

# Lecture 6: Generation IV

## 6.1 Construction

Stators experience changing magnetic field  
 $\Rightarrow$  must be laminated to prevent eddy currents.

Rotor either fitted with slip-rings and brushes,  
 or a small 'on-board' a.c. generator and rectifier  
 to supply the field current.

Two types of rotor construction used:

### Cylindrical rotor - high speed machines

Steam turbines most efficient at high speed  $\Rightarrow$   
 2 pole (3000 rpm) or 4-pole (1500 rpm)  
 generators are used. Rotors must be long and  
 thin to reduce centrifugal mechanical stresses.

Rotor experiences constant field  $\Rightarrow$  no need to  
 laminate  $\Rightarrow$  usually made from a single  
 forging of solid steel.

### Salient-pole rotor - low speed machines

Hydro-turbines most efficient at low speed  $\Rightarrow$   
 30 poles or more ( $\leq 200$  rpm). Manufactured by  
 joining separately-wound poles to rotor core.

In this lecture, we will briefly look at the construction of synchronous machines, and then see how the theory developed in the previous three lectures is applied to predict the performance of synchronous machines by considering a number of examples. We will also look at the limitations on synchronous machines using the idea of the operating chart.

The stator of the synchronous machine experiences a time-varying magnetic field at the supply frequency. It must therefore be laminated to reduce eddy current losses. In order to supply the d.c. rotor field current, the rotor is usually fitted with slip-rings, to which a d.c. voltage source may be connected via carbon brushes which are in electrical contact with the slip-rings. In very large generators, the field current can be thousands of amps, and in this case it is more usual to generate the field current 'on-board' the rotor, with a small inverted-geometry alternator. The a.c. output from this is rectified by a diode bridge which is mounted on the rotor, to provide the d.c. field current. Steam and gas turbines must be operated at high speed for maximum efficiency. Therefore, 2 and 4 pole generators are used (3000 rpm and 1500 rpm respectively). Because of the high rotational speed, a single cylindrical forging is favoured, because it is easily balanced and is mechanically robust. Also, a long, thin design is necessary to reduce mechanical stresses produced at these high speeds. Hydro-electric and also diesel generation require low speed machines, with 30 poles or more, corresponding to a rotational speed of 200 rpm. Again, this is due to the characteristics of the prime-mover. Because of the low speed, a short, wide design is

Removal of heat is important. Water cooled stator windings (hollow coils). Both stator and rotor are gas cooled.

## 6.2 Rating

Generator current limited by heating of stator coils due to resistance, voltage limited by flux (iron losses) and insulation considerations.

∴ Generators have a VA rating  $= 3V_{ph}I_{ph}$

### Example 6.1

Find the rated phase current of a 22 kV, 500 MVA three-phase star-connected generator.

$$S_{\max} = 3V_{ph}I_{ph} \quad I_{ph} = \frac{500 \times 10^6}{3 \times 22 \times 10^3 / \sqrt{3}} = 13.1 \text{ kA}$$

### Example 6.2

Generator of example 6.1 has 4 poles,  $X_s = 1.2 \Omega$ , and is connected to a 22 kV, 50 Hz infinite bus. Prime-mover power is set to 300 MW, find E and  $\delta$  for load of power factor 0.8 lag. Also find prime-mover speed and torque.

Step 1 Find speed of rotation in rad/s

$$\omega_s = \omega / p = 2\pi f / p = 100\pi / 2 = 157.1 \text{ rad / s}$$

used. Also, for ease of winding the rotor, separately-wound poles are bolted or dove-tailed to a solid steel core. These are known as salient-pole machines.

Finally, removal of heat from large generators is of great importance. For example, if a 1000 MVA generator is 98% efficient, there will be 20 MW of power dissipated as heat, which must be removed ! The stator is cooled by passing water through the hollow stator conductors. Both rotor and stator are cooled by forcing gas (usually hydrogen) in a cooling circuit which passes along the air-gap, and sometimes also through axial and radial ventilation ducts in the stator.

