4. Convective Heat Transfer (Forced Convection)

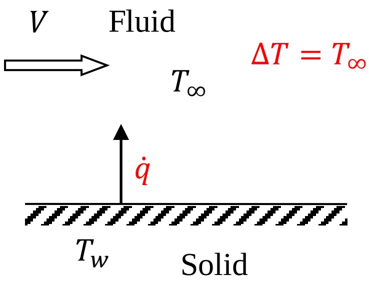
4.1. Introduction

In 1987, Cray Research Inc. released the Cray-2, then the world's fastest computer at 250MHz with 2Gb of RAM. It was contained in a cylindrical cabinet 1.15m high and 1.35m in diameter.

"We solved the problem of dissipating the 130kW it produced by immersing all the circuitry in a bath of inert liquid fluorocarbon. Computer designers are glorified cooling engineers and I am a very good plumber. "

Seymour Cray

We have already used a convective boundary condition



$\Delta T = T_\infty - T_w$

The heat transfer coefficient, *h,* is defined by:

$$\dot{q} = -\,h\,\Delta T$$

h may vary with position, e.g. x, (see later). Often we will use the averaged heat transfer coefficient,

$$\frac{\dot{Q}}{A} = -\,h_{av}\Delta T_{mean}$$

- h and h_{av} tend to get used interchangeably, depending upon the problem.
- Often, we will want the average heat transfer coefficient, and the fluid mechanics will be too complicated to allow us to calculate the local value.

We need to be able to estimate the values of h. There are two distinct situations we need to consider;

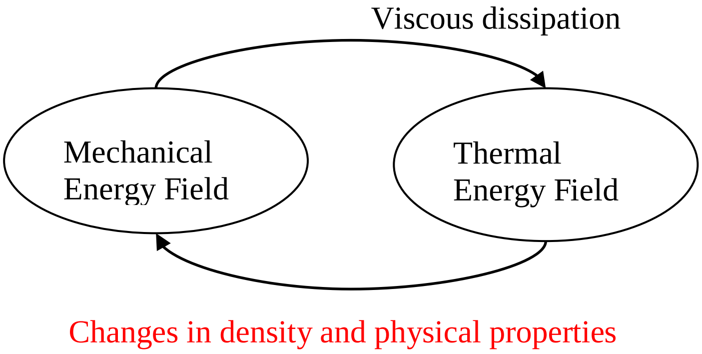
1. **Forced convection-** when the fluid is driven over the surface by e.g. a fan or pump.
2. **Free (natural) convection** – when density differences due to buoyancy drive the flow.

Forced convection is almost always the quickest way to transfer heat. In both situations, we must consider the flow of fluid over the surface.

In this lecture we will consider Forced Convection

4.2. Heat transfer and fluid mechanics

The thermal (temperature) field of a flow cannot affect the mechanical (pressure and velocity) fields if the density is assumed to be constant. We can calculate the mechanical fields first and then calculate their effect on the temperature field. The mechanical fields, however, always affect the thermal field, both by viscous dissipation and simply by convecting hot fluid through space (i.e. forced convection).



If the density cannot be assumed to be constant, the calculation requires feedback from the thermal to the mechanical field and is more complicated.

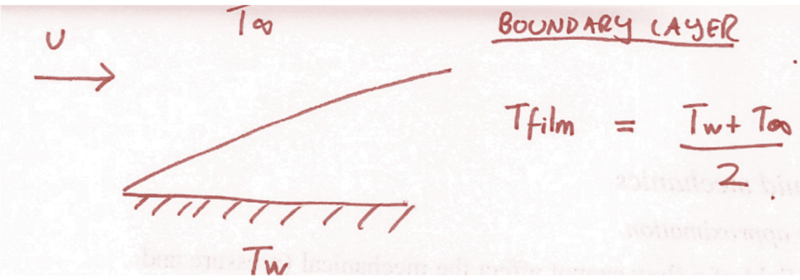
From the equation of state of ideal gases[1], $\rho = P/RT$, we see that $d\rho/\rho = -dT/T$. Incompressibility can be assumed when temperature fluctuations are less than approximately 0.1 of the background temperature. Incompressibility also requires that the Mach number is less than around 0.3.

