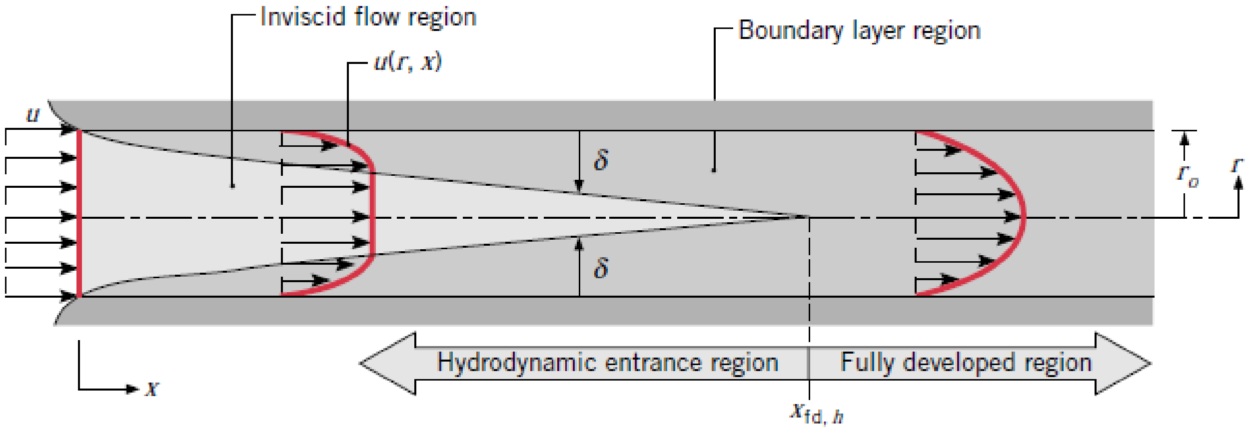
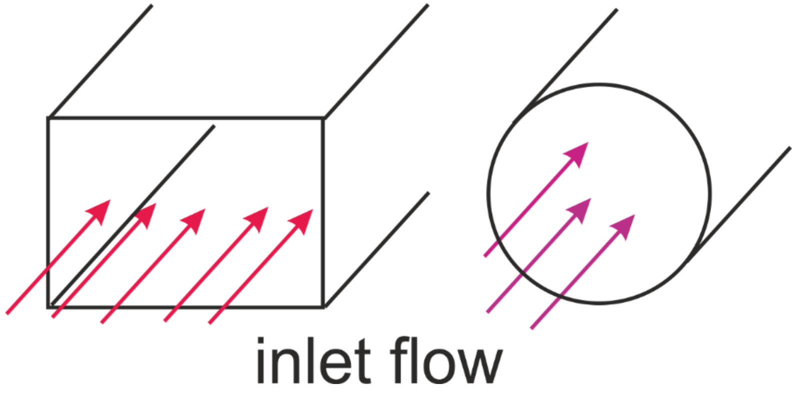
Fundamentals of Internal Flows