Like transformers synchronous generators are given a volt-amp rating, and a rated terminal voltage. The latter is limited by the flux levels in the machine (flux is roughly proportional to terminal voltage), and also insulation. The former then becomes a limit on the current which can flow in the stator windings. This is limited because the windings possess resistance, and excessive current would cause overheating due to  $I^2R$  losses. This in turn causes damage to the generator - the winding insulation starts to burn, for example. Since these losses only depend on the magnitude of the current, not on its phase w.r.t. the terminal voltage, the machine has a volt-amp, as opposed to a power rating.

In example 6.1, 22 kV is the rated line-line terminal voltage. Using the definition of generator volt-amp rating, the rated phase current may be found.

In example 6.2, the machine is connected to a 50 Hz infinite bus, and has four poles. Its speed is therefore fixed by these parameters - notice that  $p=2$ , since  $p$  means the number of pole-pairs, and the number of poles is 4.

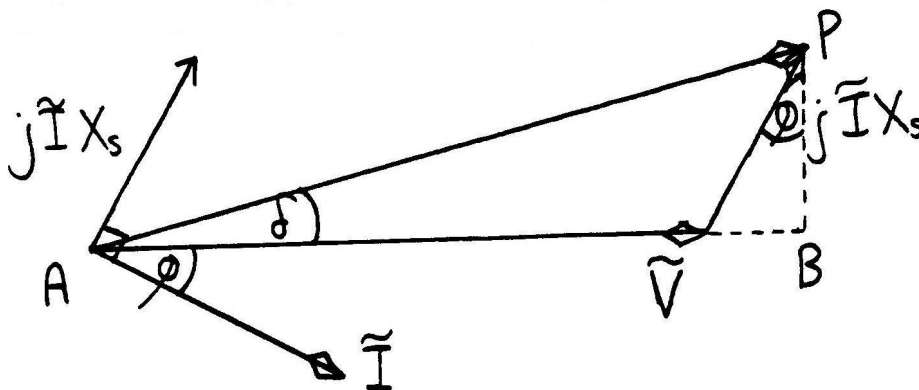
Step 2 Find the prime-mover torque

$$P_{mech} = T\omega_s \quad T = 300 \times 10^6 / 157.1 = 1910 \text{ kNm}$$

Step 3 Find phase voltage and current

$$V_{ph} = \frac{V_l}{\sqrt{3}} = \frac{22}{\sqrt{3}} = 12.70 \text{ kV} \quad (\text{star-connected})$$

$$I_{ph} = \frac{P}{3V_{ph} \cos \phi} = \frac{300 \times 10^6}{3 \times 12.7 \times 10^3 \times 0.8} = 9841 \text{ A}$$

Step 4 Draw phasor diagramStep 5 Trigonometry to find E and  $\delta$ 

$$BP = IX_s \cos \phi = 941 \times 1.2 \times 0.8 = 9448$$

$$AB = V + IX_s \sin \phi = 12700 + 9841 \times 1.2 \times 0.6 = 19.79 \times 10^3$$

$$E = \sqrt{AB^2 + BP^2} = \sqrt{9.448^2 + 19.79^2} = 21.93 \text{ kV}$$

$$\sin \delta = BP/E = 9448/21930 = 0.431 \quad \delta = 25.5^\circ$$

Since the prime-mover power is given, and the rotational speed is known, the torque may be found. Notice that for the torque to be in Nm, the rotational speed must be in rad/s.

The machine is star-connected, whereas the quoted voltage is its line-line terminal voltage. It is therefore necessary to divide by  $\sqrt{3}$  to obtain the phase voltage.

The phase current can be determined, since power, power factor and phase voltage are known.

