Part IB Paper 6: Information Engineering

LINEAR SYSTEMS AND CONTROL

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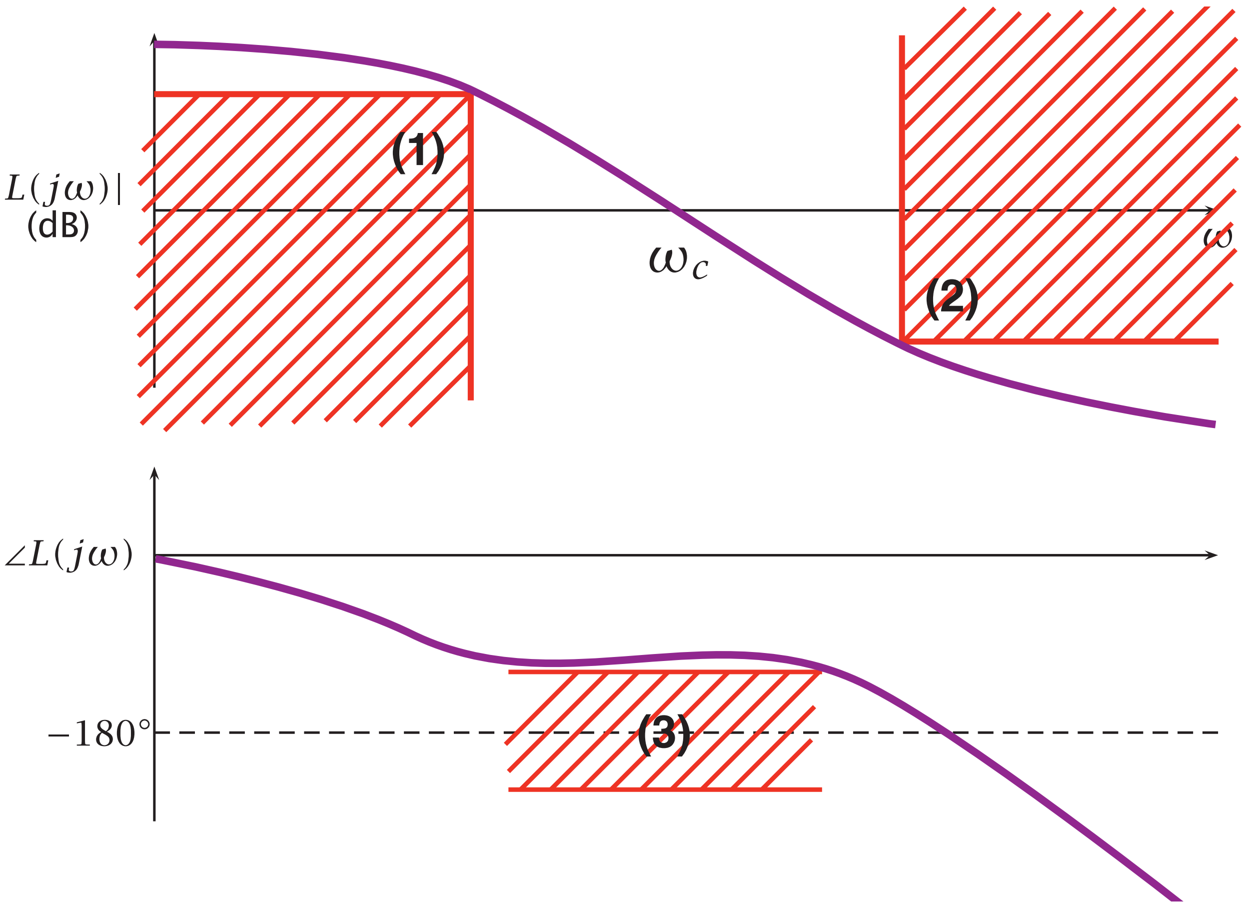
HANDOUT 7

“The design of feedback systems – an introduction”



7.1 Feedback system design, a loop-shaping approach

- This consists of choosing $K(s)$ to shape $L(s) = K(s)G(s)$ such that

1. $|K(j\omega)G(j\omega)| \gg 1$ for frequency ranges where the benefits of feedback are sought (typically $\omega < \omega_c$)

   (in order to ensure that the sensitivity function $|S(j\omega)| \ll 1$ at those frequencies.)

