Lecture 10 : Induction Motors II

10.1 Construction

Stator : Same as synchronous machine i.e. laminated, carries a balanced three-phase winding.

Rotor : Rotor winding must produce rotating magnetic field with same number of poles as stator. Must be laminated. The two options are:

Wound rotor : Three-phase windings of same pole-number as stator winding. Can be short-circuited, or provided with slip-rings and brushes, enabling external resistances to be added to rotor circuit (fig. 10.1).

Cage rotor : Single conductor per slot. Joined at ends by thick conducting *end-rings*. May be ***fabricated*** (fig 10.2) or ***die-cast*** (fig. 10.3).

Sizes : Few watts to tens of MW !

In this final lecture we will examine the construction of induction motors, and then see how the equivalent circuit can be used to predict motor performance, and in particular, the torque-speed characteristic. We will also see how this can be controlled via the rotor resistance.

The stator of an induction machine performs the same role as that of a synchronous machine - it produces a rotating magnetic field. Its construction is therefore identical (see lecture 5).
The rotor of an induction motor 'sees' a changing magnetic field, unlike that of a synchronous machine. It must therefore be laminated, to avoid excessive iron loss. Also, it must produce a rotating magnetic field of the same number of poles as the stator-driven field. This can be achieved by winding the rotor in the same way as the stator. The rotor winding can then be short-circuited, resulting in a machine with no external connection to the rotor. Alternatively, each phase of the rotor winding can be connected to slip-rings, enabling external resistances to be added to the rotor circuit. This is illustrated in fig. 10.1. This enables a degree of torque/speed control to be achieved, as we will see later.
An alternative to the wound rotor is the cage rotor, fig. 10.2. The rotor winding consists of thick conducting bars which completely fill each slot, joined at each end by an end-ring. In small machines, molten aluminium is forced under pressure into the rotor slots, fig. 10.3. In larger machines, the cage is fabricated by forcing copper bars into the slots, then brazing on end-rings. The cage rotor is far cheaper to manufacture than the wound rotor, and it is mechanically more robust, avoiding the use of rubbing contacts.