To construct the phasor diagram, it is usual to take the terminal voltage,  $V$ , as the reference phasor. The current can then be marked on, since it is known to lag the voltage by  $\cos^{-1}(0.8)$ . The phasor  $jIX_s$  is  $90^\circ$  anticlockwise w.r.t. the phasor  $I$ , and is marked on next. Finally, using  $E = V + jIX_s$ , the excitation voltage phasor,  $E$ , may be marked on.

Once the phasor diagram has been drawn, straightforward trigonometry enables  $E$  and  $\delta$  to be found.

Firstly, the lengths marked  $BP$  and  $AB$  on the phasor diagram are found.

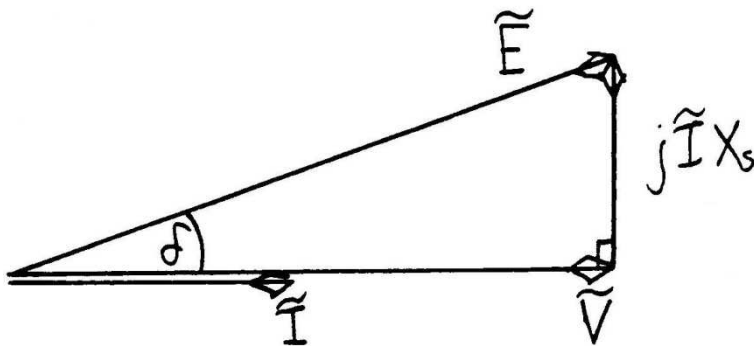
Pythagoras is then applied to give  $E$ . This will be the phase excitation voltage - the line excitation voltage would be  $\sqrt{3}$  times this value, since the generator is star-connected.

The load angle follows from trigonometry on the right-angled triangle formed by  $AB$ ,  $BP$  and  $E$ .

**Example 6.3** Load power factor increases to unity, turbine power reduces to 240 MW. Find new excitation voltage and load angle.

Step 1 Find current and draw phasor diagram

$$I_{ph} = \frac{P}{3V_{ph} \cos \phi} = \frac{240 \times 10^6}{3 \times 12.7 \times 10^3 \times 1.0} = 6298 \text{ A}$$



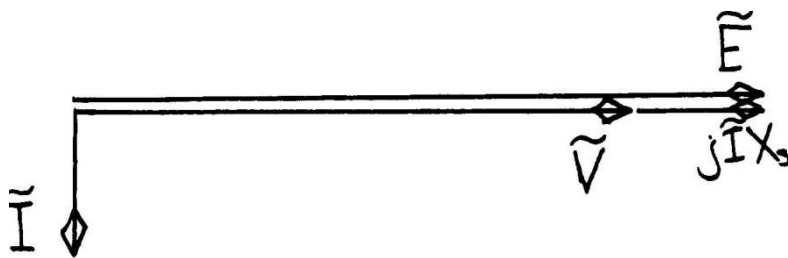
Step 2 Trigonometry to find E and  $\delta$

$$E = \sqrt{V^2 + (IX_s)^2} = \sqrt{12700^2 + 7558^2} = 14.78 \text{ kV}$$

$$\sin \delta = IX_s / E = 7558 / 14780 = 0.511 \quad \delta = 30.8^\circ$$

**Example 6.4** Find phase current if shaft power falls to zero, assuming E remains constant.

$$P = 0 \Rightarrow \sin \delta = 0 \Rightarrow \delta = 0$$



$$I = (E - V) / X_s = (14.78 - 21.7) / 1.2 = 1.73 \text{ kA}$$

Reducing the prime-mover power, and at the same time improving the load power factor to unity will reduce the current drawn from the generator.

The new current is found in the same way as before. The new phasor diagram shows V and I in phase, since the power factor is unity. The phasor  $jIX_s$  is therefore vertically upwards ( $90^\circ$  anticlockwise w.r.t. the phasor I), and finally the phasor E is obtained as the phasor sum of V and  $jIX_s$ .

Applying Pythagoras directly, since the phasor diagram forms a right-angled triangle, gives E. The load angle follows from the expression for  $\sin \delta$ .

