Lecture 4: Generation II

4.1 Magnetic field due to stator currents

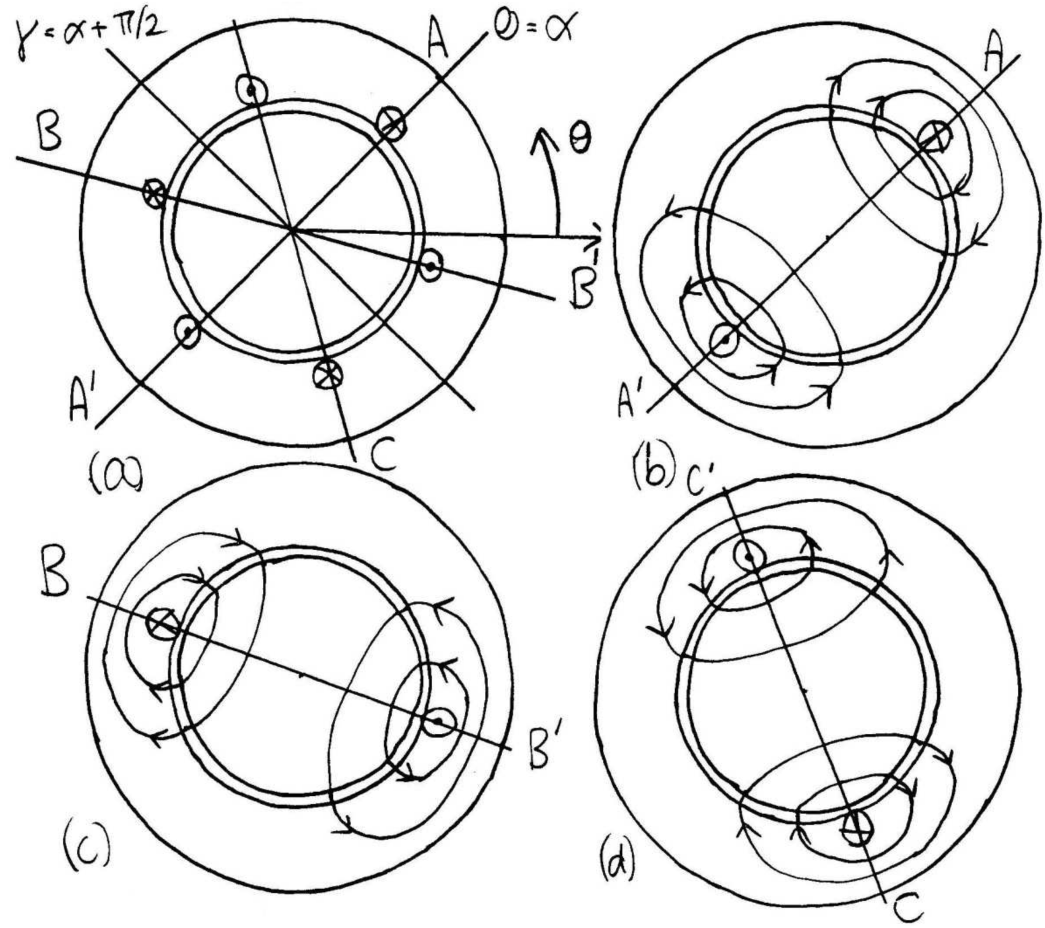


Fig. 4.1

Consider flux density due to current in phase A.

Flux crosses air-gap radially since iron is a very good conductor of magnetic flux.

Field pattern is called '2-pole' because flux density crosses air-gap once in the outward and once in the inward direction.

We examined the basic principles of operation of the a.c. generator in the last lecture, and found that the optimal method of connecting the stator coils was so as to provide a balanced three-phase voltage source. This is illustrated in Fig. 4.1(a) which shows the simplest possible three-phase generator with one coil per phase. When the generator is connected to a balanced three-phase load, currents will flow in each of the three generator phases. From lecture 1, it is clear that these currents will form a balanced three-phase set. Like the rotor coil, the stator coils will then produce a magnetic field of their own. Our first job, therefore, is to determine the form of the air-gap magnetic field due to the stator i.e. the stator-driven magnetic field.

