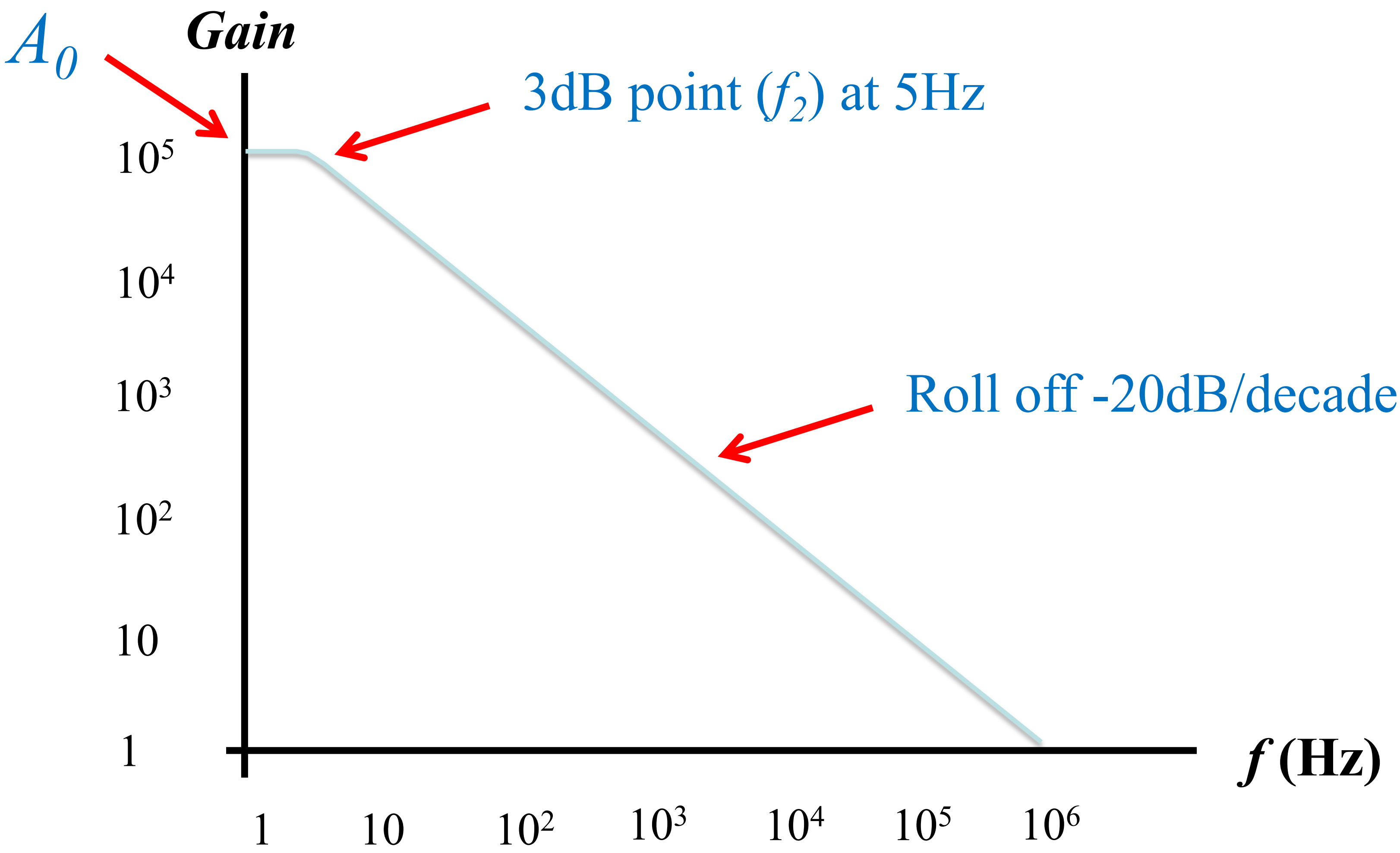
The 3 dB frequency increases by $(1 + A_o B)$

The gain-bandwidth product stays constant

This is the usual experience with feedback.

The 741 OpAmp the low frequency or dc gain is high, $> 10^5$, and there is a first order (-20 dB/decade) roll-off, characterised by a 3 dB point of $\approx$ 5 Hz.

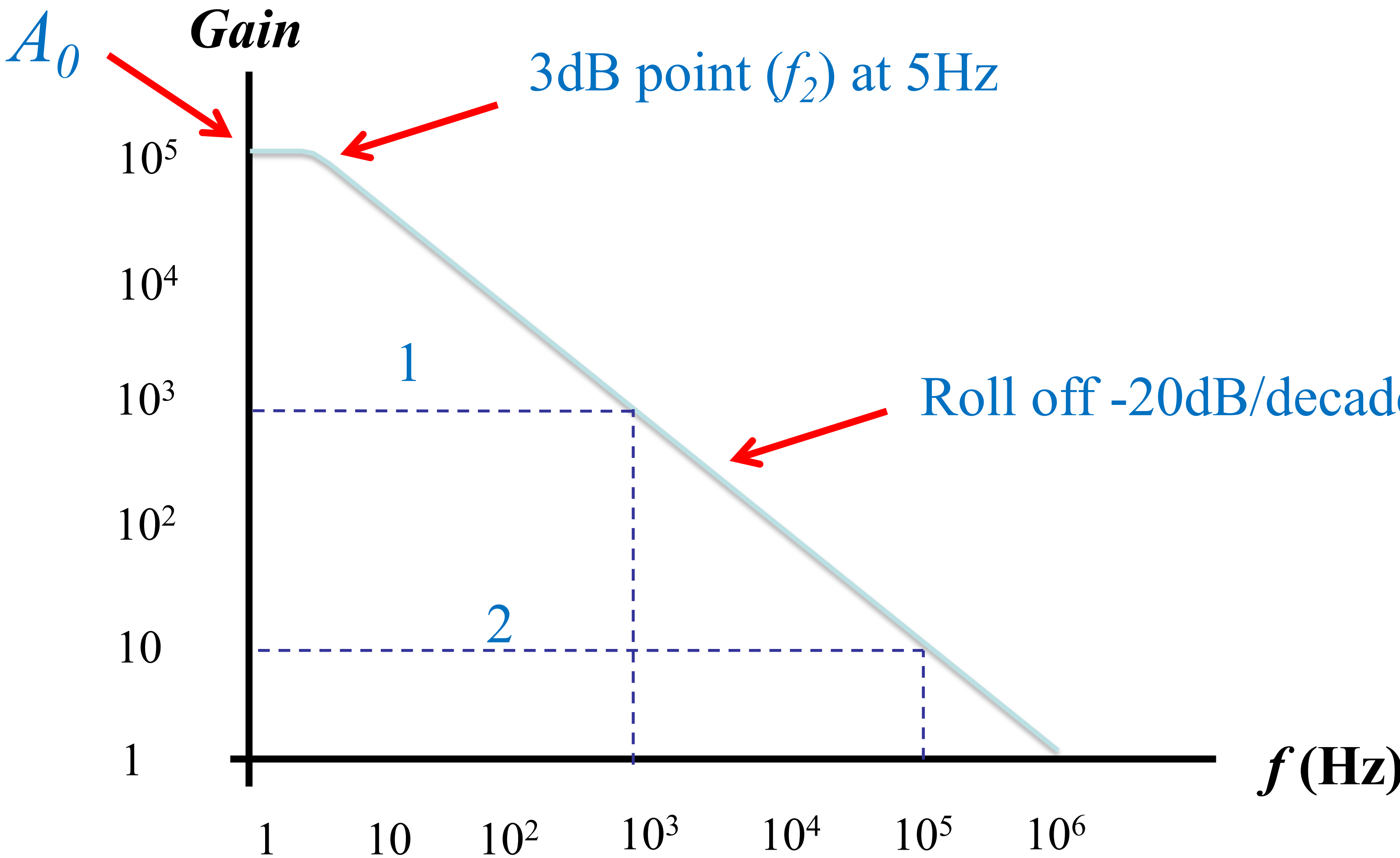
Gain-frequency plot for the 741 OpAmp.



Consider two cases:

1) Gain is set at 10^3, 3dB point is 1kHz

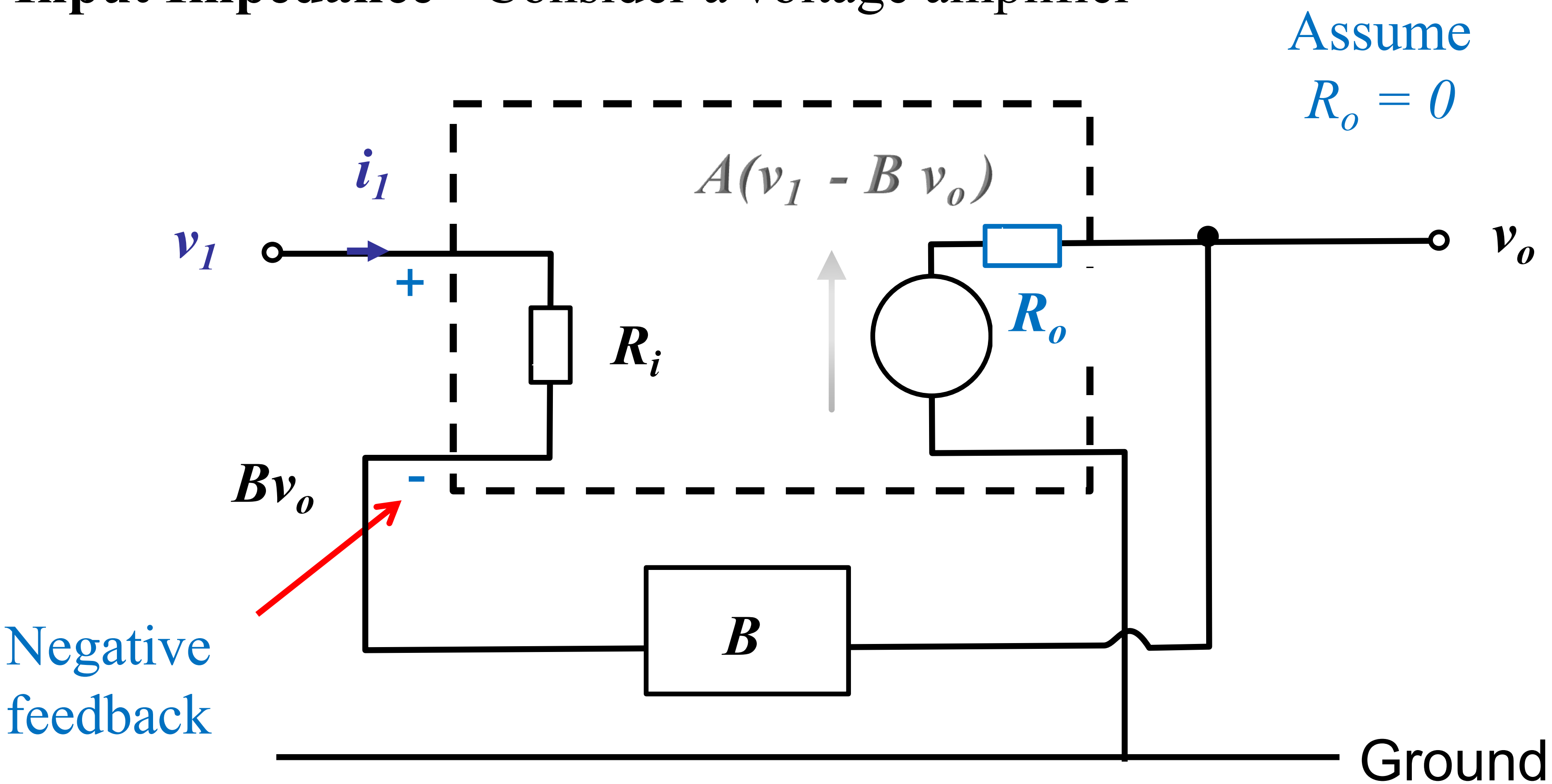
2) Gain is set at 10, 3dB point is 100kHz



Gain bandwidth product is 1MHz in both cases

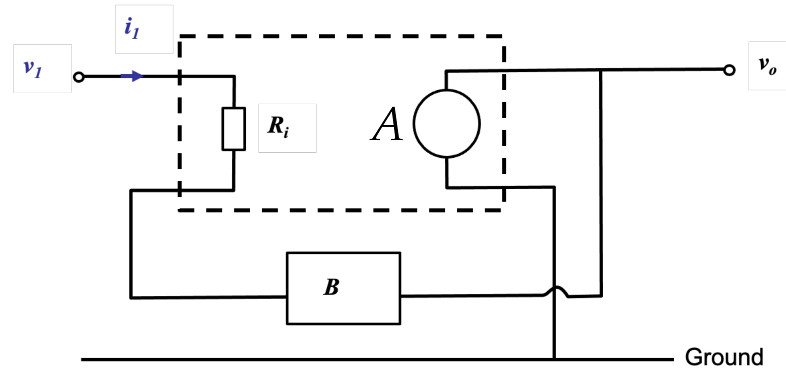
5.4 Effect of negative feedback on input and output impedances

Input Impedance - Consider a voltage amplifier



The input resistance, $R_{in} = v_1 / i_1$

Without feedback, $R_{in} = R_i$



With feedback

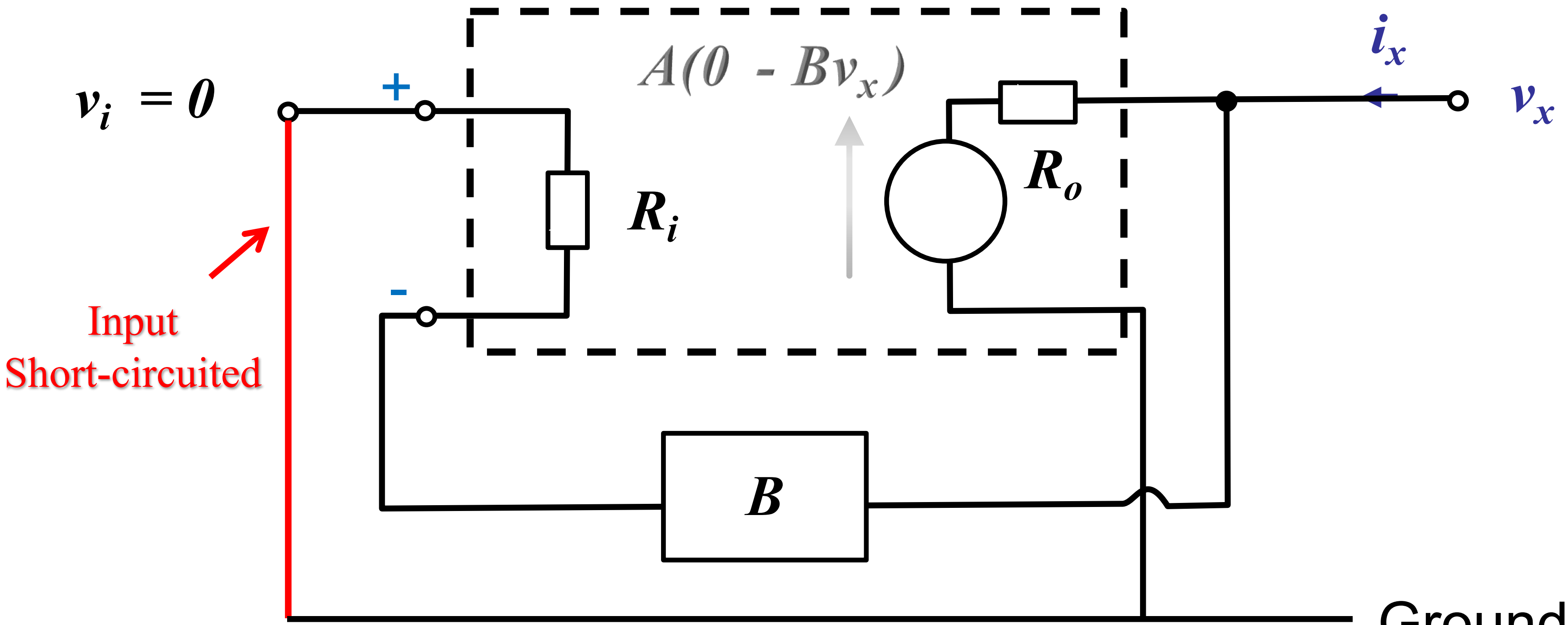
General result $$v_0 = \frac{A}{(1+AB)} v_1$$

$$i_1 = \frac{v_1 - Bv_o}{R_i}$$

Eliminate v_o $$R_{in} = \frac{v_1}{i_1} = R_i(1 + AB)$$

The input resistance is increased by a factor *(1 + AB)*

Output Impedance - Consider a voltage amplifier



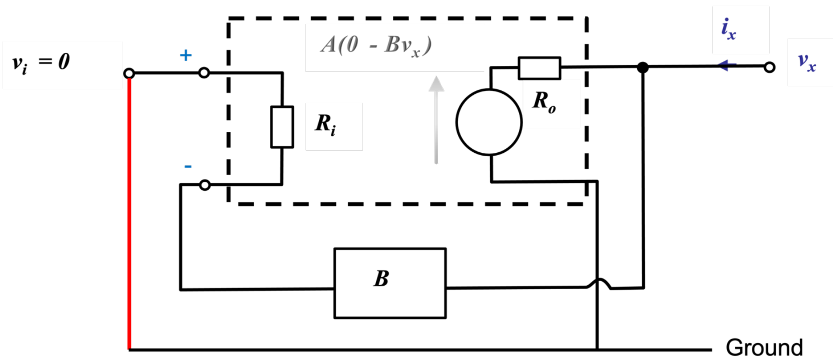
Applying a voltage v_x to the output terminals with the input shorted causes a current i_x to flow.

The output resistance $R_{out} = v_x / i_x$ (with input short-circuited)

Without feedback $R_{out} = R_o$

With feedback, analyse by summing currents at the output; take R_i to be infinite as we are only interested in the output resistance.

$$\frac{-ABv_x - v_x}{R_o} + i_x = 0$$



$$R_{out} = \frac{v_x}{i_x} = \frac{R_o}{(1 + AB)}$$

Hence the output resistance is reduced by a factor *(1 + AB)*.

Overall Summary of the Effects of Feedback

For **voltage amplifiers**, reducing the gain by $(1 + AB)$

- Increases the input resistance by $(1 + AB)$
- Decreases the output resistance by $(1 + AB)$

In addition, negative feedback

- Increases the upper 3 dB frequency
- Reduces the lower 3 dB frequency
- Reduces distortion
- Reduces the effect of a change in open loop gain

by $(1 + AB)$

However, for **current amplifiers** the effect of negative feedback is to
- Decrease the input resistance
- Increase the output resistance

Now try Example Paper 2 Q1 & Q2

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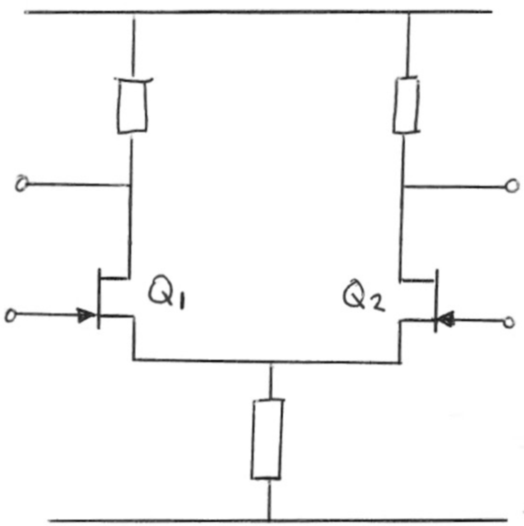
6 *Op Amps*

Operational Amplifiers

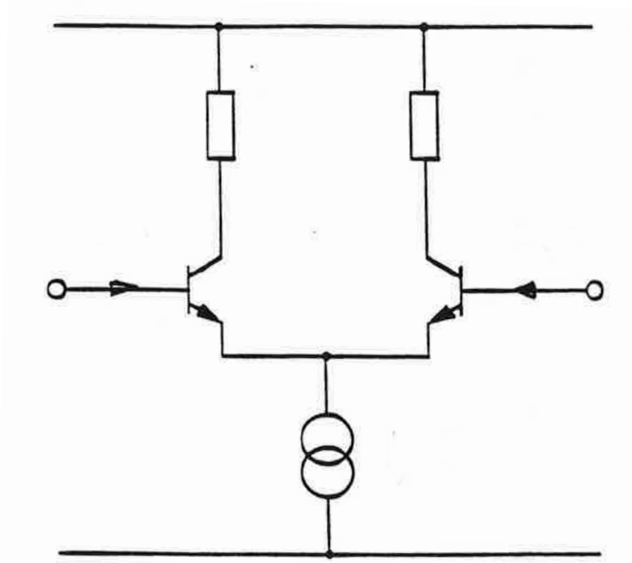
Despite the wide range of operational amplifiers (Op Amp) available, almost all use the same input circuit, namely two transistors connected as a differential pair.

The choice of input transistor (JFET, MOSFET or bipolar) has a major effect on the characteristics of an operational amplifier.

