Engineering Tripos Part IB SECOND YEAR

Part IB Paper 2: Structures

Examples Paper 2/4

Elastic structural analysis; plastic theory

*Straightforward questions are marked by †; Tripos standard questions by *.*

Elastic structural analysis: symmetry and anti-symmetry

† 1. Split the loading on the beam shown in Figure 1 into symmetric and anti-symmetric components and, hence, calculate the deflection of the centre of the beam due to the applied load. The beam has flexural rigidity EI.

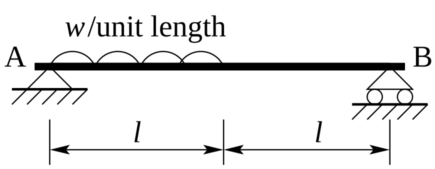

Figure 1

2. In the plane portal frame shown in Figure 2, all the joints are rigid and the flexural rigidity, EI, is uniform; the coefficient of thermal expansion is α.

   Setting the horizontal forces and bending moments at both feet to be redundant, but equal and opposite (why?), find the horizontal reaction and the bending moment induced at the column feet by a uniform temperature rise, T.

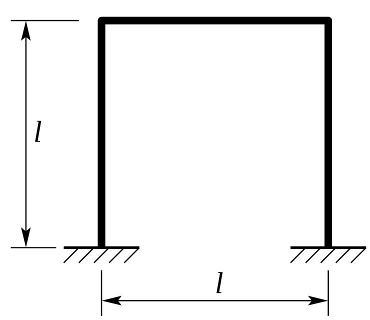

Figure 2

∗ 3. The ring structure in Figure 3 is fully fixed to its foundation and is initially stress-free. It has a uniform bending stiffness, EI. An in-plane vertical load, W, is applied at B, as shown.

(a) Sketch the ring and show all the forces acting upon it, including the reactions. Making use of symmetry and equilibrium, write all of the reactions in terms of H, the horizontal reaction at A, and the applied load, W.

(b) By enforcing compatibility, calculate the value of H.

(c) Calculate the horizontal and vertical deflection of B.

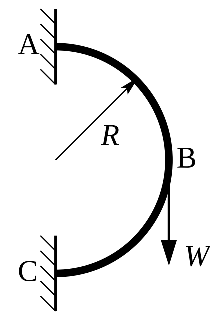

Figure 3

Plastic section modulus

† 4. Find the value of the plastic section modulus, Z_p, for bending about the axis, $x - x$, for each of the three sections shown in Figure 4. For section (b), compare your value with the corresponding section in the Structures Data book, p. 24; why is there a difference? For section (c), evaluate the fully plastic moment, M_p, given that the material yield stress is 355 N/mm^2.

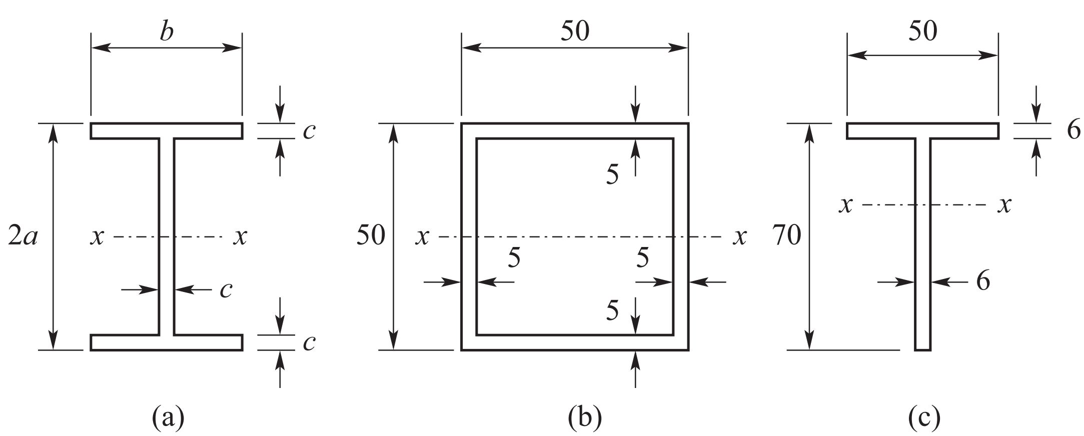

Figure 4

Plastic collapse: statically determinate beams

† 5. Figure 5 (overleaf) shows a uniform beam with fully plastic moment, M_p, simply supported at its ends. Two point loads are applied, simultaneously, as shown. Consider alternative collapse mechanisms to find the critical value of W.

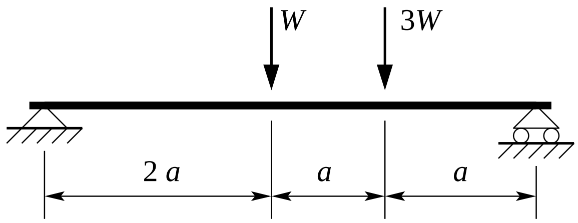

Figure 5

$*$ 6. Figure 6 shows a uniform beam, ABC, with fully plastic moment, M_p, simply supported at points A and B. The beam carries a load, $6W$, uniformly distributed over the span, AB, together with a point load, W, at the tip, C.

(a) Find the value of W at collapse based on an assumed collapse mechanism involving a single plastic hinge occurring at B.

(b) Consider an alternative collapse mechanism with a single hinge in the span, AB, occurring at an arbitrary distance, x, from A. Find the corresponding collapse load, W, in terms of x; hence, find the minimum value of the collapse load for this mechanism, and the corresponding value of x.

