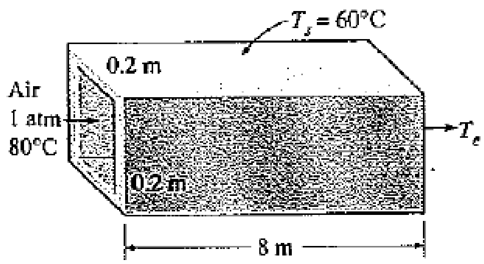
Review Problems

1 Hot air at atmospheric pressure and 80°C enters an 8-m long uninsulated square duct of cross section 0.2 m× 0.2 m that passes through the attic of a house at a rate of 0.15 m^3/s (see the figure). The duct is observed to be nearly isothermal at 60°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the attic space.



Solution: Heat loss from uninsulated square ducts of a heating system in the attic is considered. The exit temperature and the rate of heat loss are to be determined.

Assumptions

1. Steady operating conditions
2. The inner surfaces of the duct are smooth
3. Air is an ideal gas.

Properties: We do not know the exit temperature of the air in the duct, and thus cannot determine the bulk mean temperature of air, which is the temperature at which the properties are to be determined. The temperature of air at the inlet is 80°C and we expect this temperature to drop somewhat as a result of heat loss through the duct whose surface is at 60°C. At 80°C and 1 atm we read the following values from the table

$\rho = 0.9994$ kg/m^3	$c_p = 1008\ J/kg°C$
k= 0.02953 W/m.°C	Pr=0.7154
$v = 2.097 \times 10^{-5}\ m^2/s$	

Analysis: The characteristic length (which is the hydraulic diameter), the mean velocity, and the Reynolds number in this case are

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 0.2m$$

$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{0.15\ m^3/s}{(0.2\ m)^2} = 3.75\ m/s$$

$$Re = \frac{V_{avg}D_h}{v} = \frac{(3.75\ m/s)(0.2\ m)}{2.097\times10^{-5}\ m^2/s} = 35{,}765$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$L_h \approx L_t = 10D = 10 \times 0.2m = 2m$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023Re^{0.8}Pr^{0.3} = 0.023(35{,}765)^{0.8}(0.7154)^{0.3} = 91.4$$

Then,

$$h = \frac{k}{D_h}Nu = \frac{0.02953\,\frac{W}{m}\,°C}{0.2}(91.4) = 13.5\ W/m^2°C$$

$$A_s = 4aL = 4 \times (0.2m)(8m) = 6.4m^2$$

$$\dot{m} = \rho\dot{V} = \left(0.9994\,\frac{kg}{m^3}\right)\left(\frac{0.15m^3}{s}\right) = 0.150\ kg/s$$

Next, we determine the exit temperature of air from

$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}c_p)$

$= 60°C - [(60-80)°C]\exp[-\frac{(13.5\ W/m^2°C)(6.4\ m^2)}{(0.150\ kg/s)(1008\ J/kg°C)}]$

$= 71.3°C$

Then the logarithmic mean temperature difference and the rate of heat loss from the air become

$$\Delta T_{lm} = \frac{T_i - T_e}{ln\frac{T_s - T_e}{T_s - T_i}} = \frac{80 - 71.3}{ln\frac{60 - 71.3}{60 - 80}} = -15.2°C$$

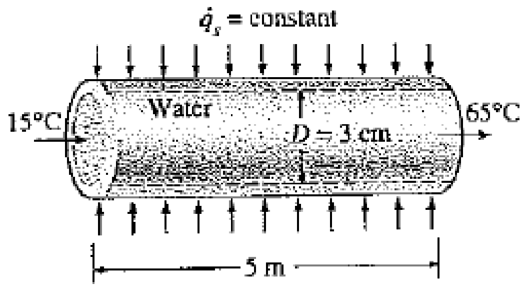
Note that this is consistent with the earlier definition of log mean temperature difference, $\Delta T_{lm} \equiv \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$, why?

$$\dot{Q} = hA_s\Delta T_{lm} = \left(13.5\,\frac{W}{m^2}\,°C\right)(6.4\ m^2)(-15.2°C) = -1313\ W$$

Therefore, air will lose heat at a rate of 1313 W, as it flows through the duct in the attic.

Discussion: The average fluid temperature is (80+71.3)/2=75.7°C, which is sufficiently close to 80°C at which we evaluated the properties of air. Therefore, it is not necessary to re-evaluate the properties at this temperature and to repeat the calculations.

2- Water is to be heated from 15^{0}C to 65^{0}C as it flows through a 3-cm-internal-diamater 5-m-long tube (see the figure below). The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10 L/min, determine the power rating of the resistance heater. Further, estimate the inner surface temperature of the tube at the exit.



