Lecture 1: Three-phase Circuits I

1.1 Overview

Three main aspects of electrical power are:

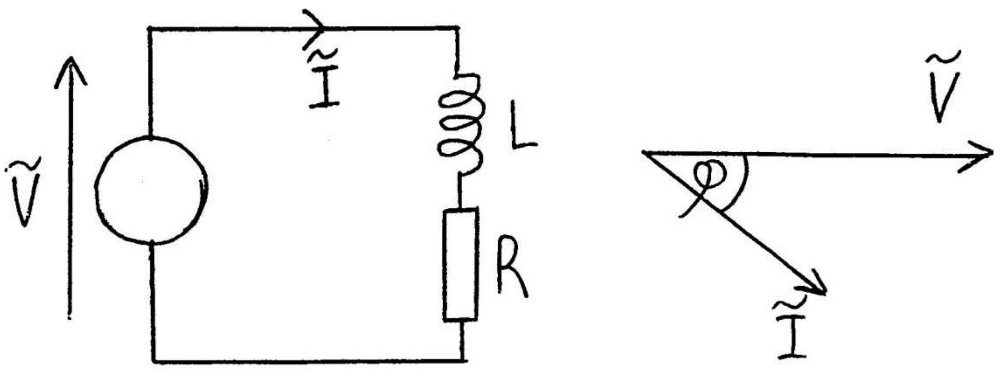
i) Generation

ii) Transmission

iii) Utilisation

Common theme is the use of balanced three-phase a.c. sources and loads.

1.1.1 Revision of 1A Electrical Power



$\overline{Z}_L = j\omega L$ (Inductor) $\overline{Z}_C = 1/j\omega C$ (Capacitor)

Real Power - Units: Watts.

Physically: Rate at which electrical energy converted into another form (heat, light etc).

Electrical power engineering is concerned with three main disciplines:

i) Generation - the conversion of various forms of source energy into electrical energy.

ii) Transmission - the process of transporting the electrical energy from where it is produced to where it is actually consumed.

iii) Utilisation - the conversion of electrical energy into industrially usable form.

In this course, each of these aspects will be considered in turn. The common thread which links all of them is the use of balanced three-phase a.c. systems, operating at 50 Hz (the reasons for these major infrastructure choices will be explained later on in the course). Consequently, we commence this course by a study of such systems.

However, before we do that, we will briefly look at the 1A material on electrical power, since it forms much of the basis of this course.

Consider the single-phase circuit shown opposite, consisting of an a.c. voltage supply and a resistor - inductor load. Inductors cause current to lag the supply voltage, hence the phasor diagram is as depicted opposite. The angle between the voltage and current is equal to the angle of the load impedance, which is $\tan^{-1}(X/R)$.

From v=Ldi/dt it may be shown that the impedance of an inductor is jωL, where ω is the angular frequency of the supply.

From i=Cdv/dt it may also be shown that the impedance of a capacitor is 1/jωC.

Only resistors can dissipate real power.

$$P = VI\cos\varphi = I^2R = V_R{}^2/R = \mathrm{Re}\left\{\tilde{V}\tilde{I}^*\right\}$$

Factor $\cos\varphi$ is known as **power factor**.

Reactive Power - Units: VARs (Volt-amps reactive).

Physically: Rate at which electrical energy is exchanged between supply and energy storage components (capacitors and inductors).

Only capacitors and inductors associated with reactive power.

$$Q = VI\sin\varphi = \mathrm{Im}\left\{\tilde{V}\tilde{I}^*\right\}$$

Sign convention

Reactive power is **consumed** by inductors and is positive $\Rightarrow$ $Q_L = I^2X_L = V_L{}^2/X_L$

Reactive power is **generated** by capacitors and is negative $\Rightarrow$ $Q_C = -I^2X_C = -V_C{}^2/X_C$

P and Q are both conserved

If the phasor current is resolved into two components, one of which is in phase with the supply voltage, and the other 90^0 out of phase, then it was shown last year that the electrical power which is converted irreversibly is given by the product of the voltage and the in-phase component of current. This quantity is referred to as real power (or just power), and is only associated with **resistors**. Notice that this gives the alternative expressions I^2R and V^2/R. It is vital to understand that the I and the V in these expressions refer to the current through and the voltage across the resistor itself, respectively.

