Lecture 5: Generation III

5.1 Generator convention

Most synchronous machines are operated as generators $\Rightarrow$ convenient to make power flowing ***out*** of the terminals positive.

$\therefore$ Reverse reference direction of current:

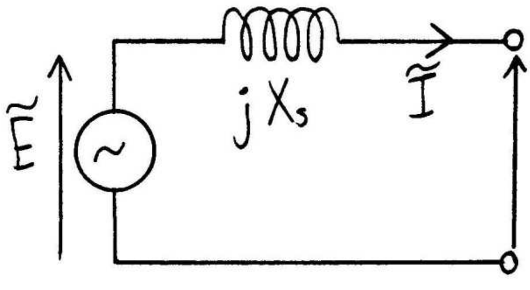

Fig. 5.1

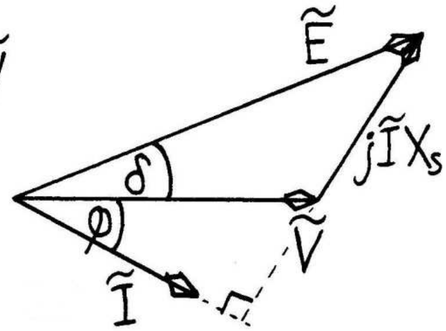

Fig. 5.2

$$\tilde{E} = \tilde{V} + j\tilde{I}X_s \quad (5.1) \quad \text{c.f.} \quad \tilde{V} = \tilde{E} + j\tilde{I}X_s \quad (5.2)$$

5.2 Stand-alone and parallel operation

Stator of a synchronous generator can be connected to a three-phase load.

Frequency of supply is then fixed by the rotor speed and can be set to any convenient value.

Vast majority of generators have stators connected to public electricity supply system.

In the last lecture, we developed an equivalent circuit to represent the synchronous machine. In this lecture, we will use that equivalent circuit to deduce the behaviour of these devices.

As we will soon see, synchronous machines are capable of converting electrical power to mechanical power (motoring) as well as generating. However, in reality most synchronous machines are used as generators, and so it is convenient to use the generator convention. In the equivalent circuit of fig. 4.7 the current was shown flowing into the generator terminals. The electrical power, given by $3VI\cos\varphi$, is then the input electrical power to the machine. The generator convention makes output electrical power positive, and so under this convention the current must be shown as flowing out of the generator terminals, fig. 5.1. This is also summed up in equation 5.1, which may be compared with equation 5.2, obtained when the motoring convention is used. Fig. 5.2 shows equation 5.1 as a phasor diagram.

The synchronous machine can be operated in a stand-alone manner, which means that a single generator supplies a single three-phase load connected to the output terminals. This situation occurs in a limited number of circumstances, usually where connection to the national grid system is prohibitively expensive. Examples are remote farms, and oil-rigs. In this situation, the prime-mover can operate at any suitable speed, and the frequency of the voltage supply will be proportional to that speed.
Most generators, however, have their stators connected to the public electricity supply system. In this system, a large

No individual generator can then affect its terminal voltage, V i.e. V is fixed magnitude, fixed frequency - termed ‘infinite bus’.

5.3 Synchronous speed

Generator rotates at synchronous speed, ω/p (in rad/s) and 60f/p (rpm). For f=50 Hz:

Poles	p	ω_S (rad/s)	N_S (rpm)
2	1	314	3000
4	2	157	1500
6	3	105	1000
:			
30	15	20.9	200

