

CAMBRIDGE UNIVERSITY ENGINEERING DEPARTMENT



PART IB Paper 4: Thermodynamics

2022/2023

Lectures 6-10

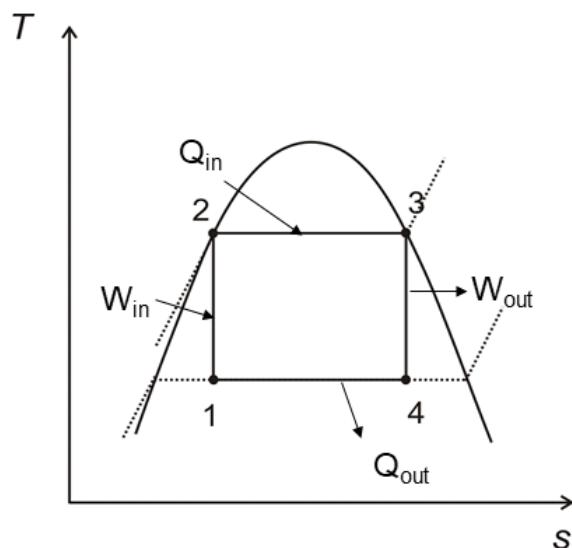
Prof R. J. Miller

Lecture 6: Power generation I

6. Power generation I

6.1 Ideal Carnot cycle?

Can we design an ‘ideal’ power plant using steam as the working fluid?



Problems:

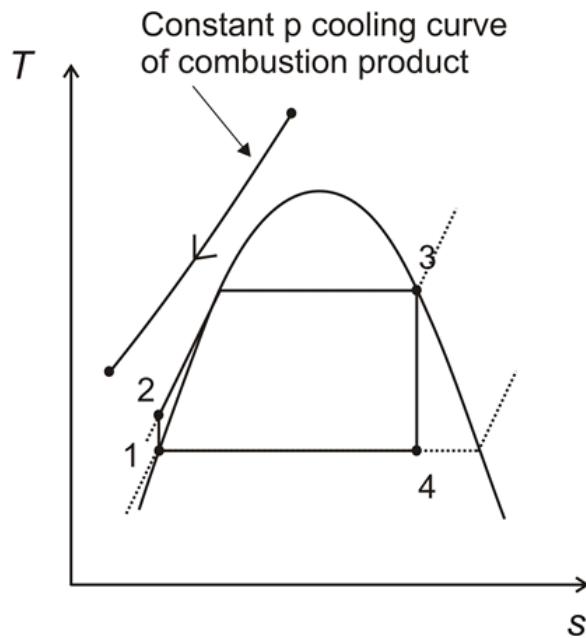
1. Significant practical problems with developing pumps that operate in the two-phase region.
2. The heat is transferred from hot combustion products that are at constant pressure.

1-2 Compressor/pump, 2-3 Boiler, 3-4 turbine, 4-1 condenser.

We mentioned repeatedly that the Carnot cycle is the most efficient cycle operating between two specified temperature limits. Thus it is natural to look at the Carnot cycle first as a prospective ideal cycle for vapour power plants. If we could, we would certainly adopt it as the ideal cycle. However, in practice there are several impracticalities:

1. In the heat addition (2-3) the temperature is limited to the critical point value, which is 374°C for water. This limits the maximum temperature of the heat addition and thus the Carnot efficiency.
2. The expansion process (3-4) is in the two-phase region. The impingement of liquid droplets onto the turbine blades causes erosion.
3. The compression process (1-2) is also in the two-phase region. It is not possible to design a pump which handles two-phase flow.

6.2 Rankine cycle



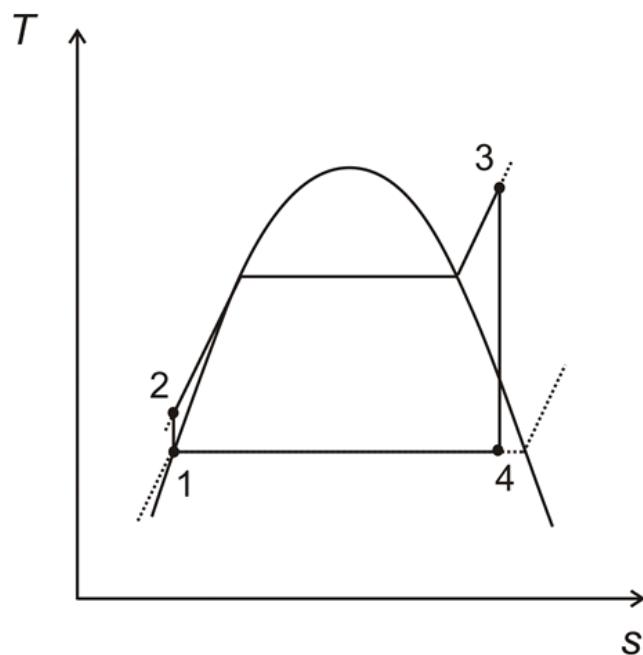
Advantages:

1. Takes advantage of heat transfer at lower temperature.
2. Pump in single phase regions.

Problem:

1. Turbine in two phase region results in high blade erosion.

6.3 Superheated Rankine cycle



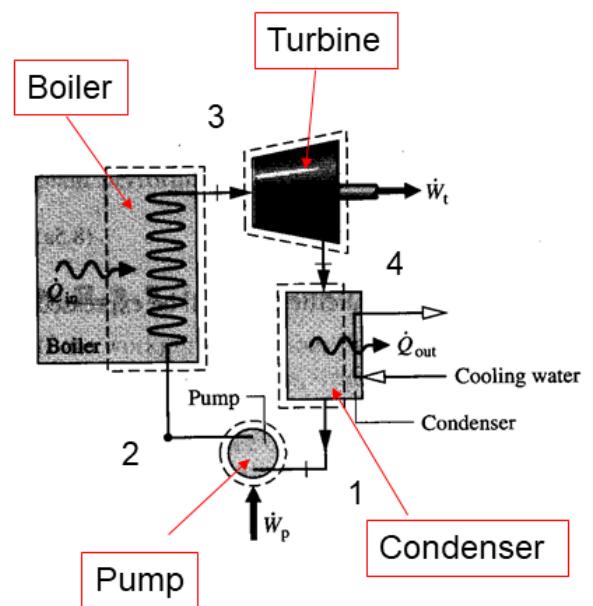
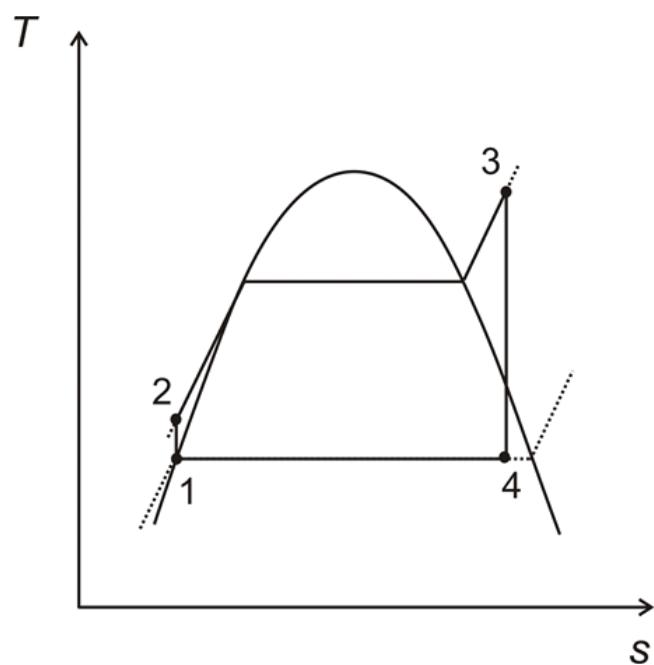
Boiler heats into superheated region.

Advantages:

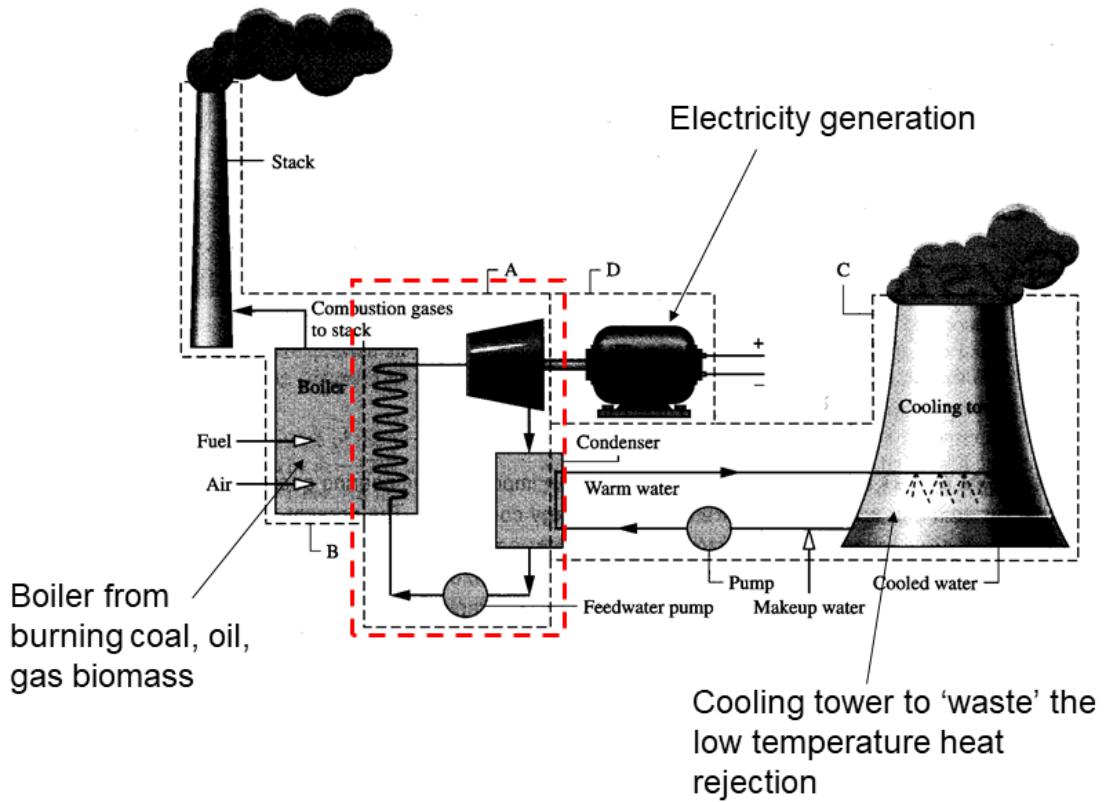
1. Turbine operates in less of the two phase region.
2. Average temperature of heat input rises and so efficiency rises.

3

6.4 Practical implementation



4

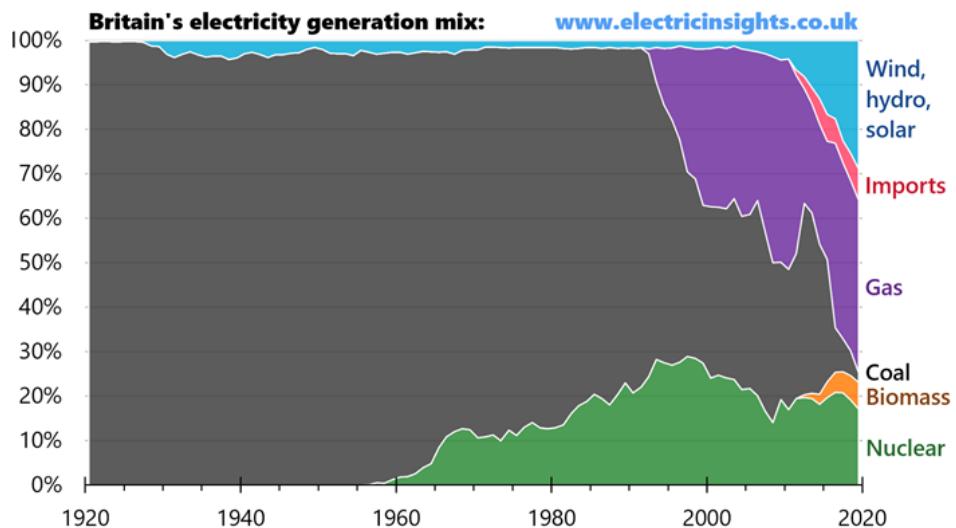


5

6.5 Background

1. Cyclic steam-based power plants are the World's biggest man-made power source.
2. The steam turbine was introduced by Sir Charles Parsons in the 1880's.
3. All steam cycles are based on the Rankine cycle which is a true thermodynamic cycle.
4. All plant uses heat to generate 50 to 2000 MW electricity from:
 1. combustion of fossil fuels (oil, coal, gas).
 2. exhaust of gas turbine.
 3. nuclear reactions.

6



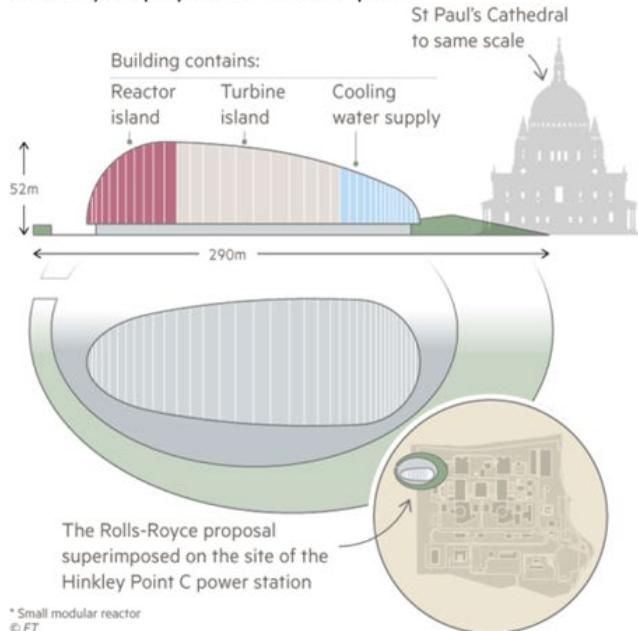
UK Fuel consumption for Electricity Production

Rapid change in the UK's energy mix will continue

7

6.6 Zero carbon power generation

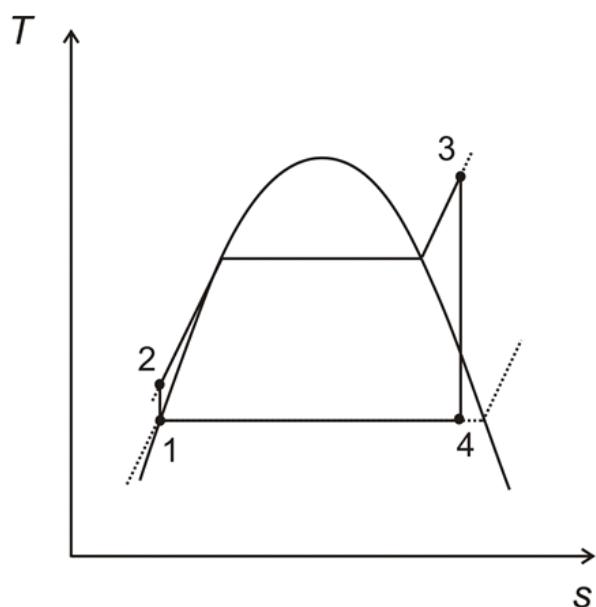
Rolls-Royce's proposal for an SMR* plant



Small Modular Reactors

8

6.6 Calculations



Pump 1-2.

$$\frac{\dot{W}_p}{\dot{m}} = h_2 - h_1$$

Boiler 2-3.

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2$$

Turbine 3-4.

$$\frac{\dot{W}_t}{\dot{m}} = h_3 - h_4$$

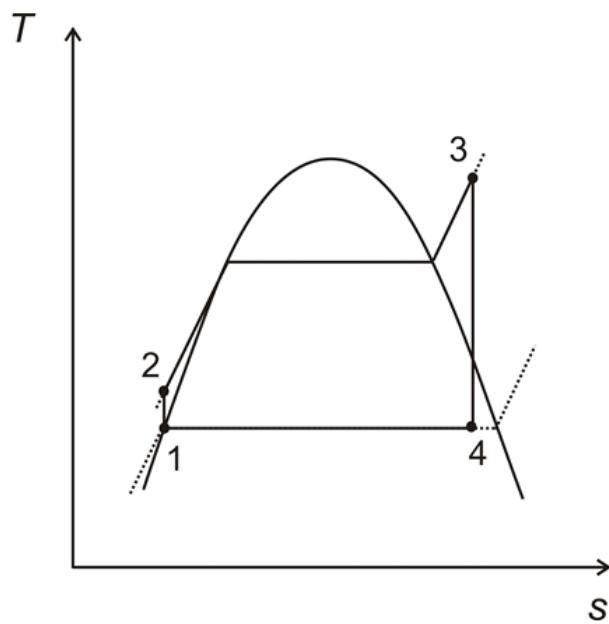
Condenser 4-1.