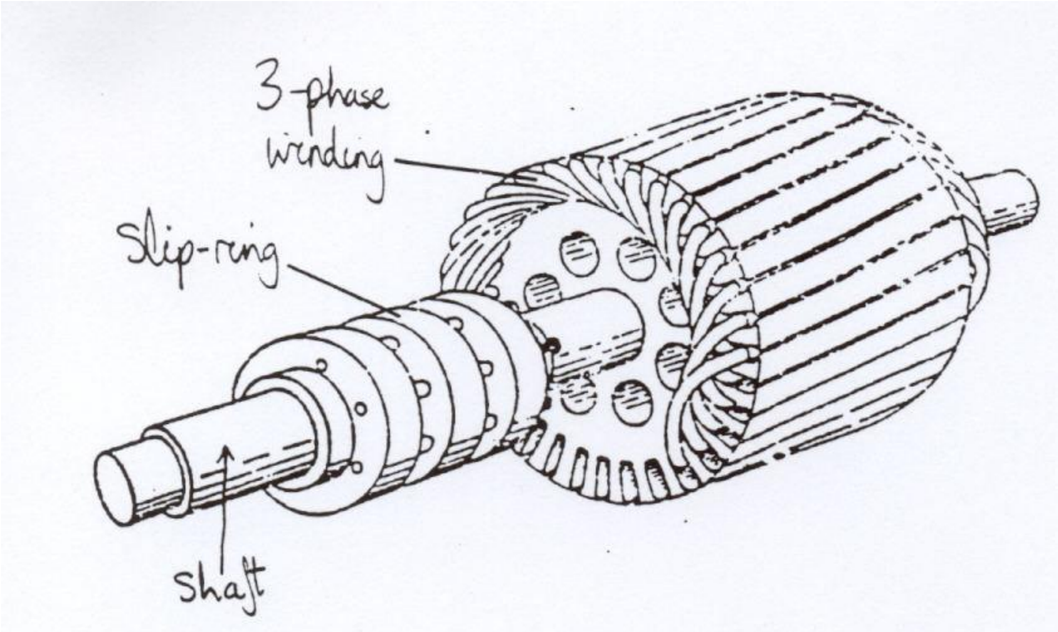


Fig. 10.1

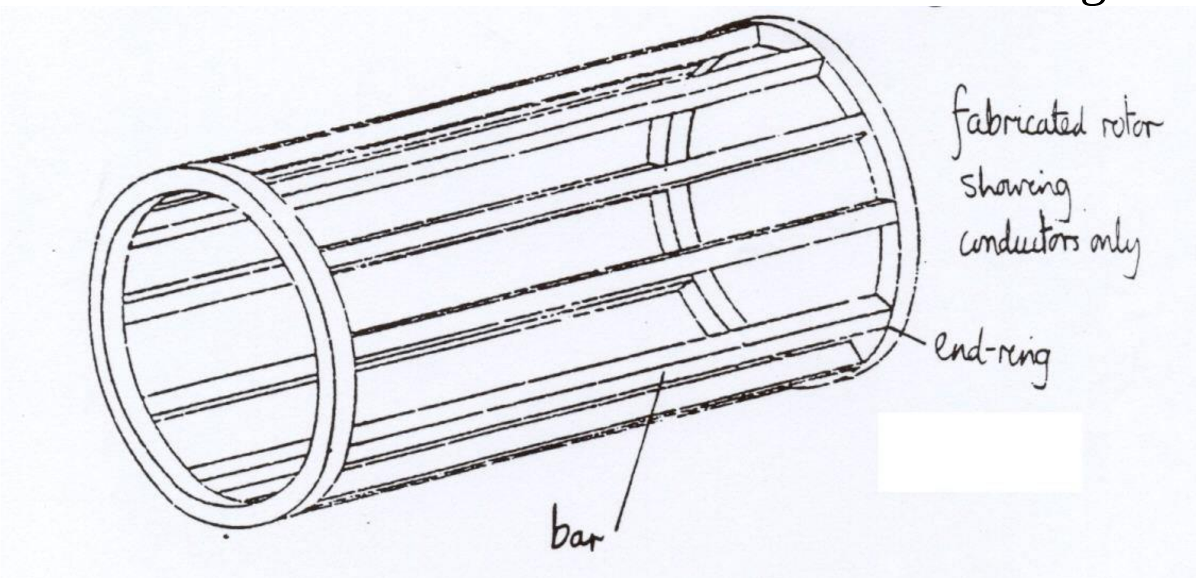


Fig. 10.2

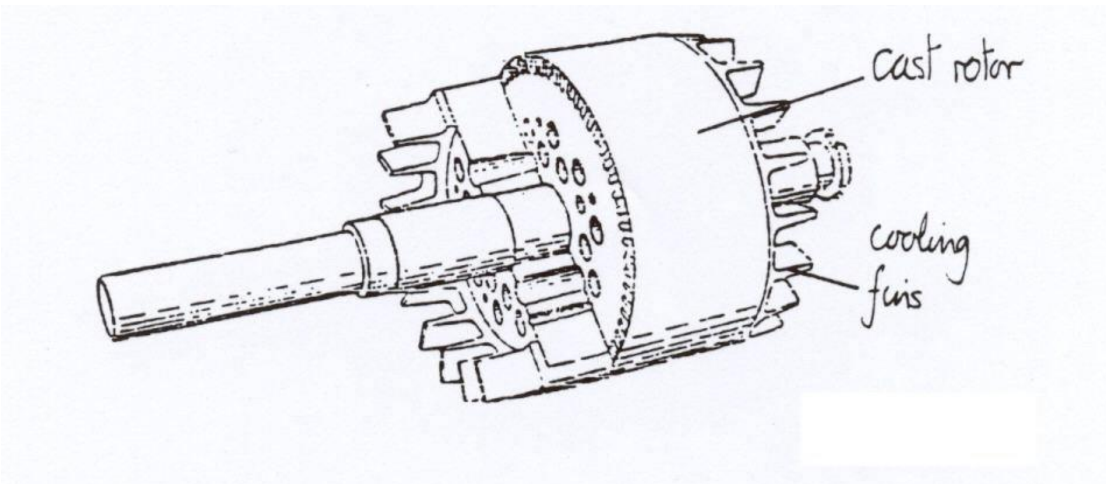


Fig. 10.3

10.2 Power and torque in induction motors

$$P_{in} = 3I_1^2 R_1 + 3I_0^2 R_0 + 3I_2'^2 R_2'/s \qquad (10.1)$$

Power losses are:

Iron loss $= 3I_0^2 R_0$ (10.2)

Stator winding loss $= 3I_1^2 R_1$ (10.3)

