

Part IB Paper 4: Thermofluid Mechanics

THERMODYNAMICS

Examples Paper 5

Heat transfer – conduction

- S1. A house has cavity walls which consist of an inner layer of “breeze blocks” 100mm thick together with an outer layer of bricks, also 100mm thick. The cavity between the bricks is 50 mm wide and may be assumed to contain still air. The thermal conductivities ($\text{Wm}^{-1}\text{K}^{-1}$) are 0.67 (breeze block), 1.32 (brick), 0.026 (air). The surface heat transfer coefficients ($\text{Wm}^{-2}\text{K}^{-1}$) are 15.2 (outer wall), 9.4 (inner wall). Calculate the heat loss per unit area when the inside temperature is 20 °C, and the outside temperature is 0 °C. Also calculate and comment on the temperature of the two brick surfaces.
- Q1. A pipeline carries oil at 50 °C through sea water at 5 °C. The pipeline is made of steel tube of 1.5 m OD and 20 mm thick, of thermal conductivity $56 \text{ Wm}^{-1}\text{K}^{-1}$. It is coated on the inside with a layer of protective paint 5 mm thick, of thermal conductivity $120 \text{ Wm}^{-1}\text{K}^{-1}$, and on the outside with two layers of insulating material each 100 mm thick, with thermal conductivities of $0.15 \text{ Wm}^{-1}\text{K}^{-1}$ (inner) and $0.07 \text{ Wm}^{-1}\text{K}^{-1}$ (outer). Calculate the heat transfer rate per unit length of pipeline, given that the surface heat transfer coefficients are $988 \text{ Wm}^{-2}\text{K}^{-1}$ (inside) and $1120 \text{ Wm}^{-2}\text{K}^{-1}$ (outside). What is the effect of swapping the layers of insulation?
- Q2. A domestic central heating system uses copper tubing of 10mm OD. The surface temperature is 60 °C, and the surface heat transfer coefficient is $10 \text{ Wm}^{-2}\text{K}^{-1}$. The air temperature is 15 °C.
- (a) Calculate the heat loss per unit length of pipe.
 - (b) It is proposed to reduce this loss by insulating the pipe with a material of thermal conductivity $0.07 \text{ Wm}^{-1}\text{K}^{-1}$. The surface heat transfer coefficient is assumed to be unaffected.
 - i) What thickness of lagging gives the worst possible result?
 - ii) What thickness is required to achieve a reduction in the heat transfer?
 - iii) What thickness of lagging would be needed to reduce the heat loss for a pipe of half the original diameter? – Comment on your answer.

Heat transfer – Transient conduction

- Q3. A sphere (initially at room temperature) is plunged into rapidly stirred water at 90 °C. The diameter of the sphere is 5mm.
- (a) Show that a lumped heat capacity model is appropriate in this case.

- (b) What is the time constant for the heating up of the 5 mm diameter sphere?
- (c) Calculate the time taken to reach 85 °C.
- (d) A much larger sphere is to be used instead. Its diameter is 50mm and it has a thermal conductivity $1/10^{\text{th}}$ of that of the smaller sphere. The heat transfer coefficient remains the same for the smaller sphere. All other properties of the larger sphere are the same. What is the time constant associated with heating up the centre of this much larger sphere?

Data:- For spheres- specific heat $\sim 3000 \text{ J kg}^{-1}\text{K}^{-1}$, thermal conductivity $\sim 0.4 \text{ Wm}^{-1}\text{K}^{-1}$, density $\sim 1000 \text{ kgm}^{-3}$. For the water/sphere heat exchange process, the heat transfer coefficient is $\sim 40 \text{ Wm}^{-2}\text{K}^{-1}$.

Heat transfer- Heat Exchangers

- Q4. In a tubular counterflow oil cooler the ID of the tubes is 12.5 mm and the wall thickness is 1.25 mm. Oil with $c_p = 1.675 \text{ kJkg}^{-1}\text{K}^{-1}$ flows at 0.63 kgs^{-1} through the tubes and is cooled from 172 °C to 60 °C. the cooling water on the outside of the tubes has a flow rate of 0.76 kgs^{-1} and enters at 5°C. The average internal and external heat transfer coefficients on the tube walls are 1.72 and $3.97 \text{ kWm}^{-2}\text{K}^{-1}$, respectively
- (a) What is the heat exchanger effectiveness?
 - (b) Neglecting the small temperature drop across the tube walls, what is the overall heat transfer coefficient based on the tube inside surface area?
 - (c) Find the log mean temperature difference, and hence the total length of tubing required.

- Q5. In certain nuclear power stations, the energy from the reactor is extracted by a water circuit of highly pressurized water (in this example at 200 bar), such that it remains a liquid when heated in the reactor. It passes into the tubes of a shell and tube heat exchanger, entering at 360 °C. These tubes are of 19 mm ID, 3 mm wall, thermal conductivity 20 Wm⁻¹K⁻¹, each 20m in length, and each with a mass flow of 1.5 kgs⁻¹. Steam is generated in the shell of the heat exchanger, from water entering as a saturated liquid at 80bar, and leaving as dry saturated steam at the same pressure.

The heat transfer coefficient at the outer surface of the tubes is 50 kWm⁻²K⁻¹. The properties of the water inside the tubes are $\rho = 670 \text{ kgm}^{-3}$, $c_p = 5.7 \text{ kJkg}^{-1}\text{K}^{-1}$, $\mu = 7.4 \times 10^{-5} \text{ kgm}^{-1}\text{s}^{-1}$, $\lambda = 0.525 \text{ Wm}^{-1}\text{K}^{-1}$. The appropriate correlation for the Nusselt number on the inside of the tube may be found in the Data Book.

