Lecture 7: Transmission I

7.1 Overview

Power systems have a high level of inter-connection and operate over a wide range of voltages.

Per-unit system enables simplification of such complex power transmission systems.

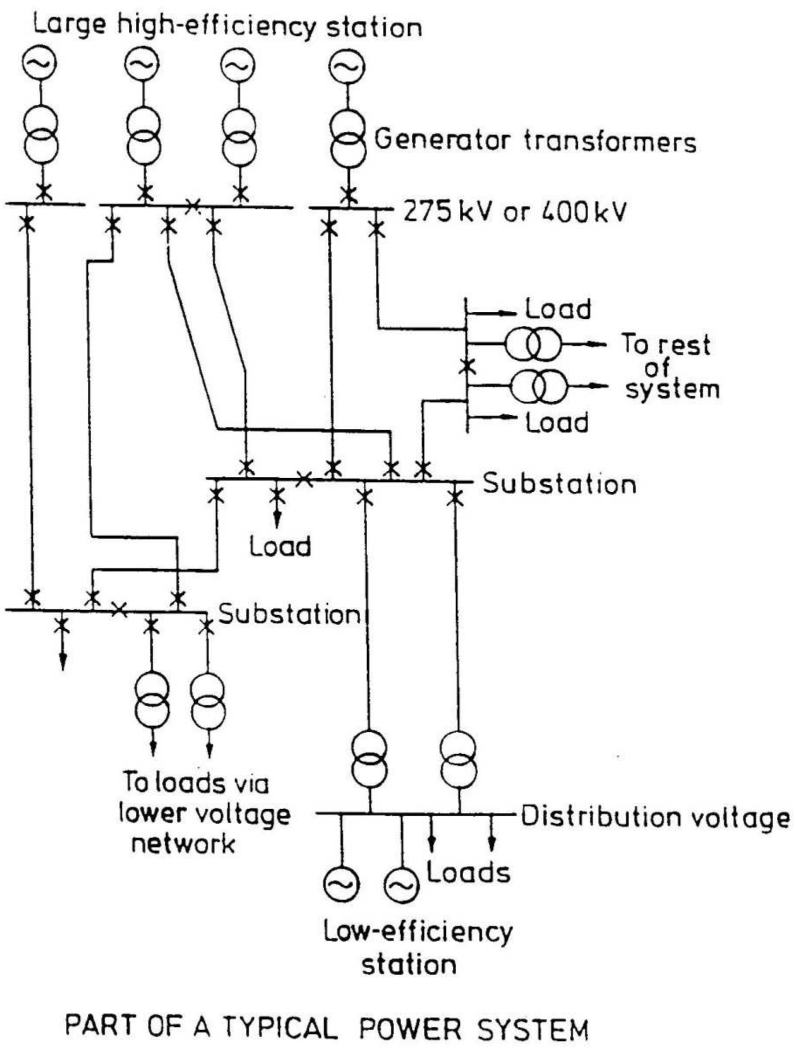


Fig. 7.1

Lectures 7 and 8 are concerned with the transmission of electricity. In this lecture, the per-unit method of analysing electrical power systems will be explained, and its advantages demonstrated by application to a number of examples.
The economics of electrical power systems taken as a whole suggests that it is best to concentrate the generation of power at a few locations (power stations) and transmit the power to the consumers. The majority of electrical power consumed in developed countries, therefore, has been transmitted via a system of high-voltage cables, or grid. Furthermore, a high level of interconnection of this system is desirable; power stations with the lowest operating costs can then always provide power irrespective of the location of the loads. Also, the system is more secure - if one part of the system fails, power can be re-routed. As we have also seen, the economics of transmission suggest the use of very high voltages - this is not compatible with the generation, or usage of electricity, hence the use of power transformers, and a wide range of system voltages. These points are illustrated in fig. 7.1, which shows part of a typical electrical power network.

The factors mentioned above make the analysis of electrical power systems very complex. Fortunately, a method known as the per-unit method simplifies things considerably. The basic idea is to replace all electrical quantities with dimensionless quantities (or, per-unit values). The advantages of doing this will become clear when we apply the method to some examples, but are stated here as:
a) all pu quantities are of similar magnitude whereas real

7.2 Per-unit system

$$\text{Per unit value} = \frac{\text{Actual value of quantity}}{\text{Base value of quantity}} \qquad (7.1)$$

Choose base values for VA and voltage; base values for current and impedance then derived:

Voltage: Base voltage, V_b, is line voltage.