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_1 - h_4$$

9

Example 6.1:

A steam power plant operates between a boiler pressure of 20MPa and a condenser pressure of 0.004MPa. The turbine entry temperature is 550°C. Calculate the net work output per unit mass of steam and the cycle efficiency.



10

1-2: Feed pump:

State 1 is saturated liquid at 4kPa = 0.04 bar. Tables: $h_1 = 121.4 \text{ kJ/kg}$.

State 2 is liquid with $s_2 = s_1$, so that h_2 can be found by interpolating or instead

$$-w_x = h_2 - h_1 = \int_1^2 vdp$$

Water is virtually incompressible, so a good approximation is $v_2 = v_{f1} = \text{constant}$

$$w_p = v(p_2 - p_1) = 0.001004 \times (20 \times 10^6 - 4 \times 10^3) = 20.1$$

$$\text{Then } h_2 = h_1 + w_p = 141.5 \text{ kJ/kg}$$

2-3 Boiler:

State 3 is superheated steam at 20MPa = 200 bar and 550°C.

Using Tables or Enthalpy-Entropy diagram have $h_2 = 3396.1 \text{ kJ/kg}$

$$q_{23} = h_3 - h_2 = 33961 - 145 = 3254.6 \text{ kJ/kg}$$

11

3-4: Turbine: State 4 is in the liquid-vapour region.

The turbine is isentropic, so that $s_4 = s_3$. Use s_4 to get x_4 .

From Tables have $s_3 = 6.339 \text{ kJ/kgK}$. Then from Tables at $p_4 = 0.04 \text{ bar}$ gives $s_f = 0.422 \text{ kJ/kgK}$ and $s_g = 8.473 \text{ kJ/kgK}$.

$$x_4 = \frac{s_4 - s_f}{s_g - s_f} = 0.73$$

Again from Tables at $p_4 = 0.04 \text{ bar}$ have $h_f = 121.4 \text{ kJ/kg}$ and $h_{fg} = 2432.3 \text{ kJ/kg}$

$$h_4 = h_f + xh_{fg} = 121.4 + 0.73 \times 2432.2 = 1897.0 \text{ kJ/kg}$$

Alternatively, use the Enthalpy-Entropy diagram - have h_4 and x_4 directly.

$$w_t = h_3 - h_4 = 3396.1 - 1897.0 = 1499.1 \text{ kJ/kg}$$

The cycle efficiency is given by

$$\eta = \frac{w_{net}}{q_{23}} = \frac{1479.0}{3254.6} = 45.4\%$$

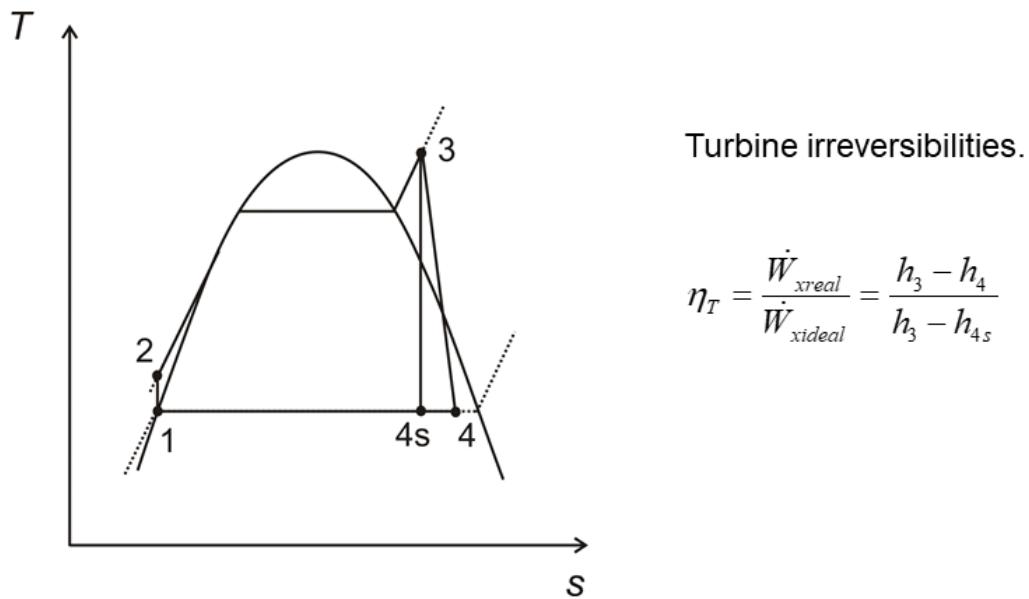
12

6.7 Notes on Rankine cycle

1. The work input required to compress the liquid water is very much less than the work output of the turbine.
2. Operating pressures are very high in the boiler and very low in the condenser. This requires high structural integrity from the enormous lengths of pipework involved. In the boiler the high temperatures and corrosive flue gases.
3. The temperature at which heat is rejected only just above ambient. This compensates for the relatively low maximum temperature, which is much lower than that achieved in a gas turbine or a Diesel engine.
4. The specific volume increases greatly through the turbine such that the volume flow rate at turbine exit is very high. This means that the turbine exit area must be very large. The final stage of a large turbine has blades of about 4m diameter.
5. The steam leaving the turbine typically has a dryness fraction of about 0.9, with the water suspended in the stream as a fog of droplets of diameter about 1 micron. These are too small to cause damage, but may deposit on blades and accumulate into larger droplets which are thrown off and may cause erosion.

13

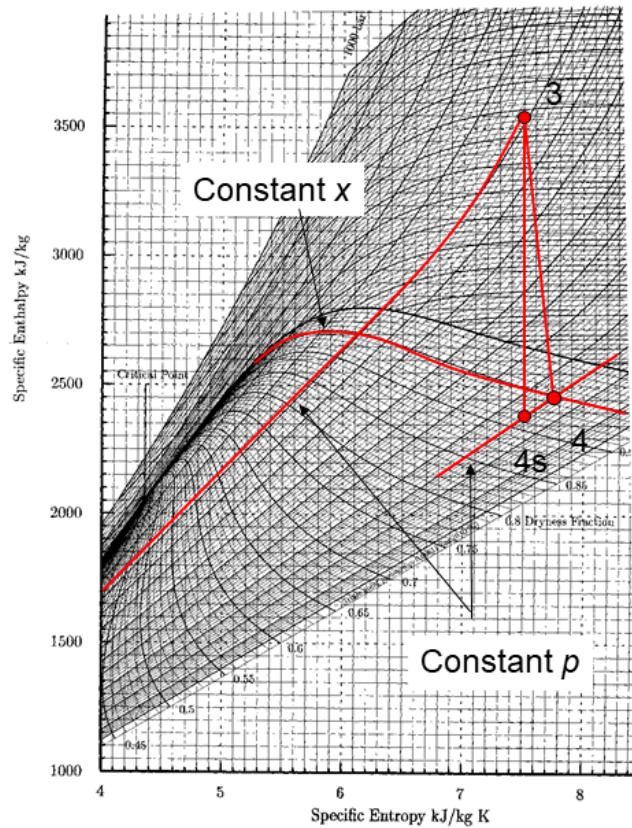
6.8 Irreversible Rankine cycle



Example 6.2:

If boiler exit state (p_3, T_3) , turbine efficiency η_T and exit pressure p_4 are known then calculate the dryness fraction at turbine exit, x_4 .

14



- 1. Ideal isentropic turbine.**
Draw vertical line down from 3 until touch exit p_4 .
- 2. Use efficiency to get real turbine enthalpy change.**

$$\eta_T = \frac{\dot{W}_{xreal}}{\dot{W}_{xideal}} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

- 3. Get dryness fraction at 4.** Use real turbine enthalpy change and turbine exit pressure p_4 . Read off the dryness fraction x_4 .

15

6.9 Zero carbon Power Generation

Concentrated solar thermal steam plants

The Ivanpah Solar Electric Generating System is a concentrated solar thermal steam plant in the Mojave Desert. The plant has a gross capacity of 392 megawatts deploying 173,500 heliostats, each with two mirrors focusing solar energy on boilers located on three centralized solar towers. The facility, costing \$2.2 billion.

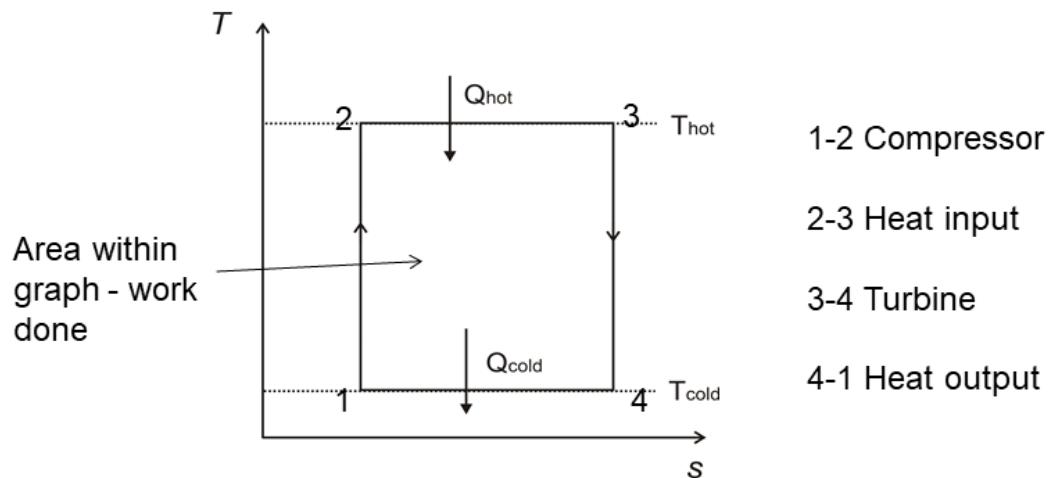


16

Lecture 7: Power generation II

7. Power generation II: How to raise efficiency

7.1 Reminder: Raising efficiency of an ‘ideal’ Carnot cycle



$$\eta = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$$

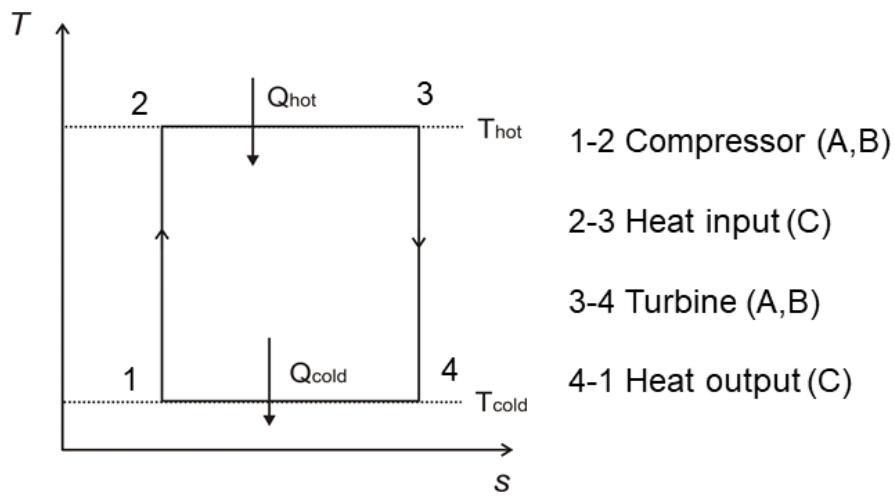
To raise efficiency

- Raise T , Q added.
- Lower T , Q rejected.

7.2 Reminder: Effect of Irreversibility

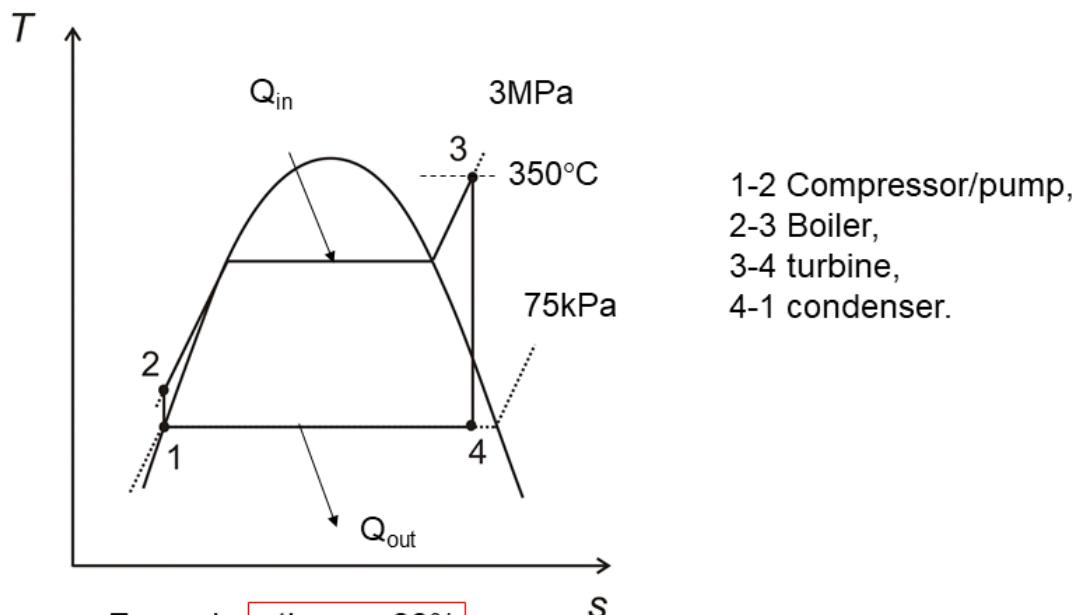
Efficiency improved by lowering irreversible entropy generation $T_0 \dot{\Delta S}_{irrev}$

- A) Boundary layer dissipation.
- B) Viscous mixing.
- C) Heat transfer across finite ΔT .



2

7.3 Baseline Rankine cycle



Example: $\eta_{therm} = 26\%$.

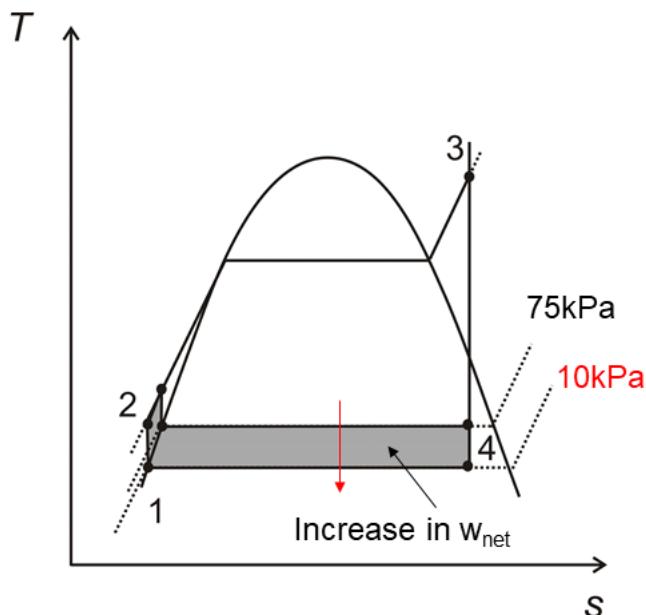
(NB calculations for all examples in Ch10, Cengel and Boles).

How can we raise efficiency?

1. Raise average T at which Q added.
2. Lower average T at which Q rejected.

3

7.4 Lowering condenser pressure



1. Lowering p lowers T .
2. Average T of Q_{out} drops so η_{therm} rises.

Practicalities

1. Vapour cycle closed loop so p can be below atmosphere.
2. If river is 15°C and condenser is 25°C then min condenser p is 3.2kPa.