Table 5.1

5.4 Variation of torque with load angle

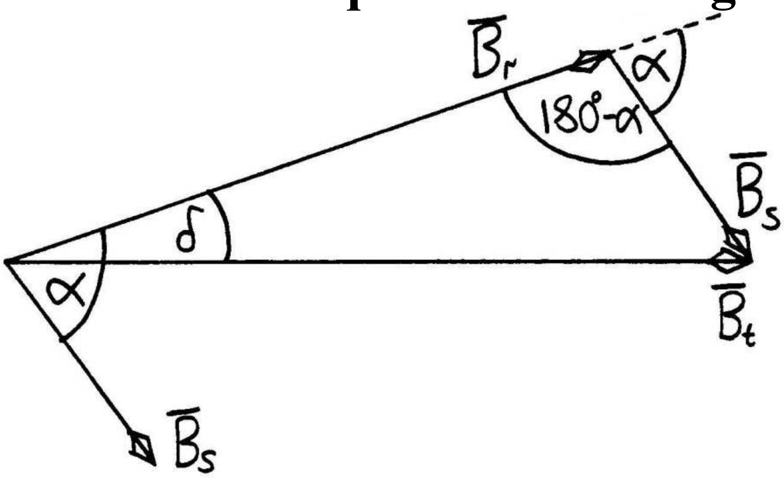

Fig. 5.3

$$T = K\hat{B}_r\hat{B}_S \sin\alpha \quad \text{(lecture 4)} \qquad (5.3)$$

number of generators are connected in parallel to feed a large number of parallel-connected loads. The system is so huge that no individual generator can influence the magnitude or frequency of its terminal voltage. In early power stations all generators were connected together by means of thick copper strips which were fixed to the walls. These went everywhere, and were consequently called ‘omnibus bars’, abbreviated to ‘bus bars’ or just ‘bus’. The term ‘bus’ is now applied in electrical engineering to any connector which connects devices together in order that they can share a common signal, such as ‘data bus’ in computers. The public supply system is referred to as an infinite bus, meaning that it is so large that it presents a fixed-frequency, fixed-magnitude voltage supply to the stator terminals of any machine connected to it. Since this is the case for most generators, we will concern ourselves only with this situation.

For any generator connected to the UK public supply system, the frequency of the terminal voltage will be 50 Hz. As we saw in the last lecture, the synchronous machine can only convert power when rotating at synchronous speed, which is now constrained to a number of values, depending only on the number of poles of the machine. These values are shown in table 5.1. Notice that the highest possible speed is 3000 rpm, for a 2-pole machine.

Fig. 5.3 shows the space phasor diagram, which we met at the end of lecture 4, in which B_r, B_s and B_t represent the rotor-driven, stator-driven and total air-gap magnetic fields, respectively. We also saw in lecture 4 that the electromagnetic torque produced through the interaction of the rotor and

Applying sine rule to fig. 5.3:

$$\frac{\sin(180-\alpha)}{\hat{B}_t} = \frac{\sin\delta}{\hat{B}_s} \quad \therefore \hat{B}_s \sin\alpha = \hat{B}_t \sin\delta \quad (5.4)$$

$$\therefore \quad T = K\hat{B}_r\hat{B}_t \sin\delta \qquad (5.5)$$

V is fixed (infinite bus) $\Rightarrow \hat{B}_t$ also fixed.

Constant excitation $\Rightarrow$ E fixed $\Rightarrow \hat{B}_r$ fixed.

$$\therefore \quad T = T_{\max} \sin\delta \qquad (5.6)$$

δ is the angle in space between the rotor-driven field and total field, known as the **load angle**.

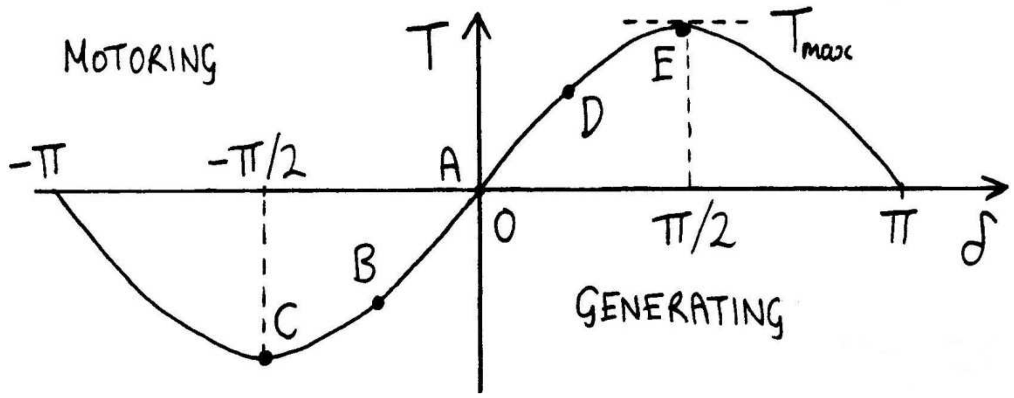

Fig. 5.4

5.5 Control of real power

Prime-mover coupled to generator, both rotating at synchronous speed, but no mechanical power transfer $\Rightarrow$ point A.

