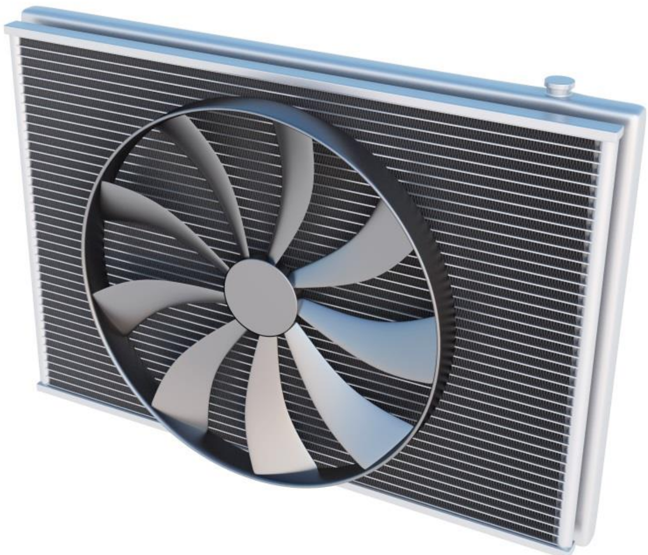
 

External Forced Convection and Natural Convection

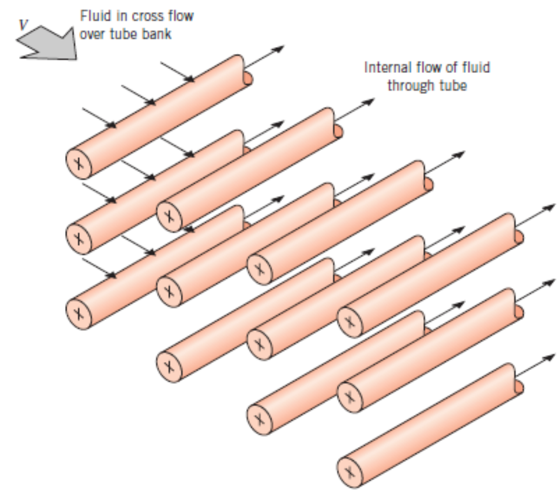
Significance of external flows and natural convection

- Flows over objects are regarded as ***external flows***, e.g. flow over a surface, flow over a sphere or cylinder.

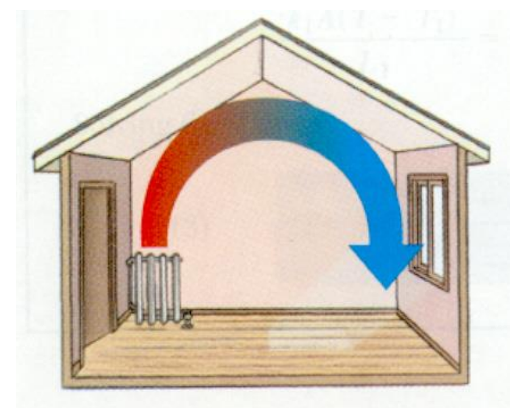
- Wide range of engineering applications for external flows in gas turbine blade cooling, heat exchangers, etc.

- In the absence of an external means of forcing the fluid to flow, such as pumps and fans, the convection is termed ***natural or free convection***.

- There are many examples of free convection in natural and manmade systems.
  - *The weather and its daily and seasonal changes are the result of natural heat convection between the surface of the earth and the atmosphere.*
  - *Any electronic device which does not have a fan relies on free convection for cooling.*
  - *Heating and air conditioning.*