FET – High R_{in}



BJT – High A, stable



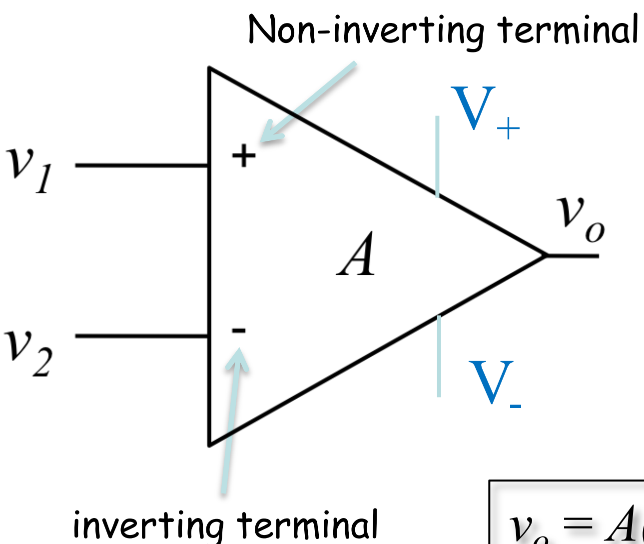
Revision: The Ideal OpAmp

As you will recall from IA lectures, the OpAmp is represented by a simple model which in its most basic form is the 'Ideal OpAmp' model. When designing OpAmp based circuits, we are always striving to use the ideal model. In order to do this we select the best possible OpAmp available for a given application and also choose suitable components (usually resistors) to accompany the OpAmp which maintain its ideal operation. Hence we have two basic rules for ensuring as ideal a performance as possible:

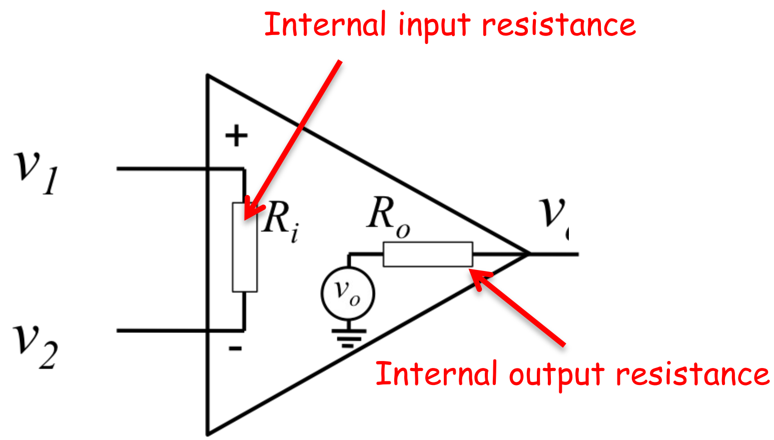
i) Choose a suitable OpAmp – Best A, R_i, R_o, bandwidth, offset voltage ... etc for a given price (£££)

ii) Choose suitable resistors – Avoid designing circuits with R values that are close to R_i or R_o

Operational Amplifiers



$$\boxed{v_o = A(v_1 - v_2)}$$



Characteristics of an ideal Op amp

1. Infinite open-loop gain, $A = \infty$
2. Infinite input resistance, $R_i = \infty$
3. Zero output resistance, $R_o = 0$

Characteristics of real Op amps:

1. Finite but very large (and usually not well-defined) open-loop gain, $A > 10^5$
2. Finite but very large input resistance, $R_i = 10^6 - 10^{12}\,\Omega$
3. Low output resistance, $R_o = 10 - 100\,\Omega$

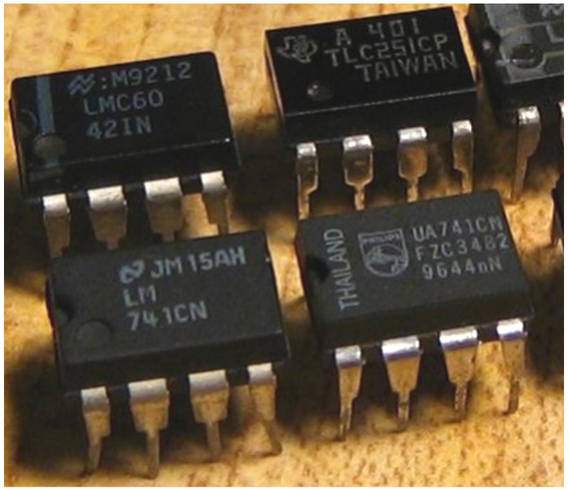
Op Amps need power!

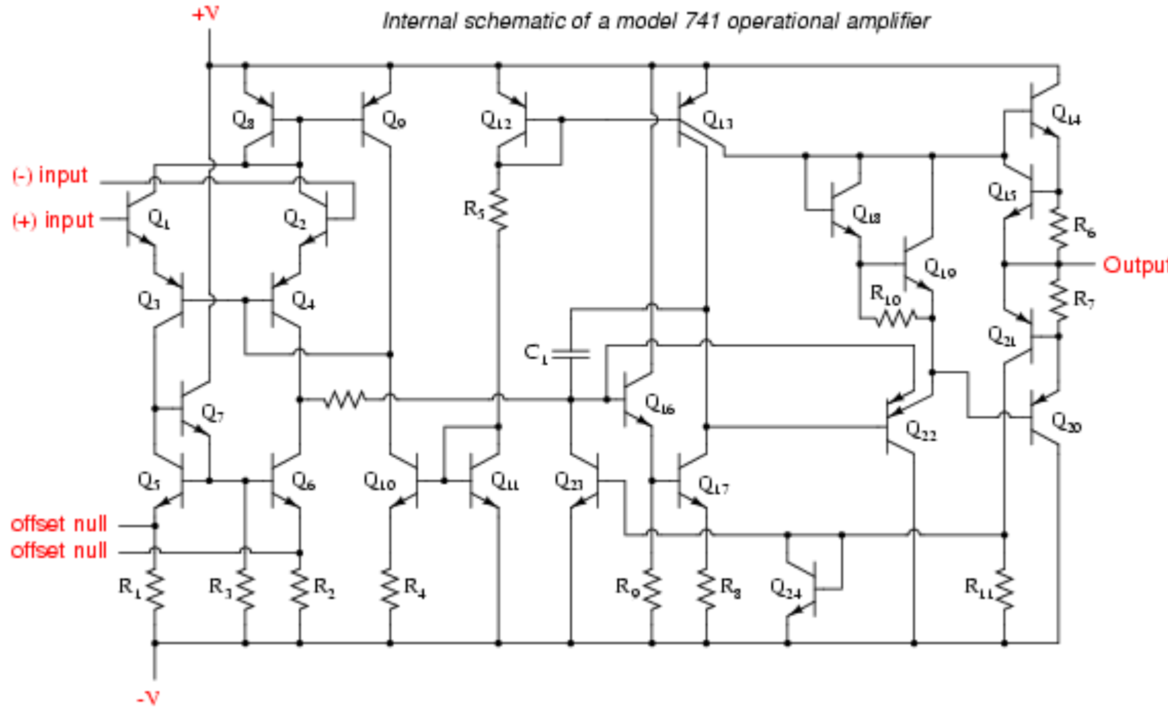
The golden rules of ideal Operational Amplifiers

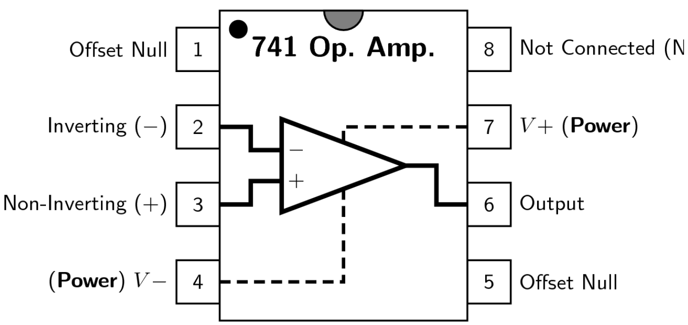
1. The output attempts to do whatever is necessary to make the voltage difference between the inputs equal to zero (*voltage rule*)

2. The inputs draw no current (*current rule*)

What do op amps look like?



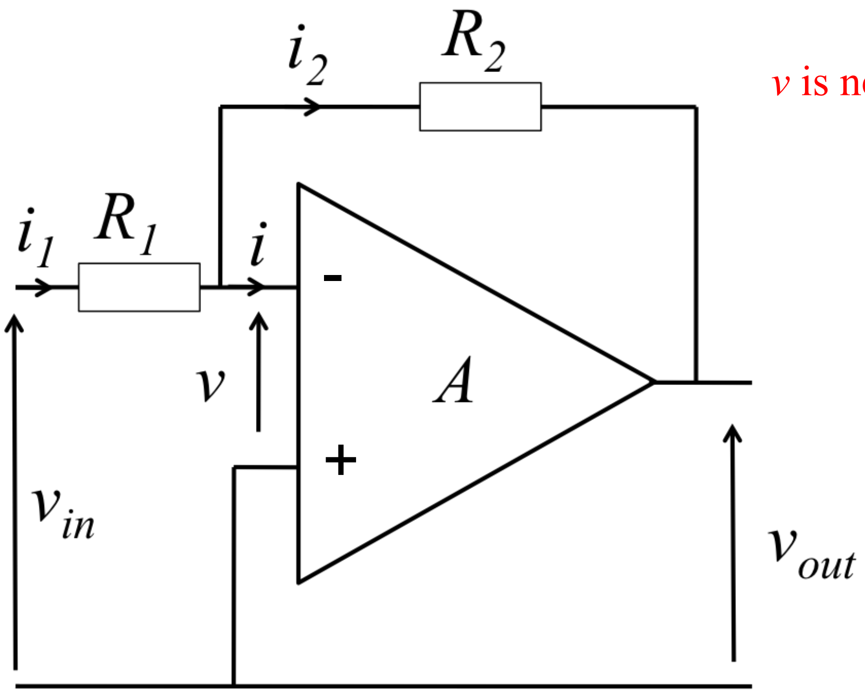




There are two fundamental operational amplifier configurations, inverting and non-inverting, which we will consider. In the case of ideal amplifiers, the small signal analysis is straightforward.

The ***power*** terminals are connected to dc supply voltages in the same way as an FET amplifier. We generally don't draw the power connections in our circuits, but they are still there! An op-amp will only work when the power is on.

The inverting Amplifier



v is negative here, as it is *defined* as (v_+ - v_-)

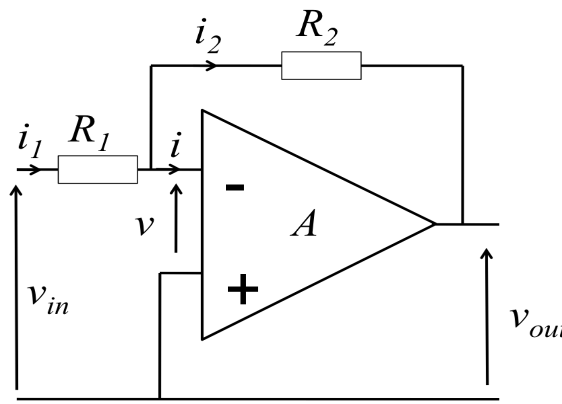
Part of the output is *fed back* to the input via the resistor R_2. The output voltage v_{out} is then

$$v_{out} = -Av$$

=> $$v = -v_{out}/A$$

If we assume that A is infinite, then $v \sim 0$, and we say that the inverting terminal is a ***virtual earth.***

Ideal op-amp => $R_i = \infty$. The current into the inverting (-) terminal will then be zero:



$$i = 0$$

Ideal op amp also => $v = 0$

=>

$$i_1 = i_2$$

Where $i_1 = (v_{in} - v)/R_1 = v_{in}/R_1$

And $i_2 = (v - v_{out})/R_2 = -v_{out}/R_2$

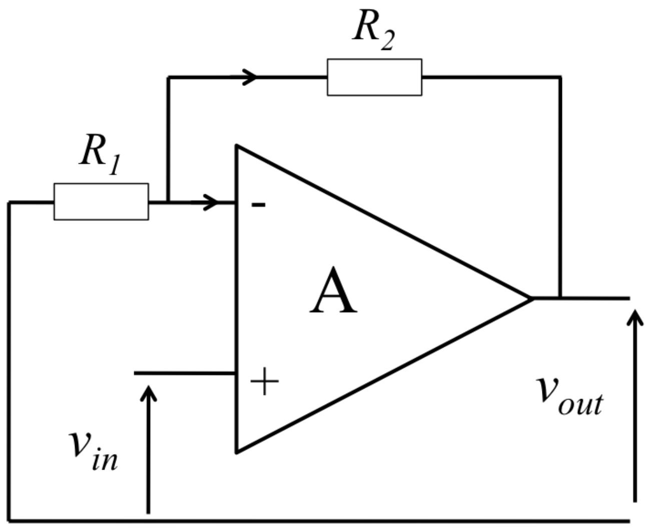
As $i_1 = i_2$, => $v_{in}/R_1 = -v_{out}/R_2$

=> **gain, $G = v_{out}/v_{in} = -R_2/R_1$**

Input resistance of the amplifier $R_{in} = R_1$

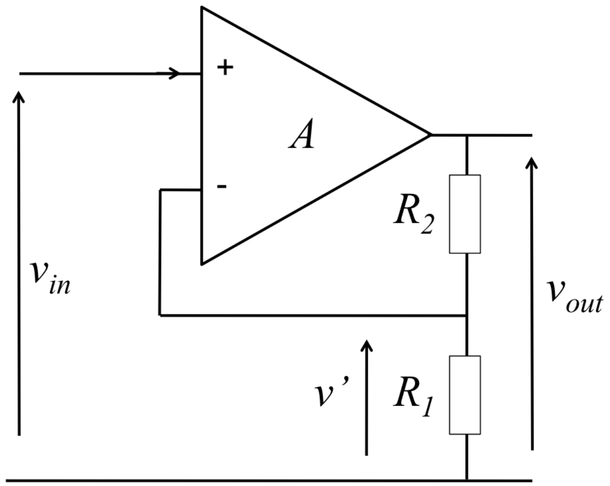
This amplifier has a negative gain, which is why we call it an inverting amplifier. Dc inputs will be inverted, as will ac ones. When an ac signal is inverted, we say that it is in anti-phase, or the phase of the output is shifted by 180° relative to the input.

The non-inverting Amplifier



i.e. the input is applied to the non-inverting terminal

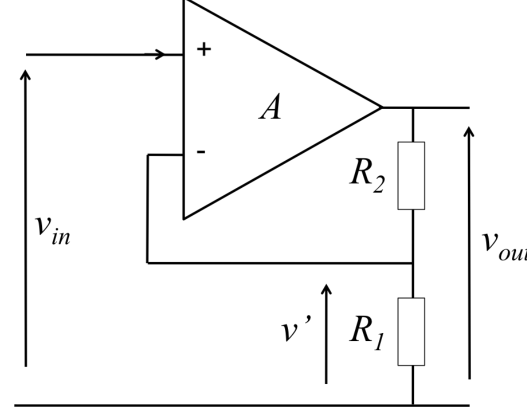
This circuit can also be shown as:



As before,

$$v_{out} = A(v_{in} - v')$$

$$v_{in} - v' = v_{out}/A$$



Now we need to determine v', using potential divider R_1 & R_2

$$v' = \frac{R_1}{R_1 + R_2} v_{out}$$

This only works if the only current flowing is from the output terminal to ground

If we can assume that $A \gg 1$, we can make the approximation that $v_{in} - v' = 0$.

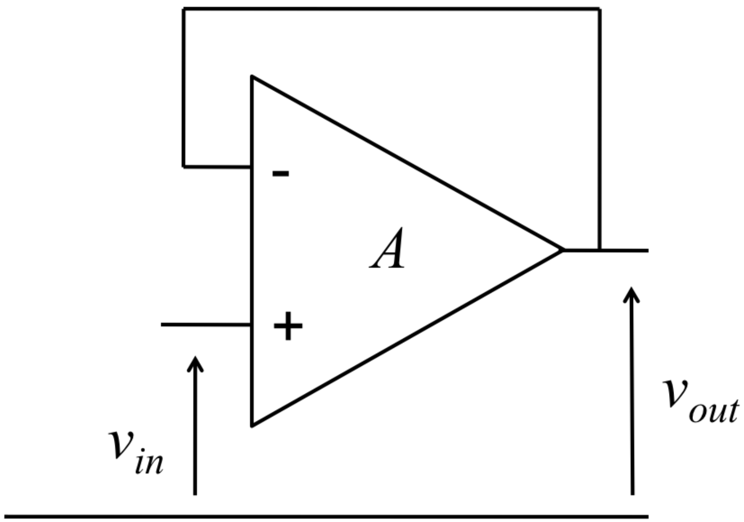
i.e. $v' = v_{in}$.

We can then substitute this in the equation relating v' to v_{out}, to find:

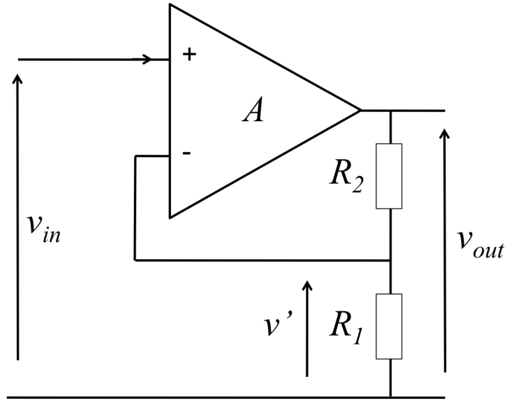
$$G = \frac{v_{out}}{v_{in}} = 1 + \frac{R_2}{R_1}$$

Now try Example Paper 2 Q3

The voltage follower



Non-inverting amplifier



$$G = \frac{v_{out}}{v_{in}} = 1 + \frac{R_2}{R_1}$$

This is essentially a non-inverting amplifier as before, but with $R_1 = \infty$ and $R_2 = 0$.

$$\text{Gain}, G = \frac{v_{out}}{v_{in}} = 1$$

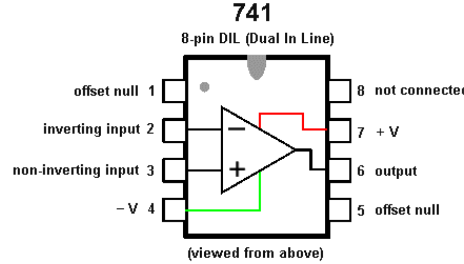
Now try Example Paper 2 Q4

$$\text{Input impedance} = AR_i \sim \infty$$

$$\text{Output impedance} = \frac{R_o}{A} \sim 0$$

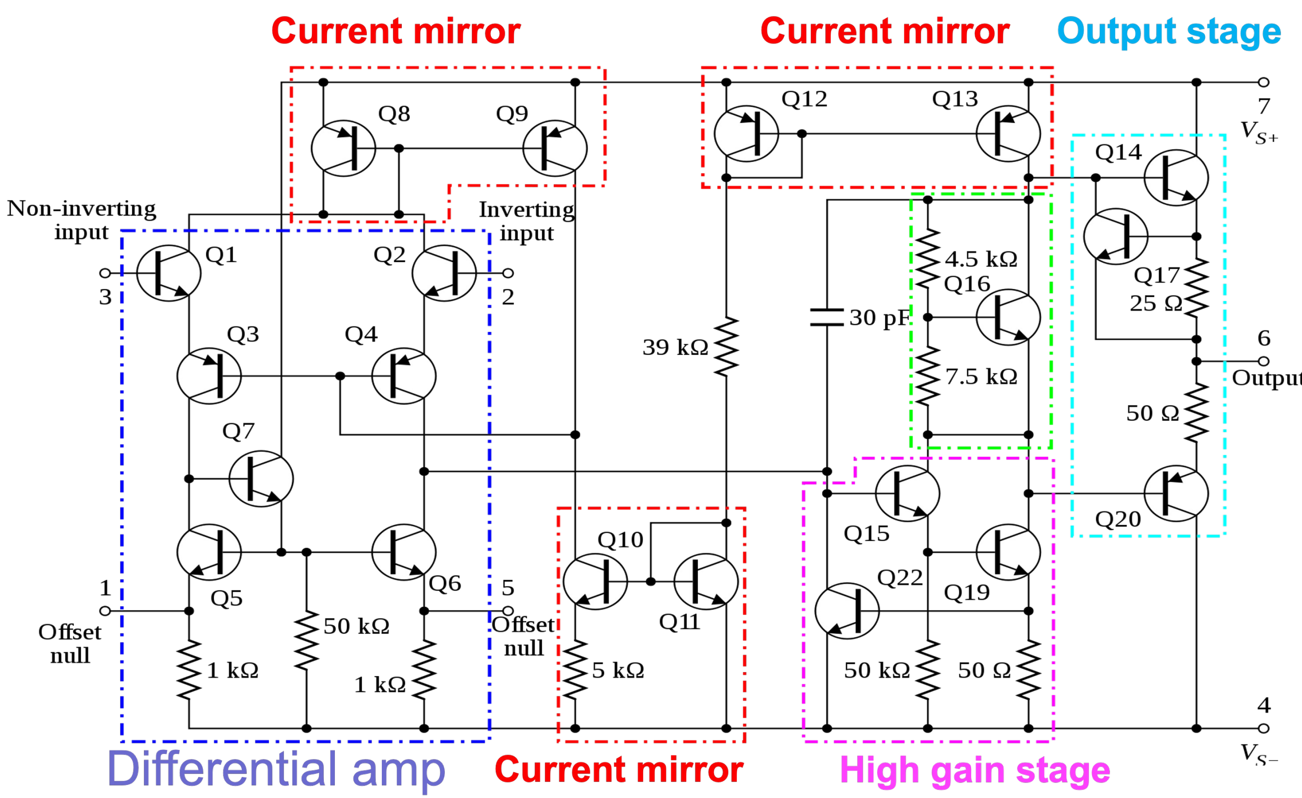
This is called a *buffer* amplifier
….source follower (FET)
….emitter follower (BJT)

The iconic 741



*http://www.talkingelectronics.com/projects/OP-AMP/OP-AMP-1.html

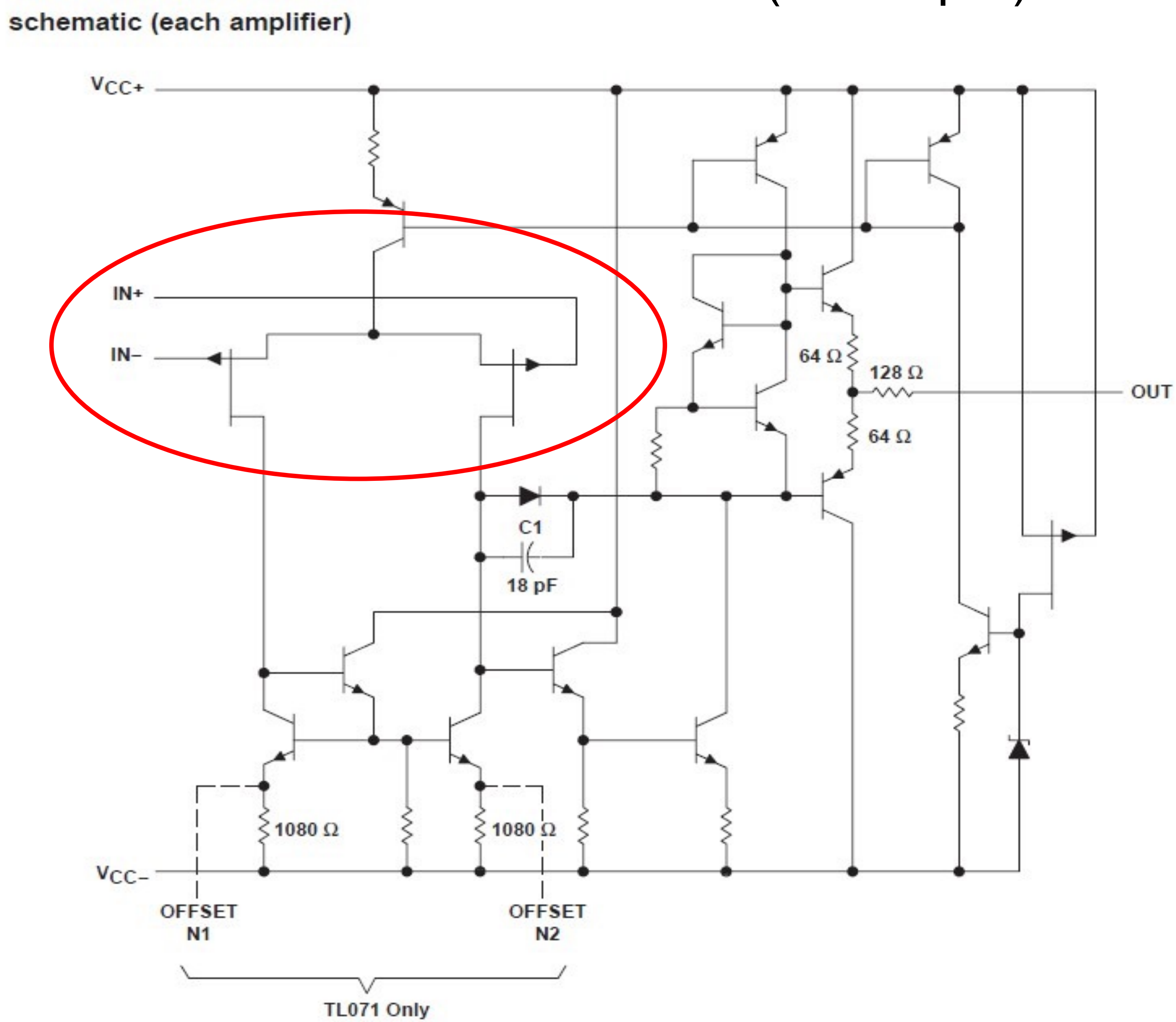
8 pin DIL



https://en.wikipedia.org/wiki/Operational_amplifier

The 741 OpAmp was the successful follow-up to the 709 which was the first really useful OpAmp. The 741 incorporated features such as an internal capacitor to set the frequency response so that it could be used with any degree of negative feedback without needing any external capacitors and short circuit protection. The 741 contains the basic circuit elements that we have studied, but in a more sophisticated form. Many operational amplifiers variants have been developed with emphasis on particular features. JFET input amplifiers are valued for low input currents and the internal circuit of the TL071 family is shown later. MOSFET amplifiers are increasingly important as part of mixed signal systems.

