Lecture 9: Induction Motors I

9.1 Principles of operation

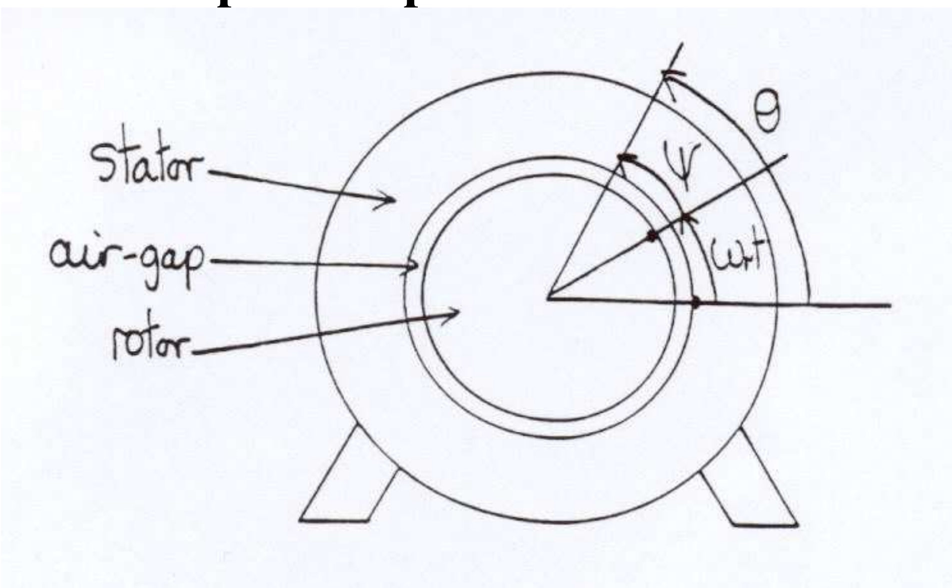

Fig 9.1

Stator wound with a balanced three-phase winding, which is excited by a balanced three-phase voltage supply $\Rightarrow$ a rotating magnetic field is established in the air-gap.

$$B_s(\theta,t) = \hat{B}_s \cos(\omega t - p\theta) \qquad (9.1)$$

Field rotates at synchronous speed $\omega_s = \omega/p$

Rotor also wound with a balanced three-phase winding - terminals are short-circuited.

Let rotor rotate at angular speed ω_r.

The induction motor is the simplest form of a.c. motor. It is used in 95 % of all industrial applications requiring an electric drive. In this lecture we will examine the basic principles of its operation, and develop an equivalent circuit for it. We will also look at the two tests used to find the equivalent circuit parameters.

The induction motor, like the synchronous machine, consists of a stationary part (the stator) and a rotating part (the rotor), which is supported in bearings concentric with the stator.

Also like the synchronous machine, the stator of the induction motor is wound with a balanced three-phase winding, as shown in fig. 9.1. When this winding is supplied from a balanced three-phase a.c. voltage source, balanced three-phase currents flow in it. As we saw in lecture 4, this will cause a rotating magnetic field to be established in the air-gap of the machine, equation 9.1.

The speed of rotation of this field is synchronous speed, fixed by the number of poles of the winding, and the supply frequency.

The rotor is also wound with a balanced three-phase winding, having the same number of poles as the stator winding. As we will see later, an alternative form of construction for the rotor is possible (cage rotor), but for now we will assume a wound rotor. It can be shown that the theory of operation in either case is identical. The rotor winding can either be short-circuited, or, as we will see later, it is also possible to connect it via slip-rings to external impedances in order to control the torque-speed characteristic of the motor.

Viewed from the rotor, the stator-driven field appears the same as when viewed from the stator, except rotating at speed $\omega_s - \omega_r$.

Defining ψ as an angular co-ordinate fixed to the rotor, the stator-driven field is therefore:

$$B_s(\psi, t) = \hat{B}_s \cos((\omega - p\omega_r)t - p\psi) \qquad (9.2)$$

Define slip: $$s = \frac{\omega_s - \omega_r}{\omega_s} \qquad (9.3)$$

Rearranging: $$\omega_r = (1-s)\omega_s = (1-s)\frac{\omega}{p} \qquad (9.4)$$

Substituting for ω_r using 9.4 in 9.2:

$$B_s(\theta, t) = \hat{B}_s \cos(s\omega t - p\psi) \qquad (9.5)$$

Field rotates at $\dfrac{s\omega}{p} = s\omega_s$ with respect to rotor.

$\therefore$ Balanced three-phase emfs of frequency $s\,\omega$ are induced in the rotor windings $\Rightarrow$ balanced three-phase currents of frequency $s\,\omega$ flow.

These set up a rotor-driven magnetic field:

$$B_r = \hat{B}_r \cos(s\omega t - p\psi - \alpha) \qquad (9.6)$$

We now consider that the rotor is rotating at speed ω_r, and imagine how the stator-driven rotating air-gap flux density will appear from the perspective of the rotor. It will appear to have the same shape (or spatial variation), but its speed will appear to be the difference between the rotor speed and its speed viewed from the stator.