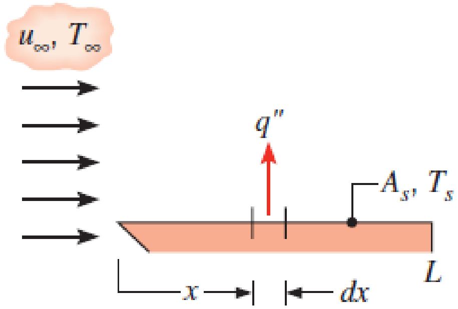
External flow over a tube bank



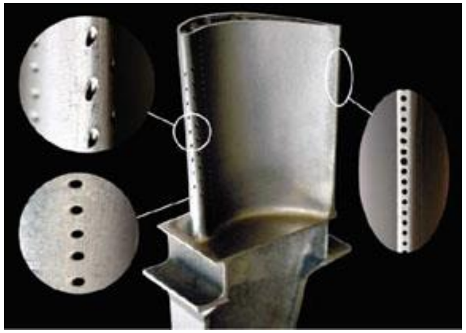
Natural convection for space cooling

External forced convection

- We can show that in all forced convection problems: $Nu = Nu(Re, Pr)$ (proof in next year heat transfer!)
- *Re* number is defined on the basis of the problem configuration.
- For flows inside tubes: $Re = Re_D = \frac{UD}{\nu}$, where D is the internal diameter of the tube.
- For flow over a flat plate: $Re = Re_x = \frac{Ux}{\nu}$, where *x* is the distance from the leading edge of the plate.
- Thermo-physical properties are evaluated at the average of the bulk flow temperature and the solid surface temperature (i.e. $T_f = (T_s + T_\infty)/2$), ***film temperature***.



Convective heat transfer from a flat plate.

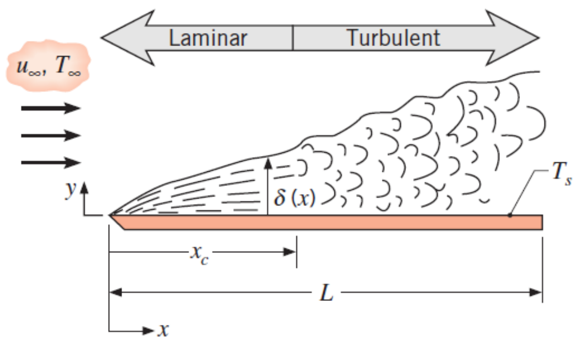


Gas turbine blade cooling, the surface of the bald can be assumed flat

Turbulent and laminar flows

- Similar to internal flows, there is a significant difference between heat transfer characteristics of laminar and turbulent regimes in external flows.
- x_c: the critical length for which the transition to turbulence begins.

  $\boldsymbol{Re_c} = \frac{\rho u_\infty x_c}{\mu} = \mathbf{5 \times 10^5}$ is the critical *Reynolds* number.
  $\boldsymbol{Re_c}$ is the value of Re_x for which transition begins, if $\boldsymbol{Re_x} < \boldsymbol{Re_c}$, flow is laminar.



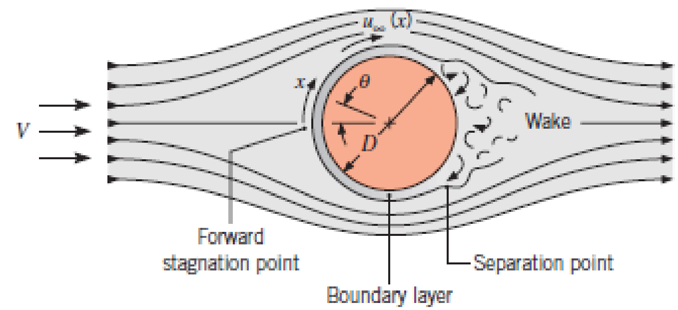
Boundary layer over a flat plate and transition from laminar to turbulent flow

  - For a laminar boundary layer over an isothermal plate: $Nu_x \equiv \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3}$.
  - For a turbulent boundary layer over an isothermal plate: $Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$ where $0.6 \le Pr \le 60$.

External flow over a single tube

The flow field over a cylinder is quite complicated and involves:

- ○ A forward stagnation point in which the flow comes to rest.
- ○ A laminar boundary layer
- ○ Transition to turbulent boundary layer
- ○ A wake region behind the cylinder, in which flow is completely chaotic and disordered.



Flow development in circular cylinder in cross flow

- Precise analysis of the flow requires investigating each of these regions.
- Experimental studies show that for the overall heat transfer from/to the cylinder:

$$\boxed{\overline{Nu_D} \equiv \frac{\bar{h}D}{k} = C Re_D^m Pr^{1/3}}$$

where $\overline{Nu}$ and $\bar{h}$ are respectively the average Nusselt number and convective heat transfer coefficients over the cross section of the tube.