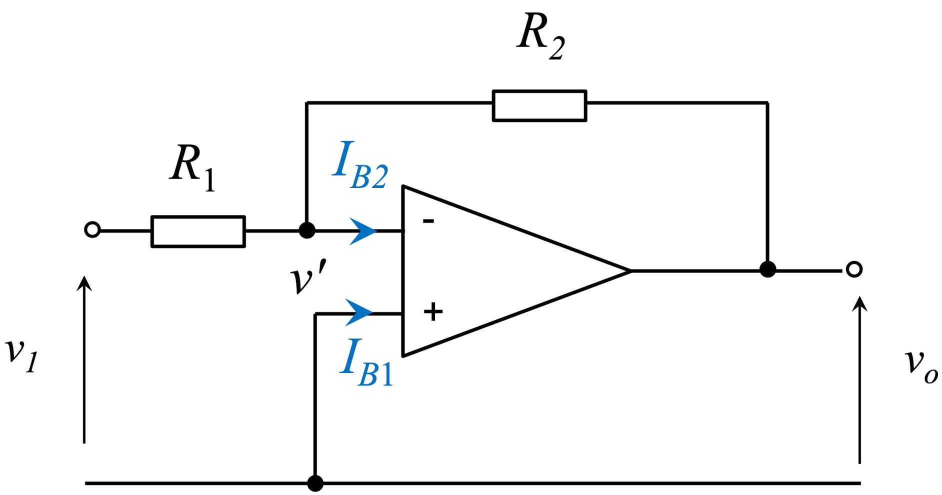
The TL071 (FET input)



Sources of non-ideality

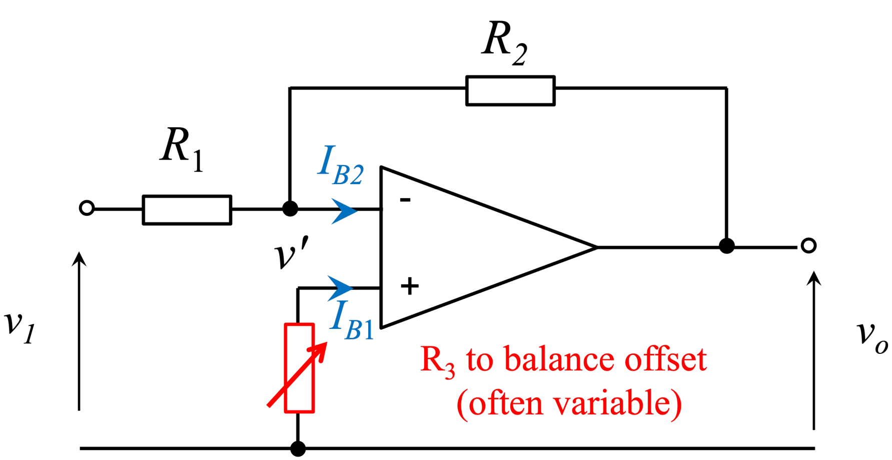
Input Bias Current

Consider an OpAmp such as the classic 741, where the input stage is made using bipolar transistors.
The base currents I_{B1} and I_{B2} are needed to set the operating points of the input transistors.
The input bias currents for the 741 amplifier are about 100 nA.
These currents can be seen in the inverting amplifier as an example.



I_{B2} flows through R_1 and R_2, so even with $v_i = 0$, v' is not zero and there is an unwanted output v_o. In the case of the 741 with R_1 = 5 kΩ and R_2 = 500 kΩ the bias currents, typically 100 nA, would give an output of ≈ 0.05 V. An alternative, and generally better, solution is to choose an operational amplifier with low bias currents.

Also, in practice, as the input transistors are never perfectly matched, the two bias currents are not always equal. It is possible to eliminate the unwanted output by adding a resistor (R_3 in the figure below) to the (+) input to ground, but this is unattractive for commercial production. In addition these currents will vary with temperature. For the 741, the offset current is typically 20 nA. Generally, it is wise to choose an operational amplifier with bias currents that are sufficiently low to avoid unacceptable unwanted output voltages – FET input operational amplifiers are attractive in this respect.



Applying zero volts to both inputs of an operational amplifier does not produce zero output because of mismatches in the input transistors.

The input offset voltage, V_{os}, is the voltage needed at the input, that is between the inverting and non-inverting inputs, to drive the output to zero.

For the 741, V_{os} is typically 2 mV and the maximum value is 6 mV.

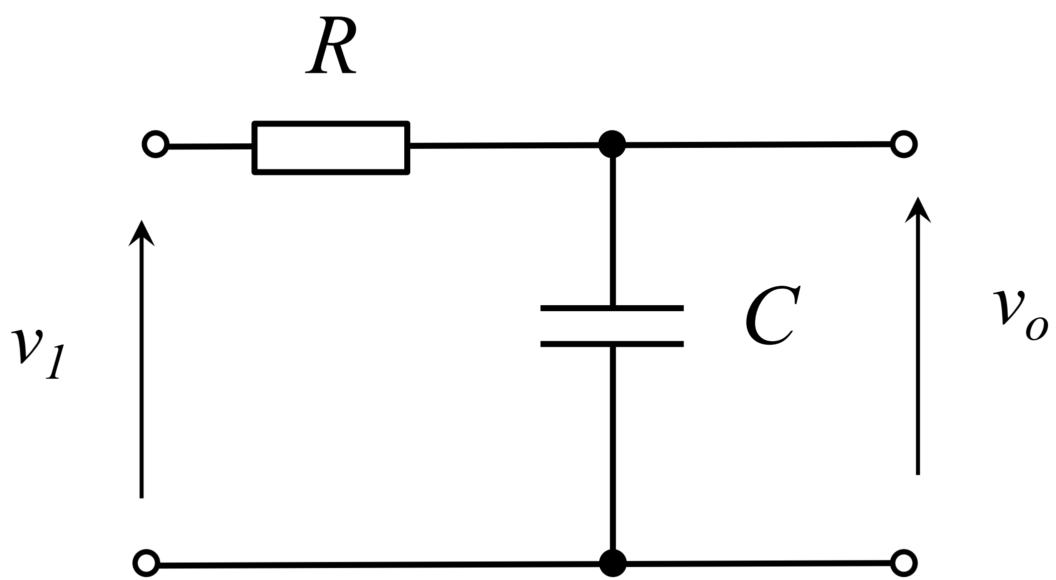
FET input operational amplifiers generally have higher input offset voltages than bipolar input operational amplifiers.

Provision is often made for nulling the input offset voltage by means of an external potentiometer (often called 'Offset Null').

The Frequency Response of Operational Amplifiers

The gain of an operational amplifier falls with frequency because of stray capacitances (*or gain compensation*).

The effect can be modelled as an *RC* network (first order).



Frequency response (v_o/v_1) is unity at dc, but falls as ω increases.

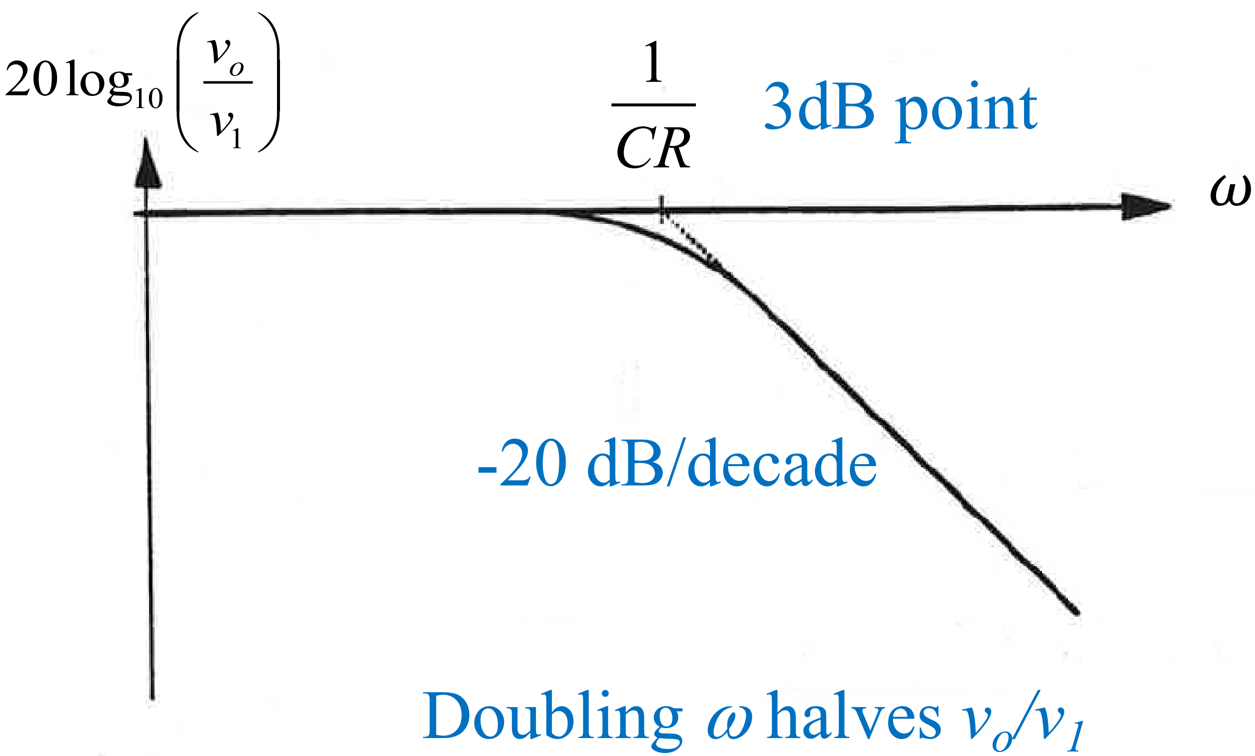
Potential divider

$$\frac{v_o}{v_1} = \frac{1}{1 + j\omega CR}$$

At low frequencies, $\omega CR << 1$, $\quad \dfrac{v_o}{v_1} \cong 1$

At high frequencies, $\omega CR >> 1$, $\quad \dfrac{v_o}{v_1} \cong \dfrac{1}{j\omega CR}$

This gives the following response



The form of the RC network is similar to that of an operational amplifier.

For example, the 741 has a high gain at dc, $\approx 10^5$, but the gain falls off above a certain frequency, $\approx$ 5 Hz, at a rate of –20 dB per decade in frequency.

This is a first order roll-off, characteristic of a single RC time constant.

The non-ideal OpAmp model

Now we move to ac

There are many ways that the problems above can manifest themselves in OpAmp circuits, however we can use three basic parameters A, R_i, R_o which we can add to the ideal OpAmp model in order to represent the effects of non ideality.

These parameters can also provide some simple design rules to help avoid designing circuits which are limited by non ideal performance.

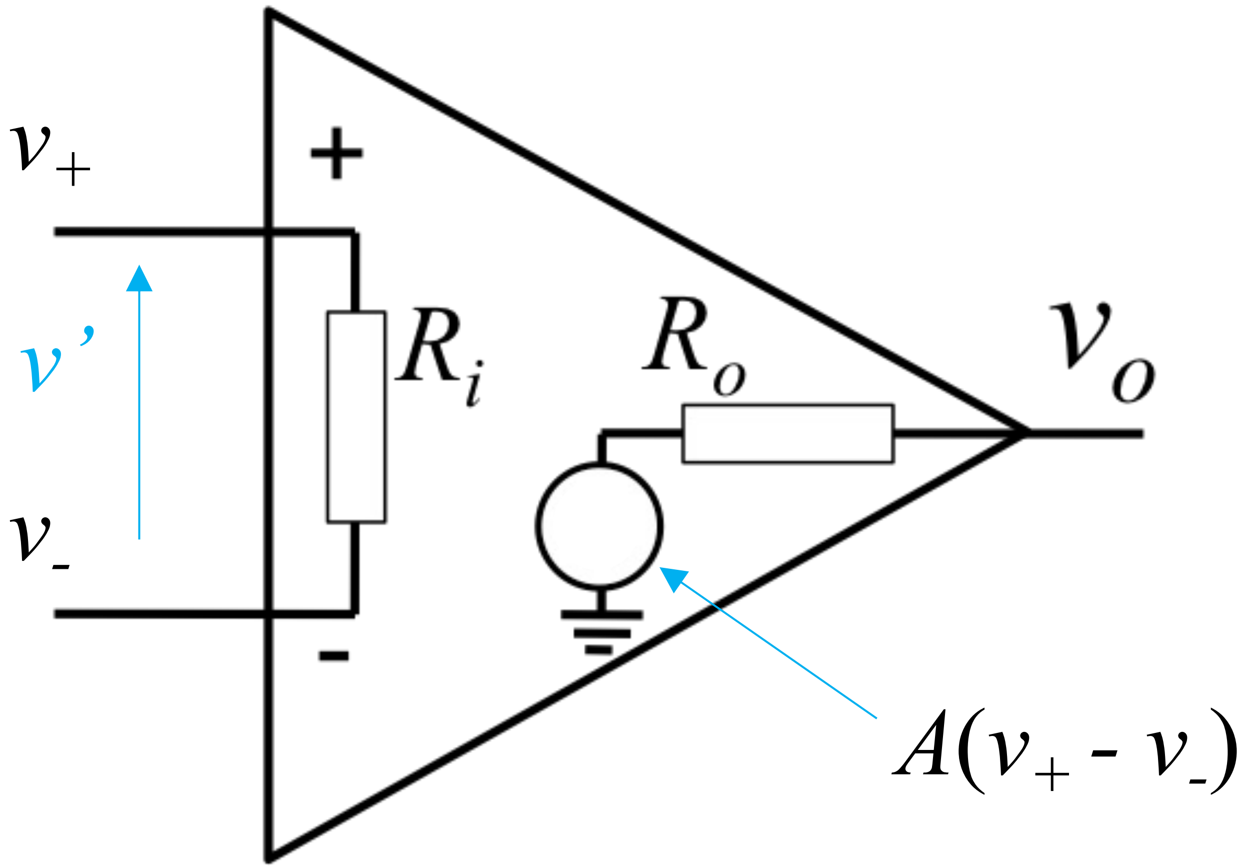
i) Select OpAmp with the highest gain/bandwidth you can afford

ii) Avoid using resistor values of the order of the OpAmp R_i

iii) Avoid using resistor values of the order of the OpAmp R_o

Useful guide – try and keep x2 from R_i and R_0

The non-ideal model is shown below (and is in the data book)

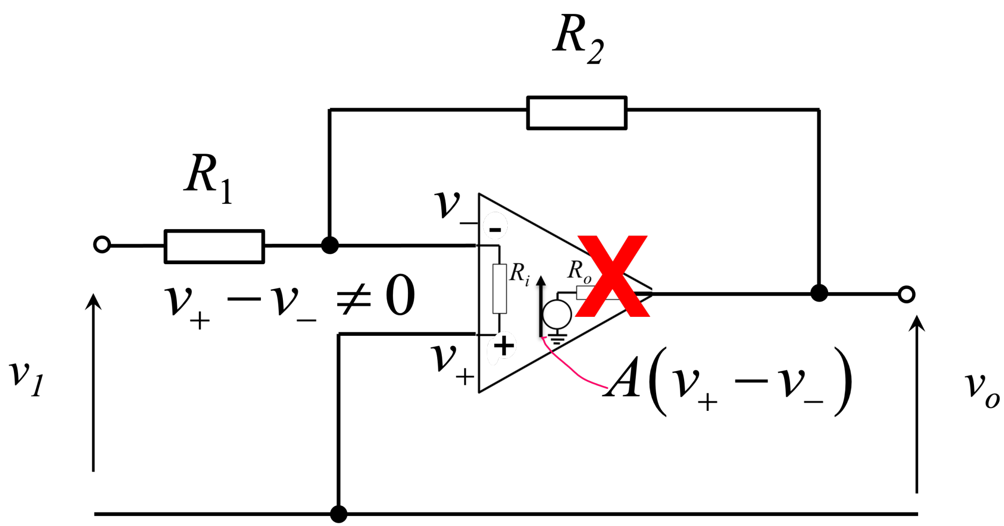


Problems can use any subset of these parameters from the fully ideal, to one or any of them as a practical constraint. The skill of the designer is to work out which effects are important and which can be neglected.

Of course the best option is always to get as close to ideal behaviour as possible.

The Inverting Amplifier with a Non-Ideal OpAmp

Practical OpAmps have finite values of gain and finite input resistance and a non-zero value of output resistance.
Therefore, we need to know how to analyse circuits to take into account these departures from ideality and, perhaps more importantly, to know when approximate solutions are valid.



Ideal gain $\dfrac{v_o}{v_1} = -\dfrac{R_2}{R_1}$

Include effects of finite input resistance, R_i and finite gain, A, but <u>not</u> the effect of output resistance (i.e. $R_o = 0$) as it will be very small by comparison.

Sum currents at the inverting input

$$R_o = 0 \;\rightarrow v_o = A(v_+ - v_-) = -Av_- \qquad (v_+ = 0)$$

$$\frac{v_1 - v_-}{R_1} + \frac{0 - v_-}{R_i} + \frac{v_o - v_-}{R_2} = 0 \qquad v_- = -\frac{v_o}{A}$$

$$Gain = \frac{v_o}{v_1} = -\frac{R_2}{R_1}\left[\frac{1}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}+\frac{R_2}{R_i}\right)}\right]$$

Non-ideal effects

When A and R_i tend to infinity, the gain is $-R_2/R_1$ as before.

Example: $A = 1000$, $R_i = 350\ \mathrm{k\Omega}$, $R_1 = 100\ \mathrm{k\Omega}$ and $R_2 = 1\ \mathrm{M\Omega}$.

Ideal gain $= -10$

Non-ideal gain $= -10\left[\frac{1}{1+0.014}\right] = -9.86$

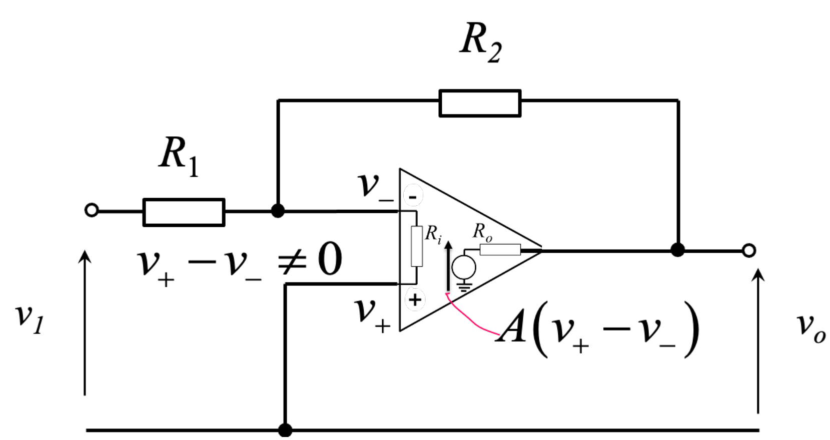
If R_i were infinite, gain would be -9.90. Most error arises from A.

If the effect of the output resistance, R_o is included, then by summing the currents at the output, the gain becomes.