stator-driven fields is given by equation 5.3. To gain some insight into the behaviour of the synchronous machine when connected to the infinite bus, the stator-driven magnetic field, B_s, is replaced in equation 5.3 in terms of the total air-gap field, B_t. This is achieved by applying the sine rule to fig. 5.3, which shows that B_s and B_t are related as shown by equation 5.4. The reason for doing this is that for operation from the infinite bus, B_t is fixed (since $B_t \propto V$, and V is fixed). Combining equations 5.3 and 5.4 gives equation 5.5. Assuming fixed terminal voltage, V, and also fixed excitation (i.e. operation with a fixed rotor field current) equation 5.5 may be written as shown by equation 5.6. This shows that the electromagnetic torque acting on the rotor of the machine varies with the sine of the angle between the rotor-driven field and the total air-gap field. This angle therefore assumes some importance, and is given the name **load angle**, symbol δ.

Fig. 5.4 opposite illustrates the variation of torque with load angle, and we can use it to deduce the physical operation of the synchronous machine, both as a generator and as a motor. Imagine that the synchronous machine is connected to the infinite bus, so that it is constrained to be rotating at synchronous speed. Also imagine that it is mechanically coupled to a prime-mover, but that no mechanical power is being transferred. Since power is torque multiplied by angular speed, the torque applied to the shaft of the generator by the prime-mover must be zero. Therefore, we are at the point marked 'A' on fig. 5.4. Equation 5.6 shows that the load angle must be zero, and so the rotor, stator and total air-gap fields are all in space phase with each other, fig. 5.5(a).

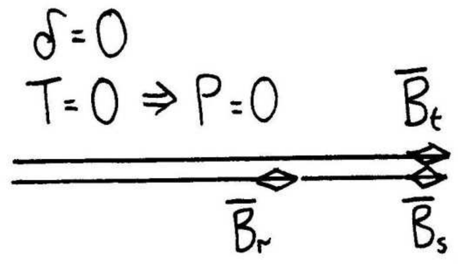

Fig. 5.5(a)

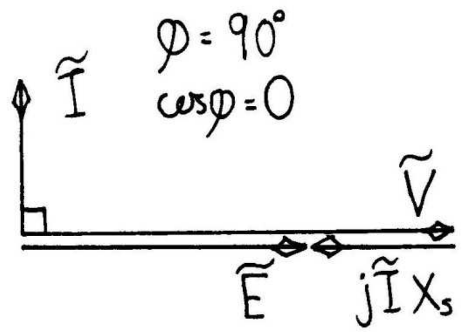

Fig. 5.5(b)

Prime-mover now exerts an accelerating torque on the generator, speed fixed to ω_S.

∴ Load angle δ increases such that generator electromagnetic torque equals prime-mover torque - point D.

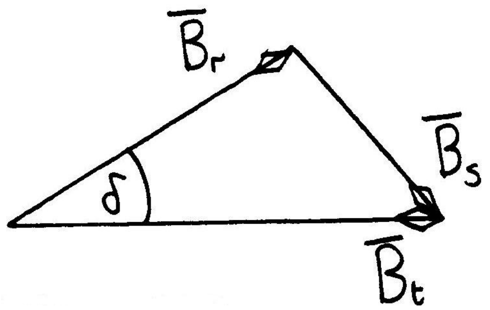

Fig. 5.6(a)

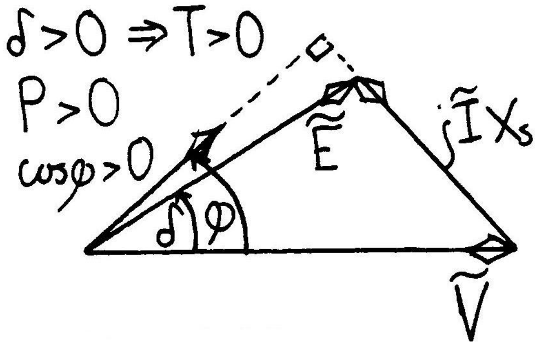

Fig. 5.6(b)

The machine is behaving as a generator, converting mechanical into electrical power.