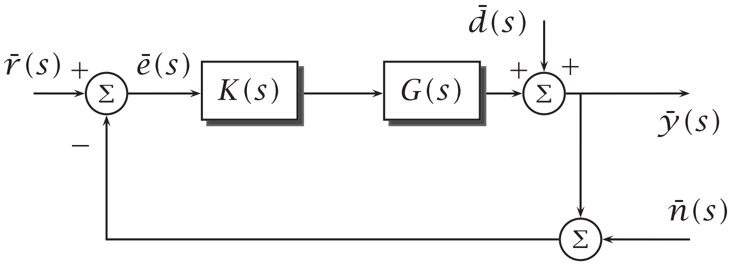
2. $|K(j\omega)G(j\omega)| \ll 1$ at other frequencies (typically high frequencies $\omega \gg \omega_c$)

   (ensuring that the complementary sensitivity function $|T(j\omega)| \ll 1$ at those frequencies.)

3. $K(j\omega)G(j\omega)$ satisfies the Nyquist stability criterion, with adequate gain and phase margins. (ensuring that neither $S(j\omega)$ or $T(j\omega)$ have a large peak in the *crossover* region in between)

Recall that $S(s) = \frac{1}{1+L(s)} = T_{d\mapsto y}$ and $T_{r\mapsto e}$

and $T(s) = \frac{L(s)}{1+L(s)} = -T_{n\mapsto y}$ and $T_{r\mapsto y}$



To achieve this, we can use combinations of phase lag and phase lead compensators. Phase lag compensators are a generalized form of P+I action and phase lead compensators are a generalized form of P+D action.

Note: Example 2 of Handout 4 is a phase lead compensator and Example 3 is a PI controller (and so a special case of a phase lag compensator).

In the *loop-shaping* approach for control design, the controller $K(s)$ is chosen such that the frequency response of the return ratio $L(j\omega)$ has an appropriate shape that leads to good closed-loop properties.

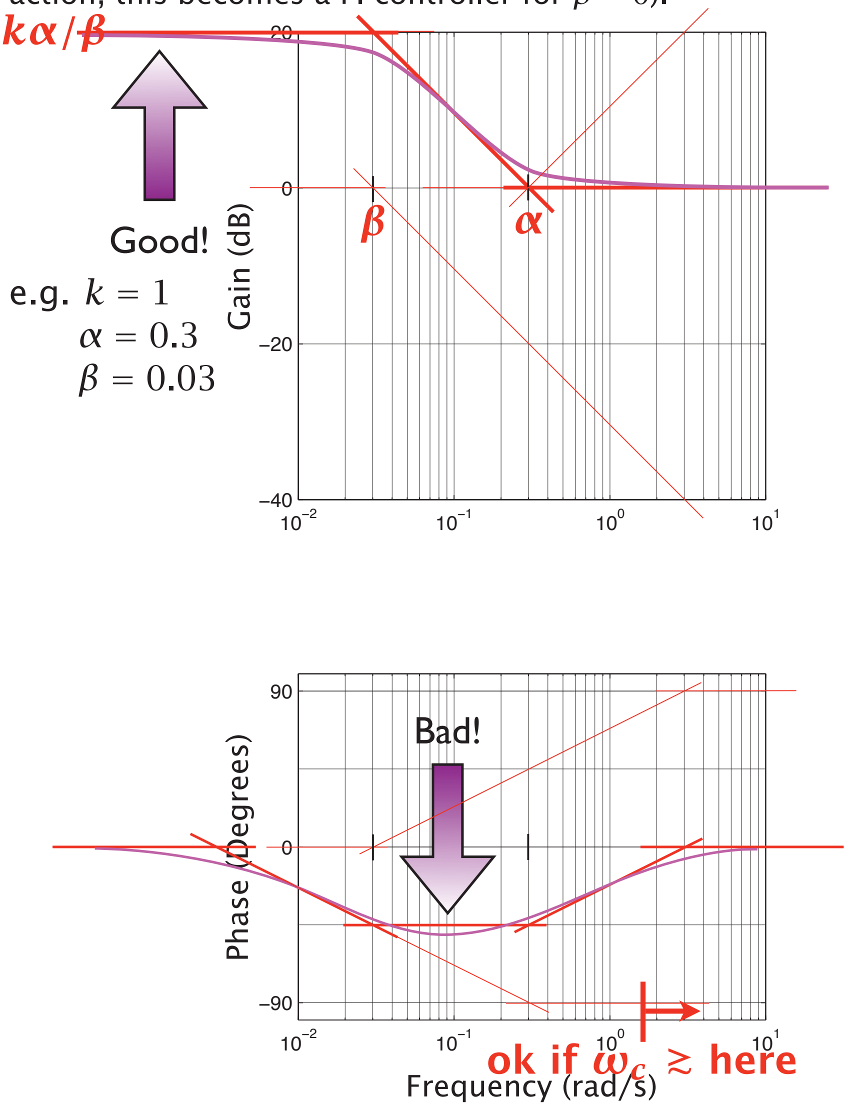
The key desired features for $L(j\omega)$ are enumerated on the left and are illustrated also graphically in the previous page (the shaded red regions are regimes to be avoided).

