6. Heat Transfer by Radiation

Heat can be transferred by Electromagnetic waves emitted by a body *as a result of its temperature.* It does not require a medium between the bodies (e.g. sun to earth), and can occur through a medium which is colder than either body.

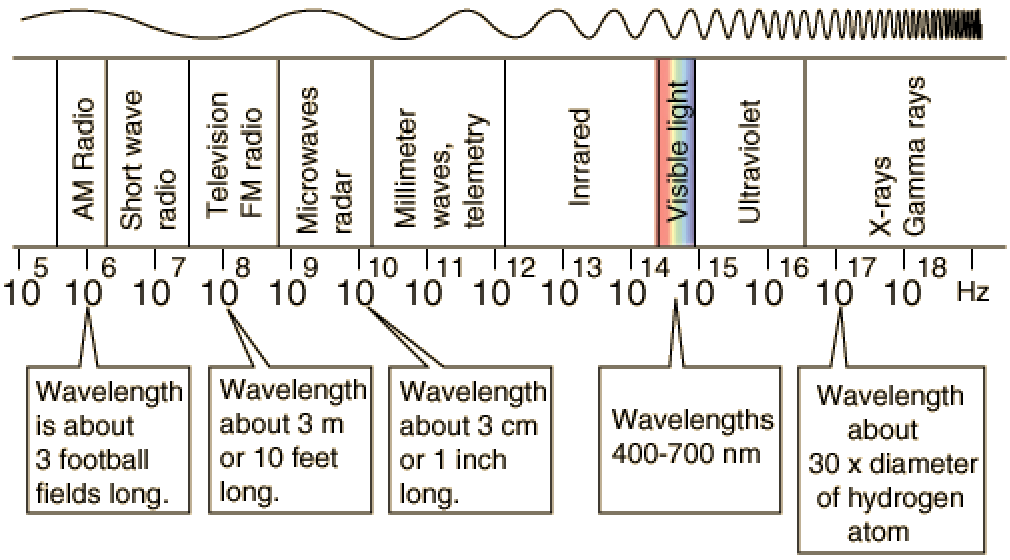


Figure 1. Electromagnetic Spectrum, source
http://en.wikipedia.org/wiki/Electromagnetic_spectrum

Thermal radiation lies approximately in the wavelength range 0.1μm to 100 μm (3.10^{15} to 3.10^{12} Hz), visible light is in the range 0.35 μm to 0.75 μm.

6.1. *The black body*

A "***black body***" is an idealised object, which will

1. completely absorb all radiation incident upon it.
2. emit the largest amount of energy per unit area possible for a given temperature

The maximum rate at which a body can radiate energy is given by the ***Stefan-Boltzmann Law***[1]

$$E_b = \sigma T^4$$

where σ (= 5.67×10^{-8} $Wm^{-2}K^{-4}$) is the ***Stefan-Boltzmann constant*** and E_b is called the *black body* emissive power.

[1] The Stefan-Boltzmann Law can be derived from statistical mechanics, which is beyond the scope of this course.

The monochromatic emissive power of a black body[2] (i.e. the power emitted between wavelengths λ and λ+dλ) is given by the relationship

$$E_{b\lambda} = \frac{c_1 \lambda^{-5}}{e^{c_2/\lambda T} - 1}$$

where λ is the wavelength, c_1 = 3.743x10^8 Wμm/m^2, and c_2 = 1.4387x10^4 μmK. If one integrates this expression over all wavelengths we obtain

$$E_b = \int_0^\infty E_{b\lambda} d\lambda = \sigma T^4$$

The figure below shows plots from the equation for $E_{b\lambda}$ for a range of temperatures.

