Lecture 2: Three-phase Circuits II

2.1 Delta-connected load

In this lecture we will consider delta-connected loads and sources, and look at the relationship between line and phase quantities for them. We will see how balanced star and delta-connected loads can be transformed (star-delta and delta-star transformations). We will also see how P, Q and S may be expressed purely in terms of line quantities, and will finish off our study of three-phase circuits with a Tripos question.

The load may also be delta-connected (like the star-connection, named after the shape formed) as shown in fig. 2.1.

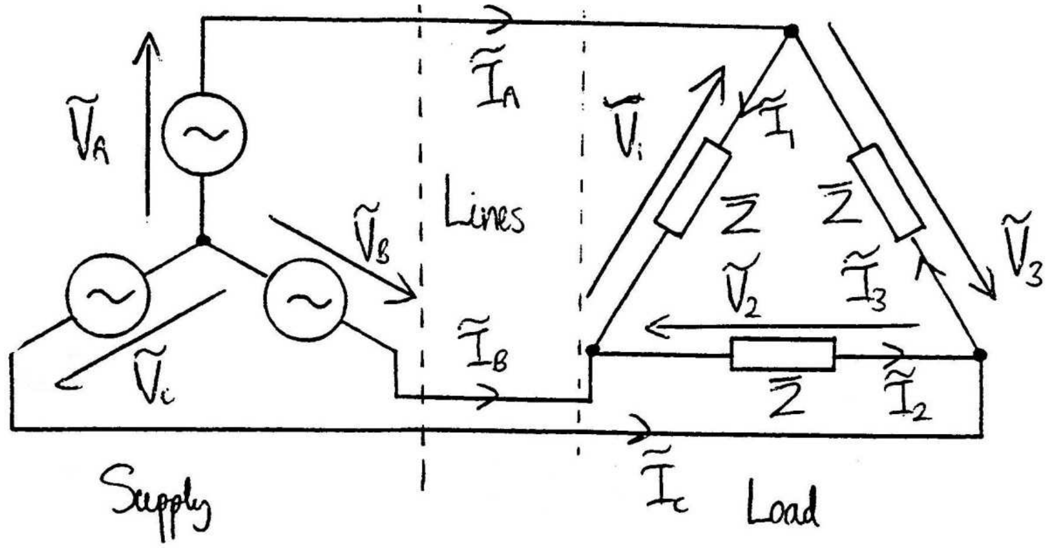


Fig 2.1

Now the voltage across each phase of the load equals the line voltage:

$$\tilde{V}_1 = \tilde{V}_{AB} \qquad \tilde{V}_2 = \tilde{V}_{BC} \qquad \tilde{V}_3 = \tilde{V}_{CA} \qquad (2.1)$$

In that case, it is clear that the voltage across each phase of the load is now equal to the **line voltage.** This contrasts with the star-connected load, in which the voltage across each phase is equal to the line-neutral voltage, and is therefore √3 times as small (for the same supply).

Load currents will form a balanced 3-phase set:

$$\tilde{I}_1 = I\,e^{j0} = I \quad \tilde{I}_2 = I\,e^{-j\frac{2\pi}{3}} \quad \tilde{I}_3 = I\,e^{j\frac{2\pi}{3}} \qquad (2.2)$$

As we saw in Lecture 1, the line voltages form a balanced three-phase set, and so for a balanced three-phase load, the load currents will also form a balanced three-phase set. This is illustrated in fig. 2.2.

However, the line current is no longer the same as the load phase current, as it was for the star-connected load.