New terms

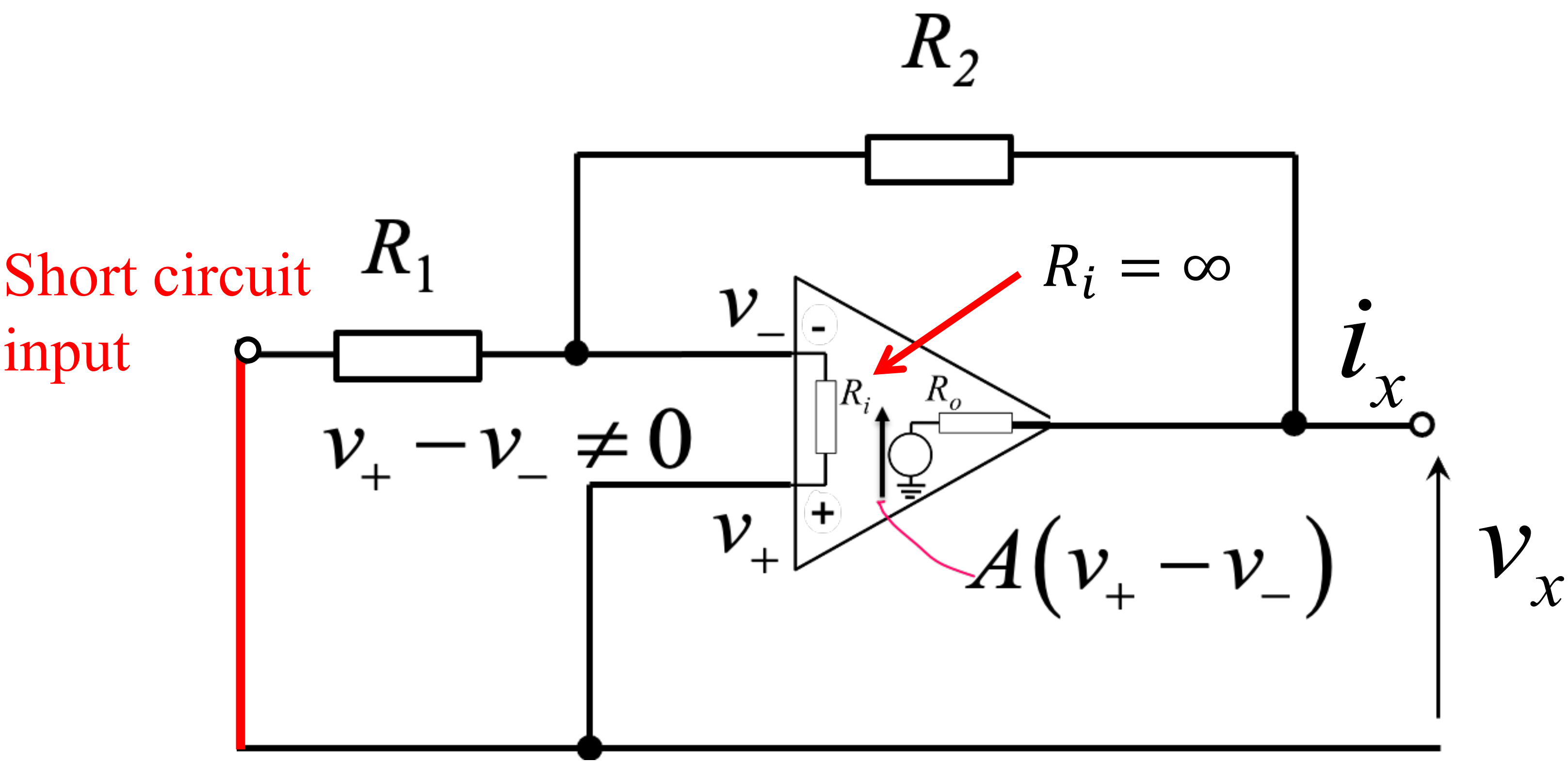
$$Gain = \frac{v_o}{v_1} = -\frac{R_2}{R_1}\left[\frac{1-\frac{R_o}{AR_2}}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}+\frac{R_2}{R_i}+\frac{R_o}{R_1}+\frac{R_o}{R_i}\right)}\right]$$

The whole exercise can be repeated for the non-inverting amplifier with similar results (*see examples paper 2, Q4 which is related*).



Output Impedance of the Inverting Amplifier

In this example we take the input resistance as infinite. With the input shorted, we apply a test voltage v_x to the output and measure a test current i_x.



$$v_+ = 0 \Rightarrow A(v_+ - v_-) = -Av_-$$

Sum currents at the inverting (-) input

$$\frac{0 - v_-}{R_1} + \frac{v_x - v_-}{R_2} = 0$$

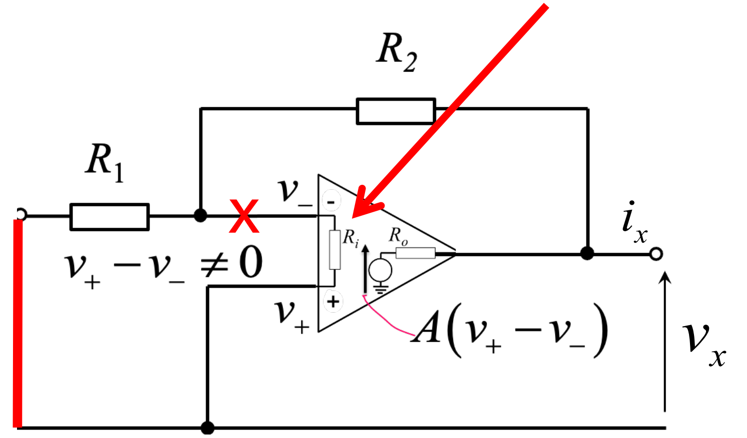
Sum currents at the output:

$$i_x + \frac{-Av_- - v_x}{R_o} + \frac{v_- - v_x}{R_2} = 0$$

Eliminate v_- and re-arrange

$$R_{out} = \frac{v_x}{i_x} = \frac{R_o}{1 + \frac{R_o + AR_1}{R_1 + R_2}}$$

$R_i = \infty$



For most operational amplifiers in circuits with moderate closed loop gains, the output resistance of the circuit is much less than the output resistance of the operational amplifier itself.

For example, with $A = 1000$, $R_1 = 100$ kΩ, $R_2 = 1$ MΩ and $R_o = 100$ Ω, R_{OUT} is 1.1 Ω.

IB Paper 5: Analysis of circuits

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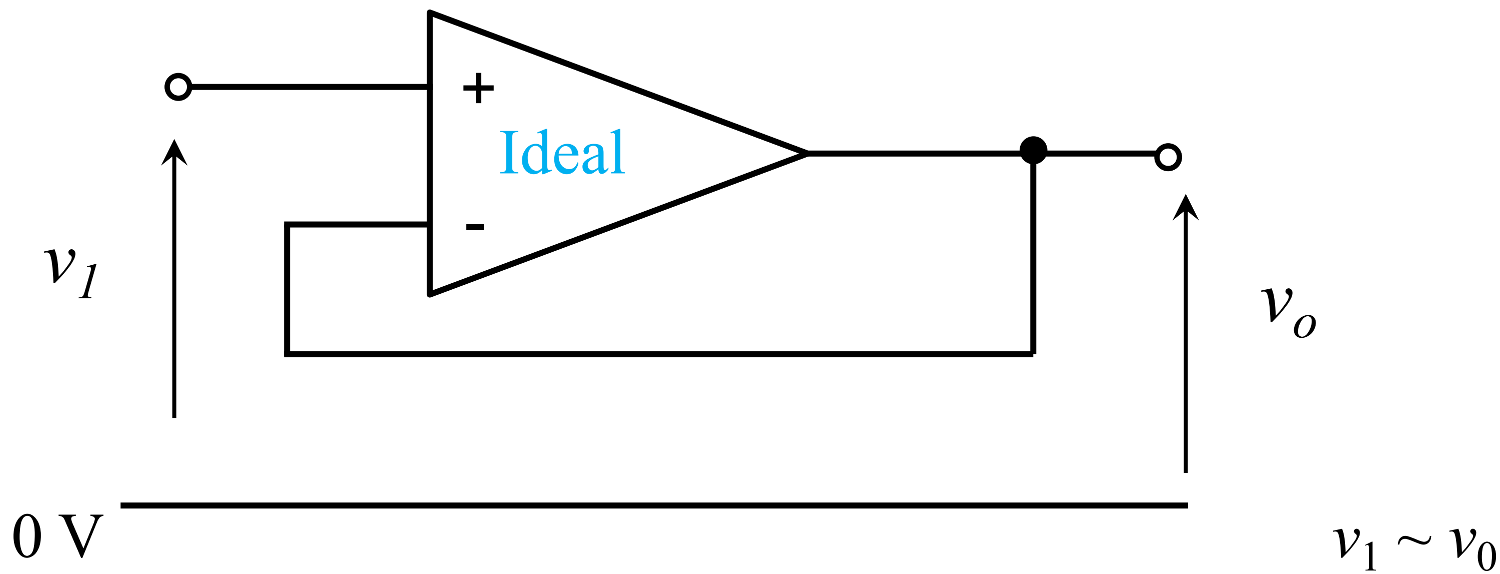
7. *Practical uses of Op amps*

Op-amp Circuits & signal processing

- Voltage follower/buffer
- Adder
- Difference amplifier
- Differentiatior
- Integrator/filter
- Transimpedance amplifier
- Gyrator

The Voltage Follower



Excellent buffer (Non-ideal in Q4, Examples paper 2)

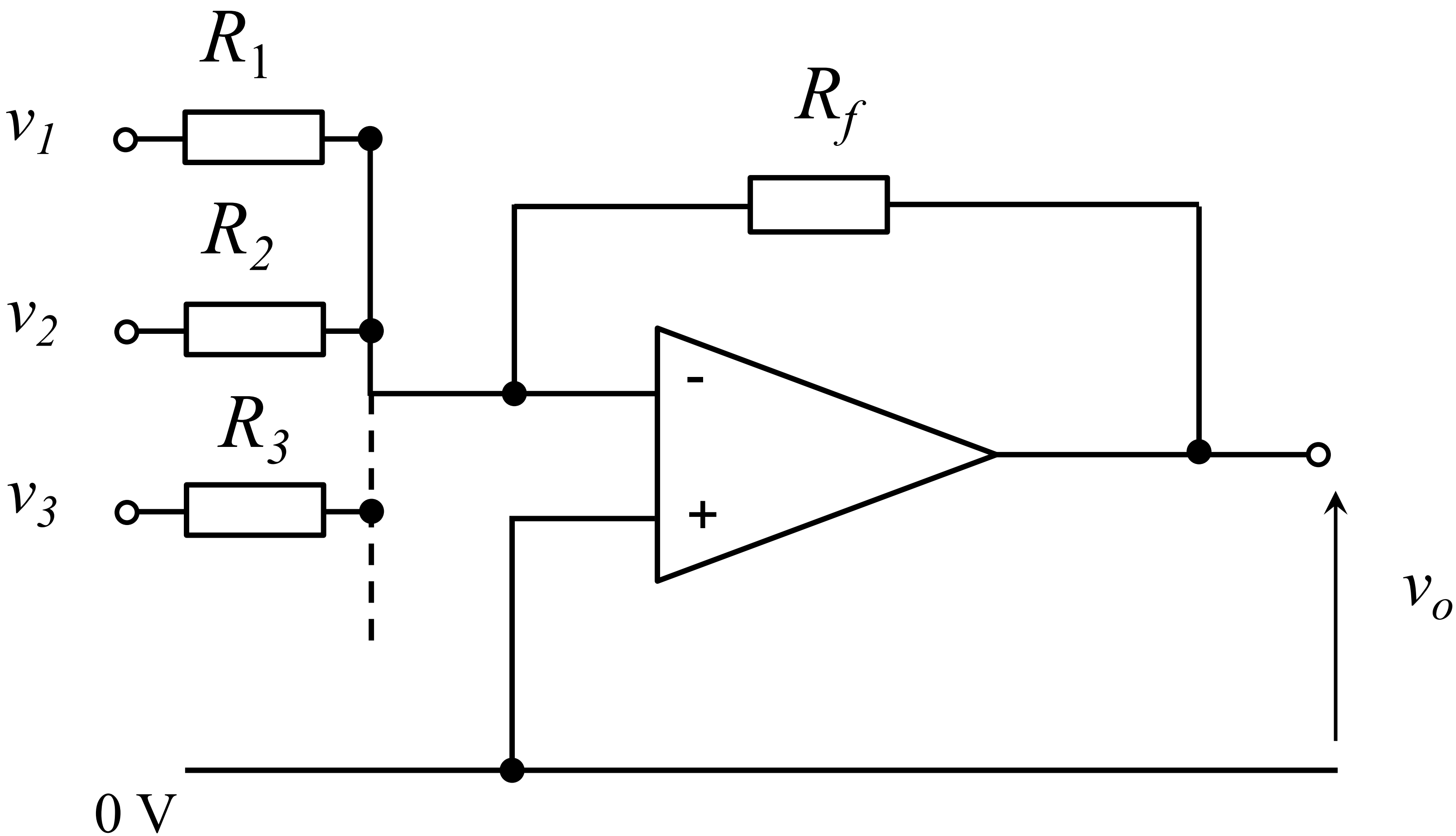
Full feedback is used in this circuit which can be considered a special case of the non-inverting amplifier with $R_4 = 0$ and R_3 omitted.

Theoretically $R_{in} > 100$ GΩ, $R_0 < 1$mΩ

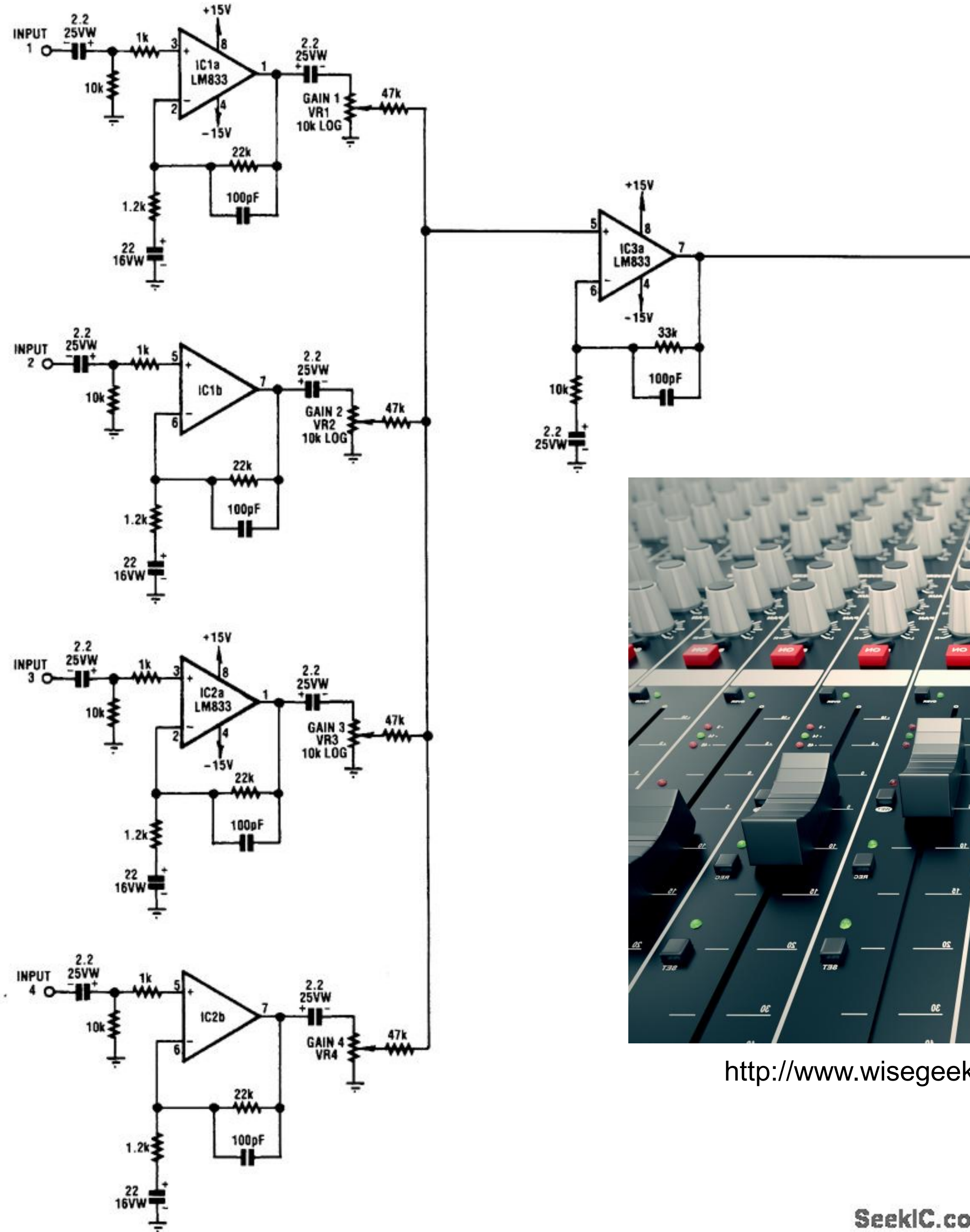
The gain is +1 (to 1 part in 10^5 at DC), the input resistance is very high and the output resistance is very low.

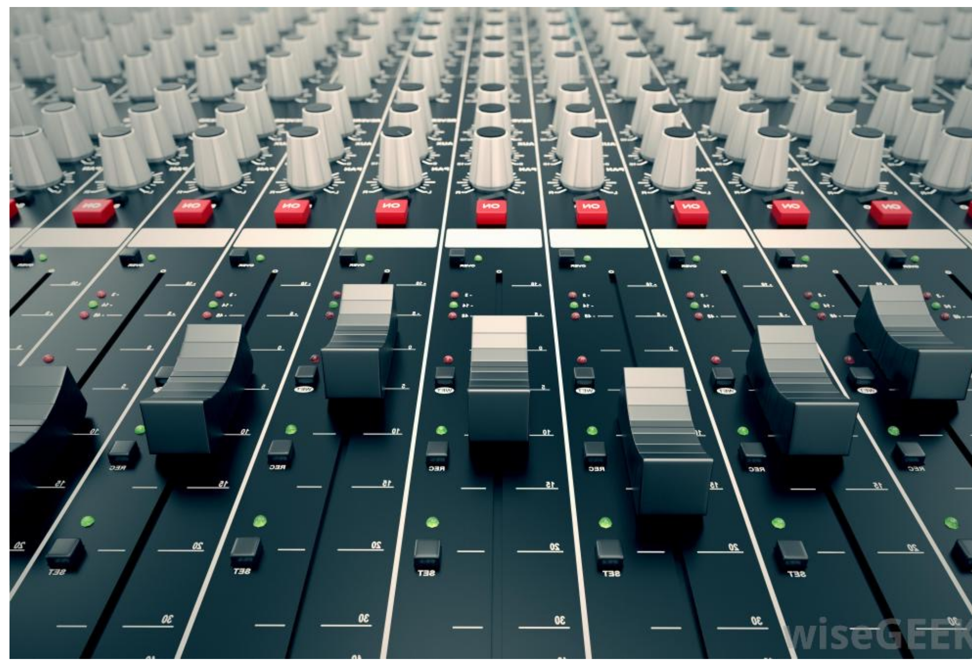
The circuit is used as a ***buffer***, for example at the output of equipment for driving long cables or the input stages of other equipment, or as the input stage to avoid loading high impedance sources.