$$M < 0.3 \implies c_p T \gg v^2/2$$

i.e the enthalpy of the fluid per unit mass, c_pT, is much greater than the kinetic energy per unit mass $v^2/2$. The convective energy transfer is then just $\dot{m}c_pT$, and the steady flow energy equation becomes

$$\dot{Q} - \cancel{\dot{W}_x} = m_{in}\left(c_pT + \cancel{\frac{1}{2}v^2} + \cancel{gz}\right)_{in} - m_{out}\left(c_pT + \cancel{\frac{1}{2}v^2} + \cancel{gz}\right)_{out}$$

The properties of the fluid, such as viscosity, thermal conductivity and specific heat capacity, depend on temperature. Although this does not provide a mechanism for energy to pass from the thermal to the mechanical field, it does allow the thermal field slightly to affect the mechanical field.

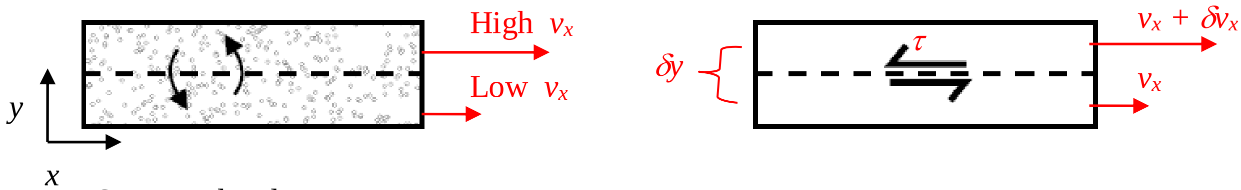


If temperature differences are small, the variations in these properties are small. Very often the properties are assumed to take the values at the mean value of the temperature, the film temperature.

[1] In other fluids, with other equations of state, we must be careful not to be near a region of sudden density change such as the boiling point.

4.3. *The analogy between heat and momentum transfer*

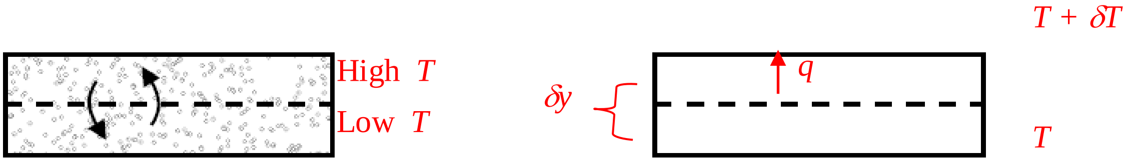
In earlier lectures you looked at the diffusion of **ordered** momentum between two fluid layers due to molecular motion. In the language of continuum models, this related the shear stress to the velocity gradient via viscosity.



Some molecules (randomly) swap places. ⇨ Momentum exchange. ⇨ Shear-stress

$$\tau = shear\ stress = \frac{shear\ force}{area} = \mu \frac{\partial v_x}{\partial y}$$

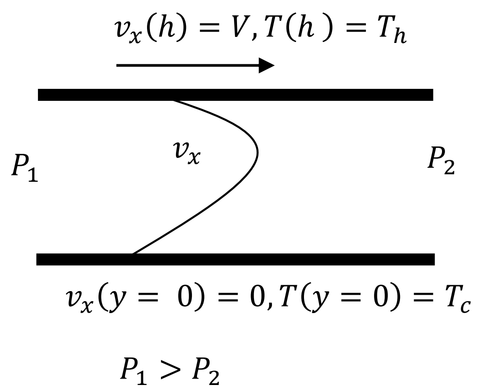
The kinetic energy of **disordered** motion diffuses in the same way. In the language of continuum models we measure this as a heat flux between the two layers if they are at different temperatures.