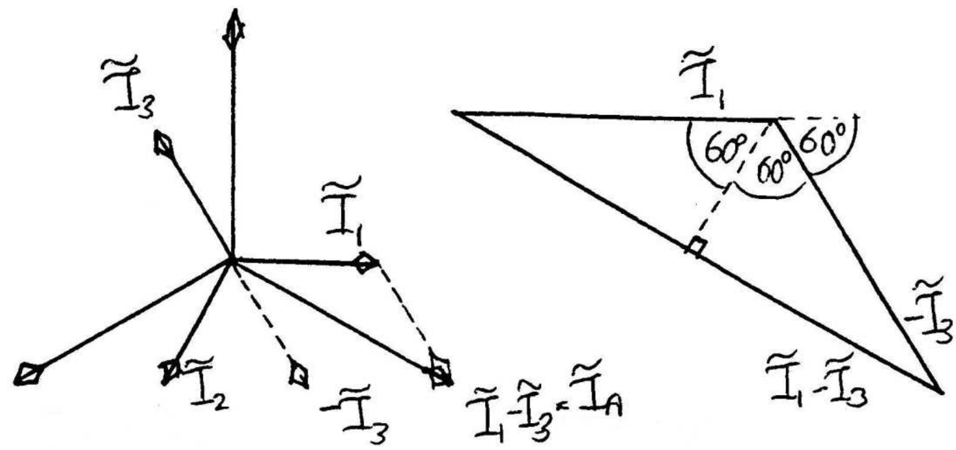


Fig. 2.2

Line currents found using Kirchoff's Current Law e.g. for line A:

$$\tilde{I}_A = \tilde{I}_1 - \tilde{I}_3 \qquad (2.3)$$

Trigonometry shows that $\left|\tilde{I}_A\right| = \sqrt{3}\left|\tilde{I}_1\right|$

Result: Line current = √3× phase current

2.2 Summary for star and delta connections

Phase quantity	**Star**	**Delta**
Voltage	$V_{line}/\sqrt{3}$	V_{line}
Current	I_{line}	$I_{line}/\sqrt{3}$

2.3 Delta-connected supply

Supply can also be connected in delta since:

$$\tilde{V}_A + \tilde{V}_B + \tilde{V}_C = 0 \qquad (2.4)$$

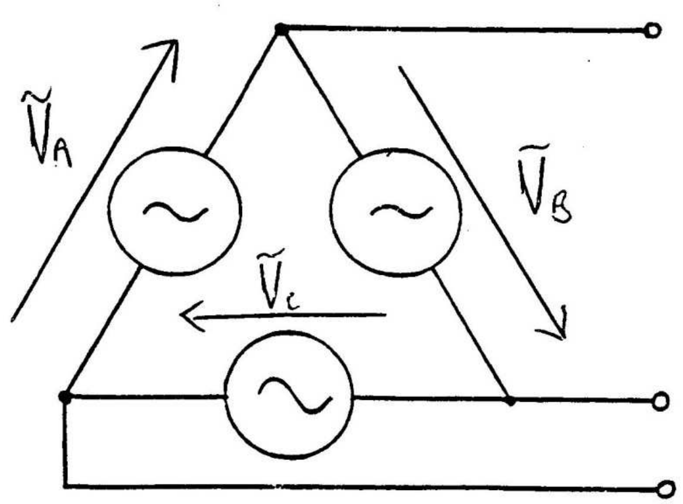


Fig. 2.3

To obtain the line current, consider summing currents at node A in fig. 2.1 (equation 2.3). By the same trigonometric arguments made in section 1.2, it is seen that the magnitude of the line current is √3 times as large as the phase current. Also, there is a 30^0 phase shift between the line and phase currents.

All these results for star and delta-connected loads are summarised in the table opposite. A common problem with analysing three-phase circuits is knowing that a factor of √3 is involved somehow, but not being sure whether it is necessary to divide or multiply by it ! Fortunately, there is an easy way of remembering, relying on some slightly dodgy arguments, as follows:
For the star-connected load, the impedances can be thought of as behaving like a potential divider connected across the line voltage. Therefore, the phase voltage should be smaller than the line voltage, and is found by dividing by √3.
For the delta-connected load, the load impedances can be thought of as acting like a current divider to the line current. Therefore, the phase current is again obtained by dividing the line current by √3.

It is also possible to connect a balanced three-phase supply in delta, fig. 2.3. However, this is far less common than connecting the supply in delta, because any small imbalance in the voltages would result in a nett voltage around the loop. In turn, this would drive a circulating current, limited only by the impedance of the voltage source. For a perfectly balanced supply, the three voltages sum exactly to zero, equation 2.4, and so there is no difficulty. The relationship between phase and line quantities for star and delta-connected supplies is the same as that for loads given in section 2.2.