The Analogue Adder



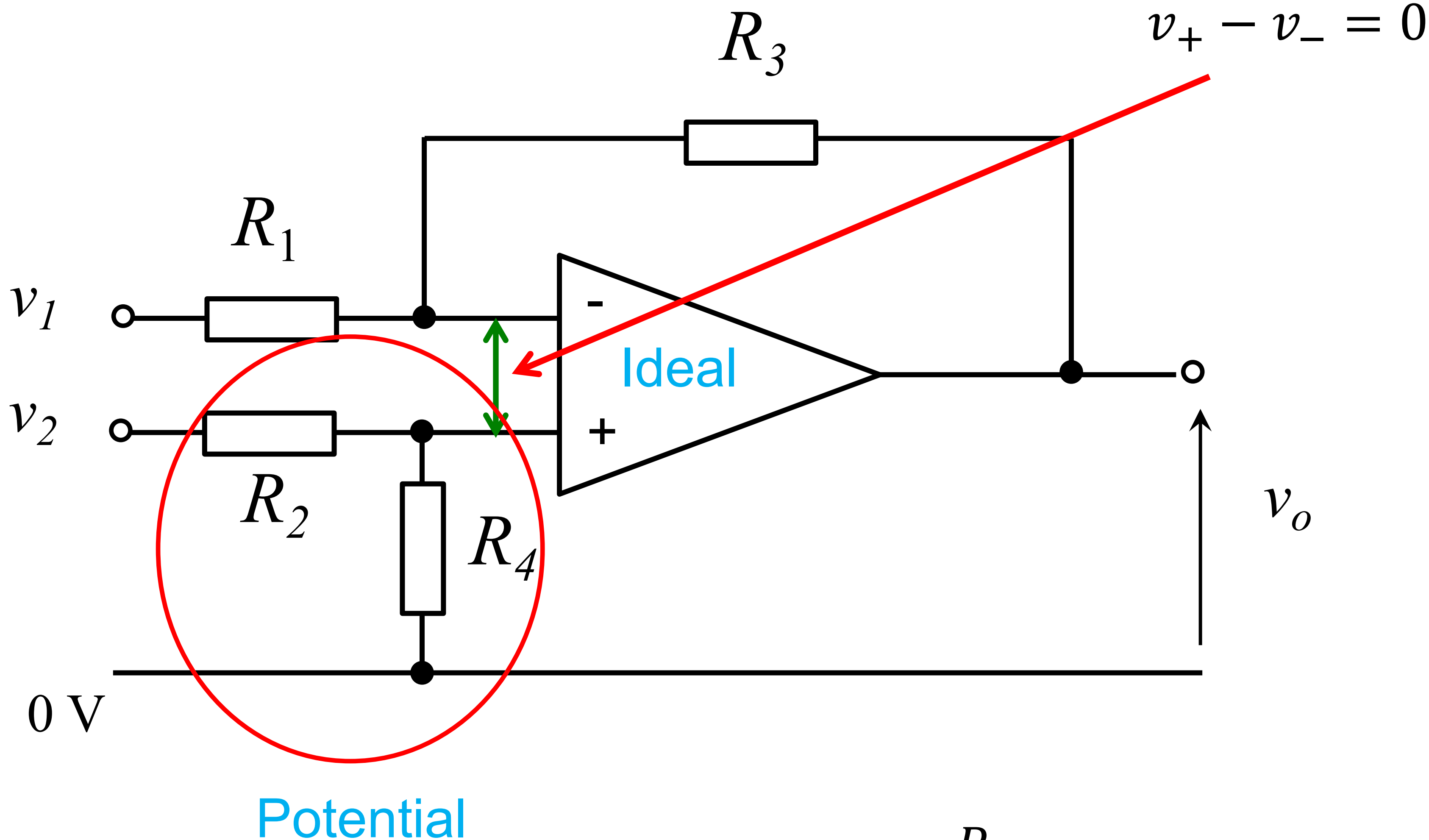
$$v_o = -\left[\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \cdots\right]$$





http://www.wisegeek.org/what-is-an-audio-mixer.htm#

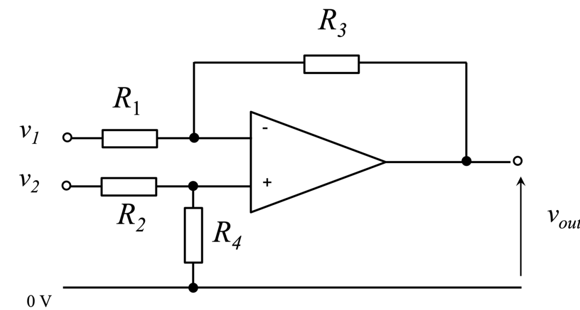
OpAmp Difference Amplifier



$$v_+ = \frac{R_4}{R_2 + R_4} v_2 = v_-$$

Sum currents at the inverting and non-inverting inputs.

$$v_o = -\frac{R_3}{R_1}v_1 + \left(\frac{R_1 + R_3}{R_1}\right)\left(\frac{R_4}{R_2 + R_4}\right)v_2$$



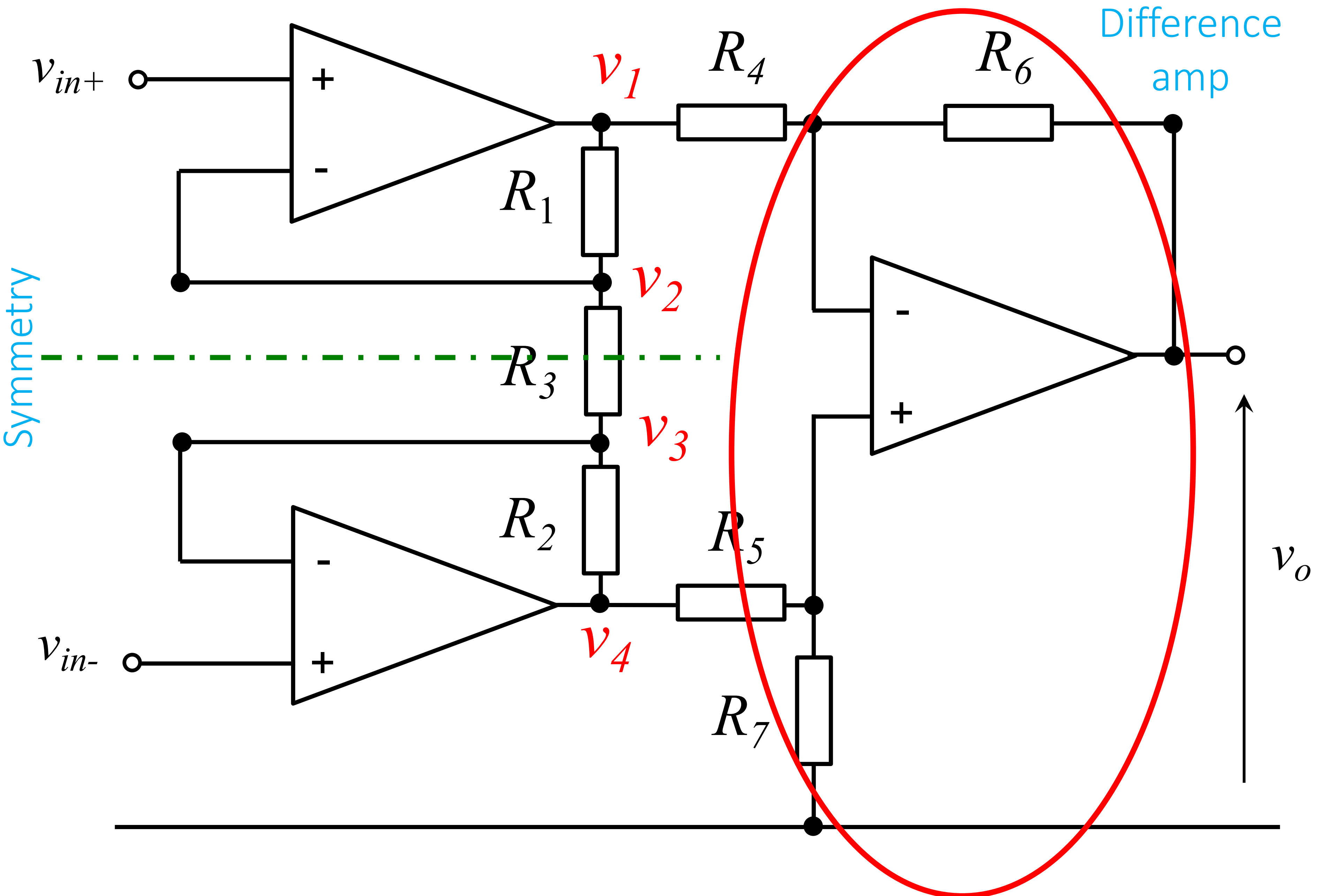
If $R_1 = R_2$ and $R_3 = R_4$ then

$$v_o = \frac{R_3}{R_1}(v_2 - v_1)$$

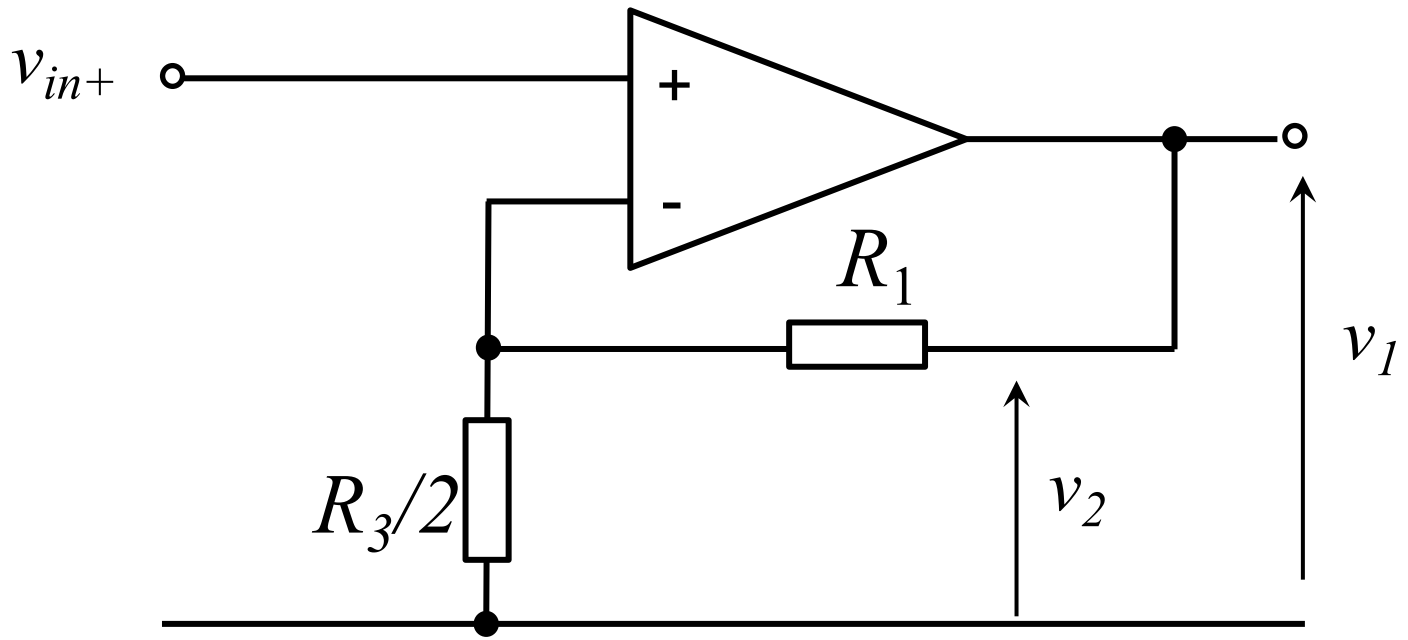
Now try Example Paper 2 Q5

There are two main snags: The input impedance is low and the CMRR is limited by the matching of resistors. The input impedance at the inverting input depends on the voltage at the non-inverting input. The input impedances could be increased by placing buffers in each input but would not improve CMRR.

Three OpAmp Difference Circuit



Using the circuit's symmetry, we can simplify the input stage



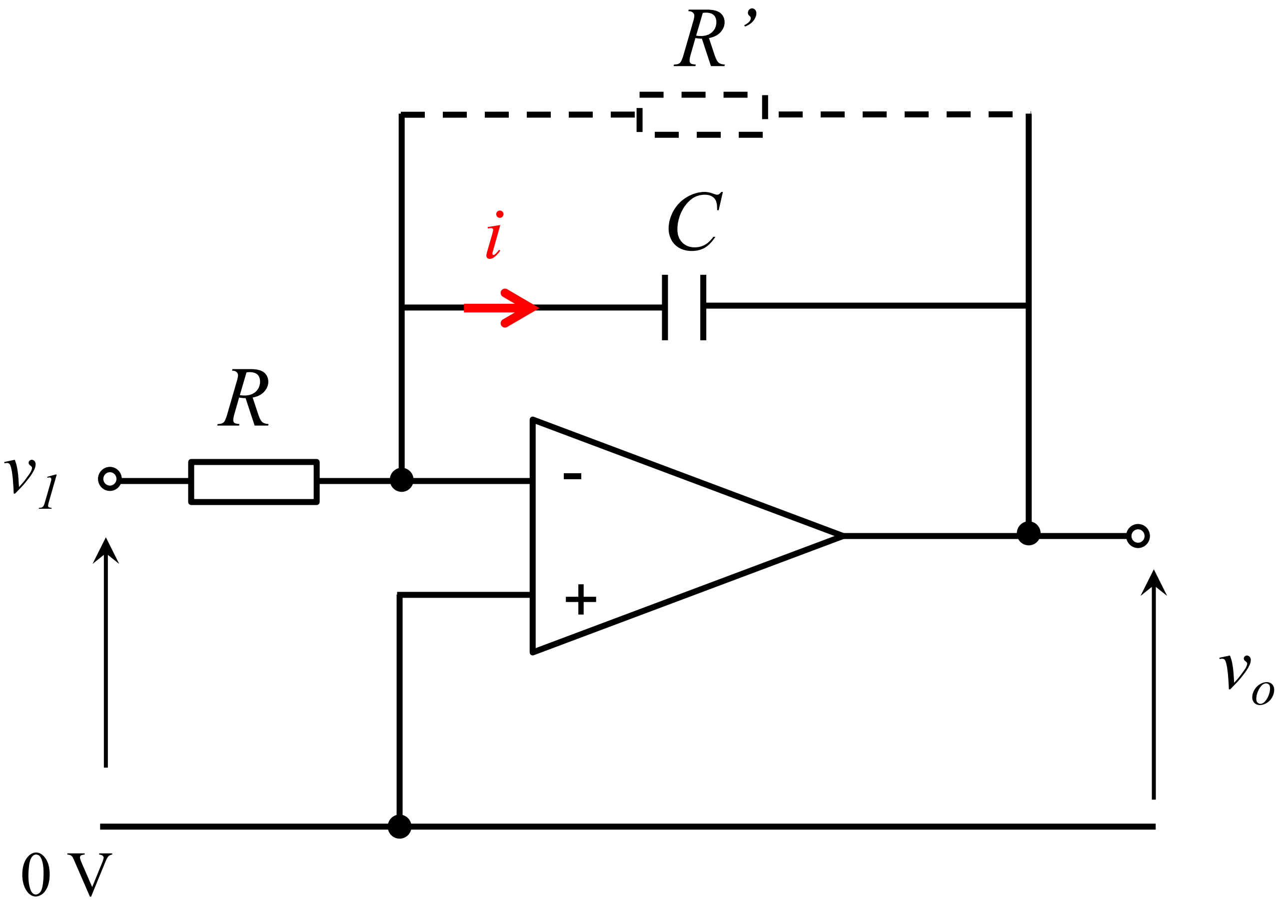
$$Ideal\ op\ amp \implies v_{in+} = v_2$$

$$v_2 = \frac{{R_3}/{2}}{R_1 + {R_3}/{2}} v_1 \implies v_1 = \frac{R_1 + {R_3}/{2}}{{R_3}/{2}} v_{in+}$$

$$Likewise, v_4 = \frac{R_1 + {R_3}/{2}}{{R_3}/{2}} v_{in-}$$

Now try Example Paper 2 Q6

The Integrator

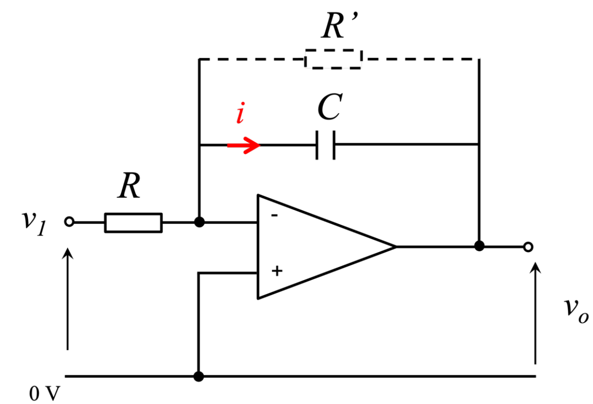


Using transient analysis

$$i = C\frac{dv}{dt}$$

Ideal OpAmp, virtual earth at the inverting input. Sum currents:

$$\frac{v_1 - 0}{R} + C\,\frac{d(v_0 - 0)}{dt} = 0$$



$$v_0 = -\frac{1}{RC}\int_{t_1}^{t_2} v_1 dt$$

Apply limits as appropriate

e.g. Square wave in – triangular wave out

- R' (dotted) can be inserted to limit the dc gain of the circuit to – R'/R.
- A **differentiator** can be made by exchanging the resistor R and capacitor C.

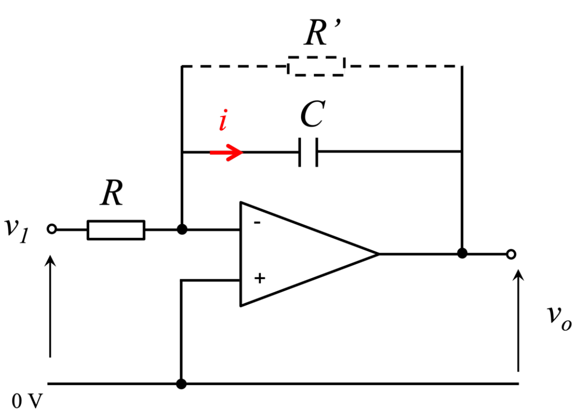
It is also a type of low pass filter

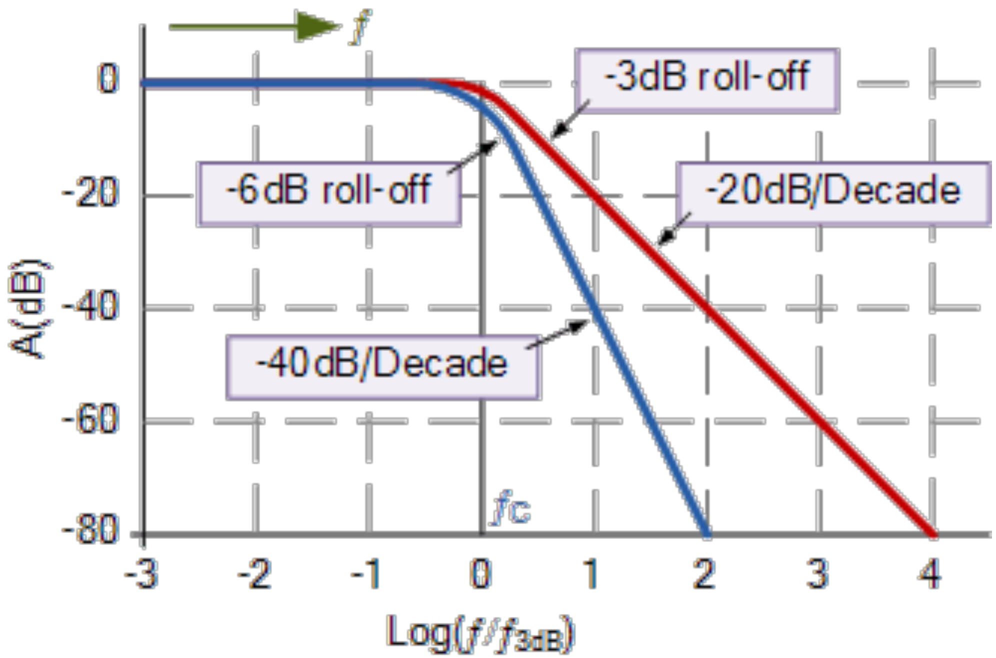
$$\frac{v_0}{v_1} = \frac{Z}{R} \quad \text{where} \quad Z = C \parallel R' = \frac{R'}{1 + j\omega CR'}$$

$$\Rightarrow \frac{v_0}{v_1} = -\frac{R'}{R}\left(\frac{1}{1 + j\omega CR'}\right)$$

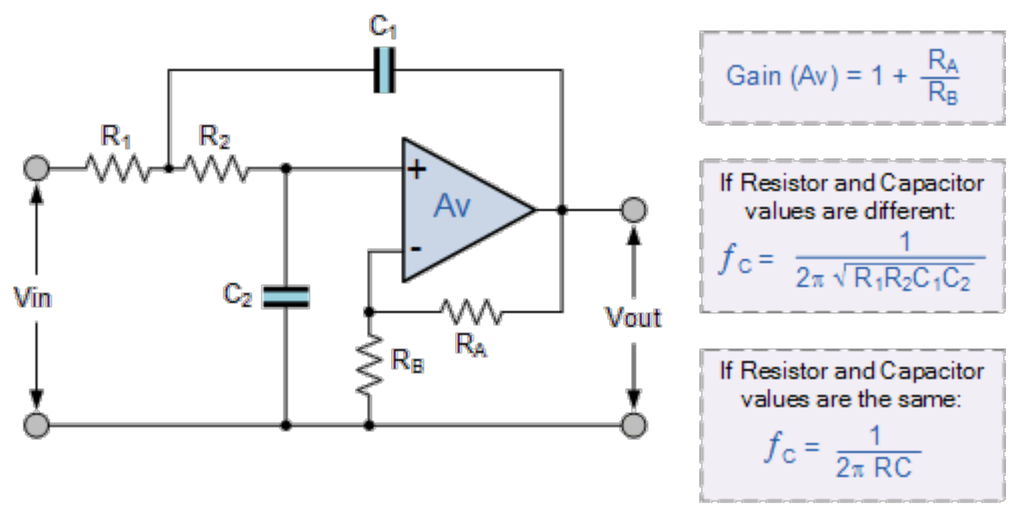
Gain

3dB point





Chebyshev filter

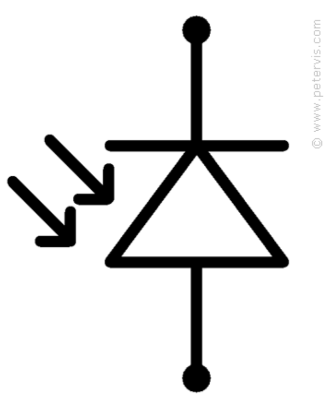


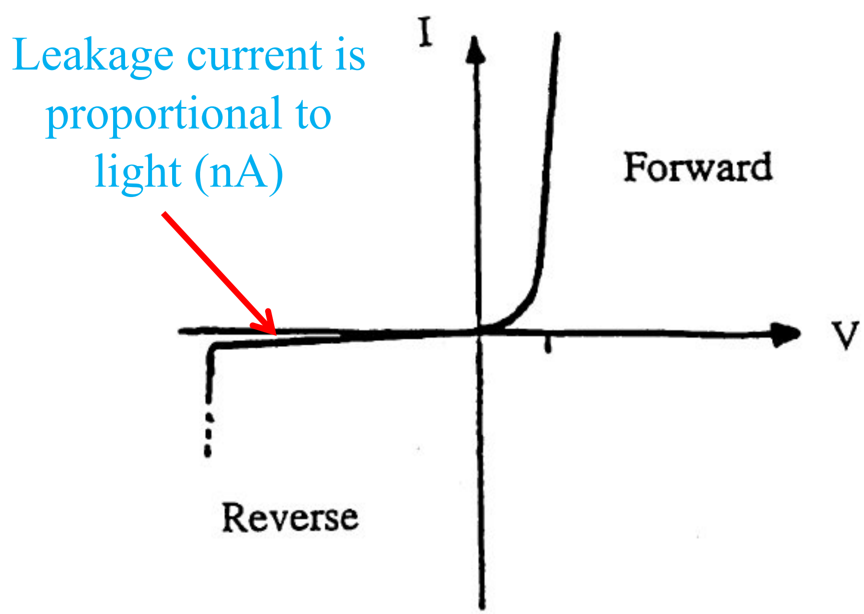
https://www.electronics-tutorials.ws/filter/second-order-filters.html

