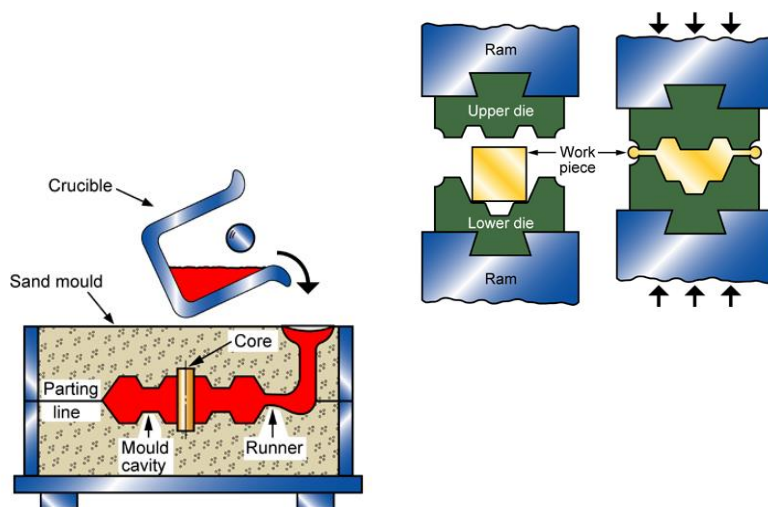IB Paper 3: Materials

Materials Processing:
Handout 2, Lectures 12-14
Shaping Processes for Metals

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Michaelmas 2023

Contents:

5. Casting

For metals, the choice of alloy is strongly linked to the shaping process. This in turn affects the microstructure and properties of the shaped part, and the possibilities for secondary processes, such as heat treatment. Two major classes of shaping process will be considered in lectures 12-14: casting (shaping by solidification in a mould) and deformation processing (shaping ductile metals using plasticity).
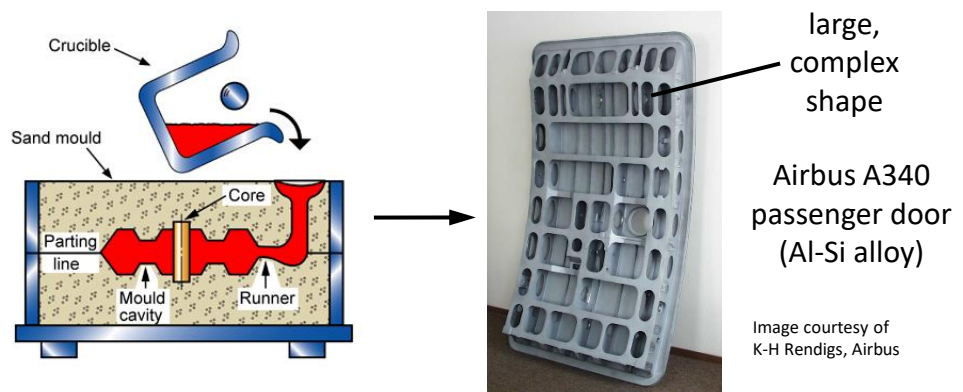
Reasons for opting for a casting route:
- **Metal alloying** (combining two or more elements): it is only practical to fix the composition in the liquid state (high solubility, rapid diffusion). Hence bulk metal for subsequent deformation processing is first cast in bulk (ingot, slab or billet).

Example: Continuous casting of steel (Fe-C alloys).

molten steel

solid billets

progressively cooled

Image: Corus media gallery
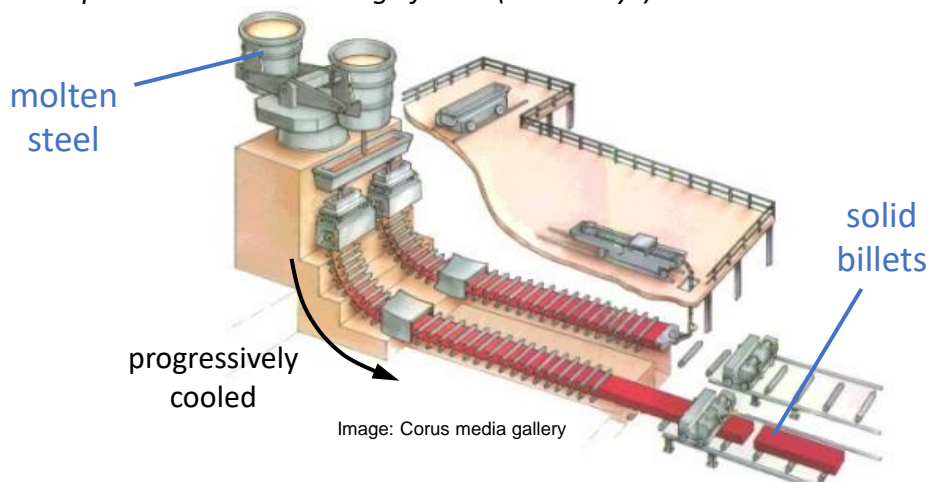
- **Complex shapes** can be cast directly to 'near-net shape', at low cost, using the high fluidity of liquid metal to fill an intricate mould.
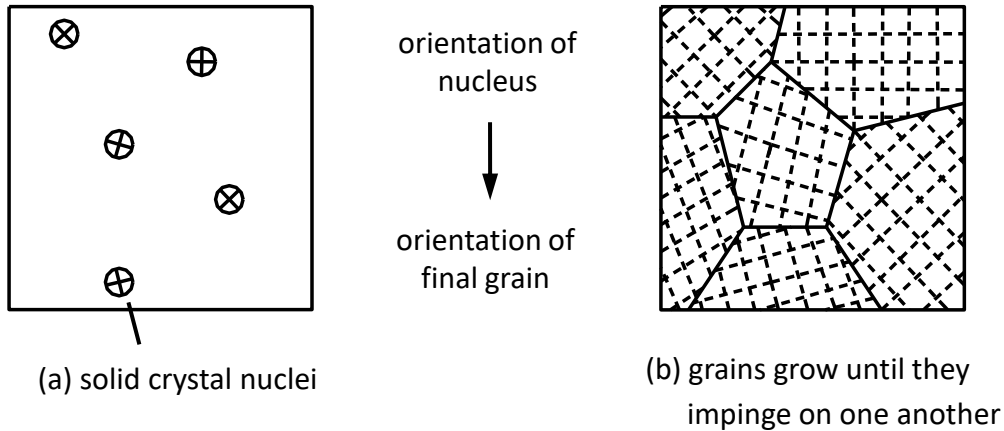 *Example: Sand casting an aluminium alloy aircraft door.*

Crucible

Sand mould

Core

Parting line

Mould cavity

Runner

large, complex shape

Airbus A340 passenger door (Al-Si alloy)

Image courtesy of K-H Rendigs, Airbus

- Some **specialist materials** depend on carefully controlled solidification (e.g. single crystals: silicon for semiconductors, Ni alloys for jet engines).
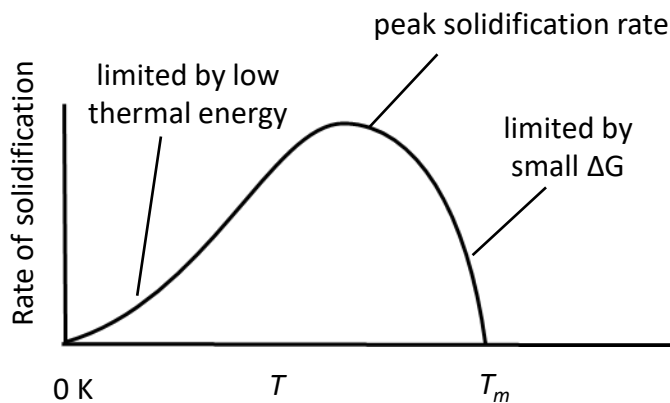
5.1 Grain structure in a casting

Solidification of metals in a casting process occurs by nucleation and growth of the solid phase in the liquid, as detailed in earlier lectures. Nearly all solidification of metals results in a **polycrystal**: grains originate from randomly oriented crystal nuclei.

orientation of
nucleus

↓

orientation of
final grain

(a) solid crystal nuclei

(b) grains grow until they
impinge on one another

The grain size and shape in a casting depends on the principles of the solidification phase transformation introduced earlier in the course:

(a) The rate of solidification depends on the undercooling, $\Delta T = T_m - T$:

peak solidification rate

limited by low
thermal energy

limited by
small ΔG

Rate of solidification

0 K T T_m

(b) Heterogeneous nucleation requires a lower undercooling than homogeneous:

heterogeneous

10^2 atoms, r = 18 nm

homogeneous

10^2 atoms, r = 1 nm

Critical radius, r^*
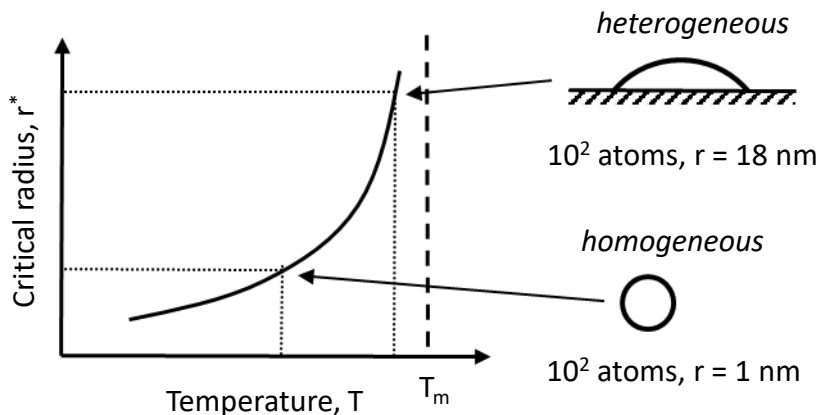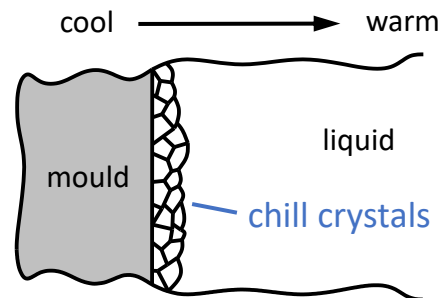
Temperature, T T_m

A typical casting grain structure is shown below. Liquid metal is poured into a mould at a temperature $T > T_m$, and cools by heat loss through the mould walls. It can be divided into three zones:

mould wall

(a) chill zone

(b) columnar zone

(c) equiaxed zone

directions of heat flow
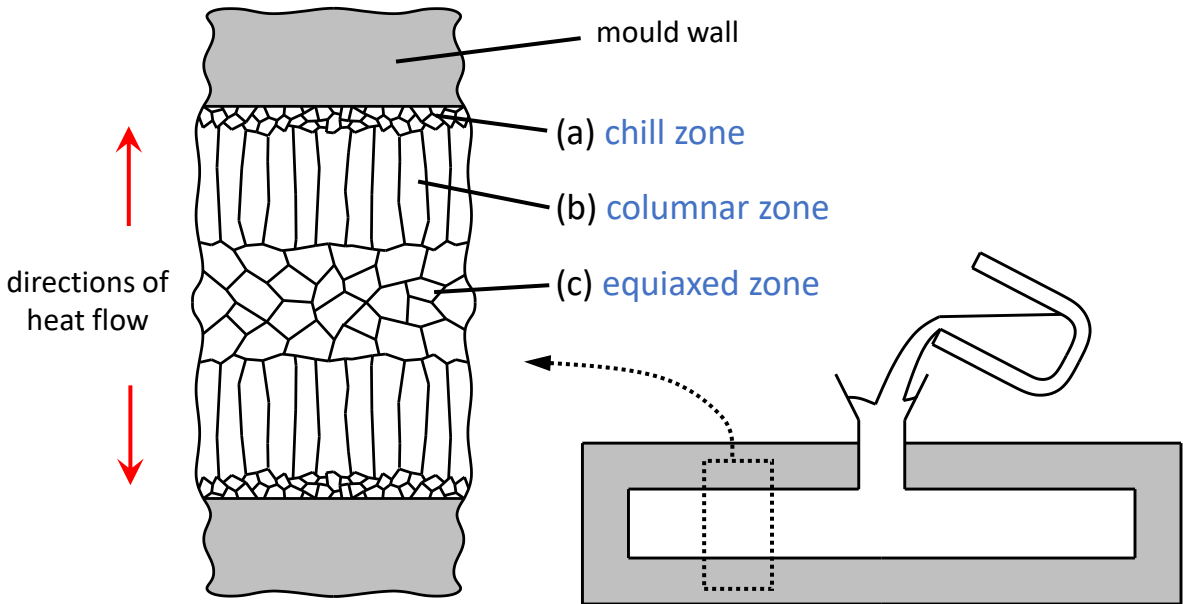
The reasons for this microstructure are as follows:

cool → warm

(a) Chill zone:

- Near to the mould wall the liquid cools rapidly.
- Nucleation starts here: larger undercooling, plus sites for heterogeneous nucleation.
- Grain sizes are small, due to the high nucleation rate.

mould

liquid

chill crystals

(b) Columnar zone:

- Next, as the temperature of the liquid falls, some preferentially orientated grains within the chill zone will begin to grow towards the centre of the casting: 'columnar grains'.
- At this stage, the undercooling in the liquid away from the wall is insufficient for new nuclei to form there.

mould

growth direction
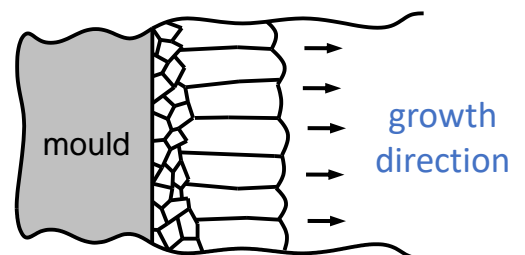
(b) Equiaxed zone:
- After the liquid has cooled further, nucleation may then occur in the remaining liquid, either homogenously, or (more likely) heterogeneously on dirt particles or other contamination within the melt.
- These grains will have random orientations (no preferred orientation = **equiaxed**) and will grow until they are impinged upon by other grains.
- Fewer nuclei form compared to the chill zone, due to the lower undercooling, and so the grain size is relatively large.
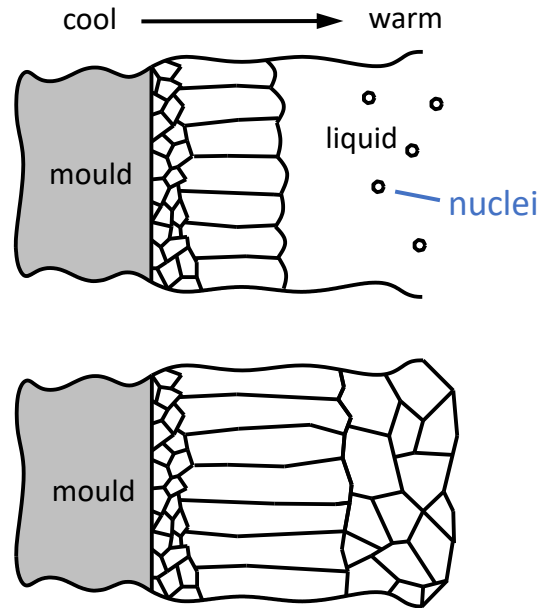

Controlling grain size: achieving fine-grained castings

A more uniform, fine-grained microstructure can be desirable for castings.

- **Grain boundary hardening** (see part IA) is an important strengthening mechanism, particularly for pure metal castings. The yield strength increases as the grain size (d) reduces:

$$\sigma_y = A + \frac{B}{\sqrt{d}}$$

grain boundaries act as obstacles to dislocations
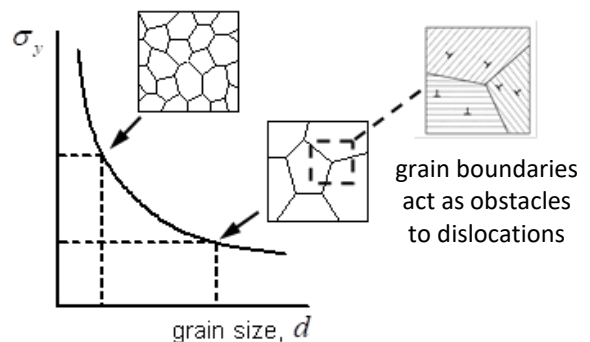
- When casting with metal alloys, a smaller grain size helps to achieve a more **uniform distribution** of alloying elements and impurities: more details on this later.

Fine-grained castings can be achieved by increasing the number of nuclei that form throughout the casting. A practical strategy for achieving this is to use **inoculants:**

- a dispersion of fine solid particles, added to the melt just before pouring into the mould
- these inoculant particles provide sites for **heterogeneous nucleation** throughout the melt (e.g. TiB$_2$ routinely added to Al casting alloys).

dispersion of small particles

many nuclei form heterogeneously
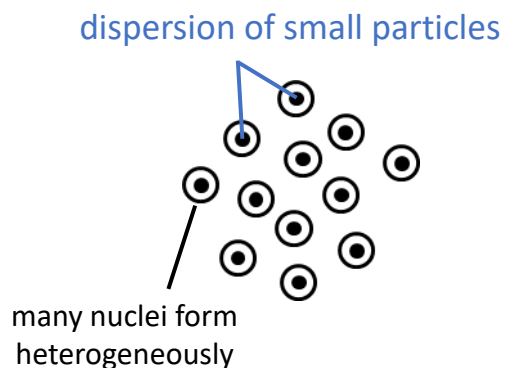
5.2 Casting alloys

In practice, castings are likely to involve alloys rather than pure metals.

- The alloy choice may be driven by **processability** considerations: how easy and reliable it is to cast with, minimising defects.
- This is particularly important for 'near net shape' castings (the mould defines the final shape of the component) with complex geometries:


- But alloy choice for castability also has consequences for the **microstructure** of the casting, and can have a negative impact on mechanical properties.

***Example 1:*** *Al-Si alloys, a common casting alloy*

Casting alloys are often **near-eutectic** in composition:

- **Low melting temperature:** reduces energy costs.

- **Low 'freezing range'** (gap between the liquidus and solidus temperatures): solid crystals in the melt increase its viscosity, and make it difficult to fill an intricate mould effectively.
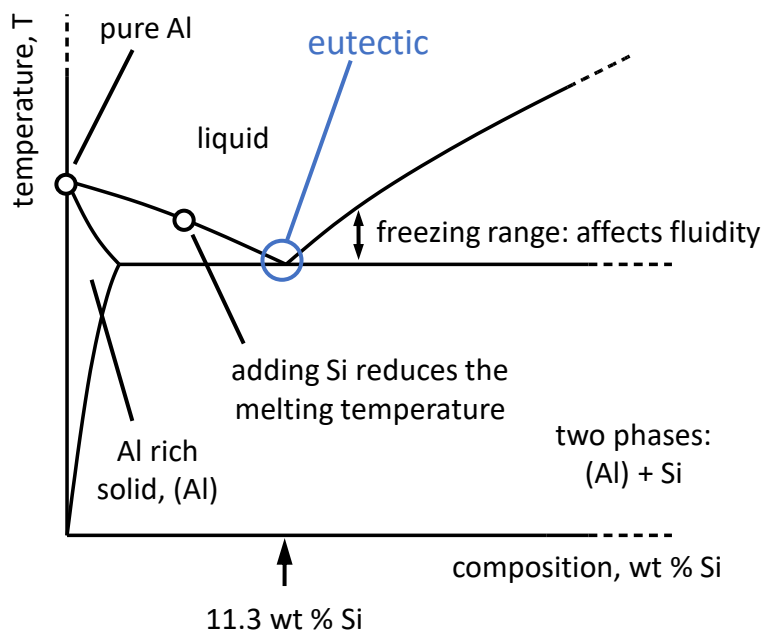

Mechanical properties of Al-Si casting alloys:

- Two-phase eutectic microstructure: **higher strength** than pure Al, but needles of brittle Si are detrimental to **toughness**.
- Commercial casting alloys often have their properties improved by (i) **heat treatment** and (ii) further **alloying**.
- This can alter the shape and size of the brittle phases, and therefore reduce the negative impact on toughness (an example is given later for Fe-C alloys).
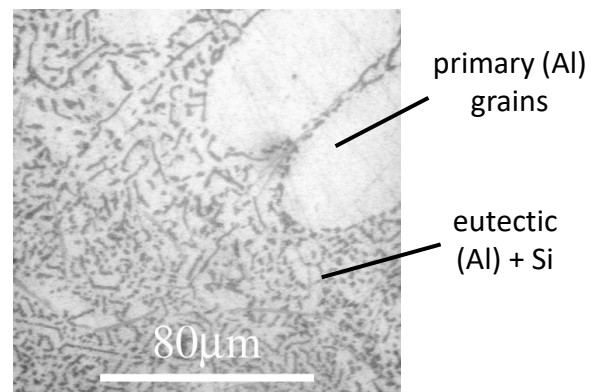
© DoITPoMS Micrograph Library, University of Cambridge (Prof T W Clyne)

Example 2: cast iron
(Fe-C alloy)
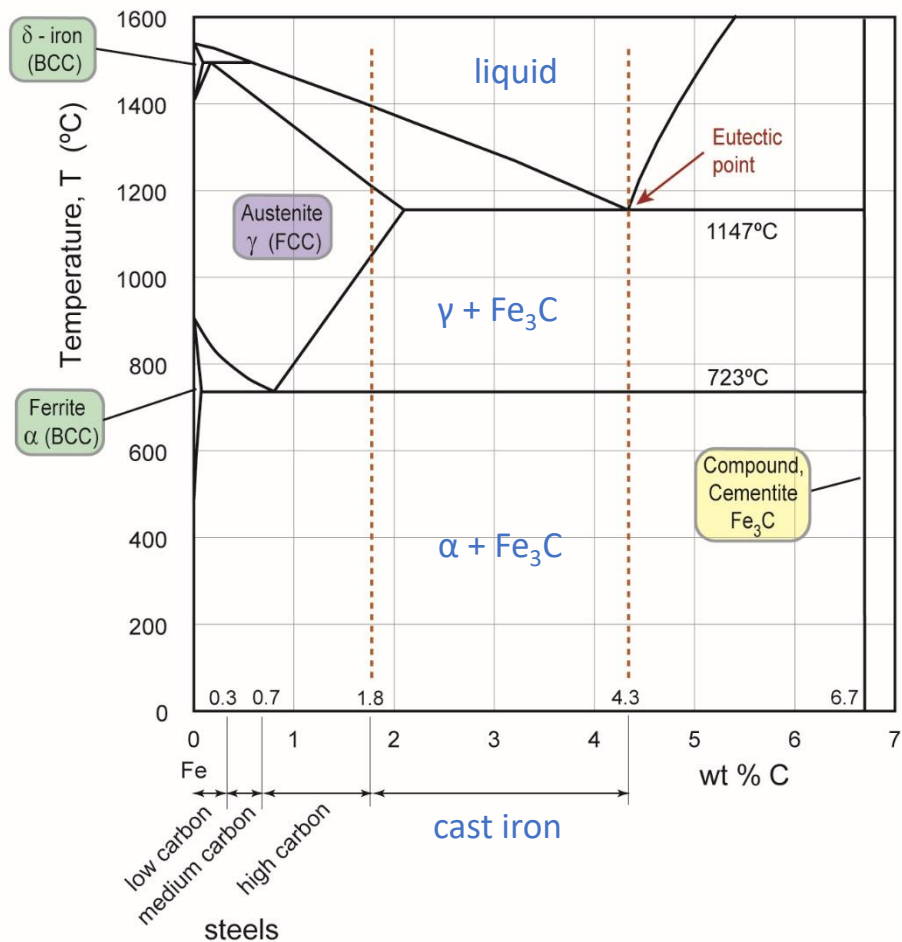
As for Al-Si alloys, cast irons are near-eutectic: easier to cast.

The microstructure is multi-phase:
- **Ferrite** and/or **pearlite** (ferrite + cementite phases), and **graphite** (pure C).
- Note that this is a **metastable** combination of phases (i.e. it doesn't follow the equilibrium phase diagram) – it is easier to form than the true equilibrium state.
- Gives strength and wear resistance, but **poor toughness**: the graphite is brittle, and it forms flakes that act like cracks

'grey' cast iron

pearlite matrix   graphite flakes
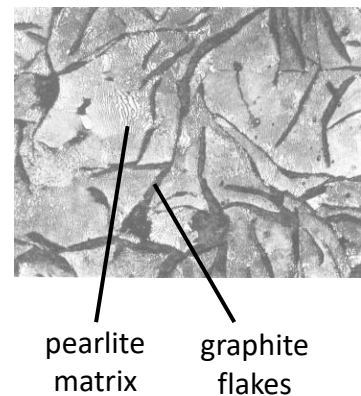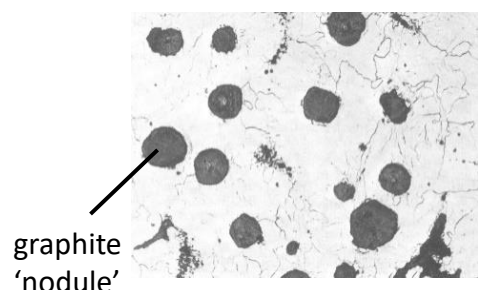
Improving the properties:
- A small alloying addition (of Mg) improves the toughness, by causing the graphite to form as spheres instead of flakes – a trick known as 'poisoning'.
- Graphite nodules act less like sharp cracks: smaller length scale, rounded.