As prime-mover torque increases, δ increases until point E is reached at $\delta = 90^0$.

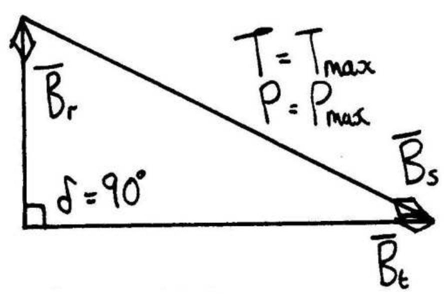

Fig. 5.7(a)

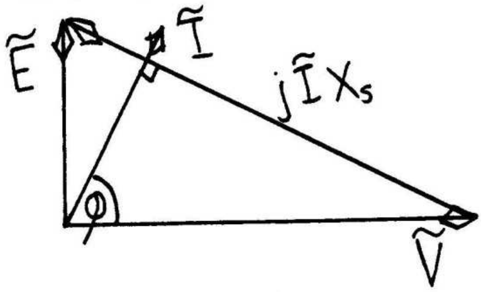

Fig. 5.7(b)

The time phasor diagram (in which $V \equiv B_t$ and $E \equiv B_r$ is therefore as shown in fig. 5.5(b). Notice that the current is 900 out of phase with the terminal voltage. This means that the power factor is zero, and therefore the output power is zero. This is consistent with the fact that the prime-mover is not supplying any mechanical power to the generator.

Now imagine that the prime-mover exerts a torque on the generator that attempts to cause it to accelerate. Because the generator is connected to the infinite bus, its speed is fixed to the synchronous speed, and is therefore constant. The nett torque acting on the rotor must be zero for constant speed ($T=Jd\omega/dt$ and $d\omega/dt = 0$). This is only possible if the load angle increases to a value such that the electromagnetic torque of the generator exactly opposes the torque from the prime-mover. This corresponds to point D in fig. 5.4. The space and time phasor diagrams for this situation are depicted in figs. 5.6(a) and 5.6(b) respectively. Fig. 5.6(b) shows that the generator is now producing electrical power, since the power factor is no longer zero.

Suppose now that the prime-mover torque continues to increase. The load angle, δ, will also continue to increase until point E in fig. 5.4 is reached (the load angle is 90^0).
This point corresponds to the maximum output power that the generator can provide, given the terminal voltage and excitation voltage. The space and time phasor diagrams for this situation are depicted in figs. 5.7(a) and 5.7(b), respectively.

This point corresponds to maximum generated power for the given values of V and E.

Any increase in prime-mover torque now causes synchronous machine to lose synchronism.

Now suppose that a mechanical load is coupled to the synchronous machine which applies a braking torque to its shaft.

$\therefore$ Load angle δ must retard such that synchronous machine electromagnetic torque equals load torque - point B.

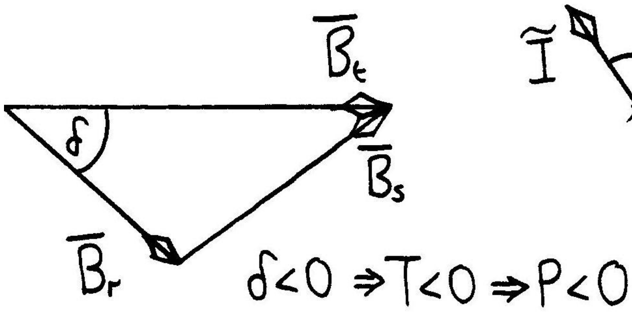

Fig. 5.8(a)

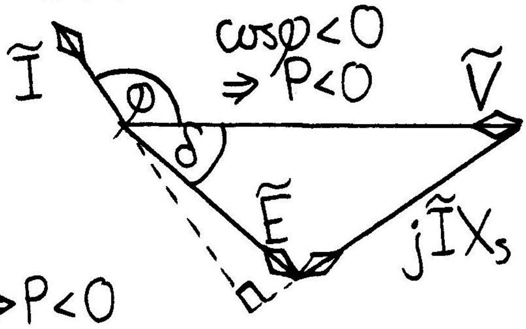

Fig. 5.8(b)

The machine is behaving as a motor, converting electrical into mechanical power.

