UNIVERSITY OF CAMBRIDGE
Department of Engineering

IB Paper 5
Electromagnetic Fields & Waves

Lecture 1
Introduction &
Transmission Lines I

https://www.vle.cam.ac.uk/course/view.php?id=70081

Electromagnetism – The Basis of Modern Living!

YEARS	
-200,000	• Appearance of *homo sapiens*
-170,000	• Clothes
-70,000	• Symbolic art
-5,000	• Bronze age
-3,200	• Iron age
-255	• Industrial revolution
-205	• **Birth of Electromagnetism**
-195	• **Electric Motor**
-155	• **Power Station**
-125	• **Wireless Radio Transmission**
-75	• **Transistor**
-55	• **Optical Fibre Data Transmission**
-50	• **Intel 4004 Microprocessor**
TODAY	• ***…life in an electromagnetic age***

Course Aims

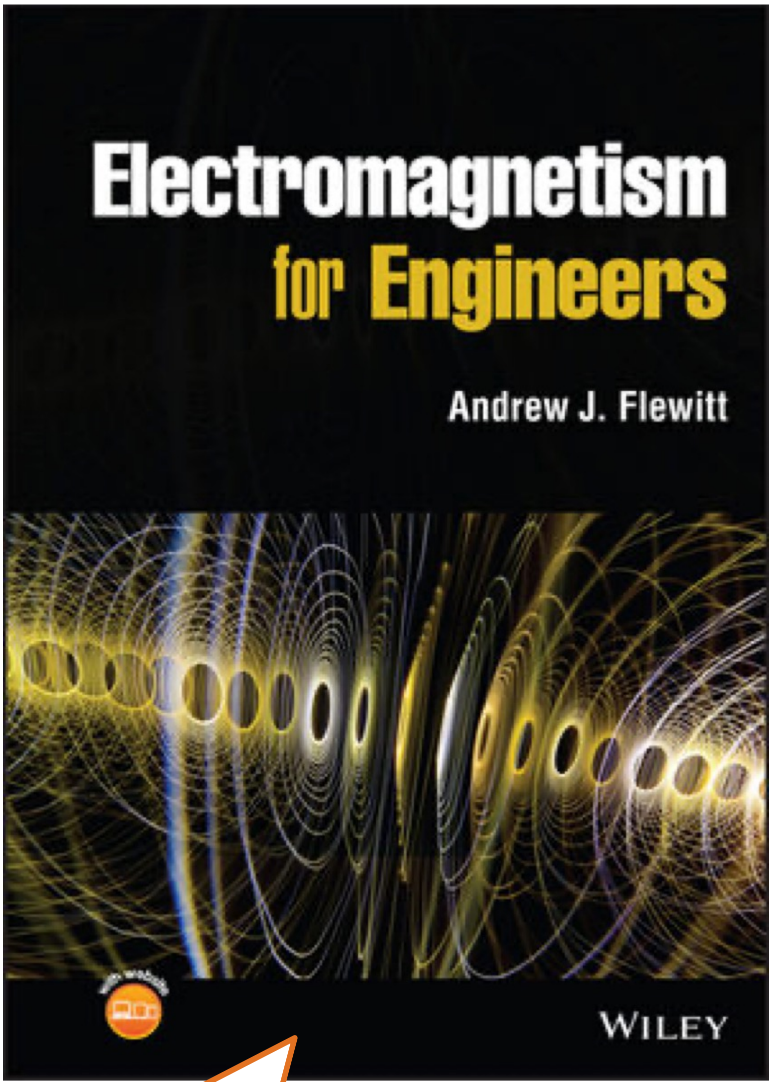
- Electromagnetism really does underpin everything we do in modern life
- Students tend not to like EM because it is invisible, but we happily accept its phenomena as being part of the natural world
- We…
  - …take our phone outside to get a better signal
  - …know that flat, metallic surfaces make good reflectors
  - …know that sunglasses 'polarise light'
  - …put food in a microwave field to heat it up
  - …avoid putting fingers between magnets
  - …know that radio waves travel at the speed of light
  - …avoid placing radios next to certain electrical appliances
  - …know that light bounces down optical fibres
  - …know that memories use charge to store data on capacitors
- ***The aim of this course is to show that the application of basic equations about the nature of electric and magnetic fields allows us to understand these phenomena and engineer them to our benefit with a focus on communications***

