Internal Flows- Heat Convection Coefficient

Thermal analysis of laminar flow in circular pips

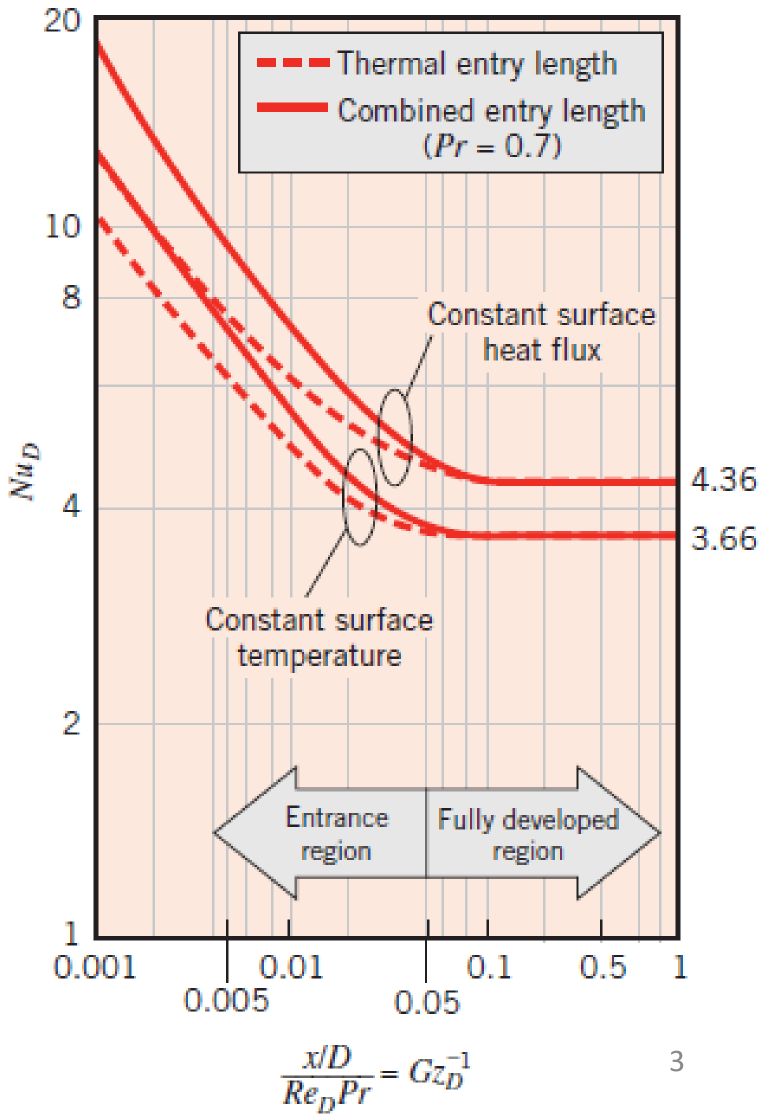
- We have so far considered the fluid mechanics and temperature variations in pipe flows. The final step is to find the appropriate convection coefficients.
- A detailed thermal analysis of the *fully developed, laminar flow in a circular tube* reveals that under these conditions Nusselt number is a constant.
  - For **constant wall heat flux**: $\boxed{Nu_D=\frac{hD}{k_f}=4.36}$
  - For **constant surface temperature**: $\boxed{Nu_D = \frac{hD}{k_f}=3.66}$

  in which D is the pipe diameter and k_f is the thermal conductivity of the fluid.
- Note that the constancy of Nusselt number is limited to the fully developed region and in the entrance region of the tube the Nusselt number does vary axially, why?

Convective heat transfer in the entrance region

- Within the entrance region the boundary layers (thermal and hydrodynamic) are growing axially and therefore the heat transfer coefficient varies with x.
- Why does the heat transfer rate decrease as x increases?
- The heat transfer coefficient in the entrance region is usually expressed in terms of Graetz number: $Gz_D \equiv (D/x)\, Re_D\, Pr$
- It has been shown that for constant surface temperature, the average Nusselt number from the entrance to point L within the entrance region, can be estimated by:

$$\boxed{Nu_D = 3.66 + \frac{0.065\left(\frac{D}{L}\right)RePr}{1+0.04[\left(\frac{D}{L}\right)RePr]^{2/3}}}$$



Convective correlations: turbulent flow in circular tubes

- Turbulent flows feature a great deal of mixing, this leads to significant increase in the rate of heat transfer. As a general rule, compared to laminar flows, turbulent flows are much more capable of convecting heat.
- Analysis of turbulent flows is extremely complicated and often requires massive computations using supercomputers.
- We, therefore, mostly rely on experimental correlations for calculation of convective heat transfer coefficient.
- For a fully developed turbulent flow in a circular tube:

$$\boxed{\begin{gathered} Nu_D = 0.027 Re_D^{\frac{4}{5}} Pr^{\frac{1}{3}} \left(\frac{\mu}{\mu_s}\right)^{0.14} \\[1em] \begin{bmatrix} 0.7 \le Pr \le 16{,}700 \\ Re_D \ge 10000 \\ \frac{L}{D} \ge 10 \end{bmatrix} \end{gathered}}$$

To use this correlation, all the conditions have to be satisfied.