In this final example the same generator is still connected to the 22 kV infinite bus and has the same excitation voltage as found in the previous example. However, the prime-mover power is now reduced to zero. In that case, the load angle must be zero, and so the phasors V and E are in phase. Since  $E > V$ , the phasor  $jIX_s$  is in phase with both V and E, and so the current phasor must lag V and E by  $90^\circ$ . This shows that the generator is supplying only reactive power, as if it were connected to a purely inductive load. It is therefore behaving as a synchronous compensator, as described in lecture 5. The phasor diagram is shown opposite, and its solution enables the phase current to be found, also shown opposite. Examples 6.1 .. 6.4 are taken from a Tripos question, Q5, Paper 5, 1994.

### 6.3 Operating Chart

Shows generator operating limitations. Scale sides of phasor diagram by  $3V/X_s$ :

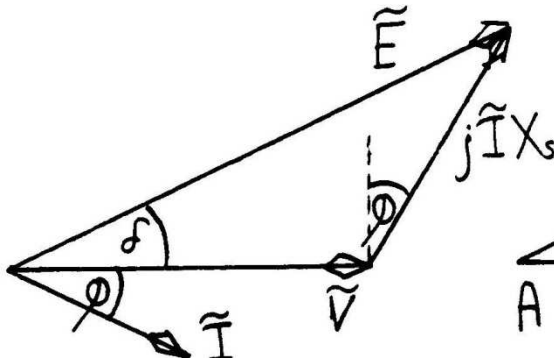


Fig. 6.1(a)

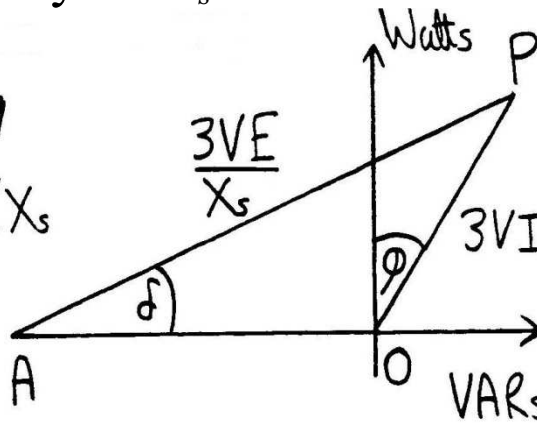


Fig. 6.1(b)

Vertical projection of  $OP = 3VI \cos \phi = P$ .

Horizontal projection of  $OP = 3VI \sin \phi = Q$ .

1. Length  $AO = 3V^2/X_s$  is fixed for operation from the infinite bus ( $V$  fixed,  $X_s$  fixed).

2. Real power is limited by prime-mover.

$\Rightarrow$  Horizontal line at  $P_{max}$  - **prime-mover limit**.

3. Length  $OP = 3VI = S$ .  $S$  is limited by the rated VA of the generator - **stator heating limit**.

$\Rightarrow$  draw a circle of radius  $VA_{rated}$ , centre  $O$ .

4. Length  $AP = 3VE/X_s$  is limited by maximum excitation emf,  $E_{max}$  - **rotor heating limit**.

$\Rightarrow$  draw a circle of radius  $3VE_{max}/X_s$ , centre  $A$ .

What limitations are there on the performance of a synchronous generator? The operating chart provides a means of illustrating these. Fig. 6.1(a) opposite shows the phasor diagram for a synchronous machine. If each side of the phasor diagram is scaled by the same factor,  $3V/X_s$ , the result is fig. 6.1(b), in which the angles of fig. 6.1(a) are preserved. The vertical projection of the point  $P$  in fig. 6.1(b) is  $3VI \cos \phi$ , whilst its horizontal projection (measured from the point marked  $O$ ) is  $3VI \sin \phi$ . Therefore, the vertical axis represents real power, and the horizontal axis reactive power.

