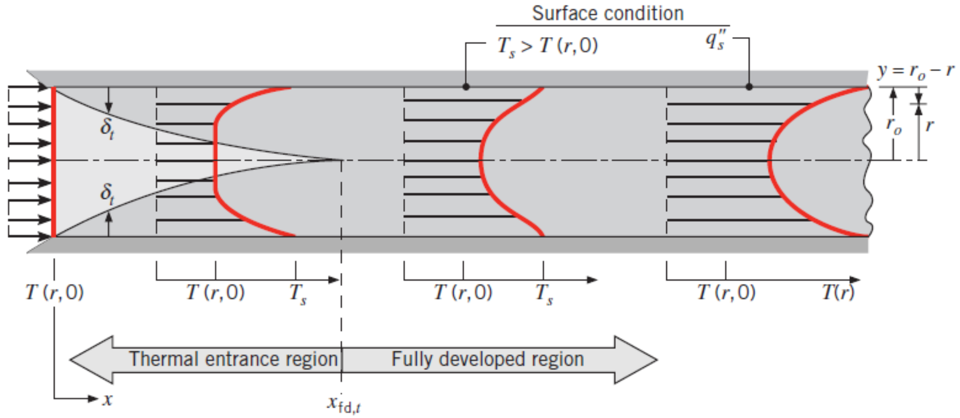
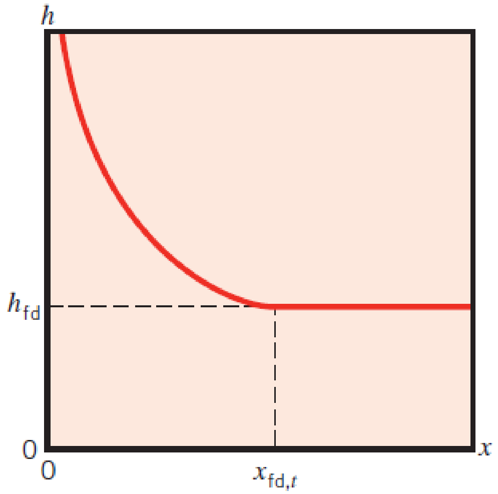
Heat Convection in Internal Flows

Some characteristics of thermally fully developed flows

- The thermal boundary layer can be seen as barrier against the convective transfer of heat.
- Along a thermally developing flow the convection coefficient drops significantly (*why*?).
- However, we can show mathematically that in the fully developed region $\frac{h}{k} \neq f(x)$.
- ***Hence, in the thermally fully developed flow of fluid with constant properties, the local convection coefficient is a constant, independent of x.***





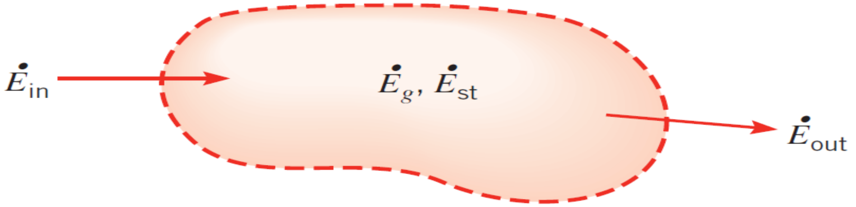
Axial variation of convection heat transfer coefficient for flow in a tube.

A quick review of thermodynamics

- Thermodynamics and heat transfer are highly complementary.
- Many heat transfer problems do need a first law analysis.
- First law of thermodynamics or balance of energy:

  The rate of increase in the amount of energy stored in a control volume must equal the rate at which energy enters the control volume, minus the rate at which energy leaves the control volume, plus the rate at which energy is generated within the control volume.

- Mathematically: $\Delta\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$



Conservation of energy in a control volume

$\Delta\dot{E}_{st}$: The rate of energy storage in the control volume
$\dot{E}_{in}$: The rate of energy transfer to the control volume
$\dot{E}_{out}$: The rate of energy removal from the control volume
$\dot{E}_g$: The rate of energy generated within the control volume

A quick review of thermodynamics, contd.