Rotor winding loss $= 3I_2^2 R_2 = 3I_2'^2 R_2'$ (10.4)

$$\therefore \quad P_{loss} = 3I_1^2 R_1 + 3I_2'^2 R_2' + 3I_0^2 R_0 \qquad (10.5)$$

Conservation of power: $P_{in} = P_{out} + P_{loss}$ (10.6)

$$\therefore \quad P_{out} = 3I_2'^2 \frac{R_2'}{s}(1-s) \qquad (10.7)$$

Mechanical power is $T\omega_r = T\omega_s(1-s)$ (10.8)

$$\therefore \quad 3I_2'^2 \frac{R_2'}{s}(1-s) = T\omega_s(1-s)$$

$$\Rightarrow \qquad T = \frac{3}{\omega_s} I_2'^2 \frac{R_2'}{s} \qquad (10.9)$$

T is the gross torque. There is also a mechanical loss torque:

$$T_{nett} = T - T_{loss} \qquad (10.10)$$

The equivalent circuit enables the performance of an induction motor to be predicted. Quantities such as the input current and rotor current can be obtained (given a value for slip) directly. However, to determine the mechanical output power, and torque produced, it is necessary to consider how conservation of power applies to induction motors.

Firstly, the input power to a real motor must be the same as that predicted by the equivalent circuit. In turn, the equivalent circuit will predict power only in the resistor components, as given by equation 10.1. The power losses in the motor (i.e. power which is converted to heat) are given by equations 10.2, 10.3 and 10.4. Notice that the rotor loss is not $3I_2^2R_2/s$, but $3I_2^2R_2$, since R_2 is the actual rotor resistance - the 1/s factor comes about because of the relative motion between rotor and stator field. The total power loss is the sum of these component losses, equation 10.5. Applying conservation of power, equation 10.6, and substituting P_{in} and P_{loss} from equations 10.1 and 10.5, and rearranging gives the output power, equation 10.7. This is the power which is converted to mechanical power.

It can be equated with mechanical power expressed in terms of torque and angular speed, equation 10.8, to give a final expression for the torque, equation 10.9.

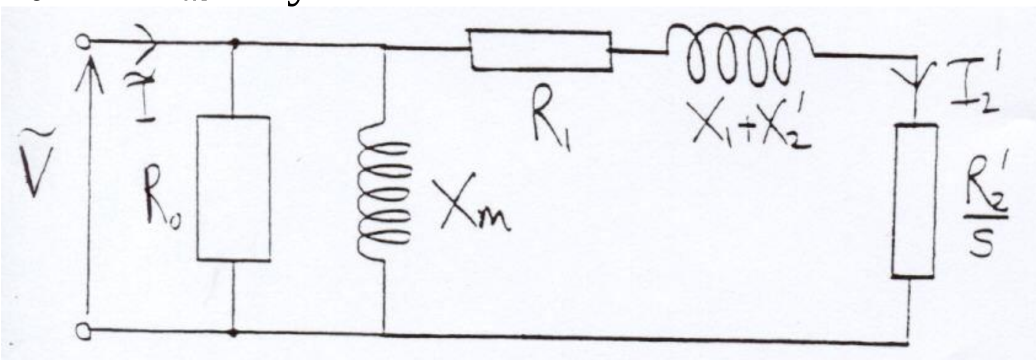
The torque predicted by this equation is the gross torque (or the electromagnetic torque) - it takes no account of mechanical loss torque owing to friction at the bearings and windage (due to aerodynamic effects). This is accommodated as a loss torque, equation 10.10.

Example 10.1

Find the torque and efficiency of the motor of example 9.1 at speed 1485 rpm. The voltage across R_1 and X_1 may be assumed small.

$$s = \frac{\omega_s - \omega_r}{\omega_s} = \frac{N_s - N_r}{N_s} = \frac{1500 - 1485}{1500} = 0.01$$