For a machine operating from the infinite bus,  $V$  is fixed. The synchronous reactance,  $X_s$ , is also fixed, and so the length  $AO$  may be determined.

The vertical projection of the point  $P$  is the real power generated. This is limited by the maximum mechanical power which the prime-mover can provide,  $P_{max}$ , and so a horizontal line can be drawn at  $P_{max}$ . This operating limit is therefore referred to as the **prime-mover limit**.

As we saw in section 6.2, the generator has a volt-amp rating given by  $3VI$ . Since  $3VI$  is the length  $OP$ , this limit is marked on the operating chart by drawing a circle about  $O$ , of radius  $3VI_{max}$ . Because this limitation is due to the heating of the stator windings owing to  $I^2R$  losses, it is known as the **stator heating limit**.

The length  $AP$  is limited by the maximum excitation voltage,  $E_{max}$  (since  $V$  and  $X_s$  are fixed). In turn, since  $E$  is proportional to the rotor field current,  $E_{max}$  is physically limited by the maximum rotor field current. This is limited by heating of the rotor winding, and is therefore known as the **rotor heating limit**. To impose this limit on the operating chart, a circle of radius  $3VE_{max}/X_s$  is drawn, centre  $A$ . These three limits are shown on fig. 6.2.

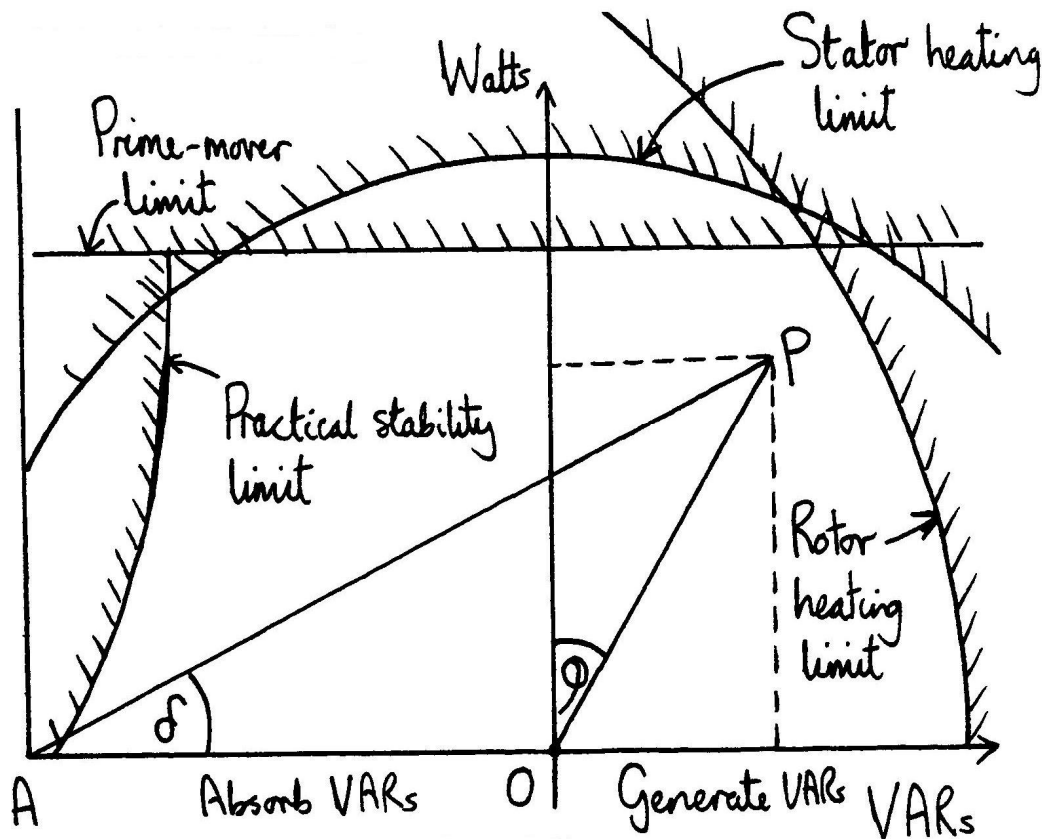


Fig. 6.2

### Example 6.5

A 250 MVA, 20 kV, star-connected generator with  $X_s = 1 \Omega$  is driven by a 200 MW steam turbine. Maximum excitation (line) is 30 kV. Draw an operating chart. Hence find the lowest lagging power factor when delivering rated power, and the excitation voltage when delivering this power at unity power factor.

Step 1 Find length  $AO$ .

$$AO = 3V_{ph}^2 / X_s = 3(20 \times 10^3 / \sqrt{3})^2 / 1 = 400 \times 10^6$$