This is a similar idea to viewing a train travelling at, say, 100 mph. If you view the train whilst sitting still by the side of the track (stationary reference frame) its speed will appear to be 100 mph. If, however, you view it from a car travelling at 70 mph in the same direction as the train (moving reference frame) its speed will appear to be 30 mph, but the appearance of the train itself remains the same.

The stator-driven field viewed in the rotating reference frame of the rotor is therefore given by equation 9.2. In order to facilitate further development of the theory, we introduce the idea of fractional slip (or just slip), s. This is given by equation 9.3, and is physically the relative speed between the stator-driven field i.e. synchronous speed and the rotor speed expressed as a fraction of the synchronous speed. Rearranging equation 9.3 to give ω_r in terms of s and ω_s, (equation 9.4) and substituting into equation 9.2, we see that the stator field appears to rotate at speed $s\omega_s$ w.r.t. the rotor. As we saw in lecture 3, a rotating magnetic field induces emfs at a frequency equal to the speed of the field w.r.t. the conductors in which the emf is induced. Therefore, slip frequency emfs are induced in the rotor three-phase winding, which cause slip frequency currents to flow. Therefore, the rotor produces its own magnetic field (the rotor-driven field) given by equation 9.6.

Rotor-driven field rotates at speed $\dfrac{s\omega}{p} = s\omega_s$ w.r.t. the rotor.

Speed of rotation in stator reference frame is:

$$\omega_r + s\omega_s = (1-s)\omega_s + s\omega_s = \omega_s \qquad (9.7)$$

Important result: Stator and rotor-driven fields have same number of poles, and rotate at the same speed. **These are the conditions required for a steady torque to be produced.**

What happens if the rotor rotates at synchronous speed ?

Rotor 'sees' a stationary field $\Rightarrow$ no emfs induced $\Rightarrow$ no currents in the rotor $\Rightarrow$ no rotor-driven field $\Rightarrow$ no torque !

At all other rotor speeds, torque is produced.

For this reason the induction motor is also known as the **asynchronous motor.**

By analogy with the stator-driven field, this field must rotate at speed $s\omega_s$ w.r.t. the rotor.

Therefore, the speed of the rotor-driven field viewed from the stator (stationary reference frame) is the sum of the rotor speed, and the speed of the rotor-driven field w.r.t. the rotor, equation 9.7. Again, this is similar to the idea that if you walk at 3 mph up the carriage of a train which is travelling at 100 mph, and in the same direction that the train is moving, you appear to be moving at 103 mph to a stationary observer sitting by the side of the track.

What we have just deduced is fundamental to the operation of the induction motor - the rotor-driven and stator-driven fields both have the same number of poles (they were wound that way) and, when viewed in the same reference frame, rotate at the same speed. These are the two conditions required for the production of steady torque (lecture 4) !

Thus far we have made no assumption about the rotor speed, ω_r, and so it might be thought that these arguments apply for all possible rotor speeds. However, consider the situation when the rotor rotates at synchronous speed. In that case, the relative speed between stator-driven field and rotor winding is zero (slip, $s=0$) and so no emf will be induced in the rotor winding. Consequently there will be no rotor current, and therefore no rotor-driven field and no torque. In other words, the induction motor produces torque at all speeds except synchronous ! Note the contrast with the synchronous machine, which only produces torque when the rotor rotates at synchronous speed. For this reason the induction motor is also referred to as the asynchronous motor.

9.2 Equivalent circuit

When stationary, induction motor is similar to a transformer with the secondary short-circuited.

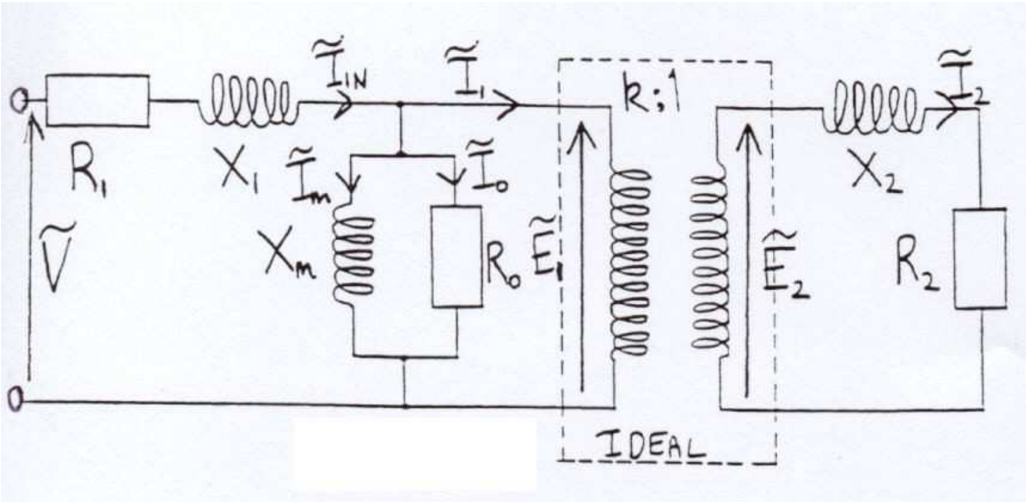

Fig. 9.2

R_1:	Stator winding resistance
X_1:	Stator leakage reactance
R_2:	Rotor winding resistance
X_2:	Rotor leakage reactance
X_m:	Magnetising reactance
R_0:	Iron loss resistance