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

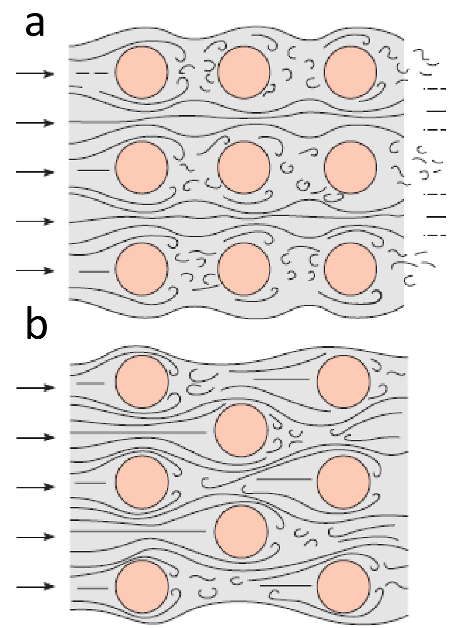
Constants in heat convection correlation for the circular cylinder in cross flow

External flow over banks of tubes- non-examinable

- Tube bundles (banks) are commonly used in heat exchangers and thermal devices.
- The flow in this problem includes all the complexities of flow over a single tube and additionally involves the effects of upstream tubes upon the downstream ones.
- The average heat transfer coefficient for the entire bank can be estimated by the following correlation

$$\left[\overline{Nu}_D = C_1\, Re_{D,\max}^m\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}\right]$$

$$\left[\begin{array}{l} N_L \geq 20 \\ 0.7 \lesssim Pr \lesssim 500 \\ 10 \lesssim Re_{D,\max} \lesssim 2 \times 10^6 \end{array}\right]$$



Flow conditions, a) aligned, b) staggered

where N_L is the number of tubes. All thermo-physical properties, with the exception of Pr_S, which is evaluated at tube surface temperature, are evaluated at the arithmetic average of the inlet and outlet fluid temperatures.

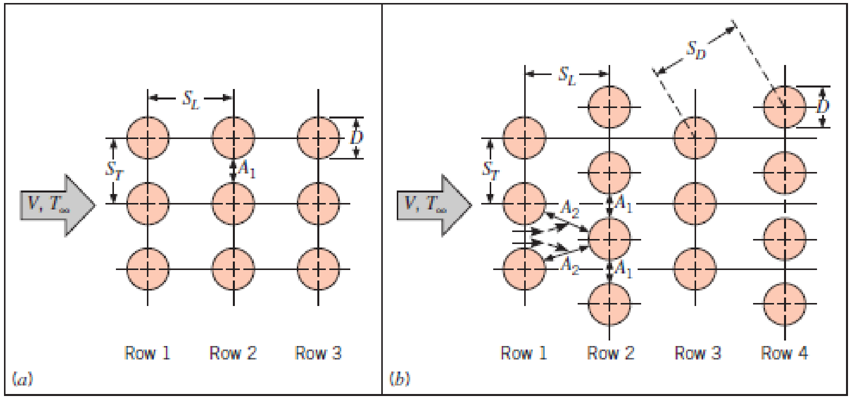
Maximum Reynolds number is defined as $Re_{D,max} \equiv \frac{\rho V_{max} D}{\mu}$, in which V_{max}

For aligned configuration is given by

$$V_{\max} = \frac{S_T}{S_T - D} V$$

For staggered configuration V_{max} is the maximum of the previous relation and

$$V_{\max} = \frac{S_T}{2(S_D - D)} V$$



Tube arrangements in a bank. (*a*) Aligned. (*b*) Staggered.