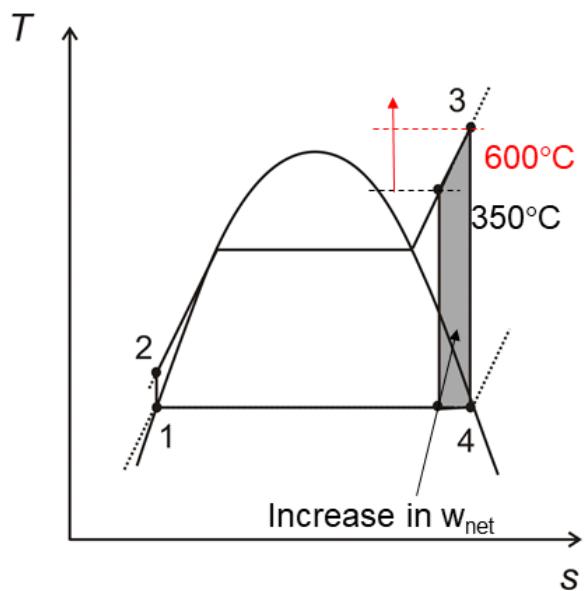
Problems:

1. Possibility of air leaking into the cycle.
2. Turbine becomes more two phase.

Example: If condenser pressure is lowered to 10kPa $\eta_{therm} = 33.4\%$

4

7.5 Superheat the steam to higher temperature



1. Average T of Q_{in} rises so η_{therm} rises.

Advantages:

1. Turbine exit becomes less two phase.

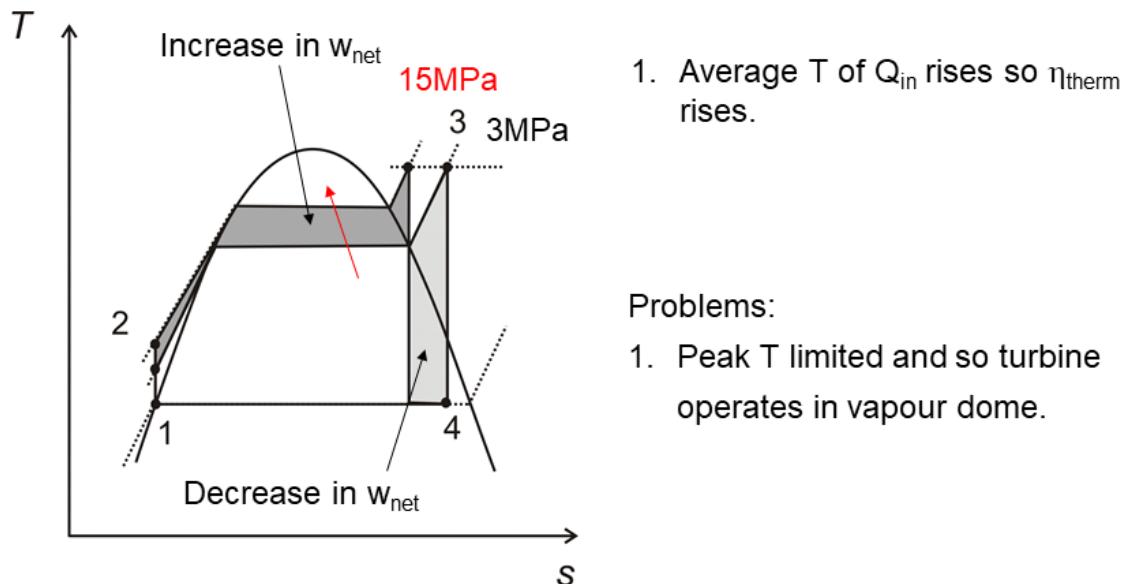
Problems:

1. Presently steam turbines limited to 620°C by metallurgical considerations. In future ceramics may be a solution?

Example: If peak T is raised to 600°C $\eta_{therm} = 37.3\%$.

5

7.6 Increasing boiler pressure

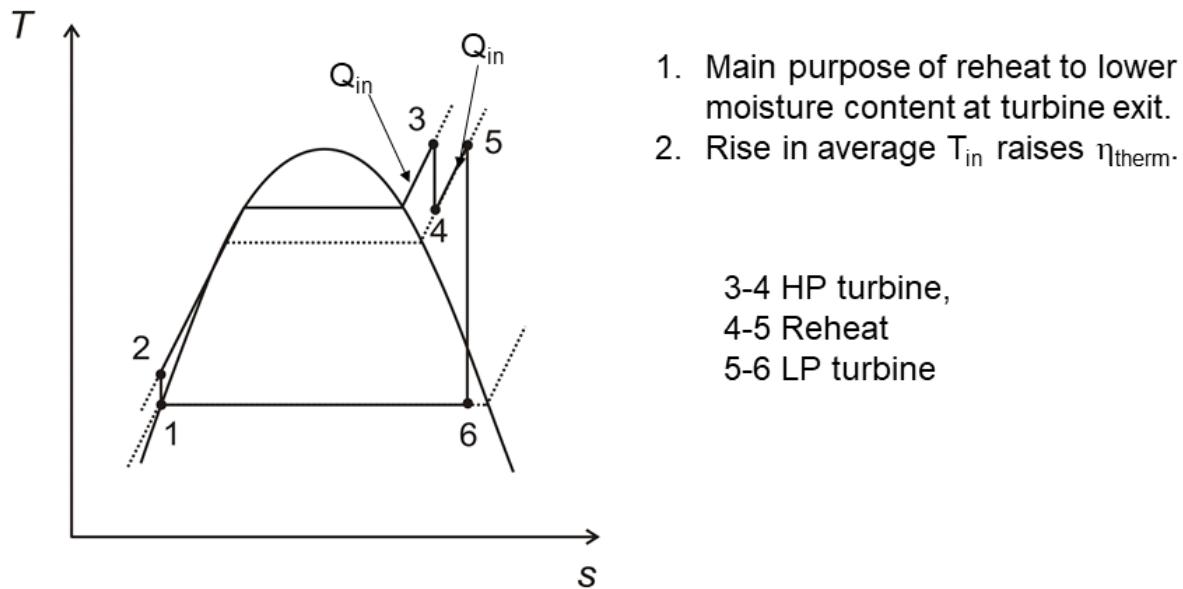


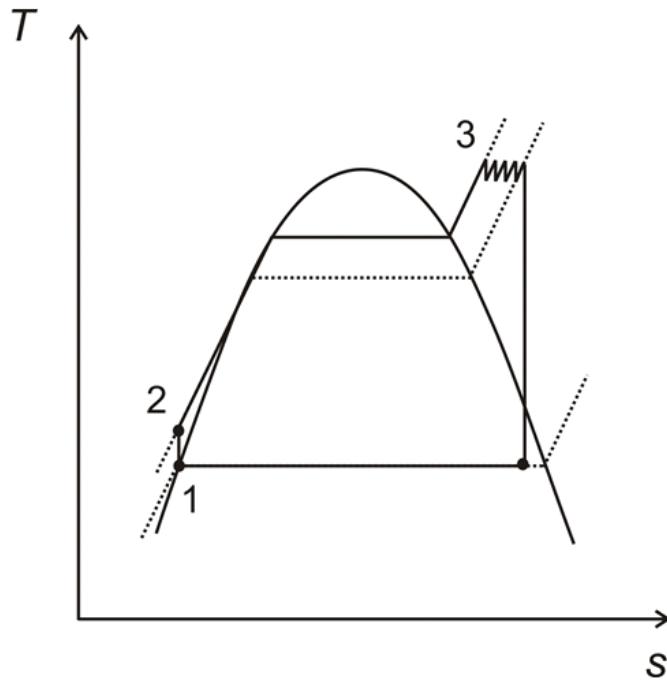
Boiler pressures have increased. In 1922 2.7MPa, to >30MPa today.

How can p_3 rise without turbine operating in vapour dome? 6

7.7 The ideal reheat Rankine cycle

How can take advantage of higher boiler pressure without excessive moisture in turbine?





1. The average T at which Q_{in} occurs raised by multiple reheat.
2. The benefit of the 2nd reheat is around half of the 1st reheat and so on.....
3. The use of more than two reheat is not practical.

Efficiency raised from 26% to 45% but what is maximum? Can we do better?

Use 2nd law analysis to find out.

8

7.8 Reminder: 2nd Law / Availability analysis

$$\dot{m}(b_e - b_i) = -\dot{W}_x + \int \left(1 - \frac{T_0}{T}\right) d\dot{Q} - T_0 \Delta \dot{S}_{irrev}$$

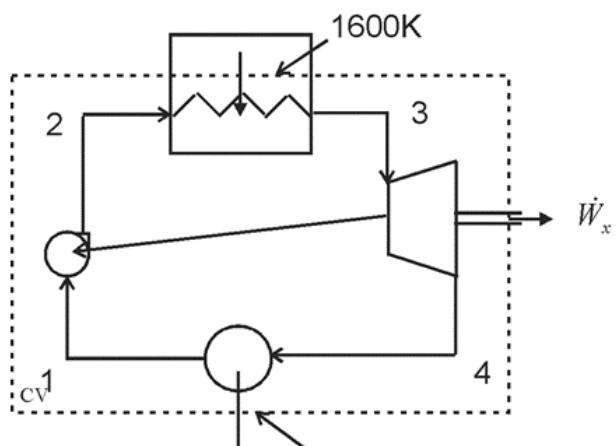
$$\int \left(1 - \frac{T_0}{T}\right) d\dot{Q}$$

Shaft power

$$\dot{W}_x = + \int \left(1 - \frac{T_0}{T}\right) d\dot{Q} - T_0 \Delta \dot{S}_{irrev}$$

Transfer of power potential due to heat transfer

Lost power potential due to irreversibilities



$$\int \left(1 - \frac{T_0}{T}\right) d\dot{Q}$$

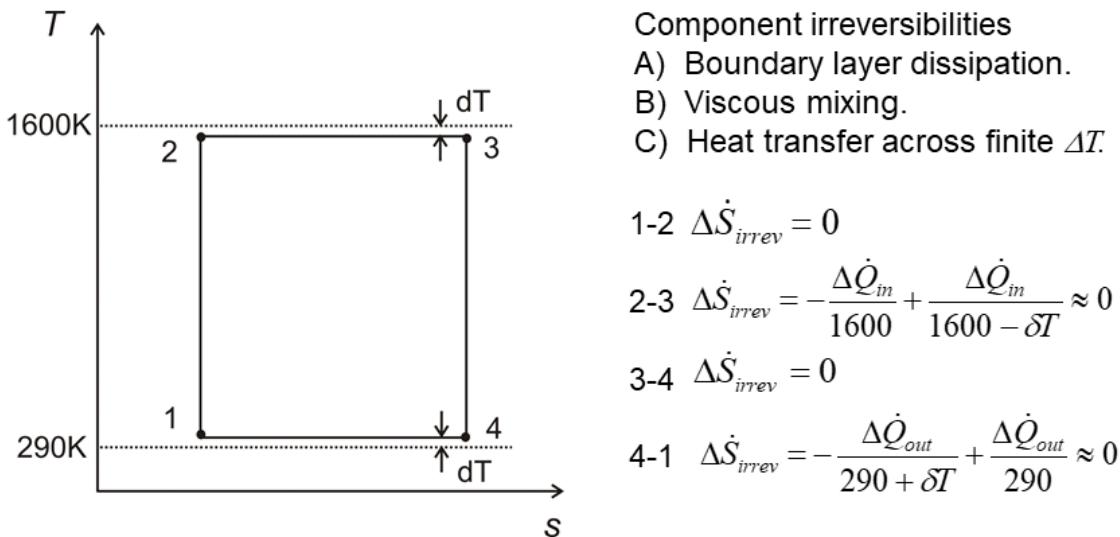
So to maximise work output minimise availability destruction.

9

7.9 2nd law analysis of Carnot cycle

An availability analysis can tell us. Lets start with the ideal Carnot cycle.

1. Q enters from a furnace at 1600K.
2. Q leaves to surroundings at 290K.



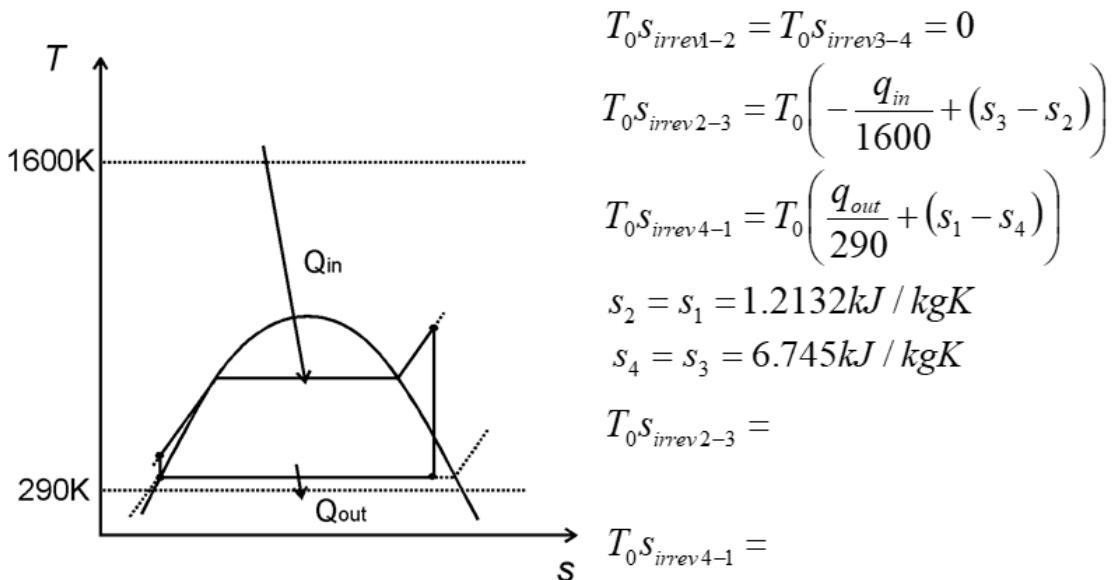
10

Example 1: 2nd law analysis of Rankine cycle

Determine the destruction of available energy (availability) in each component.

The pressures are 3MPa and 75kPa. The boiler exit T is 350°C.

$$q_{in} = 2728.6 \text{ kJ/kg}, q_{out} = 2018.6 \text{ kJ/kg}$$



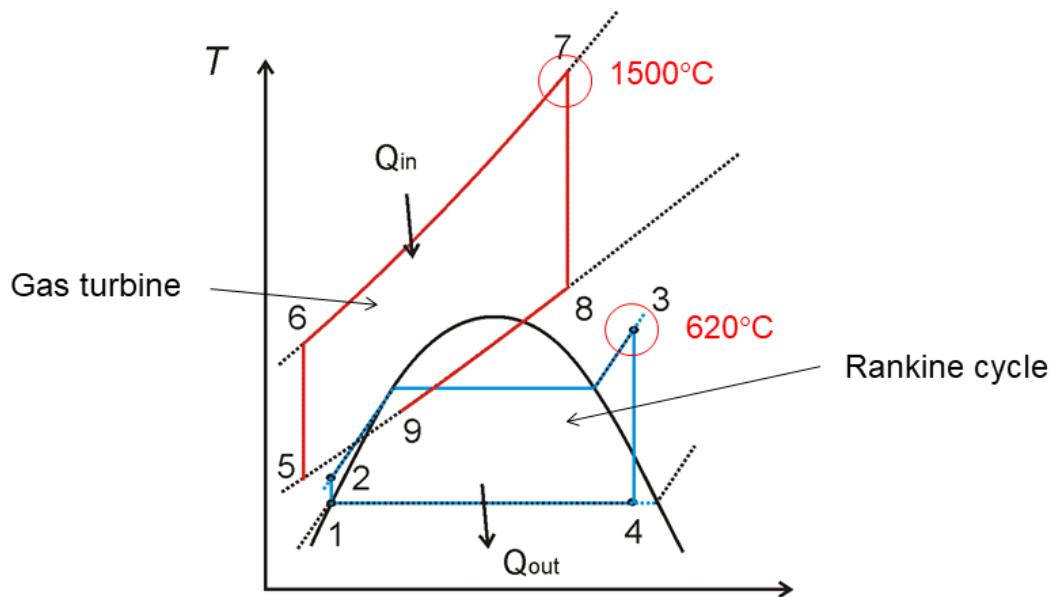
Fill in

So how do we raise efficiency of cycle even further?

