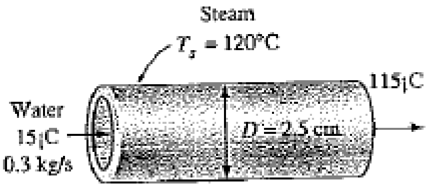
Tutorial 1

1- Water enters a 2.5 cm internal diameter thin copper tube of a heat exchanger at 15°C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120°C. If the average heat transfer coefficient is 800 $\frac{W}{m^2.°C}$, determine the length of the tube in order to heat the water to 115°C (see the figure).



Solution:
Water is heated by steam in a circular tube. The tube length required to heat the water to a specified temperature is to be determined.

Assumptions:

1- Steady operating conditions exit
2- Fluid properties are constant
3- The convection heat transfer coefficient is constant
4- The conduction resistance of copper tube is negligible so that the inner surface temperature of the tube is equal to the condensation temperature of steam.

Properties The specific heat of water at the bulk mean temperature of (15+115)/2=65°C is 4187 J/kg.°C. The heat of condensation of steam at 120°C is 2203 kJ/kg.

Analysis

Knowing the inlet and exit temperatures of water, the rate of heat transfer is determined to be

$$\dot{Q} = \dot{m}\, c_p(T_e - T_i) = \left(0.3\frac{kg}{s}\right)\left(4.187\frac{kJ}{kg°C}\right)(115°C - 15°C) = 125.6kW$$

The logarithmic mean temperature difference is

$\Delta T_e = T_s - T_e$=120°C − 115°C = 5°C

$\Delta T_i = T_s - T_i$=120°C − 15°C = 105°C

$$\Delta T_{lm} = \frac{\Delta T_e - \Delta T_i}{\ln(\frac{5}{105})} = \frac{5-105}{\ln(5/105)} = 32.85°C$$

The heat transfer surface area is

$$\dot{Q} = hA_s\Delta T_{lm} \rightarrow A_s = \frac{\dot{Q}}{h\Delta T_{lm}} = \frac{125.6\ kW}{(0.8\frac{kW}{m^2.°C})(32.85°C)} = 4.78m^2$$

Then the required tube length becomes

$A_s = \pi DL \rightarrow L = \frac{A_s}{\pi D} = \frac{4.78\ m^2}{\pi(0.025m)} = 61m$

Discussion: The bulk mean temperature of water during this heating process is 65℃, and thus the arithmetic mean temperature difference is ΔT_{am} =120-65=55℃. Using ΔT_{am} instead of ΔT_{lm} would give L= 36m, which is grossly in error. This shows the importance of using the logarithmic mean temperature in calculations.

2- Compare the hydrodynamic and thermal entry lengths of mercury (liquid metal) and a light oil flowing at 3.0 mm/s in a 25.0 mm diameter smooth tube at a bulk temperature of 75^0C. The pertinent parameters of the fluids at that temperature are: v_{Hg}=1.0×10^{-7} m^2/s, v_{oil}=6.5×10^{-6} m^2/s, $Pr_{Hg} = 0.019$, $Pr_{oil} = 85$.

Solution:

The Reynolds numbers based on the tube diameters are:

$$Re_{Hg} = \frac{VD}{v_{Hg}} = \frac{(3.0 \times 10^{-3} m/s)(25.0 \times 10^{-3} m)}{1.0 \times 10^{-7}\ m^2/s} = 750$$

$$Re_{oil} = \frac{VD}{v_{oil}} = \frac{(3.0 \times 10^{-3} m/s)(25.0 \times 10^{-3} m)}{6.5 \times 10^{-6}\ m^2/s} = 11.53$$

The hydrodynamic entry lengths are given by $(\frac{x_{fd,h}}{D}) \approx 0.05 Re_D$ so

$$x_{h,Hg} \approx (0.05)(750)(25.0mm) = 937.5\ mm$$

$$x_{h,oil} \approx (0.05)(11.53)(25.0mm) = 14.4\ mm$$

For the thermal entry lengths, $(\frac{x_{fd,t}}{D})_{lam} \approx 0.05 Re_D Pr$

$$x_{t,Hg} \approx (0.05)(750)(0.019)(25.0mm) = 17.8\ mm$$

$$x_{t,oil} \approx (0.05)(11.53)(85)(25.0mm) = 1225\ mm$$

Note the short thermal entry of mercury compared to its hydrodynamic entry lengths. What has caused this significant difference?

3- For heating water from 20°C to 60°C an electrically heated tube resulting in a constant heat flux of 10 kW/m^2 is proposed. The mass flow rate is to be such that Re_D=2000, and consequently the flow must remain laminar. The tube inside diameter is 25mm. The flow is fully developed (velocity profile). Determine (a) the length of tube required and (b) if thus proposed heating system is feasible with regard to wall temperature.