Firstly, however, consider the magnetic field produced by a single stator coil, of N turns carrying a current of I amps. Fig. 4.1(b) illustrates the flux lines for this situation. It may be shown using Ampere's Law (IA Electromagnetics) that providing the iron that forms the rotor and stator is very highly permeable, then:
a) The flux density in the air-gap crosses radially.
b) The air-gap flux density is a square wave, centered on an axis which is orthogonal to that of the coil.

Because the flux crosses the air-gap once in the radially-outward direction and once in the radially-inward direction, the air-gap field distribution is said to have two poles. We will consider other possibilities, such as four or more poles later in the lecture, and extend the ideas here to those cases.

If instead only phase B has current flowing in it, the flux lines will look as shown in Fig. 4.1(c). Fig. 4.1(d) shows the situation when only phase C has current flowing through it.

If phases A, B and C are sequentially excited, field pattern rotates !!! This is the physical basis of the rotating magnetic field.

Ampere's Law shows that air-gap flux density is a square wave of amplitude $B = \dfrac{\mu_0 NI}{2g}$.

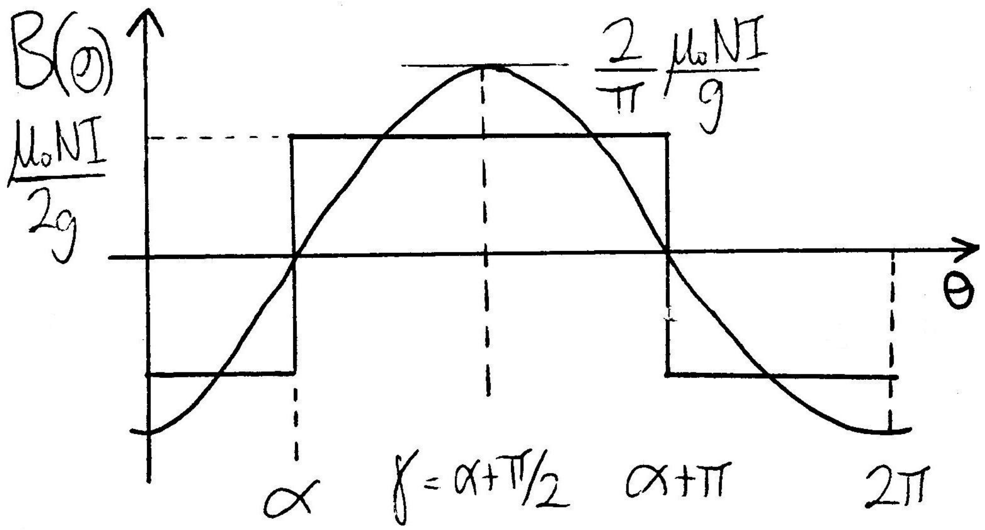


Fig. 4.2

Using Fourier analysis of a square wave and ignoring all harmonics except fundamental:

$$B(\theta) = \hat{B}\cos(\theta - \gamma) \text{ where } \hat{B} = \frac{2\mu_0 IN}{\pi g} \qquad (4.1)$$

For three coils spaced 120° apart (A B C) and supplied with balanced three-phase currents:

The important thing to note is that the whole pattern of flux lines rotates. This is the basic physical idea of the rotating magnetic field.

We now consider the case where instead of sequentially pulsing each phase, we have a balanced three-phase set of currents flowing in the phases. Ampere's Law (IA Electromagnetics) can be used to show that the air-gap flux density due to a current of I amps flowing in a single phase is a square wave of amplitude as given opposite. In this expression, N is the number of turns of wire which make up the phase, g is the air-gap length. This is illustrated in Fig. 4.2.

To facilitate the analysis, we have to make a seemingly rather severe approximation. So far in this course we have dealt exclusively with functions which are sinusoidal in space or time or both. Here we are faced with a distribution which is a square wave in space ! To get round this unfortunate situation, we resort to Fourier analysis to express the square wave as a summation of harmonics - see Mathematics Data Book for precise details. We then make the engineering approximation that all harmonics, other than the fundamental, may be ignored, giving equation 4.1 !! This, on the face of it, is pretty severe - after all, we are attempting to argue that a square wave is in fact a sine wave ! However, in real generators, the stator winding is designed to produce a flux density wave which is very close to a sine wave - the fact that we have a square wave here is because we are assuming only one coil per phase - in reality there are many. It is possible to perform a more rigorous analysis, including the effects of higher-order harmonics, but to do so is beyond the scope of this course.