Course Structure

- Part I – Transmission Lines
  - Lecture 1: Transmission line equivalent circuit
  - Lecture 2: Wave analysis of transmission lines

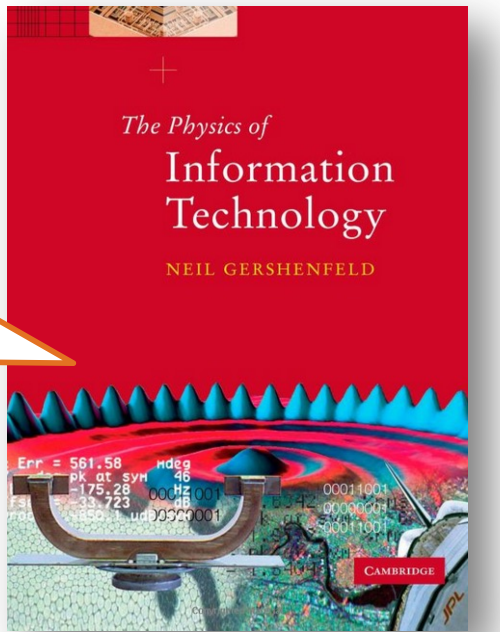
- Part II – Electromagnetic Waves
  - Lecture 3: The Maxwell Equations
  - Lecture 4: Electromagnetic Waves in Free Space & Antennas
  - Lecture 5: Reflection and Transmission of Electromagnetic Waves in Dielectrics
  - Lecture 6: Electromagnetic Waves in Conductors

Textbooks



FLE

Published in 2023 and written with this course in mind!



'No equations barred' understanding of how electromagnetics is applied in information technology

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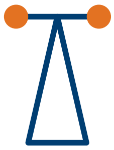
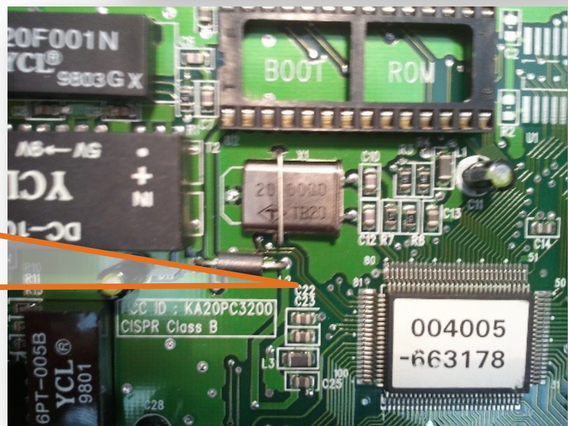
What is a Transmission Line?

- A transmission line guides the flow of energy in the form of an electromagnetic wave from one place to another
  - If energy is flowing then we can also transmit data
- Some examples on very different length scales are:

Electricity power transmission lines



Coaxial cable for low noise signals (e.g. TV signal)