VA: Any convenient choice of 3-phase VA

Current: Line current is used $\rightarrow$ $I_b = I_{line}$

$$\mathrm{S} = \sqrt{3}V_l I_l \Rightarrow VA_b = \sqrt{3}V_b I_b \quad I_b = \frac{VA_b}{\sqrt{3}V_b} \qquad (7.2)$$

Impedance: Assume star-connected:

$$Z = \frac{V_{ph}}{I_{ph}} = \frac{V_l/\sqrt{3}}{S/\sqrt{3}V_l} = \frac{V_l^2}{S} \quad \therefore \quad Z_b = \frac{V_b^2}{VA_b} \qquad (7.3)$$

With these base values:

$$\mathrm{S} = \sqrt{3}V_l I_l = \sqrt{3}V_b V_{pu} I_b I_{pu} = \sqrt{3}V_b V_{pu} \cdot \frac{VA_b}{\sqrt{3}V_b} I_{pu}$$

$$= V_{pu} I_{pu} VA_b \quad \therefore \quad S_{pu} = S/VA_b = V_{pu} I_{pu} \qquad (7.4)$$

voltages and currents vary greatly, owing to the effect of the transformers in the network.
b) gets rid of transformers by a suitable choice of base values and so the power network becomes like any other circuit.
c) gets rid of factors of 3 and √3 which otherwise would appear.
The per-unit value of a quantity is given by equation 7.1. Therefore, before it can be found, a base value for the quantity must be determined. It is possible to choose any base values for any two of the four quantities: S, V, I and Z. The other two base values are then dependent if a consistent framework is to be set up. It is usual for apparent power, S, and voltage, V, to have their base values chosen (we will see how suitable choices are made later), and for current and impedance base values to follow. The base value for voltage always means base value for the line-line voltage. The base value for VA always means three-phase VA. Notice that the numerical base value for P and Q must be the same as that for S since if:

$P_{pu} = P(\text{Watts})/VA_b$

$Q_{pu} = Q(\text{VARs})/VA_b$

$S_{pu} = \sqrt{(P/VA_b)^2 + (Q/VA_b)^2}$

$= S/VA_b$

as required. The base current means the base line current when the pu method is applied to the analysis of three-phase systems. For its value to be consistent with the chosen VA_b and V_b, equation 7.2 applies, which gives I_b in terms of VA_b and V_b. The base impedance is determined on an impedance per phase basis. Assuming that the load (or source) is star-connected, Z_b is found as shown in equation 7.3. Finally, it is important to note that three-phase apparent power, S, is given by equation 7.4 i.e. no factor of √3 is involved.

Example 7.1

A 100 MVA, 11 kV synchronous machine has pu reactance 0.15. Find base values for S, V, I and Z, and hence find X_s in Ohms.

Choose $VA_b = 100$ MVA and $V_b = 11$ kV.

$$I_b = \frac{VA_b}{\sqrt{3}V_b} = \frac{100\times 10^6}{\left(\sqrt{3}\times 11\times 10^3\right)} = 5.25\text{ kA}$$

$$Z_b = \frac{V_b^2}{VA_b} = \frac{\left(11\times 10^3\right)^2}{100\times 10^6} = 1.21\ \Omega$$

$$X_s(\Omega) = X_{s(pu)}Z_b = 0.15\times 1.21 = 0.182\ \Omega$$

7.3 Representation of transformers

X_m and R_0 are ignored, X_t usually >> R_t, so R_t is also usually ignored:

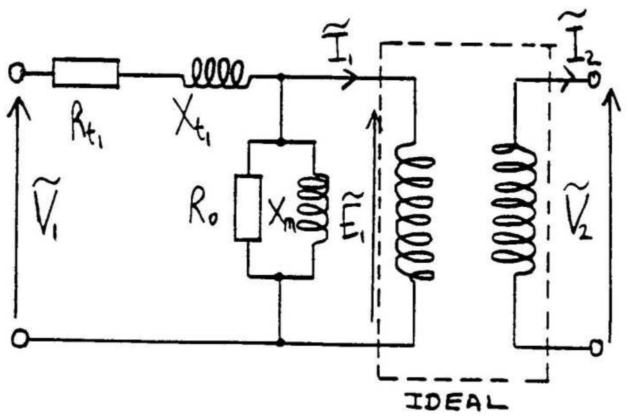


Fig. 7.2

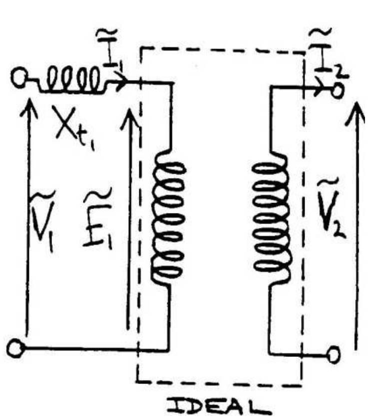


Fig. 7.3

Here we illustrate these ideas with a simple example.

Whenever a device such as a transformer or a synchronous machine is specified, the rated VA and voltage are given. Often the impedance is then quoted as a pu value - in that case, the pu value is to the base given by the rated VA and voltage. Therefore, in this example we must take VA_b and V_b as given opposite.

The values of base line current and phase impedance are given by equations 7.2 and 7.3 respectively.

Then from the definition of the per-unit value of a quantity, equation 7.1, the synchronous reactance in Ohms may be found.

Transformers play a very important role in power systems as we have already seen, and so we need to know how they can be modelled in the per-unit framework. In three-phase power systems, three-phase power transformers are employed. In Part IA we studied single-phase transformers - three-phase transformers behave in a similar way, and it will come as no surprise to find that a per-phase equivalent circuit may be used, identical in form to that of the single-phase transformer, fig. 7.2. In power systems analysis, it is usual to ignore the magnetising branch (R_0 and X_m in fig. 7.2) of transformers. This affects losses, but has no appreciable effect on voltages and currents in the system. Furthermore, the total leakage reactance is usually much larger than the total winding resistance, and so R_t is usually ignored (although it would not be difficult to include it in the model). This leaves the equivalent circuit shown in fig. 7.3, which is the transformer model used in this course when performing pu power system analysis.

Take common VA_b as transformer rated VA.

Take primary rated line-line voltage as V_{b1}.

Then choose base voltage at the secondary using transformer turns ratio:

$$\frac{V_{b2}}{V_{b1}} = \frac{E_2}{E_1} = \frac{N_2}{N_1} \qquad (7.5)$$

$$\therefore \quad \frac{E_1}{V_{b1}} = \frac{E_2}{V_{b2}} \quad \Rightarrow \quad E_{2(pu)} = E_{1(pu)} \qquad (7.6)$$

Also, can show that $I_{1(pu)} = I_{2(pu)}$ (7.7)

The transformer equivalent circuit becomes:

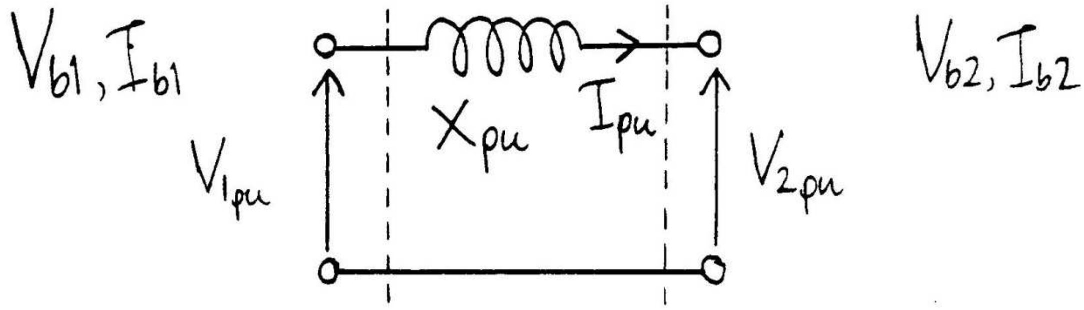


Fig. 7.4

Important result: ***Providing base voltages throughout a system are chosen in the ratio of transformer turns, the pu system gets rid of all the ideal transformers.***

Taking the rated VA of the transformer as VA_b, we then take the base voltage for the primary side of the transformer, V_{b1}, to be its rated value. If the base voltage for the secondary of the transformer, V_{b2}, is then chosen using the transformer turns ratio, equation 7.5, then it follows that in the per-unit system the voltages across the 'ideal transformer', $E_{1(pu)}$ and $E_{2(pu)}$ are the same, equation 7.6. It may also be shown that this choice of base voltages means that the per-unit primary and secondary currents are identical, equation 7.7. This is an extremely important result, because it means that the ideal transformer part of the transformer equivalent circuit can be dispensed with - in the per-unit framework, the transformer equivalent circuit is that shown in fig. 7.4. **If the base voltages throughout a power network are chosen using the transformer turns ratios, all transformers can be replaced by single reactances.** Therefore, we are free to choose any value for the base voltage at **one point** in the system; base voltages throughout the rest of the system are then fixed by the transformer turns ratios of any transformers in the system.