- Energy can be carried to or from the control volume by: heat, work and mass flow.
- For a steady state system, without internal energy generation (e.g. electrical resistance, exothermic chemical reactions):

$$\dot{\boldsymbol{m}}(\boldsymbol{h} + \mathbf{1/2}\boldsymbol{V}^{\mathbf{2}} + \boldsymbol{gz})_{\boldsymbol{in}} - \dot{\boldsymbol{m}}(\boldsymbol{h} + \mathbf{1/2}\boldsymbol{V}^{\mathbf{2}} + \boldsymbol{gz})_{\boldsymbol{out}} + \boldsymbol{q} - \dot{\boldsymbol{W}} = \mathbf{0}$$

  $\dot{m}$: mass flow rate (kg/s)
  h: specific enthalpy (J/kg)
  $1/2V^2 + gz$: specific kinetic and potential energies (J/kg)
  q : net heat transfer rate to the control volume
  $\dot{W}$: net rate of work done by the control volume

- If there is no work done and, the kinetic and potential energies are negligible and, the fluid is either a perfect gas or a liquid, then:

$$\boldsymbol{q} = \dot{\boldsymbol{m}}\boldsymbol{c}_{\boldsymbol{p}}(\boldsymbol{T}_{\boldsymbol{out}} - \boldsymbol{T}_{\boldsymbol{in}})$$

- These conditions often hold in flow inside pipes and channels.

Energy balance

- For flow of most fluids in a pipe

$$\boxed{q_{conv} = \dot{m}c_p(T_{m,0} - T_{m,i})}$$

q_{conv}: convective heat transfer rate (W/m^2)

$\dot{m}$: mass flow rate of fluid (kg/m^3)

c_p: specific heat capacity

$T_{m,0}$ & $T_{m,i}$: outlet and inlet mean temperatures of the fluid

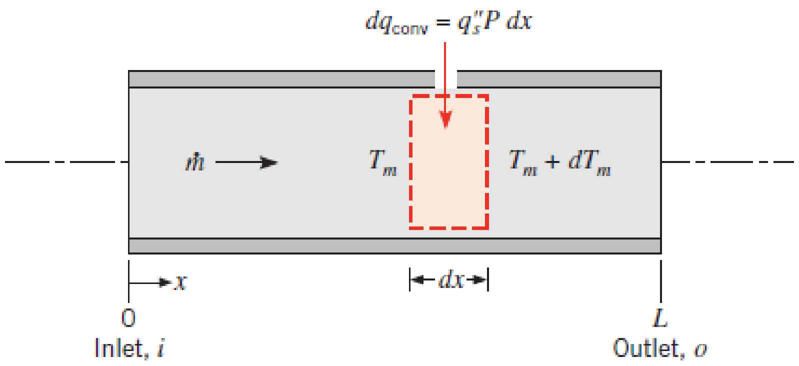
- This simple overall energy balance relates three important thermal variables (q_{conv}, $T_{m,0}$, $T_{m,i}$).

This is a general expression that applies irrespective of the nature of the surface thermal or tube flow conditions.

- Applying the first law of thermodynamics to the element shown in the figure and recalling that the mean temperature is defined such that $\dot{m}c_pT_m$ represents the true rate of thermal energy (or enthalpy) advection integrated over the cross-section

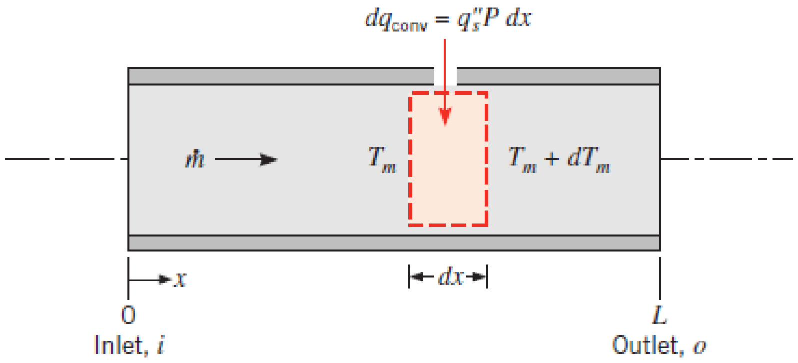
$$dq_{conv} = \dot{m}c_p[(T_m + dT_m) - T_m] \text{ or}$$

$$\boxed{dq_{conv} = \dot{m}c_p dT_m}$$



Control volume for internal flow in a tube.

- Note that $dq_{conv} = q_s'' P dx$, where P is the surface perimeter ($P = \pi D$ for a circular tube) and q_s'' is the heat transfer flux.