Internal flows-hydrodynamics

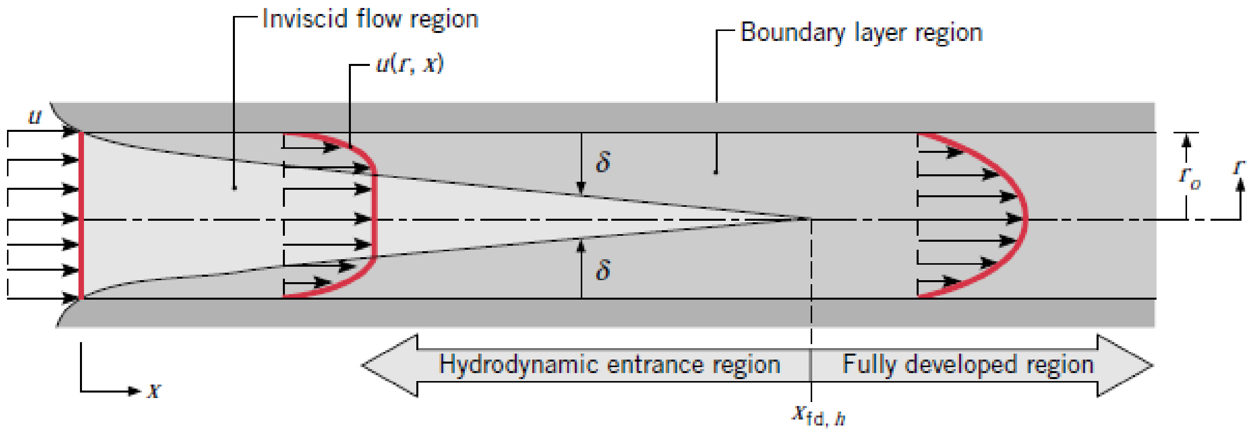
- In many applications, fluid flows inside a conduit (e.g. a pipe or channel).
- Convective heat transfer in these flows is regarded as internal flow convection.
- In internal flows a velocity boundary layer forms at the beginning of the pipe/channel.
- The flow is divided into two distinctive regions:
  - velocity boundary layer containing strong viscous effects,
  - effectively inviscid core flow.
- The boundary layer grows and the inviscid core shrinks (*entrance region*).
- In the entrance region the velocity profile is constantly changing.





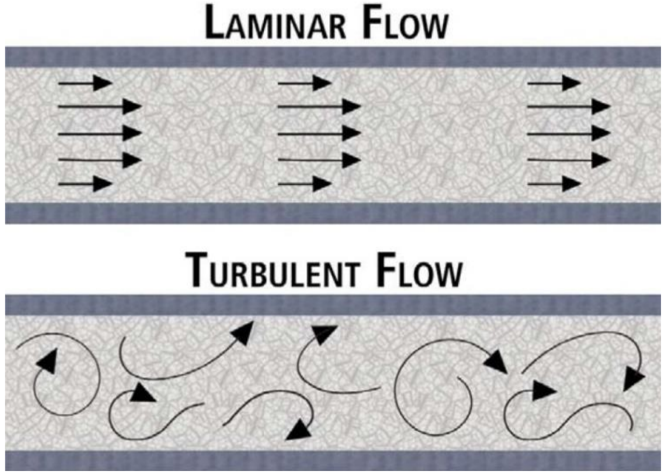
Internal flows-hydrodynamics, contd.

- Eventually the boundary layer fills the total volume of the pipe.
- Farther downstream of this point, the velocity profile remains unchanged.
- The flow is called *hydrodynamically fully developed*.
- In a hydrodynamically fully developed flow $u \neq u(x)$ and the velocity only changes transversally.
- Hydrodynamics of internal flows are characterised by the value of Reynolds number: $Re = \frac{VD}{\nu}$, where D is the pipe diameter.