It may be shown that the transformer total referred leakage reactance has the same value in the pu system irrespective of which side of the transformer it is referred to. This, of course, has to be the case for the equivalent circuit of the transformer to be replaceable in the pu method with that of fig. 7.4.

7.4 Change of base

Sometimes it is necessary to express a pu impedance to a different base:

Base 1: $VA_{b(1)}$, $V_{b(1)}$ Base 2: $VA_{b(2)}$, $V_{b(2)}$

$$Z_{pu(1)} = \frac{Z(\Omega)VA_{b(1)}}{V_{b(1)}^2} \quad Z_{pu(2)} = \frac{Z(\Omega)VA_{b(2)}}{V_{b(2)}^2} \quad (7.8)$$

Impedance in Ohms must not depend on base:

$$Z_{pu(2)} = Z_{pu(1)} \cdot \frac{VA_{b(2)}}{VA_{b(1)}} \cdot \frac{V_{b(1)}^2}{V_{b(2)}^2} \quad (7.9)$$

Often change of base only due to change of VA_b i.e. $V_{b(1)}=V_{b(2)}$:

$$Z_{pu(2)} = Z_{pu(1)} \cdot \frac{VA_{b(2)}}{VA_{b(1)}} \quad (7.10)$$

Example 7.2

A 50 MVA transformer has 0.15 pu reactance. Find the pu reactance to a 100 MVA base.

$$Z_{pu(2)} = Z_{pu(1)} \cdot \frac{VA_{b(2)}}{VA_{b(1)}} \quad Z_{pu(100)} = 0.15 \cdot \frac{100}{50} = 0.3$$

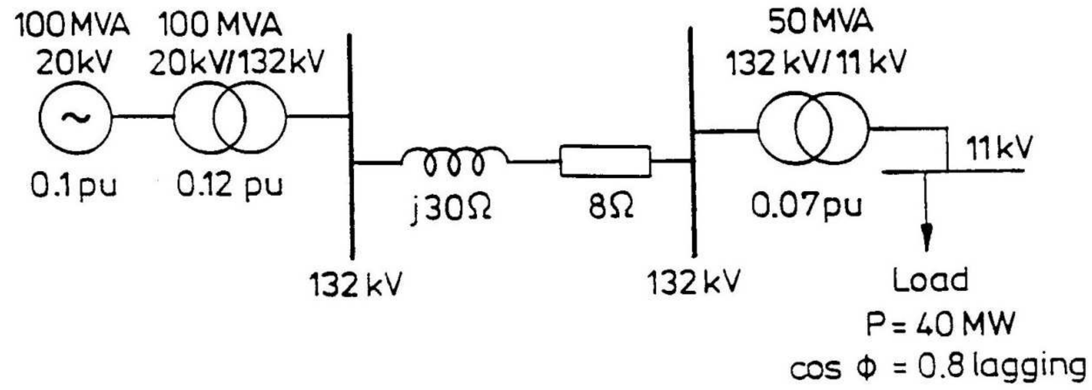
Often devices in the same power network have different VA ratings, and their pu impedances will be quoted using their rated VA and voltage as the VA_b and V_b respectively. However, the base VA used must be common throughout the whole network for the pu method to work. Therefore, it is often necessary to be able to perform a change of base.

Consider a device with pu impedance, $Z_{pu(1)}$ given to a base of $VA_{b(1)}$, $V_{b(1)}$ and suppose that it becomes necessary to convert that pu impedance to another base, $VA_{b(2)}$, $V_{b(2)}$. Equation 7.8 sums up the situation, by combining equations 7.1 and 7.3 to give Z_{pu} in both cases. The principle is that the actual impedance measured in Ohms does not depend on the base used to determine its pu value. By rearranging equations 7.8, and equating $Z(\Omega)$, and then rearranging to make $Z_{pu(2)}$ the subject of the equation gives equation 7.9. In most cases, the change of base only has to be carried out because the overall system base VA is not the same as the VA base of a particular device in the system. In that case, $V_{b(1)}=V_{b(2)}$, and equation 7.9 becomes equation 7.10.