Solution

We first calculate the average temperature which is $\bar{T}_h$=$\frac{20+60}{2}\ ^\circ C$=40°C

The thermo-physical properties should be evaluated (from the table) at this average temperature

$\rho = 994.6 \frac{kg}{m^3}$ $\qquad$ $C_p = 4.1784 \frac{kJ}{kg.K}$

Pr=4.34 k= 0.628 W/m.K

ν=0.658$\times 10^{-6}\ m^2/s$

(a) The required tube length is obtained by an energy balance, i.e.,

$$q_{conv} = \dot{m}c_p(T_{b,o} - T_{b,i}) = q''(A_s)$$

To determine $\dot{m}$ we need to know the bulk velocity. Since $Re_D = 2000$,

$\frac{DV}{\nu} = 2000;\ V = \frac{2000\,\nu}{D} = \frac{2000(0.658\times 10^{-6}\ m^2/s)}{0.025\ m} = 0.0526\ m/s$

and

$\dot{m}C_p = \rho V\left(\frac{\pi D^2}{4}\right)(c_p) = \left(\frac{994.6\ kg}{m^3}\right)\left(0.0526\frac{m}{s}\right)\left(\frac{\pi}{4}\right) \times (0.025\ m)^2\left(4178\frac{J}{kg.K}\right) = 107.3\ W/K$

Rearranging expression (i) we have

$A_s = \pi DLq'' = \dot{m}c_p(T_{b,o} - T_{b,i});\qquad L = \frac{\dot{m}c_p(T_{b,o}-T_{b,i})}{\pi D q''}$

Inserting numerical values with $\dot{m}c_p$ from expression (ii) yields

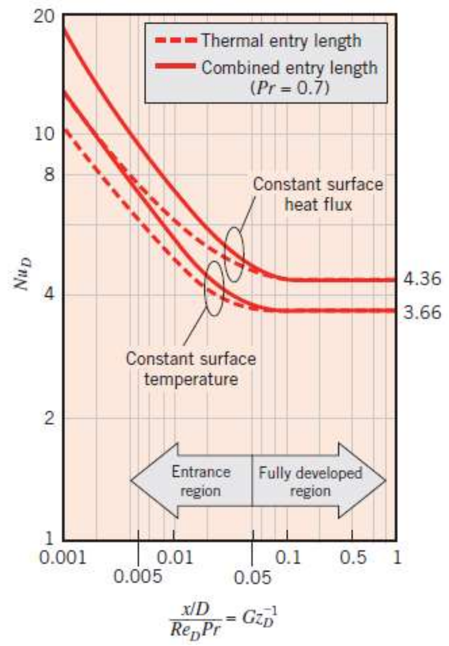
$$L = \frac{(107.3\ W/K)(60^\circ C - 20^\circ C)}{\pi(0.025m)(10000\ W/m^2)} = 5.46\ m$$

(b) Is this flow feasible? Is the wall temperature realistic?

With the flow being hydrodynamically fully developed, we check the length of the thermal entry section given by

$$x_{t,fd} = 0.05 Re_D Pr D = 0.05(2000)(4.34)(0.025\text{m}) = 10.85\text{m}$$

and clearly this is a case of constant q''; developed velocity and developing thermal profile. The Nusselt number may be obtained from the Figures below



The inverse of Graetz number at the exit is $Gz^{-1} = \left(\frac{x}{D}\right) Re_D Pr = (5.46\ m/0.025m)/(2000)(4.34) = 0.0252 = 2.52\times 10^{-2}$

and reading the dashed line for constant surface heat flux in the figure above, **Nu**=5.0

where this is the local value at the tube exit where the wall temperature will be maximum. Hence,

$$h = \frac{k}{D} Nu_D = \left(\frac{0.628 \frac{W}{mK}}{0.025\ m}\right)(5.0) = 125.6 \frac{W}{m^2.k}$$

Then with known q'', $T_{b,i}$ and *h*, the exit plane surface temperature is

$$q'' = h(T_s - T_{b,o}); \qquad T_s = \frac{q''}{h} + T_{b,o}$$

$$T_s = \frac{10\ kW}{m^2}\left(\frac{1}{125.6 \frac{W}{m^2.K}}\right) + 60°\text{C}$$

$$T_s = 139.6°\text{C} \approx 412\ K$$

This is slightly high temperature for the wall material. If necessary, the wall heat flux could be reduced thereby lengthening the tube, increasing the Greatz^{-1}number, and lowering Nu_D and h very slightly. The T_s is more affected by the q'' than the *h*, for example a 50% reduction in q'' would lower $T_{s,o}$ by approximetly 35°C.