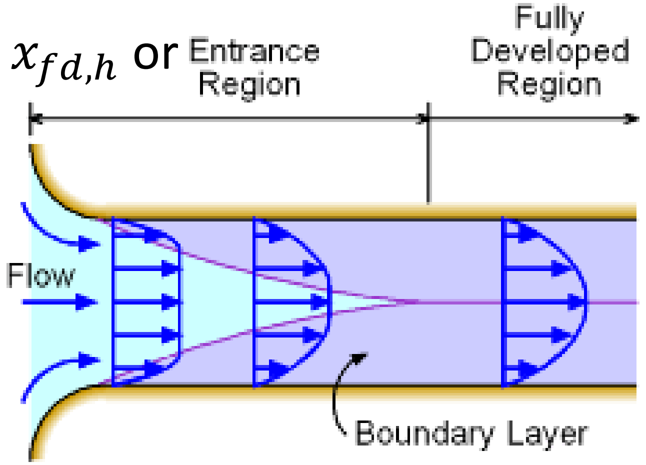


Turbulent and laminar flows

- Flow inside a pipe can be laminar or turbulent, this property of the flow significantly affects the heat transfer rate.
- If $Re_D \leq 2300$ the pipe flow is laminar, (this Re number is called ***the critical Reynolds number***)
- For greater Reynolds numbers, flow is not completely laminar and shows some turbulent features. For large enough Reynolds numbers ($Re_D \approx 10000$) it will be fully turbulent.
- The length of the entrance region depends upon the state of the flow (laminar or turbulent and the Reynolds number).
- For laminar flows $\left(\frac{x_{fd,h}}{D}\right) \approx 0.05 Re_D$, where $x_{fd,h}$ is the minimum length of the pipe downstream of that the flow is hydrodynamically fully developed (or the length of entrance region).
- For turbulent flows $10 \leq \left(\frac{x_{fd,h}}{D}\right) \leq 60$.



Schematic illustration of laminar and turbulent pipe flows



Velocity considerations

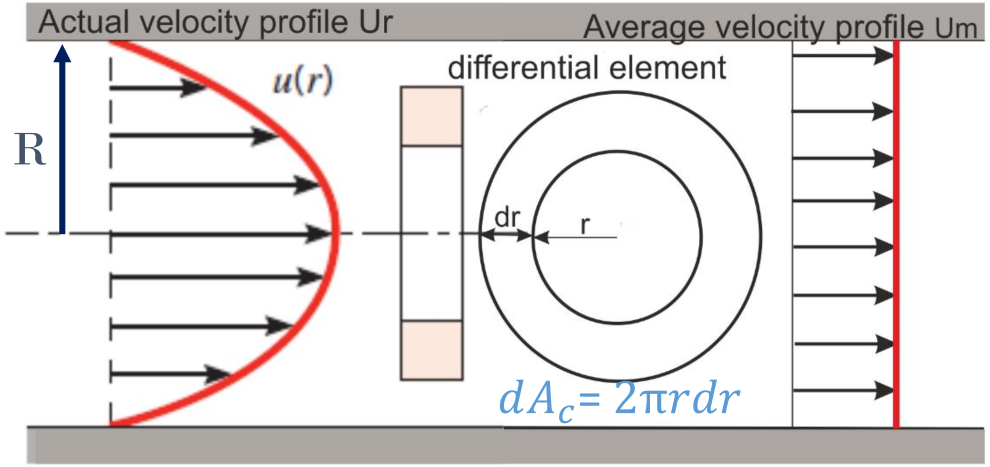
- Since velocity changes over the cross section, an area averaged velocity should be defined:

$\dot{m} = \rho u_m A_c$, where $\dot{m}$ is the mass flow rate of the fluid (kg/s), ρ is the density of fluid (kg/m^3), A_c is the cross sectional area of the pipe (m^2), and u_m is the average velocity.

- Since $\dot{m}=\int_{A_c} \rho u(r,x) dA_c$, it follows that for an *incompressible flow* (that is a flow with constant density) in a circular tube:

$$u_m = \frac{\int_{A_c} \rho u(r,x) dA_c}{\rho A_c} = \frac{2\pi\rho}{\rho\pi R^2}\int_0^R u(r,x) r dr = \frac{2}{R^2}\int_0^R u(r,x) r dr$$

- This expression may be used to determine u_m at any axial location x from the knowledge of the velocity profile at that location.



Flow velocity distribution in a duct, average flow velocity and the differential element

Hydrodynamic analysis of laminar flow

- Conservation of mass: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

  Flow is fully developed therefore $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}$=0. From conservation of mass $\frac{\partial v}{\partial y}$=0, and therefore $v = 0$.