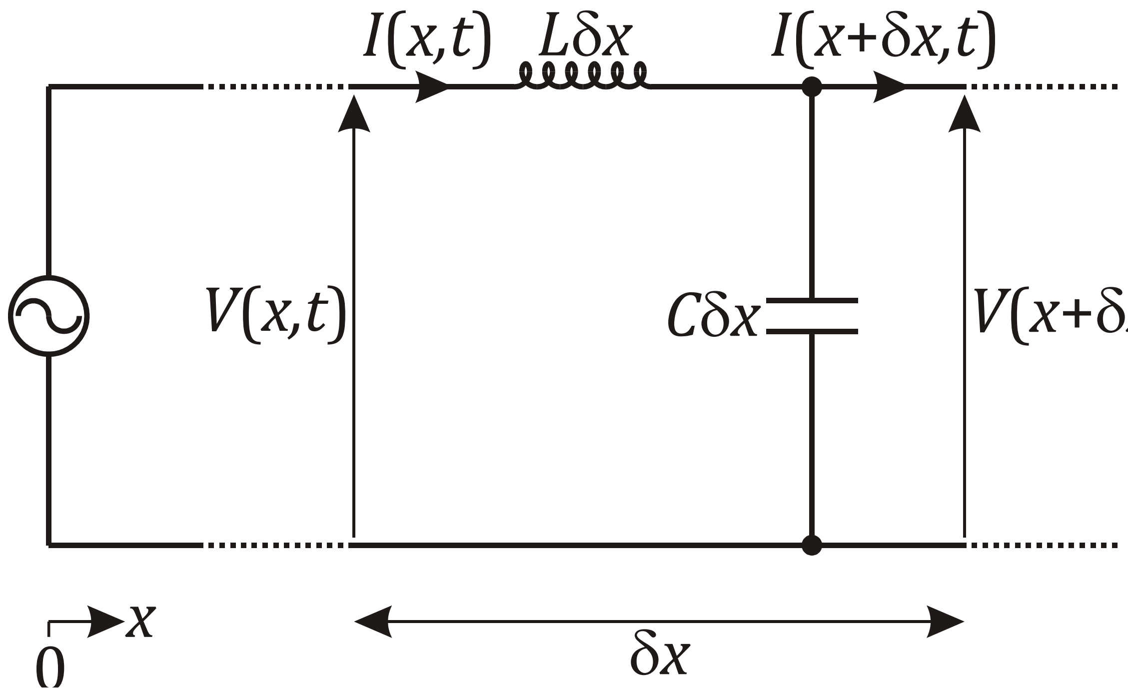
Microstrip lines for data transfer on a PCB

Ideal Transmission Line Equivalent Circuit

FLE86, GER83

- All transmission lines must consist of at least two conductors (an out and return current path) separated by an insulator (dielectric)
- We know (from Part IA) that:
  - two conductors separated by a gap have some capacitance acting between them
  - a loop of wire has some inductance
- We imaging a small length δx of a transmission line, and assign to it impedances of $C\delta x$ and $L\delta x$ where C and L are the capacitance and inductance per unit length

5/6 Q1 gets you to work this out for a coaxial cable using Gauss' Law and Ampère's Law



The Telegrapher's Equations

FLE88, GER85

- Let $\delta V = V(x+\delta x,t) - V(x,t)$
  - This is the voltage drop across the inductor, so (remembering that classical currents and voltages point in opposite directions)

$$V(x+\delta x,t) - V(x,t) = \delta V = -L\delta x\frac{\partial I}{\partial t}$$

  - This rearranges to

$$\boxed{\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}} \qquad (1.1a)$$

- Similarly, $\delta I = I(x+\delta x,t) - I(x,t)$
  - The current flowing down through the capacitor is then $-\delta I$, so

$$-\delta I = C\delta x\frac{\partial V}{\partial t}$$

  - This rearranges to

$$\boxed{\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t}} \qquad (1.1b)$$

- Eqns. 1.1a and 1.1b are collectively known as the ***Telegrapher's Equations***
  - They describe the relationship between the time- and position-dependent voltage and current at any point on a transmission line
  - Inductance and capacitance per unit length are key

Wave Equation Solution to the TE

FLE89, GER85

- If we differentiate both sides of Eqn. 1.1a with respect to x, then

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{\partial}{\partial x}\frac{\partial I}{\partial t}$$

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{\partial}{\partial t}\frac{\partial I}{\partial x} \qquad (1.2)$$

  - Substituting Eqn. 1.1b into 1.2 gives

$$\boxed{\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}} \qquad (1.3a)$$

  - Similarly, by differentiating Eqn. 1.1b with respect to x and substituting 1.1a gives

$$\boxed{\frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2}} \qquad (1.3b)$$

- Eqns. 1.3a and 1.3b have the generic form of a ***wave equation*** in one dimension
  - In general, the wave equation in one dimension for some arbitrary function ψ has the form