11

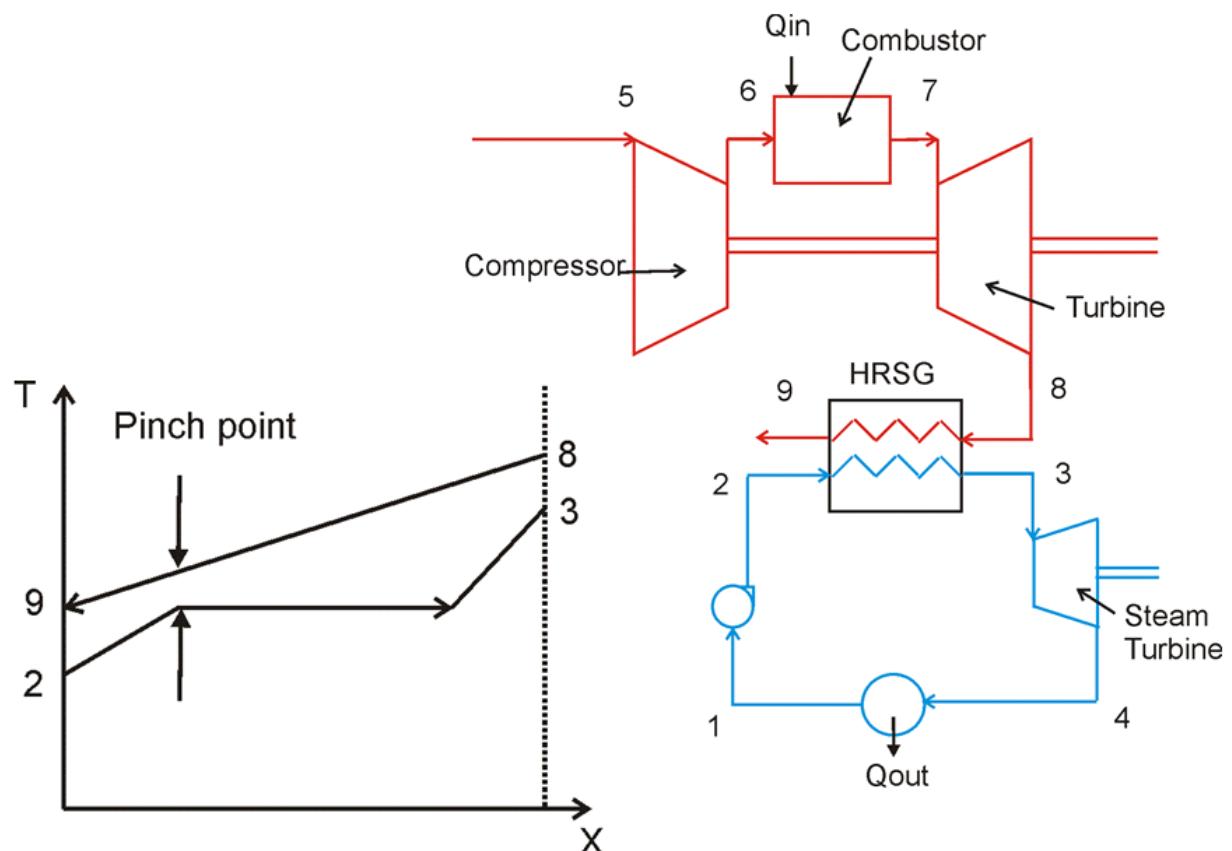
7.10 Combined Gas-vapour power cycle

Gas turbine topping a vapour cycle



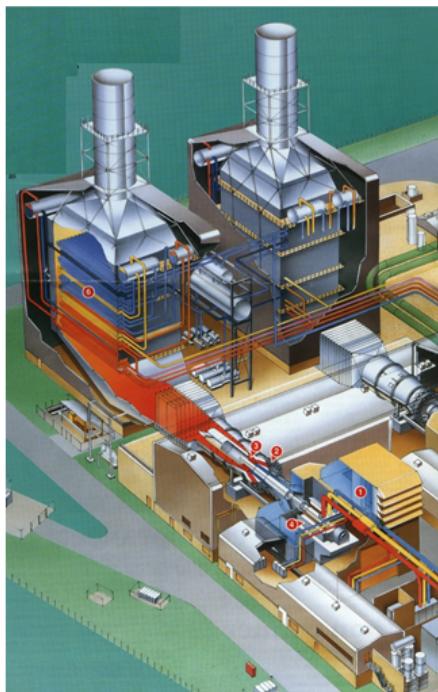
1. Gas turbines have high average T of Q_{in} but problem is exhaust T_8 high.
2. Combined cycles give $\eta_{therm} = 40-60\%$

12



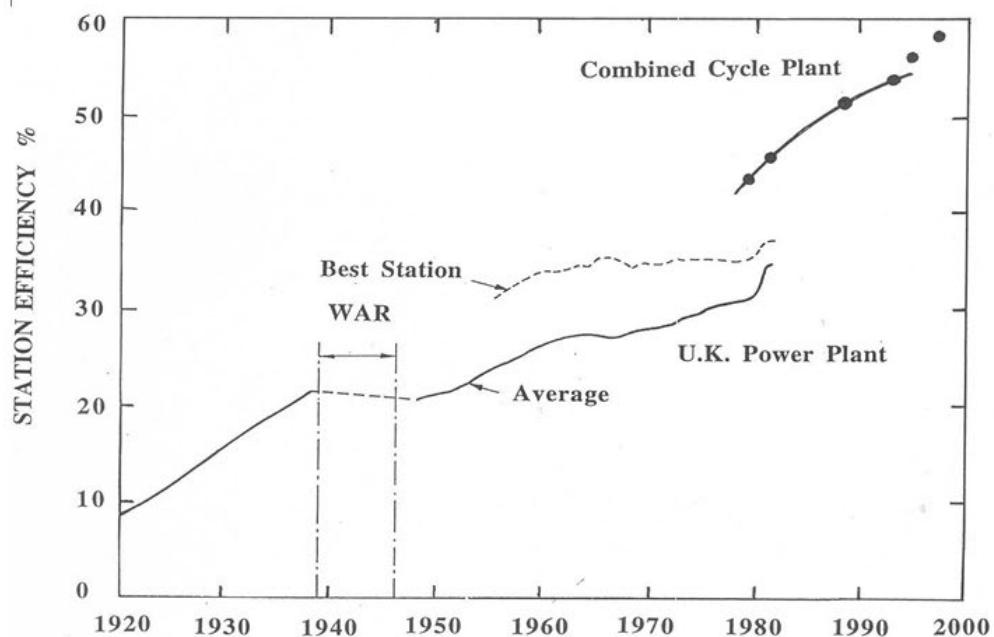
NB X is the fraction of heat transferred, 0-1.

13



1. The 1990s saw a revolution in the industry with advent of the combined-cycle, efficiency rose from 40% to 60%.
2. UK emissions from power stations fell sharply.
3. This is due to deregulation/competition.
4. Switch from coal to natural gas (natural gas is cleaner and produces less CO₂ for same heat)
5. Rapid progress in design & manufacture of large gas-powered turbines (up to 250 MW).

14

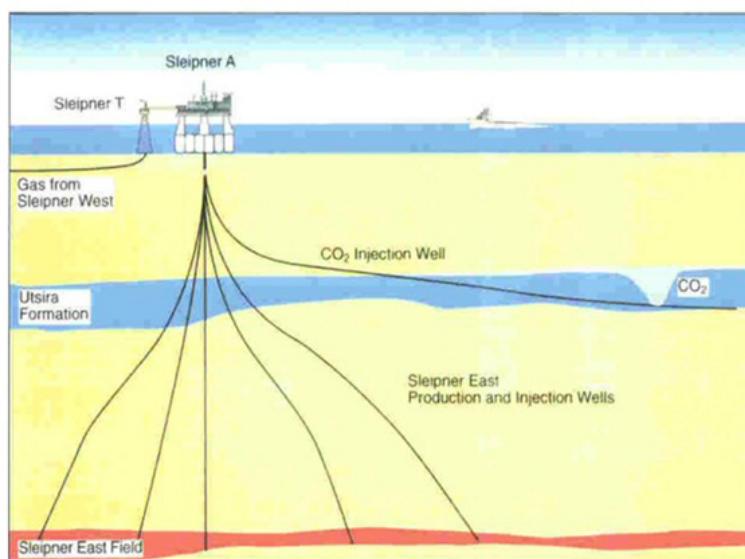


The current maximum efficiency is 63.5% with work going on to reach 70% efficiency

15

7.11 Carbon sequestration

Statoil project – CO₂ storage in a saline aquifer below the North Sea



Would reduce power output by 25%-35%.

All US coal plants - 30 million barrels of liquefied CO₂ a day (3 times daily oil import). Likely effect of carbon sequestration would be rise in price of coal based power and thus a move toward alternative forms of power generation.

16

7.12 Zero carbon power generation



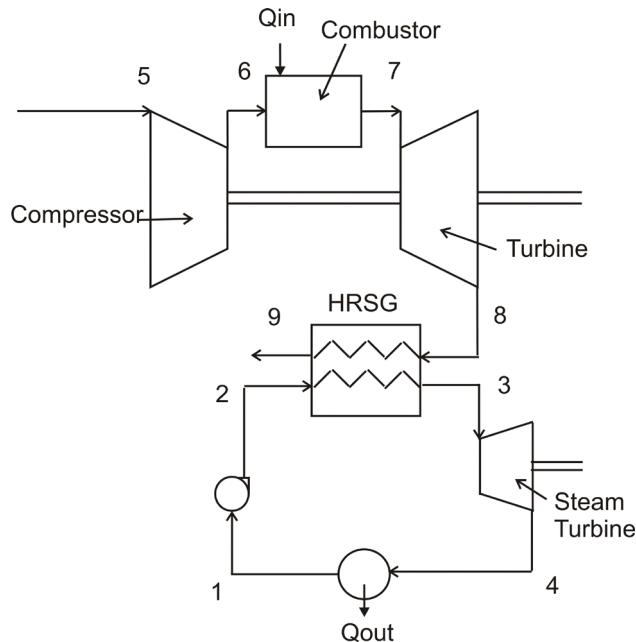
A project is underway in the Netherlands where they are planning to transform their large thermal generation into a 100% hydrogen – fired power generation plant by 2025. MHPS who is driving the practical application of large-scale hydrogen gas turbine, is taking part in this project. The hydrogen generation technology is part of the growth strategy for Japan.



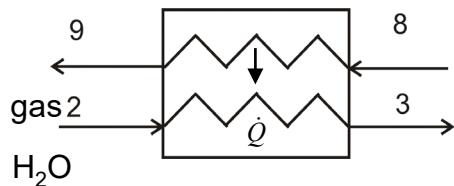
17

7.13 Analysis of HRSG

An HRSG is a heat-exchanger which exchanges heat from the exhaust gas of the gas turbine to the boiler of the Rankine cycle. The HRSG is a counter flow heat-exchanger. This means that the gas (hot side), supplied by the gas turbine exhaust, enters at one end and the water (cold side), in a steam boiler, enter at opposite end. The aim of the heat-exchanger designer is usually to minimise the temperature difference between the hot and cold flow at each point through the device. This lowers the ‘finite temperature difference’ over which the heat transfer occurs and thus lower the irreversible entropy generation.

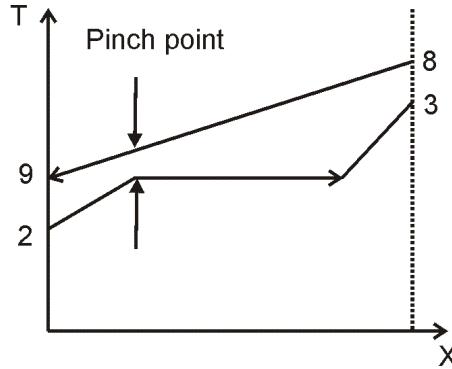


A zoom in of the HRSG is shown below.



The diagram used to analyse heat-exchangers is a distance temperature diagram. An example of this is shown below. The x axis is the fraction of heat transferred and the y axis is the temperature of each stream. On the cold side the water is first heated and then changes phase from liquid water to steam. The closest temperature between two streams is usually the point at which the water becomes saturated. This point is known as the ‘pinch point’.

An analysis of a HRSG usually involves either being given the temperatures at the four end points, 2, 3, 8, 9 and being asked to calculate the ratio of mass flows between the two streams or being given the ratio of mass flows and three of the temperatures and being asked to calculate the fourth temperature. The procedure is similar for both cases. First equate the heat flow between the two sides of the heat exchanger.



$$\dot{Q}_{outgas} = \dot{Q}_{inH_2O}$$

$$\dot{m}_{gas}(h_8 - h_9) = \dot{m}_{H_2O}(h_3 - h_2)$$

the enthalpy change of the exhaust gas can be determined using the specific heat at constant pressure. The enthalpy of the steam and water must be looked up in the CUED tables.

$$\frac{\dot{m}_{gas}}{\dot{m}_{H_2O}} = \frac{(h_3 - h_2)}{c_p(T_8 - T_9)}$$

If a question asks you to calculate the ‘pinch point’ temperature difference then a heat flow balance between the pinch point and one end of the heat-exchanger is necessary. In practice this involves drawing a control volume from the pinch point location around both sides of the heat-exchanger to one end of the heat-exchanger.

To calculate the entropy generated in a HRSG it is necessary to sum the entropy change of the two flows. The flow with heat extraction will of course drop in entropy while the flow with heat addition will rise in entropy. The sum total of these will be either positive or zero. The lost work potential in the combined flows is given by

$$T_0 \Delta \dot{S}_{irrevHRSG} = T_0 (\dot{m}_{H_2O}(s_3 - s_2) + \dot{m}_{gas}(s_9 - s_8))$$

The specific entropy of the water and steam can be obtained from tables in the CUED data book. The equation for the specific entropy change of a perfect gas can be obtained from p3 of the CUED data book.

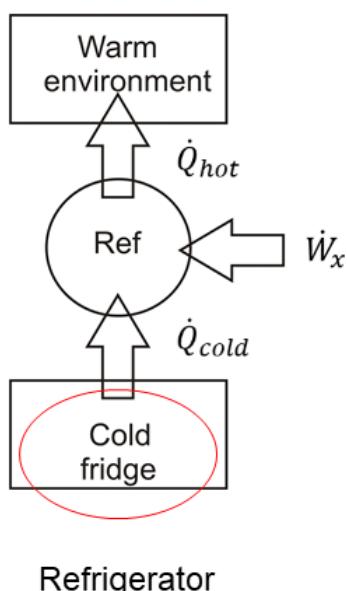
$$(s_9 - s_8) = c_p \ln\left(\frac{s_9}{s_8}\right) - R \ln\left(\frac{p_9}{p_8}\right)$$

Lecture 8: Refrigeration cycles

8. Refrigeration cycles

8.1 Refrigerators

The aim of a fridge is move \dot{Q} from cold to hot.



Why won't \dot{Q} flow from cold to hot

$$\dot{S}_{cold} = -\frac{\dot{Q}}{T_{cold}} + \dot{S}_{irrev} \xrightarrow{0}$$

$$\dot{S}_{hot} = \frac{\dot{Q}}{T_{hot}} + \dot{S}_{irrev} \xrightarrow{0}$$

$$\dot{S}_{hot} + \dot{S}_{cold} = \frac{\dot{Q}}{T_{hot}} + \frac{\dot{Q}}{T_{cold}} \leq 0$$

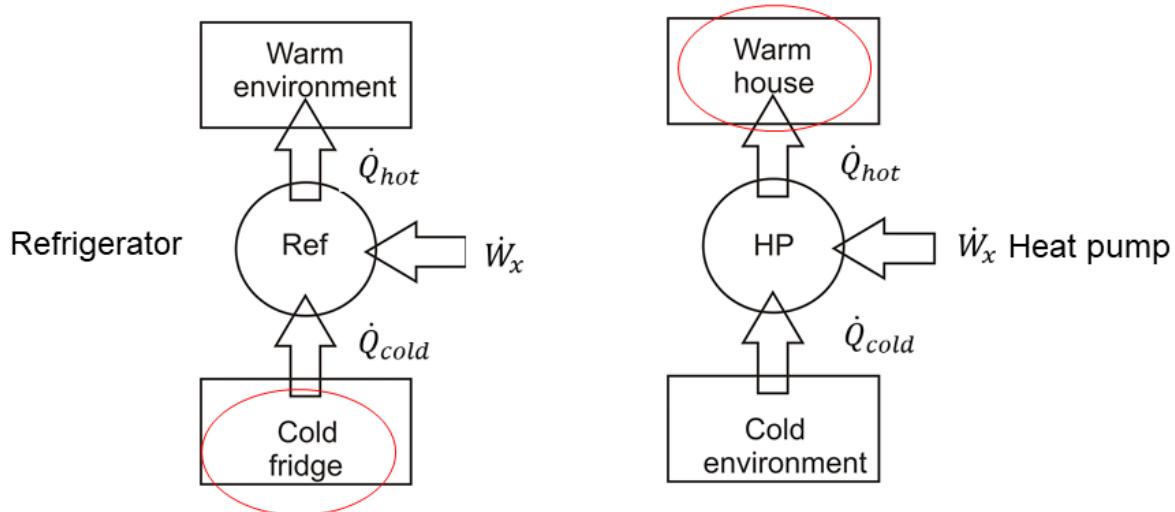
2nd law says impossible.

$$\dot{S}_{hot} + \dot{S}_{cold} = \frac{\dot{Q} + \dot{W}_x}{T_{hot}} - \frac{\dot{Q}}{T_{cold}} = 0$$