Some molecules (randomly) swap places. ⇨ Thermal energy exchange. ⇨ Heat flux

$$\dot{q} = heat\ flux = -\lambda \frac{\partial T}{\partial y}$$

Consider flow down a duct



Momentum conservation:

$$\rho \frac{Dv_x}{Dt} = \nabla.\left(\underline{\tau_x}\right) - \frac{\partial P}{\partial x}$$

Thermal energy conservation:

$$\rho C_p \frac{DT}{Dt} = -\nabla.\left(\underline{q}\right) + S_h$$

We are not interested in solving these equations here, just the form of the equations.

$$\frac{Dv_x}{Dt} = \frac{\mu}{\rho}\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) - \frac{1}{\rho}\frac{dp}{dx}$$

$$\frac{DT}{Dt} = \frac{\lambda}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{1}{\rho C_p} S_h$$

$$\nu = \frac{\mu}{\rho} \qquad \alpha = \frac{\lambda}{\rho C_p}$$

The quantities **ν** and **α** both have units m²s⁻¹ . **ν is the momentum diffusivity** (also known as the kinematic viscosity). **α is the thermal diffusivity**. The ratio ν/α is a property of the fluid called the Prandtl number (Pr).

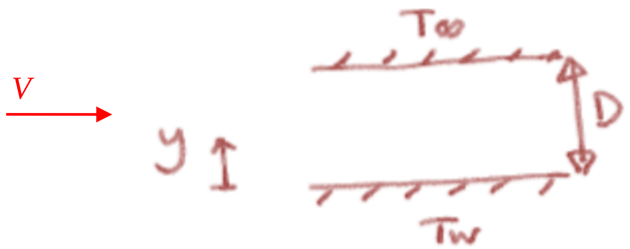
	Steam 100 °C	Air	Water
Pr $= \nu/\alpha$	0.99	0.71	9.29

4.4. Non-dimensional heat transfer coefficients

In many situations, the fluid mechanics will be too complicated to allow the heat transfer coefficient to be computed directly. We will have to resort to dimensional analysis and correlations.

Nusselt Number (commonly used)

The Nusselt Number is found by normalising the heat flux, but a conductive heat flux.



$$Nu = \frac{Heat\ flux}{Heat\ flux\ with\ no\ flow} = \frac{\dot{q}}{\dot{q}_{noflow}} = \frac{h\Delta T}{\lambda \frac{\Delta T}{D}}$$

$$Nu = \frac{hD}{\lambda}$$

[For other geometries, D will be some characteristic length scale, e.g. for spheres, the diameter is used]

Stanton Number

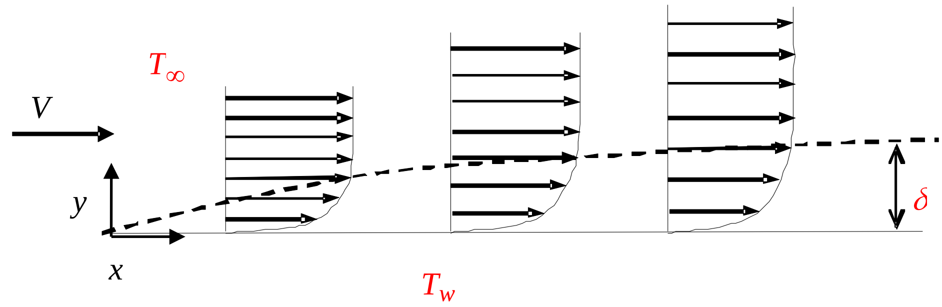
The Stanton number is found by normalising the heat flux by a characteristic convective heat flux.

$$St = \frac{Heat\ flux}{Characteristic\ convective\ heat\ flux} = \frac{h\Delta T}{\rho V C_p \Delta T} = \frac{h}{\rho V C_p}$$

4.5. *Reynolds' analogy (Prandtl number = 1)*

If the **Prandtl number equals 1**, the momentum diffusivity equals the thermal diffusivity. This means that the non-dimensional equations of heat and momentum transfer are identical. In some situations, the non-dimensional boundary conditions are also identical. This means that the non-dimensional solutions are identical and therefore that there is a direct relationship between the mechanical and thermal fields. This is Reynolds' analogy. To demonstrate this, we will revisit the flow over a flat plate.