4-Air at 1.0 atmosphere pressure and 77°C enters a 5.0 mm i.d. tube with a bulk average velocity of 2.5 m/s. The velcoity profile is developed and the thermal profile is "developing". The tube length is 1.0 m, and a constant (uniform) heat flux is imposed by the tube surface on the air over the entire length. An exit air bulk average temperature, $T_{h.o}=127^{\circ}$C, is required. Determine (a) the exit *h* value, $h_{L,}$ (b) the uniform q_s'', and (c) the exit tube surface temperature.

We will need air properties at the bulk average temperature inside the tube,

$\frac{T_{b,i}+T_{b,o}}{2} = \frac{(77+127)}{2} = 102°\text{C}$=375 K

From property tables we obtain

$\rho = 0.9403\ kg/m^3$	$c_p = 1.0115 \frac{kJ}{kg.K}$
$\mu = 2.1805 \times 10^{-5}$ Kg/m.s	$\nu = 23.33 \times 10^{-6}\ m^2/s$
k= 0.03184 W/m.K	Pr=0.693

The Reynolds number is

$$Re_D = \frac{DV}{\nu} = \frac{(0.005\ m)(2.5\ m/s)}{23.33\times10^{-6}\ m^2/s} = 536$$

and the flow is laminar. The thermal entry length is

$(x_{t,fd}/\text{D}) = 0.05 Re_D(Pr) = 26.8(0.693) = 18.6$

The L/D for this tube is (1.0/0.005) which is 200. A reasonable approach is to consider the flow, both velocity and temperature, to be fully developed over its entire tube length, since only about 10% is experiencing entry effects. For fully developed flow, constant q'', the Nusselt number is 4.36, so

$$Nu_D = \frac{hD}{k} = 4.364$$

$h = 4.364\frac{k}{D} = 4.364\frac{(0.03184\frac{W}{m.K})}{0.005\,m}$= 27.79 W/$m^2.K = h_L = h = constnt$

Use an overall energy balance

$$q_{conv} = q''(A_s) = \dot{m}c_p(T_{b,o} - T_{b,i})$$

where

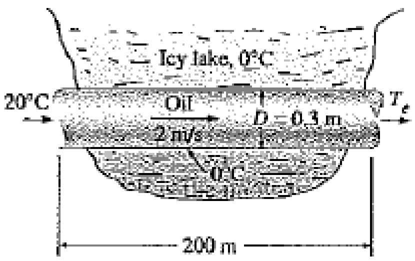
$A_s = \pi DL = \pi(0.005m)(1\,m)$=$1.57 \times 10^{-2}m^2$

5- Consider the flow of oil at 20^0C in a 30-cm- diameter pipeline at an average velocity of 2 m/s (see the figure below). A 200-m-long section of the horizontal pipeline passes through icy waters of a lake at 0^0C. Measurements indicate that the surface temperature of the pipe is very nearly 0^0C. Disregarding the thermal resistance of the pipe material, determine

(a) the temperature of the oil when the pipe leaves the lake

(b) the rate of heat transfer from the oil,

(c) the pumping power required to overcome the pressure losses and to maintain the flow of the oil in the pipe.



Oil flows in a pipeline that passes through icy waters of a lake at 0°C. The exit temperature of the oil, the rate of heat loss, and the pumping power needed to overcome pressure losses are to be determined.

Assumptions:

1- Steady operating conditions exits
2- The surface temperature of the pipe is very nearly 0℃
3- The thermal resistance of the pipe is negligible.
4- The inner surfaces of the pipeline are smooth
5- The flow is hydrodynamically developed when the pipeline reaches the lake.

Properties: we do not know the exit temperature of the oil, and thus we cannot determine the bulk mean temperature, which is the temperature at which the properties of oil are to be evaluated. The mean temperature of the oil at the inlet is 20℃, and we expect this temperature to drop somewhat as a result of heat loss to the icy waters of the lake. We evalaute the properties of the oil at the inlet temeparture, but we will repeat the calculations, if necessary, using properties at the evaluated bulk mean temperature. At 20℃ we read

$\rho = 888.1\ kg/m^3$	$v = 9.429 \times 10^{-4}\ m^2$/s	
k= 0.145 W/m.℃	c_p=1880 J/ kg.℃	Pr=10,863

Analysis:

(a) The Reynolds number is

$$Re = \frac{V_{avg}D}{v} = \frac{(2\ m/s)(0.3m)}{9.429 \times 10^{-4} m^2/s} = 636$$

Which is less than the critical Reynolds number and hence the flow is laminar. The thermal, entry length in this case is roughly