You also met the concept of reactive power last year, and saw that it is the product of the supply voltage and the 90^0 out of phase component of current (otherwise known as the quadrature component of current). Reactive power differs physically from real power in that it represents energy which is exchanged between the voltage supply and components which are capable of storing energy i.e. capacitors and inductors. Capacitors and inductors are **not** capable of dissipating real power; conversely, resistors are not capable of consuming reactive power.

You also saw last year that inductors consume reactive power, whereas capacitors generate it, giving rise to the sign convention opposite i.e. inductive reactive power is positive, capacitive reactive power is negative.

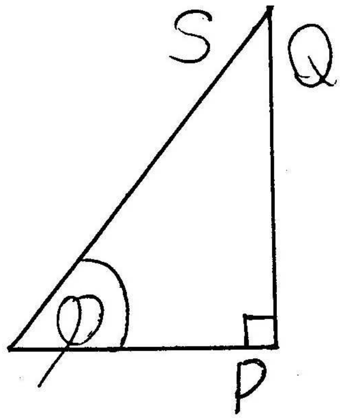
Another important point is that P and Q are both conserved. This means that all the real power consumed by an electrical load must be supplied from the voltage source in the circuit, and likewise for the reactive power.

Apparent Power - Units: VA (Volt-amps)

$$S = VI = \left((VI\cos\varphi)^2 + (VI\sin\varphi)^2 \right)^{1/2}$$

$$= \left(P^2 + Q^2 \right)^{1/2}$$

$\Rightarrow$ Power triangle

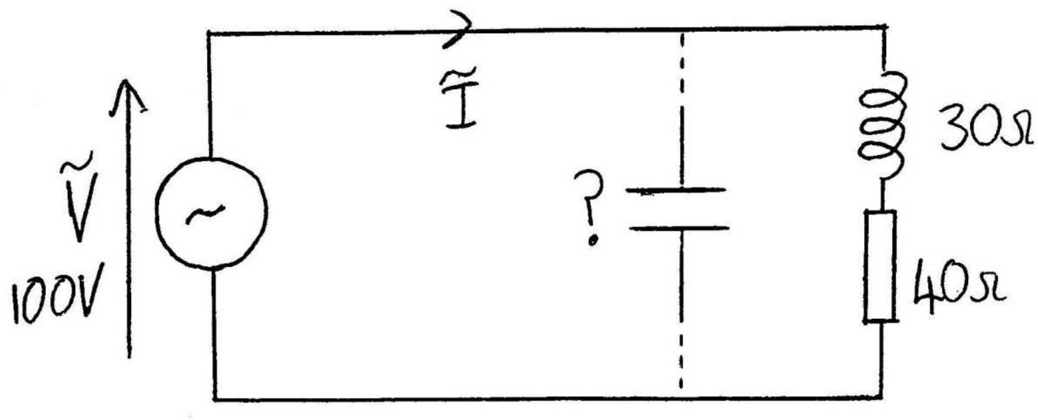


cosφ = P/S

sinφ = Q/S

tanφ = Q/P

Example - Find $|\tilde{I}|$, P, Q, S and power factor. Find capacitor such that power factor is unity.



$$|\tilde{I}| = \frac{|\tilde{V}|}{|\bar{Z}|} = \frac{100}{\left(40^2 + 30^2\right)^{1/2}} = 2\,A$$

Finally, you also met the idea of apparent power, S, which is simply the product of the magnitude of voltage and current, with no account taken of the phase angle between them. Because of the relationship (given opposite) between P, Q and S, the **power triangle** is a useful tool to help solve electrical power problems. It sums up all the relationships in an easily-remembered form.

Here we look at an example which applies most of the ideas you saw in Part 1A.

