IB Structures 2023-24

Handout 1

Thin-walled structures

Filled Version

Text and pictures in grey are omitted from the version in lectures

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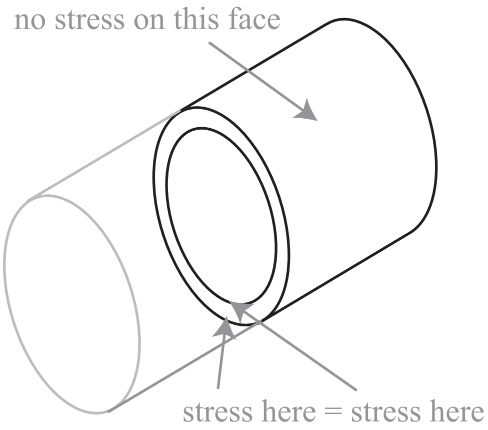
Cambridge University Engineering Department

1.1 Introduction

In this section we will consider the stresses and strains generated in *thin-walled* sections due to various loads. In general, the wall-thickness of the section is assumed to be very much less than the other dimensions of the structure, and this allows us to make a number of assumptions about the nature of the stresses:

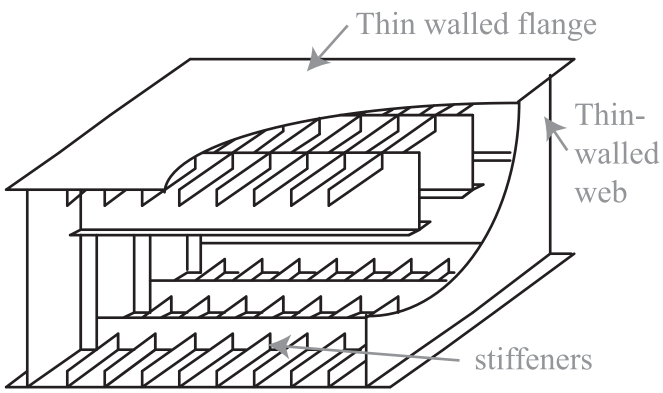
1. Through-thickness stresses are zero.
2. The stress state is uniform through the section.



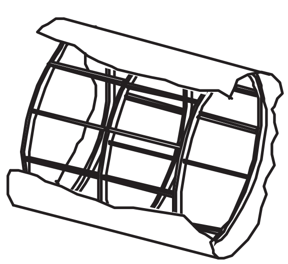
These are usually reasonable for e.g. a circular cylinder with radius/thickness ≥ 20.

We will also examine the effect of *stiffeners*. Real thin-walled structures, such as the box-girder bridge or the aircraft fuselage below, have additional stiffeners:

- to prevent local buckling of the walls;
- to carry locally concentrated loads (for instance heavy local loads on the bridge shown below);
- as a fail-safe device (see Examples Paper 2/1, q1).

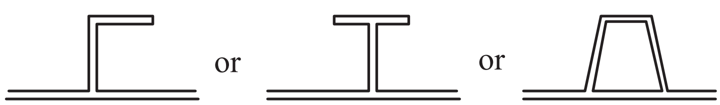


Section of a box-girder bridge



Part of an aircraft fuselage

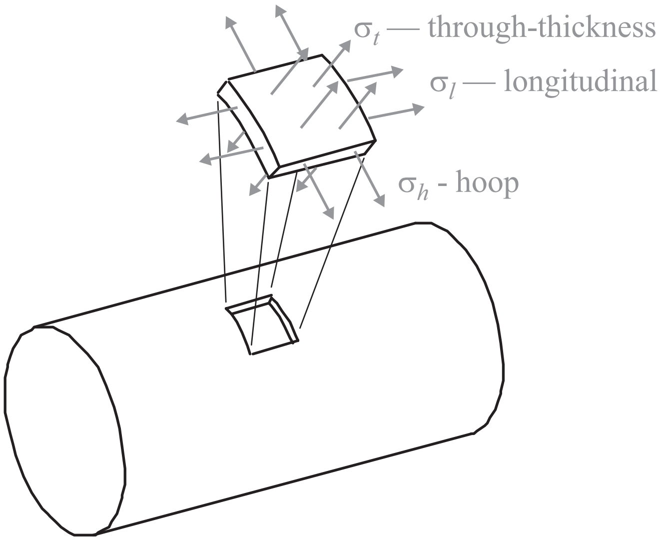
The stiffeners shown are schematic. The actual cross-section is likely to be:



1.2 Stresses

Much of this will be revision from 1A, except the section on *torsion*. It will be particularly helpful to review your 1A notes on *Bending and Shearing Stresses in Beams*.

1.2.1 Circular cylinder due to internal pressure

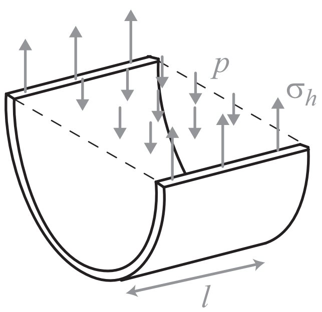


Despite our assumption, the through-thickness stress cannot be constant across the wall. If the pressure internally is p, and externally is 0, then

$$\sigma_t = -p \qquad \text{on inner face}$$
$$\sigma_t = 0 \qquad \text{on outer face}$$

However, we shall assume $\sigma_t = 0$ everywhere ($|\sigma_t| \leq p \ll \sigma_h$ for thin-walled).

Hoop stress



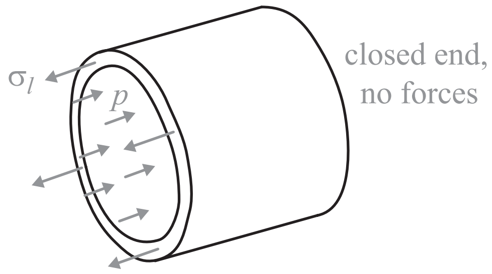
Equilibrium ↕

$$2\,p\,r\,l = \sigma_h\,l\,2t$$
$$\sigma_h = \frac{pr}{t} \qquad (\text{n.b.} \gg p)$$

Longitudinal stress

The longitudinal stress will vary, depending on the support conditions.

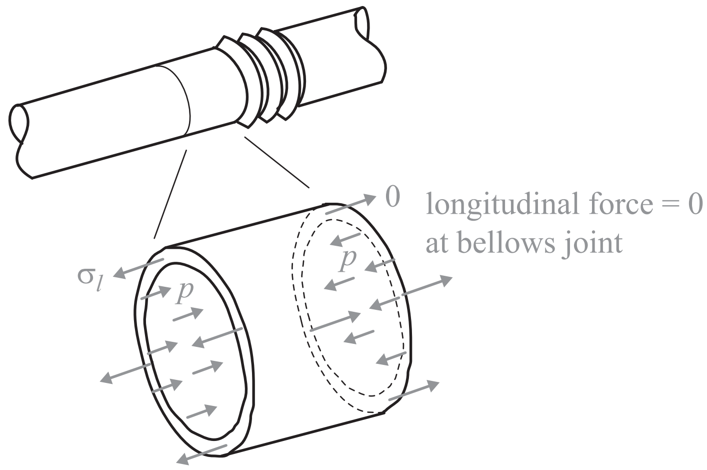
Example 1: Closed end cylinder



Equilibrium $\leftrightarrow$

$$p\,\pi r^2 = \sigma_l\, 2\pi r t$$
$$\sigma_l = \frac{pr}{2t}$$

Example 2: Pipeline with bellows expansion joint. Bellows allow expansion of the pipelines, due to e.g. temperature change.