Point C corresponds to maximum output mechanical power.

If the load torque is increased further, the machine will lose synchronism.

If the prime-mover torque is increased further, the generator torque is unable to oppose it, and the rotor will start to accelerate. Once this starts to happen, the rotor is no longer rotating at synchronous speed. As we saw in lecture 4, the generator can only produce a steady electromagnetic torque at synchronous speed, and so there is now no electromagnetic torque to oppose the prime-mover torque. Therefore the rotor will accelerate very rapidly. This situation is termed 'loss of synchronism', and can cause serious damage to a generator.

Imagine now that the synchronous machine is still connected to the infinite bus, and is therefore still rotating at synchronous speed, but that a decelerating (or braking) torque is applied to its shaft. This corresponds to applying a mechanical load which is attempting to oppose the torque produced by the synchronous machine. As before, the nett rotor torque must be zero for constant rotational speed, and so the machine must produce an electromagnetic torque which exactly counteracts the mechanical load torque. To do this, the load angle must retard to the point B in fig. 5.4. The space and time phasor diagrams for this situation are shown in figs. 5.8(a) and 5.8(b), respectively. The output electrical power, $3VI\cos\varphi$ is now negative, because φ is in the second quadrant. In other words, the ***input*** electrical power is positive, and so the machine is accepting electrical power from the infinite bus, and converting it to mechanical power to drive round the mechanical load. **It is behaving as a motor**. Again, the mechanical load torque can be increased until the load angle is 90^0, beyond which synchronism will be lost, as before. This corresponds to the point C in fig. 5.4.

5.6 Power and torque in terms of electrical quantities

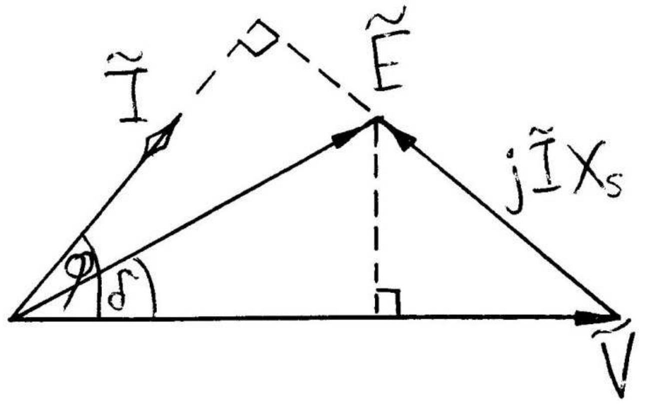

Fig. 5.9

To determine the real power produced by the generator in terms of electrical quantities, consider the phasor diagram shown in fig. 5.9.

Output electrical power is:

$$P_{out(elec)} = 3VI\cos\varphi \qquad (5.7)$$

The output electrical power is given by equation 5.7, in which V and I are phase quantities, and the factor 3 is because the machine has three phases.

Power losses neglected $\Rightarrow$ input mechanical power must equal output electrical power so output real power controlled by prime-mover.

Since the generator is modelled as being lossless, this output electrical power must be equal to the input mechanical power, giving equation 5.8, from which the torque may be found, equation 5.9.

$$P_{in(mech)} = T\omega_s = P_{out(elec)} = 3VI\cos\varphi \qquad (5.8)$$

$$\therefore \quad T = \frac{3VI\cos\varphi}{\omega_s} \qquad (5.9)$$

It is useful to put equation 5.9 into a form which eliminates I (because I varies depending on load). The phasor diagram in fig. 5.9 shows the relationship between excitation voltage, load angle and current, power factor given by equation 5.10. Combining equations 5.9 and 5.10 gives 5.11.

Phasor diagram: $\quad E\sin\delta = X_s I\cos\varphi \qquad (5.10)$

$$T = \frac{3VE}{\omega_s X_s}\sin\delta \qquad (5.11)$$

For operation from the infinite bus, and at constant excitation, and since ω_s and X_s are both constants, it is easily seen that the torque varies only with the sine of the load angle. This is to be expected, since it is what we derived by considering the torque in terms of the magnetic fields.