R_0 and X_m may be moved to the terminals:



$$I_2' = \frac{V_{ph}}{((R_1 + R_2'/s)^2 + (X_1 + X_2')^2)^{1/2}}$$

$$= \frac{240}{((0.6 + 0.5/0.01)^2 + 2.78^2)^{1/2}} = 4.74 \text{ A}$$

$$\omega_s = \frac{2\pi f}{p} = \frac{100\pi}{2} = 50\pi = 157.1 \text{ rads}^{-1}$$

$$T = \frac{3}{\omega_s} I_2'^2 \frac{R_2'}{s} = \frac{3}{157.1} \times 4.74^2 \times \frac{0.5}{0.01} = 21.5 \text{ Nm}$$

$$P_{out} = T\omega_r = 21.5 \times 155.5 = 3.343 \text{ kW}$$

Here we apply the induction motor equivalent circuit, and the ideas developed above to an example, which uses the same motor as example 9.1. There we found the equivalent parameters of this motor to be:
R_1=0.6 Ω, R_2'=0.5 Ω, X_1+X_2'=2.78 Ω, R_0=244 Ω, X_m= 91.7 Ω. The motor has a phase voltage of 240 V (415 V line, star-connected), and is 4-pole.

Firstly, given the rotor speed the slip may be found from its definition. Because the motor is a 4-pole motor, its synchronous speed in rpm, N_s, is 1500, giving a slip of 0.01.

The assumption that the voltage drop across R_1+jX_1 is small implies that the magnetising branch can be moved to the terminals with little loss of accuracy, since the voltage across it will hardly be changed by doing this. This is illustrated opposite. The referred rotor current is then readily obtained - we only require its magnitude, since it is to be used in the expression developed for torque.

The synchronous speed is evaluated as ω/p - remember that p means pole-pairs, and is therefore 2 for a 4-pole motor. We now have everything we need to evaluate the torque using equation 10.9.

$$P_{loss} = \frac{3V_{ph}^2}{R_0} + 3I_1^2 R_1 + 3I_2'^2 R_2'$$

$$= \frac{3 \times 240^2}{244} + 3 \times 4.74^2 \times 1.1 = 782.3 \text{ W}$$

$$P_{in} = P_{out} + P_{loss} = 3343 + 782 = 4.16 \text{ kW}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{3343}{4160} = 0.804 = 80.4\%$$

10.3 Variation of torque with speed

$0 < s < 1$: Motor (electrical $\rightarrow$ mechanical)
$s < 0$: Generator (mechanical $\rightarrow$ electrical)
$s > 1$: Electrical $\rightarrow$ losses (plug braking)