2.4 Star-delta and delta-star transformation

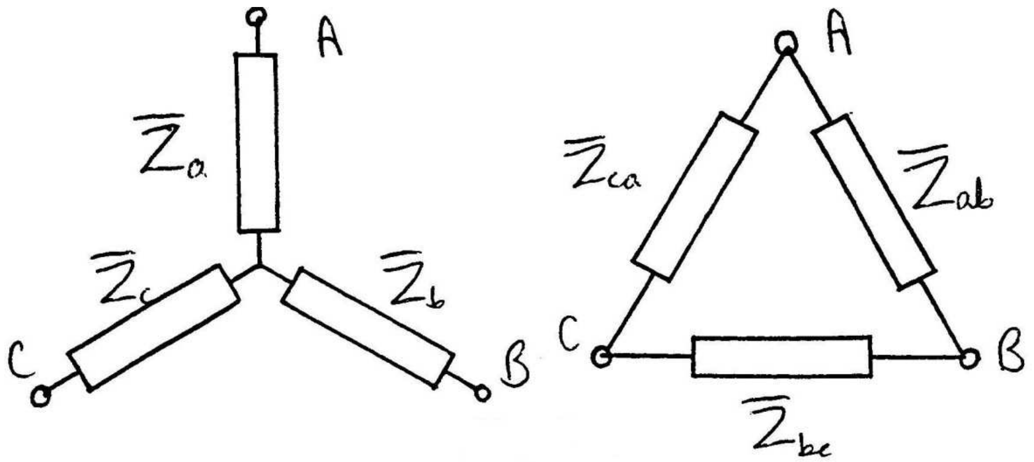


Fig. 2.4

Data book: $$\overline{Z}_{ab} = \frac{\overline{Z}_a\overline{Z}_b + \overline{Z}_b\overline{Z}_c + \overline{Z}_c\overline{Z}_a}{\overline{Z}_c} \qquad (2.5)$$

But for balanced loads: $$\overline{Z}_a = \overline{Z}_b = \overline{Z}_c = \overline{Z}_{star}$$
$$\overline{Z}_{ab} = \overline{Z}_{bc} = \overline{Z}_{ca} = \overline{Z}_{delta}$$

$$\therefore \quad \overline{Z}_{ab} = \overline{Z}_{delta} = 3\overline{Z}_{star} \qquad \overline{Z}_{star} = \overline{Z}_{delta}/3 \qquad (2.6)$$

2.5 Total three-phase P, Q and S.

$$P = 3V_{ph}I_{ph}\cos\varphi \qquad (2.7)$$

If the load (or source) is star-connected:

$$V_{ph} = V_l/\sqrt{3} \qquad I_{ph} = I_l$$

$$\therefore \quad P = 3\frac{V_l}{\sqrt{3}}I_l\cos\varphi = \sqrt{3}V_lI_l\cos\varphi \qquad (2.8)$$

Sometimes it is useful to be able to transform a star-connected load to an equivalent delta-connected load and vice-versa. For example, some three-phase circuits combine star and delta-connected loads in parallel, and the easiest way to solve them is to combine the loads in parallel. This can only be done if they are both star-connected or both delta-connected.

In the Electrical and Information Science Databook a general expression for transforming star to delta-connected loads is given (equation 2.5). This equation makes no assumption about the values of the impedances, but if we make use of the fact that for balanced three-phase loads the impedances are identical, it is easily seen that these transformations involve multiplying or dividing by 3 (equation 2.6).

In some problems no information is given regarding the connection of the loads - only the real and reactive power consumed by them is given, for example. In this section we see that expressions for real and reactive power consumed (or supplied) depends only on the line voltage and line current, irrespective of whether star or delta-connected loads or sources are used.

The expression for total three-phase power supplied to a balanced three-phase load is given by equation 2.7. This equation is valid irrespective of the type of connection.

By substituting for phase quantities in terms of line quantities, we obtain equation 2.8.

If the load (or source) is delta-connected:

$$V_{ph} = V_l \qquad I_{ph} = I_l / \sqrt{3}$$

$$\therefore \quad P = 3V_l \frac{I_l}{\sqrt{3}} \cos\varphi = \sqrt{3} V_l I_l \cos\varphi \qquad (2.9)$$