Best is a reversible fridge

1

8.2 Refrigerators and heat pumps



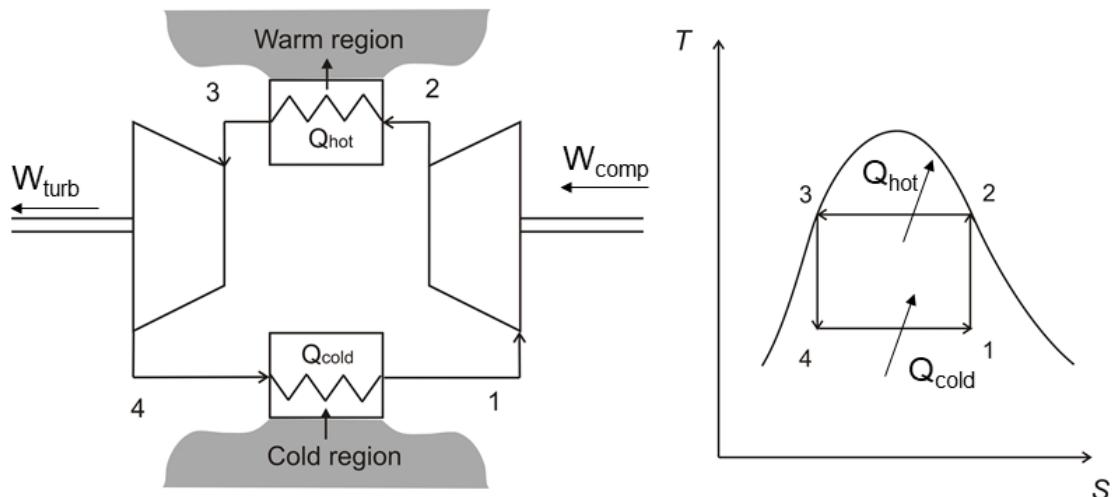
$$COP_R = \frac{\text{What you want}}{\text{What you put in}} = \frac{\dot{Q}_{cold}}{\dot{W}_x} \quad COP_{HP} = \frac{\text{What you want}}{\text{What you put in}} = \frac{\dot{Q}_{hot}}{\dot{W}_x}$$

NB $\dot{Q}_{hot} = \dot{Q}_{cold} + \dot{W}_x \quad COP_{HP} = COP_R + 1$

NB When heat pump worst, working as resistance heater $COP_{HP}=1$

2

8.3 The reversed Carnot cycle

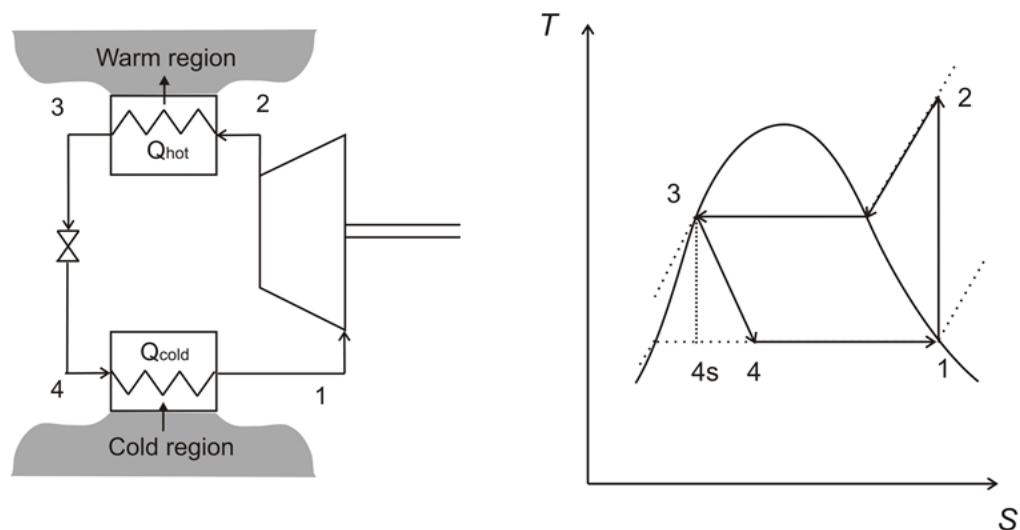


$$COP = \frac{\dot{Q}_{cold}}{\dot{Q}_{hot} - \dot{Q}_{cold}} = \frac{T_{cold}\dot{S}}{T_{hot}\dot{S} - T_{cold}\dot{S}} = \frac{T_{cold}}{T_{hot} - T_{cold}} = \frac{1}{\frac{T_{hot}}{T_{cold}} - 1}$$

$\dot{S} = \frac{\dot{Q}}{T} + \dot{S}_{irrev}$

3

8.4 Real refrigeration cycle

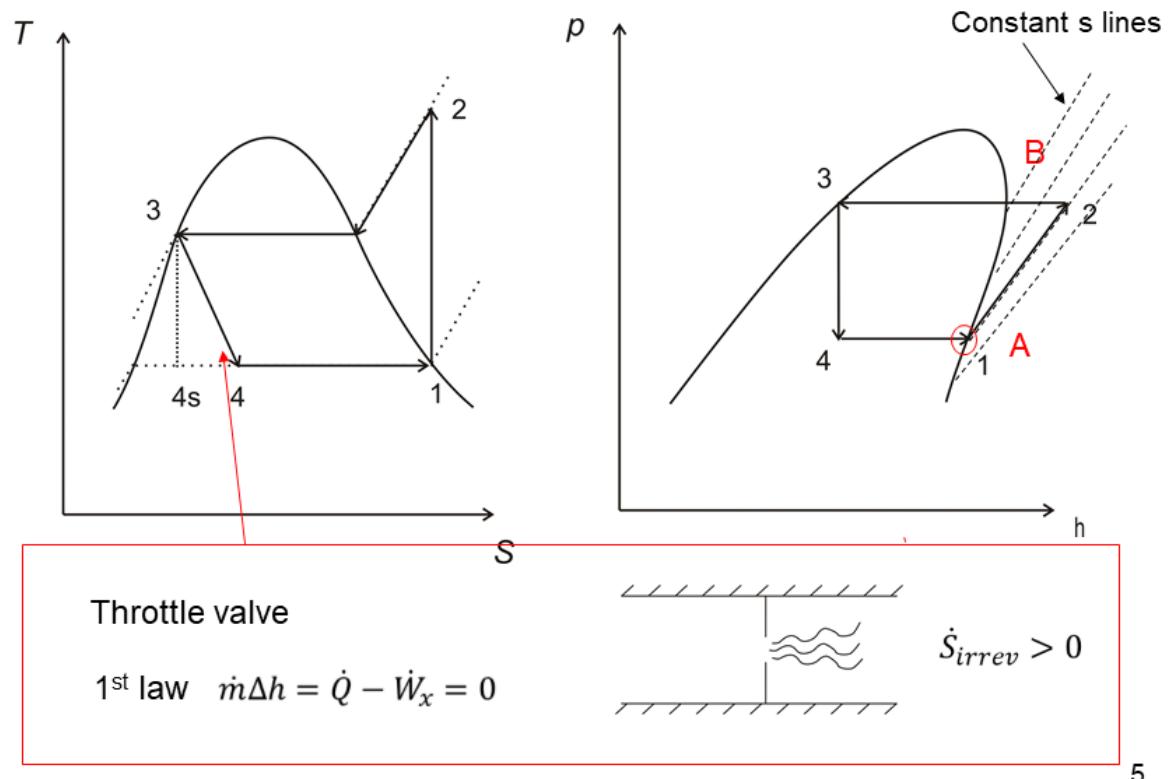


Practical alterations to cycle

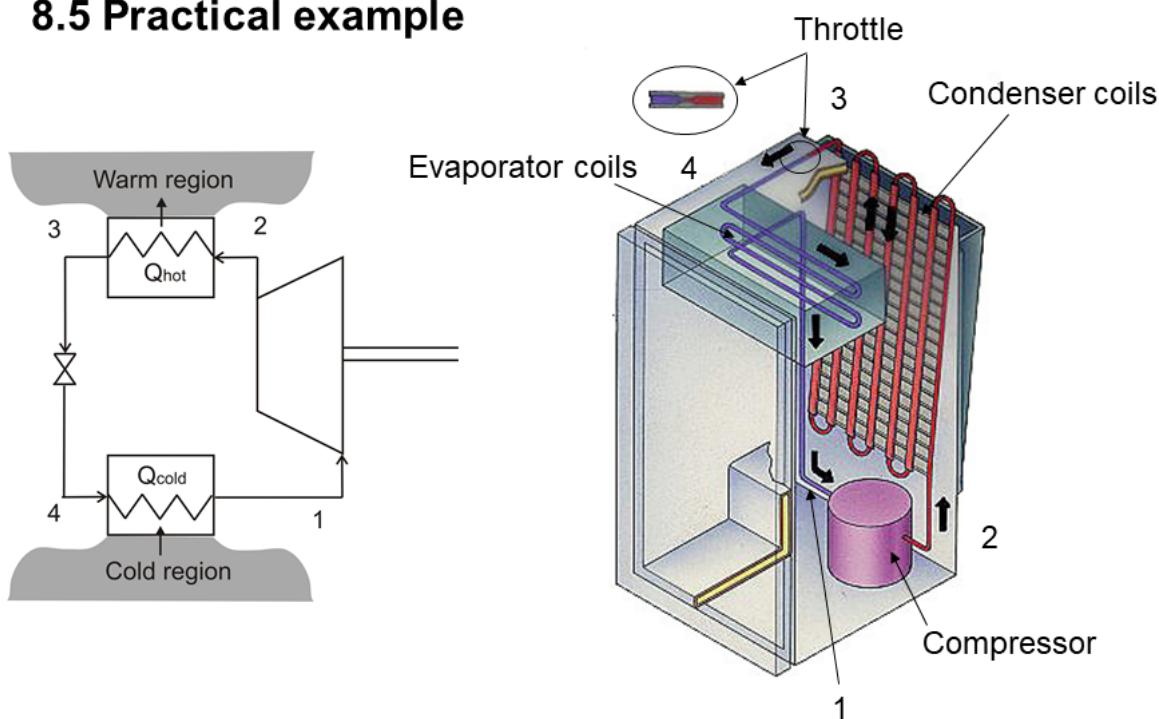
- Compression difficult in two-phase region.
- Turbine replaced with throttle valve.

4

8.5 T-S and p-h diagrams



8.5 Practical example



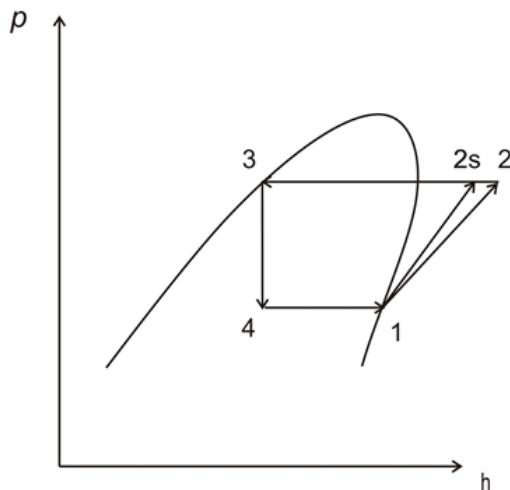
This makes for very simple robust refrigeration cycle.

6

Example 1: Refrigerator

Working fluid HFC-134a, evaporator pressure 1.06 bar, condenser pressure 21.16 bar. Compressor isentropic efficiency 0.8. Heat transfer from the cold space is 50W. Find the mass flow of the refrigerant and the COP.

Using the chart this is straightforward – In this problem we will use tables.



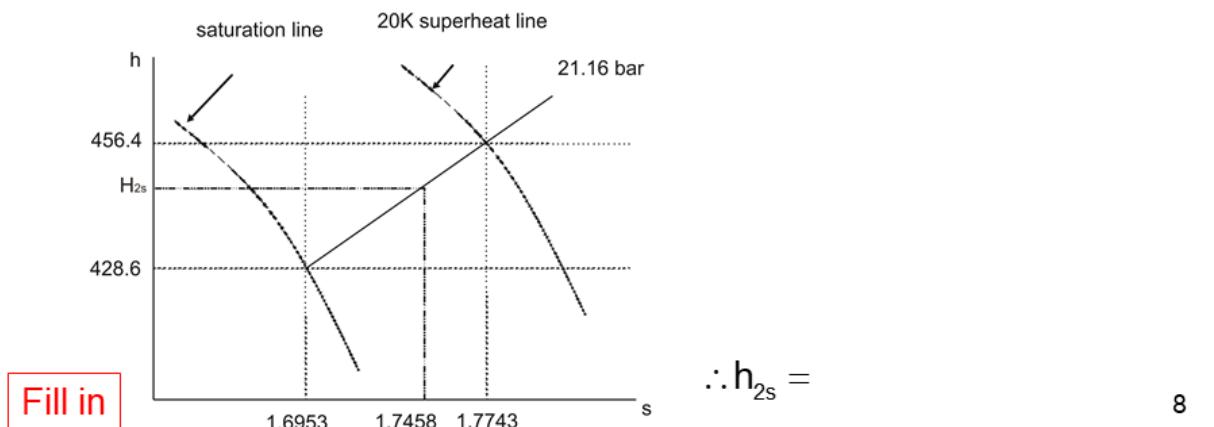
Point 1:
CUED tables
 $h_1 = 383.4 \text{ kJ/kg}$
 $s_1 = 1.7458 \text{ kJ/kgK}$

Point 2s:
 $s_{2s} = s_1 = 1.7458 \text{ kJ/kgK}$
Linear interpolation to get h_{2s}

7

PROPERTIES TABLE FOR REFRIGERANT R-134a (CH_2FCF_3)

Saturation Temp. °C	Saturation Pressure bar	Saturated						Superheated by				Saturation Temp. °C	
		Specific volume m³/kg		Specific enthalpy kJ/kg		Specific entropy kJ/kg K		20K		40K			
		v_x	v_g	h_x	h_g	s_x	s_g	h	s	h	s		
T_{sat}	P_{sat}											T_{sat}	
45	0.39	0.00070	0.46458	141.9	370.8	0.7687	1.7722	395.8	1.8348	401.3	1.8949	45	
40	0.51	0.00071	0.36094	148.1	374.0	0.7966	1.7643	389.2	1.8270	405.0	1.8869	40	
35	0.66	0.00071	0.28390	154.4	377.2	0.8221	1.7574	392.7	1.8201	408.6	1.8797	35	
30	0.84	0.00072	0.22585	160.8	380.3	0.8483	1.7512	396.1	1.8139	412.3	1.8734	30	
-25	1.06	0.00073	0.18155	167.2	383.4	0.8743	1.7458	399.5	1.8085	416.0	1.8678	-25	
60	16.82	0.00095	0.01148	287.5	426.6	1.2846	1.7022	451.8	1.7759	474.8	1.8392	60	
70	21.16	0.00101	0.00871	304.3	428.6	1.3329	1.6953	456.4	1.7743	480.7	1.8394	70	
80	26.33	0.00108	0.00654	322.4	428.8	1.3834	1.6848	460.4	1.7722	486.2	1.8395	80	



$$\eta_{Comp} = \frac{W_{ideal}}{W_{real}} \quad 0.8 = \frac{h_2 - h_1}{h_2 - h_1} = \frac{446 - 383.4}{h_2 - 383.4} \quad \therefore h_2 = 461.65 \text{ kJ/kg.}$$

CUED tables $h_4 = h_3 = 304.1 \text{ kJ/kg.}$

$$w_x = h_2 - h_1 = 461.65 - 383.4 =$$

$$q_{cold} = h_1 - h_4 = 383.4 - 304.3 =$$

$$q_{hot} = h_2 - h_3 = 461.65 - 304.3 =$$

Mass flow rate = $50/79100 = 6.32 \times 10^{-4} \text{ kg/s.}$

$$\text{COP}_R = q_{cold}/w_x = 79.1/78.25 =$$

Isentropic compression

$$\text{COP}_R = 79.1/(446-383.4) = 1.26$$

Carnot cycle

$$\text{COP}_{R\text{carnot}} = \frac{1}{\frac{T_{hot}}{T_{cold}} - 1} = 2.62$$

Fill in

NB In this problem $T_{hot} = 70^\circ\text{C}$, $T_{cold} = -25^\circ\text{C}$ what would happen if $T_{hot} = 30^\circ\text{C}$, $T_{cold} = -10^\circ\text{C}$

9

8.6 Choice of refrigerant

1. Refrigerants must be cheap, stable, inert and non-toxic.
2. The pressure range corresponding to the operating temperature range should be small (minimise the work of compression).
3. The vapour pressure should be low to reduce condenser cost, but it should be higher than atmospheric pressure to prevent air leakage into the system.
4. The latent heat of evaporation should be high in order to keep the mass flow rate low.

10

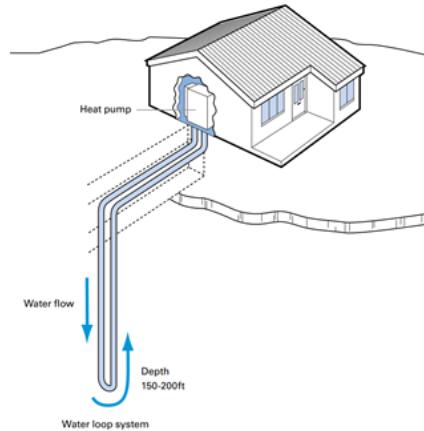
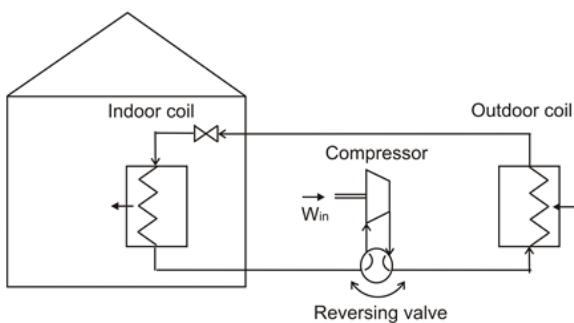
For a quasi-ideal cycle having $T_{hot} = 30^\circ\text{C}$ and $T_{cold} = -15^\circ\text{C}$, the $\text{COP}_{\text{CARNOT}} = 5.74$.