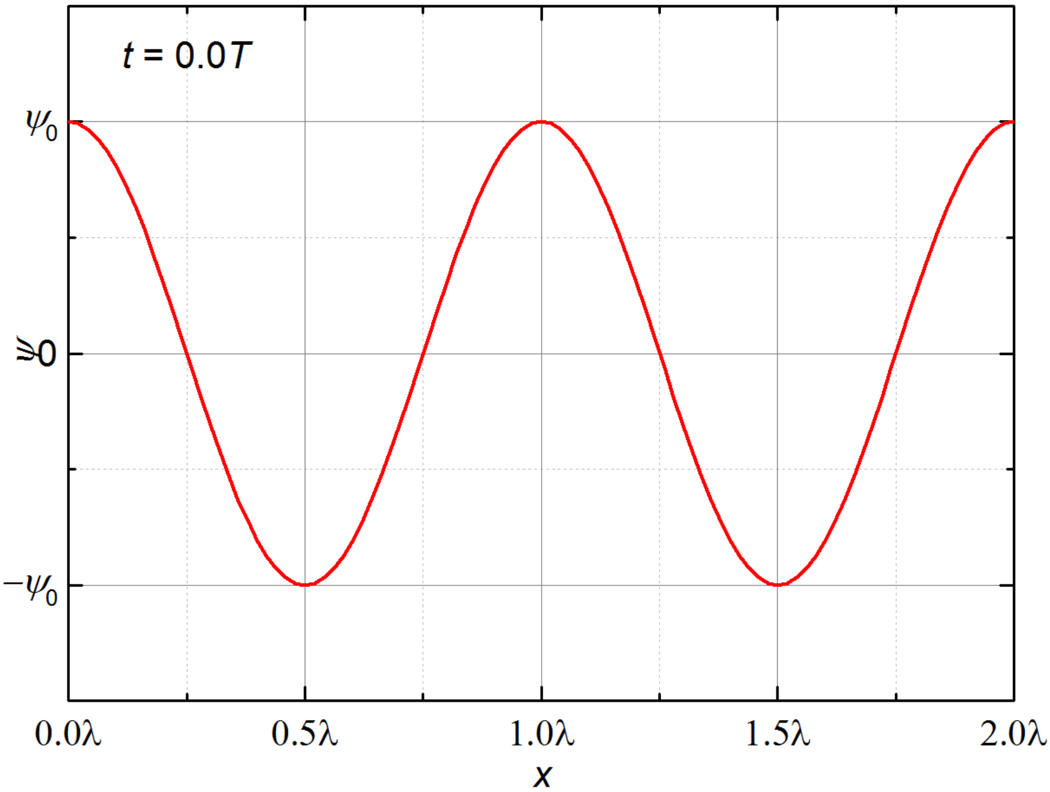
$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} \qquad (1.4)$$

  - c is a constant

- There are many solutions to the generic wave equation, but as we are considering a.c. currents and voltages, we will consider

5/6 Q2

$$\psi = \psi_0 \cos(\omega t - \beta x) \tag{1.5}$$



- This is a wave moving in the positive x-direction with time

$$\omega = 2\pi/T = 2\pi f \qquad \beta = 2\pi/\lambda$$

- ω is angular frequency and β is the propagation constant (T is time period, f is frequency and λ is wavelength)
- $\psi = \psi_0 \cos(\omega t + \beta x)$ would travel in the –ve direction

- Substituting Eqn. 1.5 into Eqn. 1.4 gives

$$\beta^2 \psi = \frac{1}{c^2}\omega^2 \psi$$

$$\therefore c = \frac{\omega}{\beta} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = \text{wave velocity} \tag{1.6}$$

- If we compare Eqns. 1.3a and 1.3b with Eqn. 1.4, we can see that the wave velocity on the transmission line is

$$\boxed{c = \frac{1}{\sqrt{LC}}} \tag{1.7}$$

  - So the inductance and capacitance per unit length determine the wave velocity
- We can substitute expressions for L and C derived from Gauss' and Ampère's Laws
  - For the case of a coaxial cable with an inner conductor of radius a and an outer conductor of radius b with an intermediate dielectric of relative permittivity ε_r and relative permeability μ_r

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(b/a)} \qquad L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{b}{a}\right) \tag{1.8}$$

  - Substituting these into Eqn. 1.7 gives a general result that

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}}} \tag{1.9}$$

  - ***The speed of the wave along the transmission line is dependent only on the property of the dielectric medium between the conductors***
  - This is always the case for transmission lines
  - If the dielectric is air ($\varepsilon_r = 1$, $\mu_r = 1$) then

$$c = (8.854\times10^{-12}.4\pi\times10^{-7})^{-1/2} = 2.998\times10^{8}\ \mathrm{m\,s^{-1}}$$