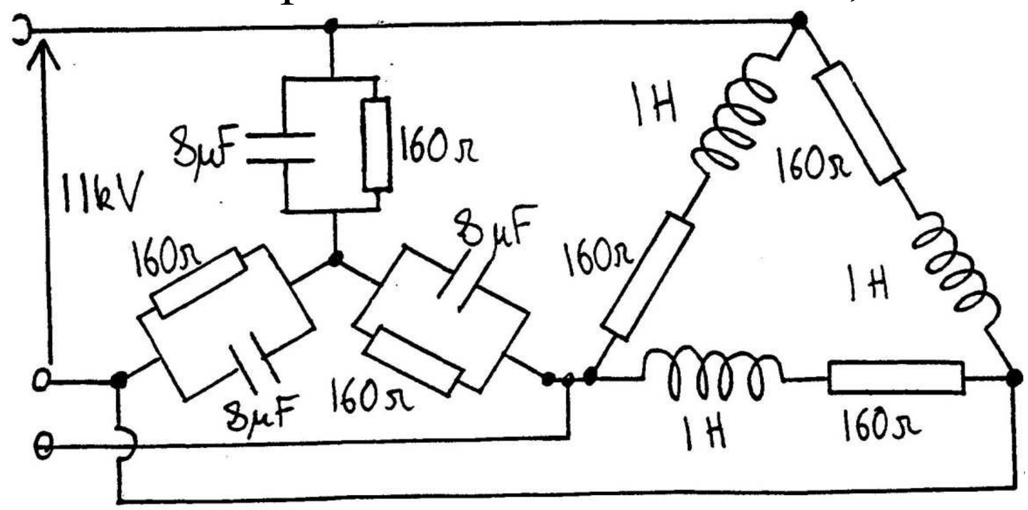
In terms of line quantities, $P = \sqrt{3} V_l I_l \cos\varphi$ for star or delta-connected source or load.

$$Q = \sqrt{3} V_l I_l \sin\varphi \quad (2.10) \qquad S = \sqrt{3} V_l I_l \quad (2.11)$$

Example 2.1
A three-phase 11 kV load draws 100 MW at power factor 0.8 lagging. Find the line current.

$$I_l = \frac{P}{\sqrt{3}\, V_l \cos\varphi} = \frac{100 \times 10^6}{\sqrt{3} \times 11 \times 10^3 \times 0.8} = 6561\,\text{A}$$

Example 2.3
For the three-phase circuit shown below, find:



Doing the same thing for the delta-connected case shows that in terms of line quantities, the expression for power is the same as that for a star-connected load, equation 2.9.

It is easily seen that these ideas can equally well be applied to reactive power, Q (equation 2.10) and apparent power S (equation 2.11).

As always, the specified voltage, 11 kV, is the line voltage. The line current is then found from equation 2.9. Notice that no information regarding the way in which the load is connected is given (or needed). Only if the currents flowing in the load itself are needed is it necessary to know whether the load is star or delta-connected.

Here we apply most of the ideas of lectures 1 and 2 to the solution of a Tripos question (1993, Paper 4, question 1).

The circuit shows a mixed load, consisting of parallel star and delta-connected loads.

a) The overall power factor, and the line current drawn from the supply.

b) The value of the star-connected capacitors which, when connected in parallel with the load, will correct the overall power factor to unity.

<u>Method 1</u> - use star-delta transformation and single-phase representation.

<u>Step 1</u> Determine load impedances.

$$\bar{Z}_{star} = R \Big\| \tfrac{1}{j\omega C} = \frac{1}{\frac{1}{R} + j\omega C} = \frac{R}{1 + j\omega CR}$$

$$= \frac{160}{1 + j2\pi \times 50 \times 8 \times 10^{-6} \times 160} = 137.7 - j55.38\ \Omega$$

$$\bar{Z}_{delta} = R + j\omega L = 160 + j2\pi \times 50 \times 1.0$$

$$= 160 + j314.2\ \Omega$$

<u>Step 2</u> Transform delta load to star.

$$\bar{Z}_{star} = \frac{\bar{Z}_{delta}}{3} = \frac{160 + j314.2}{3} = 53.33 + j104.7\ \Omega$$

We have met the idea of power factor correction using capacitors in the 1A course on power. To recap, in order for the power factor to be corrected to unity, the total reactive power consumed must be zero. Therefore, the capacitors must generate exactly the same reactive power as that consumed by the original load.

To illustrate all of the ideas which we have met, we will do the question two ways.