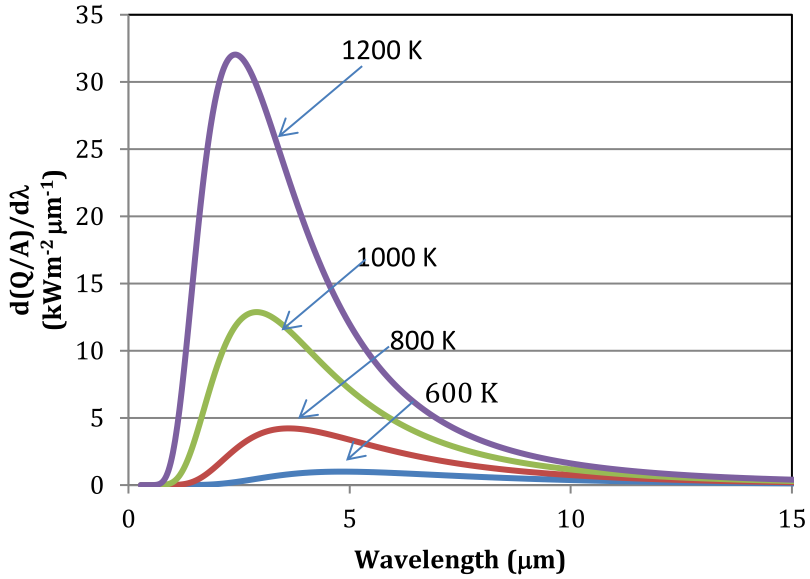


Figure 2. Spectrum of radiation from a black body

Note that the peak moves to shorter wavelengths with increasing temperature – the visible spectrum is between 0.4 μm (violet) and 0.7μm (red), so we would expect a body being heated to appear dull red first, then a colour change through orange to yellow etc.

The peak value of $E_{b\lambda}$ is at a wavelength $\lambda_{\max} = \dfrac{2897.6}{T} \mu m$

How hot is the surface of the sun?

The sun's surface behaves almost as a black body, with a temperature of about 5780K – from the relationship above $\lambda_{max} = \frac{2897.6}{5780} = 0.5\ \mu m$ -not surprisingly, in the visible range.

[2] This is a result from statistical thermodynamics and its derivation is beyond the scope of this course.

6.2. *Real surfaces*

Real surfaces emit less radiation than a black body. The monochromatic emissivity ε_λ characterises the surface

$$\varepsilon_\lambda = \frac{\text{radiation emmited per unit area between } \lambda \text{ and } \lambda\text{+d}\lambda}{E_{b\lambda}}$$

The monochromatic emissivity is a function of the wavelength of radiation, λ. The total emissivity (usually just called the emissivity) is given by

Emission from real surface

$$\varepsilon = \frac{E}{E_b} = \frac{\int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda}{\sigma T^4}$$

Emission for black body at same T

In general the emissivity is a function of temperature.

Surface	**T(K)**	**Emissivity ε**	**Surface**	**T(K)**	**Emissivity ε**
Polished aluminium	500, 1000	0.039, 0.066	Polished copper	353	0.018
Polished iron	450	0.052	Oil paints, all colours	373	0.92-0.96
Oxidized iron	373	0.74	Water	273	0.95

6.2.1. *The Grey Body*

A grey body is defined such that the monochromatic emissivity, ε_λ is independent of wavelength, i.e. $\varepsilon_\lambda$is constant $\Longrightarrow \varepsilon_\lambda = \varepsilon$