*'spheroidal' cast iron*

graphite 'nodule'

5.3 Segregation: concentration gradients in cast alloys

One consequence of casting with alloys (rather than pure metals) is that a non-uniform distribution of alloying elements (and impurities) can develop on solidification. From the phase diagram, changing the alloy composition affects both T_m and the temperature range over which solidification occurs: "partition" of the liquidus and solidus.

Cooling though this two-phase region:
- The first solid to form is purer (i.e. has a **lower concentration**) than the alloy composition.
- If the alloy follows equilibrium, as the temperature falls the proportion of solid increases, and the equilibrium composition of the solid changes, **increasing in concentration**.
- When solidification is complete, the equilibrium composition of the solid matches that of the alloy.
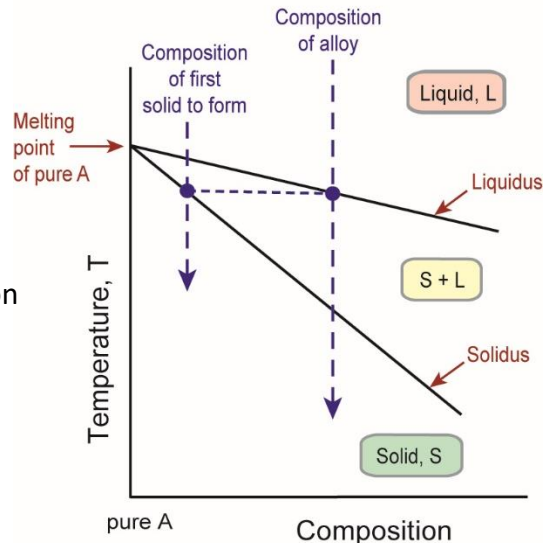
However, in practice, real alloy solidification differs from this equilibrium scenario:
- Equilibrium solidification requires solute to **diffuse** into the solid, increasing its concentration as the temperature falls (following the solidus composition).
- In practice, **diffusion is too slow** for this to happen, and a gradient in concentration is "locked in", across each grain.
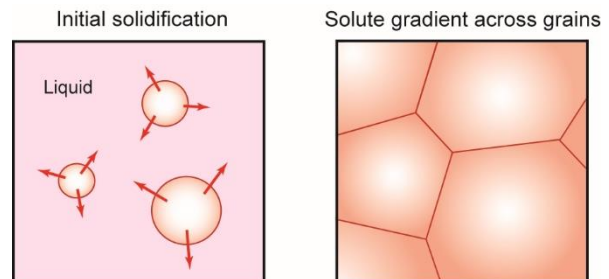
This is known as **segregation**. Segregation leads to concentration gradients across two length scales, with higher solute concentrations in the last solid to form:

     (i) across a grain, from the centre to the boundary: microsegregation

     (ii) across a whole casting, from the mould to the centre: macrosegregation

A similar process also leads to a non-uniform distribution of **impurities**:
- Metals always contain impurities: e.g. S and O in steels, left over from the original extraction processes.
- These often dissolve readily in the liquid, but **not in the solid**.
- During solidification, the impurities are therefore rejected into the liquid, concentrating in the last liquid to solidify: i.e. at grain boundaries (microsegregation), and the centre of the casting (macrosegregation).

Consequences of segregation:

(a) Non-uniform distribution of main alloying elements:
- Leads to a variation in **yield strength** (by solid solution or precipitation hardening).

(b) Concentration of impurities on grain boundaries:
- The impurities may form solid phases (a problem if these are brittle) or trapped bubbles of gas (giving porosity).
- This is damaging to properties – notably **toughness**.

Solutions to segregation:

(1) **Reduce the grain size** of the casting (e.g. using inoculants – see above). A smaller grain size means a large grain boundary area over which to distribute the impurities / porosity that accumulate there, and so lower local impurity concentrations.

(2) Add small alloying additions that **react with impurities**. This forms a fine dispersion of solid particles during solidification, trapping the impurity in a harmless state. Examples: Mn and Al in carbon steels, trapping impurity S and O, as MnS and Al_2O_3.

(3) '**Homogenise**' the casting: heating up the casting, to enable redistribution of the solute by diffusion.

Modelling homogenisation:

We can use diffusion analysis to give a first order model of the redistribution of solute atoms by diffusing during homogenisation.

Solute gradient across grains

x-direction

To achieve this, we need to make some modelling simplifications. These are not completely realistic, but it gives a first order estimate of the key parameter dependencies. We will assume:

1. Diffusion occurs in 1D, along an x-direction traversing the grain.
2. All grains have the same size, d.
3. The initial solute concentration is a sinusoidal function of x, with average C_0 and peak C_1 at the grain boundaries.
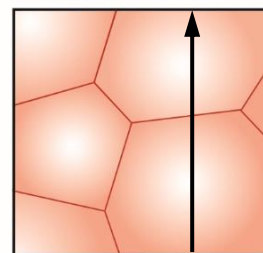
Taking $x = 0$ at a grain boundary, we have initial conditions:

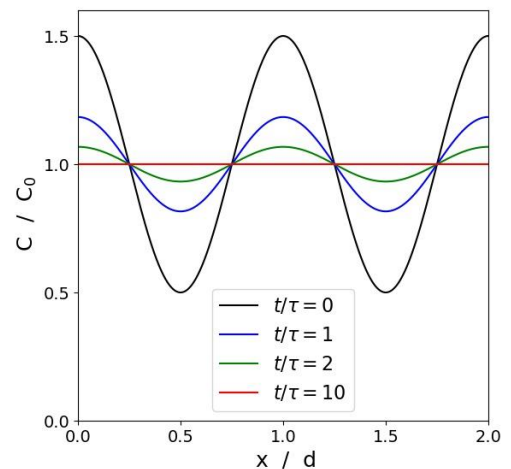$$C(x, 0) = C_0 + (C_1 - C_0) \cos\left(\frac{2\pi x}{d}\right)$$

Following the method outlined previously, this leads to the solution:

$$C(x, t) = C_0 + (C_1 - C_0) \cos\left(\frac{2\pi x}{d}\right) \exp\left(-\frac{4\pi^2 D t}{d^2}\right)$$

Although approximate, this solution indicates some important parameter sensitivities for the homogenisation process.

- Homogenisation will be considered complete once the amplitude of concentration fluctuations drops below a specified level. The time constant governing the rate of decay of the amplitude:

$$\tau = \frac{d^2}{4\pi^2 D}$$

- There is therefore strong temperature dependence on the rate of homogenisation, via the diffusion coefficient:

$$D = D_0 \exp\left(-\frac{Q}{RT}\right)$$

- There is also sensitivity to the grain size, due to the d^2 dependency. Inoculants (see above) can be used to achieve a fine-grained casting, and reduce the homogenisation time.

5.4 Summary of the learning outcomes: casting

After completing section 5 you will be able to do the following:

1. Describe the microstructure of a casting, and explain how it can be controlled.
2. Explain the use of alloying to improve the castability of a metal.
3. Explain 'segregation' in casting alloys.
4. Analyse the homogenisation of a casting using diffusion theory.

5.5 Quiz W6.2: casting

The microstructure of a casting often consists of a fine-grained structure near to the mould wall. Which of the following correctly explain this observation (more than one can be correct)?

1. The undercooling is larger near the mould wall.
2. There are sites for heterogeneous nucleation at the mould wall.
3. The casting alloy is near the eutectic composition.
4. Segregation occurs during solidification.

Additional worked examples: casting

For further explanation, a video will be available with the following worked examples:
- Tripos Question 2008 Q2(a), which helps with learning outcome 1.
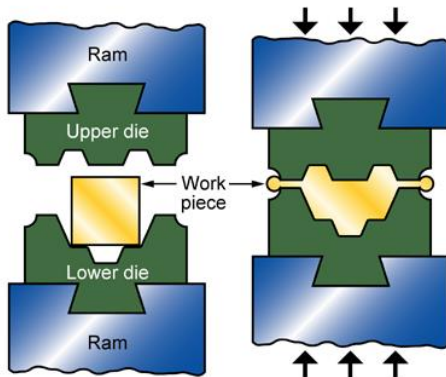- Tripos Question 2019 Q2, which helps with Examples Paper 4 Q8.

Now attempt: Examples Paper 4, Q.7-8
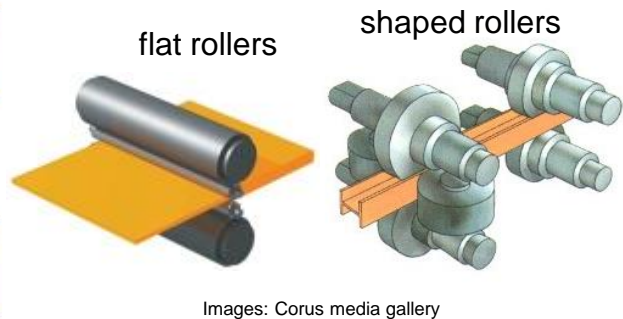
6. Deformation processing: microstructure control

Metals and alloys that have been shaped by deformation processes are known as 'wrought'. They must be sufficiently **ductile** to undergo large amounts of plastic straining without failing. Deformation processes are mostly done in **compression**, to avoid necking and tensile failure at large strains.
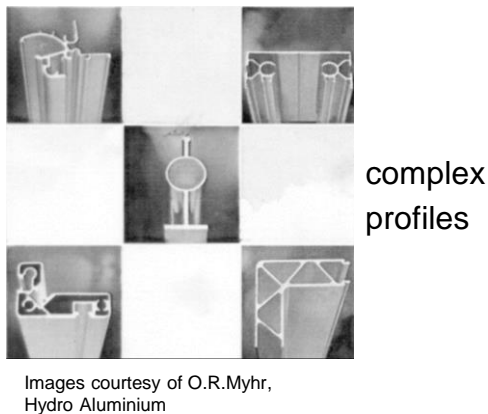
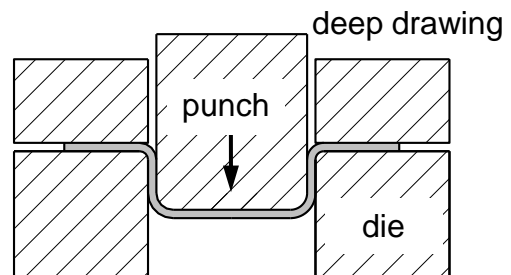Some typical deformation processes are shown below:

(1) forging

(2) rolling

flat rollers

shaped rollers

Ram

Upper die

Work piece

Lower die

Ram

Images: Corus media gallery

(3) extrusion

(4) sheet metal forming

complex profiles

Images courtesy of O.R.Myhr, Hydro Aluminium

deep drawing

punch

die

There are advantages to shaping components by deformation, rather than casting:

- Improved mechanical properties: alloy choice, microstructure control.
- Lower temperatures: lower energy consumption.
- Better surface finish and dimensional accuracy.

This section focuses on the microstructural changes that occur during deformation processing of metals, and how properties and microstructure can be controlled.