Firstly the magnitude of the current is found. For circuits with components in series, it is always advisable to find the current and then use I^2R, I^2X for P and Q respectively. One of the most common mistakes is using V^2/R, V^2/X for circuits with components in series, and taking V as the total series voltage. This is incorrect, since the voltage required in these expressions is the voltage across the resistor and inductor respectively, which is not known without making a separate calculation. However, the current is common to all components in a series-connected circuit.

$$P = I^2 R = 2^2 \times 40 = 160W$$

$$Q = +I^2 X_L = 2^2 \times 30 = 120VAR$$

$$S = VI = 100 \times 2 = 200VA$$

Check using power triangle:

$$S = \left(P^2 + Q^2\right)^{1/2} = \left(160^2 + 120^2\right)^{1/2} = 200 \text{ VA}$$

$$\cos\varphi = P/S = 160/200 = 0.8\, lagging$$

For unity power factor, total reactive power = 0

$$\therefore Q_{CAP} + Q_{LOAD} = 0 \quad \Rightarrow \quad Q_{CAP} = -120VAR$$

$$Q_{CAP} = -\frac{V^2}{X_C} = -\frac{100^2}{X_C} = -120VAR$$

$$X_C = 100^2/120 = 83.3\Omega = 1/2\pi f C$$

$$\therefore C = 1/\left(2\pi \times 50 \times 83.3\right) = 38.2\mu F$$

Using the facts that only resistors dissipate real power, and inductors reactive power, we find P and Q as shown opposite. Notice that the reactive power is given a positive sign, since it is associated with the inductor.

The apparent power may be found simply as the product of V and I, or by using the power triangle.

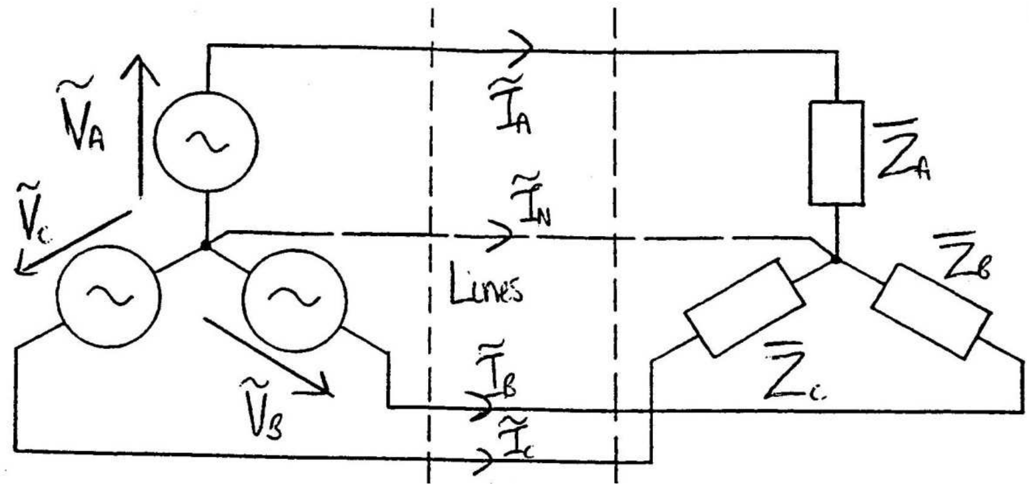
The power factor is given by the power triangle as P/S. Note that it is quoted as **lagging**, since inductive loads are associated with lagging power factors (current lags voltage). Capacitive loads, on the other hand, are associated with **leading** power factors, since current leads voltage.

For the overall power factor to be unity, the total reactive power must be zero (unity power factor means φ=0 and so Q=0). Therefore, the capacitor must generate the same reactive power as the inductor consumes.

Since the voltage across the capacitor is known, we use $Q=V^2/X_C$.

Finally, we find C from X_C using $X_C=1/\omega C$.

1.2 Star-connected three-phase supply



Supply Fig. 1.1 Load

A balanced three-phase voltage supply consists of three voltages of equal magnitude, but differing in phase by 120^0 with respect to each other. When these voltages are connected together as shown in fig. 1.1, the resulting three-phase supply is **star-connected**., because of the shape formed. The common point at the centre is known as the star point, and is always at 0 volts. Because of this, it is also known as the neutral point, and if there is a conductor connected to this point, it is known as the neutral conductor (or just neutral). The voltages w.r.t. this point are known as the phase voltages.

Balanced three-phase supply $\Rightarrow$ three voltages of equal magnitude differing in phase by 120^0.