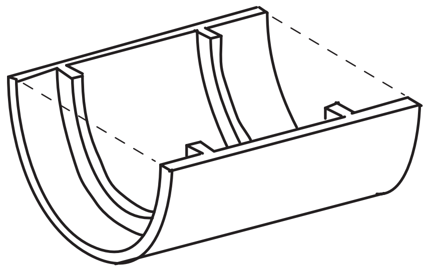


Equilibrium $\leftrightarrow$

$$\sigma_l = 0$$

Effect of stiffeners

Circumferential stiffeners must reduce the average hoop stress:



Consider an average hoop stress σ_{av}

$$\sigma_{\text{av}} A = p\,2rl;$$
$$A > 2lt \quad \Rightarrow \quad \sigma_{\text{av}} < \frac{pr}{t}$$

where A is the wall area *including* stiffeners. But a shorter free body that excluded stiffeners would give

$$\sigma_{\text{av}} = \frac{pr}{t}.$$

Clearly the hoop stress must vary along the section. However, locally, away from stiffeners, the hoop stress may well reach its familiar value — $\sigma_h = \frac{pr}{t}$ is a useful conservative estimate. ***Longitudinal stiffeners*** (if connected to the end of the cylinder) will carry the same load as the skin, and hence the longitudinal stress will be reduced.

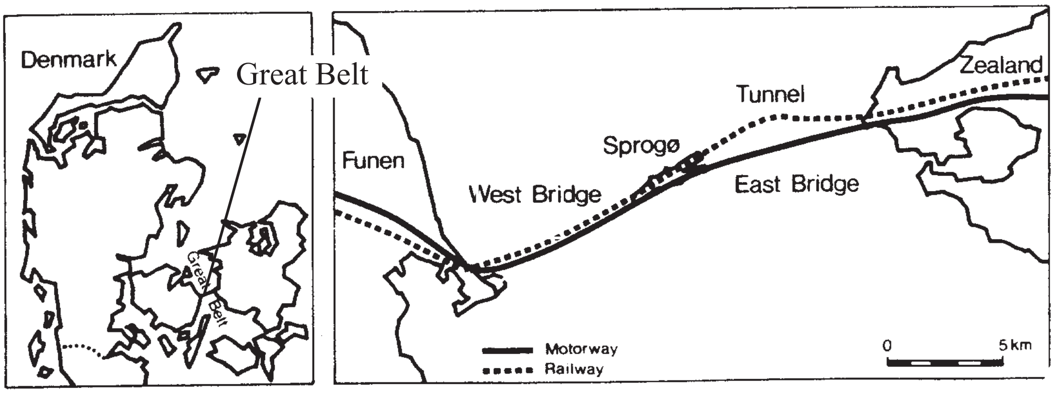
$$\sigma_l A = p\,\pi r^2$$

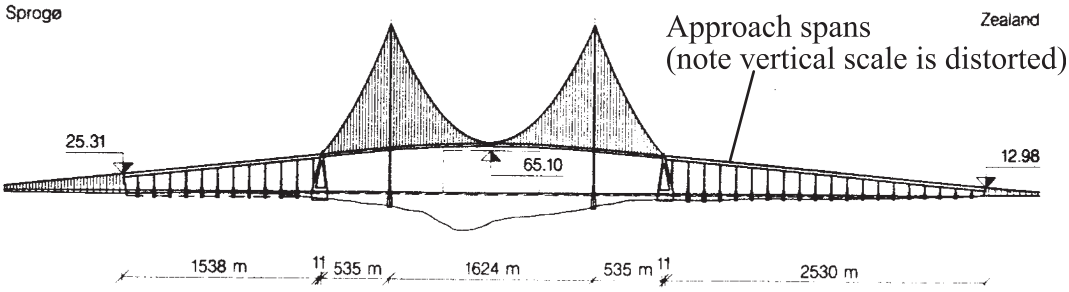
where A is the wall area *including* stiffeners — σ_l will not vary around the section.