In example 7.2, the 0.15 pu reactance is quoted to a VA base of 50 MVA i.e. $VA_{b(1)}$=50 MVA. The new base to which the pu reactance is to be quoted is 100 MVA, so $VA_{b(2)}$=100 MVA. No change of base due to a change in base voltage is required, so equation 7.10 is used. When choosing the system VA base for a problem, therefore, a major consideration is avoiding too many base changes, or at least making them as painless as possible. Either choose a VA base which is i) shared by the majority of devices in the system, or ii) is a multiple of their VA ratings.

Example 7.3

Find real and reactive power supplied by the generator, power dissipated in the feeder and voltage at the generator.



In example 7.3 we combine all of the ideas met so far to solve most of a Tripos question (Q5 Paper 5, 1996). The question is illustrated opposite - basically, a 100 MVA, 20 kV generator supplies an 11 kV, 40 MW load of 0.8 lagging power factor via a 132 kV feeder, and so step-up and step-down transformers are required as illustrated.

The first step is to choose the system VA base - this value is common to the whole of the system. A good choice here is 100 MVA because the generator and step-up transformers are both 100 MVA devices.

We can only choose the base voltage at one point in the system - all other base voltages are then fixed by the transformer turns ratios. The obvious choice is 20 kV at the generator (otherwise changes of base for all devices will be required due to voltage), giving base voltages of 132 kV for the whole of the system between the secondary of the step-up transformer, and the primary of the step-down transformer. The base voltage at the secondary of the step-down transformer must be 11 kV.

<u>Step 1</u> Choose system VA base

VA_b = 100 MVA (minimises changes of base)

<u>Step 2</u> Choose system base voltages

Must be in transformer turns ratios. Choose 20 kV, 132 kV and 11 kV.

<u>Step 3</u> Change of base on 50 MVA transformer

Only the 50 MVA transformer requires a change of base, and this is achieved using equation 7.11, with $VA_{b(1)}$=50 MVA and $VA_{b(2)}$=100 MVA.

$$Z_{pu(100\,MVA)} = Z_{pu(50\,MVA)} \cdot \frac{100}{50} = 0.07 \times 2 = 0.14$$

Step 4 Convert feeder impedance to pu

$$Z_b = \frac{V_b^2}{VA_b} = \frac{\left(132\times10^3\right)^2}{100\times10^6} = 174\ \Omega$$

$$\therefore \quad Z_{pu} = \frac{\overline{Z}(\Omega)}{Z_b} = \frac{8+j30}{174} = 0.0459 + j0.172$$

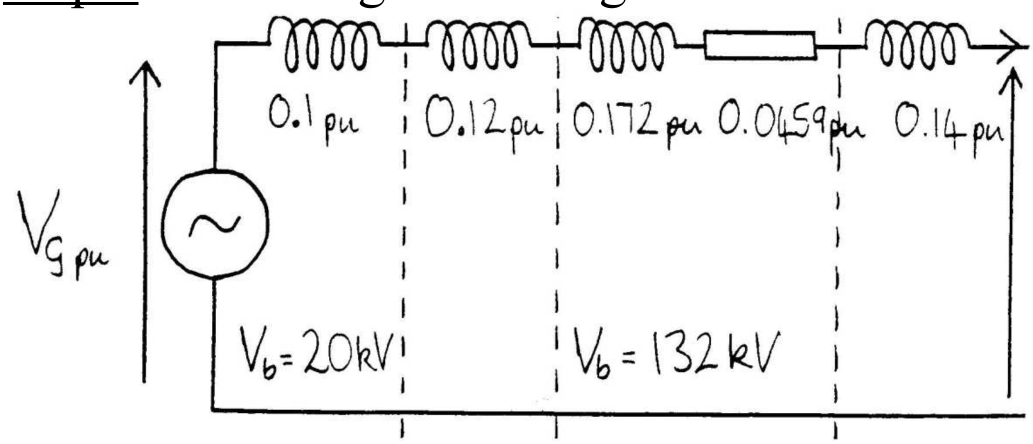
Step 5 Find pu load current and impedance

$$S = P/\cos\varphi = 40/0.8 = 50\ \text{MVA}$$

$$S_{pu} = S/VA_b = 50/100 = 0.5$$

$$S_{pu} = V_{pu}I_{pu} \quad \Rightarrow \quad 0.5 = 1\times I_{pu} \quad I_{pu} = 0.5$$