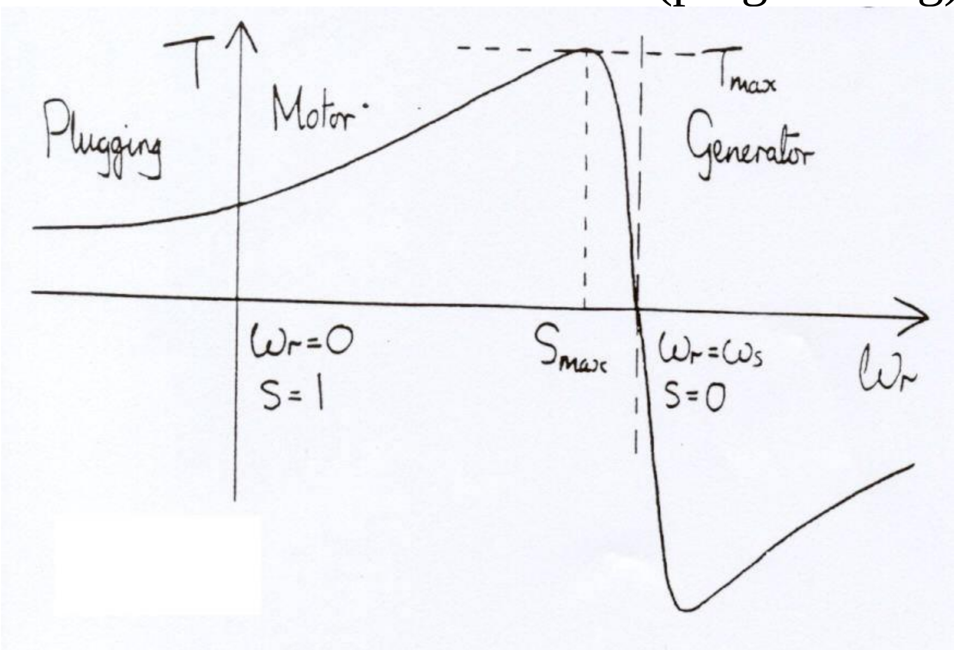


Fig. 10.4

The mechanical output power is obtained as shown opposite - the actual speed of the rotor is required, and is $(1\text{-}s)\omega_s = 0.99\omega_s$ giving 155.5 rad/s.
The total power losses are found by summing iron loss and rotor and stator copper loss.
The input power is found by summing output power and losses from which the efficiency may be determined.
The torque-speed characteristic of induction machines is of great significance, enabling a suitable motor to be chosen given the required torque-speed curve of the mechanical load. By varying s (and hence speed) and using the equivalent circuit, I_2' can be found for each value of s. In turn torque can be calculated, equation 10.9. Torque and speed can then be plotted. Note that slip decreases as speed increases - a slip of 1 corresponds to zero speed, whereas a slip of zero corresponds to synchronous speed. A typical characteristic is shown in fig. 10.4 opposite. There are three regions:
$0<s<1$: For values of s in this range, the torque is always positive, meaning that it acts in the direction of rotation. The machine is therefore behaving as a motor.
$s<0$: The rotor speed is now greater than synchronous speed, and the torque is negative. This means that the output mechanical power is also negative i.e. an external source of mechanical power is driving the rotor round, and so the machine must be generating. Induction generators are used when the prime-mover speed cannot accurately be controlled - for example, in wind generation.
$s>1$: The rotor is rotating in the opposite direction to the stator-driven magnetic field. This only occurs if two of the stator phases are swapped to reverse the direction of rotation. It is used as a way of rapidly stopping an induction motor, known as plug braking.

10.4 Maximum torque

Move R_0 and X_m to terminals.

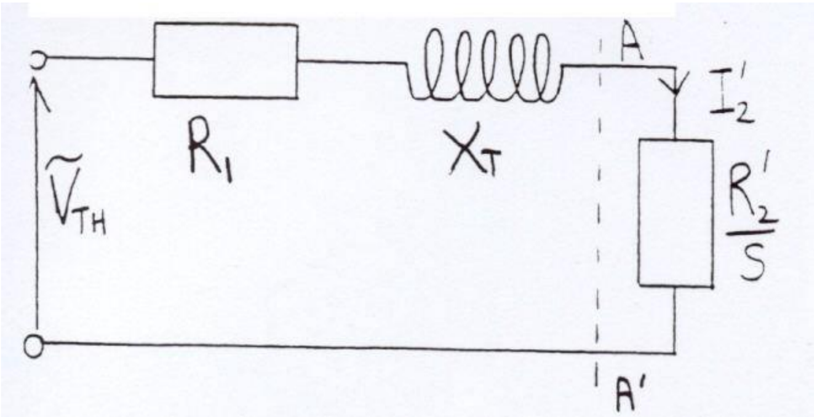


Fig. 10.5

Maximum torque → maximum 'loss' in R_2'/s.
Use maximum power transfer theorem:

$$R_2'/s_m = Z_S \quad \Rightarrow \quad s_m = R_2'/Z_S \qquad (10.11)$$

where $Z_S = \sqrt{R_1^2 + X_T^2}$ and $X_T = X_1 + X_2'$

$$I_2' = \frac{V_{TH}}{\sqrt{\left(\frac{R_2'}{s} + R_{TH}\right)^2 + X_2'^2}} \qquad (10.12)$$

$$\therefore \quad I_{2m}'^{\,2} = \frac{V^2}{(Z_S + R_1)^2 + X_T^2} = \frac{V^2}{Z_S^2 + 2Z_S R_1 + R_1^2 + X_T^2} = \frac{V^2}{2Z_S(Z_S + R_1)} \qquad (10.13)$$

$$T_{\max} = \frac{3}{\omega_s}\frac{I_{2m}'^2 R_2'}{s_m} = \frac{3V^2}{2\omega_s(Z_S + R_1)} \qquad (10.14)$$

From the torque-speed curve of fig. 10.4, it is clear that there is a speed (or slip) value for which the torque of an induction motor is maximised. Here we investigate how that slip, and the maximum torque value, may be found.

The basic idea is to maximise the 'power dissipated' in the R_2'/s parameter, since torque is proportional to that power, equation 10.9. The simplest approach is use the maximum power transfer theorem. This states that for maximum power transfer to a load, the load impedance should be equal to the source impedance. Here we are attempting to maximise the power in R_2'/s, and so R_2'/s should be made equal to the source impedance, Z_S. In turn, this gives the slip, s_m, at which maximum torque is developed, equation 10.11.