  - The wave travels at the speed of light

Expressions for Current and Voltage Waves

- The general solution to Eqn. 1.3a for voltage is

$$V = V_F \cos(\omega t - \beta x + \phi_F) + V_B \cos(\omega t + \beta x + \phi_B) \quad (1.10)$$

  - V_F and V_B are the amplitudes of the forward and backwards voltage waves
  - ϕ_F and ϕ_B are their respective phases
  - We can re-express this using complex notation as

$$V = \overline{V_F} e^{j(\omega t - \beta x)} + \overline{V_B} e^{j(\omega t + \beta x)} \quad (1.11)$$

  - $\overline{V_F}$ and $\overline{V_B}$ are complex numbers representing both the amplitude and phase offset of the voltage wave, where

$$\overline{V_F} = V_F e^{j\phi_F} \qquad \overline{V_B} = V_B e^{j\phi_B}$$

- Likewise the general solution to Eqn. 1.3b for current is

$$I = I_F \cos(\omega t - \beta x + \phi_F) + I_B \cos(\omega t + \beta x + \phi_B) \quad (1.12)$$

  - We can re-express this in complex notation as

$$I = \overline{I_F} e^{j(\omega t - \beta x)} + \overline{I_B} e^{j(\omega t + \beta x)} \quad (1.13)$$

- We now have expressions for voltage and current waves that can pass in either direction along a transmission line
  - The speed of the wave is determined by the speed of electromagnetic waves in the dielectric

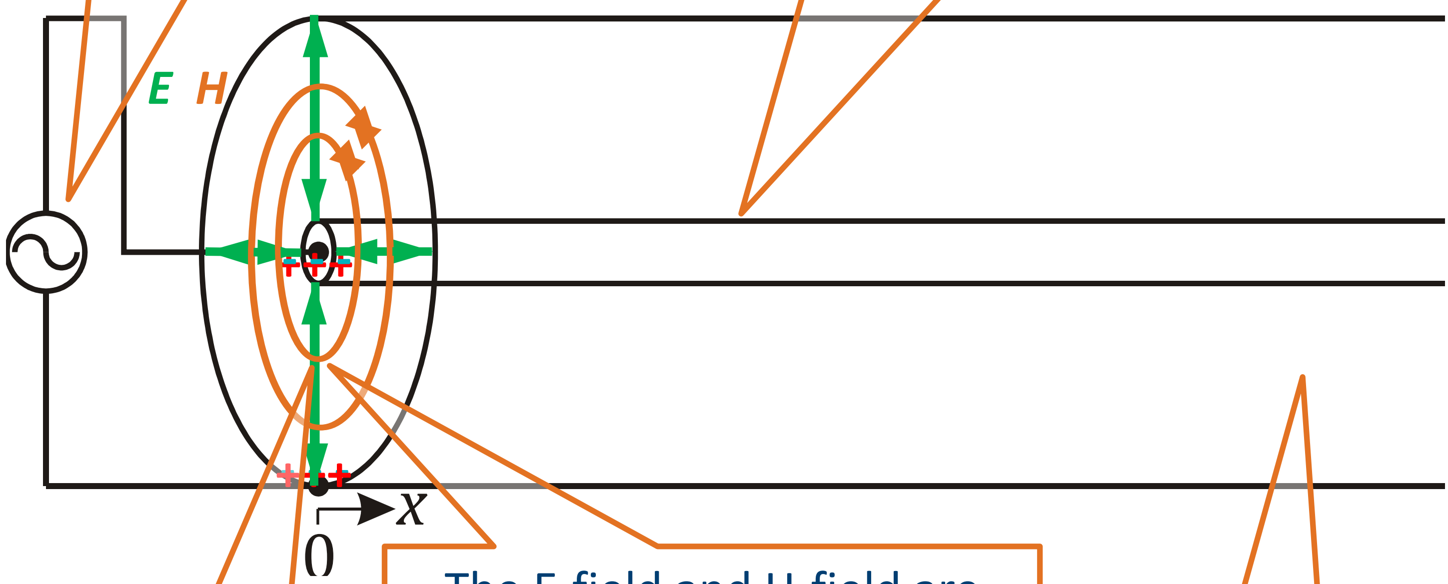
5/6 Q1

How Do Transmission Lines Work?