4.5.1. *Recap of the momentum boundary layer over a flat plate*



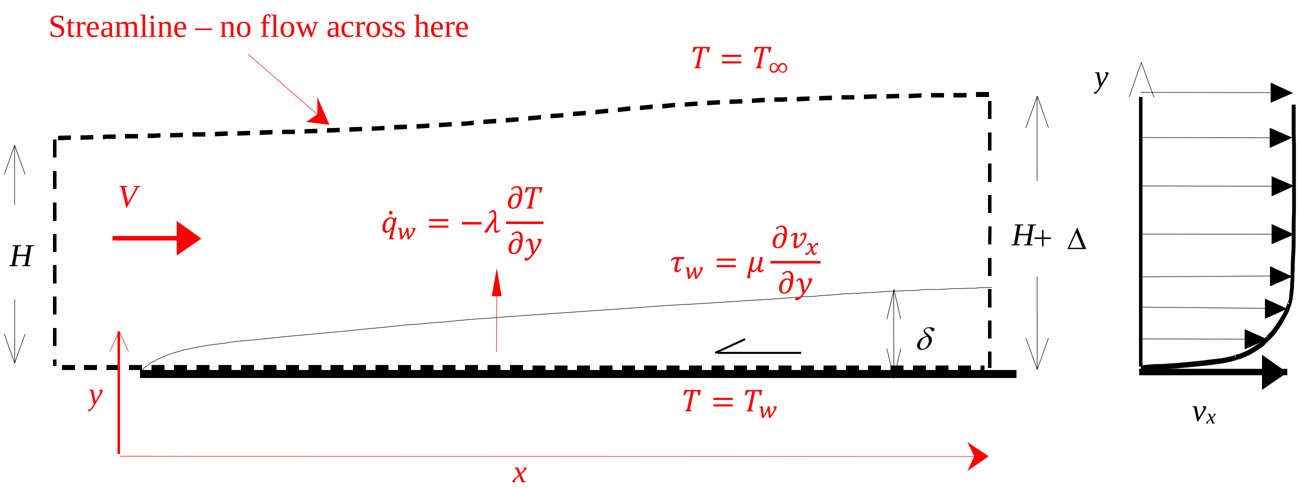
The velocity profile for the laminar boundary layer was given (approximately) by

$$\frac{v_x(y)}{V} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

We also saw that the thickness of the boundary layer grows with distance from the leading edge as

$$\frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}$$

4.5.2. *Heat transfer from a flat plate*



- **Conservation of mass**

$$\dot{m}_{out} = \dot{m}_{in} \Longrightarrow \underbrace{\rho V(H+\Delta-\delta) + \int_0^{\delta} \rho v_x dy}_{\text{mass flow out}} = \underbrace{\rho V H}_{\text{mass flow in}}$$

- ***Conservation of x*-momentum** (The pressure is effectively uniform throughout the flow and its effect cancels)

Force on CV = Net Flux of Momentum Out

$$-\int_0^{x} \underbrace{\tau_w}_{\mu\frac{\partial v_x}{\partial y}}\, dx = \underbrace{\rho V(H+\Delta-\delta)V + \int_0^{\delta} (\rho v_x) v_x dy}_{\text{Momentum flow out}} - \underbrace{(\rho V)HV}_{\text{Momentum flow in}}$$

Substituting in for τ_w and dividing by V

$$-\mu \int_0^{x} \left.\frac{\partial\left(\frac{v_x}{V}\right)}{\partial y}\right|_{wall} dx = \rho V(H+\Delta-\delta) + \int_0^{\delta} (\rho v_x)\left(\frac{v_x}{V}\right) dy - (\rho V)H$$

Enthalpy = $h^{\circ} + c_p(T - T^{\circ})$

$T^{\circ} = T_w$ and $h^{\circ} = 0$

- **Conservation of energy**

Heat flow = Enthalpy flow OUT – Enthalpy flow IN

$$\int_0^{x} \underbrace{\dot{q}_w}_{-\lambda\frac{\partial T}{\partial y}}\, dx = \underbrace{\rho V(H+\Delta-\delta)c_p(T_\infty - T_w) + \int_0^{\delta} (\rho v_x) c_p (T - T_w) dy}_{\text{enthalpy flow out}} - \underbrace{(\rho V)H c_p(T_\infty - T_w)}_{\text{enthalpy flow in}}$$