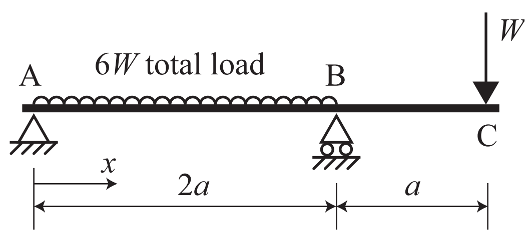

Figure 6

Plastic collapse: statically indeterminate beams and frames

† 7. Figure 7 shows a uniform beam simply supported at three points, A, B and C. Find the value of the load, W, for plastic collapse to occur, given that the fully plastic moment is M_p. The support at C provides total restraint to vertical movement: is this a relevant consideration? Does the length of the span, BC, affect the limiting value of W?

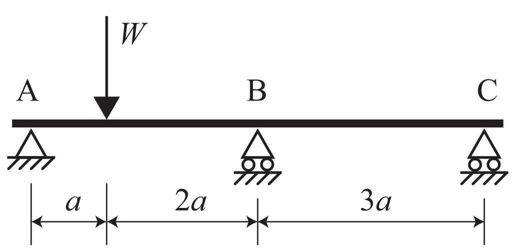

Figure 7

∗ 8. Figure 8 shows a uniform beam, ABC, of fully plastic moment, M_p, which is built in at A and simply supported at B. The beam carries a total load of magnitude, $5W/4$, uniformly distributed over the length, AC.

(a) Consider, initially, only collapse mechanisms that have a plastic hinge at one or more of the points; A, B, mid-span of AB. Find a good upper bound on the value of W at collapse.

(b) Make an improved upper bound estimate by allowing the position of the hinge near the mid-span to vary. What is the percentage change in the estimate from (a)?

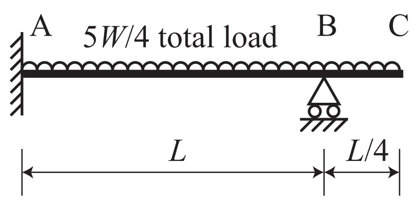

Figure 8

9. The light portal frame shown in Figure 9 has full strength joints and fixed feet. the fully plastic moment, M_p, is 60 kNm for the beam and 40 kNm for each column. The frame is to carry working loads as shown. Determine the load factor, λ, that, when applied to both loads together, will cause the frame to collapse plastically. Use a work calculation for various hypothetical mechanisms.

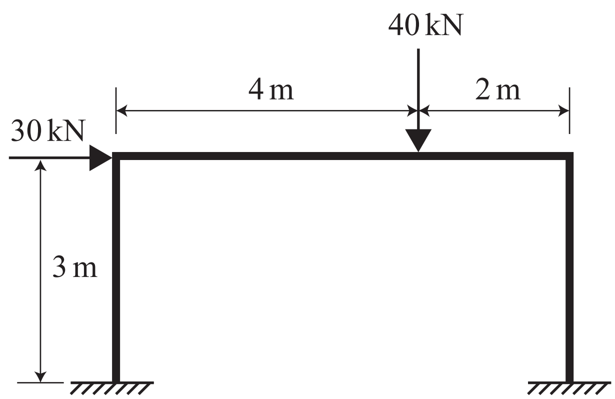

Figure 9

∗ 10. Draw an Interaction Diagram showing the collapse load of the 2-D structure shown in Figure 10 (overleaf) for any positive values of V and H, and calculate the collapse load if $V = H$. The fully plastic moment is M_p throughout. Joint A is fully clamped: B, C and D are full-strength connections; E is pinned.

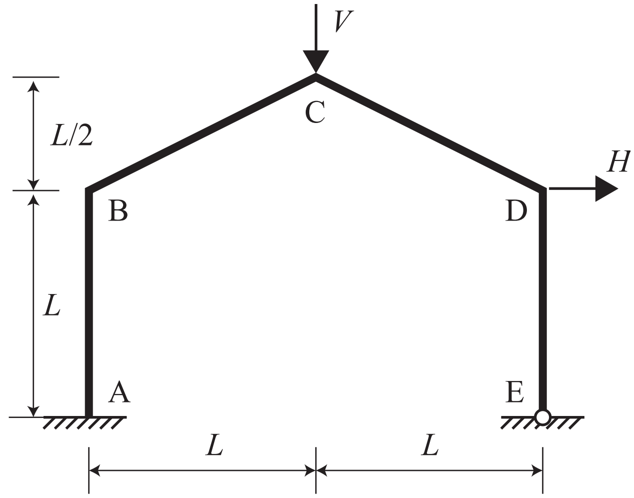

Figure 10

Suitable Tripos questions

Part IB Paper 2: 2021/2,5; 2019/3; 2018/5; 2017/3,5; 2016/2,3,4.

ANSWERS

1. $5wl^4/48EI$.
2. $3EI\alpha T/l^2$ (inwards), $-2EI\alpha T/l$ (giving tension on the inside of the frame).
3. (b) $0.318W$ to the left at A. (c) Zero horizontal deflection, vertical deflection $0.019WR^3/EI$ (downwards).
4. (a) $bc(2a-c)+(a-c)^2c$. (b) 15.3 cm^3 (SDB: 14.5 cm^3). (c) 12.9 cm^3, 4.58 kNm.
5. $0.364M_p/a$.
6. (a) M_p/a. (b) $2M_pa/(x(5a-3x))$, $24M_p/25a$ for $x=5a/6$.
7. $2M_p/a$.
8. $13.71M_p/L$, $13.60M_p/L$ with hinge at $0.542L$ from A, 0.8%.
9. 1.52, other mechanisms give 1.78 or 1.875.
10. Sway, $H=3M_p/L$; beam, $H+V=5M_p/L$; combined, $2H+V=6M_p/L$; at collapse, $H=V=2M_p/L$.

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