Try Questions 1 and 2, Examples Sheet 2/1

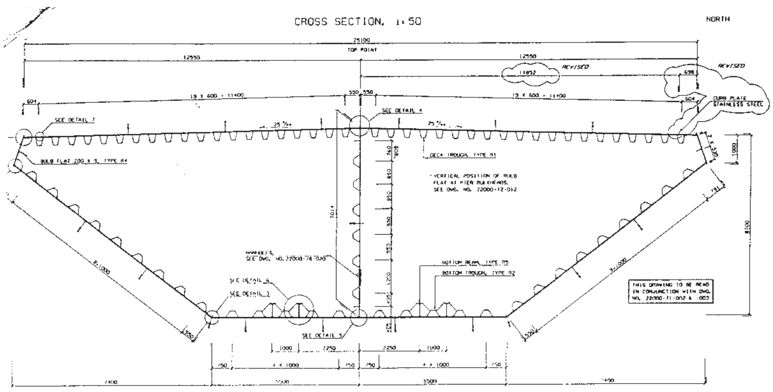
1.2.2 Case study introduction — lifting Storebælt approach spans

The Storebælt link is an 18 km combined bridge and tunnel that forms part of a road and rail link connecting the mainland of Denmark to the island of Zealand. The link opened in 1998. The most spectacular part of the link is a suspension bridge making up part of the East Bridge, which has a main span of 1624 m. This study, however, will concentrate on the approach spans, a series of steel box girder bridges, each 193 m in length. More details of the project will be found on the world-wide web at http://www.storebaelt.dk





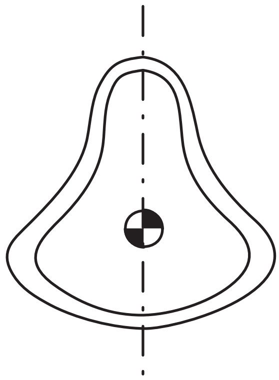
The cross-section used for the box-girder bridges is shown below



The approach spans were lifted into place in one piece. We will calculate the stresses that this generated in the bridge — but first some theory.

1.2.3 Stresses due to axial forces, bending moments and shear forces

We will consider general, non-circular cylinders that have a vertical axis of symmetry, under the action of vertical loads:



Why the axis of symmetry? It ensures that vertical loads and bending moments (symmetric applied loads):

Cause vertical deflections, curvatures in a vertical plane (symmetric deformations)

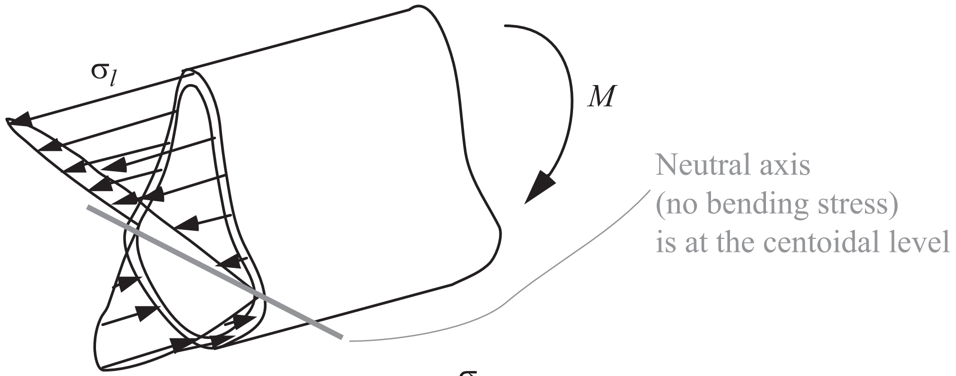
Do not cause sideways deflections, twist (antisymmetric deformations)

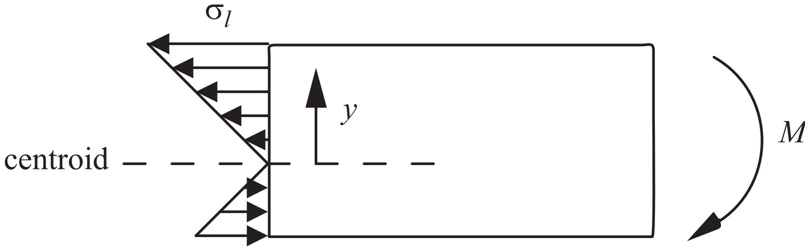
There will be more about symmetry and antisymmetry in Handout 3.