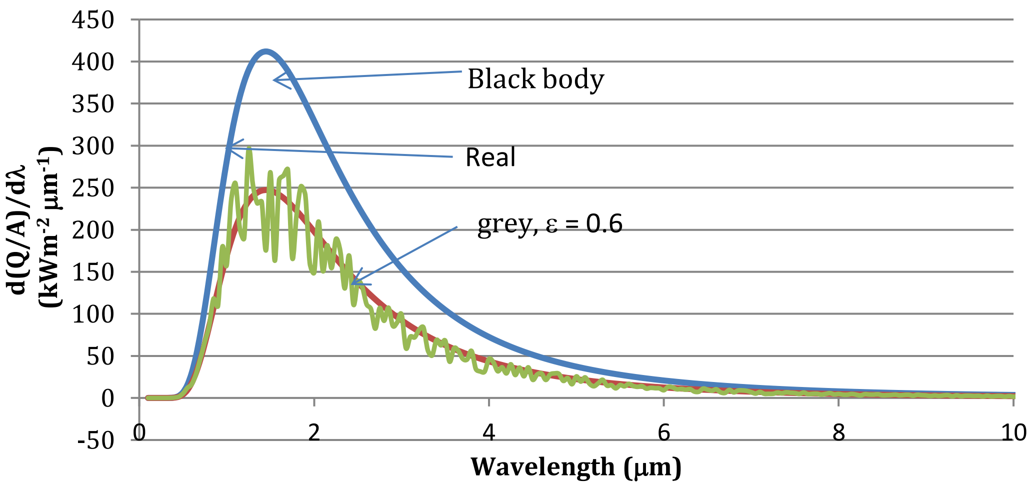
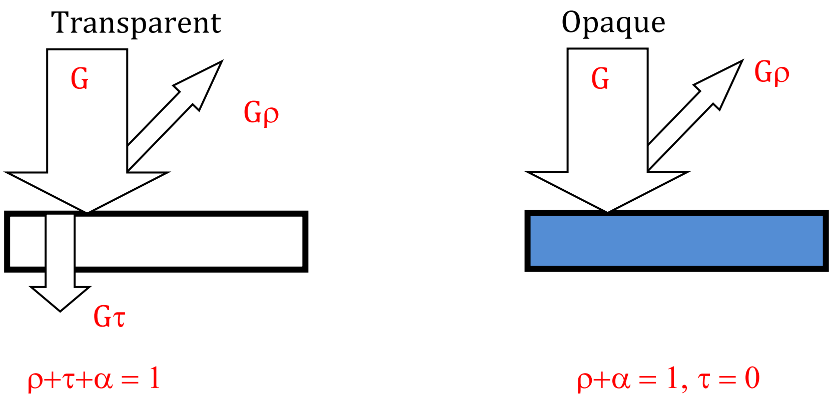


Figure 3. Spectrum of emissive power from a black, grey (ε=0.6) and a real surface at 2000 K

6.2.2. *Absorption, transmission and reflection*

When radiation strikes a surface, a fraction (= ρ, the reflectivity) will be reflected, a fraction (= α, the absorbtivity) will be absorbed, and a fraction (= τ, the transmissivity) will be transmitted. Thus $\rho+\alpha+\tau=1$. Most solid bodies do not transmit thermal radiation, so the transmissivity can be taken as zero, hence $\rho+\alpha=1$.



The reflected part of the radiation may be "specular" (i.e. as from a mirror), diffuse (i.e. scattered equally in all directions), or some combination of these. ***Most surfaces are predominately diffuse reflectors.***

6.2.3. *Kirchhoff's identity*

Due to processes occurring at the quantum scale (very very small!), the probability of a photon being absorbed on hitting the surface, is equal to the probability of a photon being emitted. This gives rise to Kirchhoff's identity.

Kirchhoff's identity, $\varepsilon = \alpha$, i.e. the absorptivity is equal to the emissivity.

There is a question on the example sheet which "proves" this must be true for a body exchanging radiation with black enclosure, at equilibrium. However, Kirchhoff's identity is generally true.

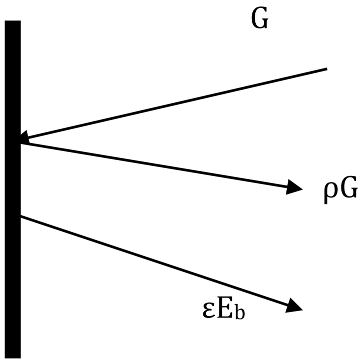
6.2.4. *Radiation to and from a non -black bodies.*

The total radiative emission from a non-black body surface not only has an εE_b component (the radiation leaving a surface due to its temperature), but also a ρ times the incident radiation' component.