Substituting in for $\dot{q}_w$ and dividing by $c_p(T_\infty - T_w)$

$$-\frac{\lambda}{c_p}\int_0^{x} \left.\frac{\partial\left(\frac{(T-T_w)}{(T_\infty - T_w)}\right)}{\partial y}\right|_{wall} dx = \rho V(H+\Delta-\delta) + \int_0^{\delta} (\rho v_x)\frac{(T-T_w)}{(T_\infty - T_w)} dy - (\rho V)H$$

Writing $v' = \frac{v_x}{V}$ and $\theta = \frac{(T-T_w)}{(T_\infty - T_w)}$, we can now see the similarity between the momentum and the energy equation

$$-\mu \int_0^x \frac{\partial v'}{\partial y}\bigg|_{wall} dx = \rho V(H + \Delta - \delta) + \int_0^\delta (\rho v_x) v' dy - (\rho V)H$$

$$-\frac{\lambda}{c_p} \int_0^x \frac{\partial \theta}{\partial y}\bigg|_{wall} dx = \rho V(H + \Delta - \delta) + \int_0^\delta (\rho v_x) \theta dy - (\rho V)H$$

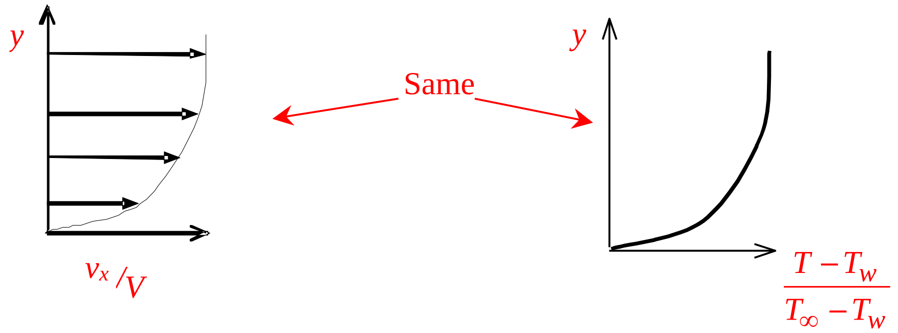
For $\mathrm{Pr} = \dfrac{\mu c_p}{\lambda} = 1$ we conclude

$v' = \dfrac{v_x}{V}$ and $\theta = \dfrac{(T-T_w)}{(T_\infty - T_w)}$, satisfy the same equation.

Moreover ,

$\dfrac{v_x}{V} = 0$ for $y = 0$ $\qquad \dfrac{v_x}{V} = 1$ for $y \geq \delta$

$\dfrac{T-T_w}{T_\infty - T_w} = 0$ for $y = 0$, $\qquad \dfrac{T-T_w}{T_\infty - T_w} = 1$ for $y \geq \delta$



i.e. $\dfrac{(T-T_w)}{(T_\infty - T_w)} = \dfrac{v_x}{V}$ is a solution of the problem for temperature!

The above relationship between v_x and T enables us to relate directly the gradient of T at the wall to that of v_x at the wall.

$$\frac{T - T_w}{T_\infty - T_w} = \frac{v_x}{V} \quad \Longrightarrow \quad \frac{1}{(T_\infty - T_w)} \frac{\partial T}{\partial y}\bigg|_{wall} = \frac{1}{V} \frac{\partial v_x}{\partial y}\bigg|_{wall}$$

$$h = \frac{\lambda \left.\frac{\partial T}{\partial y}\right|_{wall}}{(T_\infty - T_w)} = \frac{\lambda}{V} \frac{\partial v_x}{\partial y}\bigg|_{wall} = \frac{\lambda}{V} \frac{\tau_w}{\mu}$$

τ_w can be related to a friction factor, c_f i.e.

$$h = \frac{\lambda}{V} \frac{\tau_w}{\mu} = \frac{\lambda}{V} \frac{\frac{1}{2} \rho V^2 c_f}{\mu}$$

Dividing $\rho\, c_p V$ (and noting that we are assuming Pr = 1)