With V and E fixed, $T = T_{\max}\sin\delta$ as before.

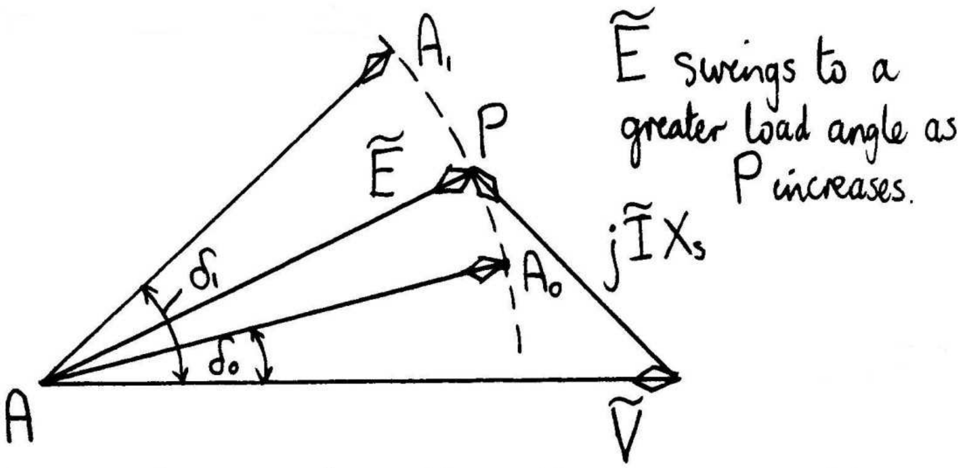

Fig. 5.10

Here we consider what happens to the phasor diagram for a synchronous machine when the torque due to the prime-mover increases. This might correspond to the opening of the steam valve in a turbine, for example. An increase in torque means an increase in the real power being generated, say from P_0 to P_1. Assuming constant excitation, so E is fixed, the length AP in fig. 5.10 is fixed. Therefore, the point P is constrained to move in an arc about A. The operating point A_0 in fig. 5.10 must therefore move to A_1, so that the load angle increases (since real power is proportional to the sine of the load angle). Using equation 5.11, the relationship between the new and old load angles, δ_1 and δ_0, is given by equation 5.12.

Power changes from P_0 to P_1 at constant E.
V fixed (infinite bus) $\Rightarrow$ point P constrained to move in an arc about A.

$$\frac{P_1}{P_0} = \frac{T_1\omega_s}{T_2\omega_s} = \frac{\left(3VE\sin\delta_1\right)/X_s}{\left(3VE\sin\delta_0\right)/X_s} = \frac{\sin\delta_1}{\sin\delta_0} \qquad (5.12)$$

5.7 Control of reactive power

Now we look at the control of reactive power. Consider a generator operating from the infinite bus, and generating a fixed amount of real power, P. Equation 5.13 shows that for P to be fixed, Esinδ must be constant. But, Esinδ is simply the projection of the point marked A_0 in fig. 5.11 on to an axis which is perpendicular to the terminal voltage phasor, V. The point A_0 corresponds to the generator producing power at unity power factor, since V and I are in phase. Suppose now that the excitation is increased, but the prime-mover power is held constant. The point A_0 must move to the right along the horizontal dashed line of fig. 5.11, to the new point A_1. The phasor diagram for this situation shows that I lags V, so the generator is producing reactive power. By continuing to increase E, the magnitude of the reactive power produced will increase.
The machine in this situation is described as being 'over-excited', because E is greater than its normal value.
Suppose now that the excitation is decreased from the value corresponding to A_0.

$$P = T\omega_s = 3VE\sin\delta \qquad (5.13)$$

$\therefore$ Constant P $\Rightarrow$ $E\sin\delta$ constant $\Rightarrow$ vertical projection of point A_0 constant i.e. changes in E cause A_0 to move along horizontal line.