6.1 Cold working

Deformation processes taking place at **room temperature** are referred to as "cold working". The following microstructural changes occur:

(1) Grains change shape, following the overall imposed strain, because plastic deformation in metals occurs at **constant volume**. This is due to the microstructural origins of metal plasticity: deformation occurs by dislocations gliding through the material in shear (refer to your IA notes for a reminder on this).
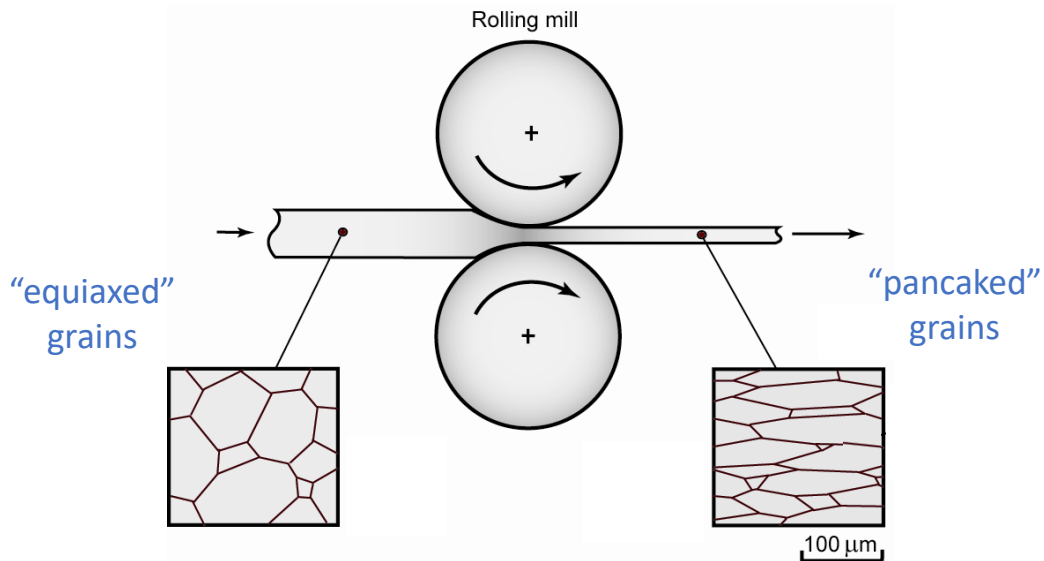
(2) Work hardening occurs, increasing the yield stress. Recall from IA that this is related to an increase in dislocation density (ρ_d) during plastic straining, and the interactions between these dislocations.

- As ρ_d increases, the spacing between dislocations (s) reduces:
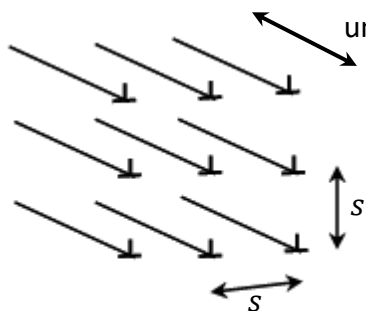

Per unit length of dislocation:

$$\rho_d = \frac{1}{s^2} \quad \Rightarrow \quad s = \frac{1}{\sqrt{\rho_d}}$$

- Dislocations pin each other: the shear stress required to overcome obstacles is inversely proportional to the obstacle spacing ($= s$).

- So, the yield strength increases as ρ_d increases: $\sigma_y \propto \frac{1}{s} = \sqrt{\rho_d}$

A useful and realistic model for the work hardening stress-strain curve for metals is given by the power law relationship:

All curves:
σ = 100 MPa at
ε = 0.002

$\sigma = a\,\varepsilon^{n}$

a, n = constants

n = 0.1 – 0.3 (typically)
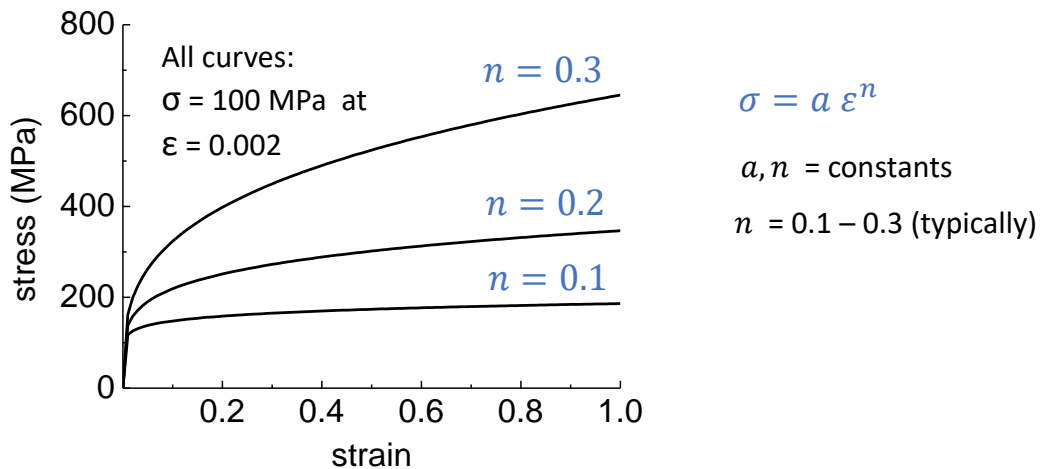
Work hardening is useful in the final deformation steps, to increase the yield strength. However, earlier in the deformation processing, it brings problems:

- As deformation progresses: applied forces must increase
- Ductility reduces: risk of fracture

Deforming to large plastic strains therefore becomes difficult (e.g. rolling aluminium foil). To make large strain processes possible, we need to **anneal** the material periodically.

6.2 Annealing

Annealing is a form of heat treatment. The material is heated in order to reverse the effects of work hardening:

- reduce the yield strength
- increase the ductility.
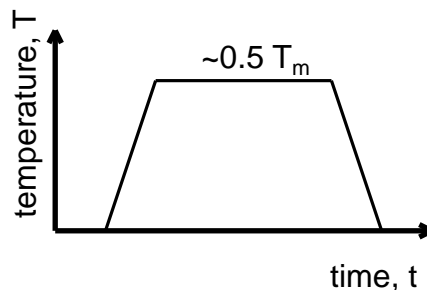
The **driving force** for the microstructural changes that take place during annealing is the elastic **stored energy** within the material that results from the increased **dislocation density** during cold working. During plastic deformation, the work done is given by the area under the stress-strain curve:

plastic work:
not recovered

elastic energy
recovered when
material breaks
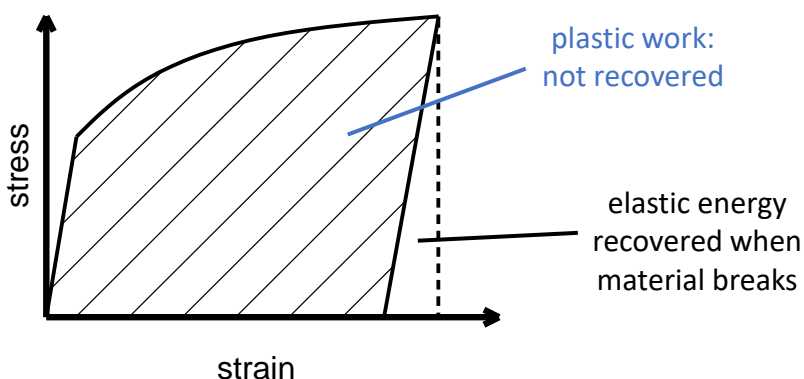
Considering the plastic work done (the energy not recovered on fracture):
- ~ 95% is dissipated as heat
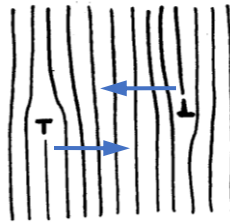- ~ 5% remains stored in the material in dislocations

Recall from IA that dislocations induce elastic strain in the lattice around them, which can be expressed as a line energy per unit length. This is the source of the stored energy as the dislocation density (length/unit volume) increases.

During annealing, two microstructural changes occur, which reduce this stored energy associated with dislocations. Both depend, to some extent, on **diffusion of atoms**, and are therefore temperature dependent.
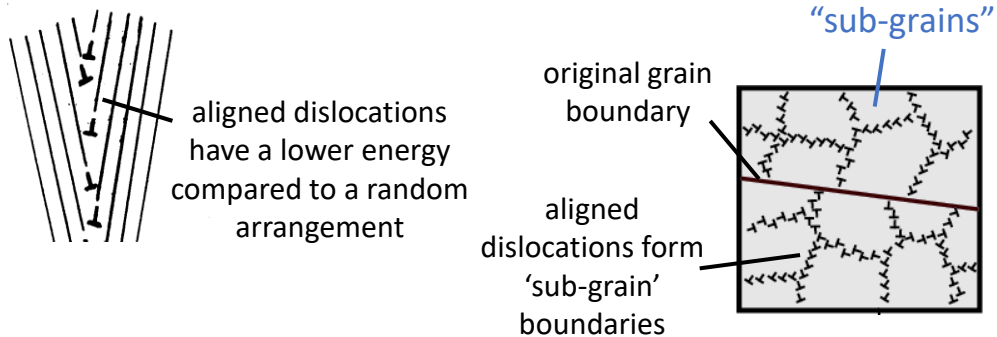
Mechanism 1: Recovery
One route is for **dislocations to rearrange** into a pattern that reduces the strain in the crystal lattice around them: this is called **recovery**.

(1) dislocations of opposite orientation can annihilate each other:

(2) dislocations of similar orientations can align and form **subgrains**:

aligned dislocations have a lower energy compared to a random arrangement

"sub-grains"

original grain boundary

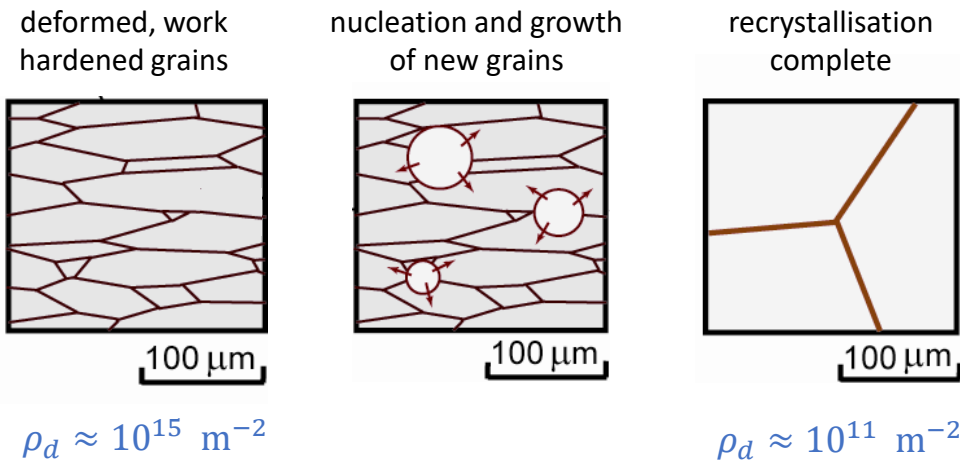aligned dislocations form 'sub-grain' boundaries

- These processes depend on (i) glide of dislocations and (ii) local rearrangements of the crystal structure near to the dislocations by diffusion of atoms.
- Minimum annealing temperature for recovery to occur: $T_{\min} \approx 0.1\, T_m$
- Only a small drop in dislocation density occurs: small drop in σ_y

Mechanism 2: Recrystallisation

Another route to relieve stored dislocation strain energy is to nucleate and grow **new crystals** within the work hardened grains:

- New grains: much lower ρ_d $\Rightarrow$ large drop in σ_y

deformed, work hardened grains	nucleation and growth of new grains	recrystallisation complete

$\rho_d \approx 10^{15} \ \mathrm{m}^{-2}$ $\rho_d \approx 10^{11} \ \mathrm{m}^{-2}$

- Recrystallisation requires more significant atomic rearrangement by diffusion than recovery, so higher temperatures are required.
- Minimum annealing temperature for recrystallisation to occur: $T_{\min} \approx 0.3 \, T_m$

When to anneal? Example: cold rolling annealing schedule.

In cold rolling, the thickness of the sheet is progressively reduced by **many passes** through a rolling mill.

The decision on when to perform intermediate annealing steps is based on two factors:

- The yield stress remaining **below a critical value** during rolling set by the load limits on the rolling mill.
- Meeting a **final yield stress** requirement, to ensure the rolled material is within its design strength range.
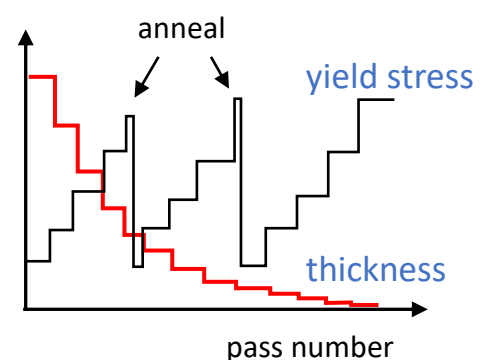
To work out the annealing schedule, start by calculating the total number of passes required to achieve the required thickness reduction. Next, calculate the strain per pass.