Refrigerant	COP	COP/COP _{Carnot}
Dichloroethane	5.14	0.894
R-21	5.09	0.888
Methyl chloride	4.90	0.853
Ammonia	4.85	0.845
Sulphur dioxide	4.73	0.825
R-12	4.72	0.822
R-22	4.56	0.794
HFC-134a	4.08	0.711
CO ₂	2.56	0.446

1. Early fridges used sulphur dioxide and methyl chloride (Banned in 1920s after a few leaks caused deaths).
2. General Motors developed R-12, R-21 in late 1920's (CFC family). R-12 trademarked 'Freon'.
3. In mid 1970's realised that CFC's destroy ozone layer and thus contribute to increased skin cancer and global warming.
4. R-22 has only 5% of ozone-depleting capacity. Recently developed R-134a is chlorine free and has largely replaced R-12.

11

8.7 Heat Pumps

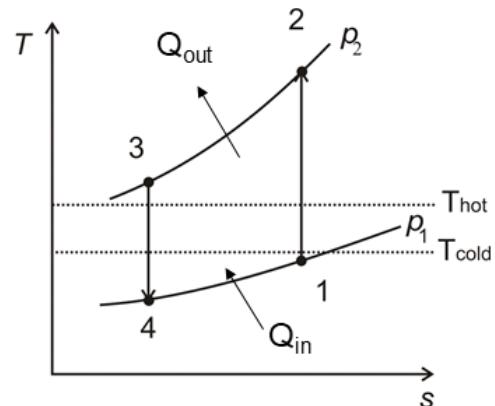
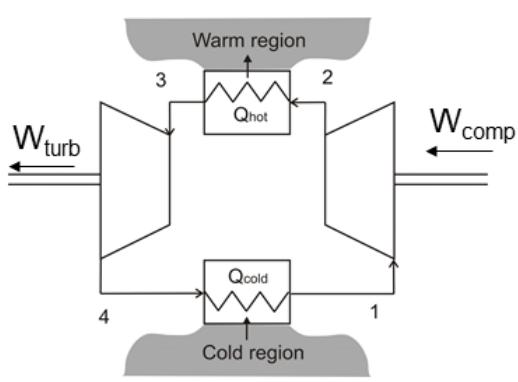


1. Heat pumps are more expensive to purchase and install than other heating systems.
2. Can save money in the long run.
3. In last decade ~1/3 of single family homes built in US fitted with one.
4. Most common heat source atmosphere but water and soil also used.
5. Biggest problem with using atmosphere is frost (in humid climates below 2-5°C). Frost disrupts heat transfer into coils. Defrosted by reversing.
6. Practical COPs are 1.5 to 4 depending on temperature of source.

12

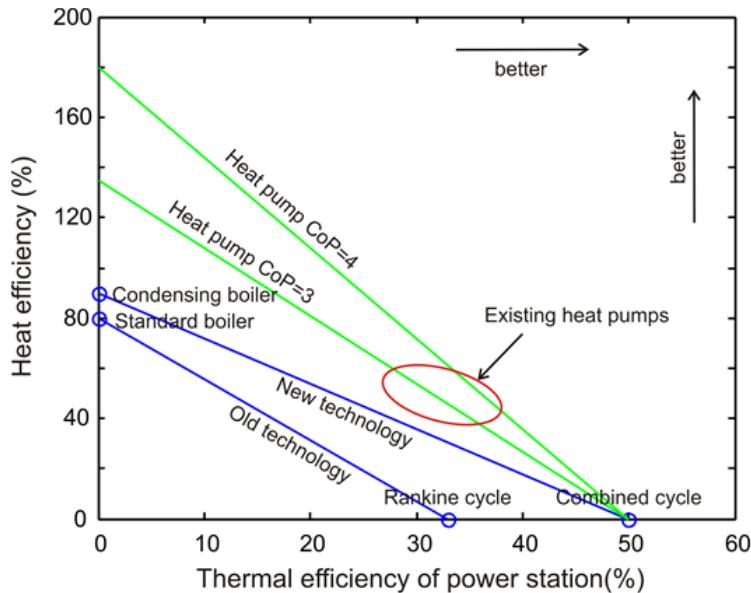
8.8 Gas refrigeration

1. All refrigerators so far involve phase change. Refrigerators can also be designed which use only the gas phase.
2. Used to achieve very low temperatures for liquefaction of gases.
3. Specialist uses such as aircraft cabin cooling.



8.9 Zero carbon heating

Using high efficiency combined cycle power stations to run heat pumps locally can produce heat efficiencies of 185% (NB condensing boiler 90%)



Heat efficiency = Heat provided to house / energy in fuel

If the electricity to power the heat pump comes from renewable power then heating can be made zero carbon

14

The graph above shows an analysis of heat pumps taken from 'Sustainable Energy' by David MacKay. The horizontal axis shows the thermal efficiency of a Rankine cycle and a combined cycle burning gas. The vertical axis shows the heat efficiency of two types of home boiler, a standard type and a condensing type. The lines joining each pair of points show mixes of heat and electricity from burning gas. By fixing the combination of heat and electricity you require your gas supply to provide you can sit anywhere on one of the two lines. The bottom line represents old technology and top line new technology.

The two lines above (green in the lecture) represent taking electricity from the average combined cycle power plant in the UK and choosing for a fraction of it to operate a heat pump. The bottom line is a heat pump with a COP of 3 and top line a COP of 4. The analysis allows for transmission line losses of 8%. The analysis shows that if all the electricity is used to operate a heat pump, with COP of 4, the heat efficiency would be 185%. This is a surprising result as burning the fuel in a condensing boiler results in a heat efficiency of 90%, half the value.

NB There is no rule that says that the maximum heat efficiency that you can get from a chemical energy source is 100%. The limit is set by the 2nd law. If you consider a reversible case, the best the 2nd law tells you can occur, the maximum heat efficiency is set by the temperatures across which the heat engine and the heat pump operate. Consider the case of fuel burnt at 1500k in an environment of 300k. The Carnot efficiency is 80%. If this was then used to power a heat pump operating between the environment at 300k and a reservoir at 310k the Carnot COP would be 31. This would give a theoretical heat efficiency of 2480%.

In future heat pumps will probably get even better and become much more common. In Japan, thanks to strong legislation favouring efficiency, heat pumps are now available with COP of 4.9. By powering heat pumps using renewable energy we have a method of zero carbon heating and cooling.

Lecture 9: Properties of mixtures

9. Properties of mixtures

9.1 Introduction

The objective of this lecture is to show how we can determine the properties of arbitrary gas mixtures

We have treated air as a single substance.

Component	Symbol	Volume
Nitrogen	N ₂	78.084%
Oxygen	O ₂	20.947%
Argon	Ar	0.934%
Carbon Dioxide	CO ₂	0.033%
Neon	Ne	18.2 parts per million
Helium	He	5.2 parts per million
Krypton	Kr	1.1 parts per million
Sulfur dioxide	SO ₂	1.0 parts per million
Methane	CH ₄	2.0 parts per million
Hydrogen	H ₂	0.5 parts per million
Nitrous Oxide	N ₂ O	0.5 parts per million
.....

9.2 Describing mixture composition

In any fraction of a mixture the number of moles n_i is given by

$$n_i = \frac{m_i}{M_i}$$

Where m_i is the mass of the fraction and M_i is the molecular mass

The total mass and number of moles in a mixture is given by

$$m = \sum_{i=1}^j m_i \quad n = \sum_{i=1}^j n_i$$

The mass fraction and mole fraction x_i of any fraction of a mixture is given by

$$\frac{m_i}{m} \quad \frac{n_i}{n} = x_i$$

2

9.3 Dalton's Law

Let's imagine a mixture of gases separated into its constituents such that each occupies a volume equal to that of the mixture, and is at the same temperature as the mixture. The Dalton Law states:-

'The pressure of a mixture of gases are equal to the sums of the pressures of the individual components when each occupies a volume equal to that of the mixture at the temperature of the mixture.'

Fill in

$$\boxed{} = \boxed{} + \boxed{}$$

Symbolically

$$P = \sum_{i=1}^j p_i$$

NB this is because each component fills such a small volume, i.e. molecules of each are free to roam through whole volume.

Where p_i is called the partial pressure of component i .

3

9.4 Relating p, V and T for an ideal gas

$$p_i = \frac{n_i \bar{R} T}{V} \quad \text{and} \quad p = \frac{n \bar{R} T}{V} \quad \text{so} \quad p_i = p \frac{n_i}{n}$$

$$V_i = \frac{n_i \bar{R} T}{p} \quad \text{and} \quad V = \frac{n \bar{R} T}{p} \quad \text{so} \quad V_i = V \frac{n_i}{n}$$

Therefore if the pressure in a tank with a gas mixture is known and the mole fractions are known the partial pressure and partial volume can be calculated.

4

9.5 Evaluating U, H and S

Total internal energy equals the sum of the internal energy's in the components etc.

$$U = m u = \sum_{i=1}^j U_i = \sum_{i=1}^j m_i u_i \quad \text{therefore} \quad u = \sum_{i=1}^j \frac{m_i}{m} u_i$$

$$H = m h = \sum_{i=1}^j H_i = \sum_{i=1}^j m_i h_i \quad \text{therefore} \quad h = \sum_{i=1}^j \frac{m_i}{m} h_i$$

$$S = m s = \sum_{i=1}^j S_i = \sum_{i=1}^j m_i s_i \quad \text{therefore} \quad s = \sum_{i=1}^j \frac{m_i}{m} s_i$$

Similarly

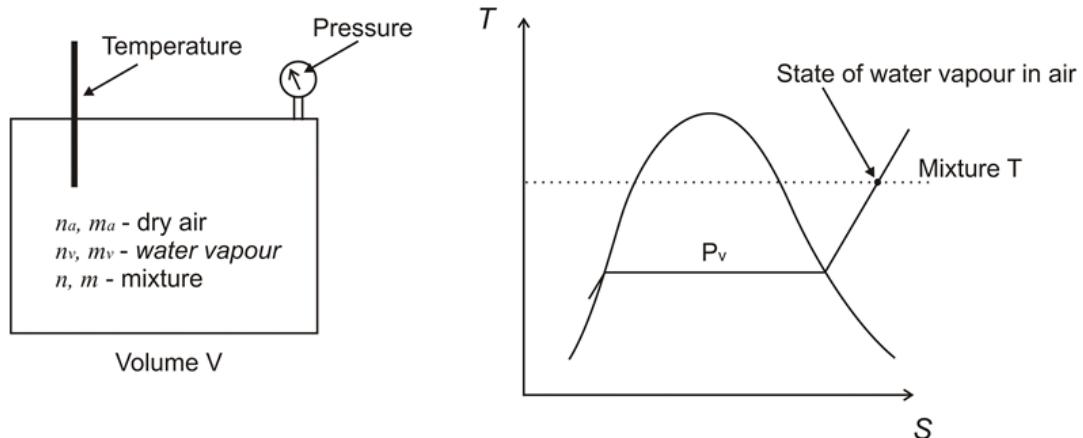
$$c_v = \sum_{i=1}^j \frac{m_i}{m} c_{v_i} \quad \text{and} \quad c_p = \sum_{i=1}^j \frac{m_i}{m} c_{p_i} \quad \text{and} \quad R = \sum_{i=1}^j \frac{m_i}{m} R_i$$

5

9.6 Mixtures of gases and vapours

A mixture of dry air and water vapour can be treated as a mixture of two component gases.

What is the state of the water vapour?



$$p = \frac{n\bar{R}T}{V} = \frac{(n_a + n_v)\bar{R}T}{V} \quad \text{Partial p's are} \quad p_a = p \frac{n_a}{n} \quad \text{and} \quad p_v = p \frac{n_v}{n}$$

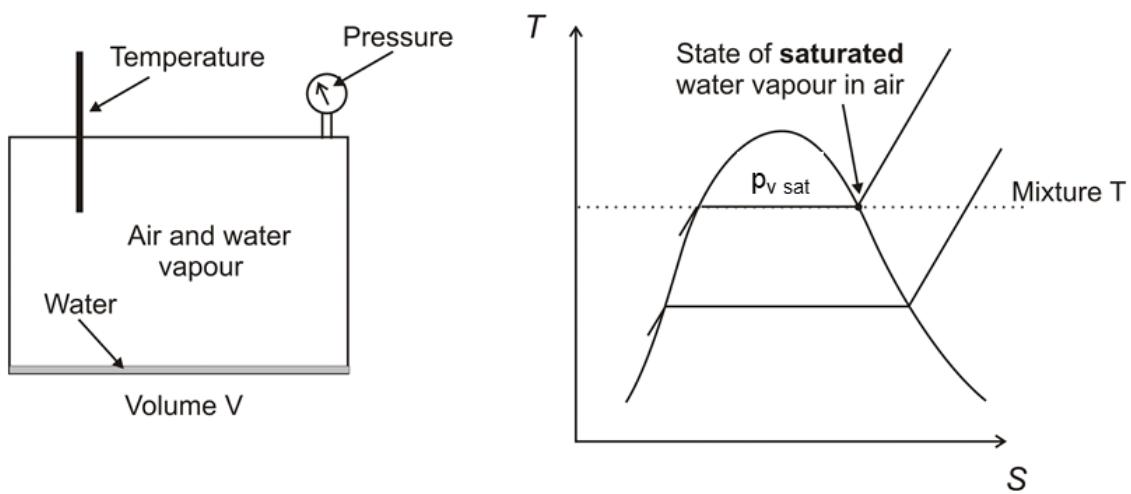
The water vapour is superheated steam at T and p_v

NB subscripts: v – water vapour, w – liquid water, a – air.

6

9.7 Saturated mixture

If there is a liquid layer in the bottom of the tank then the water will evaporate until the air becomes saturated and equilibrium is reached.

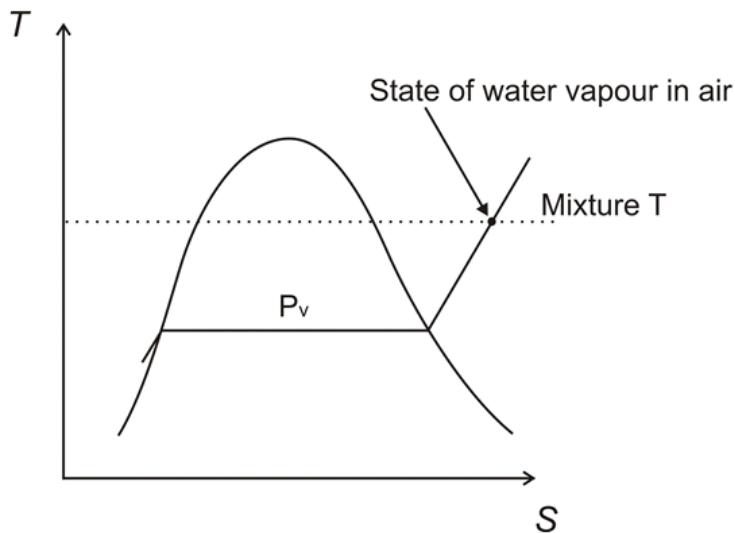


1. The partial pressure of the water vapour will rise as more water evaporates.
2. It rises until it reaches the saturation pressure of water at that T
3. $P_{v\text{ sat}}$ can be looked up in the CUED tables for water at T .

7

9.8 How can a mixture become saturated?

Fill in



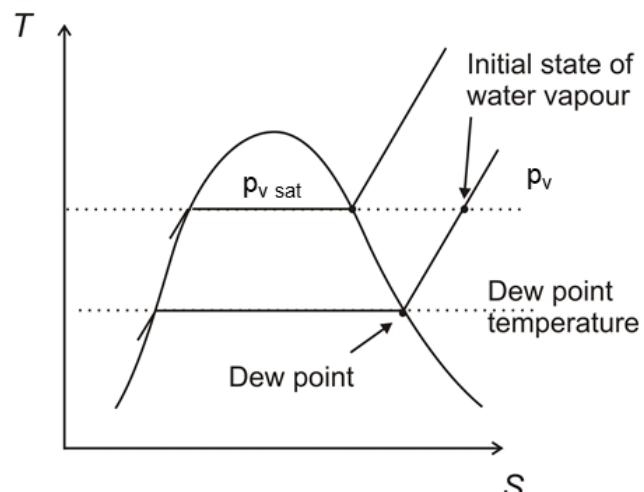
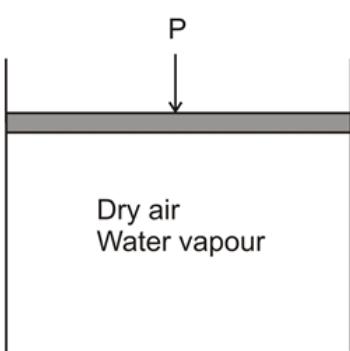
- 1) Evaporation of liquid water raises partial pressure.
- 2) Raising pressure of mixture raises partial pressure.
- 3) Lowering the temperature of the mixture.

$$p_v = p \frac{n_v}{n}$$

8

<http://www.youtube.com/watch?v=f0RNZnTmKzg> <http://www.youtube.com/watch?v=qvUSkIRqUGg>

9.9 Dew point temperature



The water vapour is originally superheated. As the tank is cooled the pressure remains constant.