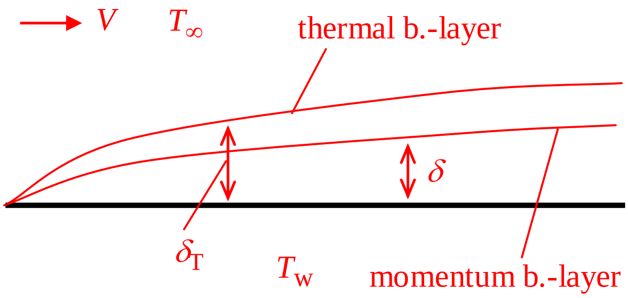
$$\frac{h}{\rho\, c_p V} = \frac{1}{2}\, c_f \frac{\lambda}{\mu c_p} = \frac{1}{2} c_f \frac{1}{Pr}$$

$$St = \frac{1}{2} c_f$$

This result indicates that we cannot have high heat transfer without having high wall friction. This is not surprising because both heat and momentum transfer have the same underlying mechanism: molecular motion or, in a turbulent flow, eddy motion

4.6. *Heat transfer if the Prandtl number ≠1*

If the Prandtl number is not equal to 1, the transport equations are no longer identical and we have two types of boundary layer: a momentum boundary layer and a thermal boundary layer. The relative thicknesses of the boundary layers are determined by the Prandtl number



$Pr \ll 1$ for liquid/molten metals $\Rightarrow \delta \ll \delta_T$

Pr about 1 for air, other gases, water $\Rightarrow \delta \approx \delta_T$

$Pr \gg 1$ for thick oils, etc. $\Rightarrow \delta \gg \delta_T$

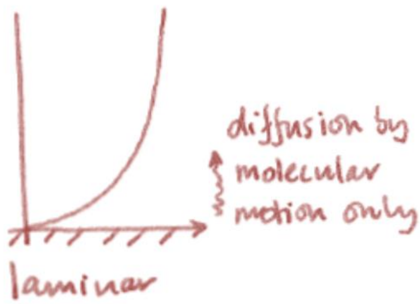
We find from experiments that:

$$\frac{h}{\rho c_p V} = St = \frac{1}{2} c_f \, \mathrm{Pr}^{-2/3}$$

(For gases usually have $Pr \approx 0.7$ the correction is small and usually ignored).

4.7. *Turbulent boundary layers*

If a boundary layer becomes turbulent, there is additional transport of momentum and thermal energy due to the turbulent eddies. This can greatly increase the heat transfer.



Reynolds analogy still applies

4.8. Evaluation of St and Nu for a flat plate boundary layer

The velocity profile for the laminar boundary layer was given (approximately) by

$$\frac{v_x(y)}{V} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

with the corresponding shear stress at the wall of,

$$\tau_w = \mu \left.\frac{\partial v_x}{\partial y}\right|_{y=0} = \mu \frac{3V}{2\delta}$$

and **local** friction factor

$$c_{fx} = \frac{\tau_w}{\frac{1}{2}\rho V^2} = \mu \frac{3}{\rho \delta V}$$

From the above, $St_x = \frac{1}{2} c_{fx} Pr^{-\frac{2}{3}}$

$$St_x = \mu \frac{3}{2\rho V \delta} Pr^{-\frac{2}{3}}$$

We also saw that the thickness of the boundary layer grows with distance from the leading edge as $\delta = \frac{4.64x}{\sqrt{Re_x}}$

$$St_x = \frac{3\mu}{2\rho x V} \frac{\sqrt{Re_x}}{4.64} Pr^{-\frac{2}{3}} = 0.323 Re_x^{-\frac{1}{2}} Pr^{-\frac{2}{3}}$$

Or for the Nusselt number

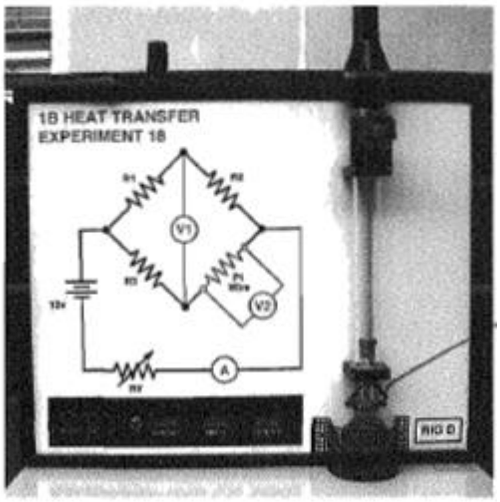
$$Nu_x = 0.323 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

These expressions give a value for the **local value** of *Nu*, *St* and *h* as a function of *x*. Suitable averages would have to be taken to give the average heat transfer coefficient over a plate of length L.