If a pass through the rollers reduces the thickness from t_0 to t_1, the true strain is:

$$\varepsilon_t = \ln(1 + \varepsilon_n) = \ln\left(1 + \frac{t_1 - t_0}{t_0}\right) = \ln\left(\frac{t_1}{t_0}\right)$$

Note the sign convention here is tensile strains are positive. True strain is convenient to use as, unlike nominal strain, it is **additive**. For N passes, the total true strain is equal to the sum of the true strains in each pass:

$$\varepsilon_t = \ln\left(\frac{t_1}{t_0}\right) + \ln\left(\frac{t_2}{t_1}\right) + \ ... \ \ln\left(\frac{t_N}{t_{N-1}}\right) = \ln\left(\frac{t_1}{t_0}\frac{t_2}{t_1}...\frac{t_N}{t_{N-1}}\right) = \ln\left(\frac{t_N}{t_0}\right)$$

Whereas the total nominal strain is not:

$$\varepsilon_n = \frac{t_N - t_0}{t_0} \neq \frac{t_1 - t_0}{t_0} + \frac{t_2 - t_1}{t_1} + \ ...$$

As the magnitude of the strain increases, so does the yield stress, due to work hardening :
- A power law relationship can be used to model this, as outlined above: $\sigma = a\,\varepsilon^n$
- The **magnitude** of the strain can be used in the hardening law – the sign convention adopted above can be dropped.
- Each anneal has the effect of **"resetting" the strain** to zero (though the thickness reduction will be preserved).

The annealing schedule is set by determining:
(1) The thickness reduction (number of passes) between the final anneal and the final product, to meet the design spec on product strength.
(2) The maximum thickness reduction between anneals, set by the rolling mill force limit. This determines how many additional annealing steps are required.

### *Example of a cold rolling annealing schedule (Examples Paper 5 Q2)*

- Cold rolling: $t_0 = 0.4$ mm $\rightarrow$ $t_f = 6.7\ \mu$m
- Thickness reduction per pass: 40 % $\rightarrow$ 8 passes required (regardless of annealing)
- Strain: $\varepsilon = \ln(t_N/t_0)$
- Yield stress (MPa): $\sigma = 100|\varepsilon|^{0.3}$ (strain accumulated since last anneal)
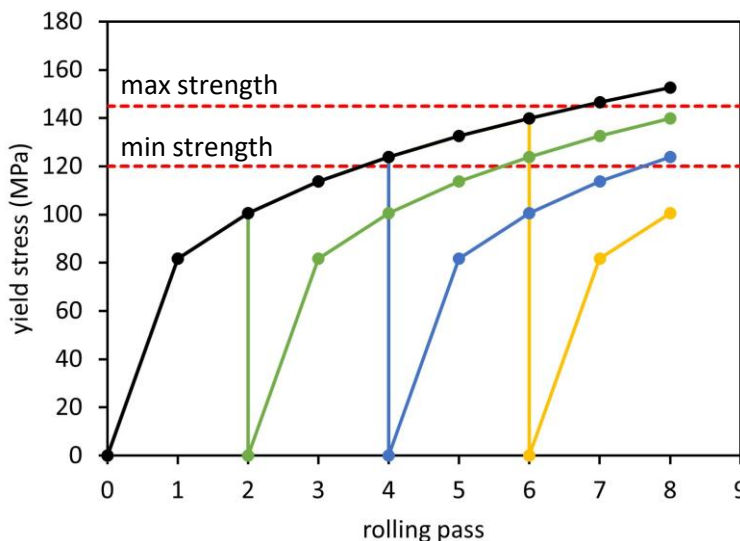
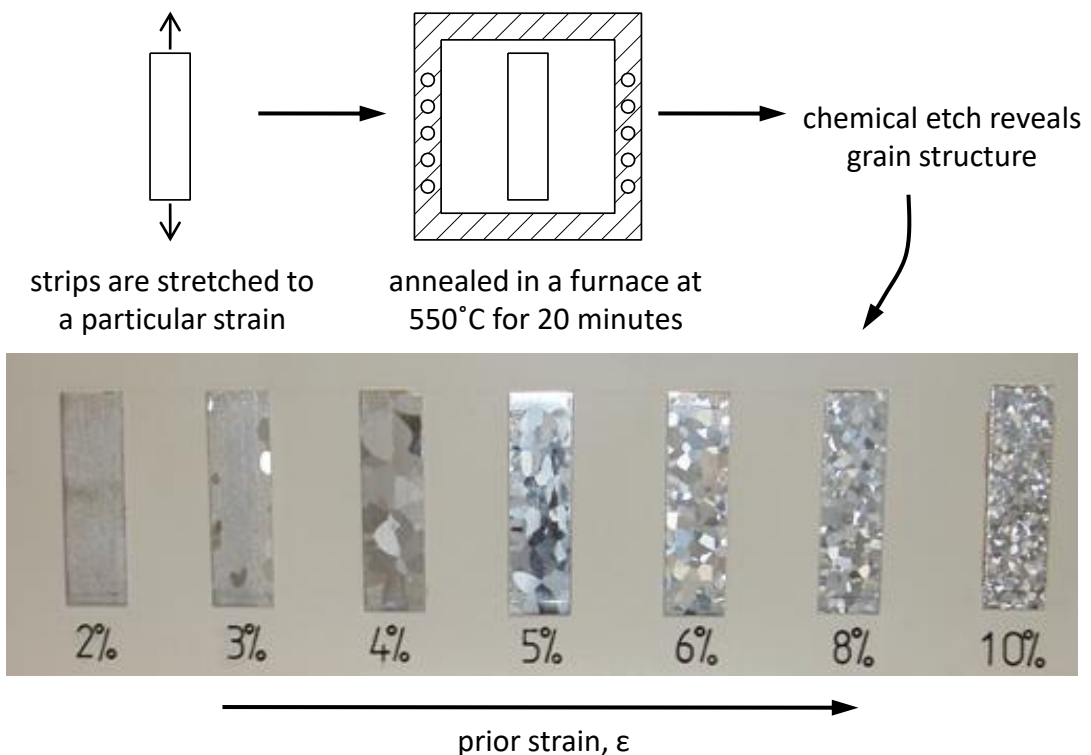## 6.3 Using recrystallisation to control grain size

Just as nucleation can be used to control grain size in a casting, similar principles apply to recrystallisation. In casting, the thermodynamic driving force for nucleation is the Gibbs free energy change, ΔG. In recrystallisation, the driving force for nucleation of new grains is the dislocation strain energy.

The dislocation strain energy depends on the dislocation density ρ_d. This in turn is related to the amount of plastic strain ε. No recrystallisation occurs below a critical level of plastic strain, ε_{crit}. The final grain size depends on $(\varepsilon - \varepsilon_{crit})$.
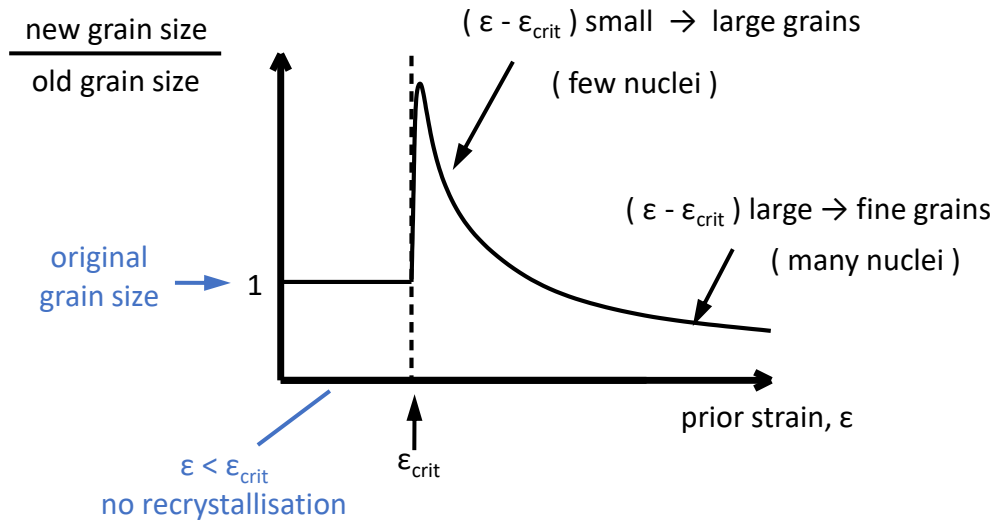
Example: Aluminium recrystallisation

Illustrated below is an experiment carried out to identify the effect of prior straining on the recrystallised grain structure of aluminium sheet:

strips are stretched to a particular strain

annealed in a furnace at 550˚C for 20 minutes

chemical etch reveals grain structure

prior strain, ε

Grain growth after recrystallisation

If the material is held at a high temperature after recrystallisation has completed, grain growth will occur.

- Grain boundaries, as discontinuities in the crystal structure, carry an **energy penalty** (similar to the interface energy when a solid nucleus forms in a liquid).
- If atoms are able to continue to rearrange by diffusion, the grain size will tend to increase, thus reducing the **surface area to volume ratio** of the grains.

A larger grain size (d) reduces the grain boundary hardening: $\sigma_y = A + \dfrac{B}{\sqrt{d}}$

6.4 Hot working

In practice, the processes of deformation and annealing are often **combined** by hot working the material. The material is deformed at high temperature ($\sim$ 0.7 T_m).

Image: Corus media gallery

This has a number of advantages:
- The material yield strength remains low, due to a lack of strain hardening, and the deformed material remains ductile:
  - lower tool forces required
  - larger strains in one pass
- No need for separate heat treatment.

6.5 Summary of the learning outcomes: deformation processing and microstructure

After completing section 6 you will be able to do the following:

1. Compare the attributes of wrought alloys and casting alloys.
2. Explain the role of annealing during hot or cold working of a wrought alloy.
3. Describe the annealing mechanisms of recovery and recrystallisation.
4. Explain the use of recrystallisation to control grain size.

'Wrought' alloys are suitable for shaping by deformation processes. A wrought mild steel (0.15 wt% C) is compared with cast iron (3.1 wt% C).

Which of the following statements are correct (more than one can be correct)?

1. Cast iron contains more carbon than mild steel in order to increase its toughness.
2. Mild steel is less suited to near net shape casting because it is further from the eutectic composition.
3. Mild steel is more suited to deformation processing because it is more ductile.
4. The strength of mild steel can be increased by annealing.

Now attempt: Examples Paper 5, Q.1-2

7. Deformation processing: process modelling

In this section we will analyse the forces required to shape components via deformation processes. We will consider two examples: forging and rolling.
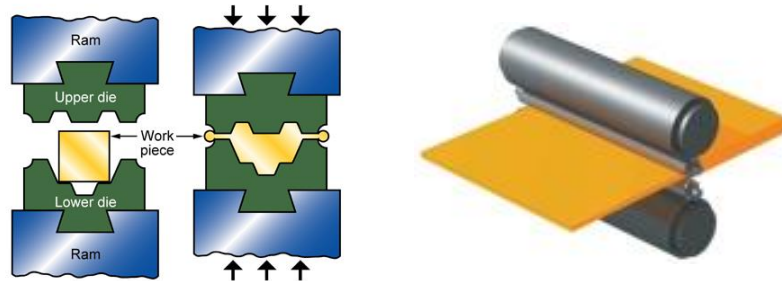
These process models help to show the key parameter sensitivities within the manufacturing process. For example, the relationship between forces and:
* material properties (such as σ_y),
* processing conditions (such as friction),
* component shape (thickness, aspect ratio).

The models rely on the following key principles:
* equilibrium of stresses,
* material laws: yield criteria for metals,
* fiction laws: boundary conditions between the tool and workpiece.

7.1 Modelling yielding in metals

For multiaxial stress states, the combination of stresses at the onset of yielding can be modelled using yield criteria, introduced in more detail in the IB Structures course.