So p_i remains constant. $p_i = p \frac{n_i}{n}$

The temperature drops to the **dew point temperature** and water starts to condense. 9

9.10 Specific humidity, relative humidity

The specific humidity is $\omega = \frac{m_v}{m_a}$ (NB also known at humidity ratio)

$$\omega = \frac{m_v}{m_a} = \frac{M_v p_v V / \bar{R}T}{M_a p_a V / \bar{R}T} = \frac{M_v p_v}{M_a p_a} = 0.622 \frac{p_v}{p_a} = 0.622 \frac{p_v}{p - p_v}$$

18.02
28.966
 $p = p_a + p_v$

Molecular mass of water to that of dry air

The relative humidity is $\phi = \frac{n_v}{n_{v,sat}}$ (Ratio of moles of vapour to moles of vapour necessary to saturate)

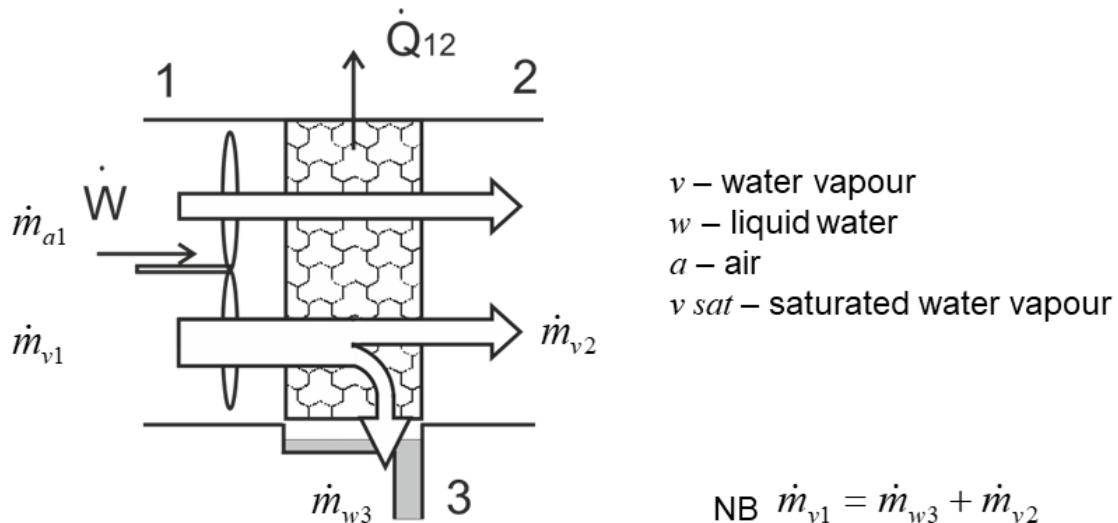
$$\phi = \frac{n_v}{n_{v,sat}} = \frac{p_v V / \bar{R}T}{p_{v,sat} V / \bar{R}T} = \frac{p_v}{p_{v,sat}}$$

ϕ in temperate climates are in the range 30% – 60%

10

Example 1: Air conditioning

An air conditioning passing 4kg s⁻¹ of moist air over a cooling coil which removes water vapour by condensation. The inlet air is at an atmospheric pressure of 1.013 bar, a temperature of 30°C and has a relative humidity of 45%. The outlet air is at 10°C and is saturated (relative humidity 100%). Calculate \dot{m}_{w3} and \dot{Q}_{12} . NB Ignore fan work.



Calculate \dot{m}_{w3}

Inlet

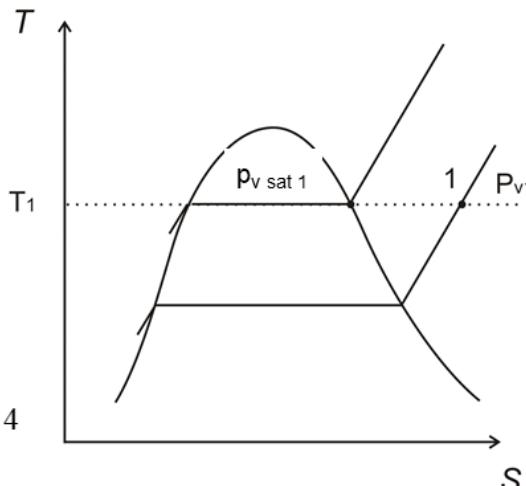
$$\phi_1 = 0.45 = \frac{P_{v1}}{P_{vsat1}} \quad p_{vsat1} = 0.0425 \text{ bar}$$

$$\therefore P_{v1} = 0.0191 \text{ bar}$$

$$\omega_1 = 0.622 \frac{P_{v1}}{p - P_{v1}} = 0.0119 = \frac{\dot{m}_{v1}}{\dot{m}_a}$$

$$\dot{m}_{v1} + \dot{m}_{a1} = 4 \text{ kg s}^{-1} \Rightarrow (1 + 0.0119)\dot{m}_a = 4$$

$$\dot{m}_{v1} = 0.0470 \text{ kg s}^{-1} \quad \dot{m}_{a1} = 3.953 \text{ kg s}^{-1}$$



Outlet

$$\phi_2 = 1 = \frac{P_{v2}}{P_{vsat2}}$$

$$p_{v2} = 0.01228$$

$$\omega_2 = 0.622 \frac{P_{v2}}{p - P_{v2}} = 0.0076 = \frac{\dot{m}_{v2}}{\dot{m}_{a2}}$$

$$\dot{m}_{v2} = 0.0300 \text{ kg s}^{-1}$$

$$\text{Mass of water condensed} = \dot{m}_{v1} - \dot{m}_{v2} = 0.0170 \text{ kg s}^{-1}$$

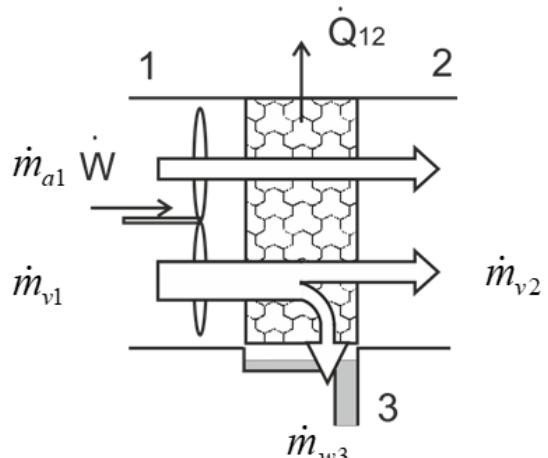
12

Calculate \dot{Q}_{12}

$$\dot{Q}_{12} = \dot{m}_{v1}h_{v1} - \dot{m}_{w3}h_{w3} - \dot{m}_{v2}h_{v2} + \dot{m}_{a1}c_{pa}(T_2 - T_1)$$

c_{pa} transport properties of air p28 CUED

$$\begin{aligned} \dot{m}_a c_p (T_2 - T_1) &= 3.953 \times 1.013 \times (-20) \\ &= -80.09 \text{ kW} \end{aligned}$$



$$h_{v2} = 2519.2 \text{ kJ kg}^{-1}, h_w = 42 \text{ kJ kg}^{-1} \text{ at } 10^\circ\text{C}$$

$$h_{v1} = 2555.5 \text{ kJ kg}^{-1} \text{ at } 30^\circ\text{C}$$

$$\therefore \dot{Q}_{12} = [0.03 \times 2519.2 + 0.017 \times 42 - 0.047 \times 2555.5] - 80.09 = -122.90 \text{ kW}$$

13

Example 2: How do you ensure the correct humidity in an aircraft in flight?

$T_{\text{outside}} = 217\text{K}$, $p_{\text{outside}} = 0.194 \text{ bar}$

$T_{\text{inside}} = 20^\circ\text{C}$, $p_{\text{inside}} = 0.75\text{bar}$

$$\phi_{\text{outside}} = 1.0$$

$$\omega_{\text{outside}} = \frac{m_v}{m_a} = \omega_{\text{inside}}$$

$$\phi_{\text{inside}} = 0.005$$

Fill in

$$\phi_{\text{ideal}} = 0.3$$

14

Humidifying the air with water spray?

$$\phi_{\text{ideal}} = 0.3$$

↓

Mass of water = Mass of air $\times \omega_{\text{ideal}} = 500\text{kg}$

↑

Mass of air in plane $\sim 500\text{kg}$ hours of flight ~ 10 changes per hour ~ 20

Human humidification?

$$\phi_{\text{breathed}} = 1.0$$

↓

Mass of water breathed = Mass of air $\times \omega_{\text{breathed}} = 130\text{kg}$

↑

Breathing mass flow $\sim 0.06\text{kg/h}$ hours of flight ~ 10 Passenger No. ~ 400

$$\phi_{\text{inside}} = 0.1$$

from the web:-“In 1999, Boeing admitted that aircraft “typically operate in the 10-15 percent relative humidity range.” Unfortunately, studies have shown that attempting to increase the humidity of cabin air by reducing the flow of dry air from outside the plane leads to increased health complications.”

15

A recent study by the US National Research Council concluded that flying in an aircraft is as likely to make you sick as any other enclosed environment, such as a cinema or office. “There is always an increased risk of infection whenever you enter a confined space, but an aircraft cabin is no worse an environment than the office you sit in every day. Modern aircraft refresh air 15 times an hour, compared to fewer than 12 times hour in an office building.” states Dr Mark Grendau. “On most transatlantic jets the air is filtered through hospital-grade filters that are designed to remove over 99% of bacteria and particles. The cabin is also separated into ventilation systems covering typically every seven rows, limiting the spread of germs through the plane.”

Travellers, however, often complain about picking up colds and viruses on flights. The research shows that this is unlikely to be true and that symptoms similar to those caused by a cold or virus are brought on by the low relative humidity. Aircraft manufacturers seem to differ in the reason they give for the low relative humidity in aircraft. Sometimes it is stated that the low relative humidity is deliberate and is used to prevent the fuselage from corroding. It seems unlikely though that the airlines would be prepared to carry the excess weight of water required to humidify the air. It will be interesting to see whether the next generation of planes with composite fuselages have a raised cabin relative humidity.

9.11 Lowering emission – Evaporative cooling

Air conditioning accounts for about 12% of US homes energy use



The Concept: In dry climates that don't get too hot, evaporative coolers can be effective at lowering the temperature while consuming a fraction of the energy used by vapour-compression devices. The problem is that the cooling method—, which makes use of the fact that the evaporation of water absorbs heat, doesn't quite do enough cooling in extremely hot climates like Arizona. And it can actually make things worse in humid areas by adding water vapour to the air.

Researchers at the National Renewable Energy Laboratory in the US have developed a new kind of a/c system that combines evaporative cooling with the properties of hydrophobic (water-resistant) membranes and a desiccant, which absorbs moisture from the air. The device produces cool, dry air, using no refrigerants and much less energy than traditional vapour-compression methods. The researchers will spend the next few years optimizing the system, called a Desiccant-Enhanced Evaporative air conditioner (DEVap).

16

Lecture 10: Combustion processes

10. Combustion processes

10.1 Introduction

The combustion of fossil fuels continues to be the dominant method of generating electricity and fuelling personal transport.

First we will look at setting up the chemical equations, then use the 1st law to solve combustion problems.

NB predicting noxious emissions from combustion processes (NO_x, CO, particulates) lies outside the scope of this course – the quantities of these pollutants is usually a very small component of the products of combustion, though their effects on health can be severe.

1

10.2 Chemical equations

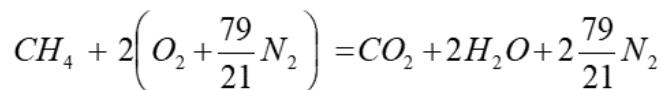
Stoichiometric combustion: Methane (CH_4) / Air.

NB Assume air 21% oxygen and 79% nitrogen.

Step 1: Fuel / Fuel-derived products



Step 2: Use number of oxygen molecules on RHS to set air on LHS



Nitrogen is carried over to the RHS (takes no part in the combustion).

NB Equation written on a molar basis.

The equation says “1 kmol of fuel combines with $2*(1+79/21)= 9.52\text{kmols of air}$ to form.....”

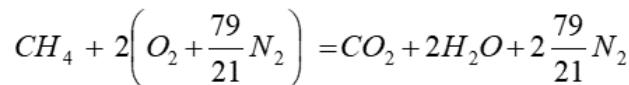
10.3 Calculating mass fraction and air fuel ratio

Air fuel ratio is defined as ratio of masses (not moles)

$$AFR = \frac{\dot{m}_a}{\dot{m}_f}$$

Convert molar chemical equation to masses

NB Molar masses found in the CUED data book



Mass of each fraction $m_i = n_i M_i$

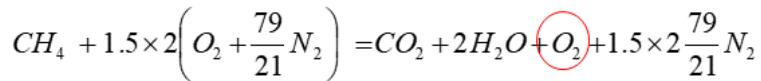
$$(12 + 4 \times 1) + 2 \times (32 + \frac{79}{21} \times 28) = 44 + 2 \times 18 + 2 \times \frac{79}{21} \times 28$$

$$16\text{kg CH}_4 + 274.67\text{kg air} = 44\text{kg CO}_2 + 36\text{kg H}_2\text{O} + 210.67\text{kg N}_2$$

Air fuel ratio is thus $AFR = \frac{274.67}{16} = 17.2$

10.4 Calculations with excess air

If there were more air supplied, say “50% excess air” then we would have



$$(12 + 4 \times 1) + 3 \times (32 + \frac{79}{21} \times 28) = 44 + 2(18) + 32 + 3 \times \frac{79}{21} \times 28$$

$$16kg CH_4 + 412kg air = 44kg CO_2 + 36kg H_2O + 32kg O_2 + 316kg N_2$$

The air fuel ratio is thus $AFR = \frac{412}{16} = 25.75$

4

10.5 Lambda and equivalence ratio

Lambda (λ) is commonly used to express the stoichiometry,

$$\lambda = \frac{AFR}{AFR_{stoichiometric}}$$

$\lambda > 1$ the combustion is lean.

$\lambda = 1$ the combustion is stoichiometric.

$\lambda < 1$ the combustion is rich.

Equivalence ratio (ϕ) is also used

$$\phi = \frac{1}{\lambda}$$

5

Example 1:

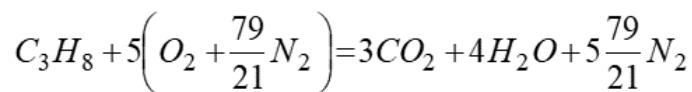
Propane, C_3H_8 , burnt with $\lambda = 0.8$. Pressure is 1bar. Calculate the mass fraction of each of the products of combustion. NB Temperature of combustion product 1800°C.

Water vapour – **wet** basis.
Water liquid – **dry** basis.

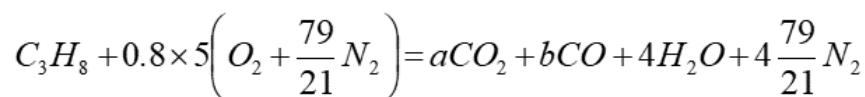
NB Assume **wet** when the resulting combustion product > 100°C.
Usually the case for combustion problems.

6

Step 1: Stoichiometric relationship



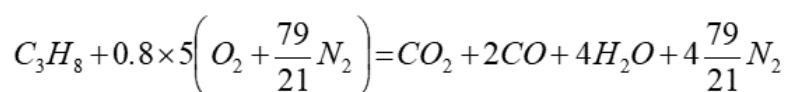
Step 2: $\lambda = 0.8$ we can now write



Assume hydrogen burns first, then remaining carbon as carbon monoxide.