In particular, we are aiming for $L(j\omega)$ to have a high gain at low frequencies, low gain at high frequencies and at the frequency ω_c where the gain is 1, the phase should be larger than $-180°$. The latter ensures that the Nyquist stability criterion is satisfied (with a sufficient gain and phase margin), and therefore the closed-loop system is asymptotically stable.

These specifications for $L(j\omega)$ can be achieved by means of phase lag and phase lead compensators, which will be discussed extensively in the rest of the handout.

- $\boxed{K(s) = k\dfrac{s+\alpha}{s+\beta}}$ for $\beta < \alpha$ ($< \omega_C$ typically) $= \dfrac{k\alpha}{\beta}\dfrac{1+s/\alpha}{1+s/\beta}$

– Phase lag compensator (a generalized form of proportional+integral action, this becomes a PI controller for $\beta = 0$).



– improves low frequency gain (and so reduces steady-state errors) at the expense of introducing phase lag at frequencies between $\omega \approx \beta$ and $\omega \approx \alpha$ (although this is not an issue if $\alpha \ll \omega_C$) .

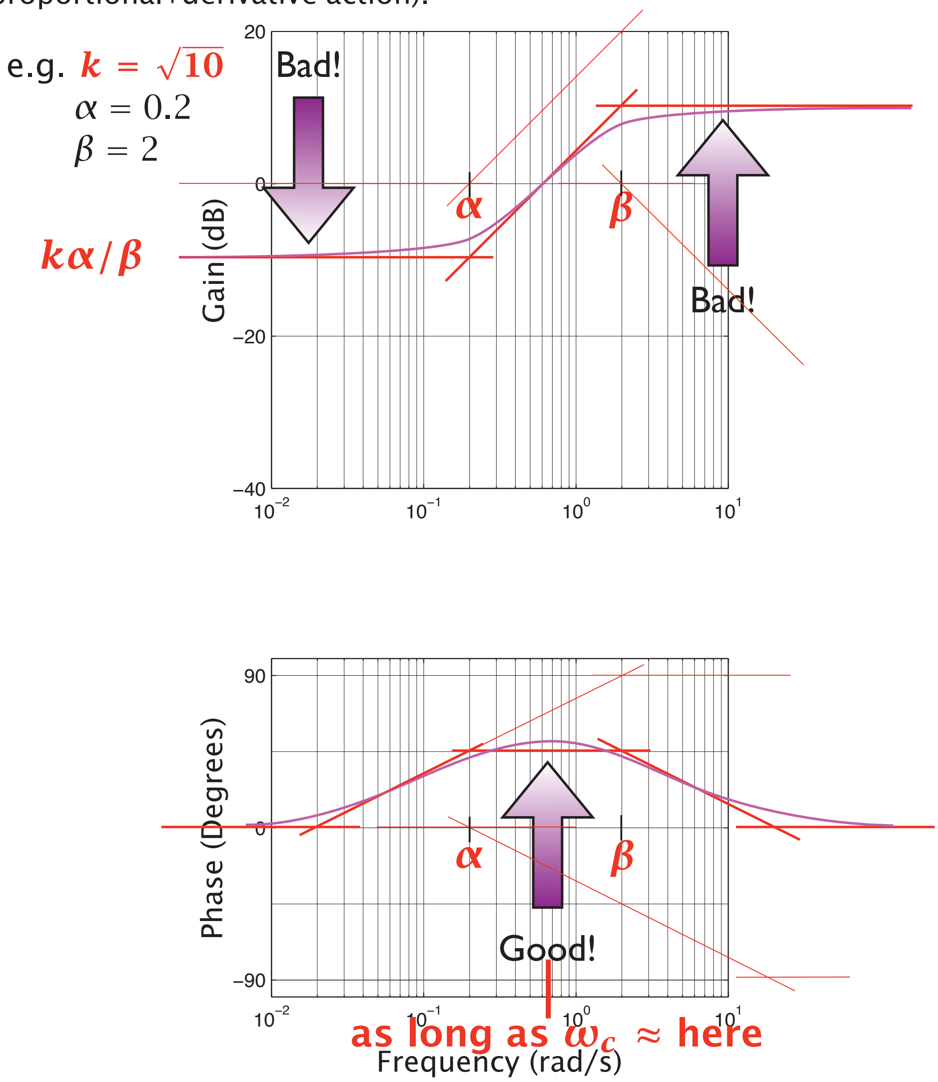
A *phase lag compensator* has a transfer function of the form specified on the left. A typical Bode plot of such a transfer function is also illustrated.