- In this correlation all properties, with the exception of μ_s, are evaluated at $\overline{T_m}$ (average temperature of the fluid inside the tube, i.e. $(T_{m,i}+T_{m,o})/2$).
- When the temperature difference is not large the following correlation can be also used:

$$\boxed{Nu_D = 0.023 Re_D^{4/5} Pr^n \text{ where } n = 0.4 \text{ for heating and } 0.3 \text{ for cooling.}}$$

- Note that this equation has been experimentally validated within the following range of parameters: $0.7 \leq Pr \leq 160$, $Re_D \geq 10{,}000$, $\frac{L}{D} \geq 10$.
- These correlations (above and the one on the last slide) may be applied to both uniform surface temperature and heat flux conditions.
- They give an ***estimation*** of the convective heat transfer coefficient and can include errors up to 25%! Yet, they are still widely used in engineering calculations.
- To reduce the error margin to less than 10% more complex correlations should be used (see the textbook for examples of these).

Example 1

Hot air flows with a mass rate of $\dot{m} = 0.050$ kg/s through an uninsulated sheet metal duct of diameter $D = 0.15$ m, which is in the crawlspace of a house. The hot air enters at 103°C and, after a distance of $L = 5$ m, cools to 85°C. The heat transfer coefficient between the duct outer surface and the ambient air at $T_\infty = 0°C$ is known to be $h_o = 6\ \mathrm{W/m^2 \cdot K}$.

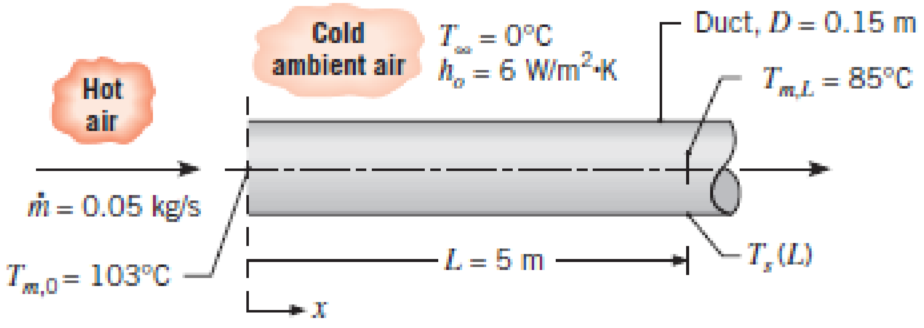
1. Calculate the heat loss (W) from the duct over the length L.
2. Determine the heat flux and the duct surface temperature at $x = L$.

Known: Hot air flowing in a duct.

Find:

1. Heat loss from the duct over the length L, q(W).
2. Heat flux and surface temperature at $x = L$.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Constant properties.
3. Ideal gas behavior.
4. Negligible viscous dissipation and negligible pressure variations.
5. Negligible duct wall thermal resistance.
6. Uniform convection coefficient at outer surface of duct.
7. Negligible radiation.

Properties: Table A.4, air ($\overline{T}_m = 367$ K): $c_p = 1011$ J/kg·K. Table A.4, air ($T_{m,L} = 358$ K): $k = 0.0306$ W/m·K, $\mu = 211.7 \times 10^{-7}$ N·s/m^2, $Pr = 0.698$.

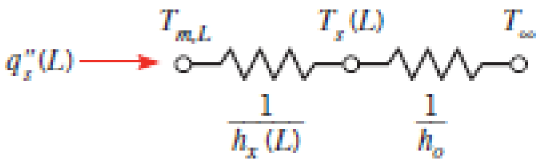
Analysis:

1. From the energy balance for the entire tube,

$$q = \dot{m}c_p(T_{m,L} - T_{m,0})$$

$$q = 0.05 \text{ kg/s} \times 1011 \text{ J/kg} \cdot \text{K}(85 - 103)°\text{C} = -910 \text{ W} \qquad \triangleleft$$

2. An expression for the heat flux at $x = L$ may be inferred from the resistance network



where $h_x(L)$ is the inside convection heat transfer coefficient at $x = L$. Hence

$$q_s''(L) = \frac{T_{m,L} - T_\infty}{1/h_x(L) + 1/h_o}$$

The inside convection coefficient may be obtained from knowledge of the Reynolds number.