- Substituting from this equation into the differential energy balance ($dq_{conv} = \dot{m} c_p dT_m$), yields

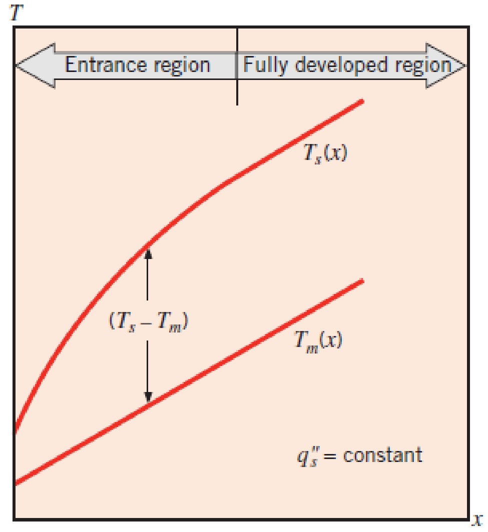
$$\boxed{\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = \frac{P}{\dot{m} c_p} h[T_s - T_m(x)]}$$

- Note that if $T_s > T_m$, heat is transferred to the fluid and T_m increases with x and if $T_s < T_m$ the opposite is correct.
- For constant heat flux: $q_{conv} = q_s''(P.L)$ and

therefore $\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} \neq f(x)$

- Integration from $x = 0$, it follows that

$$\boxed{T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m} c_p} x \qquad q_s = constant}$$



Axial temperature variations for heat transfer in a tube under constant heat flux

- For constant surface temperature, we define $\Delta T = T_s - T_m$, then

$$\frac{q_s'' P}{\dot{m} c_p} = \frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m} c_p} h \Delta T$$

- Separating the variables and integrating, $\int_{\Delta T_i}^{\Delta T_o} \frac{d(\Delta T)}{\Delta T} = -\frac{P}{\dot{m} c_p} \int_0^L h dx$
- Or, $\ln \frac{\Delta T_0}{\Delta T_i} = -\frac{PL}{\dot{m} c_p} (\frac{1}{L} \int_0^L h dx)$,
- Recall that we defined the average coefficient of convections as $\overline{h_L} = \frac{1}{L} \int_0^L h dx$, so

$\ln \frac{\Delta T_0}{\Delta T_i} = -\frac{PL}{\dot{m} c_p} \overline{h_L}$ $\quad T_s$=constant, where $\overline{h_L}$, or simply $\bar{h}$ is the average value of h for the entire tube. By rearranging we can readily show that

$$\boxed{\frac{\Delta T_0}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp(-\frac{PL}{\dot{m} c_p} \bar{h}) \qquad T_s\text{=constant}}$$

Had we integrated from the tube inlet to some axial position x within the tube, we would have obtained the similar, but more general results that

$$\boxed{\frac{\Delta T_0}{\Delta T_i} = \frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp(-\frac{Px}{\dot{m} c_p} \bar{h}) \qquad T_s\text{=constant}}$$

where $\bar{h}$ is now the average value of h from the tube inlet to x.

- Determination of an expression for the total heat transfer rate q_{conv} is complicated by the exponential nature of temperature distribution. We can rewrite the overall energy balance as

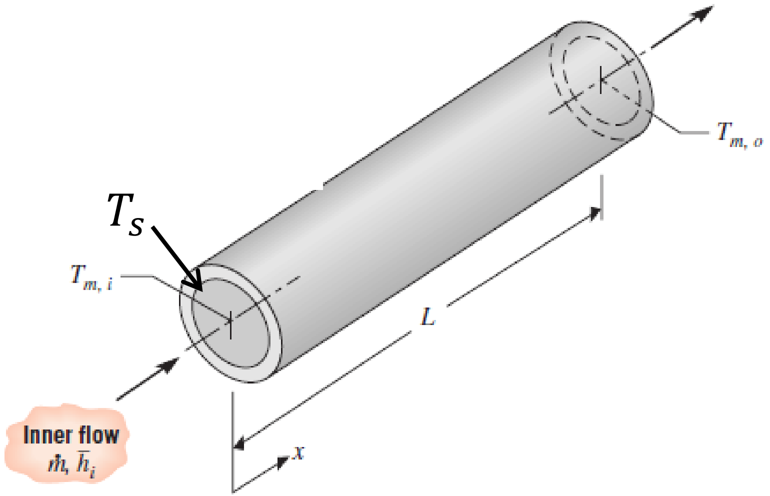
$$q_{conv} = \dot{m}c_p\left[\left(T_s - T_{m,i}\right) - \left(T_s - T_{m,o}\right)\right] = \dot{m}c_p(\Delta T_i - \Delta T_o)$$

and substituting for $\dot{m}c_p$ from the previous equations, we obtain

$$\boxed{q_{conv} = \bar{h}A_s\Delta T_{lm} \qquad T_s = constant}$$

where A_s is the tube internal surface area (A_s=$P.L$) and ΔT_{lm} is the *log mean temperature difference*,