The key benefit provided by a phase lag compensator is the fact that it *increases the low frequency gain.* This is achieved, however, at the expense of introducing a *phase lag.*

If the compensator is designed such that the maximum phase lag occurs at a frequency well below ω_c, then this phase lag will not affect significantly the phase margin of the system.

- $\boxed{K(s) = k\dfrac{s+\alpha}{s+\beta}}$ for $\alpha < \beta$ (and $\alpha < \omega_c < \beta$ typically)

– Phase lead Compensator (a generalized form of proportional+derivative action).



– can improve gain and phase margins (improving robustness and damping) at the expense of decreasing the low frequency gain (increasing steady-state errors) and increasing the high frequency gain (increasing sensitivity to noise).

NOTE: WE CAN USE A COMBINATION OF PHASE LEAD AND PHASE LAG.

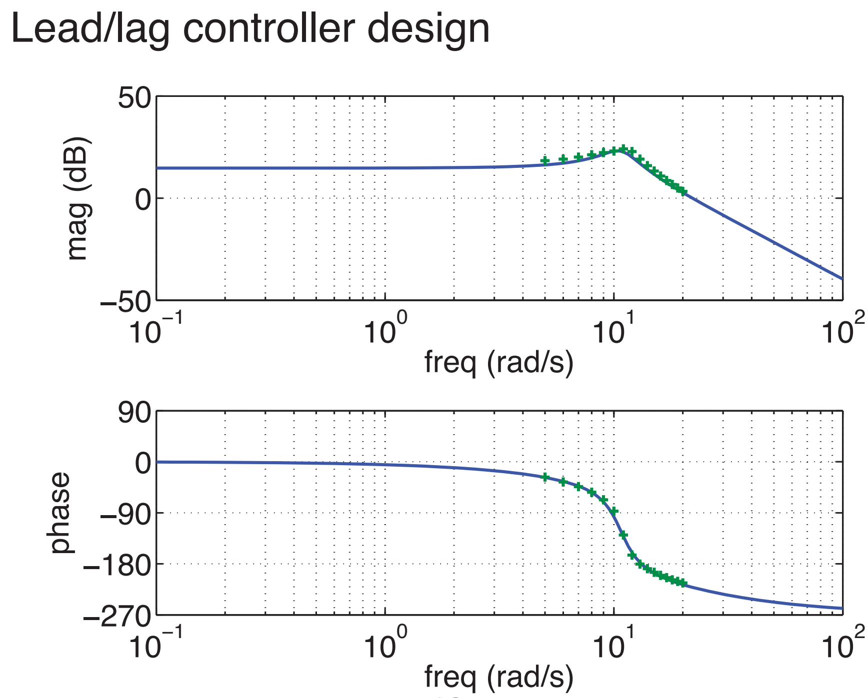
A *phase lead compensator* has a transfer function of the form shown on the left. A typical Bode plot of such a transfer function is also illustrated.

A phase lead compensator results in an increase in the phase in a certain frequency range (unlike a phase lag compensator that introduces a phase lag).