- (a) Calculate the overall heat transfer coefficient U based on the inner surface area of the tube.
- (b) Show that the variation of temperature T of the water in the tubes is given by

$$\frac{T - T_s}{T_{in} - T_s} = \exp\left(-\frac{\pi d U x}{\dot{m} c_p}\right)$$

Where T_{in} is the tube inlet temperature, T_s is the temperature of the water–steam mixture in the shell, d is the tube ID. \dot{m} is the mass flow rate inside the tube and x is the distance along the tube.

- (d) Calculate the rate of steam generation per tube. (The temperature of saturated steam at 80 bar is 295 °C, and the enthalpy difference between water and steam at this condition (h_{fg}) is 1442.8 kJkg⁻¹)

Convective Heat Transfer

Note that energy transport properties of air (thermal conductivity, Prandtl Number, etc.) are given in the Thermofluids Data Book, page 28. Density values should be found assuming that air is a perfect gas.

- Q6. Air at atmospheric pressure and 5 °C flows at a speed of 10 m/s over a surface with a constant temperature of 45 °C. At a particular point on the surface, the coefficient of friction c_f is found to have a value of 0.01. Find the value of film temperature appropriate to this point and, using Reynolds' analogy (for $Pr = 1$), estimate the local value of heat flux.

By how much would your estimate change if you took account of the fact that $Pr \neq 1$?

- Q7. For a particular process plant design it is necessary to know the heat transfer coefficient for benzene flowing at a speed of 1 m/s in a pipe of 10 mm bore. An experiment is set up to measure the required heat transfer coefficient by using water in a pipe of 20 mm bore and applying the principle of similarity.

- (a) Specify the flow speed and the temperature of the water.
- (b) The experiment yields a measured value of $3.0 \text{ kW m}^{-2} \text{ K}^{-1}$ for the heat transfer coefficient in water. What is the corresponding value for benzene ?

Data for benzene:

$$c_p = 1.66 \text{ kJ kg}^{-1} \text{ K}^{-1}; \quad \rho = 880 \text{ kg m}^{-3};$$

$$\lambda = 0.16 \text{ W m}^{-1} \text{ K}^{-1}; \quad \mu = 6.7 \times 10^{-4} \text{ kg m}^{-1} \text{ s}^{-1}$$

Data for water: use Thermofluids Data Book page 27.

- Q8. The following empirical correlations apply to natural convection driven by a heated vertical plate:

$$Nu = 0.590 (Gr Pr)^{0.25} \quad 10^4 < Gr Pr < 10^9$$

$$Nu = 0.021 (Gr Pr)^{0.40} \quad 10^9 < Gr Pr < 10^{13}$$

where the Nusselt number is based on the mean heat transfer coefficient over the height of the plate. Explain why you would expect different correlations to apply for different Gr number ranges.

A domestic central heating radiator may be treated as a flat plate 1 m high and 2 m long, with a uniform surface temperature of 60°C . The temperature of the room is 20°C . Using the appropriate correlation, calculate the rate of heat transfer by convection. Ignore the effect of any nearby walls (i.e. assume that the radiator has two surfaces, each transferring heat to an effectively infinite fluid).

- Q9. A solar panel 1 m long and 0.35 m wide is exposed to the sun so that 860 W m^{-2} is absorbed. Most of this power is used to heat water for domestic use but 25 % of it is removed *uniformly* (i.e. at uniform heat flux) from the surface of the panel by the wind, blowing in a direction parallel to the length of the panel. The air temperature is 25°C and the wind speed is 6 m/s. The uniform heat flux empirical correlation for *local* convective heat transfer at a distance x from the leading edge of the panel is:

$$Nu_x = 0.453 Re_x^{1/2} Pr^{1/3}; \quad Re_x < 5 \times 10^5 \quad (\text{laminar})$$

Check that the correlation is appropriate (since the temperature of the panel is unknown, the film temperature may be taken as 25°C), then calculate

- (a) the heat flux
- (b) the local temperature at the trailing edge of the panel
- (c) the average temperature over the whole panel $\bar{T} = \frac{1}{L} \int_0^L T dx$

ANSWERS

- S1. 8.62 Wm^{-2} , 0.57°C , 1.22°C
- Q1. 116.5 Wm^{-1} , 4.1% decrease in heat loss.
- Q2. (a) 14.14 Wm^{-1} , (b, i) 2 mm, (b, ii) 5.22 mm, (b, iii) 30.8 mm (!)
- Q3. (a) $\text{Bi} = 0.083$, (b) 62.5s, (c) 160s (d) 5208 s
- Q4. (a) 0.67, (b) $1.264 \text{ kWm}^{-2} \text{ K}^{-1}$, (c) 27.3 m.
- Q5. (a) $6.0 \text{ kWm}^{-2} \text{ K}^{-1}$, (c) 0.22 kgs^{-1}
- Q6. 25°C , 2400 W m^{-2} , increase by approximately 25%
- Q7. (a) 0.66 m/s , 20°C
(b) $1592 \text{ W m}^{-2} \text{ K}^{-1}$
- Q8. 575.4 W
- Q9. (a) 215 W m^{-2} ,
(b) 57.7°C ,
(c) 46.8°C

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