$$\boxed{\Delta T_{lm} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)}}$$



Convective heat transfer in fluid flow in a constant temperature tube

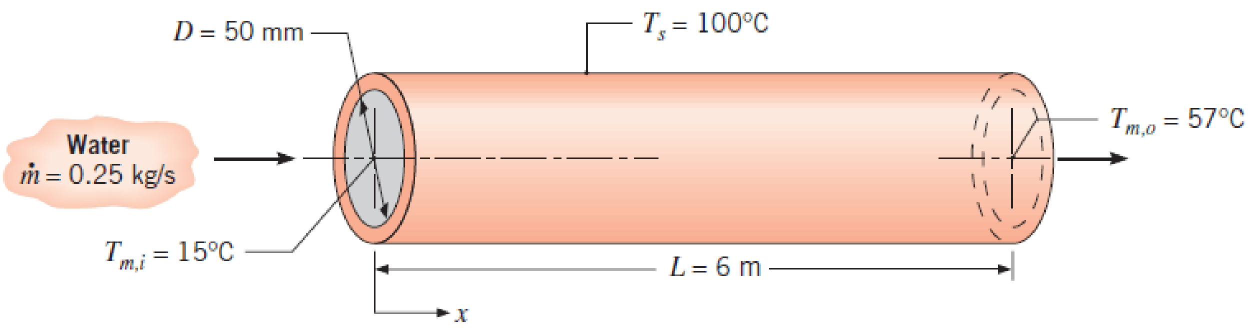
Example 1

Steam condensing on the outer surface of a thin-walled circular tube of diameter $D = 50$ mm and length $L = 6$ m maintains a uniform outer surface temperature of 100°C. Water flows through the tube at a rate of $\dot{m} = 0.25$ kg/s, and its inlet and outlet temperatures are $T_{m,i} = 15°C$ and $T_{m,o} = 57°C$. What is the average convection coefficient associated with the water flow?

Known: Flow rate and inlet and outlet temperatures of water flowing through a tube of prescribed dimensions and surface temperature.

Find: Average convection heat transfer coefficient.

Schematic:



Assumptions:

1. Negligible tube wall conduction resistance.
2. Incompressible liquid and negligible viscous dissipation.
3. Constant properties.

Properties: Table A.6, water ($\bar{T}_m = 36°C$): $c_p = 4178\ \text{J/kg}\cdot\text{K}$.

Analysis: We showed that $q_{\text{conv}} = \dot{m}c_p(T_{m,o} - T_{m,i})$ also, for constant surface temperature tube $q_{\text{conv}} = \bar{h}A_s\Delta T_{\text{lm}}$
Combing the two:

$$\bar{h} = \frac{\dot{m}c_p}{\pi DL}\,\frac{(T_{m,o} - T_{m,i})}{\Delta T_{\text{lm}}}$$

$$\Delta T_{\text{lm}} = \frac{(T_s - T_{m,o}) - (T_s - T_{m,i})}{\ln[(T_s - T_{m,o})/(T_s - T_{m,i})]}$$

$$\Delta T_{\text{lm}} = \frac{(100 - 57) - (100 - 15)}{\ln[(100 - 57)/(100 - 15)]} = 61.6°\text{C}$$

Hence

$$\bar{h} = \frac{0.25\ \text{kg/s} \times 4178\ \text{J/kg}\cdot\text{K}}{\pi \times 0.05\ \text{m} \times 6\ \text{m}}\,\frac{(57 - 15)°\text{C}}{61.6°\text{C}}$$

or

$$\bar{h} = 755\ \text{W/m}^2\cdot\text{K} \qquad \triangleleft$$

Comments: If conditions were fully developed over the entire tube, the local convection coefficient would be everywhere equal to $755\ \text{W/m}^2\cdot\text{K}$.