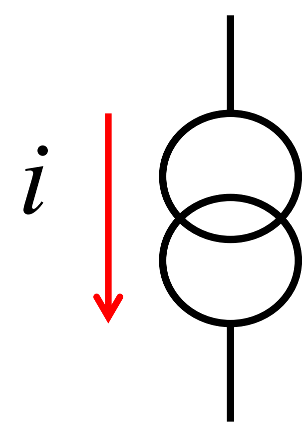
The Transimpedance Amplifier

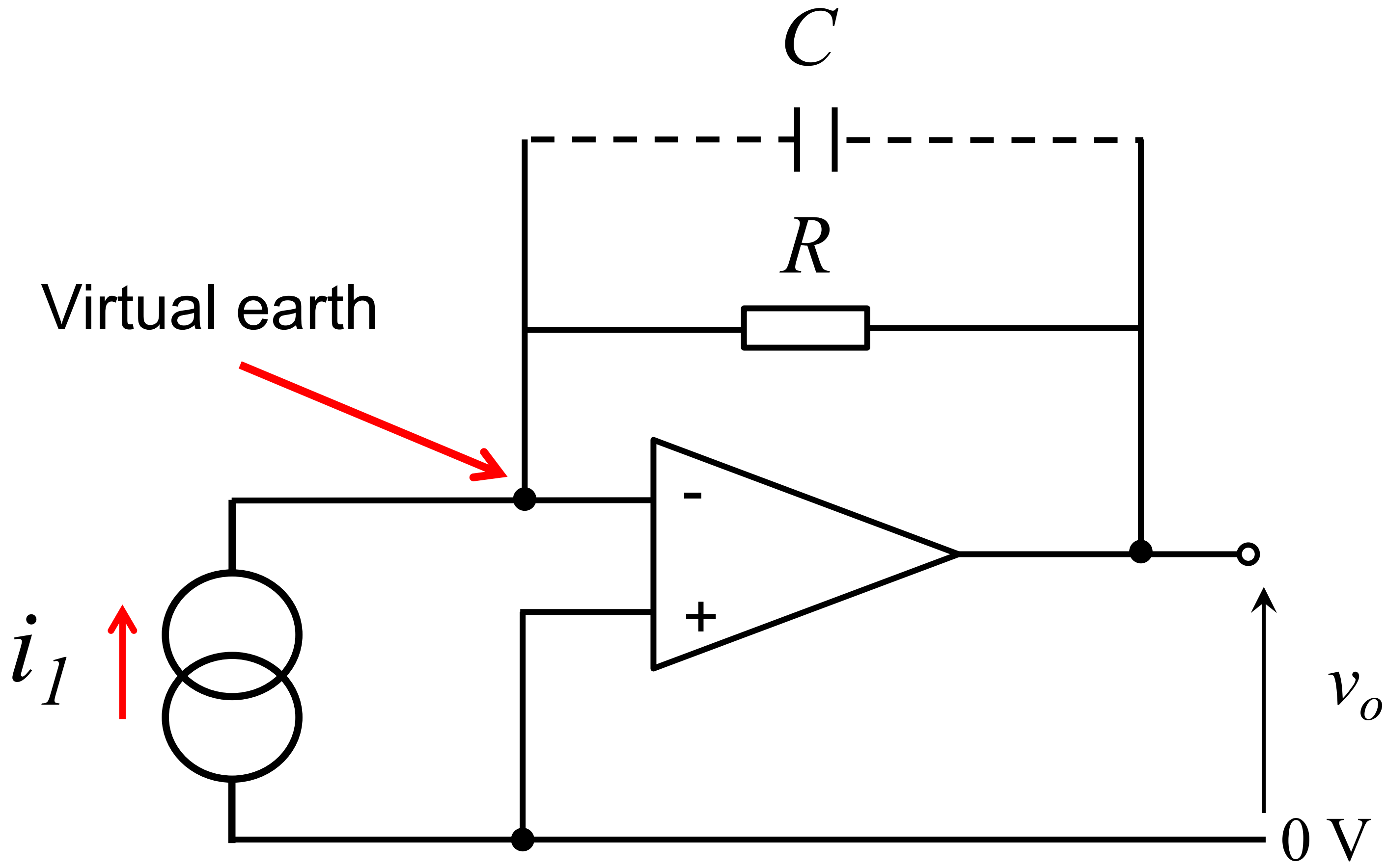
Some signal sources are in fact currents and not voltages. If we have an input current i_1 and an output voltage v_o, then the ratio i_1/v_o is in fact an impedance.

This is a transimpedance amplifier and is often used when in applications such as optical communications, where the input signal comes from a reverse biased photodiode









$$i_1 = \frac{0 - v_0}{R} \qquad \Rightarrow \frac{v_0}{i_1} = -R$$

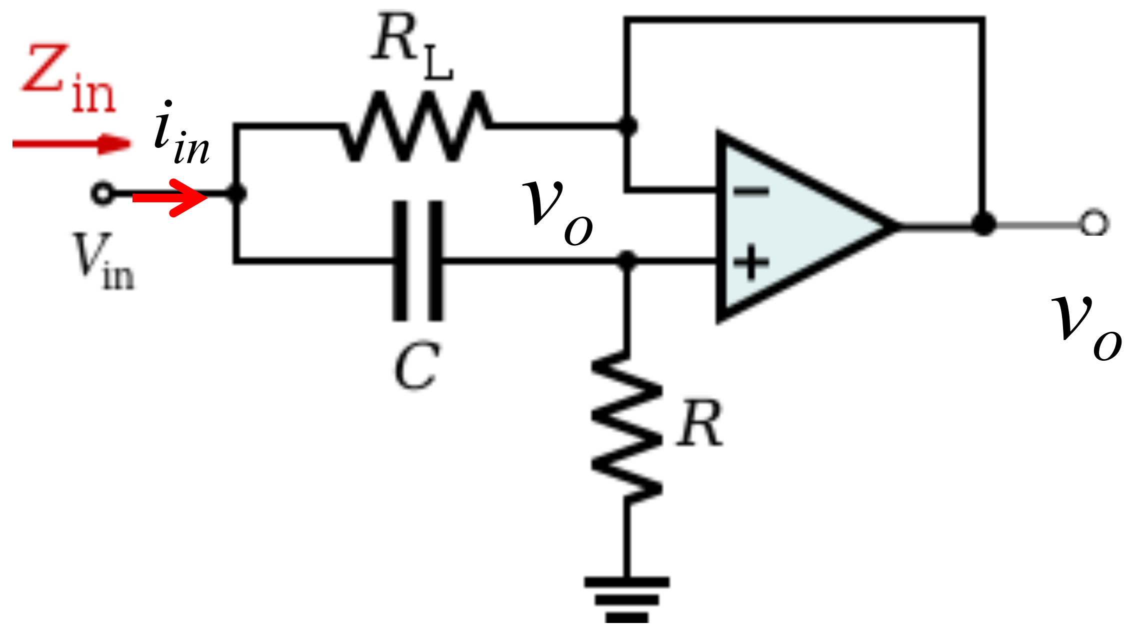
Transimpedance

i_1 = nA, want large R but parasitic C reduces high freq response

The Gyrator

The gyrator is particularly useful in applications where an inductor is required, but the actual physical size of the component is not practical, such as making an inductor on a silicon chip.

An Op-Amp gyrator can be used to make the same value of L but using a capacitor C and resistors instead.



$$Z_{in} = \frac{v_{in}}{i_{in}}$$

Potential divider at + input of OpAmp

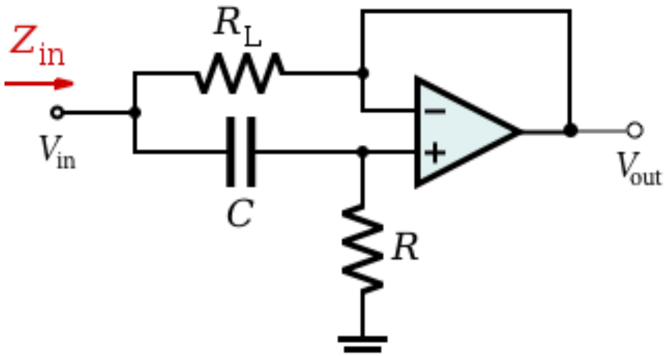
$$v_o = \frac{v_{in} j\omega CR}{1 + j\omega CR}$$

$$i_{in} = \frac{v_{in} - v_o}{R_L} + \frac{v_{in}}{R + \frac{1}{j\omega C}} = v_{in} \frac{(1 + j\omega CR_L)}{R_L(1 + j\omega CR)}$$

$$\Rightarrow Z_{in} = \frac{R_L + j\omega CRR_L}{(1 + j\omega CR_L)}$$

If R is large, then the numerator dominates and looks like an inductor set by C and R_L:

$$L \approx CRR_L$$



IB Paper 5: Analysis of circuits

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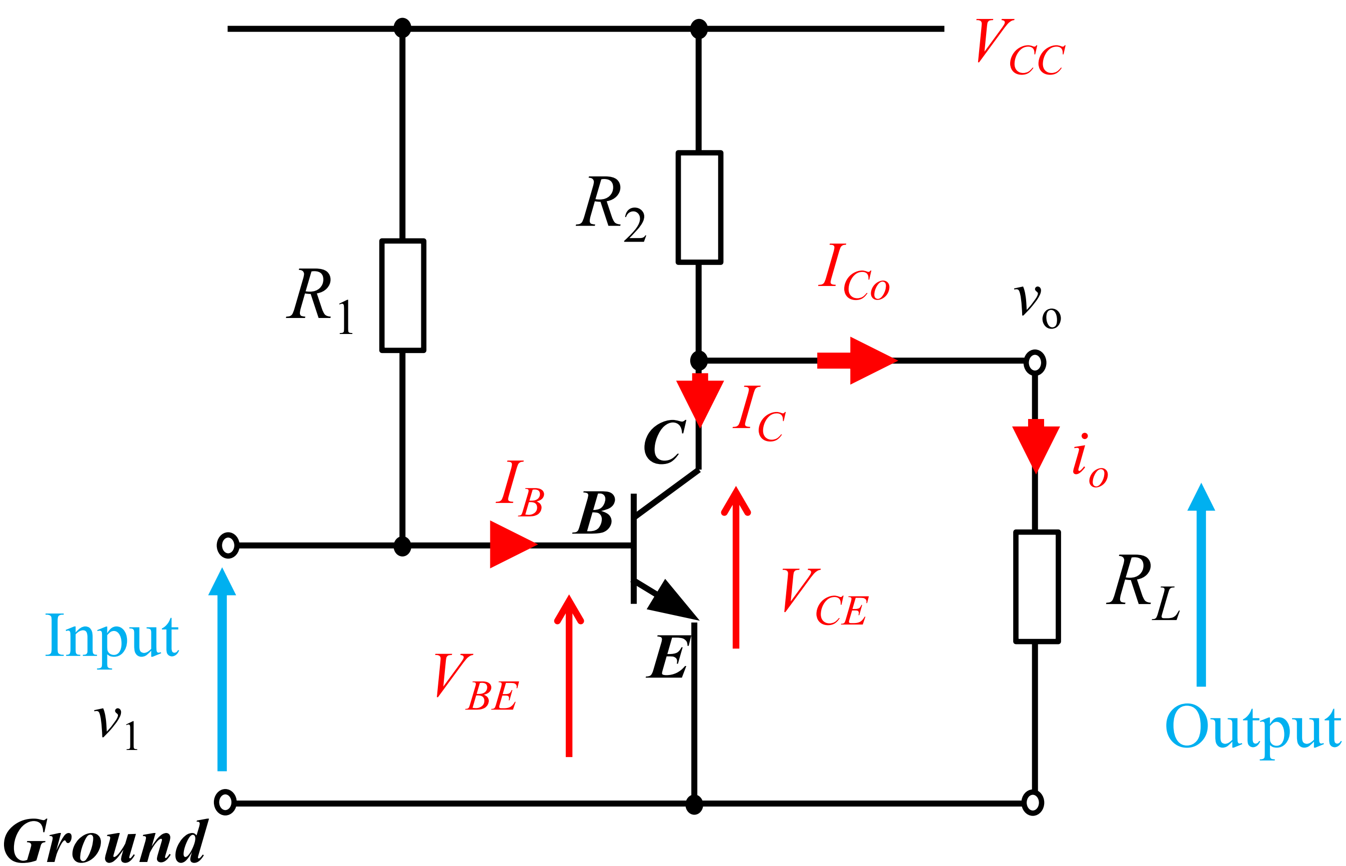
8. *Power amplifiers & Oscillators*

The bipolar transistor as a power amplifier

Power amplifiers can be constructed with both FETs and bipolar transistors.

The simple circuit shown below has a resistive load, R_L in the collector of a bipolar transistor



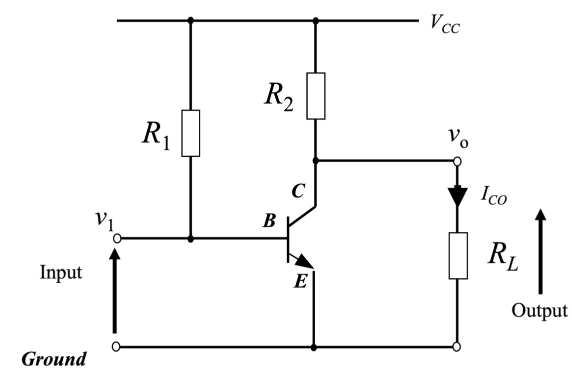
- DC current I_{Co} flows through R_L
- Small-signal input v_1 modulates I_{Co} by an amount i_o
- This leads to an output i_oR_L

It is often usual to choose an operating point at half the maximum current and midway between the maximum and minimum output voltages.

The operating region will be defined by the maximum permissible values of collector current, collector to emitter voltage and power dissipation.

Bias circuits are used to set the operating point

$$V_{CE}, I_B, I_C$$

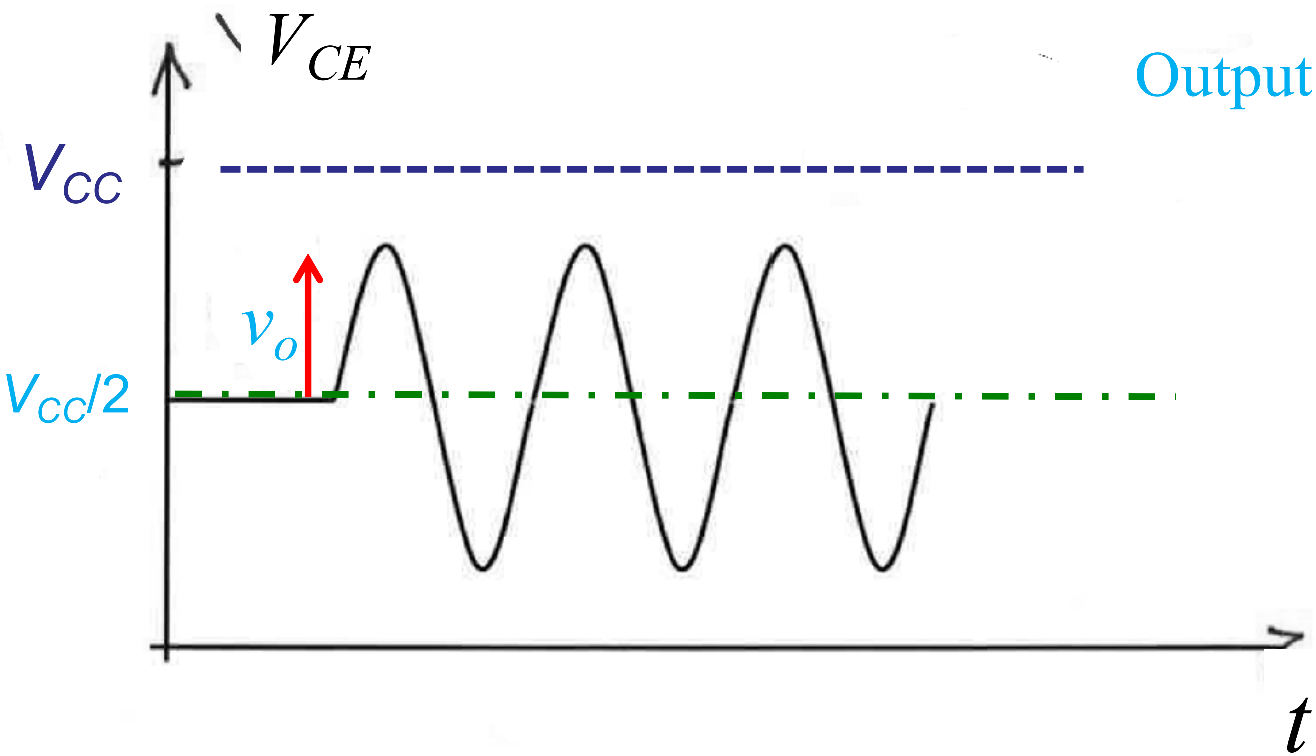


$$V_{CE} = \frac{V_{CC}}{2}$$

$$I_{C0} = \frac{V_{CC}}{2R_L}$$

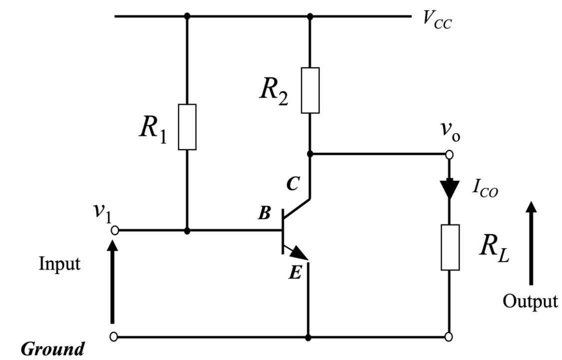
DC current in R_L

Apply v_1 to input, see resulting modulation at output, across R_L



Output: voltage v_o

current $i_o = v_o/R_L$



The ac output power is obtained by considering the voltage swing v_o and the current swing i_o.

Ignoring the effects of saturation, the max voltage excursion is $V_{CC}/2$ and the max current is $V_{CC}/2R_L$, for an operating point at $V_{CC}/2$.

For a sine wave output with these amplitudes the peak output power is:

$$P = \frac{v_o}{\sqrt{2}} \frac{i}{\sqrt{2}} = \frac{v_0^2}{2R_L}$$

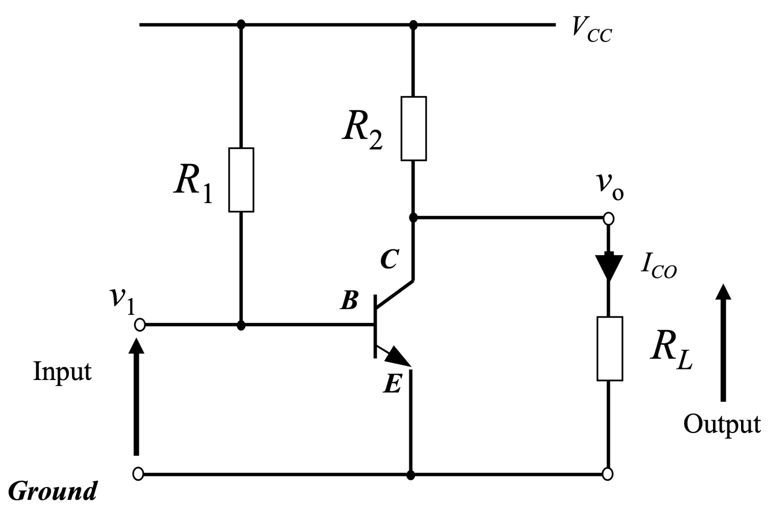
RMS value from peak

with a maximum value of

$$P_{max} = \frac{V_{CC}^2}{8R_L}$$

when

$$v_o = \frac{V_{CC}}{2}$$



The DC input power is the average current drawn multiplied by the supply voltage V_{CC}.

- The steady state current at the operating point is I_{CO} and that current is always drawn, whatever the signal level, as the average value of a sine wave is zero.

The power efficiency of amplifiers is defined as:

$$\eta = \frac{AC\ power\ out}{DC\ power\ in} = \frac{\frac{v_0^2}{2R_L}}{V_{CC} I_{C0}} = \frac{v_0^2}{V_{CC}}$$

Remember

$$I_{C0} = \frac{V_{CC}}{2R_L}$$

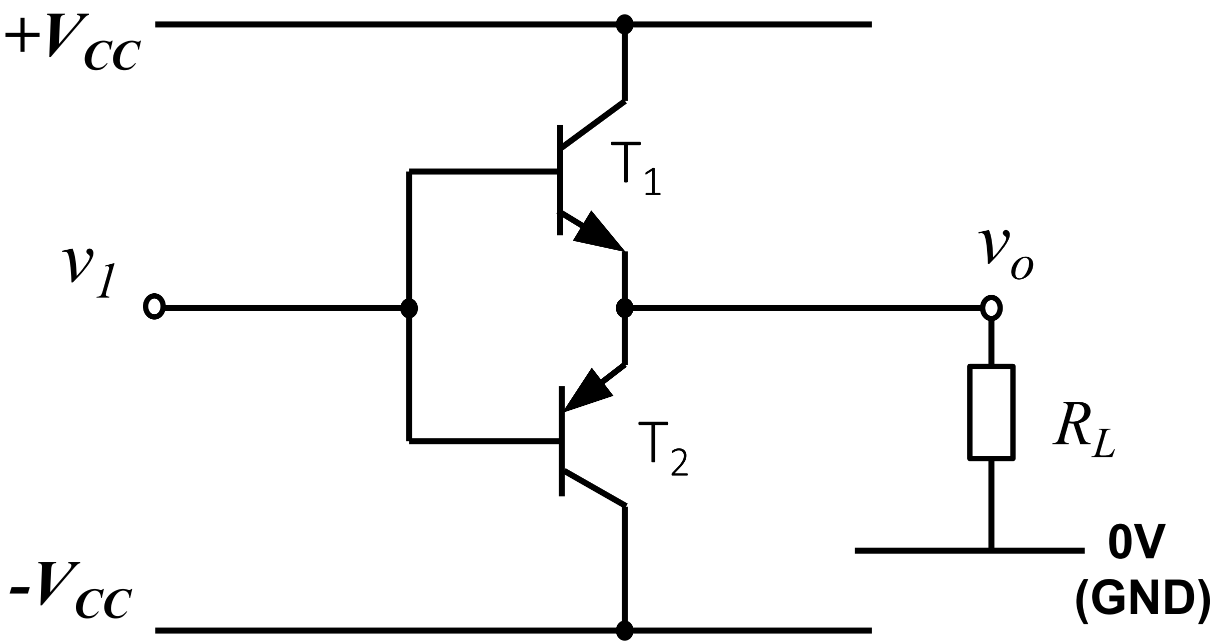
e.g. For an operating point of $V_{CC}/2$, the ***maximum efficiency*** is therefore 25%. This scenario is known as *Class A operation*

- Class A means all transistors have (DC) currents flowing through them at all times (in order to establish their operating point)

The Complementary Source or Emitter Follower

A circuit which overcomes these problems is the *complementary follower (Also known as a **push-pull amplifier**)*

- This can use an *npn* and a *pnp* bipolar transistor or an *n*-channel and a *p*-channel FET.