Axial force

$$\sigma_l = \frac{P}{A} \qquad = \frac{\text{force}}{\text{area}} \qquad \text{Resultant force acts through centroid}$$

Bending moments

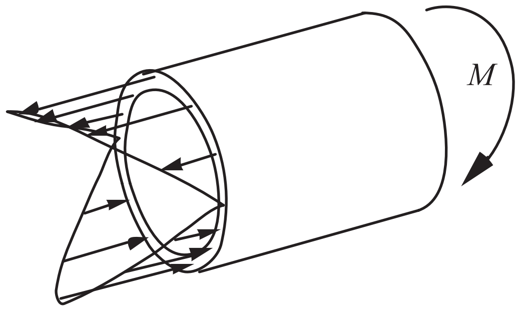


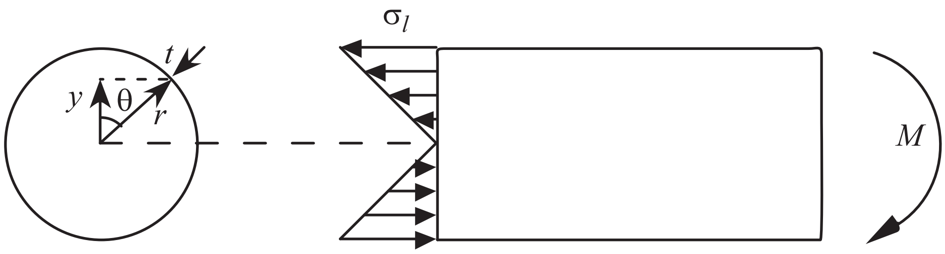


$$\sigma_l = \frac{My}{I} \qquad \text{from 1A, structures data book p.5}$$

Example: Longitudinal stresses in a circular section due to applied moments

Calculate the distribution of longitudinal stresses in a circular section due to an applied moment.





$$\sigma_l = \frac{My}{I}$$

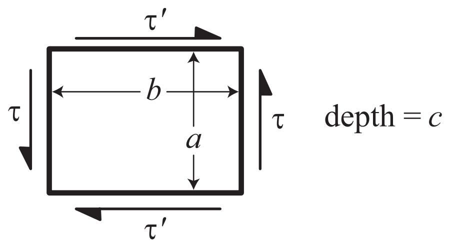
$$y = r\cos\theta$$

$$I = \int y^2\, dA = \pi r^3 t \qquad \text{(From the mechanics data book, or by integration)}$$

$$\sigma_l = \frac{M\cos\theta}{\pi r^2 t}$$

Shear Stress

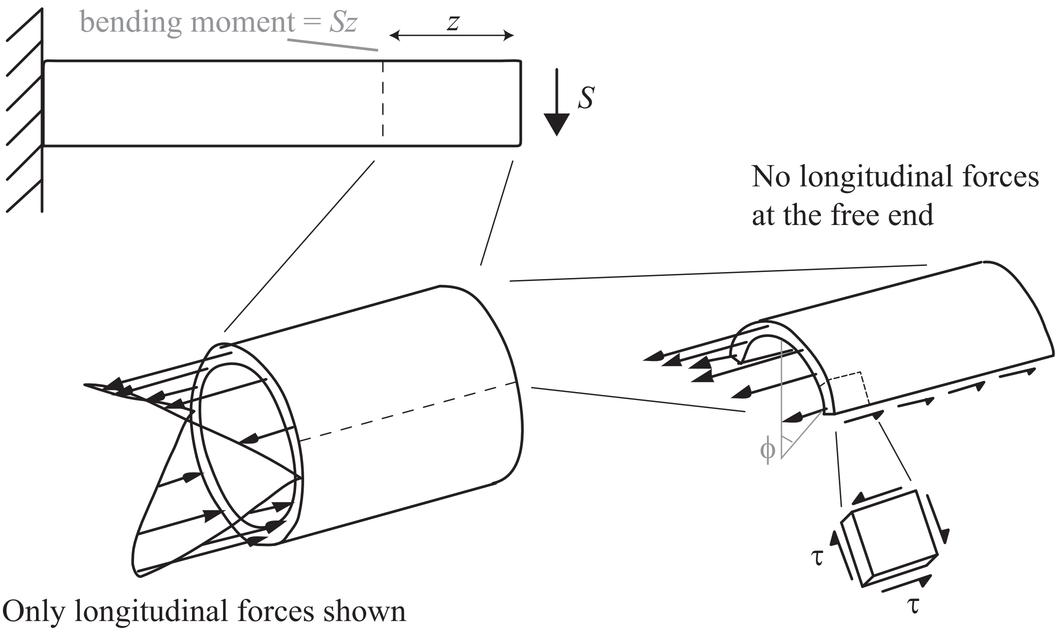
A Reminder About Complementary Shear Stress If a shearing stress τ is applied to a block of material, what is the shearing stress τ'?