Phase	γ	I(t)
A	0	$\hat{I}\cos\omega t$
B	120°	$\hat{I}\cos\left(\omega t - 120^\circ\right)$
C	-120°	$\hat{I}\cos\left(\omega t + 120^\circ\right)$

Superpose the fields that they produce:

$$B(\theta,t) = B(\theta,t)_A + B(\theta,t)_B + B(\theta,t)_C \qquad (4.2)$$

$$= \frac{2\mu_0 \hat{I} N}{\pi g}\left\{\begin{array}{l}\cos\theta\cos\omega t \\ +\cos(\theta - 120)\cos(\omega t - 120) \\ +\cos(\theta + 120)\cos(\omega t + 120)\end{array}\right\}$$

$$= \frac{\mu_0 \hat{I} N}{\pi g}\left\{\begin{array}{l}\cos(\omega t - \theta) + \cos(\omega t + \theta) \\ +\cos(\omega t - \theta) + \cos(\omega t + \theta - 240) \\ +\cos(\omega t - \theta) + \cos(\omega t + \theta + 240)\end{array}\right\}$$

$$= \frac{3\mu_0 \hat{I} N}{\pi g}\cos(\omega t - \theta) \qquad (4.3)$$

This is the equation for a rotating field !

<u>Important result:</u> Balanced three-phase currents flowing in a balanced three-phase winding produce a rotating magnetic field.

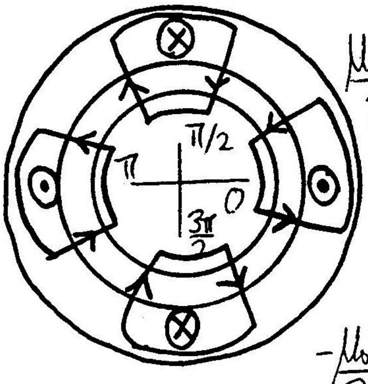
Having determined, and approximated the spatial distribution of the air-gap flux density due to one stator coil, we now consider the combined effect of all three stator coils. It is assumed that balanced three-phase currents flow in the stator coils, and that the coils themselves are 120^0 apart geometrically. The currents, and the angles of the coils are summed up in the table opposite.

To find the field produced, we apply the principle of superposition. This states (in the case of electromagnetic fields) that the field due to several sources (i.e. currents in coils) is the sum of the fields which would be produced individually by those sources, equation 4.2.

Substituting for I and γ from the table into equation 4.1, and summing the flux densities produced by each phase as shown in equation 4.2 gives the second line of equation 4.2. The terms in this line are then re-written using the trig. identity: cosAcosB=(cos(A-B)+cos(A+B))/2. This gets to the third line of equation 4.2. This shows three terms cos(ωt-θ) to be added, and also three terms cos(ωt+θ), cos(ωt+θ-240) and cos(ωt+θ+240). It may be shown that the latter sum to zero, giving the final result, equation 4.3. This is the equation for a rotating magnetic field ! This extremely important result is the basis for three-phase generators and also motors (the induction motor). Physically stated, it shows that balanced three-phase currents flowing in a balanced three-phase winding produce a rotating magnetic field.

4.2 Multi-pole windings

Two coils per phase gives a four pole field:



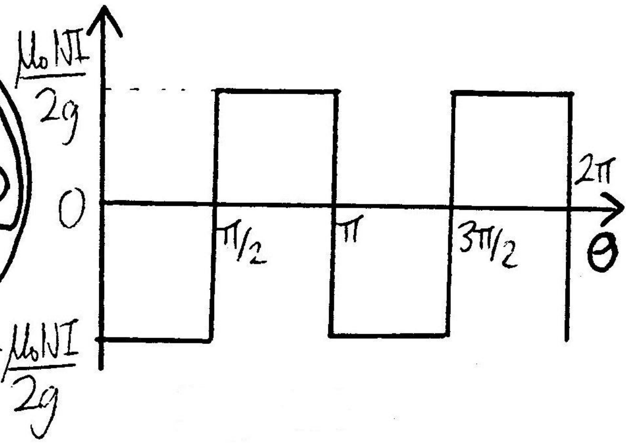


Fig. 4.3 Fig. 4.4

$\therefore$ p coils per phase gives a 2p pole field.