Solution: water is to be heated in a tube equipped with an electrical resistance heater on its surface. The power rating of the heater and the inner surface temperature at the exit are to be determined.

Assumptions

1 Steady flow conditions exist
2 The surface heat flux is uniform
3 The inner surfaces of the tube are smooth

Properties: The properties of water at the bulk mean temperature of are $T_b = \frac{T_i+T_e}{2} = \frac{15+65}{2} = 40°C$

$\rho = 992.1\ kg/m^3$	$c_p = 4179\ J/kg°C$
k=0.631 W/m.°C	Pr=4.3
$v = \frac{\mu}{\rho} = 0.658 \times 10^{-6}\ m^2/s$	

Analysis The cross sectional and heat transfer surface areas are

$$A_c = \frac{1}{4}\pi D^2 = \frac{1}{4}\pi(0.03m)^2 = 7.069 \times 10^{-4} m^2$$

$$A_s = \pi DL = \pi(0.03m)(5m) = 0.471\ m^2$$

The volumetric flow rate of water is given as $\dot{V} = 10\frac{L}{min} = 0.01\ \mathrm{m}^3/\mathrm{min}$. Then the mass flow rate becomes

$$\dot{m} = \rho\dot{V} = \left(992.1\frac{kg}{m^3}\right)\left(0.01\frac{m^3}{min}\right) = 9.921\frac{kg}{min} = 0.1654\ kg/s$$

To heat the water at this mass flow rate from 15°C to 65°C, heat must be supplied to the water at a rate of

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = \left(0.1654\frac{kg}{s}\right)(65-15)^{\circ}C = 34.6\frac{kJ}{s} = 34.6\,kW$$

All of this energy must come from the resistance heater. Therefore, the power rating of the heater must be 34.6 kW.

The surface temperature T_s of the tube at any location can be determined from

$$\dot{q} = h(T_s - T_m) \rightarrow T_s = T_m + \frac{\dot{q}}{h}$$

where h is the heat transfer coefficient and T_m is the mean temperature of the fluid at that location. The surface heat flux is constant in this cases, and its value can be determined from

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{34.6\,kW}{0.471m^2} = 73.46\,kW/m^2$$

To determine the heat transfer coefficient, we first need to find the mean velocity of water and the Reynolds number:

$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{0.010\,m^3/min}{7.069\times10^{-4}m^2} = 14.15\frac{m}{min} = 0.236\,m/s$$

$$Re = \frac{V_{avg}D}{v} = \frac{(0.236\,m/s)(0.03\,m)}{0.658\times10^{-6}\,m^2/s} = 10{,}760$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry length is roughly

$$L_h \approx L_t \approx 10D = 10\times0.03 = 0.3\,m$$

which is much shorter than the total length of the tube. Therefore, we can assume fully developed turbulent flow in the entire tube and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 0.023Re^{0.8}Pr^{0.4} = 0.023(10760)^{0.8}(4.32)^{0.4} = 69.4$$

Then,

$$h = \frac{k}{D}Nu = \frac{0.631\frac{W}{m}{}^{\circ}C}{0.03m}(69.4) = 1460\,W/m^2{}^{\circ}C$$

And the surface temperature of the pipe at the exit becomes

$$T_s = T_m + \frac{\dot{q}}{h} = 65^{\circ}C + \frac{73460\frac{W}{m^2}}{1460\frac{W}{m^2}{}^{\circ}C} = 115^{\circ}C$$

Discussion: Note that the inner surface temperature of the tube will be 50 °C higher than the mean water temperature at the tube exit. This temperature difference of 50 °C between the water and the surface will remain constant throughout the fully developed flow region.

3- Engine oil enters at 35°C into a tube of 10 mm diameter at the rate of 0.05 kg/s and is to be heated to 45°C. The tube wall is maintained at 100°C by condensing steam on the external surface of the tube. The conduction thermal resistance of the tube is negligible.

Answer the following questions

a) Is the flow of oil through this tube laminar or turbulent and why?
b) Determine the length of hydraulic entrance regions.
c) Assuming a thermally and hydrodynamically fully developed flow in a very long tube, what is the convection coefficient?
d) Determine the length of the tube required.
e) Schematically sketch the variation of the flow temperature along the pipe and the temperature of pipe surface. Briefly explain what happens to these two temperatures of if the pipe is very long.