Equation 10.12 expresses the referred rotor current in terms of the phase voltage. Substituting this value of slip gives the referred rotor current at this particular value of slip, denoted I_{2m}'. However, since the expression for torque involves $I_2'^2$, we find $I_{2m}'^2$ instead, equation 10.13.

This expression is finally substituted into the equation for torque (10.9) to give the result, equation 10.14.

Example 10.2

For motor of previous example, find maximum torque and the speed at which it occurs.

$$V_{ph} = 240 \text{ V} \qquad Z_S = \left| R_1 + j\left(X_1 + X_2' \right) \right|$$

$$= \left(0.6^2 + 2.78^2 \right)^{1/2} = 2.84 \ \Omega$$

$$T_{\max} = \frac{3}{2\omega_s} \frac{V^2}{R_1 + Z_S} = \frac{3}{2 \times 157.1} \times \frac{240^2}{0.6 + 2.84}$$

$$= 159.7 \text{ Nm}$$

$$s_m = \frac{R_2'}{Z_S} = \frac{0.5}{2.84} = 0.176$$

$$N = (1 - s) N_s = (1 - 0.176) \times 1500 = 1236 \text{ rpm}$$

10.5 Rotor resistance torque/speed control

i) $T_{\max}$ is independent of R_2' ii) $s_m \propto R_2'$

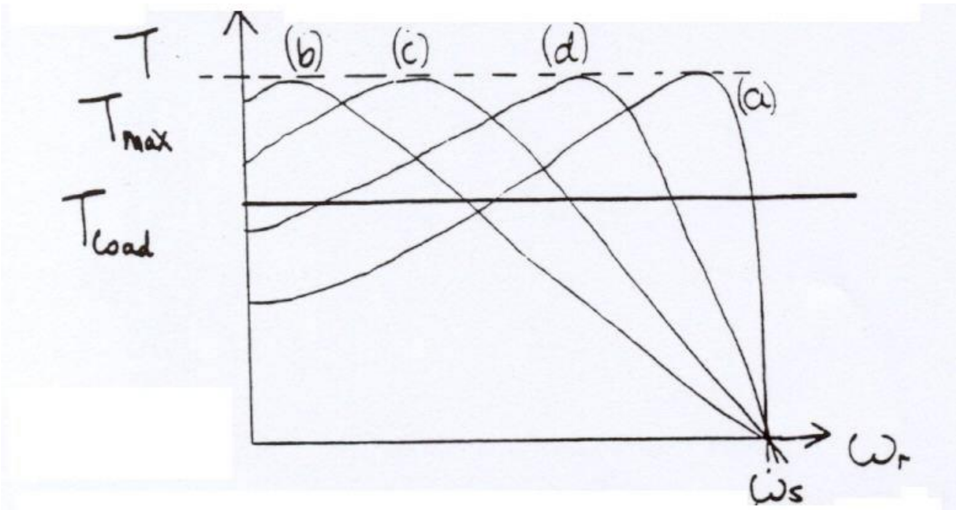


Fig. 10.6

Adding extra rotor resistance at starting will increase torque and reduce starting current.

In this example we apply the above ideas to finding the maximum torque of the motor used in example 10.1.

Since the magnetising branch has been moved to the terminals, the source impedance, Z_S, is the magnitude of the complex impedance $R_1+j(X_1+X_2')$.

Substituting all the known values into equation 10.14 gives the maximum torque, and equation 10.11 gives the slip at which it occurs, and hence the speed.

We saw in the previous section that maximum torque, and the slip at which it occurs are given by equation 10.14 and 10.11 respectively. These equations show that:
i) the maximum value of torque is independent of R_2'
ii) the slip which it occurs at is proportional to R_2'.
The torque-speed curve shown in fig. 10.6, curve (a) is typical, and shows that T_{max} occurs at a very small value of slip i.e. at a speed which is close to synchronous. Now suppose that the rotor resistance could be varied. This would not alter the maximum torque, but would alter the speed at which it occurred. For example, increasing R_2' would increase s_m, and so reduce the speed corresponding to T_{max}. Effectively, we have a means for controlling the torque-speed characteristic. This can be exploited in starting induction motors. Suppose that an induction motor is to be used to drive a constant-torque load, with the characteristic marked 'T_{load}' in fig. 10.6, and that the motor has to be able to start with the load coupled. Without additional rotor resistance, the motor would be unable to start, since $T_{motor} < T_{load}$ (curve (a) in fig. 10.6). By adding in extra rotor resistance, T_{max} can be made to occur at a slip of 1 (curve (b) in fig. 10.6) and now the motor-load will accelerate.