Let us define two new terms:

G = Irradiation. The total radiation incident on a surface per unit time and area
J = Radiosity. The total radiation leaving a surface per unit time and area.

We further assume that these quantities are uniform over each surface: this needs to be kept in mind when applying the results to come.



$$J = \varepsilon E_b + \rho G$$

$$J = \varepsilon E_b + \rho G$$

The radiation leaving a surface is the sum of the radiation originating from the body due to its temperature, and the reflected part of the incoming radiation.

Since $\rho = 1 - \alpha = 1 - \varepsilon$ (the surfaces are assumed to be non-transmissive), the expression for J may be written

$$J = \varepsilon E_b + (1-\varepsilon)G$$

$$G = \frac{J - \varepsilon E_b}{1-\varepsilon}$$

Now the net radiation leaving a surface per unit area per unit time $\left(\dot{Q}/A\right)$ is the difference between the radiosity and the irradiation, so

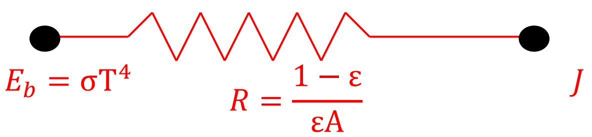
$$\frac{\dot{Q}}{A} = J - G = \varepsilon E_b + (1-\varepsilon)G - G$$

If we substitute for G, we obtain

$$\dot{Q} = \frac{E_b - J}{(1-\varepsilon)/\varepsilon A}.$$

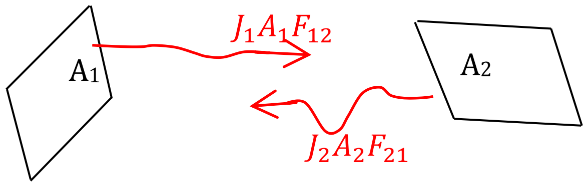
The expression above can be interpreted using an "ohm's law" analogy, with $E_b - J$ as a potential difference, and $\frac{1-\varepsilon}{\varepsilon A}$ as a 'surface' resistance.

i.e.



6.3. *Radiative exchange between surfaces*

Let us examine the heat transfer between two surfaces A_1 and A_2 (both diffuse reflectors)



We define

F_{12} **as the fraction of energy leaving surface 1 that arrives at surface 2**

F_{21} **as the fraction of energy leaving surface 2 that arrives at surface 1**

- $F_{i,j}$ is called the shape factor (or sometimes the view factor)
- $F_{i,j}$ is purely geometric and is related to how much of surface j can be seen by surface i

Since electromagnetic rays travel in straight lines, if radiation can travel from 1 to 2 along a path, it can also travel from 2 to 1 along the same path. This means that view factors are linked by the ***reciprocity relationship***.

$$A_1F_{12}=A_2F_{21}$$

sinceit is ***purely geometric*** this does not depend the nature of the radiation emitted or reflected from the surface.

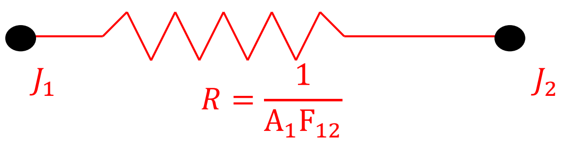
Of the total radiation that leaves surface 1 (J_1A_1), a quantity $J_1A_1F_{12}$ reaches surface 2. Likewise from 2 to 1, a quantity $J_2A_2F_{21}$ reaches surface 2 from surface 1. The net exchange is thus given by

$$\dot{Q}_{12}=J_1A_1F_{12}-J_2A_2F_{21}$$

But $A_1F_{12}=A_2F_{21}$, so

$$\dot{Q}_{12}=\frac{J_1-J_2}{1/A_1F_{12}}$$

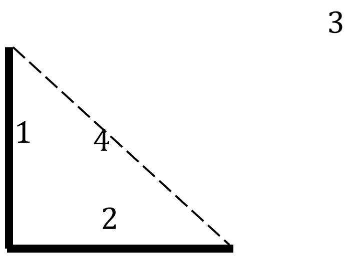
We interpret this expression in an "Ohm's Law" way, regarding $\dot{Q}_{12}$ as a current flow, J_1-J_2 as a potential difference, and $\frac{1}{A_1F_{12}}$ as a 'space' resistance.