The rotating field is then given by:

$$B(\theta,t) = \hat{B}\cos(\omega t - p\theta) \tag{4.4}$$

Consider a point on the wave and imagine that it moves an angular distance $\Delta\theta$ in time Δt:

$$\hat{B}\cos(\omega t - p\theta) = \hat{B}\cos(\omega(t+\Delta t) - p(\theta+\Delta\theta)) \tag{4.5}$$

$$\therefore \quad \omega t - p\theta = \omega(t+\Delta t) - p(\theta+\Delta\theta)$$

$$0 = \omega\Delta t - p\Delta\theta \quad \Rightarrow \quad \Delta\theta/\Delta t = \omega/p \tag{4.6}$$

Speed of rotation of a 2p-pole field = ω/p.

The machine considered in the previous section had one coil per phase, and the coils were placed 120^0 apart from each other. This produced a 2-pole field pattern. If instead there were two coils per phase, with each coil subtending an angle of 90^0 at the centre of the machine as shown in fig. 4.3, a 4-pole field would result, fig. 4.4. To generalise, each phase could have p coils, each subtending an angle of $180/p^0$ at the centre of the machine, resulting in an air-gap field with 2p poles.

If we went back to the analysis of the rotating magnetic field, and replaced the spatial distributions of the form cosθ, cos(θ+120) and cos(θ-120) with cospθ, cos(pθ+120) and cos(pθ-120), we would find that the form of the air-gap field distribution was that given by equation 4.4 opposite i.e. we still have a rotating magnetic field, except now if has 2p poles. It is important to remember that in the analysis, p means the number of pole-pairs, and not the number of poles e.g. an 8-pole generator has 8 poles, and so p=4.

Apart from making the spatial variation of the field appear different to that of the 2-pole field, the number of poles affects the speed of rotation of the field. To deduce this, imagine choosing any point on the flux density wave, say $B(\theta,t)=B_1$, and determine how far it has moved, Δθ, in time Δt. For the point to remain of value B_1, the arguments inside the cosine terms of equation 4.5 must remain the same. A small amount of algebra reveals that the speed of the field is ω/p, equation 4.6. This speed is known as the synchronous speed. Therefore, for the same supply frequency, a four-pole field rotates at half the speed of a 2-pole speed. We will see how this impacts on practical generators in the next lecture.

4.3 Conditions for steady torque production

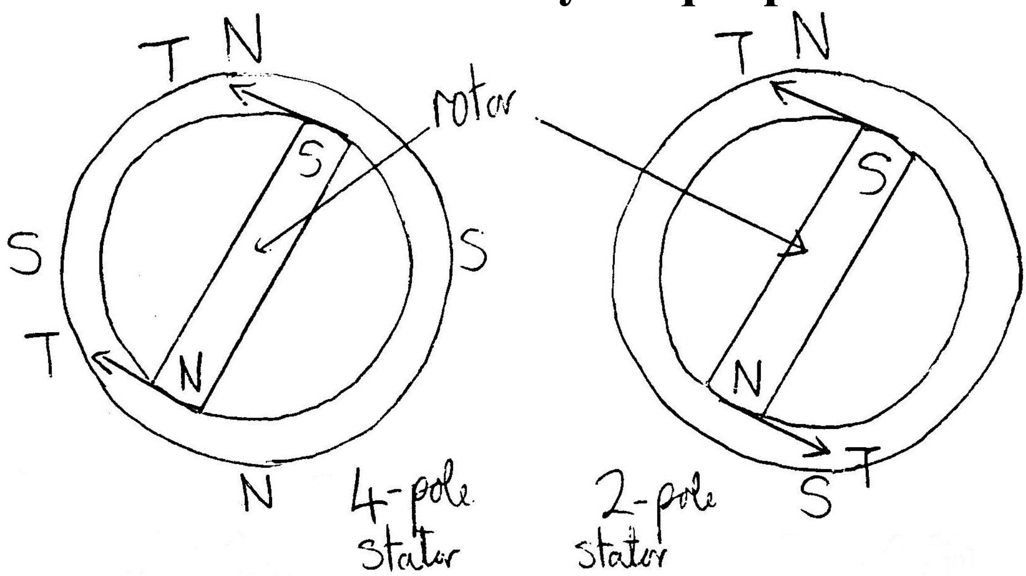


Fig. 4.5a Fig. 4.5b

Rotor and stator pole numbers different $\Rightarrow$ no nett torque acting on the rotor (fig. 4.5a).