Values of C_1 and m are listed in table below

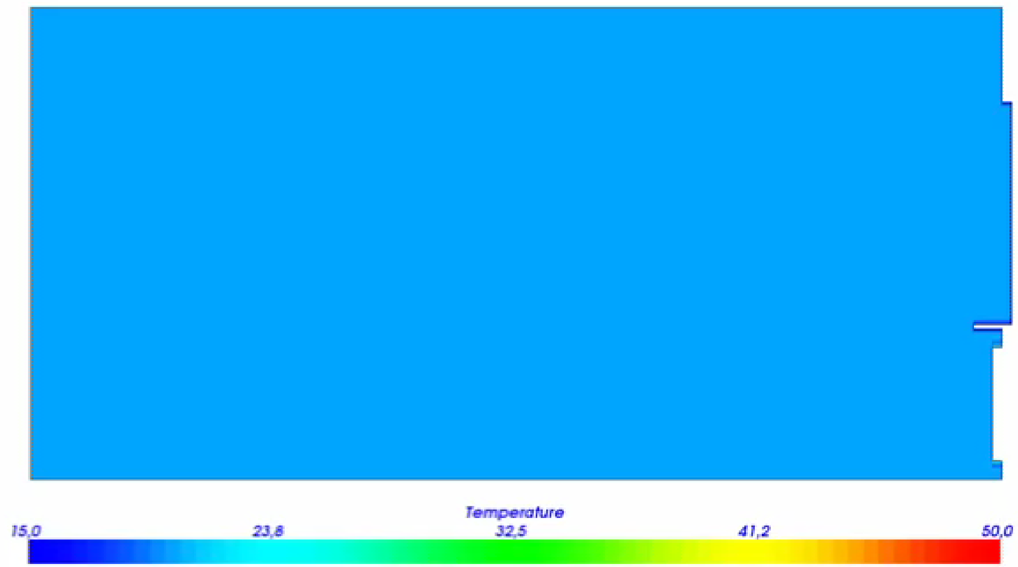
Configuration	$Re_{D,max}$	C_1	m
Aligned	10–10^2	0.80	0.40
Staggered	10–10^2	0.90	0.40
Aligned	10^2–10^3	Approximate as a single	
Staggered	10^2–10^3	(isolated) cylinder	
Aligned ($S_T/S_L > 0.7$)[a]	10^3–2×10^5	0.27	0.63
Staggered ($S_T/S_L < 2$)	10^3–2×10^5	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered ($S_T/S_L > 2$)	10^3–2×10^5	0.40	0.60
Aligned	2×10^5–2×10^6	0.021	0.84
Staggered	2×10^5–2×10^6	0.022	0.84

[a]For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.

Natural convection

- From everyday life we know that hot fluid tends to rise and cold fluid always sinks.
- This rise and sink of fluids with different temperatures sets convection currents (fluid motion) without any external means of pushing the fluid.
- The required force for motion is supplied by buoyancy due to the difference in density of hot/cold fluid and the surrounding fluid.

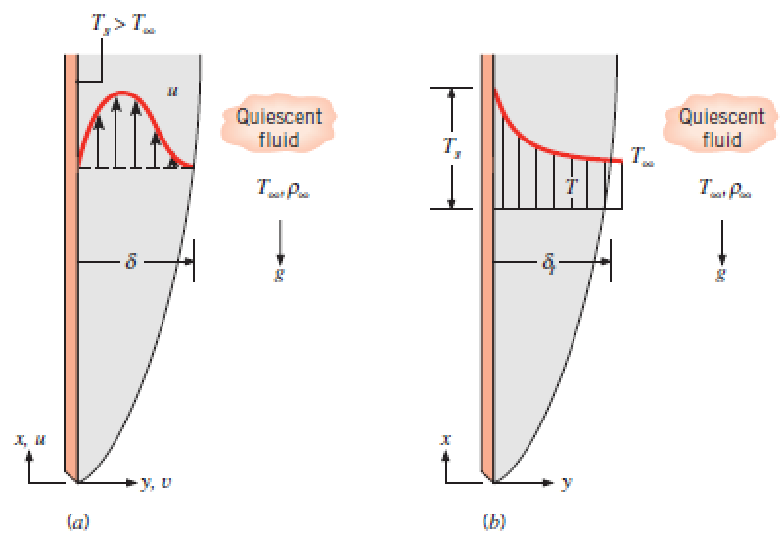
Numerical simulation of air motion and temperature in a room with a heater.