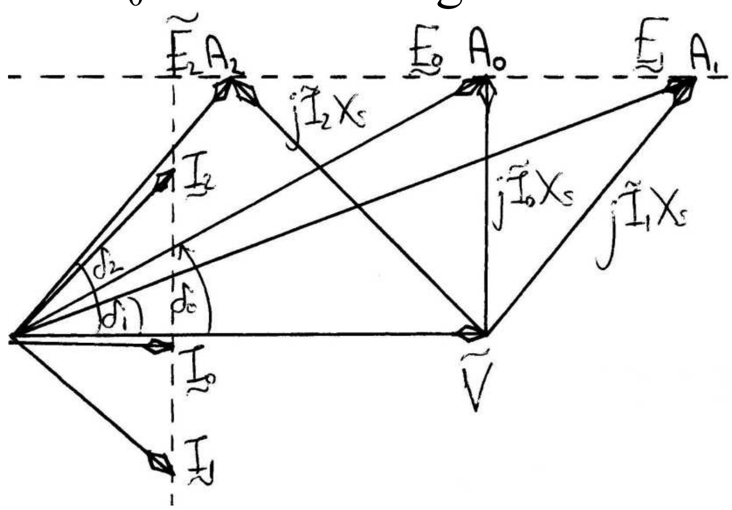

Fig. 5.11

Increasing E $\Rightarrow$ point A_0 moves to A_1 $\Rightarrow$ current lags voltage $\Rightarrow$ machine supplies VARs. The machine is '**over-excited**'.

Decreasing E $\Rightarrow$ point A_0 moves to A_2 $\Rightarrow$ current leads voltage $\Rightarrow$ machine absorbs VARs. The machine is '**under-excited**'.

Controlling E enables VARs to be controlled.

If synchronous machine is neither generating or motoring, P = 0 $\Rightarrow$ $\delta = 0$ so V and E in phase.

Phasor diagram for E >V (over-excited) shows that the machine is ***supplying*** reactive power.

Phasor diagram for E <V(under-excited) shows that the machine is ***absorbing*** reactive power.

Synchronous machines can be used to correct power factor - **synchronous compensator**.

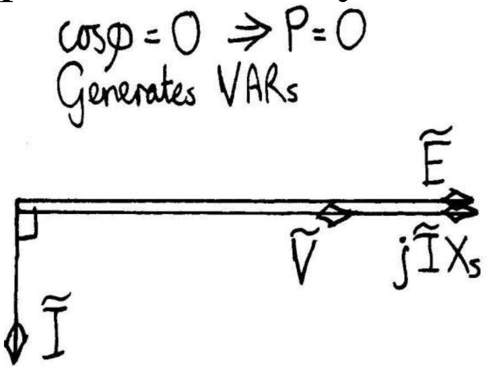

Fig. 5.12(a)

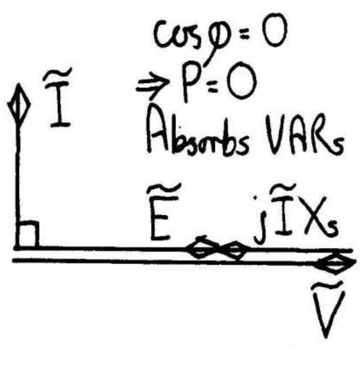

Fig. 5.12(b)

The operating point must then move to the left along the horizontal dashed line, to the point A_2. The phasor diagram for this situation shows that I leads V, and so the generator is receiving reactive power. By further decreasing the excitation, the amount of reactive power absorbed will increase. In this situation the machine is described as being 'under-excited', because E is less than its usual value.

What all of this clearly demonstrates is that by controlling the excitation voltage E (by varying the rotor field current), it is possible to control the reactive power produced by a synchronous generator.

Suppose now that a synchronous machine is neither producing or receiving any real power. In that case, the load angle must be zero, and so V and E are in phase. If E is greater than V, so that the machine is over-excited, the current must lag the voltage by 90^0, fig. 5.12(a). In that case the machine is supplying only reactive power. It can therefore be used to compensate for the reactive power absorbed by an inductive load i.e. it can be used in the same way that capacitors are used for power factor correction. It has the additional advantage that the reactive power supplied can be set to any value, whereas capacitor banks can only generate discrete amounts of reactive power.

If the machine is under-excited, the current leads the voltage by 90^0, as shown in fig. 5.12(b). It will then absorb only reactive power, and can compensate for the reactive power generated by a leading power factor load. When the synchronous machine is used in this manner, it is referred to as a synchronous compensator.