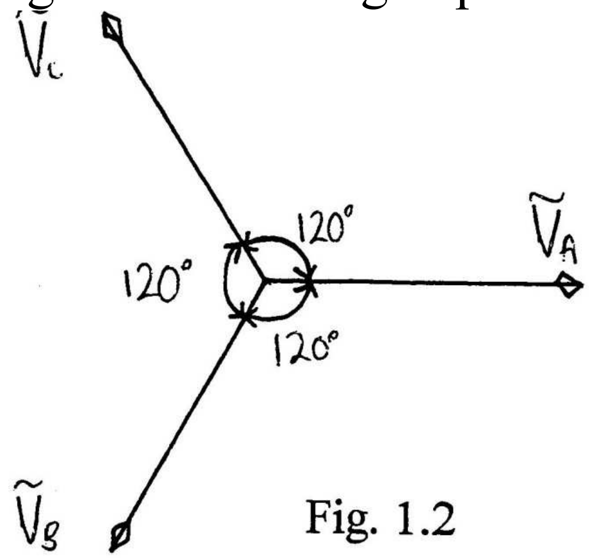


Fig. 1.2

These ideas are summed up in the phasor diagram of figure 1.2, and mathematically in equation 1.1.

Phase voltages are voltages w.r.t. ground i.e. $\tilde{V}_A$, $\tilde{V}_B$ and $\tilde{V}_C$.

Mathematically:
$$\tilde{V}_A = Ve^{j0} = V$$
$$\tilde{V}_B = V\,e^{-j\frac{2\pi}{3}} \quad \tilde{V}_C = V\,e^{j\frac{2\pi}{3}} \qquad (1.1)$$

Line voltages - voltages between pairs of lines:

$$\tilde{V}_{AB} = \tilde{V}_A - \tilde{V}_B,\ \ \tilde{V}_{BC} = \tilde{V}_B - \tilde{V}_C\ ,\ \ \tilde{V}_{CA} = \tilde{V}_C - \tilde{V}_A.$$

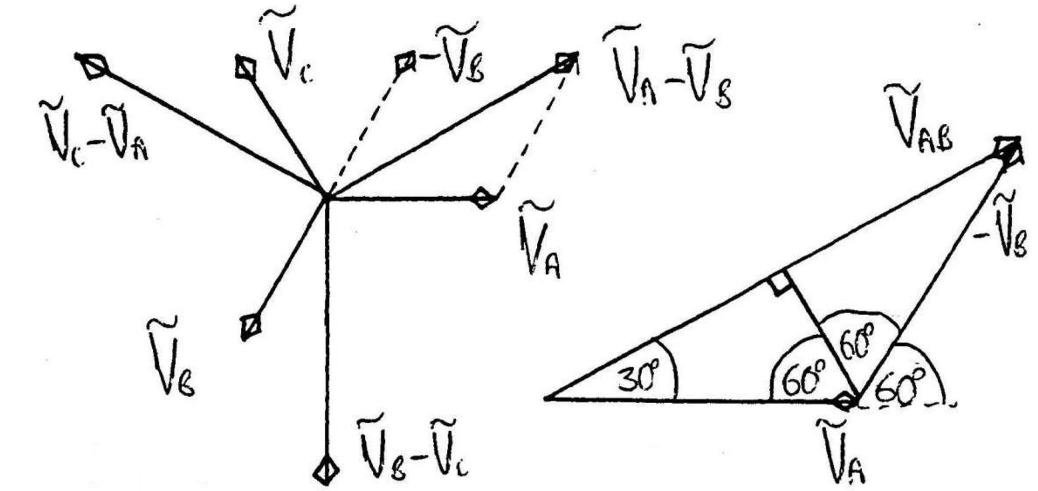


Fig. 1.3

Trig:
$$\left|\tilde{V}_{AB}\right| = 2\left|\tilde{V}_A\right|\sin 60^0 = \sqrt{3}V$$
$$\angle\tilde{V}_{AB} = \angle\tilde{V}_A + 30^0 = 30^0 \qquad (1.2)$$