Step 3: Simultaneous equations to calculate a & b

$$\begin{array}{ll} C: & 3 = a + b \\ O: & 8 = 2a + b + 4 \end{array} \quad \text{Therefore } a = 1, b = 2$$



7

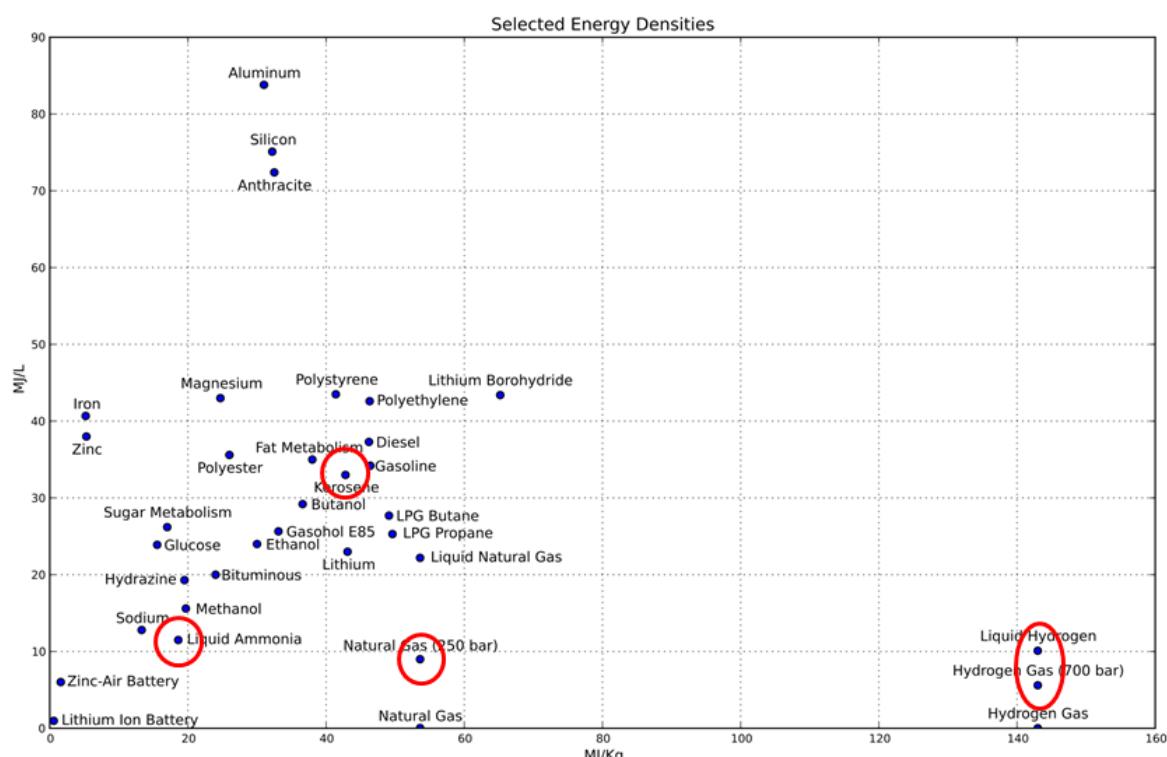
Now we need to convert molar fraction into a mass fraction.

$$\frac{m_i}{m} = \frac{n_i M_i}{\sum n_i M_i} = \frac{x_i M_i}{\sum x_i M_i} \quad NB \ x_i = n_i/n$$

Gas	n_i	M_i	$x_i = n_i/n$ wet	$x_i M_i$ wet	$\frac{m_i}{m} = \frac{x_i M_i}{\sum x_i M_i}$ wet
CO ₂	1	44	0.045	2.00	0.074
CO	2	28	0.091	2.54	0.094
H ₂ O	4	18	0.18	3.27	0.121
N ₂	15.05	28	0.68	19.11	0.710
Sum	22.05		1	26.9 = M	1

8

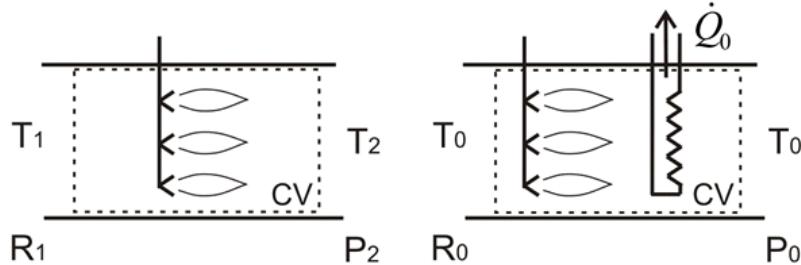
10.6 Zero carbon combustion



Hydrogen (H₂) and ammonia (NH₃) are zero carbon fuels

9

10.7 First law applied to combustion



Stoichiometric Equation	Molar Mass M of Fuel kg/kmol	Phase	Calorific Value MJ/kg	
$C + \frac{1}{2}O_2 \rightarrow CO$	12	solid	9.190	
$C + O_2 \rightarrow CO_2$	12		32.760	
$CO + \frac{1}{2}O_2 \rightarrow CO_2$	28		10.100	
			Higher: H_2O to water	Lower: H_2O to steam
$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$	2 [†]	gas	142.000	
$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$	16	gas	55.500	50.010
$C_2H_6 + 3.5O_2 \rightarrow 2CO_2 + 3H_2O$	30	gas	51.870	47.470
$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$	44	gas	50.360	46.360

NB pressure of 1 bar

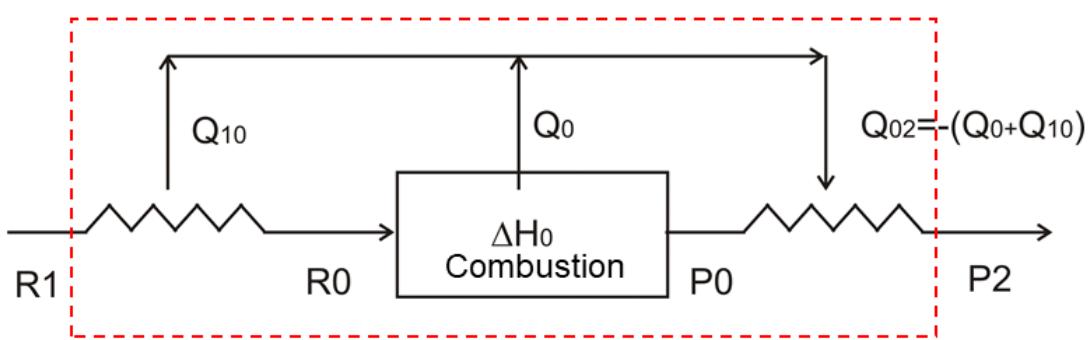
10

Calorific values/Enthalpy of reaction, table from p11 CUED thermo data book

How can we determine the temperature of a gas after combustion?

Split combustion into three processes.

- 1) Heat removed so reactants brought from initial state (R1) to 25°C (R0).
- 2) Heat removed so that combustion occurs at constant 25°C (R0 to P0).
- 3) Heat added so combustion products raised to final T (P0 to P2).

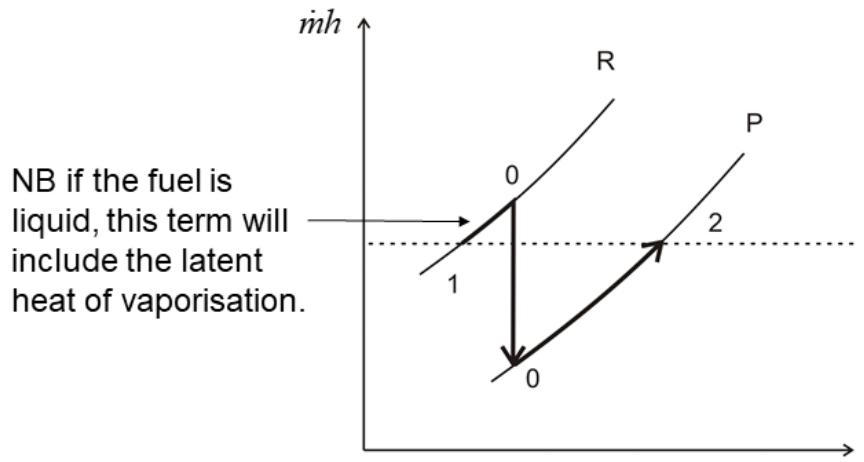


$$1^{\text{st}} \text{ law} \quad \dot{Q} - \dot{W}_x = \dot{m}(h_{P2} - h_{R1}) = \dot{m}[(h_{P2} - h_{P0}) + (h_{P0} - h_{R0}) + (h_{R0} - h_{R1})]$$

How do we determine the enthalpy of reaction at constant 25°C?

11

$$0 - 0 = \dot{m} (h_{P2} - h_{P0}) + \dot{Q}_0 + \dot{m} (h_{R0} - h_{R1})$$



For low temperatures use $\dot{m}_i c_{pi} (T_{R0} - T_{R1})$ for each component in mixture.

For high temperatures (typically the products of combustion) we use tabulated data, as the gases are “very” semi-perfect. “Molar enthalpies at low pressure” in data book.

12

Molar enthalpies, written \bar{h} , means that h is expressed per kmol (in this case MJ/kmol), rather than per kg basis

MOLAR ENTHALPIES AT LOW PRESSURES

At low pressures, and over the temperature range quoted, the gases listed in this Table behave as semi-perfect gases. That is, while having the molar equation of state $pv = RT$, their specific heat capacities c_p and c_v are not constant but are functions only of temperature.

Gas	Air	N_2	O_2	H_2	CO	CO_2	H_2O	Gas
Molar Mass kg/kmol	29	28	32	2^\dagger	28	44	18	Molar Mass kg/kmol
Temperature K	Molar enthalpy MJ/kmol							Temperature K
200	5.79	5.81	5.79	5.69	5.81	5.96	6.62	200
298.15=25°C	8.64	8.67	8.66	8.46	8.67	9.37	9.90	25°C=298.15
300	8.70	8.72	8.71	8.52	8.72	9.44	9.96	300
400	11.62	11.64	11.68	11.42	11.64	13.37	13.35	400
500	14.57	14.58	14.74	14.34	14.60	17.67	16.82	500
600	17.59	17.56	17.90	17.27	17.61	22.27	20.39	600
700	20.66	20.61	21.16	20.21	20.69	27.12	24.09	700
800	23.81	23.72	24.50	23.16	23.85	32.18	27.90	800
900	27.03	26.89	27.90	26.13	27.07	37.41	31.83	900
1000	30.30	30.14	31.37	29.14	30.36	42.78	35.90	1000
1100	33.64	33.44	34.88	32.18	33.71	48.27	40.09	1100
1200	37.02	36.79	38.43	35.26	37.11	53.87	44.41	1200
1300	40.44	40.19	42.01	38.38	40.54	59.55	48.84	1300

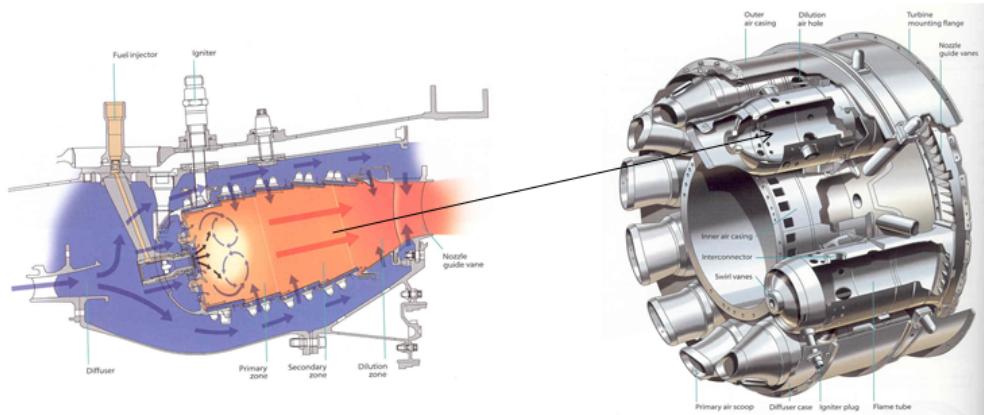
It is best to demonstrate the use of the 1st law with an example.

13

Example 2:

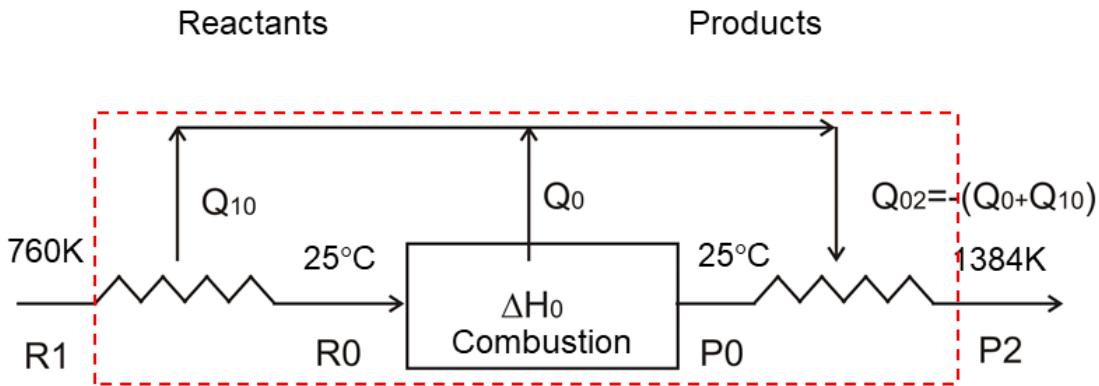
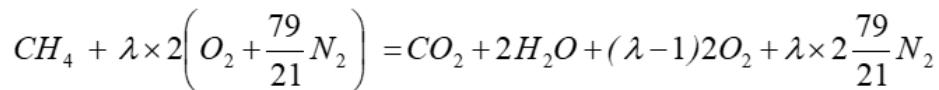
A Rolls Royce RB211-6556 is a stationary power generation unit, based on their aviation engines. The fuel for these units is generally diesel, kerosene or natural gas (CH_4). In a set of tests natural gas was used.

The compressor exit temperature was 760K and the turbine inlet temperature was 1384K. Calculate the equivalence ratio ϕ of the combustion process. Assume fuel enters at 25°C.



14

NB all enthalpy's are calculated on the basis of 1kmol of fuel. NB $\lambda = \frac{1}{\phi}$



$$0 = \dot{m}(h_{P2} - h_{P0}) + \dot{Q}_0 + \dot{m}(h_{R0} - h_{R1})$$

$$\sum n_{ip} (\bar{h}_{1384} - \bar{h}_{298}) \quad M_{\text{CH}_4} CV \quad \sum M_{iR} c_{pRi} (T_{298} - T_{1384}) \quad 15$$

It is strongly recommended that you use a tabular approach to these questions.

	n_{iP}	\bar{h}_{298}	\bar{h}_{1384}	$n_{ip} (\bar{h}_{1384} - \bar{h}_{298})$	n_{iR}	M_{iR}	$n_{iR} * M_{iR} * c_p * \Delta T$
CH4	-	-	-		1	16	0
Air	-	-	-		9.52λ	29	$9.52\lambda * 29 * 1.04 \times 10^{-3} * (298 - 760)$
CO ₂	1	9.37	64.39	55.02	-		
H ₂ O	2	9.90	52.66	85.52	-		
O ₂	$2(\lambda-1)$	8.66	45.05	$72.78 * (\lambda-1)$	-		
N ₂	$2\lambda 79/21 = 7.52 \lambda$	8.67	43.07	$258.83 * \lambda$	-		
Sum	9.52λ+1			$67.76 + 331.61 * \lambda$			

1. CUED Molar enthalpies 2. C_p calculated at the average temperature

NB assume **wet** basis

16

$$0 = \dot{m}(h_{P2} - h_{P0}) + \dot{Q}_0 + \dot{m}(h_{R0} - h_{R1})$$

$$0 - 0 = [67.76 + 331.61\lambda] - [50.01 \times 16] + [29 \times 9.52\lambda \times 1.04 \times 10^{-3} (298 - 760)]$$

Solving above gives $\lambda = 3.68$ or and equivalence ratio $\phi = 0.272$

17

Depending on the temperature and composition of the products the change in enthalpy can be calculated in a number of ways. If the combustion is undertaken with a large excess of air (very lean) the products after combustion may still be relatively cold. This is the case in large industrial space heaters. In such cases the c_p of the combined combustion products are often found to be relatively constant. In such problems the reactant and products enthalpy changes can be calculated simply, by using $c_p\Delta T$. If the combustion occurs without a large excess of air (not very lean) the final temperature of the products will be high and c_p of each of the products will vary non-linearly with temperature. In these cases the molar enthalpies on p13 of the CUED data book must be used.

In cases likely the above problem of a gas turbine combustor the temperature of the reactants is initially high. The reactants are made up of air and fuel. The fuel is normally supplied at 25°C and therefore its enthalpy change is zero. The air's enthalpy change can usually be calculated with relative accuracy by using $c_p\Delta T$. The c_p value at the average temperature should be used. In the case above

$$T_{mean} = \frac{(298 + 760)}{2} - 273 \sim 250^{\circ}C$$

The value of c_p can be found in the transport properties table on p28 of the CUED data book.

Care should always be taken in deciding which of the above methods to use. Variations in c_p can be significant at high temperatures.

NB Be careful when applying the 1st law to combustion. In the above analysis each term is on the basis of 1kmol of fuel. It should be noted that the enthalpy of reaction is a negative quantity (by definition), and is in MJ/Kg (not /kmol) in the data book.