The final limitation is on the load angle,  $\delta$ . We have already seen that if the load angle exceeds  $90^\circ$ , synchronism is lost, and so a theoretical limit is the line  $\delta=90^\circ$  i.e. a vertical line through A.

In practice, it is usual to allow for a margin of safety, known as the stability margin. This is because if the machine were operating at  $\delta=90^\circ$  and there was a sudden increase in demand for real power, synchronism would be lost. However, the inclusion of this is beyond the scope of this course.

Fig. 6.2 illustrates how all of these constraints on the generator operation may be marked on to a chart, known as an **operating chart**.

Here we consider an example showing how the operating chart is constructed from data provided, and then used to derive information about other aspects of the prime-mover/generator behaviour.

The first step in constructing an operating chart is to determine the lengths  $AO$ ,  $OP_{\max}$ ,  $AP_{\max}$  and  $P_{\max}$ . This enables a suitable scale for the chart to be determined.

The length  $AO$  is given opposite. For operation off the infinite bus,  $AO$  is fixed, since  $V$  and  $X_s$  are fixed. Notice that all quantities in the expressions for  $AO$ ,  $OP_{\max}$  and  $AP_{\max}$  are phase quantities. Therefore, since the generator is star connected, the rated voltage given must be divided by  $\sqrt{3}$ .

Step 2 Find maximum lengths for  $OP$  and  $AP$ .

$$OP_{\max} = 250 \times 10^6 \text{ (VA rating - stator heating)}$$

$$\begin{aligned} AP_{\max} &= 3V_{ph} E_{ph(\max)} / X_s \\ &= 3(20 \times 10^3 / \sqrt{3})(30 \times 10^3 / \sqrt{3}) / 1 = 600 \times 10^6 \end{aligned}$$

Step 3 Choose scale and mark  $AO$ ,  $OP_{\max}$ ,  $AP_{\max}$  and prime-mover limit on to diagram.

Choose  $1 \text{ cm} \equiv 50 \times 10^6$ .