Result: Line voltage =$\sqrt{3}\times$phase voltage

Balanced three-phase load: $\bar{Z}_A = \bar{Z}_B = \bar{Z}_C = \bar{Z}$

Ohms' law:
$$\bar{I}_A = \frac{\bar{V}_A}{\bar{Z}} \qquad (1.3)$$

$$\bar{I}_B = \frac{\bar{V}_B}{\bar{Z}} = \frac{\bar{V}_A e^{-j\frac{2\pi}{3}}}{\bar{Z}} = \bar{I}_A e^{-j\frac{2\pi}{3}} \qquad (1.4)$$

$$\bar{I}_C = \frac{\bar{V}_C}{\bar{Z}} = \frac{\bar{V}_A e^{j\frac{2\pi}{3}}}{\bar{Z}} = \bar{I}_A e^{j\frac{2\pi}{3}} \qquad (1.5)$$

Another balanced three-phase set of voltages is obtained by taking the potential difference between pairs of lines. This is known as the line-line voltage, or simply, the line voltage.

To determine the relationship between line and phase voltages, phasor subtraction is performed on the phase voltages to determine the line voltages, as shown in fig. 1.3.

Phasor V_{AB} forms the base of an iscolese triangle; the other two sides are the phasors V_A and V_B, and the vertex angle is 120^0. Dividing the isosceles triangle into two equal triangles, it is seen that half the length of its base is $V_A\sin60^0$, and therefore the magnitude of the line voltage is given by equation 1.2. Furthermore, if the vertex angle is 120^0, the other two angles in the isosceles triangle must be 30^0, showing that the line voltages lead the phase voltages by 30^0.

This result is an extremely important one, which crops up in the analysis of three-phase circuits over and over again.

Now consider what happens when the three-phase supply is connected to a balanced three-phase star-connected load. The term 'balanced' when applied to the load simply means that the impedance of each phase is identical.

Each phase of the load will have the supply phase voltage across it, and so current will flow in each phase according to Ohm's Law, equations 1.3, 1.4, 1.5. By substituting for phase B and phase C voltages in terms of the phase A voltage, it is seen that the currents in each phase of the load will be equal in magnitude, and have a phase difference between each other of 120^0. In other words, the load currents also form a balanced three-phase set.

$\therefore$ Currents form balanced three-phase set.

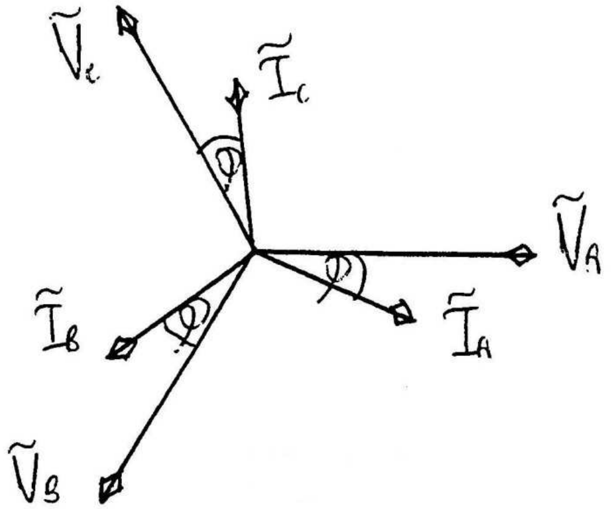


Fig. 1.4

Also, line current = phase current.

Neutral current found by summing currents at star point:

$$\tilde{I}_A + \tilde{I}_B + \tilde{I}_C + \tilde{I}_N = 0 \qquad (1.6)$$

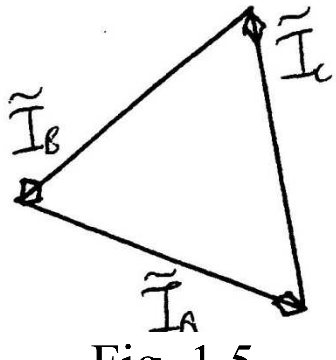


Fig. 1.5

$$\therefore \quad \tilde{I}_N = 0$$

Conclusion: For a balanced 3-phase load it doesn't matter whether or not the neutral conductor is connected.

This fact is illustrated in fig. 1.4, which shows the phase voltages and currents on a phasor diagram.

It is also clear that for a star-connected load, the phase current and the line current (line current meaning the current which flows in the supply lines which connect the voltage source to the load) are identical.

What about the current flowing in the neutral conductor ? This is obtained by applying Kirchoff's Current Law to the star point, equation 1.6.