Note:
- 2 transistors
- 2 supplies

Here T_1 is an *npn* transistor and T_2 is a *pnp* transistor.

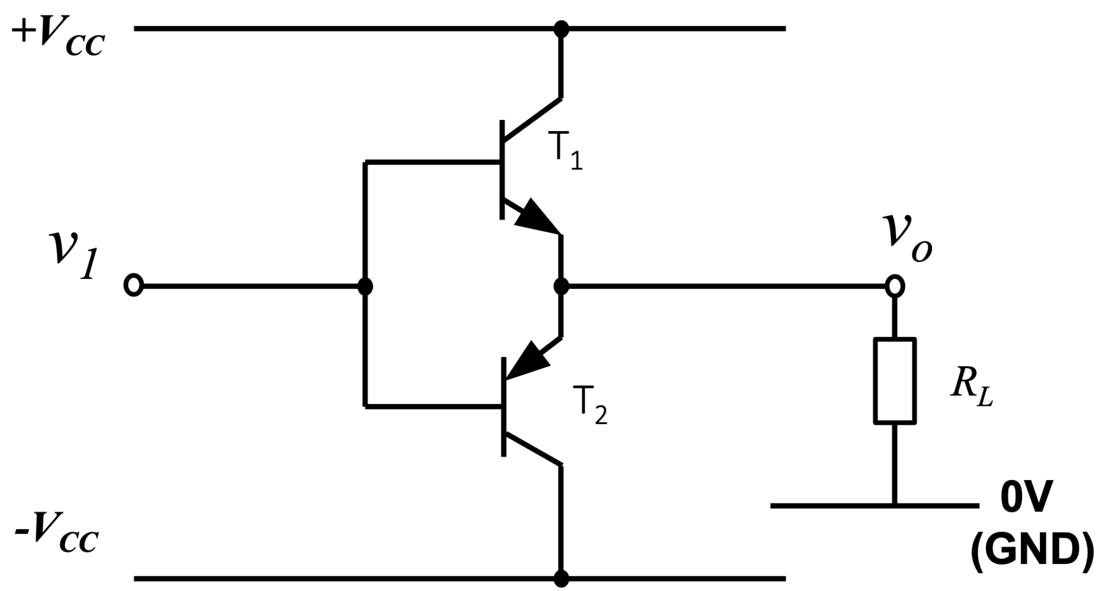
With no input signal both transistors are non-conducting. As v_1 becomes more positive T_1 starts to conduct as an emitter follower, T_2 remains non-conducting.

As v_1 becomes more negative T_2 starts to conduct and behaves as an emitter follower. T_1 remains non-conducting.

Note that for a positive going v_1 rising from zero there is essentially no output until v_1 reaches ~ 0.7 V when T_1 starts to conduct.

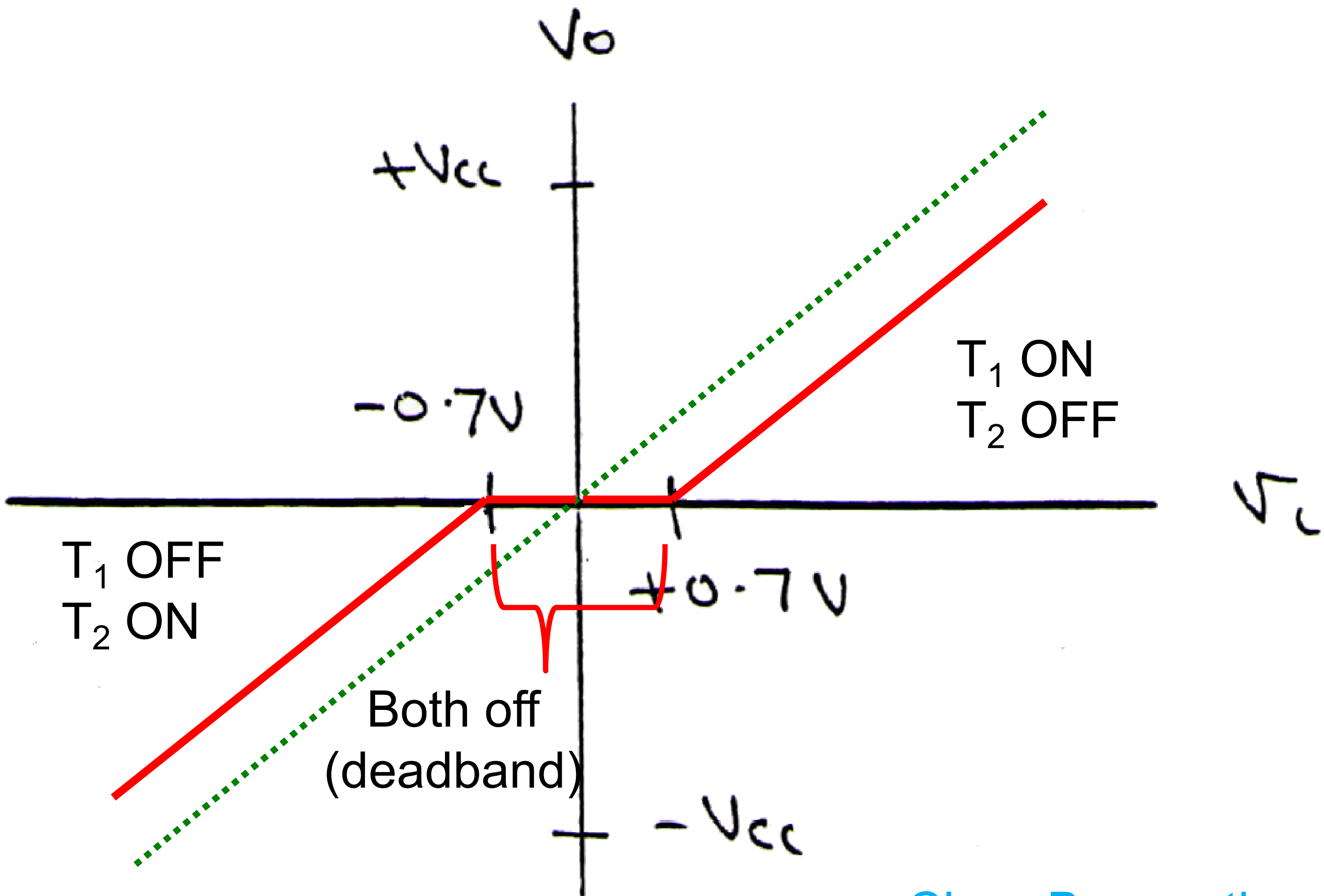
Above this voltage the output rises linearly until T_1 saturates and v_o is close to $+V_{CC}$.

The same follows for a negative input voltage and T_2



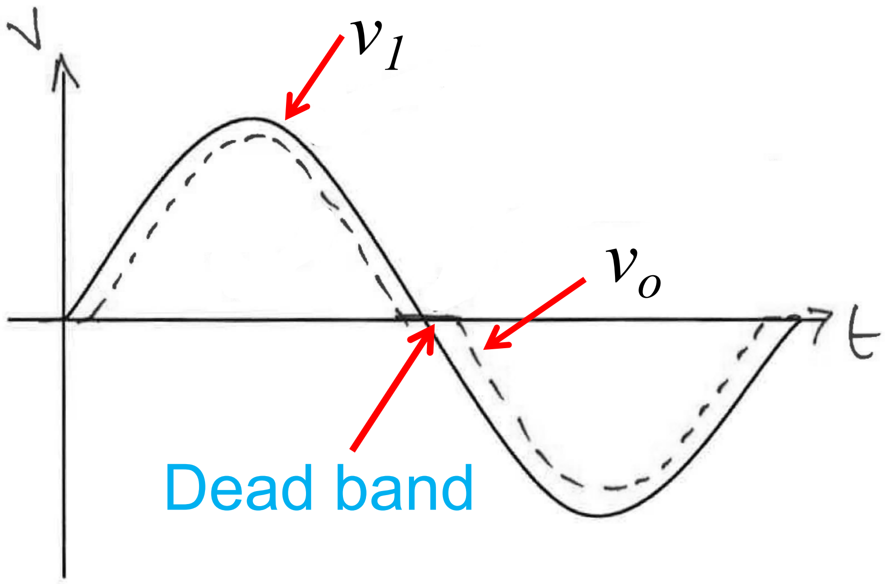
AC properties can be solved with SSM

Transfer characteristic



Class B operation

The effect on a sine wave input is as follows:



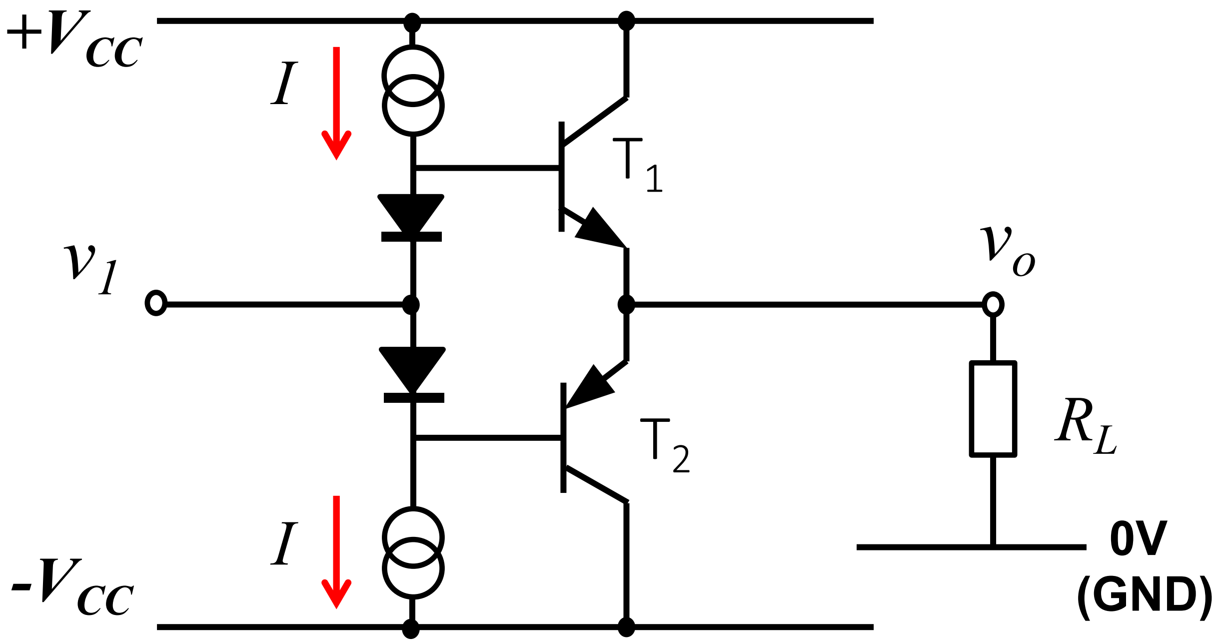
There is an absence of output for -0.7 V < v_1 < +0.7 V, an effect known as ***crossover distortion***.

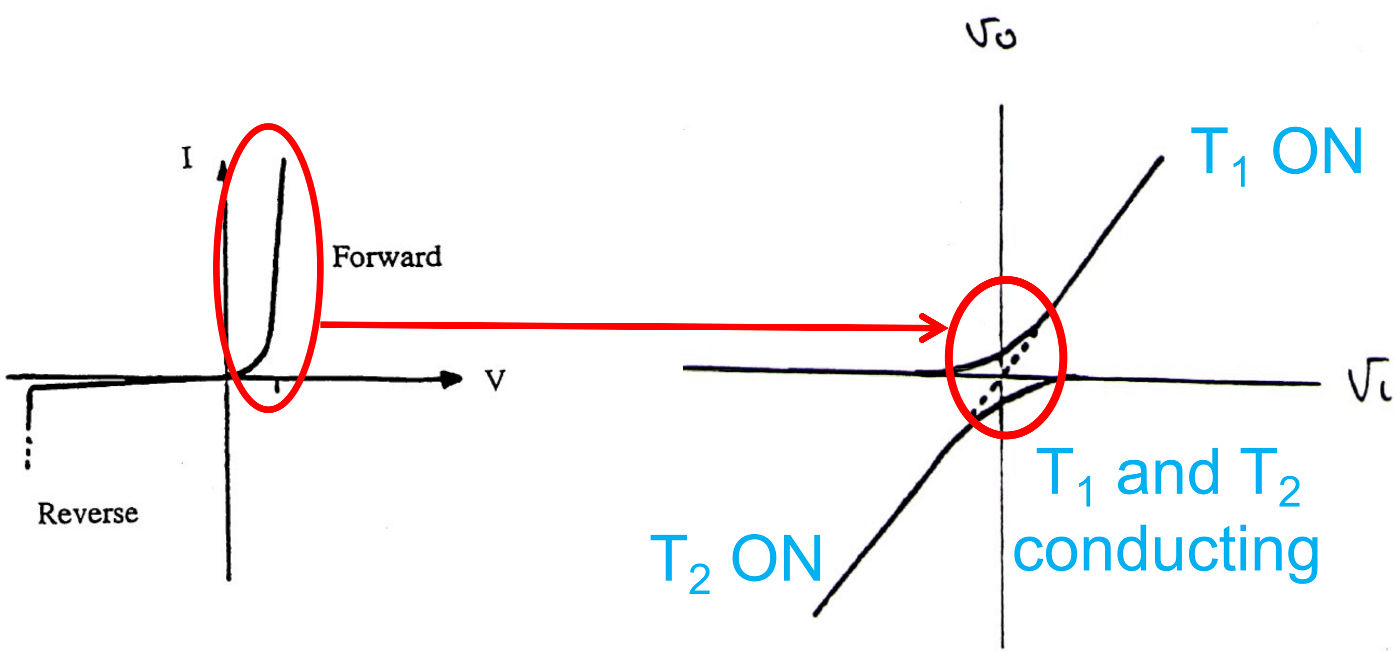
This is particularly unattractive as it leads to high order harmonics, e.g. 15th, 17th etc.

A solution to this problem is to bias the circuit to the edge of conduction in the state of zero input signal.
A circuit which performs this uses the voltage drop across two diodes to set the bias for the two transistors.
The voltage drop across the two diodes should provide the correct voltage to bias the two base-emitter junctions into conduction.
In practice more complex circuits are often used to ensure greater stability of the bias.

Complementary Emitter Follower with Biasing

The current sources shown could be current mirrors as in the differential amplifier section.





Class AB operation

Amplifier classes

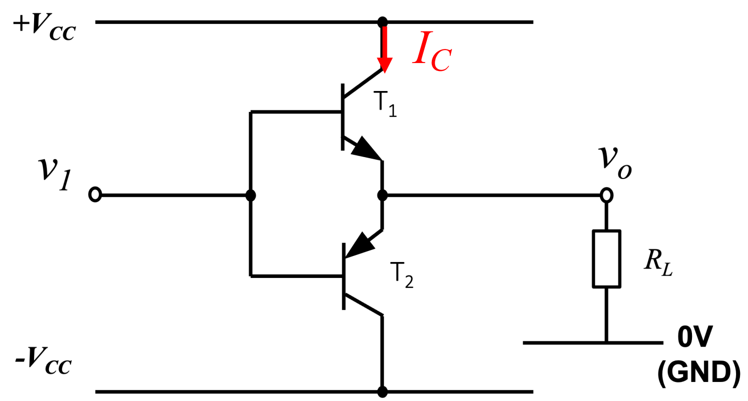
Class A is characterised by the drawing of a constant current from the supply, irrespective of signal level. Most small signal amplifiers are class A

In the complementary emitter follower each transistor conducts for half the cycle - this is known as class B operation. Higher efficiency and zero power dissipation are characteristics of this mode of operation. The problem is crossover distortion.

Operating the transistors so that they conduct for more than half the cycle but less than the whole is called class AB operation. Class AB operation gives a compromise between linearity and power output and efficiency. Class AB are widely used in the output stages of amplifiers from Op-amps through to audio power amplifiers.

Efficiency of the complementary-emitter follower?

$$\eta = \frac{AC\ power\ out}{DC\ power\ in}$$

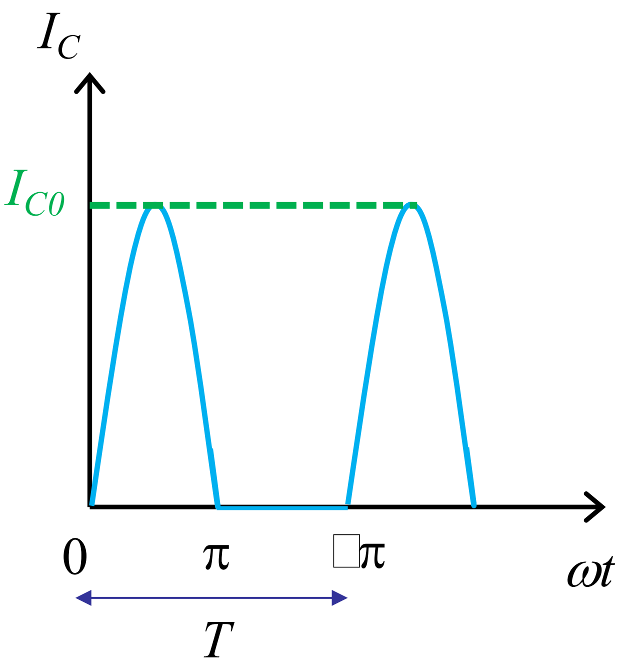


Power dissipated in the load: $\dfrac{v_o^{\ 2}}{2R_L}$

Power drawn from the supply:

$$P_{Supply} = \frac{1}{T}\int_0^{\frac{T}{2}} V_{CC}\, I_C dt$$ (Where $I_C = v_o/R_L$)

Assuming v_1 is sinusoidal, $v_0(t) = v_0\sin(\omega t)$ = $v_0\sin(2\pi t/T)$



$$\Longrightarrow P_{Supply} = \frac{1}{T}\int_0^{\frac{T}{2}} V_{CC}\,\frac{v_0 sin\left(\frac{2\pi t}{T}\right)}{R_L}\, dt = 2V_{CC}\,\frac{v_0}{\pi R_L}$$

$$\Longrightarrow \eta = \frac{v_0^2 \big/ 2R_L}{2V_{CC}\,\frac{v_0}{\pi R_L}} = \frac{\pi}{4}\,\frac{v_0}{V_{CC}}$$

e.g. Where the operating point is such that $v_o = V_{CC}$, the efficiency = $\pi/4$ ~78%

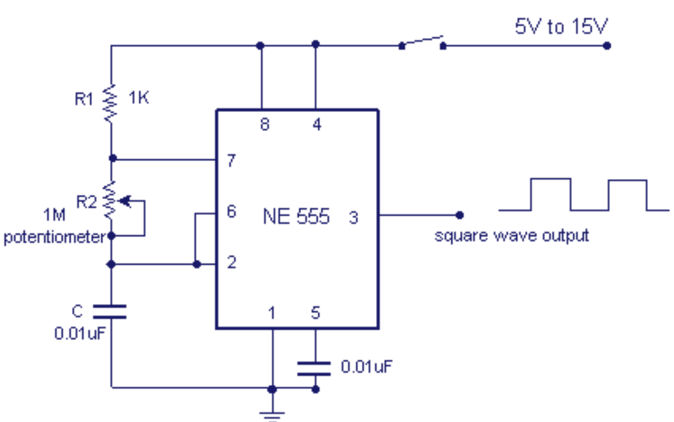
Now try Example Paper 2 Q7

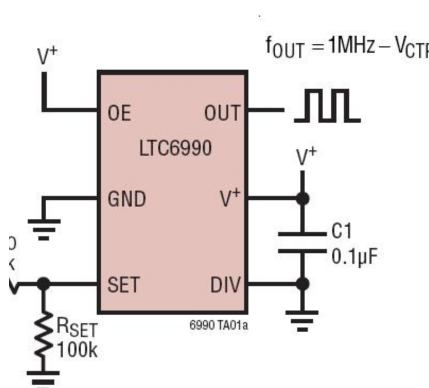
We frequently need sources of signals, for example sine waves or square waves, in electronics. In all cases we desire a stable frequency and a controlled amplitude.

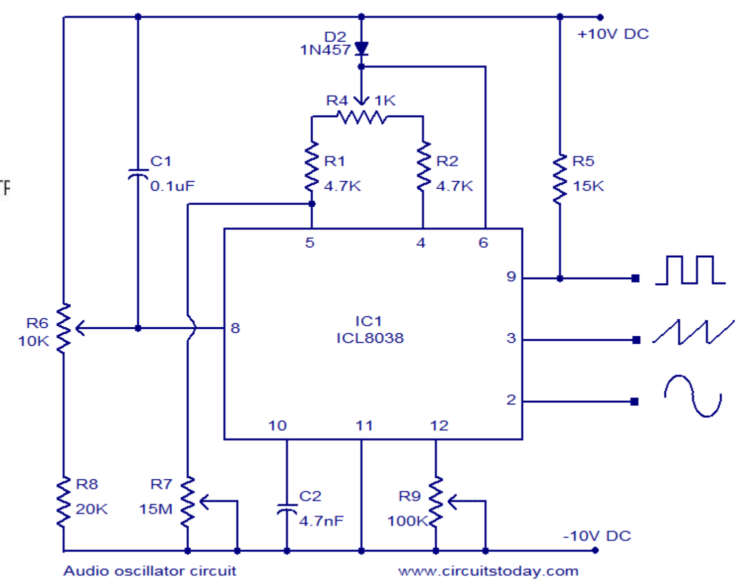
One way of generating sine waves is to use a feedback network with a frequency dependent response (i.e. a filter), combined with a linear amplifier.

This approach is convenient up to moderate frequencies, say 10 MHz, but at higher frequencies other circuits are preferred.

There are also special integrated circuits available.







So far, it has been assumed that the feedback that is applied in circuits is negative, that it is anti-phase to the output.

However, it is possible either by design or through unexpected phase shifts for the feedback to become positive, i.e. in phase with the input.