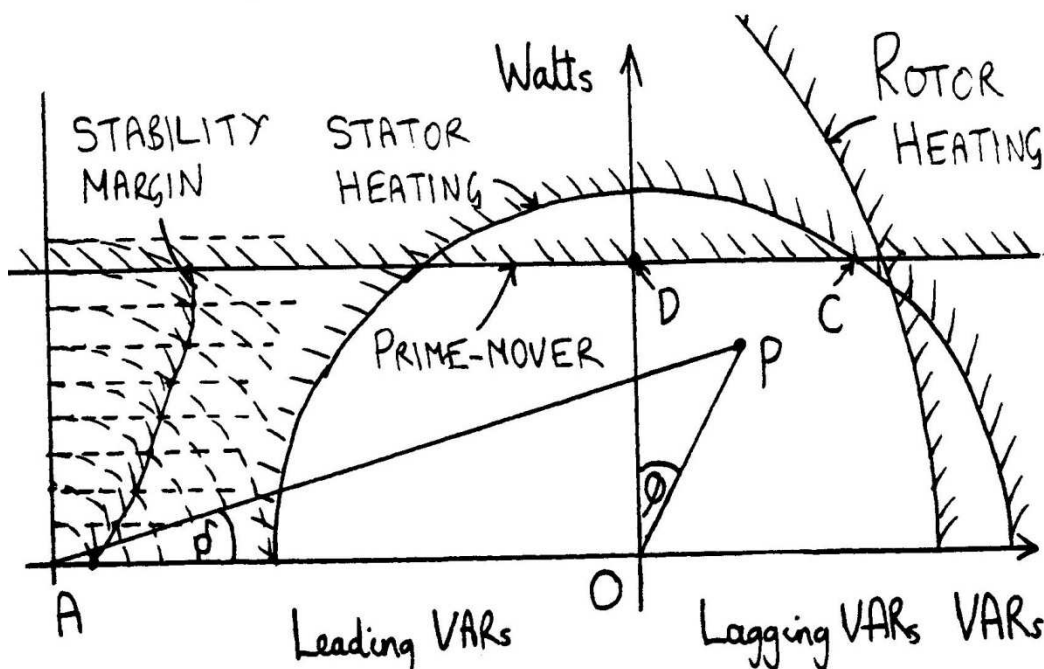


Fig. 6.3

The maximum length of  $OP$  is the VA rating of the generator, since  $OP=3VI$ . Therefore, a circle of radius 250 is drawn about the point  $O$ , and labelled 'stator heating limit'.

The maximum length of  $AP$  is given opposite. Again, notice that the maximum excitation emf is converted from the line value given to the phase value. A circle of radius 600 is drawn about the point  $A$ , and labelled 'rotor heating limit'.

These limits, together with the prime-mover limit (which is a horizontal line passing through the vertical axis at 200) are now drawn, as shown over the page. A useful tip when drawing operating charts is to think about the scale carefully before starting. In this case, the vertical scale is from 0 to 200 (due to the prime-mover limit). The horizontal scale has to go from  $AO+OP_{\max}$  or  $AP_{\max}$ , whichever is the smaller (since we don't care about regions which are outside any of the chart limits). In this case,  $AP_{\max}$  is 600,  $AO+OP_{\max}$  is 650, so the horizontal scale must cover 600. The available width of these notes for the drawing is 14 cm, so a suitable scale is  $1 \text{ cm} \equiv 50$ .

**Step 4** Use operating chart to determine required range of power factors.

Lowest lagging p.f. at 200 MW is point C:

$$S = 250 \text{ MVA}, \quad P = 200 \text{ MW} \quad \cos \phi = P/S = 0.8$$

Unity p.f. and maximum power is point D:

$$AP = \sqrt{AO^2 + P_{\max}^2} = \sqrt{400^2 + 200^2} \times 10^6 \\ = 447 \times 10^6$$

$$\therefore 3V_{ph}E_{ph}/X_s = V_lE_l/X_s = 447 \times 10^6$$

$$E_l = 447 \times 10^6 \times 1/20 \times 10^3 = 22.4 \text{ kV}$$

Lagging power factors are to the right of the vertical axis, and so the lowest lagging power factor for which rated power can be delivered is given by the point marked D (the point on the prime-mover limit which is as far to the right as possible before another limit is met). The limit which is met is the stator heating limit. Therefore, the output real power is 200 MW, and the output apparent power is 250 MVA, so the power factor may be obtained from the power triangle as shown opposite.

If the generator is generating at unity power factor, the point P must be on the vertical axis (otherwise some reactive power is being produced). If it is generating at rated real power, then the point P must be on the prime-mover limit, and so must be at the point marked D shown on the operating chart. The length AP for this situation may be found by Pythagoras, and is then equated with  $3V_{ph}E_{ph}/X_s$ . By replacing  $V_{ph}$  and  $E_{ph}$  in terms of the respective line quantities, which are both  $\sqrt{3}$  times as big because the generator is star-connected, the line excitation voltage may be found directly, as shown opposite.