The determination of shape factors for other than the simplest geometries is tedious and normally done by computer or taken from graphical data. However we shall now see, certain useful results can be deduced by physical reasoning.

6.3.1. *Shape factor algebra*

Consider the radiation exchange situation shown in the figure – the L-section is extends infinitely into the page, and for simplicity, let us assume surfaces 1 and 2 are of equal length in the plane of the page. Typically we will be interested in the radiation exchange between the three surfaces, 3 representing the environment. Ignore for the moment the dashed line (surface 4).



Shape factors for a surface add up to unity

By definition, the shape factor is the fraction of energy leaving one surface and hitting another, therefore

$$F_{11} + F_{12} + F_{13} = 1$$

Note that we have included the self shape factor, F_{11}, since a surface may be able to see itself.

Convex surfaces cannot see themselves – self view factor = 0

In the above figure, surface 1 cannot see itself, so $F_{11} = 0$.

Reciprocity

As demonstrated above, for example, $F_{12}A_1 = F_{21}A_2$, where $A_1 = A_2$ in this example.

Using these rules (and simple logic), we can for example, calculate F_{12}. By drawing the imaginary surface 4, we note that $F_{14} + F_{12} = 1$, so if we can find F_{14} the problem is solved.

By reciprocity: $A_1 F_{14} = A_4 F_{41}$

And by symmetry $F_{41} = 0.5$

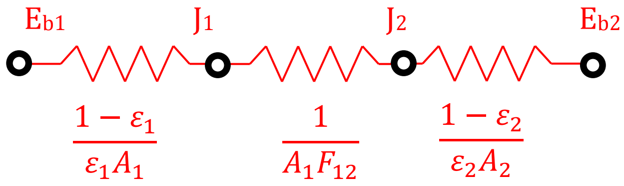
$$F_{14} = 0.5 \frac{A_4}{A_1} \Longrightarrow F_{14} = \frac{\sqrt{2}}{2}$$

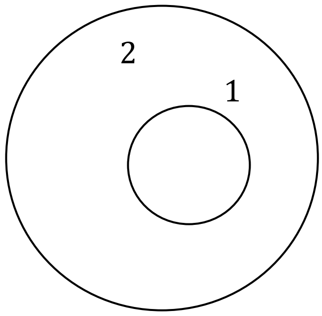
$$F_{12} = 1 - F_{14} = 1 - \frac{\sqrt{2}}{2} = .293$$

Note that this relationship applies to black and non-black bodies, but the radiation must be diffuse.

6.3.2. *Example – radiation exchange between concentric cylinders.*

The radiation network is





Thus, the overall "resistance" is

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1}+\frac{1}{\mathrm{A}_1\mathrm{F}_{12}}+\frac{1-\varepsilon_2}{\varepsilon_2 A_2}$$

So that the heat exchanged between 1 and 2 is,

$$\dot{Q}=\frac{(\sigma T_1^4-\sigma T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1 A_1}+\frac{1}{\mathrm{A}_1\mathrm{F}_{12}}+\frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right)}$$

Since $F_{12}=1$, and multiplying top and bottom by A_1,

$$\dot{Q}=\frac{A_1(\sigma T_1^4-\sigma T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1}+1+\left(\frac{\mathrm{A}_1}{\mathrm{A}_2}\right)\frac{(1-\varepsilon_2)}{\varepsilon_2}\right)}$$

Note that this expression is valid whether the cylinders are concentric or not; in fact either cylinder can be any shape as long as $F_{11}=0$ - i.e. the inner cylinder cannot see itself.