Example 10.3

What extra rotor resistance must be added to the motor of the previous example to give maximum starting torque ?

$$s_m = \frac{R_2'}{Z_{TH}} = 1 \quad \Rightarrow \quad R_2' = Z_{TH} = 2.84$$

$$\therefore \quad 2.84 = 0.5 + \mathrm{R}'_{\mathrm{extra}} \quad \Rightarrow \quad \mathrm{R}'_{\mathrm{extra}} = 2.34\ \Omega$$

Wound-rotor machines: extra rotor resistance can be connected via slip-rings. For cage-rotor machines, skin-effect is exploited:

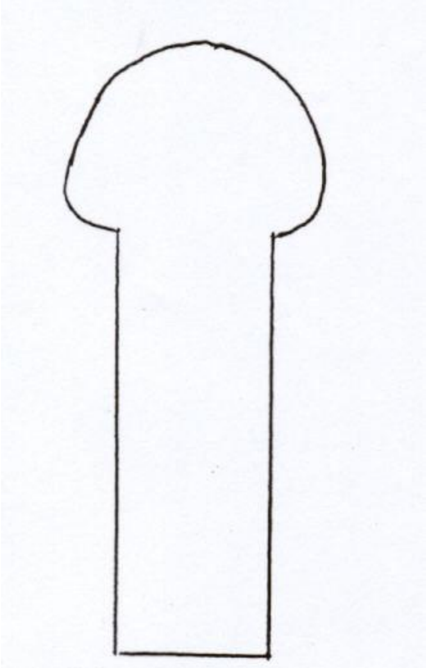


Fig. 10.7

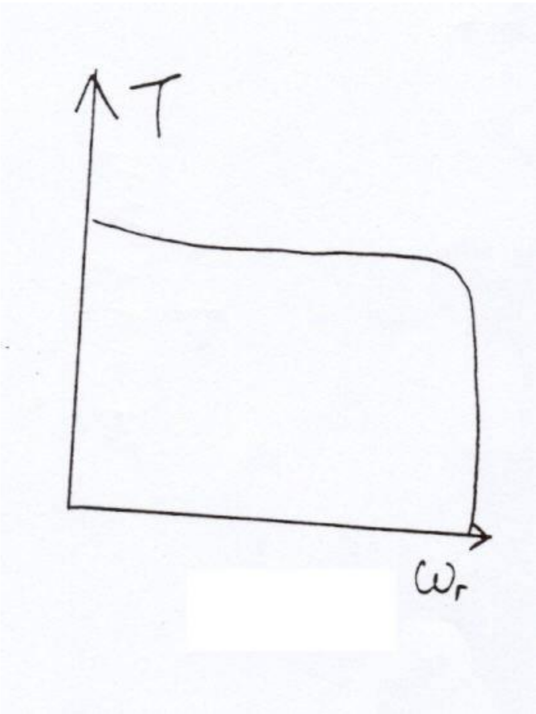


Fig. 10.8

By progressively reducing the extra rotor resistance (curves (c) and (d)) as the motor speeds up, the load can be accelerated to the required speed as quickly as possible, by always ensuring that the motor is operating at maximum torque. The additional benefit is that starting current is reduced.

These ideas are illustrated in the example opposite, in which it is desired to maximise the starting torque. Since the speed is zero at starting, the slip is 1, and so by equation 10.11, we require $R_2'=Z_S$. This is acheived by adding extra rotor resistance equal to the difference between that already present and that required.

Wound rotor machines (fig. 10.1) frequently have the windings connected to slip-rings. This enables additional resistance to be added to the rotor simply by connecting resistance to the brushes which rub on the slip-rings.

For cage rotor machines a similar effect can be obtained by using specially shaped slots. These slots exploit skin-effect - the tendency of current at high frequency to flow in the surface of a conductor, thereby increasing the resistance. Since high frequencies only occur at low speed, this creates the desired effect. Such a slot shape is shown in Fig. 10.7, known as a Boucherot slot. At starting the majority of the current flows in the smaller cross-section at the top of the slot, so with a reduced conductor area the resistance is naturally large. At low slips, the current flows over the entire slot area, thus the effective resistance becomes reduced. The result is the torque-speed curve shown in Fig. 10.8, so that maximum torque is achieved over most of the speed range.