Moment equilibrium: $\tau'(bc)(a) = \tau(ac)(b) \quad \Rightarrow \quad \tau' = \tau$
A state of simple shear requires equal shear stress on all four faces of an arbitrary small block. Stresses τ' are *complementary* to τ (and vice-versa).

Example (from first principles): Cylindrical cantilever subject to end load

Calculate from first principles the stresses for this simple case to show how the shear stress formula was derived.



From earlier example,

$$\sigma = \underbrace{Sz}_{\text{moment}} \frac{r\cos\theta}{I}$$

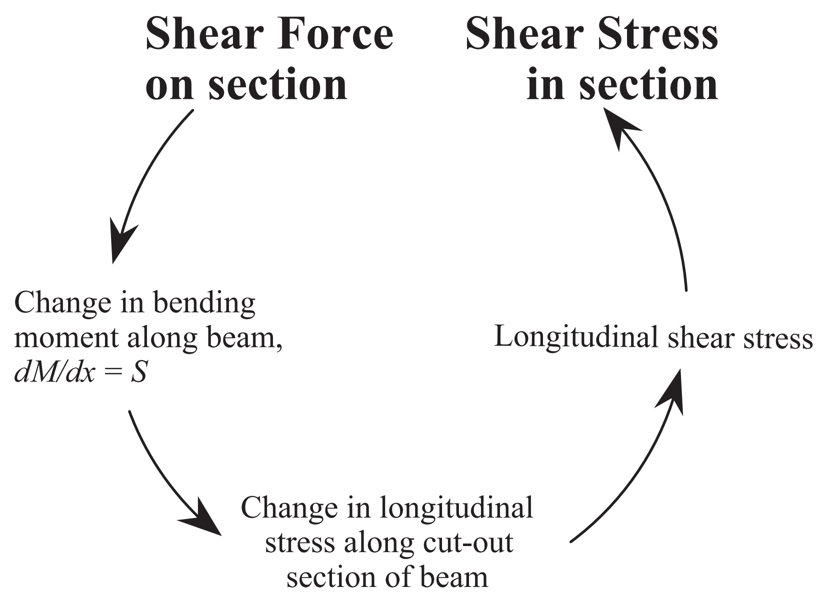
Shear forces acting along the cut edge must balance the end stresses.

$$
\begin{aligned}
2\,\tau z t &= \int_{-\phi}^{\phi} \sigma_l\, t\, r\, d\theta \\
\text{shear stress} \times \text{area} &= \text{total force due to longitudinal stresses} \\
&= \frac{Sz}{I}\int_{-\phi}^{\phi} r^2 t \cos\theta\, d\theta \\
&= \frac{Szr^2t}{I} 2\sin\phi
\end{aligned}
$$

$$
\begin{aligned}
\text{Total shear force/unit length} &= \frac{\text{shear stress} \times \text{area}}{z} \\
&= \frac{S}{I} 2r^2 t \sin\phi \\
&= \frac{S}{I} A_s\bar{y} \quad \text{— as data book}
\end{aligned}
$$

where $A_s\bar{y} = 2r^2 t \sin\phi$ is the *first moment of area of the cut out section*. You can check this in the Mechanics Data Book.