A very useful result is obtained when $A_2 \to \infty$ (or $\varepsilon_2 \to 1$), which is that

$$\dot{Q}=A_1\varepsilon_1(\sigma T_1^4-\sigma T_2^4)$$

This expression is quite general – it says (in words) that when an object is in a large enclosure, the net radiation exchange is independent of the emissivity of the enclosure, and it is as if the enclosure were a black body.

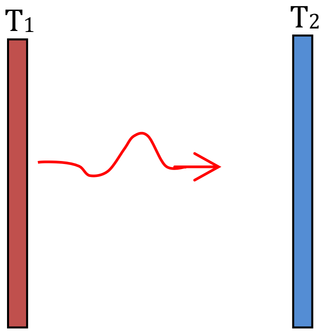
A couple of lines of manipulation of the relationships above shows that $G_1 \approx E_{b2}$ for $A_2 >> A_1$, i.e. all the radiation striking surface 1, appears as if it has come from a black body.

6.3.3. Example - Radiation Shields.

Two very large parallel planes with emissivities of 0.3 and 0.8 exchange heat. Find the reduction in heat transfer per unit area, if a shield with emissivity 0.04 is placed between them.

[Note that all $A_1 = A_2 = 1$ and all the shape factors are unity, $F_{12} = F_{21} = 1$ etc..]

No shield



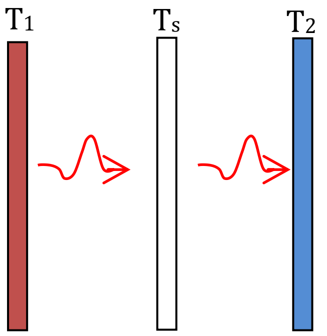
$E_{b1} = \sigma T_1^4$ $\quad$ $E_{b2} = \sigma T_2^4$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} \qquad \frac{1}{F_{12}A_2} = 1 \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2}$$

$$R_{noshield} = \frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2}$$

$$R_{noshield} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 = 3.584$$

Shield



$E_{b1} = \sigma T_1^4$ $\quad$ $E_{bs} = \sigma T_s^4$ $\quad$ $E_{b2} = \sigma T_2^4$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} \qquad \frac{1}{F_{1s}A_1} \qquad \frac{1-\varepsilon_s}{\varepsilon_s A_s} \qquad \frac{1-\varepsilon_s}{\varepsilon_s A_s} \qquad \frac{1}{F_{2s}A_2} \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2}$$

$$R_{shield} = R_{noshield} + 2\left(\frac{1-\varepsilon_s}{\varepsilon_s}\right) + 1$$

$$R_{shield} = 3.584 + 49 = 52.584$$

Since

$$\dot{Q} = \frac{\sigma(T_1^4 - T_2^4)}{R}$$

The ratio of heat fluxes is

$$\frac{\dot{Q}_{shield}}{\dot{Q}_{noshield}} = \frac{R_{nosheld}}{R_{shield}} = \frac{3.584}{52.584} = 0.068$$

a reduction of 93.2%

6.4. *Radiation in the environment*

Solar radiation is very different to most terrestrial radiation, in that it is at much shorter wavelengths, and many important phenomena that affect us directly relate to this.

