Paper 1: Mechanics

Examples Paper 0

Revision

This examples paper should be completed over the Christmas vacation as a warm-up for Part IB Mechanics. Many of the questions come from past Part IA Tripos papers. You should be prepared to discuss this material at your first Mechanics supervision in the Lent Term.

§ indicates that the question is venturing onto slightly new ground, but nothing is difficult.

1 A uniform rectangular lamina of mass m is centred on x-y axes and has side length a along the x axis and b along the y axis.

(a) Show from first principles that the radius of gyration of the lamina about the x axis is $b/\sqrt{12}$. [you might need to remind yourself what ”radius of gyration” means - look at Section 5 of the Mechanics Data Book]

(b) The rectangular lamina now rotates about an axis that is normal to the plane and passing through one corner of the lamina. Find the moment of inertia of the lamina about this axis.

2 The uniform rod AB shown in Figure 1 is of length l and mass m. A particle of mass m is attached to the rod at A and a second particle of mass $2m$ is attached at point B. The rod rotates about fixed point C which is located at distance x from B.

(a) Find the mass moment of inertia of the assembly about point C.

(b) If x can be varied find the minimum possible moment of inertia of the assembly and the value of x at which this occurs.

(c) For what value of x will C be at the centre of mass of the assembly? Does your answer fit with your understanding of the parallel-axis theorem?

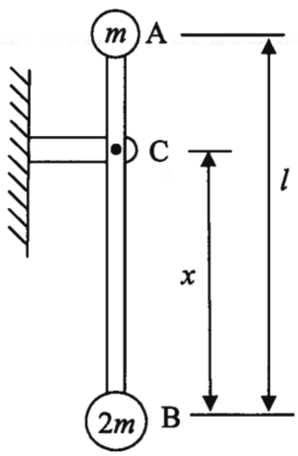

Figure 1

3 A simple model of a cyclist's leg is shown in Figure 2. The upper leg is AB, the lower leg is BC and the bicycle's crank is CD, with lengths as shown. At the instant shown the angle between AB and BC is $90°$ and the angle between CD and an imaginary line joining A and C is also $90°$. CD rotates with angular velocity ω about D in the direction shown.

(a) Using the method of instantaneous centres (you know the directions of the velocities of B and C) find the angular velocity of BC, and hence of AB. Sketch a velocity diagram.

(b) § A torque T at D resists the motion of crank CD - this is the torque propelling the bicycle. By considering a balance of power on the mechanism, find the torque that the rider's leg must be exerting on AB at A.

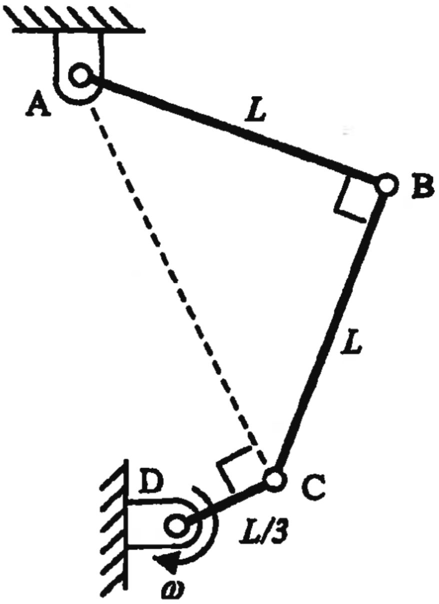

Figure 2

4 A uniform rod pendulum is shown in Figure 3. It has length L and mass m and it is free to pivot about A. It is swinging in a vertical plane and its motion is described by the angle θ measured from the downward vertical. The rod is released from rest when $\theta = \pi/2$.

(a) What is the initial angular acceleration of the rod?

(b) Find the angular acceleration and angular velocity of the rod as functions of θ.

(c) Find the maximum vertical force at A and the angle θ at which it occurs.

(d) Find the maximum horizontal force at A and the angle θ at which it occurs.