- Let’s take the coaxial cable as an example

The voltage source pushes an oscillating current into one end of the coaxial cable

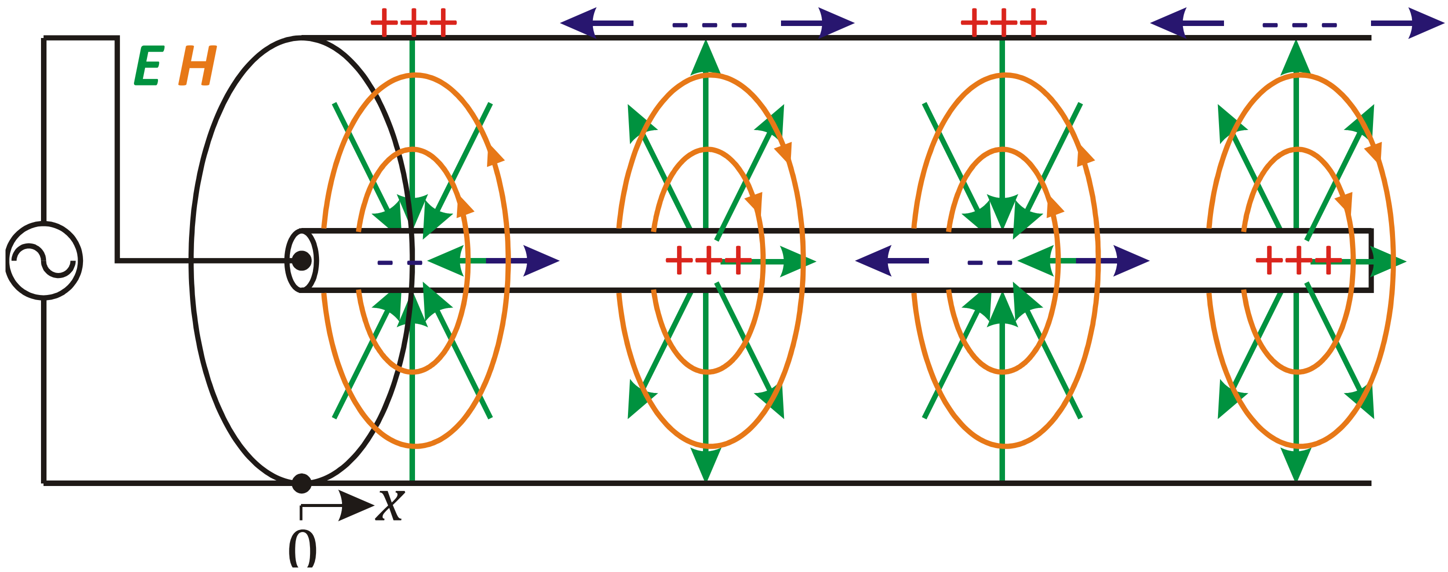
As the wave propagates along in the dielectric, it induces charge on the conductors



The E-field and H-field are perpendicular and this produces an electromagnetic wave perpendicular to both

The charge pushed in produces an electric field whilst the current produces a magnetic field

The induced charge sustains the E- and H-fields, guiding it along the transmission line