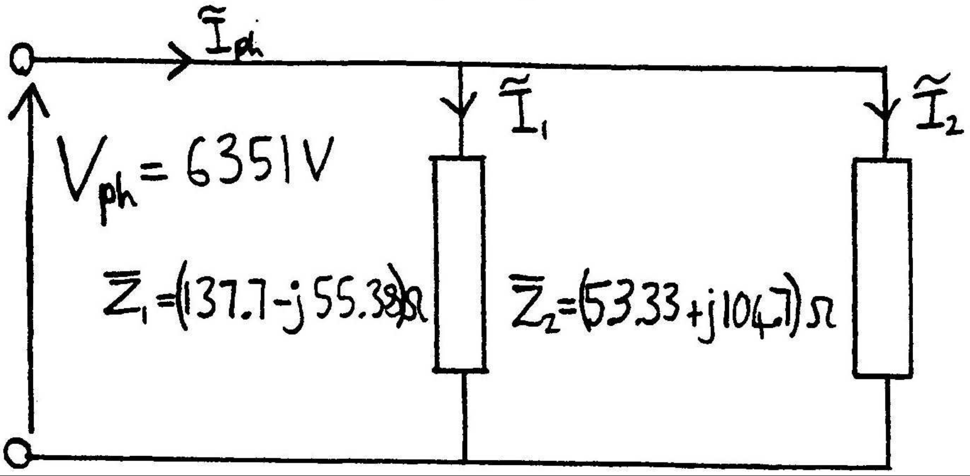
In the first method, we will transform the delta-connected load into a star-connected load. The two star-connected loads can then be combined in parallel, and represented as a single-phase circuit.

In the second method, the ideas of conservation of real and reactive power will be used to obtain the total P and Q consumed by the load. Then, using earlier expressions for P and Q in terms of line quantities, it will be possible to determine the line current.

The first step is to determine the impedances of the star and delta-connected loads. The ideas for doing this have been covered at Part 1A. Notice that the star-connected load has a negative imaginary part, showing that it is resistor-capacitor, whereas the delta-connected load has a positive imaginary part, showing that it is resistor-inductor.

Whilst in principle it would now be possible to determine the line current due to each of these loads, and add them as phasors, it is easy to make mistakes doing this, because of the phase shifts involved between line and phase quantities. It is safer to transform the delta-connected load to star-connected using equation 2.6.

Step 3 Draw single-phase circuit.



Step 4 Determine the phase voltage.

$$V_{ph} = V_l / \sqrt{3} = 11000 / \sqrt{3} = 6351 \text{ V}$$

Step 5 Determine the line current.

$$\tilde{I}_l = \tilde{I}_{ph} = \frac{\tilde{V}_{ph}}{\bar{Z}_1} + \frac{\tilde{V}_{ph}}{\bar{Z}_2}$$

$$= \left(\frac{6351}{137.7 - j55.38} + \frac{6351}{53.33 + j104.7} \right)$$

$$= 64.23 - j32.19 = 71.84\angle -26.6^0 \text{ A}$$

Step 6 Hence find P, Q, S and power factor.

$$\text{Power factor} = \cos\varphi = \cos\left(-26.6^0\right) = 0.894 \text{ lag}$$

$$S = \sqrt{3} V_l I_l = \sqrt{3} \times 11 \times 10^3 \times 71.84 = 1.37 \text{ MVA}$$

$$P = S\cos\varphi = 1.37 \times 0.894 = 1.22 \text{ MW}$$

Since the delta-connected load has been replaced by its star-connected equivalent, we now have two star-connected loads in parallel. Since the star points of both loads are at 0 V, they can be thought of as being connected together, and a single-phase equivalent circuit may now be drawn, as shown opposite. This illustrates how it is now possible to combine the two loads into a single equivalent star-connected load by determining their parallel impedance.

However, it is slightly easier to determine the phasor currents flowing into each load, and sum them. Before this can be done, the phase voltage must be found, and since the loads are star-connected, we need to divide the line voltage by √3 to achieve this.

Since the loads are star-connected, the line and phase currents are the same thing. The total line current is therefore obtained by phasor summation of the phase currents flowing in each load. To sum phasor quantities, it is best to leave the complex numbers in rectangular form i.e. Real+jImaginary. Having found the sum, the current is put into polar form to facilitate the determination of real, reactive and apparent power, and power factor.

The power factor is the cosine of the angle of the phase current w.r.t. the phase voltage, as shown opposite.

Using equation 2.11 the apparent power may be found from the line current and voltage. The power triangle is then used to find real and reactive power.

$$Q = \sqrt{S^2 - P^2} = \sqrt{1.37^2 - 1.22^2} = 613 \text{ kVAR}$$

<u>Step 7</u> Find expression for capacitor VARs.

$$Q_{cap} = 3\frac{V_{ph}^2}{X_C} = 3\frac{\left(V_l/\sqrt{3}\right)^2}{1/\omega C} = \omega C V_l^2$$

<u>Step 8</u> Capacitor VARs = load VARs.

$$2\pi \times 50 \times C \times 11000^2 = 613 \times 10^3 \quad C = 16.1\ \mu\text{F}$$

<u>**Method 2**</u> - Use conservation of P and Q.