$x_{t,fd} \approx 0.05RePrD = 0.05 \times 636 \times 10863 \times (0.3m) \approx 103600m$

which is much greater than the total length of the pipe. This is typical of fluids with high Prandtl numbers. Therefore, we assume thermally developing flow and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 3.66 + \frac{0.065\left(\frac{D}{L}\right)RePr}{1+0.04\left[\left(\frac{D}{L}\right)RePr\right]^{\frac{2}{3}}} =$$

$$3.66 + \frac{0.065\left(\frac{0.3}{200}\right) \times 636 \times 10{,}863}{1 + 0.04[(0.3/200) \times 636 \times 10863]^{2/3}} = 33.7$$

Note that this Nusselt number is considerably higher than the fully developed value of 3.66. Then

$$h = \frac{k}{D}Nu = \frac{0.145\,\frac{W}{m°C}}{0.3\ m}(33.7) = 16.3\ W/m^2°C$$

Also,

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$

$$= 0°C - [(0-20)°C]\exp\left[-\frac{\left(163\,\frac{W}{m^2°C}\right)(188.5\ m^2)}{\left(125.6\,\frac{kg}{s}\right)\left(1881\,\frac{J}{kg}°C\right)}\right] = 19.74°C$$

Thus, the mean temperature of oil drops by a mere 0.26°C as it crosses the lake. This makes the bulk mean oil temperature 19.87°C, which is practically identical to the inlet temperature of 20°C. Therefore, we do not need to re-evaluate the properties.

(b) The logarithmic mean temperature difference and the rate of heat loss from the oil are

$$\Delta T_{lm} = \frac{T_i - T_e}{ln\frac{T_s - T_e}{T_s - T_i}} = \frac{20 - 19.74}{ln\frac{0 - 19.74}{0 - 20}} = -19.87°C$$

$$\dot{Q} = hA_s\Delta T_{lm} = \left(16.3\,\frac{W}{m^2}°C\right)(188.5m^2)\left(-19.87^\circ C\right) = -6.11 \times 10^4 W$$

Therefore, the oil will lose heat at a rate of 61.1 kW as it flows through the pipe in the icy waters of the lake. Note that ΔT_{ln} is identical to the arithmetic mean temeparture in this case, since $\Delta T_i \approx \Delta T_e$.

(C) The laminar flow of oil is hydrodynamically developed. Therefore, the friction factor can be determined from

$$f = \frac{64}{Re} = \frac{64}{636} = 0.1006$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho V^2}{2} = 0.1006\frac{200\ m}{0.3\ m}\frac{(888.1\ kg/m^3)(2\ m/s)^2}{2} = 1.19\times 10^5 N/m^2$$

$$\dot{W}_{pump} = \frac{(125.6\frac{kg}{s})(1.19\times 10^5 N/m^2)}{888.1\ kg/m^3} = 16.8\ kW$$

Discussion We need a 16.8 kW pipe just to overcome the friction in the pipe as the oil flows in 200 m long pipe through the lake.

6- Air at 2 atm and 200℃ is heated as it flows through a tube with a diameter of 25.4 mm at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant-heat-flux condition is maintained at the wall and the wall temperature is 20℃ above the air temeparture, all along the length of the tube. How much the bulk temperature increase over a 3-m length of the tube?

Solution

We first evaluate the Reynolds number at the inlet bulk temperature to determine the flow regime. The properties of air at 200℃ are

$\rho = \frac{P}{RT} = \frac{(2)(1.0132\times 10^5)}{(287)(473)} = 1.493\ kg/m^3$	$c_p = 1.025\ kJ/kg℃$
$\mu = 2.57\times 10^{-5}\ \frac{kg}{m.s}$	$Pr = 0.681$
$k = 0.0386\ \ W/m℃$	

$$Re_d = \frac{\rho u_m d}{\mu} = \frac{(1.493)(10)(0.0254)}{2.57\times 10^{-5}} = 14{,}756$$

so that the flow is turbulent.

$$Nu_d = \frac{hd}{k} = 0.023Re_d^{0.8}Pr^{0.4} = (0.023)(14756)^{0.8}(0.681)^{0.4} = 42.67$$

$$h = \frac{k}{d}Nu_d = \frac{(0.0386)(42.67)}{0.0254} = 64.85\ W/m^2℃$$

The heat flow per unit length is then

$$\frac{q}{L} = h\pi d(T_s - T_b) = (64.85)\pi(0.0254)(20) = 103.5\ W/m$$

We can now make an energy balance to calculate the increase in bulk temperature in a 3.0 m length of tube:

$$q = \dot{m}c_p\Delta T_b = L(\frac{q}{L})$$

We also have

$\dot{m}=\rho u_m\frac{\pi d^2}{4} = (1.493)(10)\pi\frac{(0.0254)^2}{4}=7.565\times 10^{-3}\ kg/s$

So that we insert the numerical values in the energy balance to obtain

$$(7.565\times 10^{-3})(1025)\Delta T_b = (3.0)(103.5)$$

and

$$\Delta T_b = 40.04℃$$