The databook gives correlations for the **averaged/overall** Nusselt number, for various geometries, from which the overall heat transfer coefficient can be derived.

4.9. The forced convection experiment

In most situations, the flow field is too complicated to be calculated by hand. Computational Fluid Dynamics (CFD) can be used but this is often expensive and may not be accurate. It can be easier to perform an experiment on a geometrically-identical situation and then scale up with the relevant non-dimensional parameters. You do this in the 1B heat transfer experiment of forced convection over a platinum wire.



Platinum wire.

air's physical properties.

ρ, μ, λ, c_p

1. List the dependent and independent variables

Dependent	Independent					
h	D	V	ρ	μ	λ	c_p.
$\frac{M}{T^3\Theta}$	L	$\frac{L}{T}$	$\frac{M}{L^3}$	$\frac{M}{LT}$	$\frac{ML}{T^3\Theta}$	$\frac{L^2}{T^2\Theta}$

2. Count the dimensions

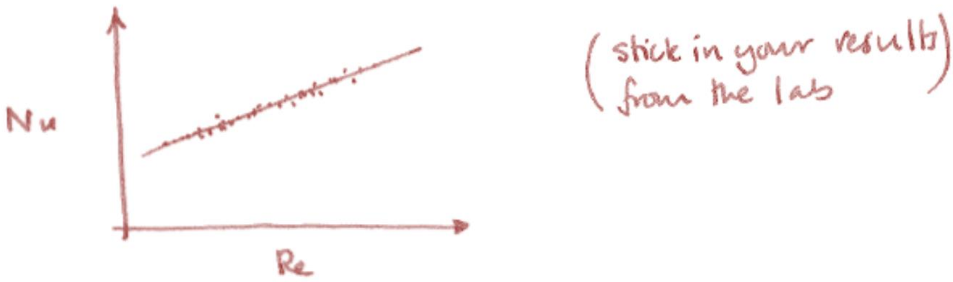
7 Variables, 4 Dimensions $\Rightarrow$ 3 groups

3. Create dimensionless numbers

$$\frac{hD}{\lambda} \left(\text{or } \frac{h}{\rho V c_p} \right), \quad \frac{\rho V D}{\mu} = Re, \quad \frac{\mu c_p}{\lambda} = Pr$$

$$Nu \ (\text{or } St) = f(Re, Pr).$$

When you plot the Nusselt number as a function of Reynolds number (given that air has a fixed Prandtl number) you find that your results collapse to a single line.



The physics is encapsulated in this line. For a gas with the same Prandtl number as air, you can use the chart you created to calculated the heat transfer from a geometrically similar situation of any size, between the Reynolds number ranges that you studied. In heat transfer problems you usually have to rely on dimensional analysis and scaling.

4.10.Key Points for Forced Convection

- Heat transfer takes place in boundary layers and is strongly influenced by the state, size, etc. of the local boundary layer.
- There is a strong analogy between heat transfer and momentum transfer leading to Reynolds Analogy $\dfrac{h}{\rho c_p V} = St \quad = \quad \dfrac{1}{2} c_f$
- The local heat transfer coefficient for a flat plate can be derived from a simple model of the velocity profile
- Turbulent boundary layers involve higher heat transfer than laminar ones.
- Correlations (i.e. curve fits to experimental data) will be different for different states of boundary layers (laminar, transitional, turbulent) and it is important to check.
- For more general problems $Nu_{av} = \dfrac{h_{av} D}{\lambda} = fn(\mathrm{Re}, \mathrm{Pr})$