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.05 \text{ kg/s}}{\pi \times 0.15 \text{ m} \times 211.7 \times 10^{-7} \text{ N} \cdot \text{s/m}^2} = 20{,}050$$

Hence the flow is turbulent. Moreover, with $(L/D) = (5/0.15) = 33.3$. it is reasonable to assume fully developed conditions at $x = L$.

$$Nu_D = \frac{h_x(L)D}{k} = 0.023\, Re_D^{4/5}\, Pr^{0.3} = 0.023(20{,}050)^{4/5}\,(0.698)^{0.3} = 56.4$$

Note that n=0.3 for this cooling problem.

$$h_x(L) = Nu_D \frac{k}{D} = 56.4 \frac{0.0306 \text{ W/m} \cdot \text{K}}{0.15 \text{ m}} = 11.5 \text{ W/m}^2 \cdot \text{K}$$

Therefore

$$q_s''(L) = \frac{(85 - 0)°\text{C}}{(1/11.5 + 1/6.0)\text{m}^2 \cdot \text{K/W}} = 335 \text{ W/m}^2$$

Referring back to the network, it also follows that

$$q_s''(L) = \frac{T_{m,L} - T_{s,L}}{1/h_x(L)}$$

in which case

$$T_{s,L} = T_{m,L} - \frac{q_s''(L)}{h_x(L)} = 85°\text{C} - \frac{335 \text{ W/m}^2}{11.5 \text{ W/m}^2 \cdot \text{K}} = 55.9°\text{C} \quad \triangleleft$$

Non-circular cross sections

- Many engineering applications involve convection transport in non-circular tubes.
- In such configurations an effective diameter is defined which is often regarded as *hydraulic diameter* and is defined as

$$D_h \equiv \frac{4A_c}{P}$$

where $\boldsymbol{A_c}$ and $\boldsymbol{P}$ are the flow cross-sectional area and the wetted perimeter, respectively.

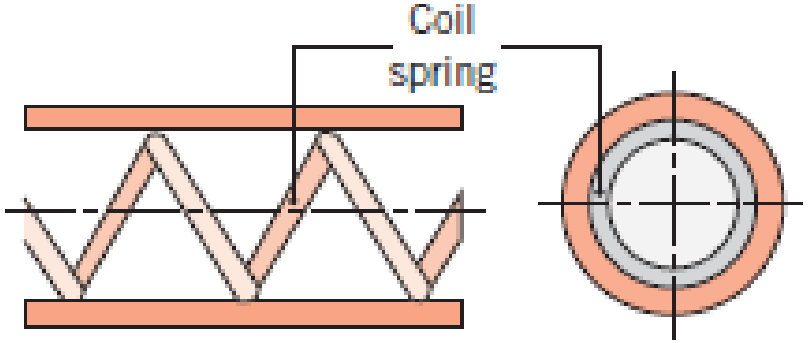
- For turbulent flows in non-circular ducts, D in the previous correlations should be replaced by D_h. No other modification is necessary.
- Note that the output of the calculation is the average Nusselt number over the cross section of the tube.
- The previous laminar flow relations, however, cannot be used in a non-circular tube. The values of Nusselt number should be extracted from the table.

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q''_s)	(Uniform T_s)
	—	4.36	3.66
	1.0	3.61	2.98
	1.43	3.73	3.08
	2.0	4.12	3.39
	3.0	4.79	3.96
	4.0	5.33	4.44
	8.0	6.49	5.60
	∞	8.23	7.54
Heated / Insulated	∞	5.39	4.86
	—	3.11	2.49

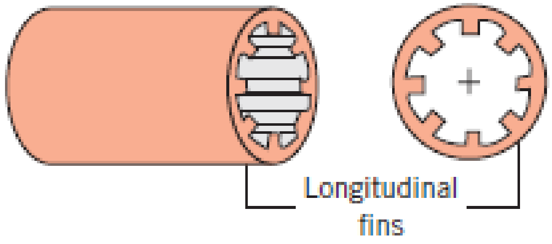
Nu for fully developed laminar flow in tubes of differing cross section

Heat transfer enhancement

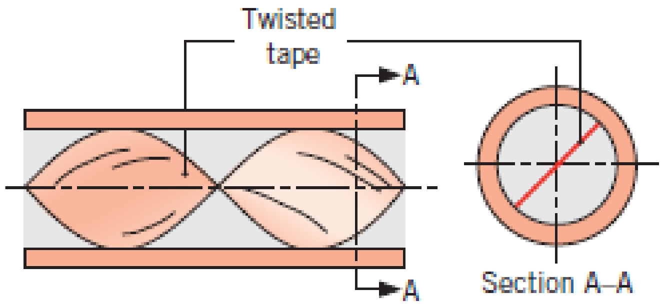
- It follows from the Newton's law of cooling ($Q_s = hA_s(\Delta T)$) that to enhance the rate of heat transfer either of the convective heat transfer coefficient or heat transfer surface area should increase. (Note that ΔT is usually specified by the application and cannot be changed).
- Enhanced heat transfer leads to more efficient and usually smaller equipment and is often economically attractive.
- A few methods of heat transfer enhancement are:
  - Introducing more turbulent flow and therefore increasing the convective heat transfer coefficient, such as increasing the surface roughness to enhance turbulence, by machining the surface or through insertion of coil-spring wire.



- Increasing the heat transfer surface area, which can be done by internal fins



- Introducing a swirl through insertion of a twisted tape. The insert consists of a thin strip that is periodically twisted through 360 degrees. Introduction of a tangential velocity component increases the fluid velocity, particularly near the tube internal wall.



- What problem these techniques may cause?