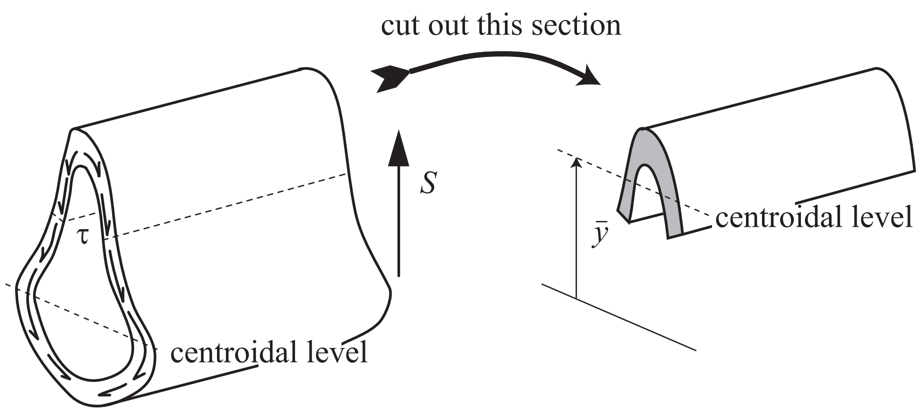
How to derive the formula for shear stress



Shear stress due to applied shear force

$$\text{shear force/unit length} = \frac{SA_s\bar{y}}{I}$$

(1A, and structures data book p6)



A_s cross-sectional area of the ‘cut out’ section, shown shaded.

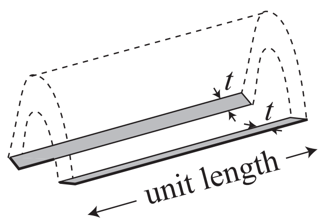
$\bar{y}$ distance from the centroid of the whole section to the centroid of the ‘cut out’.

$A_s\bar{y}$ first moment of area of the ‘cut out’ section.

S applied shear force.

I second moment of area of *whole* section.

The shear stress can then be found by dividing the shear force by the area of the longitudinal cut, shown shaded below.



$$\tau = \frac{\text{shear force}}{1 \times 2t} \qquad \text{in this case}$$

The *maximum* shear stress will be found with the *minimum* area, so the cut should be perpendicular to the wall

Effect of stiffeners

Longitudinal stiffeners will increase the cross-sectional area of the structure, and hence reduce the stress due to axial loads

$$\underbrace{\sigma_l}_{\text{reduced}} = \frac{P}{\underbrace{A}_{\text{increased}}}$$

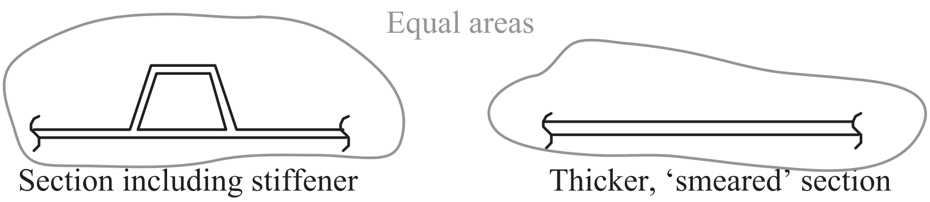
The stiffeners will also increase the second moment of area for the section, and hence also reduce the stresses due to bending.

$$\underbrace{\sigma_l}_{\text{reduced}} = \frac{My}{\underbrace{I}_{\text{increased}}}$$

The shear stresses due to shear-loading will not be changed significantly. The increase in I will generally be matched by an increase in $A_s\bar{y}$. The stiffeners may slightly redistribute the stresses.

$$\text{shear force/unit length} = \frac{S\ \overbrace{A_s\bar{y}}^{\text{increased}}}{\underbrace{I}_{\text{increased}}} \qquad \text{negligible overall effect}$$