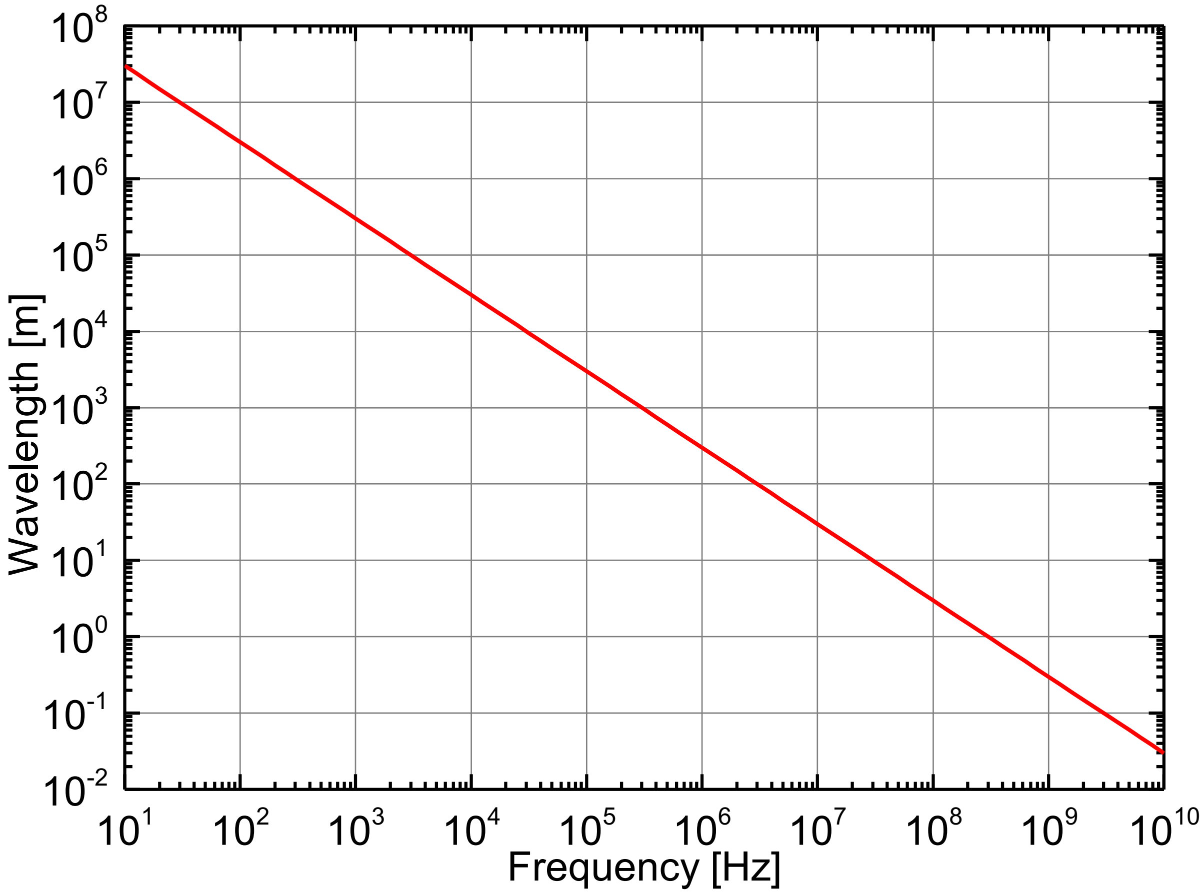


- Electrons in the conductors are only oscillating locally, moving at the drift velocity
  - This creates local regions of net positive and negative space charge close to the conductor surface
  - Eqns. 1.11 and 1.13 are describing this charge oscillation
- The free charge in the conductors is guiding the surrounding electromagnetic wave
  - This is why the wave (and energy) is able to travel at high speeds determined by the dielectric
- While some power is associated with the charge motion in the conductors, a proportion is associated with the surrounding electromagnetic wave

Wavelength

- Why have we not had to worry about this before?
  - We have always assumed the flow of charge in conductors to be like an incompressible fluid with no spatial phase difference
  - We can use $\lambda = c/f$ to work out wavelength
  - This is shown assuming an air/vacuum dielectric