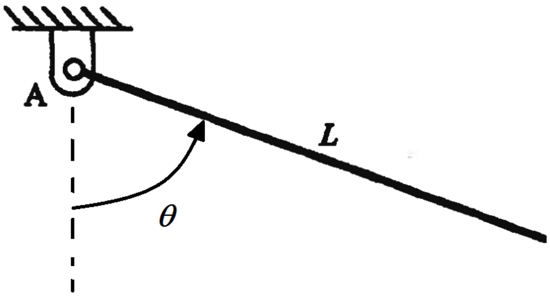

Figure 3

5 A system of frictionless pulleys, cables and masses is shown in Figure 4 with masses m, $2m$ and M as indicated. The two masses A and B are resting on the ground and the combined mass of pulleys and cables is small compared with m and M. The mass M is released from rest.

(a) What is the condition on M for the system to remain motionless when M is released?

(b) § Show that M would have to be more than $8m$ in order to lift both the resting masses off the ground when released.

(c) § If $M = 4m$ find the vertical acceleration of mass A.

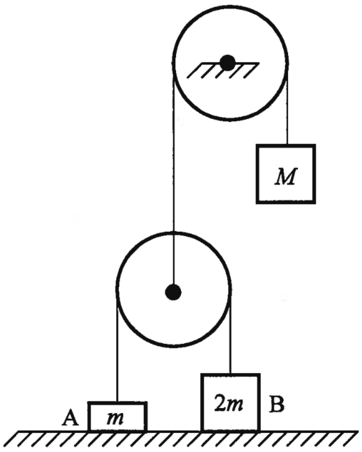

Figure 4

6 A hollow cone with cone angle of $90°$ is held with its axis vertical as shown in Figure 5. A small particle of mass m slides freely around the frictionless inner surface of the cone and its height is observed to vary between height h and $2h$ above the apex.

(a) What are the maximum and minimum speeds of the particle?

(b) The orbiting particle collides with a second stationary particle when it is at the lowest point of its orbit. The two particles fuse together and the subsequent motion is a circular orbit at constant height h. What is the mass of the second particle?

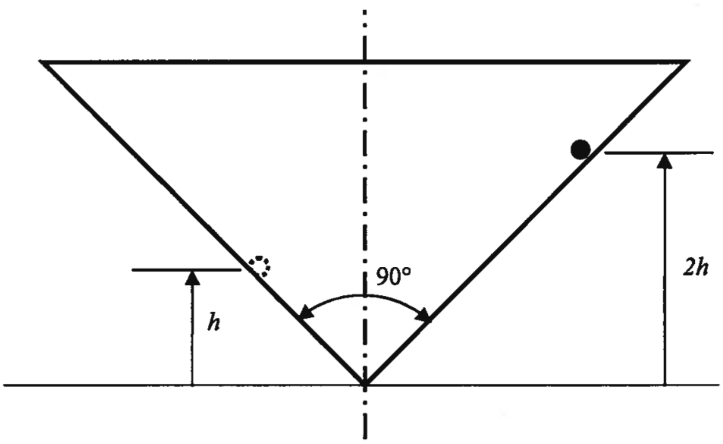

Figure 5

Suitable past Tripos questions

Revision: As this is a revison examples paper all past Part IA Mechanics Tripos papers can be considered listed here.

Answers

1(b). $m(a^2 + b^2)/3$

2(a). $m(4x^2 - 3lx + 4l^2/3)$

2(b). $\frac{37}{48}ml^2, 3l/8$

2(c). $3l/8$. Yes. Moment of inertia can only be increased by moving away from G.

3(a). $\frac{\omega}{3\sqrt{2}}, \frac{\omega}{3\sqrt{2}}$

3(b). $3\sqrt{2}T$

4(a). $\frac{-3g}{2L}$

4(b). $\ddot{\theta} = \frac{-3g}{2L}\sin\theta, \dot{\theta}^2 = 3g/L\cos\theta$

4(c). $5mg/2, \theta = 0$

4(d). $9mg/8$,$\theta = \pi/4$

5(a). $M \leq 2m$

5(c). $g/2$

6(a). $\sqrt{8gh/3}, \sqrt{2gh/3}$

6(b). $(\sqrt{8/3} - 1)m$