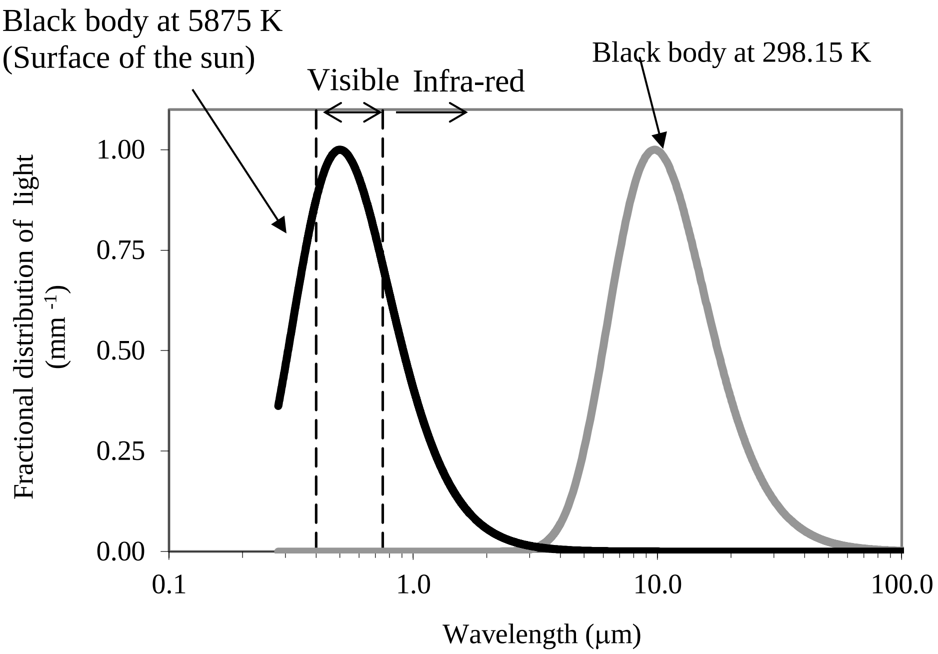


Figure 4. Spectrum for black bodies at 5780 K (i.e. the sun) and 298.15 K (i.e. the earth). Note that the spectrums have been normalised so that the area under each curve is unity.

Table 1. Major contributors to the greenhouse effect and which wavelengths of radiation they absorb

Gas	Main Absorption bands
Carbon Dioxide, CO_2	4.3 – 4.4 μm, 14 – 16 μm
Water Vapour, H_2O	2.6 -2.8 μm, 5.2 – 7.4 μm, 21 – 116 μm,
Methane, CH_4	3.2-3.5 μm, 7.4 – 8.1 μm
Nitrous Oxide, N_2O	4.5-4.6 μm, 7.6-8 μm

- Incoming UV rays are absorbed by O_3 and O_2.
  OZONE LAYER – PROTECTS US FROM HARMFULL UV RADIATION

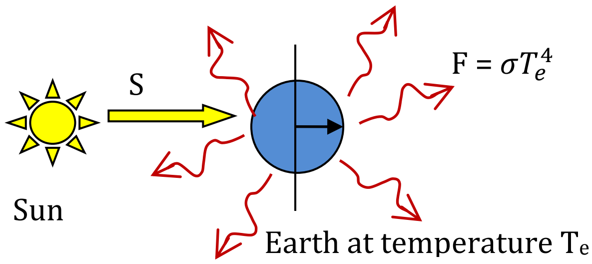
- Outgoing infra-red radiation is absorbed by gases such as CO_2, H_2O and CH_4.
  GREEN HOUSE EFFECT.

6.4.1. Example- a simple model of the greenhouse effect

A very simple model of the green house gas effect shows how the wave length difference between solar and terrestrial radiation is central. Previously we assumed that the absorbance, reflectivity and emissivity were not functions of wavelength, when considering the greenhouse effect, this approximation is no longer valid.

Albedo, A = the fraction of the incident (i.e. short wave) radiation which is reflected

First consider a planet, radius R, with a solar flux (short wave radiation) of S Wm^{-2}, and an albedo of A. For long wave radiation, the planet can be taken to be black.