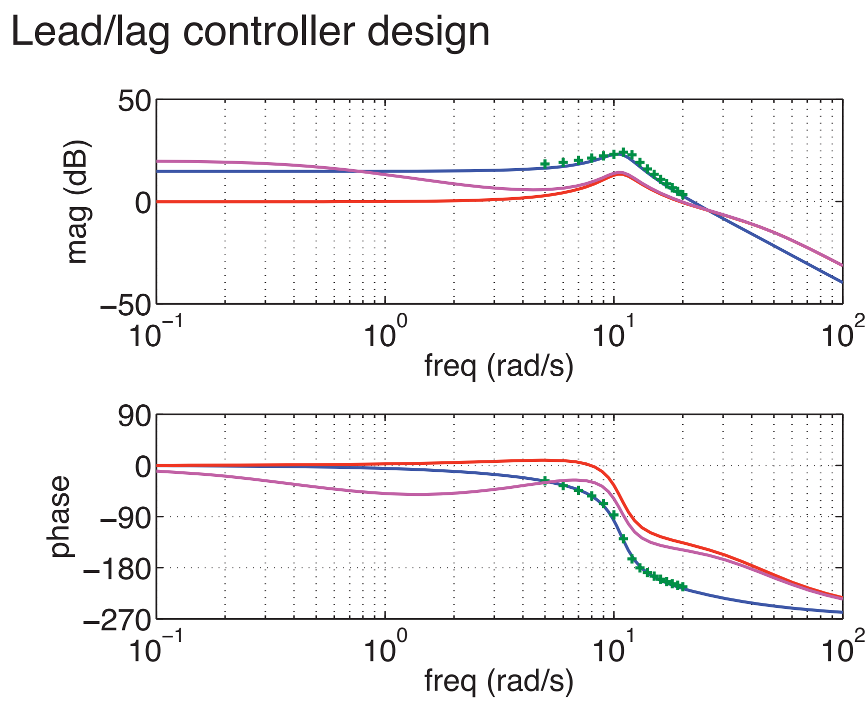
If this increase in phase occurs at a frequency close to ω_c then this will improve the gain margin of the system, as follows from the Nyquist stability criterion.

On the other hand, a phase lead compensator decreases the low frequency gain and increases the high frequency gain, which are undesirable properties.

Phase lead and phase lag compensators can be combined together (i.e. the transfer function of the controller is the product of the transfer functions of the two compensators). This can lead to some very powerful control designs, and one such example is illustrated in the next page.



$$G(s) = \frac{5.431}{\left(\frac{s^2}{10.745^2} + 2\times 0.1613\frac{s}{10.745} + 1\right)(s/16.54+1)}$$



The figure on the top illustrates the Bode plot of a plant $G(s)$ and the red/purple lines in the figure below illustrate the Bode plot of the return ratio $L(s) = K(s)G(s)$ in two cases. The red line shows the Bode plot of $L(s)$ when $K(s)$ is a phase lead compensator. We observe in the plot that this improves the phase margin of the system at the expense of reducing the low frequency gain.

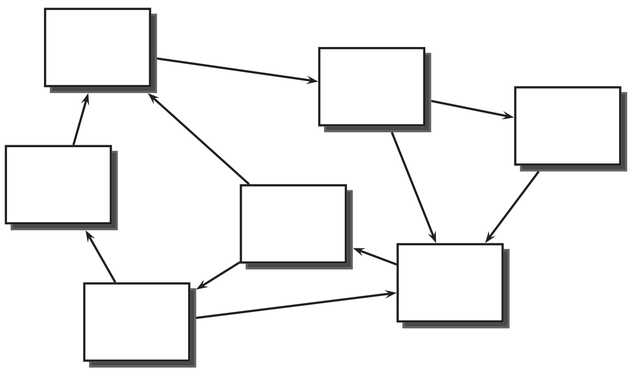
The purple line corresponds to the case where a phase lag compensator has been addiditonally included in $K(s)$. This increases the low frequency gain of $L(j\omega)$ but introduces a phase lag. Nevertheless, the phase lag compensator has been designed such that the maximum phase lag occurs well below ω_c, hence the phase margin is not much affected.

What the course was about:

The keys to understanding the course are the following *two* relationships:

- The relationship between the time and frequency (Laplace) domains:
  - Steady state responses (to both constant and sinusoidal inputs)
  - Pole locations.
- The relationship between open and closed loop properties (i.e. predicting properties of the feedback system from its return ratio).
  - Nyquist stability criterion (predicting stability of the feedback system).
  - Gain and phase margins (predicting the robustness of that stability and, indirectly, closed loop pole locations).
  - Understanding the map $L(j\omega) \mapsto L(j\omega)/(1 + L(j\omega))$ and $L(j\omega)/(1 + L(j\omega))$ (predicting the *performance* of the feedback system – which involves reading off $L(j\omega)$ and $1 + L(j\omega)$ from the Nyquist diagram).

What the course is *really* about



- Understanding complex systems as an interconnection of simpler subsystems.
- Relating the behaviour of the interconnected system to the behaviour of the subsystems.