Yield criteria are expressed in terms of the **principal stresses:**

$$(\sigma_1, \ \sigma_2, \ \sigma_3)$$

- For any element of material, an orientation of the faces exists where the shear stresses vanish.
- These are the principal stress directions, and the direct stresses at these orientations are the principal stresses.
- They can be determined by rotating around Mohr's circle.
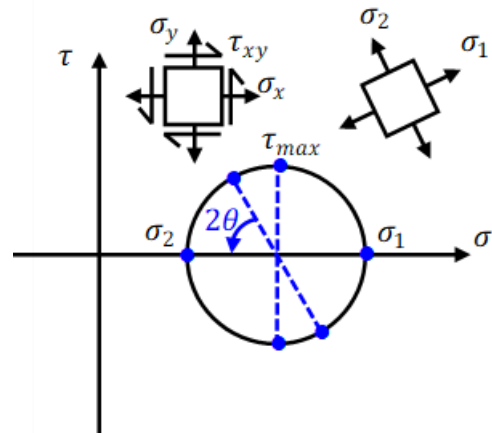
Note also that the maximum shear stress acting on an element in a particular plane is given by the radius of Mohr's circle: i.e. $\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_2)$ (in the 1-2 plane)

Two yield criteria have been introduced:

- Tresca yield criterion: $\max\{ |\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\} = Y$

  physical basis: critical maximum shear stress

- von Mises yield criterion: $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2$

  physical basis: critical elastic shear strain energy

Here, Y = uniaxial yield stress (also commonly given the symbol σ_y).

In both cases, a key feature of the model is that **shear** governs yielding:
- A purely **hydrostatic** state of stress has $\sigma_1 = \sigma_2 = \sigma_3$. It induces **no shear** on any material plane (the Mohr's circle is just a point on the σ axis), and does not cause yielding, according to either yield criterion.
- This is supported by experimental observation: metal yielding is insensitive to hydrostatic stress (though other materials are - e.g. foams).
- This is due to the **microstructural basis** of plasticity: metal plasticity occurs by dislocation movement, driven by shear stresses.
- Note that this mechanism of plastic deformation does not require a volume change (confirmed experimentally).
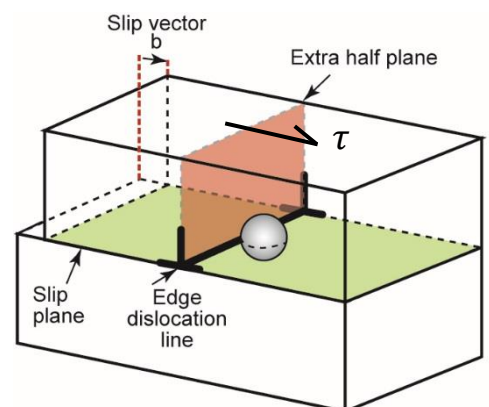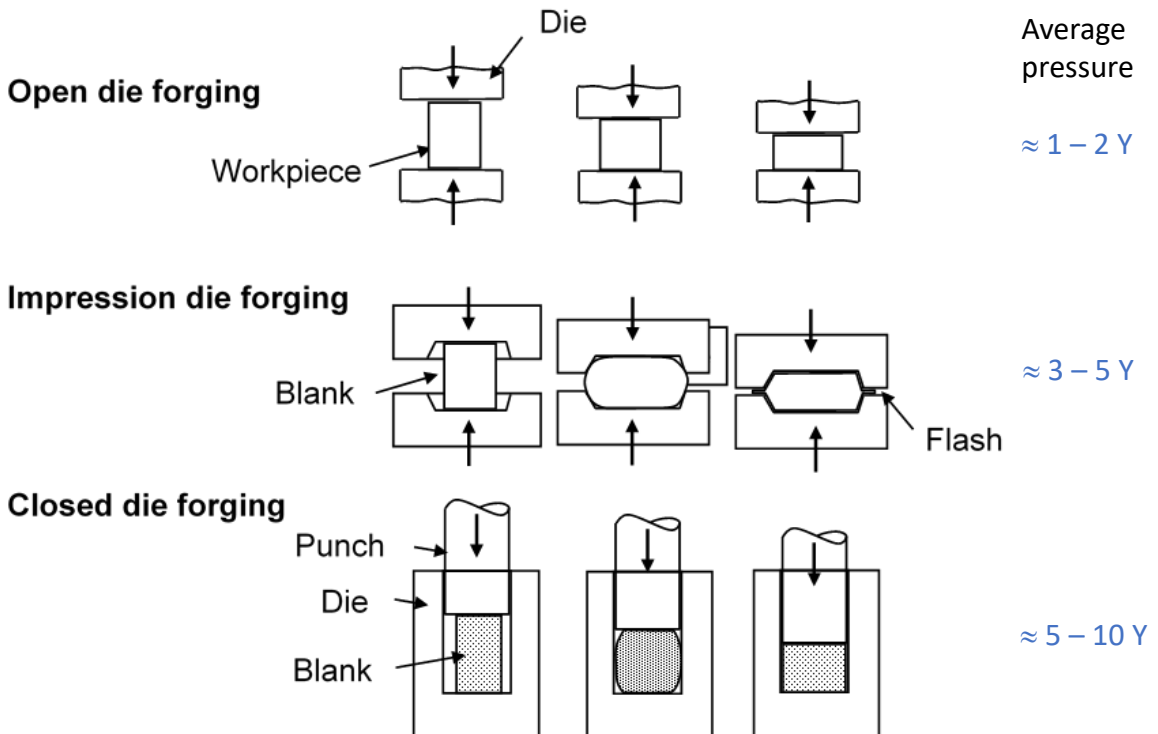
7.2 Analysis of forging

Forging involves compressive plastic deformation of a solid between two dies. Over 90% of forging is done hot:

* lower yield stress $\Rightarrow$ lower forces required, greater complexity of shape possible

Forging parts with complex shapes will often be carried out incrementally, over a number of forging stages. For example, for a con-rod:


many forging stages
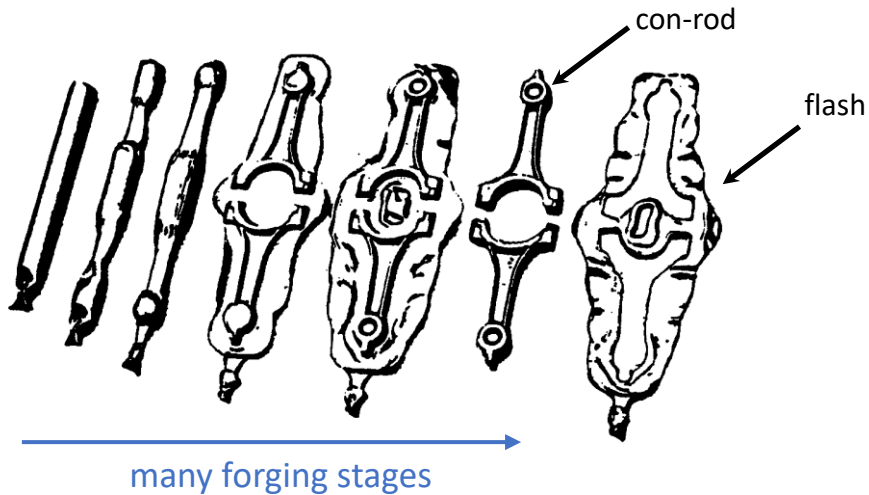
Friction plays an important role in the forging loads. Its contribution is sensitive to the degree of contact between the workpiece and the dies:

Average pressure

Open die forging $\approx 1 - 2\,Y$

Impression die forging $\approx 3 - 5\,Y$

Closed die forging $\approx 5 - 10\,Y$

We will analyse the open-die forging of a slab between parallel dies.
- Dimensions: slab height = $2h$, width = $2h$, depth (into page) = d (>> h)
- The dies are lubricated to minimise the forging load and reduce die wear, but some friction between the slab and dies is unavoidable.
- The direction of the friction shear stress will oppose the flow of material away from the centre line.
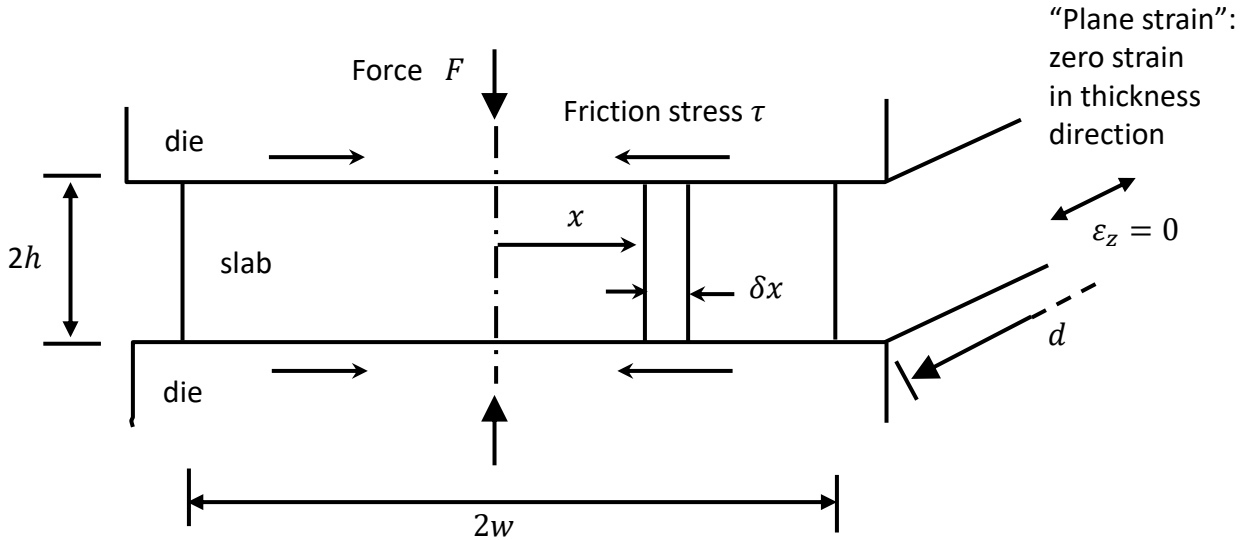

Aim of the analysis:

  (i) Calculate the pressure $p(x)$ between the slab and dies.
  (ii) Integrate $p(x)$ to give the forging force F.

Step 1 – Consider equilibrium of a small element of material

Consider an element of material at a distance x ($x > 0$) from the centre line of the forging, with width δx and height $2h$. Note that the pressure p, friction stress τ and horizontal stress in the material σ_x will, in general, vary with position x.
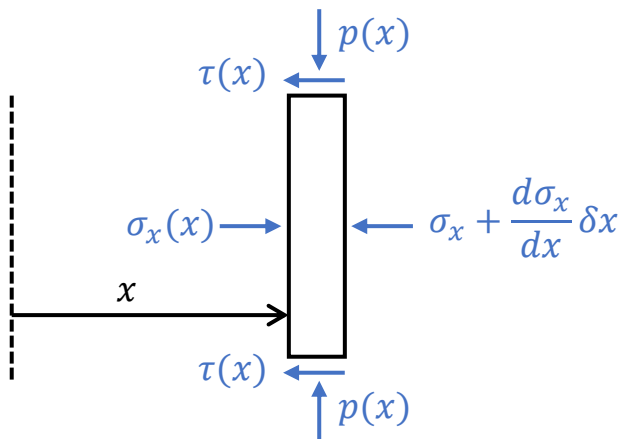
Consider unit depth into the page.

Take *compressive* $p(x)$ and $\sigma_x(x)$ as positive

Resolving forces horizontally:

$$2h\sigma_x - 2h\left(\sigma_x + \frac{d\sigma_x}{dx}\delta x\right) - 2\tau\delta x = 0 \qquad \therefore \frac{d\sigma_x}{dx} = -\frac{\tau}{h} \qquad (1)$$

Step 2 – Apply the yield criterion

This will allow us to relate σ_x and p (the pressure is what we want to find). The material must be yielding everywhere, so the stresses must satisfy a yield criterion.

- We need to identify the principal stress. To simplify the analysis, we will assume that the frictional shear stress τ is small compared to σ_x and p. So we will take σ_x and p as **approximately** the principal stress.

- Note also that the magnitude of $p > \sigma_x$. To first order, the pressure $p \sim Y$, whereas $\sigma_x \sim \tau$. The frictional shear stresses will often be much less than the yield stress.
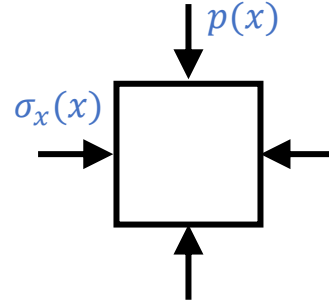
Using the Tresca criterion (algebraically simpler): $p - \sigma_x = Y$

Substituting for σ_x into equation (1), and assuming the yield stress Y is constant (i.e. no work hardening – a reasonable approximation in hot forming) :

$$\frac{dp}{dx} = -\frac{\tau}{h} \qquad (2)$$

Step 3 – Friction law

We can choose from a number of models to represent the effect of friction. The best choice would have to be determined by experiment.