5/6 Q3

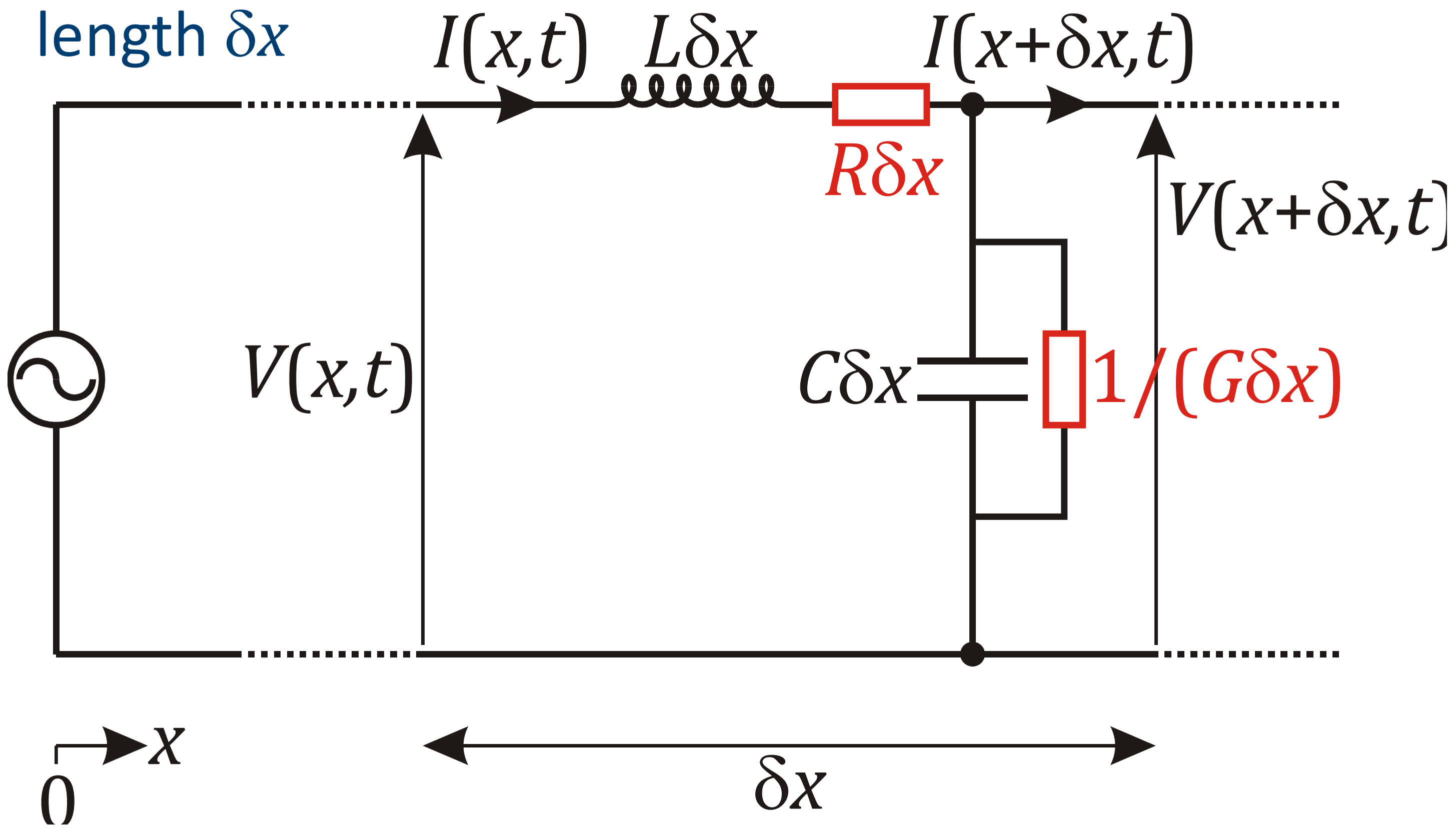


- If the wavelength is long compared with the physical length of the system we are looking at, then we can consider the current to be behaving as an incompressible fluid
  - We do not need to think in transmission line terms

‘Lossy’ Transmission Lines

FLE95

- The conductors will have a resistance per unit length, R, and the dielectric will not be a perfect insulator, but have a conductance per unit length G, giving a new equivalent circuit for the small length δx



- Eqns. 1.11 and 1.13 gain a new term which causes the current and voltage amplitude to decay with distance

$$V = \overline{V_F} e^{j(\omega t - \beta x)} e^{-\alpha x} + \overline{V_B} e^{j(\omega t + \beta x)} e^{\alpha x} \quad (1.14a)$$

$$I = \overline{I_F} e^{j(\omega t - \beta x)} e^{-\alpha x} + \overline{I_B} e^{j(\omega t + \beta x)} e^{\alpha x} \quad (1.14b)$$

- α is known as the ***attenuation constant***
- The ***propagation constant*** is then defined as

5/6 Q4

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (1.15)$$

- for a full analysis of the equivalent circuit element