From section 5, the closed loop gain, *G*, is given by

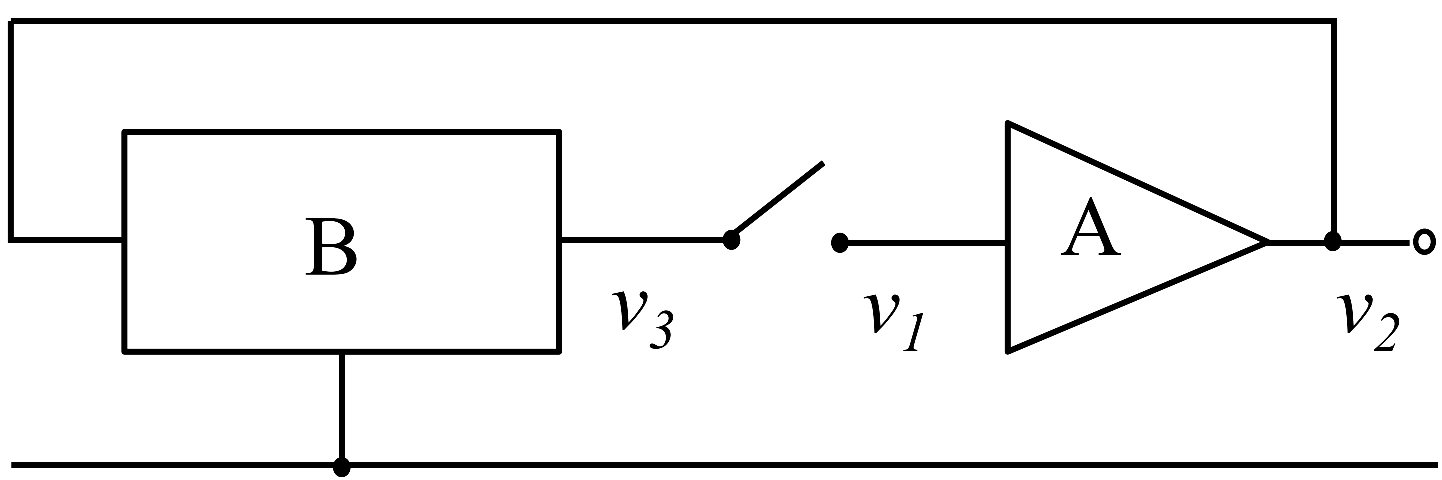
$$Gain = \frac{v_0}{v_1} = \frac{A}{(1 + AB)}$$

The criterion for stability of a system was defined by Nyquist. It is that the phase shift never exceeds 180° relative to true negative feedback for frequencies at which the loop gain AB is greater than 1

If AB = -1, then the gain becomes infinite, and oscillations will occur.

The circuit can produce an output without any external input.

This can be exploited to make oscillators using circuits which give AB = -1 at a particular frequency

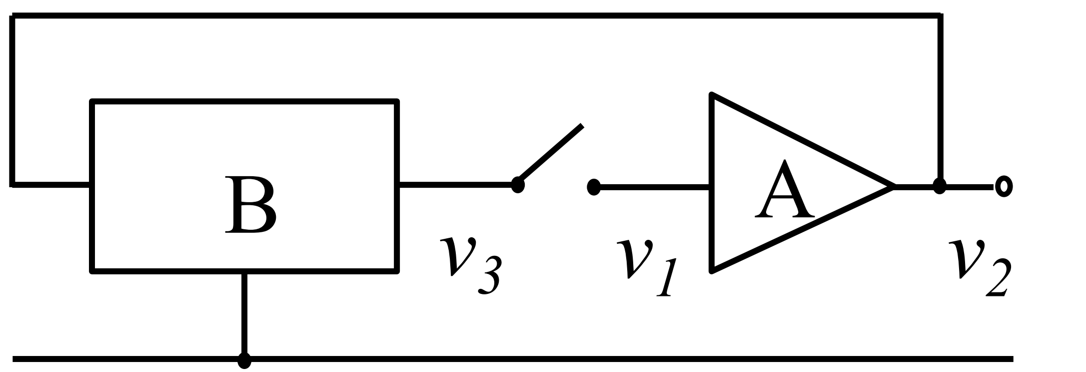


A is an amplifier, and B is a passive network of resistors, capacitors and perhaps inductors.

Assume that the gain of A is constant, and it shows no change in phase over the frequency range of interest.

The amplifier may be inverting or non-inverting

$$A = \frac{v_2}{v_1}$$



The passive network varies with frequency and is of the form

$$\frac{v_3}{v_2} = B(j\omega)$$

Suppose an external sinusoidal input is applied to the amplifier input.

Noise

If the voltage at the output of the passive network is the same as this voltage, the switch can be closed and an output will be maintained with the original input removed.

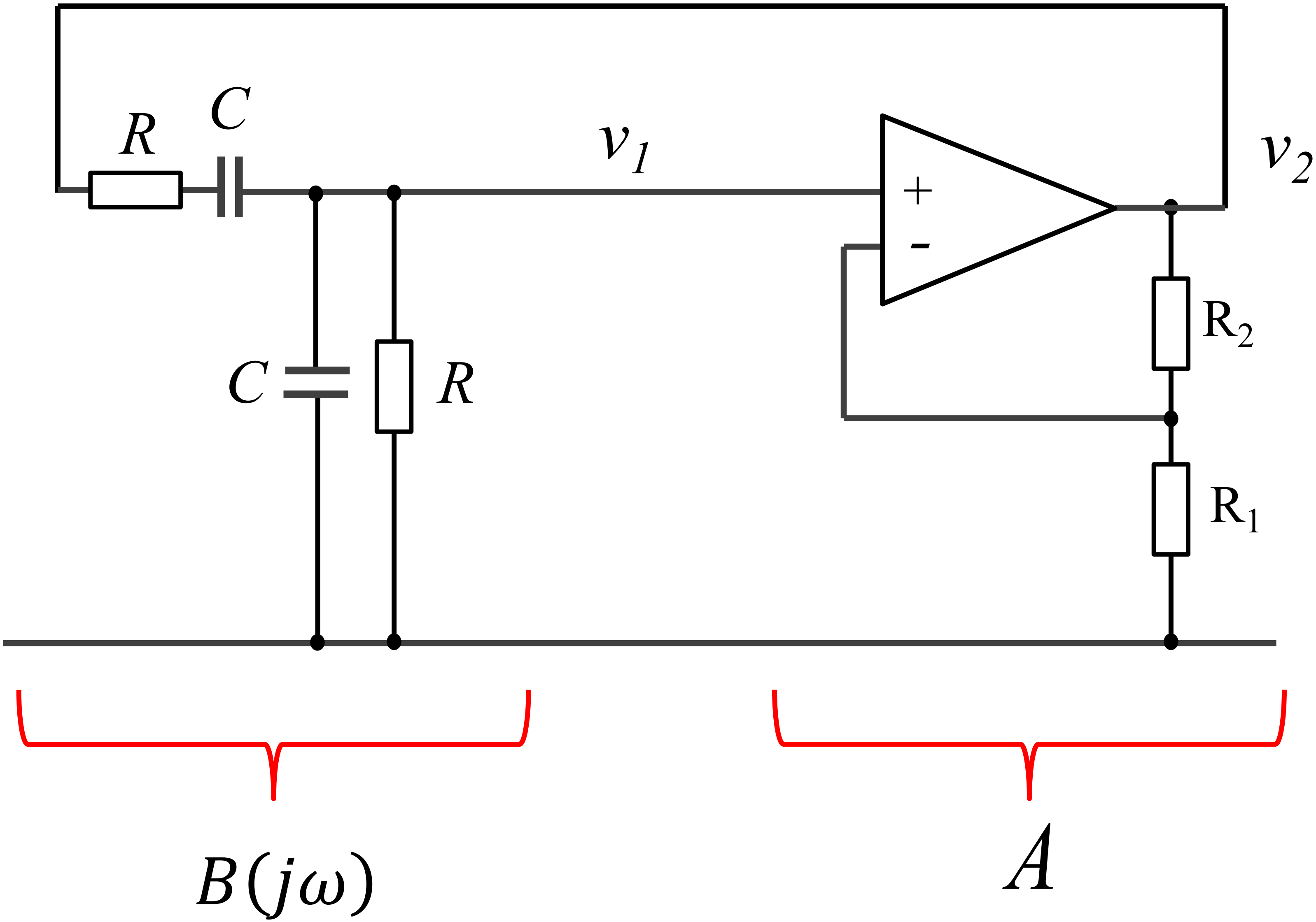
Conditions for oscillation are:

$$\angle AB(j\omega) = 0 \text{ or } 2\pi \quad \text{and} \quad |AB(j\omega)| = 1$$

Loop phase of 0 or 2π

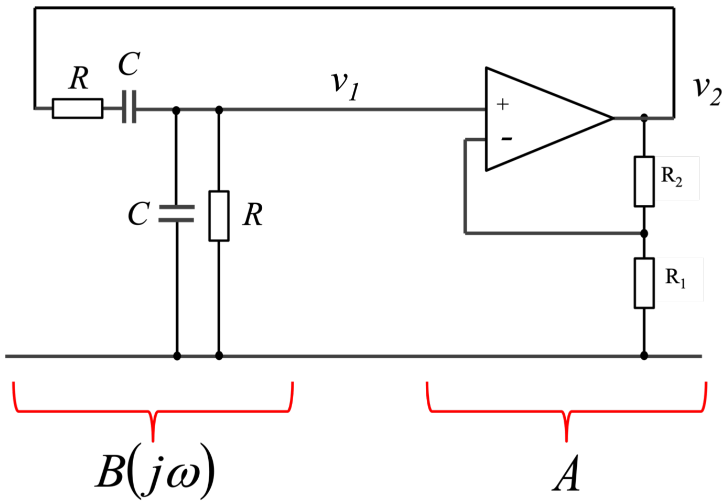
Loop gain of unity

Wien Bridge Oscillator



The amplification is provided by an operational amplifier configured in non-inverting mode.
The passive network consists of two resistors and two capacitors.
Assuming the operational amplifier is ideal, the gain of the amplifier circuit is

$$A=\frac{v_2}{v_1}=1+\frac{R_2}{R_1}$$



Analysing the passive network as a potential divider:

$$B(j\omega)=\frac{v_1}{v_2}=\frac{1}{3+j(\omega RC-1/(\omega RC))}$$

See examples
Paper 2 Q8

The conditions for oscillation are satisfied if:

$$\angle B(j\omega) = 2n\pi \;\ldots\ldots n = 0, 1, 2, \ldots$$

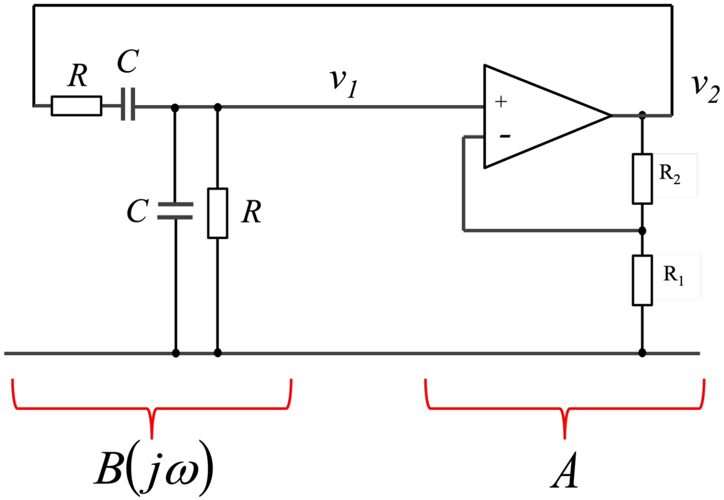
This occurs at an angular frequency of:

$$\omega = \frac{1}{RC}$$

Then $B = 1/3$

Therefore the gain of the amplifier circuit A must be:

$+3$

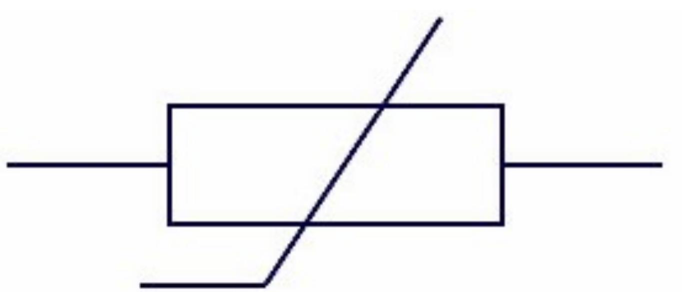


In actual circuits, it is necessary to stabilise the amplitude of oscillations to avoid overloading of the amplifier and consequent distortion.

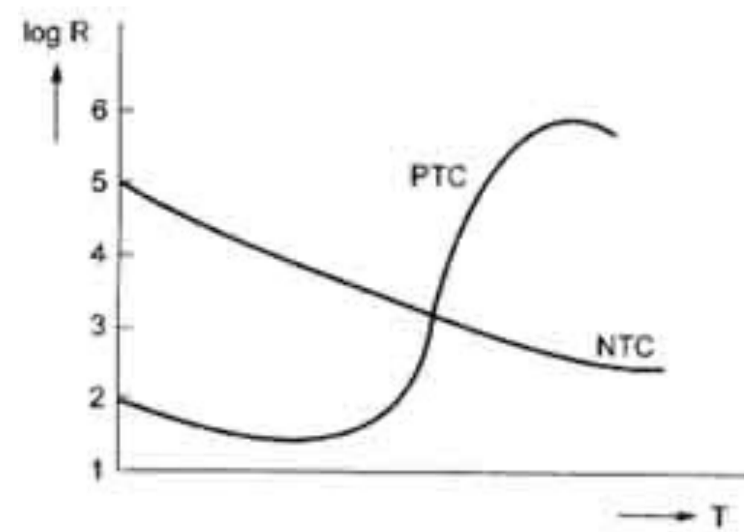
An effective method is to use a thermistor as one of the two resistors which define the gain.

such as R_1 or R_2

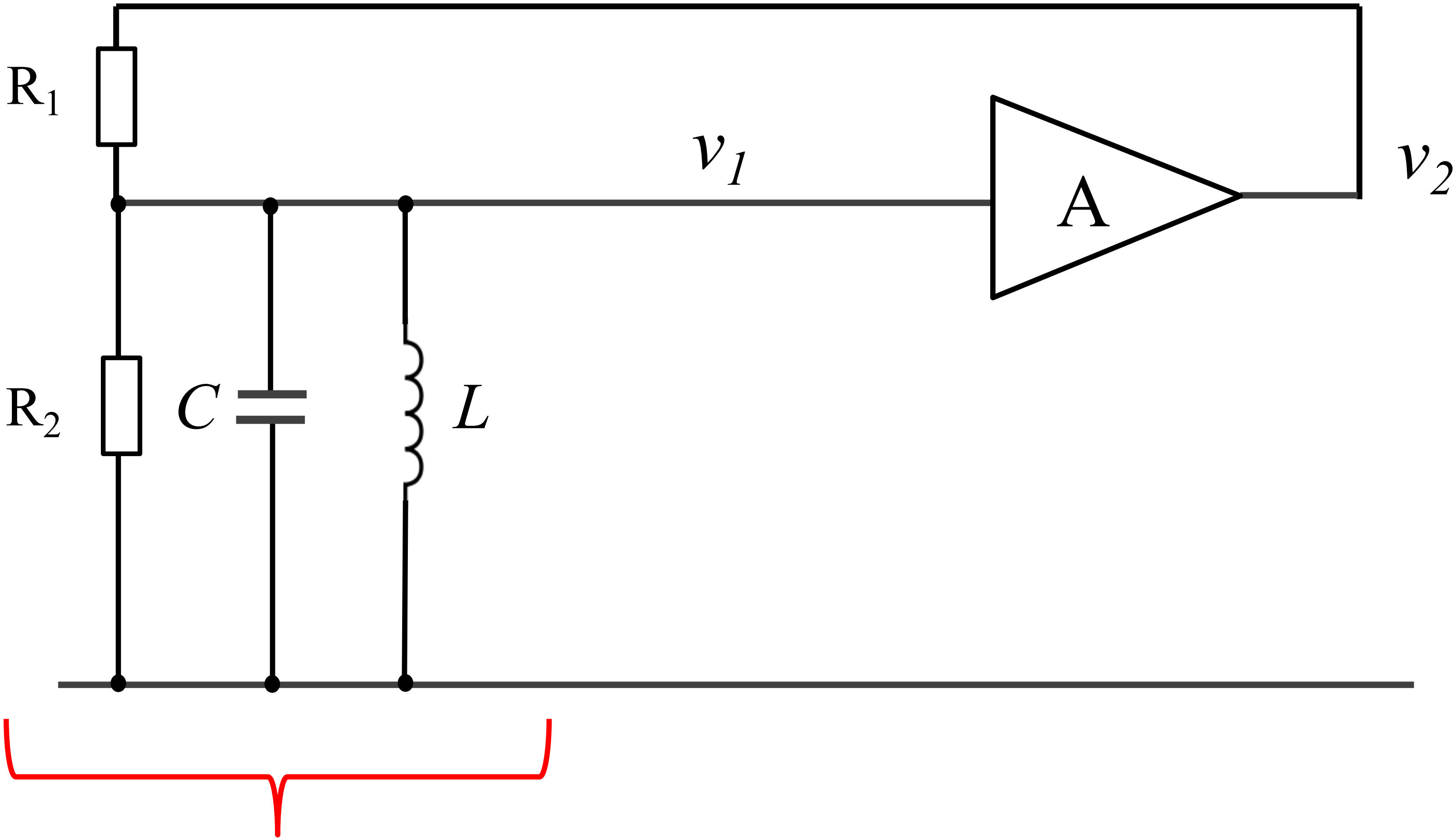
If a thermistor with a negative temperature coefficient of resistance is used for R_i, any increase in output increases the power in the resistance, which heats it up, and reduces its resistance. This reduces the circuit gain, and so equilibrium is reached



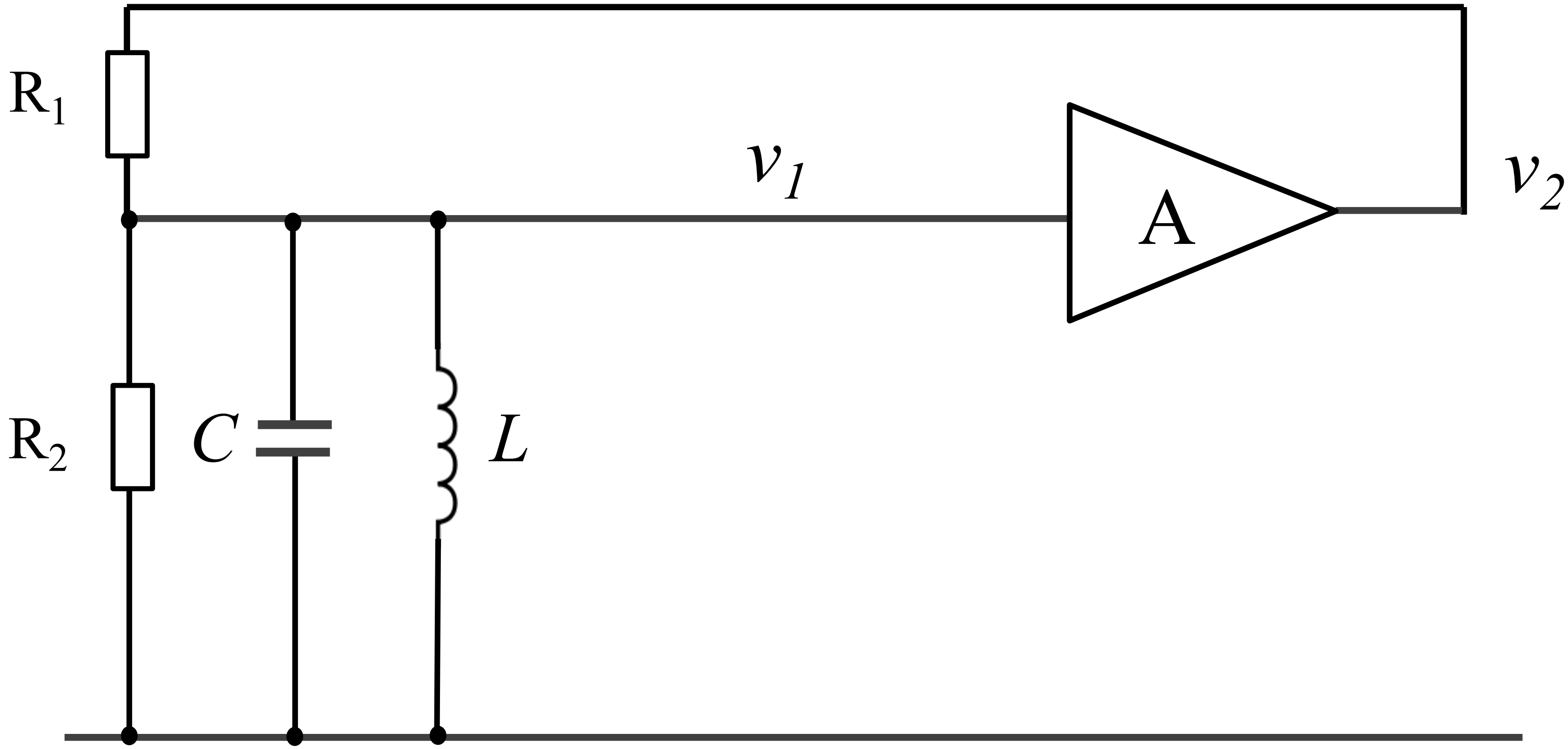
Thermistor Symbol



Oscillators using tuned circuits



$$B(j\omega) = \frac{v_1}{v_2} = \frac{1}{\left(1 + \frac{R_1}{R_2}\right) + jR\left(\omega C - \frac{1}{\omega L}\right)}$$


R1
v1
v2
A
R2
C
L

If the amplifier is non-inverting, the condition for oscillation is:

$$\angle B(j\omega) = 0 \text{ or } 2\pi$$

hence

$$\left(\omega C - \frac{1}{\omega L}\right) = 0 \qquad \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

At this angular frequency the gain of the network is

$$B = \frac{R_2}{R_1 + R_2} \qquad = \frac{1}{A}$$

Now try Example Paper 2 Q8