Example 2

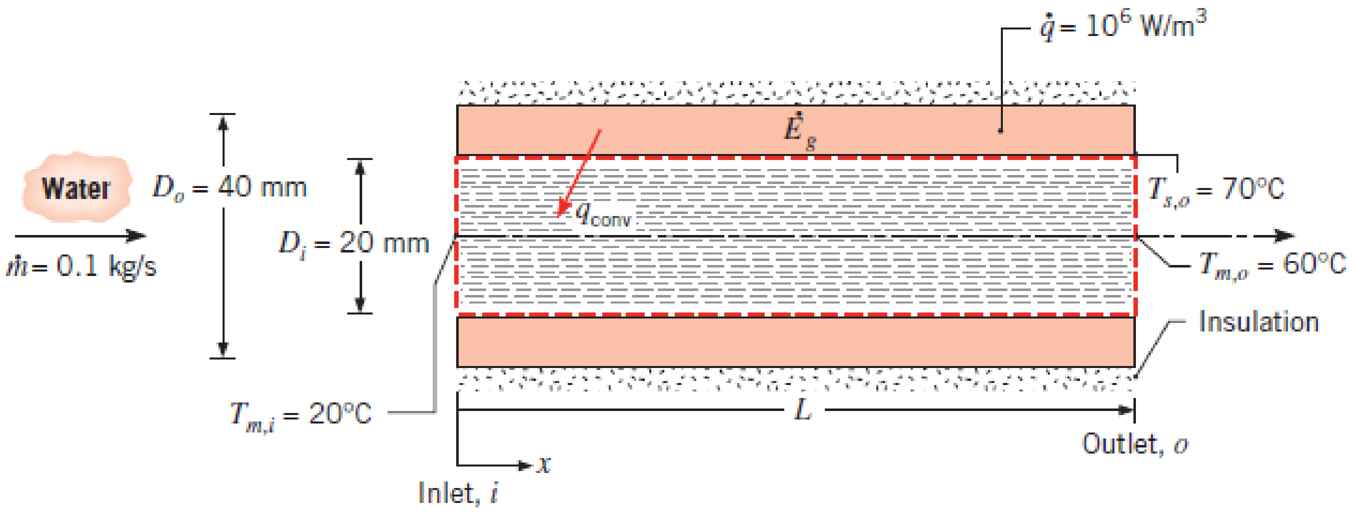
A system for heating water from an inlet temperature of $T_{m,i} = 20°C$ to an outlet temperature of $T_{m,o} = 60°C$ involves passing the water through a thick-walled tube having inner and outer diameters of 20 and 40 mm. The outer surface of the tube is well insulated, and electrical heating within the wall provides for a uniform generation rate of $\dot{q} = 10^6 \text{ W/m}^3$.

1. For a water mass flow rate of $\dot{m} = 0.1$ kg/s, how long must the tube be to achieve the desired outlet temperature?
2. If the inner surface temperature of the tube is $T_s = 70°C$ at the outlet, what is the local convection heat transfer coefficient at the outlet?

Known: Internal flow through thick-walled tube having uniform heat generation.

Find:

1. Length of tube needed to achieve the desired outlet temperature.
2. Local convection coefficient at the outlet.



Assumptions:

1. Steady-state conditions.
2. Uniform heat flux.
3. Incompressible liquid and negligible viscous dissipation.
4. Constant properties.
5. Adiabatic outer tube surface.

Properties: Table A.6, water ($\overline{T}_m = 313$ K): $c_p = 4179$ J/kg·K.

Analysis:

1. Since the outer surface of the tube is adiabatic, the rate at which energy is generated within the tube wall must equal the rate at which it is convected to the water.

$$\dot{E}_g = q_{\text{conv}}$$

With

$$\dot{E}_g = \dot{q}\frac{\pi}{4}(D_o^2 - D_i^2)L$$

it follows from the overall energy balance ($q_{conv} = \dot{m} c_p (T_{m,0} - T_{m,i})$) that

$$\dot{q} \frac{\pi}{4} (D_o^2 - D_i^2) L = \dot{m} c_p (T_{m,o} - T_{m,i})$$

or

$$L = \frac{4\dot{m} c_p}{\pi (D_o^2 - D_i^2) \dot{q}} (T_{m,o} - T_{m,i})$$

$$L = \frac{4 \times 0.1 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K}}{\pi (0.04^2 - 0.02^2) \text{ m}^2 \times 10^6 \text{ W/m}^3} (60 - 20)°\text{C} = 17.7 \text{ m} \qquad \triangleleft$$

2. From Newton's law of cooling, $h_o = \dfrac{q_s''}{T_{s,o} - T_{m,o}}$

Assuming that uniform heat generation in the wall provides a constant surface heat flux, with

$$q_s'' = \frac{\dot{E}_g}{\pi D_i L} = \frac{\dot{q}}{4} \frac{D_o^2 - D_i^2}{D_i}$$

$$q_s'' = \frac{10^6 \text{ W/m}^3}{4} \frac{(0.04^2 - 0.02^2) \text{ m}^2}{0.02 \text{ m}} = 1.5 \times 10^4 \text{ W/m}^2$$

it follows that

$$h_o = \frac{1.5 \times 10^4 \text{ W/m}^2}{(70 - 60)°\text{C}} = 1500 \text{ W/m}^2 \cdot \text{K}$$