- ***In the fully developed region, velocity field has only one axial component which remains unchanged along the pipe.***
- Consider flow in a cylinder of radius r, momentum conservation:

$$\pi r^2 dp - 2\pi r dx\tau = 0,$$
$$\tau = \mu \, {du}/{dr}$$

- Combining these two gives:

$$du = \frac{r}{2\mu}\left(\frac{dp}{dx}\right)dr \;\Rightarrow\; u(r) = \frac{r^2}{4\mu}\left(\frac{dp}{dx}\right) + C_1.$$

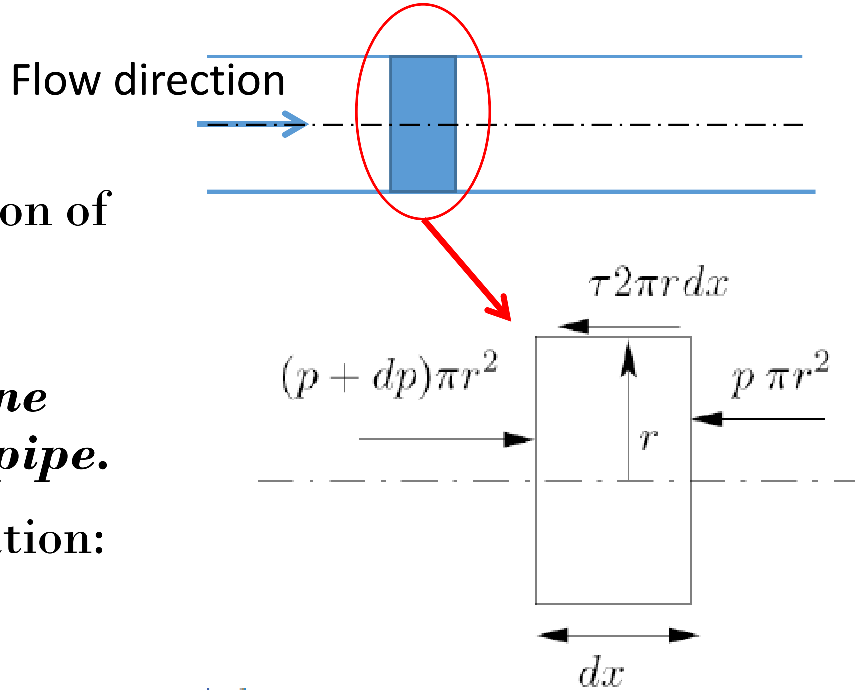


Illustration of the forces acting on an element of fluid in a pipe flow

- The condition of no-slip velocity at the wall yields ($u_{y=R}$=0)

$$u(r) = \frac{1}{4\mu}(\frac{dp}{dx})(r^2 - R^2).$$

- Now the mass flow rate is

$$\dot{m}=2\pi\rho \int_0^R ru(r)dr,$$

- We defined the bulk flow velocity as $U_m = \dot{m}/_{\rho A_c}$.

Thus:

$$\boxed{U_m = \frac{\dot{m}}{\rho A_c} = -\frac{R^2}{8\mu}(\frac{dp}{dx})}$$

$$\Rightarrow \boxed{\frac{u(r)}{U_m} = 2\left[1 - (\frac{r}{R})^2\right].}$$

- $\tau_s = \mu(\frac{du}{dr})\Big|_{r=R} = \frac{8\mu U_m}{D}$ and therefore, the skin friction coefficient for the pipe flow, C_f, which is also known as Fanning friction coefficient is