Solution:

a) Bulk mean temperature=($T_{m,i}$+$T_{m,o}$)/2=$\frac{35+45}{2} = 40℃$

At this temperature the values of thermo-physical properties are

$$\rho = 867\,\frac{kg}{m^3}, C_p = 1964\,\frac{J}{kg.K}, \mu = 0.210\,\frac{Ns}{m^2}, k = 0.144\,\frac{W}{mK}, Pr = 2870$$

$$Re = \frac{\rho UD}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \times 0.05}{\pi \times 0.01 \times 0.210} = 30.32$$

Since this value is below the critical Reynolds number, the flow inside the tube remains laminar.

b) $x_{fd,h} = 0.05 \times D \times Re_D = 0.05 \times 0.01 \times 30.32$ =0.015m, length of the hydraulic entrance region.

c) Assuming fully developed flow in this constant wall temperature problem, Nu=3.66 and therefore $h = \frac{3.66 \times 0.144}{0.01} = 52.7\ W/m^2K$

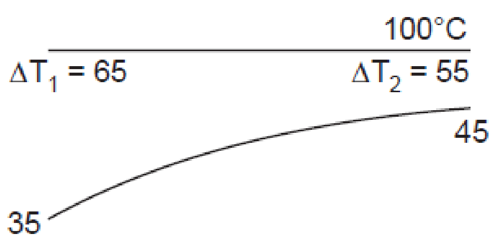
d) $q = mc_p\Delta T = 1966 \times 0.05 \times (45 - 35) = 983\ W$

LMTD=$\frac{65-55}{\ln(\frac{65}{55})} = 59.9℃$

$$\pi DLh(LMTD) = Q\ \rightarrow\ L = \frac{Q}{\pi Dh(LMTD)} = \frac{983}{\pi \times 0.01 \times 52.7 \times 59.9} = 9.92\ m$$

So, the tube length required is 9.92 m or almost 10m.

e) The temperature variation along the tube looks schematically as the figure below. If the tube is infinitely long the fluid temperature approaches that of the wall and eventually reaches thermal equilibrium with the tube.



4- A parallel flow heat exchanger cools oil from 60°C to 30°C using water available at 20°C. The outlet temperature of the water is 26°C. The rate of flow of oil is 10 kg/s. The specific heat of the oil is 2200 J/kg K and the overall heat transfer coefficient is U = 300 W/m2 K.

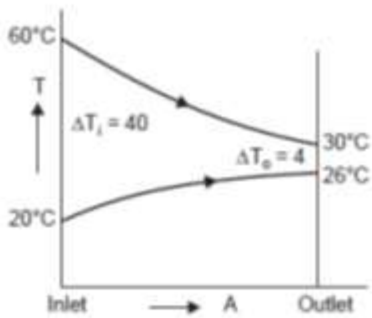
a) Schematically sketch the temperature variation in the heat exchanger.
b) Calculate the required surface area of the heat exchanger.

The heat exchanger is now reconfigured to operate under counter flow arrangement

c) For the new arrangement, schematically sketch the temperature variation in the heat exchanger.
d) Calculate the required surface area of the heat exchanger.
e) Compare your results in sections (b) and (d) and briefly discuss why they are different.

Solution:

a) Temperature variation in the parallel flow arrangement looks like this



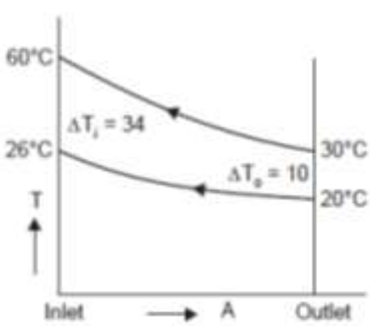
b) $Q = m_h c_h (T_{hi} - T_{ho}) = 10 \times 2200(60 - 30) = 660000\ W$

$Q = UA(LMTD)$

$$LMTD = \frac{40 - 4}{ln\frac{40}{4}} = 15.635℃$$

$6{,}60{,}000 = 300 \times A \times 15.635 \quad \therefore \quad \mathbf{A = 140.71\ m^2}$

c) The counter-flow arrangement will be like this:



d)

$$\text{LMTD} = \frac{34 - 10}{\ln \frac{34}{10}} = 19.611°\text{C}$$

$$A = 112.18 \text{ m}^2$$

This is about 20% less compared to parallel flow arrangement

e)

The counter flow arrangement provides more uniform temperature difference along the flow and hence a better rate of heat flow. The counter flow type can also be used to cool or heat over a wider range of temperatures. In the above case by increasing the area or by reducing flow the hot oil can in the limit be cooled to 20°C. Manipulation in the opposite direction can get the water heated to 60°C. This is not possible in the parallel flow where the