(1) Coulomb friction: the shear stress is taken to be proportional to the pressure:

$$\tau = \mu p$$

The coefficient of friction μ will depend on the materials and the lubrication.

(2) Tresca friction: the shear stress is taken to be proportional to the shear yield stress, k:

$$\tau = mk$$

Note that the shear yield stress: $k = Y/2$ (Materials Databook; Tresca yield criterion). The friction factor m varies in the range $0 \leq m \leq 1$ depending on the lubrication. Taking $m = 1$ is known as 'sticking friction'.

Assume Tresca friction, and substitute for τ into equation (2):

$$\frac{dp}{dx} = -\frac{mk}{h} = -\frac{mY}{2h} \qquad (3)$$

Step 4 – Integrate, identifying suitable boundary conditions

At the edge of the strip, $x = \pm w$:

- The surface is stress free: $\sigma_x = 0$
- Therefore, applying the Tresca yield criterion: $p = \sigma_x + Y = Y$

Integrating equation (3), applying these boundary conditions:

$$\int_{p(x)}^{Y} dp = -\frac{mY}{2h}\int_{x}^{w} dx \quad \Rightarrow \quad p(x) = Y + \frac{mY}{2h}(w - x) \quad [x > 0]$$

If we repeat the analysis for an element of material on the **left-hand side** of the centre line ($x < 0$), the equations are identical apart from a change in the sign of τ, because friction acts in the opposite direction here (material flows to the left):

$$\frac{d\sigma_x}{dx} = +\frac{\tau}{h} \quad \rightarrow \quad \frac{dp}{dx} = +\frac{mY}{2h} \quad \rightarrow \quad p(x) = Y + \frac{mY}{2h}(w + x) \quad [x < 0]$$

The solution for $p(x)$ is plotted below for different values of the friction factor m. This is known as the **"friction hill"**:


$$p_{\max} = Y\left(1 + \frac{mw}{2h}\right)$$
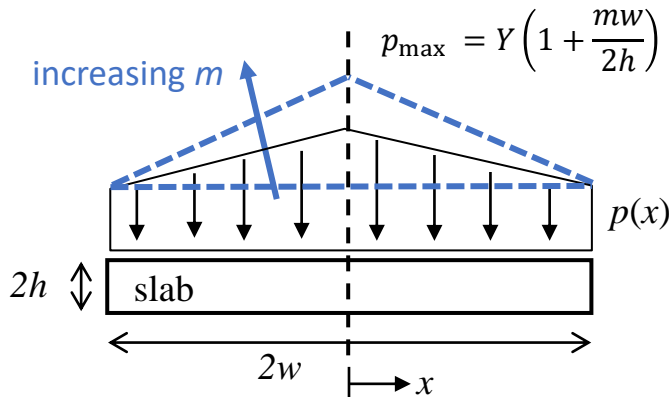
The total forging load (per unit depth) can be found by integrating $p(x)$ over the width of the strip. This can also be expressed as an average pressure, $\bar{p}$:

$$F = \int_{-w}^{+w} p(x)\ dx = 2wY\left(1 + \frac{mw}{4h}\right) \quad \Rightarrow \quad \bar{p} = \frac{F}{2w} = Y\left(1 + \frac{mw}{4h}\right)$$

The model, though approximate, helps to identify the key parameters that influence the forging load:

- *Material parameter* – yield stress, Y: $\uparrow$ temperature $\Rightarrow \downarrow Y \Rightarrow \downarrow F$
- *Process parameter* – friction factor, m: $\uparrow$ lubrication $\Rightarrow \downarrow m \Rightarrow \downarrow F$
- *Design parameter* – aspect ratio, w/h : high w/h $\Rightarrow$ high F

The dependence on w/h means that the
'flash' should be minimised – thin
sections can contribute significantly to
the forging load.

flash
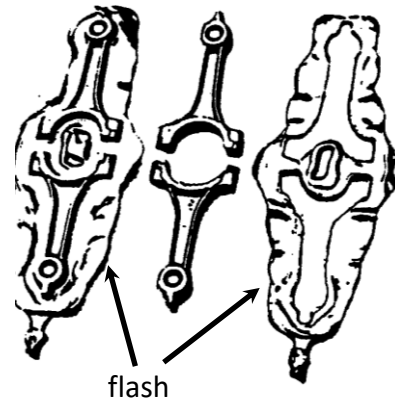
7.3 Analysis of rolling

The detailed analysis of rolling is beyond the scope of this course. However, the
pressure variation between the rollers also follows a **"friction hill"** behaviour, very
similar to the forging problem. For Coulomb friction, $\tau = \mu p$:
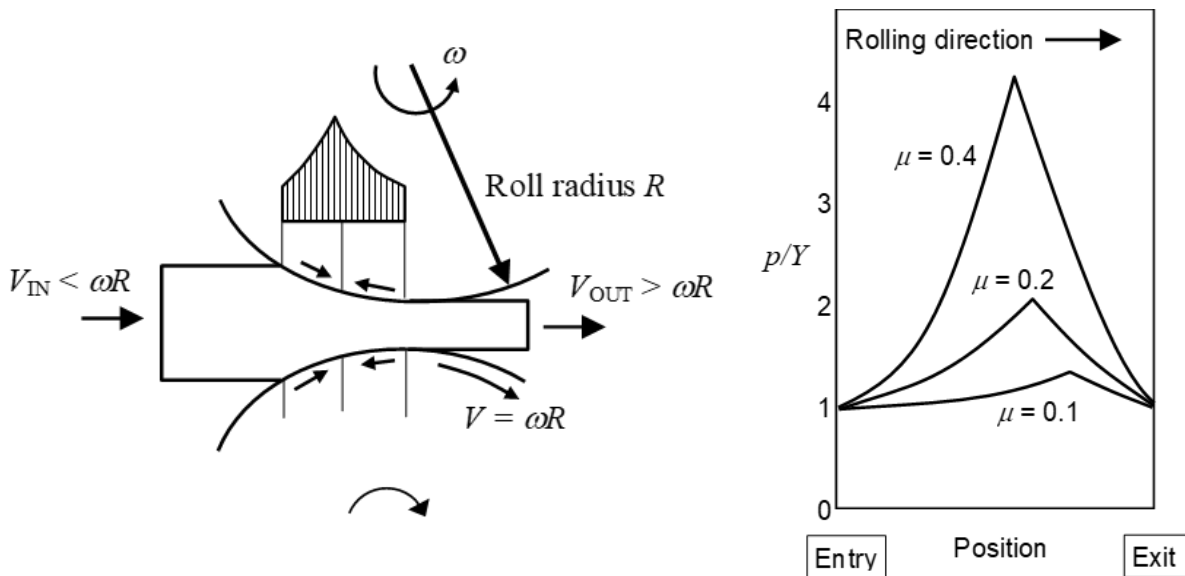
This similarity in the pressure distribution between rolling and forging is due to a
similarity in the frictional shear stress distribution across the width of the contact patch:

- At the inlet, the roll speed (ωR) is faster than the strip speed (V_{IN}). The frictional
  shear stress pulls the strip into the roll gap. (Some friction is therefore essential for
  this process, even though it increases the rolling forces).

- Conservation of mass means that $V_{OUT} > V_{IN}$. At the outlet, the roll speed is less
  than the strip speed, so the friction now **opposes** the strip motion.

- The position in the roll gap where the roll and strip have the same speed (ωR) is the
  neutral plane. At this position, the frictional shear stress **changes direction**, and the
  pressure is a **maximum** (as derived in the forging analysis, above).

The rolling force is given by the integral of the pressure distribution: $F = \int p(x)\ dx$

A first estimate could be obtained by taking $p(x)$ from the forging analysis above, taking the strip thickness in the forging model to be the average strip thickness in the rolling process.

Note that the force F on the strip does not act through the roll axis $\Rightarrow$ a **torque** T must also be applied.

The rolling torque:
$$T = F \times \alpha R$$
where $\alpha \approx 0.1 - 0.5$ needs to be obtained from a more detailed calculation (beyond the scope of this course).
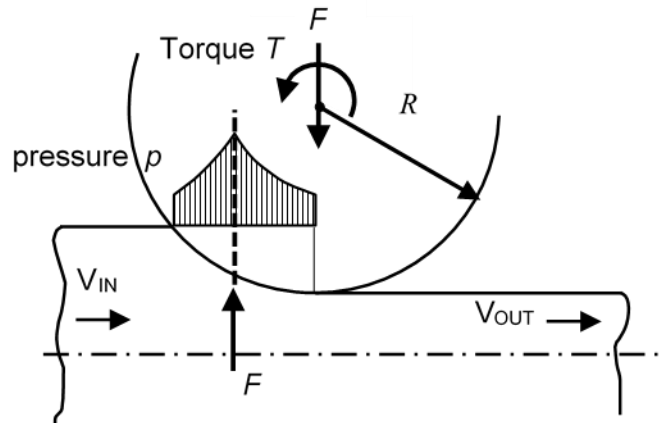
## 7.4 Case study: wrought vs cast alloys in torsion

The typical composition of a wrought alloy (shaped by deformation, and so ductile) leads to contrasting mechanical properties versus casting alloys (chosen near eutectic, for processibility). This case study describes an experimental investigation using **torsion testing** to comparing their properties.

Tensile testing was introduced in the IA Materials course. Torsion theory has been introduced in IB Structures. A torsion test loads the material in **shear**.
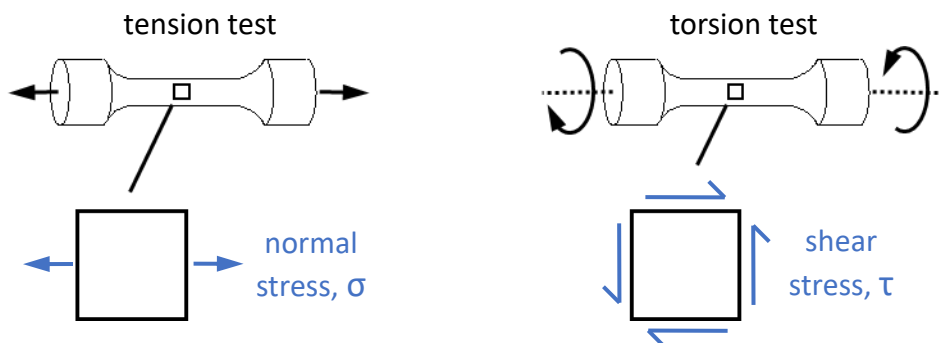

Torsion testing has some advantages over tensile testing:

- **Necking** does not occur in torsion. The necking instability in a tensile test limits the amount of plastic straining that a test specimen can be subjected to.
- A torsion test is therefore a good way to investigate strain hardening behaviour, up to **large plastic strains**.
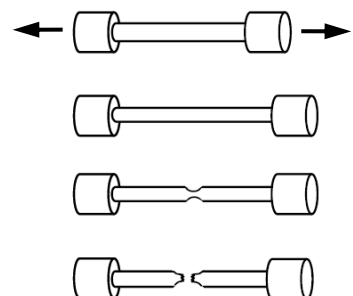
But torsion testing has the disadvantage that the stress state in the material is more complicated than in a tension test.

- Firstly, this is because a stress state of pure shear gives one tensile and one compressive principal stress, from Mohr's circle. The principal stresses will be at $\pm 45°$ to the bar axis:
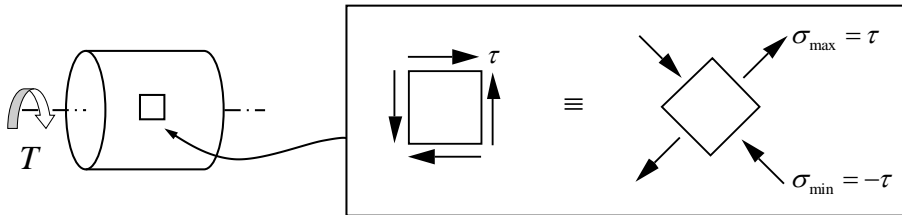
- Secondly, the magnitude of the shear stress will vary with radial position (r) in the bar. Recall from the Structures course, for an elastic bar with a circular cross section:
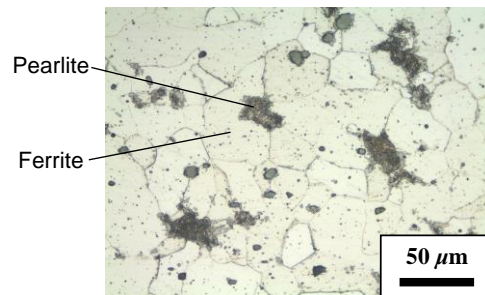
$$\tau(r) = \frac{Tr}{J}$$

T = torque, $J = \frac{\pi R^4}{2}$ for a bar of outer radius R

Test method

Test specimens, solid bars with circular cross-sections, were manufactured from two contrasting alloys:

Wrought alloy: mild steel with composition 0.15 wt% C.
- Its microstructure is mostly soft and ductile ferrite, with a small fraction of harder pearlite (ferrite + cementite, Fe_3C).