<u>Step 1</u> Find P and Q for star load.

Since components in parallel, use V^2/R, V^2/X.

$$P_{star} = 3\frac{V_{ph}^2}{R} = 3\frac{6351^2}{160} = 756.3 \text{ kW}$$

$$Q_{star} = -3\frac{V_{ph}^2}{X_C} = -3\frac{6351^2}{397.9} = -304.1 \text{ kVAR}$$

<u>Step 2</u> Find P and Q for delta load.

Phase voltage = line voltage = 11 kV
Series components, so find I and use I^2R, I^2X.

The plan for finding the star-connected capacitors required to correct the power factor to unity is to first of all write down an expression for the reactive power generated by such capacitors. Since they are to be connected in parallel with the loads, and as they are star-connected, the voltage across them is known to be the line voltage/√3. The reactive power generated is then found as $3V^2/X_C$.

This expression can then be equated with the reactive power consumed by the load. This is because for the power factor to be unity, the total reactive power consumed must be zero.

A simpler approach to this question is to use the ideas of conservation of real and reactive power. This avoids the use of complex algebra, and delta-star transformations.

Since the resistor and capacitor of the star-connected load are in parallel, the voltage across them is known to be the line-neutral voltage i.e. $V_{line}/\sqrt{3}$. This was found earlier to be 6351 V.

The real and reactive power can then be found using $P=V^2/R$, $Q=V^2/X_C$ as shown opposite. Notice that the reactive power is assigned to be negative. This is consistent with the idea that capacitors generate reactive power.

For the delta-connected load, the phase voltage is equal to the line voltage. Since the resistor and inductor are in series, the voltage across them individually is not known, so we cannot use V^2/R, V^2/X directly. Instead, it is easier to find the phase current flowing in them, and use I^2R and I^2X.

$$I = \frac{V_{ph}}{|\overline{Z}|} = \frac{11000}{\sqrt{160^2 + 314.2^2}} = 31.2 \text{ A}$$

It is only necessary to find the magnitude of the phase current - the angle is not required for the calculation of P and Q. This fact saves on complex algebra.

$$P_{delta} = 3I^2 R = 3 \times 31.2^2 \times 160 = 467.3 \text{ kW}$$

$$Q_{delta} = 3I^2 X_L = 3 \times 31.2^2 \times 314.2 = 917.4 \text{ kVAR}$$

Now we apply the ideas met last year regarding conservation of real and reactive power. The total real power flowing into the star and delta-connected loads must be the sum of the powers consumed by them individually, and likewise for the total reactive power consumed. In the case of the reactive powers, notice that they tend to cancel each other to some extent, since the star-connected load is resistor-capacitor, whilst the delta-connected load is resistor-inductor. However, since the overall reactive power turns out to be positive, the combined load must appear to be resistor-inductor.

Step 3 Find total P and Q.

$$P_t = P_{star} + P_{delta} = 756.3 + 463.7 = 1.22 \text{ MW}$$

$$Q_t = Q_{star} + Q_{delta} = -304.1 + 917.4 = 613 \text{ kVAR}$$

From a knowledge of P and Q, the apparent power, S, may be found from the power triangle, as can the power factor. Notice that the power factor is quoted as lagging, since the total reactive power turned out to be positive.

Step 4 Use power triangle to find S and cosφ

$$S_t = \sqrt{P_t^2 + Q_t^2} = \sqrt{1.22^2 + 0.613^2} = 1.37 \text{ MVA}$$

$$\cos\varphi = P/S = 1.22/1.37 = 0.894 \text{ lag}$$

The line current can be found from a knowledge of the apparent power and the line voltage from equation 2.10. The rest of the question is tackled in exactly the same way as method 1.

You should now be able to tackle Examples Sheet 5/3.

Step 5 Find line current

$$S_t = \sqrt{3} V_l I_l = 1.37 \text{ MVA} \quad \Rightarrow \quad I_l = 71.9 \text{ A}$$

Power factor correction capacitor value found in the same way as method 1.