Step 6 Draw single-line diagram



Step 7 Find total series pu reactance

$$X_{T(pu)} = 0.1 + 0.12 + 0.172 + 0.14 = 0.532$$

All impedances are in per-unit form except for the feeder, which is in Ohms/phase. The base impedance which applies to the section of the network of which the feeder is a part is found from equation 7.3 with V_b=132 kV (VA_b is 100 MVA for all of the system).

The per-unit feeder impedance may then be found by dividing the actual impedance by the base value, equation 7.1.

The apparent power taken by the load is found from a knowledge of the real power and the power factor using the power triangle. The per-unit apparent power is then determined by dividing it by VA_b, to give 0.5. This can be equated with $V_{pu}I_{pu}$ (equation 7.4). The actual load voltage is 11 kV, so its pu value is 11 kV/V_b, with V_b=11 kV i.e. 1 pu. Therefore, I_{pu}=0.5.

We could proceed to find the equivalent star-connected load impedance, and draw a complete single-line diagram. However, it is just as easy to use conservation of P and Q.

The single-line diagram marks in all the system devices and their pu impedances. At the left-hand side is the generator voltage, which is one of the quantities to be found, and on the right-hand side we simply show a voltage of 1 pu feeding a load of P_{pu}=0.4, Q_{pu}=0.3. It is a good idea to match points from the actual system onto the single-line diagram, and hence mark on the separate regions of the system, and the relevant base voltages. This helps to keep track of how the pu representation of the system relates to the actual system.

From the single-line diagram the total series pu reactance is found by adding all the individual pu reactances.

Step 8 Apply conservation of P and Q

$$P_{f(pu)} = I_{pu}^2 R_{f(pu)} = 0.5^2 \times 0.0459 = 0.0115$$

$$P_f = P_{f(pu)} \times VA_b = 0.0115 \times 100 = 1.15\,\text{MW}$$

$$P_{G(pu)} = P_{f(pu)} + P_{load(pu)} = 0.0115 + 0.4 = 0.4115$$

$$\therefore \quad P_G = P_{G(pu)} VA_b = 0.4115 \times 100 = 41.15\ \text{MW}$$

$$Q_{G(pu)} = I_{pu}^2 X_{T(pu)} + Q_{load(pu)}$$
$$= 0.5^2 \times 0.532 + 0.3 = 0.433$$

$$\therefore \quad Q_G = Q_{G(pu)} VA_b = 0.433 \times 100 = 43.3\ \text{MVAR}$$

Step 9 Find the generator voltage

$$S_{G(pu)} = \sqrt{Q_{G(pu)}^2 + P_{G(pu)}^2}$$
$$= \sqrt{0.4115^2 + 0.433^2} = 0.597 = V_{G(pu)} I_{pu}$$

$$\therefore \quad 0.597 = V_{G(pu)} \times 0.5 \qquad V_{G(pu)} = 1.195$$

$$V_G = V_{G(pu)} V_b = 1.195 \times 20 = 23.9\ \text{kV}$$

The total pu feeder power dissipated, $P_{f(pu)}$, is given opposite (only resistors dissipate power) and this is converted to power in Watts by multiplying by the base VA.

Conservation of P_{pu} and Q_{pu} is now applied. The total pu power supplied by the generator, $P_{G(pu)}$ must equal that consumed by the load (0.4) plus that consumed by the feeder (0.0115). The total pu reactive power supplied is the sum of the load pu reactive power (0.3), and the reactive power taken by all other reactances, $I_{pu}^2X_T$.

The generator real power (in Watts) and reactive power (in VARs) can now be found by multiplying the respective pu values by VA_b.

The total pu apparent power supplied by the generator can then be found from the power triangle, and equated with the pu generator voltage ($V_{G(pu)}$) $\times I_{pu}$. This enables $V_{G(pu)}$ to be found. Multiplying by the relevant base voltage (20 kV) gives the actual generator voltage.

This example illustrates the strength of the pu method - it effortlessly takes account of the effects of transformers by applying a simple set of rules. If you are yet to be convinced, try this example using conventional methods !