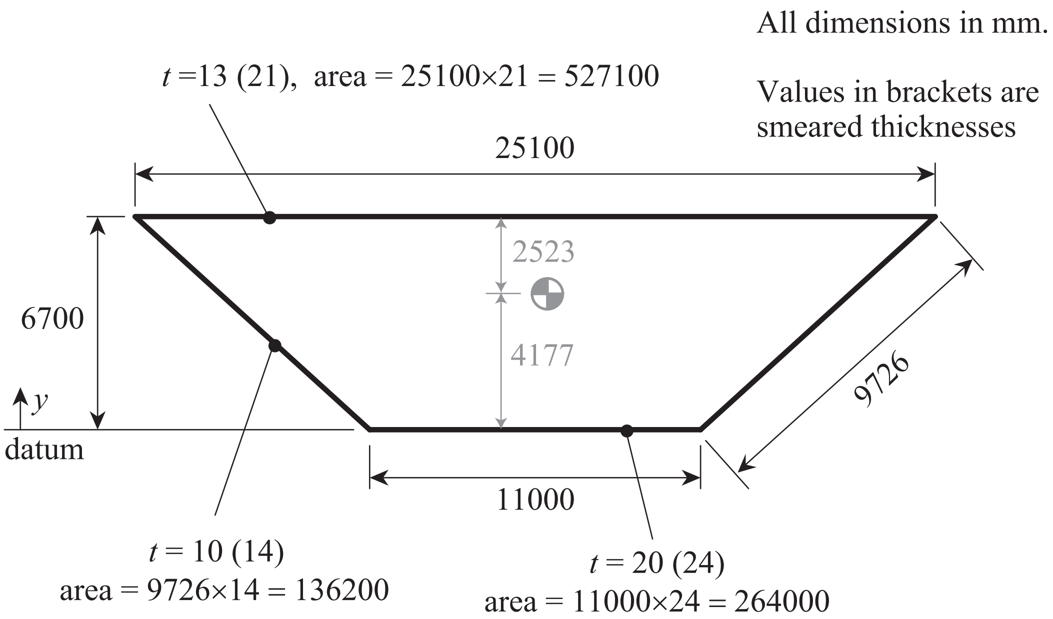
To avoid the full complications of calculating exact section properties including stiffeners, the stiffeners are often 'smeared' out by making the thin walls a little thicker.



It is necessary to have a good understanding of the problem to know when to use the smeared, and when to use the actual thickness. The case study should help to clarify with examples.

1.2.4 Case study part 1 — Stresses during the lifting of a Storebælt approach span

The exact cross-section used for the spans was shown earlier. Below is a slightly simplified cross-section that we will use, which omits the central diaphragm. The stiffeners have been ‘smeared’. The actual, and the smeared thickness, is shown for each part of the cross-section.



Section properties

Total Area

$$
\begin{aligned}
A &= 527100 + 2 \times 136200 + 264000 \quad \text{all mm}^2 \\
&= 1.064 \times 10^6 \text{ mm}^2 = 1.064 \text{ m}^2
\end{aligned}
$$

Mass/Unit length

$$\rho A = 7800 \text{ kg/m}^3 \times 1.064 \text{ m}^2 = 8300 \text{ kg/m}$$

Centroid

$$
\begin{aligned}
A\bar{y} &= \sum A_s \bar{y}_s \quad \text{for each section} \\
&= 527100 \times 6700 + 2 \times 136200 \times 6700/2 \quad \text{mm}^3 \\
&= 4.444 \times 10^9 \text{ mm}^3
\end{aligned}
$$

$$\therefore \bar{y} = 4177 \text{ mm}$$

2nd moment of area Consider a section at a time (around the centroid!), ignoring the 2nd moment of area of the flanges about their own centroid, (the $bd^3/12$ terms) as negligible.
Top flange

$$I_{\text{top}} = 527100 \times 2523^2 = 3.355 \times 10^{12} \text{ mm}^4$$

Bottom flange

$$I_{\text{bottom}} = 264000 \times 4177^2 = 4.606 \times 10^{12} \text{ mm}^4$$