$$C_f = \frac{2\tau_s}{\rho U_m^2} = \frac{16}{Re_D} ,$$

Friction factor in fully developed flow

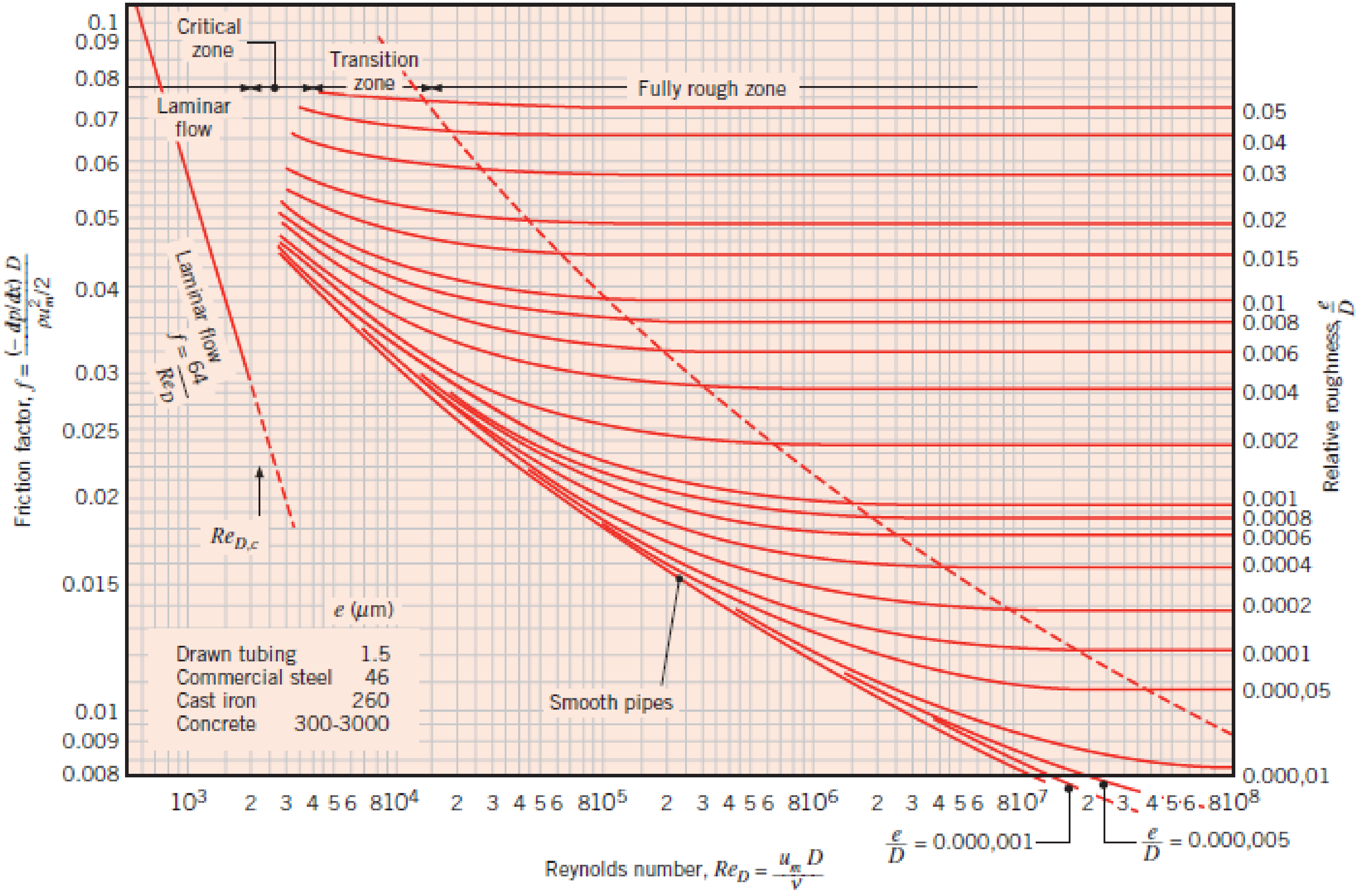
- Pressure drop in pipe flow is a very important engineering issue,
  - increases the pumping power,
  - increases the required energy and cost of the process.
- To evaluate this, a friction factor is often used. This is a dimensionless parameter defined as