The overall energy (short wave) absorbed is projected area×S × fraction absorbed, i.e.

$$\dot{Q}_s = S\pi R^2(1-A)$$

If the planet's surface is at T_e, then the (mainly long wave) energy emitted is

$$\dot{Q}_p = 4\pi R^2 \sigma T_e^{\,4}.$$

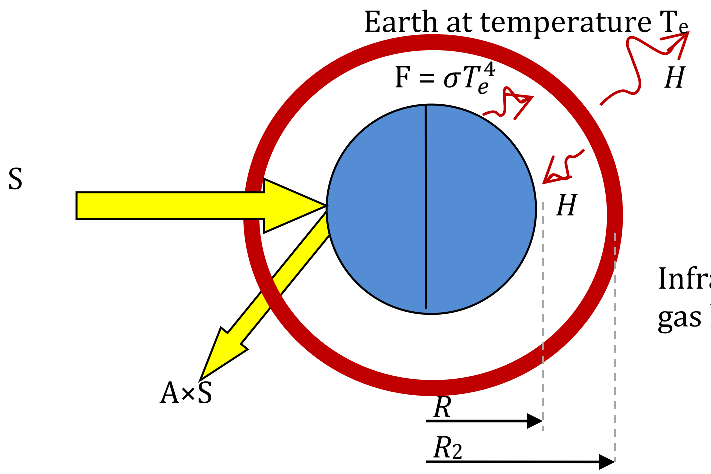
At steady state $\dot{Q}_s = \dot{Q}_p$ giving

$$\sigma T_e^4 = \frac{S(1-A)}{4}$$

For S = 1000 Wm^{-2}, and average value of the albedo, A = 0.1, we get T_e = 250 K, **too cold to support life on earth**!

We have neglected fact that the Earth's atmosphere contains gases that absorb the longwave radiation emitted from the Earth's surface.

Consider the (simplified) situation shown below, where we have (very crudely) assumed that infra-red absorbing gases form a layer of some thickness above the surface, spread equally over the entire planet.



Note that the diagram exaggerates the distance of the layer form the planet, in practice $R_2 \approx R$

The overall energy (short wave) absorbed is projected area×S × fraction absorbed, i.e.

$$\dot{Q}_s = S\pi R^2(1-A)$$

The gas layer absorbs F, and itself emits a radiative (infra-red) flux of *H*. Since the layer is elevated from the surface, it emits radiation through both its upper and lower surface in equal amounts. Thus overall the layer emits a total flux of *2H W m*$^{-2}$. In this new situation the outer surface of the absorbing layer becomes the outer surface of the planet**,** so that the amount of energy radiated by the planet becomes,

$$\dot{Q}_p = 4\pi R^2 H\ .$$

At steady sate $\dot{Q}_s = \dot{Q}_p$ giving

$$H = \frac{(1-A)S}{4} \qquad (*)$$

At the surface, the energy balance is:

$$4\pi R^2 F = H\,4\pi R^2 + (1-A)S\pi R^2$$

i.e. $$F = H + \frac{(1-A)S}{4}$$

Using this result to eliminate *H* from (*) gives:

$$F = \sigma T_e{}^4 = \frac{(1-A)S}{2}$$

The addition of one IR absorbing layer has thus changed the balance of energy at the ground. Using the same values of *S* (=1000 Wm^{-2}) and *A* (=0.1) as before, we now get T_e = 298.5 K.

Without the greenhouse effect, life on earth would not exist!

6.5. *Key points for Radiative heat transfer*

- The power per unit area emitted by a black body is $E_b = \sigma T^4$
- Emissivity is the ratio of actual power emitted to that emitted by a black body at the same temperature.
- A grey surface has a constant emissivity which does not depend on wavelength. For a real (grey) surface the power emitted is $E = \varepsilon\sigma T^4$
- $\rho+\tau+\alpha = 1$ (reflectivity + transmissivity + absorptivity = 1)
- The absorptivity is equal to the emissivity (Kirchhoff's law), $\varepsilon = \alpha$
- The irradiation, G, is the total power arriving on a unit area of a surface
- The radiosity, J, is the total power leaving a unit area of a surface. $J = \rho G + E$
- The shape/view factor between two surfaces is a geometric quantity which gives the amount of radiation leaving one surface and hitting the other.
- We can construct an electrical circuit analogy for a radiation problem.
- Without the greenhouse effect, the earth would be too cold to support life. Too much infra-red absorbing gas in the atmosphere will cause global warming.