Grashof number

- Motion of fluid induced by natural convection forms a boundary layer over the solid surface.
- This boundary layer is characterised by the so called Grashof number, which is defined as $Gr_x \equiv \frac{g\beta(T_s - T_\infty)x^3}{v^2}$ where β is the volumetric thermal expansion coefficient and is a thermo-physical property of the fluid (for ideal gases $\beta = \frac{1}{T}$).
- Grashof number is a measure of relative strength of buoyancy and viscous forces.

Natural convection over a vertical isothermal, a) velocity boundary layer, b) thermal boundary layer



- Re number is a measure of relative strength of inertia and viscous forces.
- Gr number in natural convection is equivalent to Re number in forced convection.
- Thus, in natural convection $Nu = Nu(Gr, Pr)$.

Natural Convection-Empirical Correlations

- In engineering, we often use empirical correlations to obtain an estimation of natural convection coefficient.
- The product of *Gr* and *Pr* is termed Rayleigh number, which appears frequently in natural convection correlations $Ra_x \equiv Gr_x Pr = \frac{g\beta(T_S - T_\infty)x^3}{v\alpha}$.
- For free convection from a vertical plate, the average Nusselt number over the plate is given by $\boxed{\overline{Nu_L} = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2}$
- Similar correlations exist for other configurations, e.g. flow over or under a flat plate.
- Similar to external flow the thermo-physical properties are evaluated at the average temperature of stagnant fluid and surface temperature (film temperature).

Example

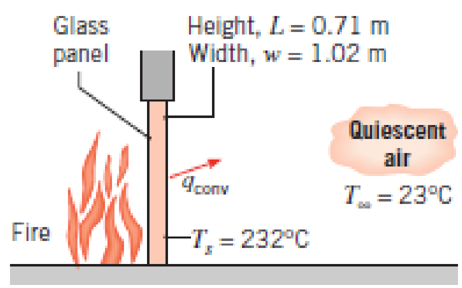
A glass-door firescreen, used to reduce exfiltration of room air through a chimney, has a height of 0.71 m and a width of 1.02 m and reaches a temperature of 232°C. If the room temperature is 23°C, estimate the convection heat rate from the fireplace to the room.

Solution

Known: Glass screen situated in fireplace opening.

Find: Heat transfer by convection between screen and room air.

Schematic:



Assumptions:

1. Screen is at a uniform temperature T_s.
2. Room air is quiescent.
3. Ideal gas.
4. Constant properties.

Properties: Table A.4, air ($T_f = 400$ K): $k = 33.8 \times 10^{-3}$ W/m·K, $\nu = 26.4 \times 10^{-6}$ m^2/s, $\alpha = 38.3 \times 10^{-6}$ m^2/s, $Pr = 0.690$, $\beta = (1/T_f) = 0.0025$ K^{-1}.

Analysis: The rate of heat transfer by free convection from the panel to the room is given by Newton's law of cooling

$$q = \overline{h} A_s (T_s - T_\infty)$$

where $\overline{h}$ may be obtained from knowledge of the Rayleigh number.

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\alpha\nu}$$

$$= \frac{9.8 \text{ m/s}^2 \times 0.0025 \text{ K}^{-1} \times (232 - 23)^\circ\text{C} \times (0.71 \text{ m})^3}{38.3 \times 10^{-6} \text{ m}^2/\text{s} \times 26.4 \times 10^{-6} \text{ m}^2/\text{s}} = 1.813 \times 10^9$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387(1.813 \times 10^9)^{1/6}}{[1 + (0.492/0.690)^{9/16}]^{8/27}}\right\}^2 = 147$$

Hence

$$\overline{h} = \frac{\overline{Nu}_L \cdot k}{L} = \frac{147 \times 33.8 \times 10^{-3} \text{ W/m} \cdot \text{K}}{0.71 \text{ m}} = 7.0 \text{ W/m}^2 \cdot \text{K}$$

and

$$q = 7.0 \text{ W/m}^2 \cdot \text{K}\ (1.02 \times 0.71) \text{ m}^2 (232 - 23)^\circ\text{C} = 1060 \text{ W} \qquad \triangleleft$$