Summing the phase currents to obtain the neutral current is illustrated in the phasor diagram of fig. 1.5, which shows that the neutral current is zero !

The conclusion from this is that for a balanced three-phase voltage supply connected to a balanced star-connected three-phase load, it doesn't matter whether or not the neutral is connected.

1.3 Three-phase real and reactive power

Consider the complex volt-amps in each phase.

$$\overline{S}_A = P_A + jQ_A = \widetilde{V}_A \widetilde{I}_A^* \tag{1.7}$$

$$\overline{S}_B = \widetilde{V}_B \widetilde{I}_B^* = \widetilde{V}_A e^{-j\frac{2\pi}{3}} \left(\widetilde{I}_A e^{-j\frac{2\pi}{3}} \right)^* = \widetilde{V}_A \widetilde{I}_A^* = \overline{S}_A \tag{1.8}$$

$$\overline{S}_C = \widetilde{V}_C \widetilde{I}_C^* = \widetilde{V}_A e^{j\frac{2\pi}{3}} \left(\widetilde{I}_A e^{j\frac{2\pi}{3}} \right)^* = \widetilde{V}_A \widetilde{I}_A^* = \overline{S}_A \tag{1.9}$$

For a balanced 3-phase load, P, Q and S are identical in each phase.

$$\overline{S}_{TOTAL} = \overline{S}_A + \overline{S}_B + \overline{S}_C = 3\overline{S}_A \tag{1.10}$$

Total 3-phase P = 3× (P per phase)

Total 3-phase Q = 3× (Q per phase)

Total 3-phase S = 3× (S per phase)

The real and reactive power consumed by the three-phase load can be obtained by summing the real and reactive power consumed by each phase of the load. Recalling from the First Year course that complex volt-amps are given by the product of the complex voltage and the conjugate of the complex current, it is seen that the real and reactive power supplied to phase A is given by equation 1.7. Applying the same ideas for phases B and C, and then substituting for the phase B and C currents and voltages in terms of the respective phase A quantities give equation 1.8 and 1.9.

These demonstrate that the complex volt-amps supplied to phases B and C are identical to those supplied to phase A.

The total three-phase complex volt-amps supplied to the load is obtained by summing the complex volt-amps supplied to each phase, equation 1.10.

Since the complex volt-amps supplied to each phase are identical, the total three-phase real, reactive and apparent power are obtained by multiplying the respective per-phase quantities by three.

1.4 Single-phase representation

Since each phase behaves identically, only need to analyse one of them.

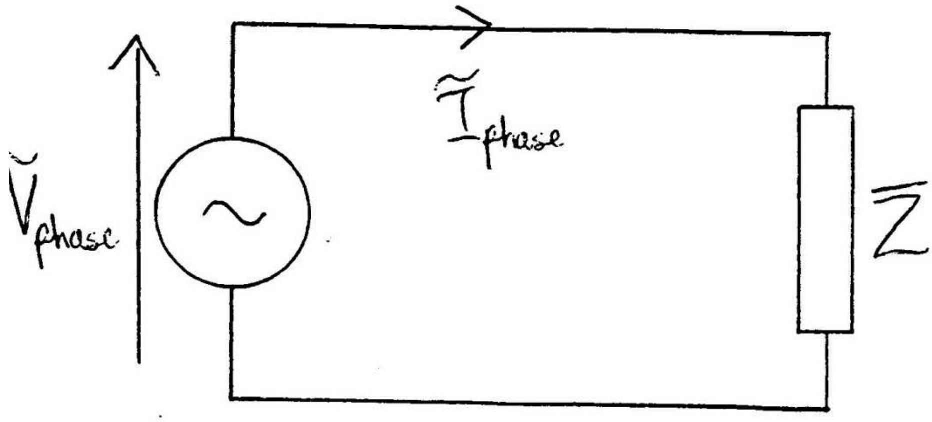


Fig 1.6

It should now be clear that for this situation, there is no need to analyse each phase. It is sufficient to consider only one of the phases, since what is happening in the other two phases in terms of P, Q and S is identical, and in terms of V and I is related by phase shifts of $\pm 120^0$. It is therefore possible to use the single-phase representation shown in fig 1.6, but remembering that to obtain total real power, reactive power and apparent power, it is necessary to multiply the respective per-phase quantities by 3.