Rotor and stator pole numbers same $\Rightarrow$ nett torque acting on the rotor (fig. 4.5b).

Can generalise these arguments to machine with p_r rotor pole-pairs and p_s stator pole-pairs.

***Condition 1*: For torque to be produced, the rotor and stator-driven fields must have the same number of poles i.e. $p_r = p_s$**

Under steady-state conditions, a generator rotates at some fixed speed, ω_r. Therefore, the nett torque acting on the rotor must be zero. The prime-mover must be applying torque to the rotor in the direction of rotation for there to be a conversion from mechanical to electrical power (the power supplied by the prime-mover is $T\omega_r$). Therefore, there must be a steady electromagnetic torque, produced by the interaction of the rotor and stator fields, which exactly opposes the torque from the prime-mover. Consequently we need to study the conditions required for this constant electromagnetic torque to be produced.

It is possible to derive the conditions required in a purely mathematical manner, using the principle of **virtual work**, which you met last year in the 1A Electromagnetics course. However, the derivation is long and complex, and so here we will consider only physical arguments.

Firstly, consider fig. 4.5a, which shows a two-pole rotor and a four pole stator. The north pole of the rotor will be repelled from the north pole of the stator, giving rise to a torque in the anticlockwise direction. The south pole of the rotor will be attracted to the north pole of the stator, giving rise to an equal torque, but which acts in the clockwise direction. Hence, there will be no nett torque. This argument is true regardless of the precise rotor position.

Now consider fig. 4.5b, which shows a two pole rotor and a two pole stator. Now there is a nett torque acting on the rotor because the torques acting on both north and south pole both act in the anticlockwise direction, and reinforce each other.

Consider stator producing a rotating magnetic field, speed of rotation ω/p_s (section 4.2).

Assume rotor rotating at some speed which is **not** ω/p_s.

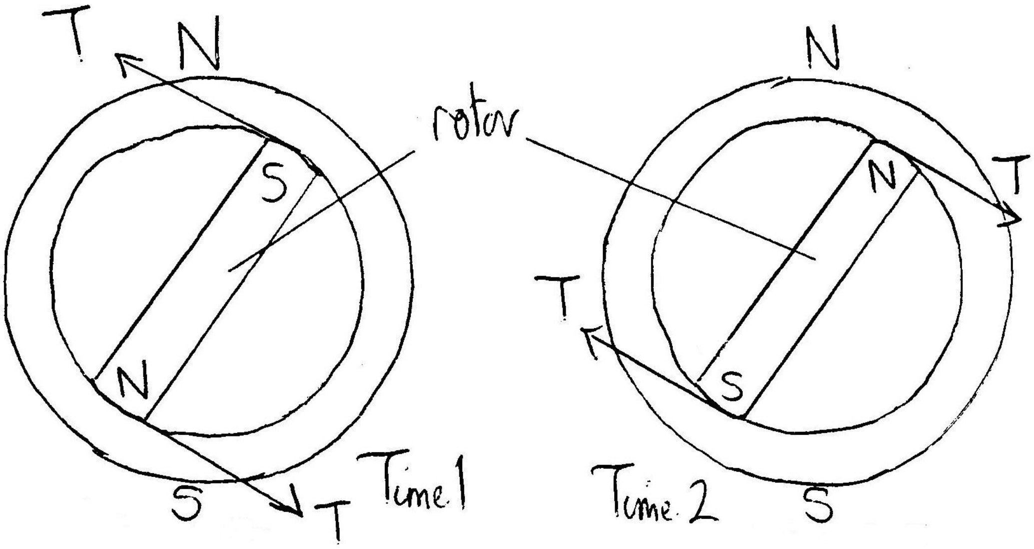


Fig. 4.6a Fig. 4.6b

At time 1, torque on rotor is anticlockwise.