$$f \equiv \frac{-\left({dP}/{dx}\right)D}{\rho u_m^2/2}$$

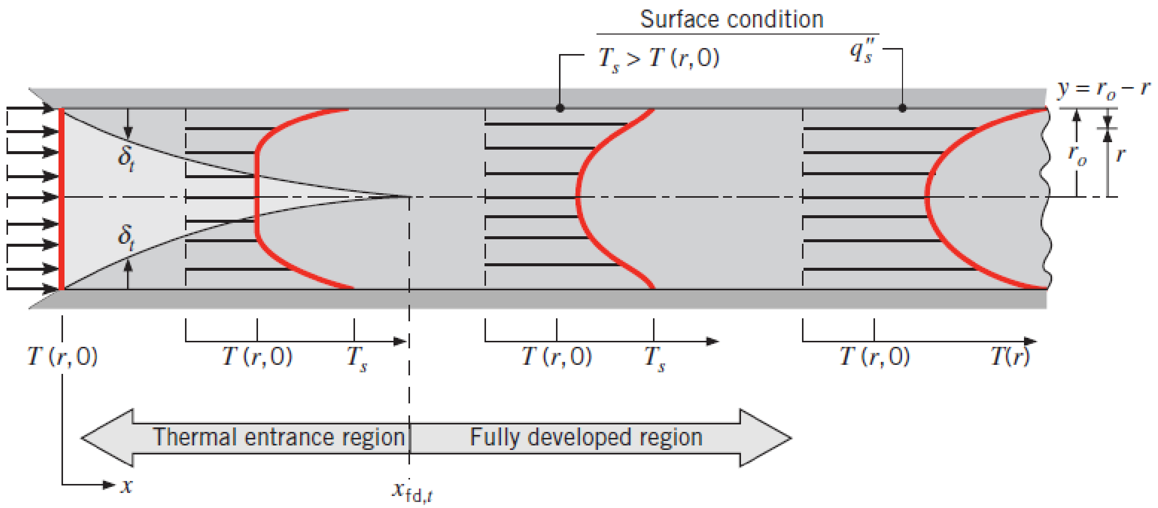
- Note that, friction factor is different to friction coefficient defined as $C_f = \frac{2\tau_s}{\rho U_m^2}$
- Since $\tau_s = \mu\left(\frac{du}{dr}\right)\Big|_{r=R}$ we can show that $C_f = \frac{f}{4}$.
- For fully developed laminar flow: $f = \frac{64}{Re_D}$ (derive this relation!)

Water pumps used in a power station. Friction losses have a strong impact upon the electricity that these pumps consume.

- For turbulent flows the analysis is much more complicated and empirical results are frequently used (Moody diagram).
- Note that in practice pressure drop is dominated by the Reynolds number and surface roughness of the pipe.



Thermal considerations



- If the pipe surface temperature is different to that of the fluid then a thermal boundary layer develops.
- Similar to velocity boundary layer, the thermal boundary layers grow and eventually meet.
- This leads to the formation of a thermally fully developed region.
- Note that the surface of the pipe can be under constant temperature or constant heat flux.
- The shape of the temperature profile in the fully developed region depends upon the relative temperature of the duct and fluid.
- For laminar flow: $(\frac{x_{fd,t}}{D})_{lam} \approx 0.05 Re_D Pr$

- Similar to velocity, the temperature field varies across the pipe cross section.

- An average temperature is defined on the basis of the total thermal energy transported.

- The rate of internal energy transported: $\dot{E}_t = \int_{A_c} \rho u c_v T dA_c$.

- If the mean temperature is defined such that $\dot{E}_t = \dot{m} c_v T_m$ then, $T_m = \frac{2}{u_m r_0^2} \int_0^{r_0} uTrdr$.

- If the surface temperature of the pipe is T_s then the local heat flux is

$$q'' = h(T_s - T_m) \text{ (note that } q'' = q/\text{A}),$$

where h is the local convection coefficient and T_s is the local temperate of the pipe surface.

- Note that T_m is never a constant and varies along the pipe, *why?*