Example 1.1

Each phase of a balanced three-phase star-connected load consists of a resistance of 30 Ω in series with an inductive reactance of 40 Ω. Find the phase voltage, phase current, line current, the real, reactive and apparent power consumed, and the power factor when connected to a three-phase 415 V supply.

It is normal practise to quote the line voltage when specifying the voltage of a three-phase supply. This is because the line voltage can always easily be measured, whereas measuring the phase voltage relies on having access to a star point. Therefore, the 415 V specified in this example is the line voltage at the load, and since the load is star-connected, the phase voltage i.e. the voltage across each phase of the load is obtained by dividing the line voltage by √3.

$$V_{phase} = \frac{415}{\sqrt{3}} = 240 \text{ V} \qquad \text{(star-connected)}$$

$$\bar{Z} = (30 + j40) = \sqrt{30^2 + 40^2}\angle \tan^{-1} 40/30$$
$$= 50\angle 53.1^o \ \Omega$$

The impedance is resistor-inductor in series, and is therefore equal to (30+j40) Ω (notice the '+j' term, because inductive reactance was specified).

To find the phase current, the load impedance is converted into polar form.

Use single-phase representation

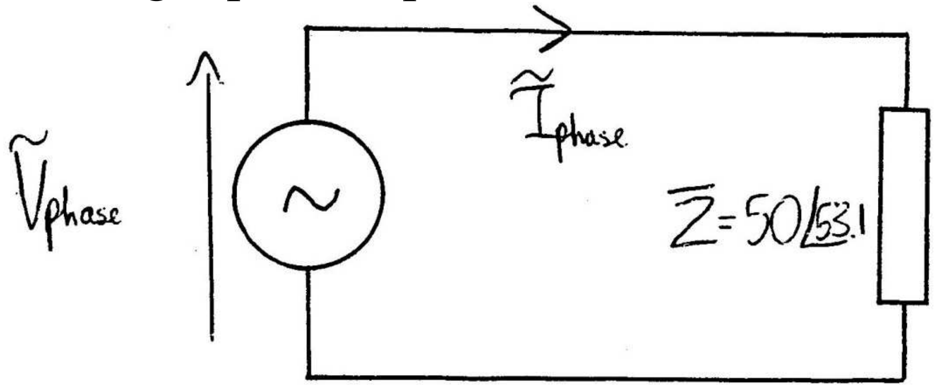


Fig. 1.7

As seen in section 1.4, a single-phase representation is possible, since each phase behaves identically except for phase shifts in voltage and current. This is illustrated in fig. 1.7.

Ohm's law is now applied to the circuit shown in fig. 1.6 to obtain the phase current i.e. the current which actually flows through the load impedance.

$$\bar{I}_{phase} = \frac{\bar{V}_{phase}}{\bar{Z}} = \frac{240}{50\angle 53.1^o} = 4.8\angle -53.1^o\ A$$

For a star-connected load, line and phase currents are identical.

Star-connected: $\bar{I}_{line} = \bar{I}_{phase} = 4.8\angle -53.1^o\ A$

The real power per phase can be found from $P=I^2R$, and the reactive power per phase can be found from $Q=I^2X$.

To obtain the total three-phase real and reactive power, we simply multiply the respective per-phase quantities by 3.

P per phase = $I^2_{phase}R = 4.8^2 \times 30 = 691.2\ W$

$\therefore$ Total P = 3 × 691.2 = 2073.6 W

Q per phase = $I^2_{phase}X = 4.8^2 \times 40 = 921.6\ VAR$

$\therefore$ Total Q = 3 × 921.6 = 2764.8 VAR

The total apparent power and the power factor can be found from the power triangle.

The power factor can be obtained directly from the power triangle as P/S. We could also have used the idea that the power factor is the cosine of the load impedance angle. Notice that the power factor is quoted as lagging, since the load is inductive.

Use power triangle to find S and cosφ.

$$S = \sqrt{P^2 + Q^2} = \sqrt{2073.6^2 + 2764.8^2} = 3456\ VA$$

$$\cos\varphi = {}^{P}\!/_{S} = 2073.6/3456 = 0.6\ lagging$$