At time 2, it is clockwise.

∴ Rotor torque varies with time, and has zero average value.

If rotor speed is equal to ω/p_s torque is constant.

***Condition 2*: For steady torque rotor speed must be equal to speed of the stator-driven field i.e. $\omega_r = \omega/p_s$.**

This speed is termed **synchronous speed**.

These arguments can be generalised to any number of rotor and stator poles, and the condition for any nett torque to exist is that the number of stator poles and rotor poles must be identical.

Assuming this condition is met, now imagine that the stator is producing a rotating magnetic field (henceforth referred to as the stator-driven field). As shown in section 4.2, the speed of rotation will be ω/p_s. Suppose the rotor rotates at some constant speed which is not equal to the speed of the stator-driven field. Fig. 4.6a shows one possible instant in time, at which the torque acting on the rotor will be in the anticlockwise direction, whereas fig. 4.6b shows another instant in time at which the torque will be the same as it is in fig. 4.6a, but in the clockwise direction. Therefore, the torque acting on the rotor merely pulsates, and has zero average value.

However, if the rotor was to rotate at exactly the same speed as the stator-driven field, then the torque acting on it would be the same at all points in time, giving rise to a non-zero average torque.

What this has shown is that a synchronous machine can only produce a constant electromagnetic torque at one particular speed, which is fixed by the supply frequency of the stator winding and the stator pole number. This speed is known as the **synchronous speed.**

Torque at synchronous speed is:

$$T = K\hat{B}_s\hat{B}_r \sin\alpha \tag{4.7}$$

***Summary*: For steady torque the rotor must rotate in synchronism with the stator field, and rotor and stator must possess the same number of poles.**

4.4 Development of the equivalent circuit

Rotor-driven, stator-driven and total air-gap fields can be shown on a **space** phasor diagram.

Time phase of induced emf equals space phase of inducing magnetic field, and magnitude of induced emf $\propto$ magnitude of inducing field.

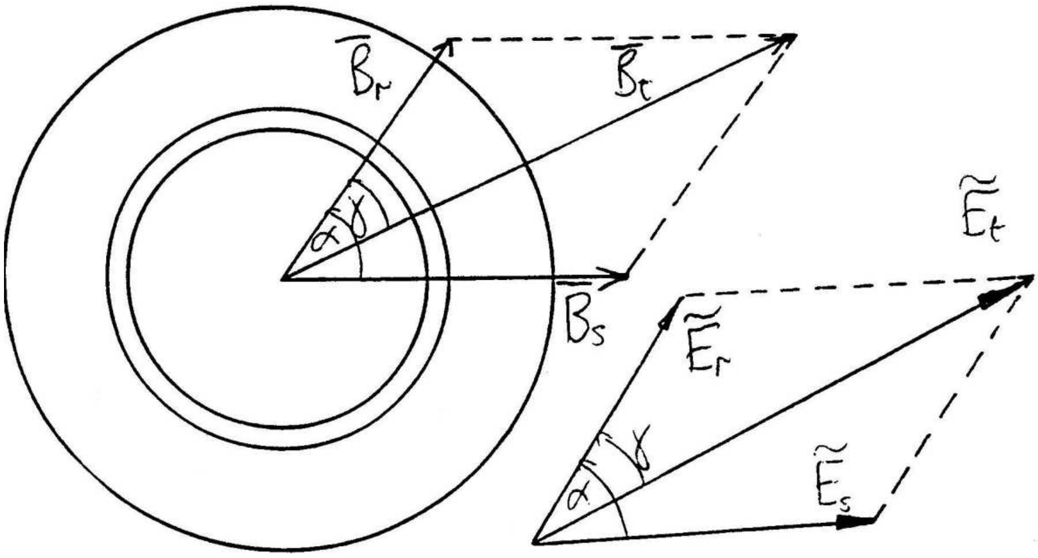


Fig. 4.7 Fig. 4.8

The torque produced when conditions 1 and 2 are met can be shown to be as given by equation 4.7. Putting this physically, steady torque is only produced if the rotor speed is equal to the speed of rotation of the rotating magnetic field produced by the stator (synchronous speed). Only under these conditions can a steady transfer of energy from mechanical to electrical occur, and for this reason this type of machine is referred to as a **synchronous machine**.

