UNIVERSITY OF CAMBRIDGE
Department of Engineering

Bernoulli Distribution

- Trial with binary output: $\mathrm{X} \in \mathbb{X} = \{0, 1\}$, and $p \in [0, 1]$
- $\mathrm{X} \sim \mathrm{Ber}(p) \quad \Leftrightarrow \quad P_{\mathrm{X}}(k) = \begin{cases} p & \text{if } k = 1, \\ 1-p & \text{if } k = 0, \\ 0 & \text{otherwise.} \end{cases}$
- $\mathbb{E}[\mathrm{X}] = p$ and $\mathrm{Var}[\mathrm{X}] = p(1-p)$
- $\mathbb{H}[\mathrm{X}] = -p \log_2 p - (1-p) \log_2 (1-p) = \mathcal{H}_2(p)$

Geometric Distribution

- How many trials do I need to be successful? $\mathrm{X} \in \mathbb{X} = \{1, 2, \dots\}$, and $p \in [0, 1]$
- $\mathrm{X} \sim \mathrm{Geo}(p) \quad \Leftrightarrow \quad P_{\mathrm{X}}(k) = \begin{cases} p(1-p)^{k-1} & \text{if } k \in \{1, 2, \dots\}, \\ 0 & \text{otherwise.} \end{cases}$
- $\mathbb{E}[\mathrm{X}] = 1/p$ and $\mathrm{Var}[\mathrm{X}] = (1-p)/p^2$
- $\mathbb{H}[\mathrm{X}] = \mathcal{H}_2(p)/p$

Binomial Distribution

- How many times was I successful after n trials? $\mathrm{X} \in \mathbb{X} = \{0, 1, 2, \dots n\}$ with $n \in \{1, 2, \dots\}$, and $p \in [0, 1]$
- $\mathrm{X} \sim \mathrm{B}(n, p) \quad \Leftrightarrow \quad P_{\mathrm{X}}(k) = \begin{cases} {}^{n}\mathrm{C}_k \, p^k (1-p)^{n-k} & \text{if } k \in \{0, 1 \dots, n\}, \\ 0 & \text{otherwise.} \end{cases}$ where ${}^{n}\mathrm{C}_k = \dfrac{n!}{k!(n-k)!}$
- $\mathbb{E}[\mathrm{X}] = np$ and $\mathrm{Var}[\mathrm{X}] = np(1-p)$
- No simple formula for $\mathbb{H}[\mathrm{X}]$. For large n, $\mathbb{H}[\mathrm{X}] \overset{n \gg 1}{\approx} \log_2 \sqrt{2\pi e n p(1-p)}$

Poisson Distribution

- How many times was I successful given a success rate λ? $\mathrm{X} \in \mathbb{X} = \{0, 1, 2, \dots\}$, and $\lambda \in (0, \infty)$
- $\mathrm{X} \sim \mathrm{Pois}(\lambda) \quad \Leftrightarrow \quad P_{\mathrm{X}}(k) = \begin{cases} \dfrac{\lambda^k e^{-\lambda}}{k!} & \text{if } k \in \{0, 1, 2, \dots\}, \\ 0 & \text{otherwise.} \end{cases}$
- $\mathbb{E}[\mathrm{X}] = \lambda$ and $\mathrm{Var}[\mathrm{X}] = \lambda$
- No simple formula for $\mathbb{H}[\mathrm{X}]$. For large λ, $\mathbb{H}[\mathrm{X}] \overset{\lambda \gg 1}{\approx} \log_2 \sqrt{2\pi e \lambda}$

Links with the Binomial Distribution

- With $\{\mathrm{X}_j \sim \mathrm{Ber}(p)\}_{j=1\dots n}$ independent, $\displaystyle\sum_{j=1}^{n} \mathrm{X}_j \sim \mathrm{B}(n, p)$
- $\mathrm{B}(n, \lambda/n) \xrightarrow{n \to \infty} \mathrm{Pois}(\lambda)$