Casting alloy: cast iron with composition 3.1 wt% C.
- The higher C concentration brings it closer to the eutectic composition (see earlier lectures).
- Its microstructure in this case is mostly pearlite, with large flakes of brittle graphite.
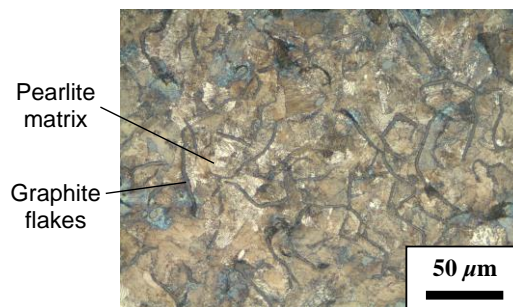
The bars were tested to failure using one of the CUED torsion testing machines. This allows a plot of torque versus angle of twist (per unit length of bar) to be produced.
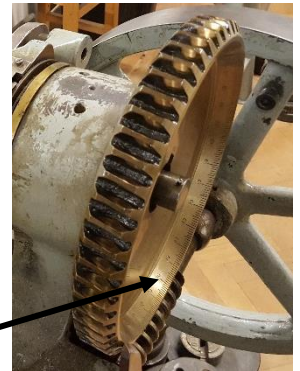

(1) Specimens have square ends, to allow them to be gripped by carrier plates

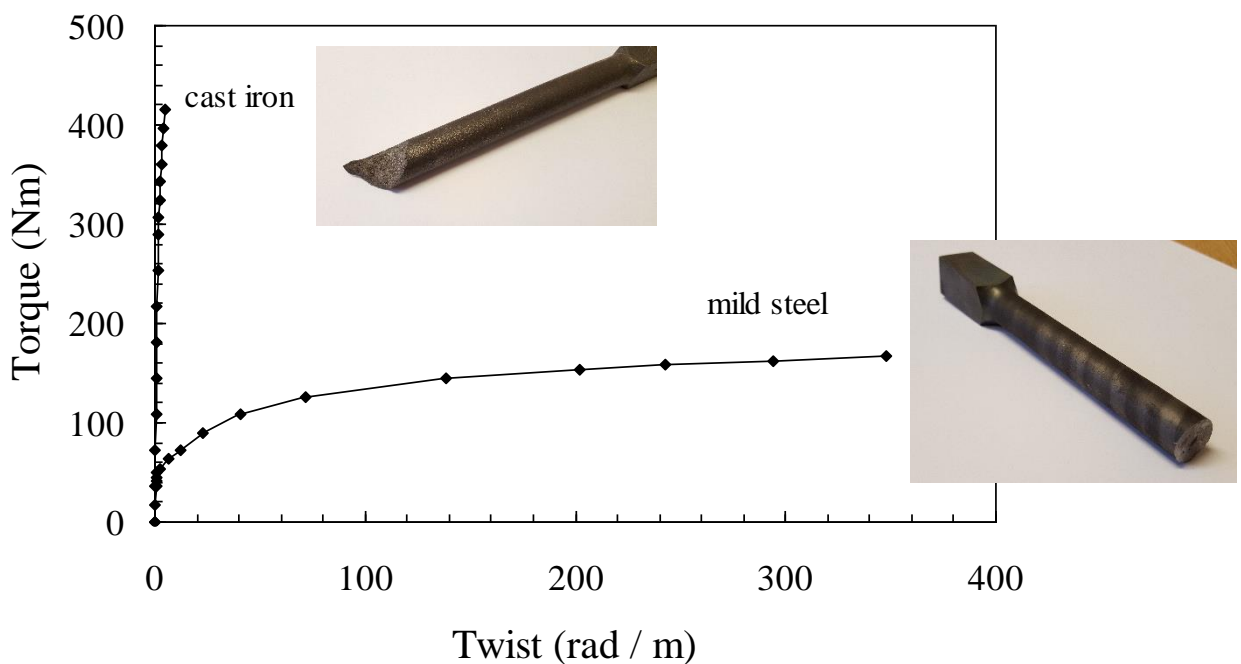(2) The torque is applied by hanging weights from a lever arm

(3) For each value of applied torque, the hand wheel is turned to twist the specimen, until the lever arm is raised to the horizontal

(4) The angle of twist can then be read off a protractor on the hand wheel.

Torsion test results

The torsion test results up to fracture for the two specimens are shown below:

Observations:

- The mild steel behaves in a ductile manner, yielding at a torque of 50 Nm. The bar, of length 150 mm, breaks after nearly 10 complete revolutions. It fractures perpendicular to the axis of the bar.
- The cast iron doesn't show any significant plastic deformation. It breaks in a brittle manner at a torque of 415 Nm. It fractures in a spiral shape.

Analysis: mild steel bar

At the beginning of the test, the mild steel bar will deform elastically. During elastic torsion of a circular bar, the shear stress increases linearly from the centre to the surface of the bar. The maximum shear stress at $r = R$ is:

$$\tau_{max} = \frac{TR}{J} = \frac{TR}{\left(\frac{\pi R^4}{2}\right)} = \frac{2T}{\pi R^3}$$

Yielding will occur here first. The bar has a diameter of 12.6 mm. Using the measured torque at the onset of yielding (T_y = 50 Nm) the **shear yield stress (k)** can be calculated:

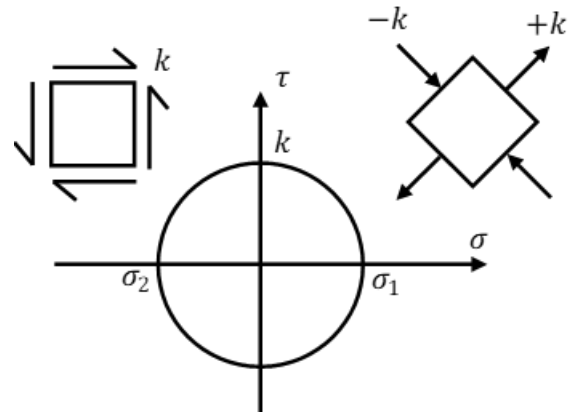$$\tau_{max} = k = \frac{2T_y}{\pi R^3} = \frac{2(50)}{\pi(6.3 \times 10^{-3})^3} = 128 \ \text{MPa}$$

We need to use the yield criteria to relate k to the uniaxial yield stress Y for the steel.

From the Mohr's circle for an element of material in pure shear, the in-plane principal stresses at yield are:

$$\sigma_1 = k, \ \sigma_2 = -k$$

In pure shear, the out-of-plane direction is stress free:

$$\sigma_3 = 0$$

We can plot the two yield criteria on principal stress axes, to show the combination of (σ_1, σ_2) at the onset of yielding.
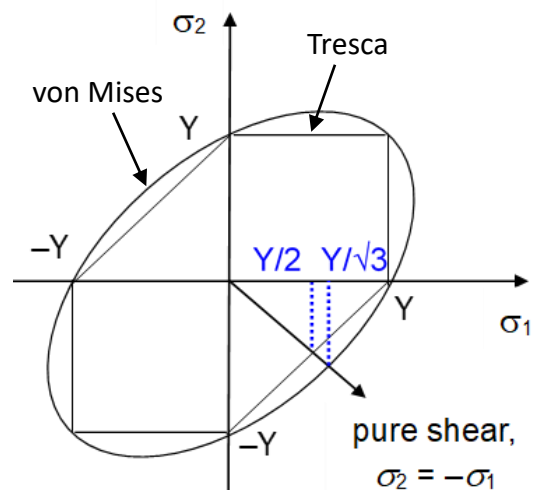
Substituting the principal stresses into the yield criteria:

- Tresca: $k = \dfrac{Y}{2}$
- von Mises: $k = \dfrac{Y}{\sqrt{3}} \approx 0.58Y$

The predicted uniaxial yield stresses are therefore:

- Tresca: $Y = 2k = 256 \ \text{MPa}$
- von Mises: $Y = \sqrt{3}k = 222 \ \text{MPa}$

These are consistent with expected values for EN1A mild steel.

Sources of uncertainty?
- **Choice of yield criterion:** For stress states other than uniaxial loading, the two yield criteria give slightly different predictions for the relationship between Y and k.
- **Detecting the onset of yielding:**
  - In a tension test, the stress state is uniform, so yielding occurs simultaneously throughout the cross section.
  - In a torsion test, the stress state is non-uniform. It may therefore be difficult to detect precisely the first onset of yield, which occurs only at the surface.
  - Further increasing the torque will cause the region of plastically deforming material to propagate towards the centre of the bar, until it is fully plastic. There is therefore no clear single 'yield point'.

Mode of failure?
The specimen finally fails along a plane perpendicular to the axis of the bar.
- This is a plane of maximum shear stress, from Mohr's circle.
- This is a characteristic of ductile fracture. Recall that shear stresses are key for plastic deformation of ductile metals.

Analysis: cast iron bar
The cast iron bar was observed to fail in a brittle manner. Yield criteria therefore won't be applicable.

But we can use the mode of failure to deduce a more appropriate criterion.
- spiral fracture, 45° to bar axis ⇒ failure governed by the **maximum tensile stress** (from Mohr's circle - see above)

Assuming the bar remains elastic up to failure, from Mohr's circle (see above) the maximum tensile stress:

$$\sigma_{\max} = \tau_{\max} = \frac{2T}{\pi R^3}$$

Using the measured torque at fracture T_f = 415 Nm, and the cast iron bar diameter $2R$ = 17.7 mm, the tensile stress at fracture:

$$\sigma_f = \frac{2T_f}{\pi R^3} = \frac{2(415)}{\pi (8.85 \times 10^{-3})^3} = 381 \ \text{MPa}$$

Using linear elastic fracture mechanics (refer to your IA Materials notes, and see also Examples Paper 4 Q7), we can use the fracture stress σ_f to estimate the **fracture toughness** of the material.

- At fracture, the stress intensity: $K \approx \sigma_f \sqrt{\pi a} = K_{IC}$

- Assuming the critical flaw is a graphite flake:
    flaw size $a \approx 100$ µm $\Rightarrow K_{IC} \approx 7$ MPa $\sqrt{m}$

- Assuming the critical flat is a casting defect (e.g. air bubble)
    flaw size $a \approx 1$ mm $\Rightarrow K_{IC} \approx 21$ MPa $\sqrt{m}$

The Data Book gives $K_{IC} = 20$ - 40 MPa $\sqrt{m}$ for cast iron, indicating that casting defects with size of the order of mm may initiate failure for these torsion specimens.

Pearlite
matrix

Graphite
flakes

50 µm
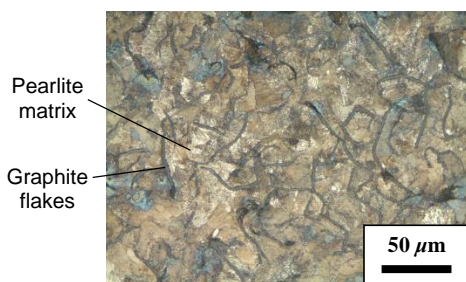
7.5 Summary of the learning outcomes: deformation process modelling

After completing section 7 you will be able to do the following:

1. Analyse the stresses within a 2D plane strain forging process to calculate the forging pressure distribution and forging load.
2. Use the forging analysis to describe how forging loads depend on material, process and component geometry parameters.
3. Describe the forces in a rolling process, and the analogy with forging.

7.6 Quiz W7.2: deformation process modelling

Yield criteria relate the stress components required to yield a metal undergoing a forging process. The Tresca and von Mises yield criteria are both related to the maximum shear stresses in the material.

Which of the following correctly explain this (more than one can be correct)?

1. Frictional shear stresses are important in forging processes.
2. Forging processes involve plane strain deformations.
3. Shear stresses are related to dislocation motion in metals.

Additional worked example: deformation processing

For further explanation, a video will be available with the following worked example:
- Tripos Question 2016 Q6, which helps with Examples Paper 5 Q3.

Now attempt: Examples Paper 5, Q.3