In order to predict the behaviour of the synchronous machine, we need to develop an equivalent circuit for it.

Firstly, the rotor-driven and stator-driven magnetic fields can be represented on a space phasor diagram, fig. 4.7. The space phasors point in the direction of the positive peak of the sinusoidal space distribution of those fields. The total air-gap field is the sum of the rotor and stator-driven fields, and is obtained by addition of the space phasors, giving B_t in fig. 4.7. In lecture 3 we saw that the emf induced in a stator coil by a rotating field is proportional to the magnitude of that field. This idea is also true for a complete stator phase winding. $\therefore$ If B_r induces emf $E_r = KB_r$ in one phase of the stator then B_s induces emf $E_s = KB_s$ in that same phase. Equally the resultant emf, E_t will be proportional to the sum of the two air-gap fields (B_t) i.e. $E_t = KB_t$. Furthermore, it was also seen that the time phase of an induced emf equals the space phase of the inducing magnetic field. Therefore, the time phasor diagram for the emfs due to B_r (E_r), B_s (E_s) and B_t (E_t) looks the same as the space phasor diagram as shown in fig. 4.8.

Ideal stator winding $\Rightarrow E_t = V$.

$E_s \propto B_s \propto I = jX_sI$, since stator consists of coils so it must be inductive.

X_s is called the synchronous reactance.

E_r is the emf induced by rotor-driven field and is known as the excitation voltage, E. **It is assumed to be proportional to the rotor field current (or rotor excitation).**

$$\therefore \quad \overline{E}_t = \overline{E}_r + \overline{E}_s \rightarrow \overline{V} = \overline{E} + j\,\overline{I}\,X_s \qquad (4.8)$$

Equation 4.8 represented by equivalent circuit:

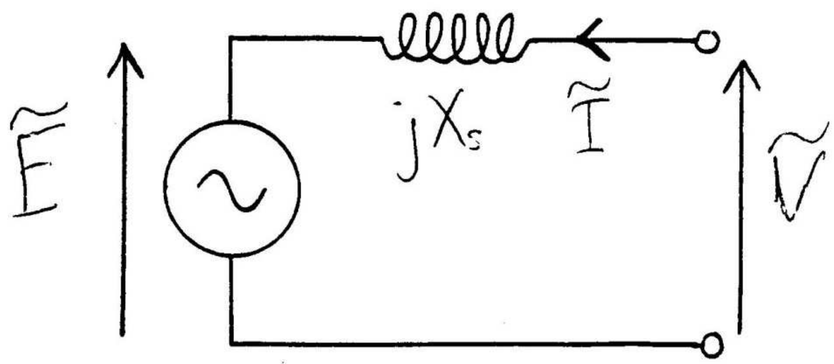


Fig. 4.9

Assuming an ideal stator winding i.e. a winding with zero resistance, the total induced emf, E_t , is the voltage that is measured at the machine terminals. This must be equal to the supply voltage (since the stator windings are connected to the supply). i.e.E_t=V.

E_S is the back emf induced in a stator phase due to the field set up by its own currents. We have not taken any account of the resistance of the stator windings, which must therefore look like **inductance** and so $E_s=jX_sI$. X_S is termed the **synchronous reactance** of the machine.

E_r is the emf induced in the same stator phase by the rotor-driven field. It is generally referred to as the generated emf or the excitation and given the symbol E. The magnitude of E depends on the (d.c.) field current which flows in the rotor winding, and it is usual to assume that E $\propto$field current.

Therefore, the phasor equation suggested by the time phasor diagram of fig. 4.8 becomes that given by equation 4.8. In turn, equation 4.8 may be represented by the equivalent circuit of fig. 4.9. It is important to appreciate that:
i) this equivalent circuit relates to one phase of the stator, in which all voltages and currents alternate at the same frequency.
ii) the other two phases will behave in the same way, except that all voltages and currents differ in phase from their counterparts by $\pm 120^0$.
iii) the only effect that the rotor has on this equivalent circuit is to alter the magnitude of the excitation voltage, E. It is always assumed in this course that the excitation voltage is directly proportional to the rotor field current.