k is the ***effective*** turns ratio, stator : rotor turns, so $\widetilde{E}_1 / \widetilde{E}_2 = k$ and $k\widetilde{I}_1 = \widetilde{I}_2$.

Induced emf per phase in the stationary rotor is:

$$\widetilde{E}_2 = \widetilde{I}_2\left(R_2 + jX_2\right) \qquad (9.8)$$

Consider a stationary induction motor. When currents flow in the stator, they produce a magnetic field which links the rotor conductors. This causes emf to be induced in the rotor winding, and since this is short-circuited, currents flow. This is identical to the action of a transformer with a short-circuited secondary; the stator is then equivalent to the primary winding, and the rotor is equivalent to a short-circuited secondary winding . Therefore, under stationary conditions, the equivalent circuit for the induction motor is identical to that of the transformer, except with the secondary short-circuited, fig. 9.2. The other two phases give rise to identical equivalent circuits, and so a per-phase analysis is sufficient. The equivalent circuit parameters are analogous to those of transformers: R_1 and R'_2 represent power losses which occur in the stator and rotor windings (copper losses), X_m represents the current which flows in the stator winding to establish the air-gap magnetic field (magnetising reactance), R_0 represents iron losses in the stator and rotor laminations, and X_1 and X_2' represent stator and rotor leakage reactances. The effective turns ratio, k, is similar to the turns ratio of a transformer and represents the ratio of stator:rotor winding turns.

The obvious main difference between transformers and induction motors is the relative movement between rotor and stator, and we now see how this can be accommodated within the equivalent circuit of fig. 9.2. The induced emf in the stationary rotor is found by analysing the rotor circuit of fig. 9.2, giving equation 9.8.

Now suppose that the rotor rotates at a speed so that the slip is s.

Now allow the rotor to rotate with slip s.

$\tilde{E}_2 \rightarrow s\,\tilde{E}_2$ because $\omega \rightarrow s\omega$ and emf $\propto \omega$ where ω is the speed of the field relative to the rotor.

$X_2 \rightarrow sX_2$ because frequency of rotor currents is $s\omega$ and X_2 is calculated using frequency ω.

$$\therefore \qquad s\,\tilde{E}_2 = \tilde{I}_2\left(R_2 + j\,s\,X_2\right) \qquad (9.9)$$

Divide by s:
$$\tilde{E}_2 = \tilde{I}_2\left(\frac{R_2}{s} + jX_2\right) \qquad (9.10)$$

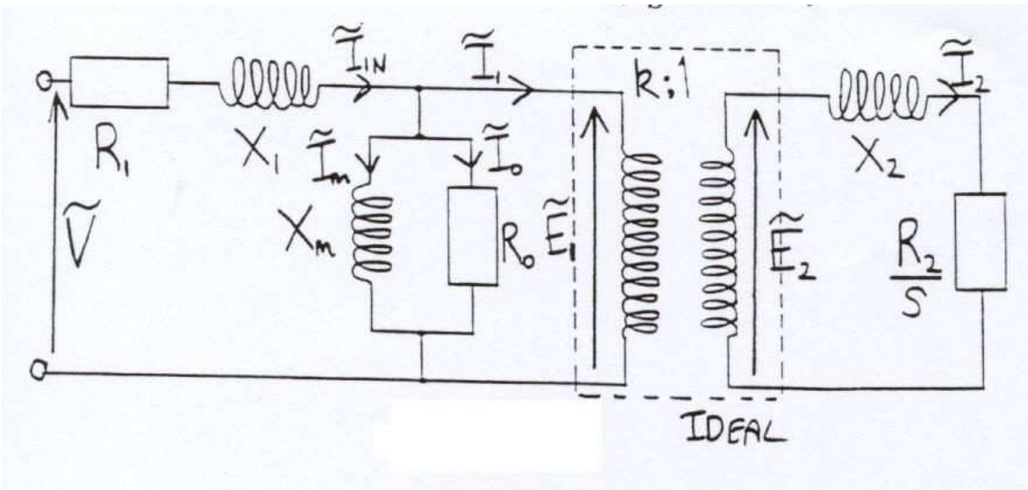

Fig. 9.3

Refer rotor parameters to the stator:

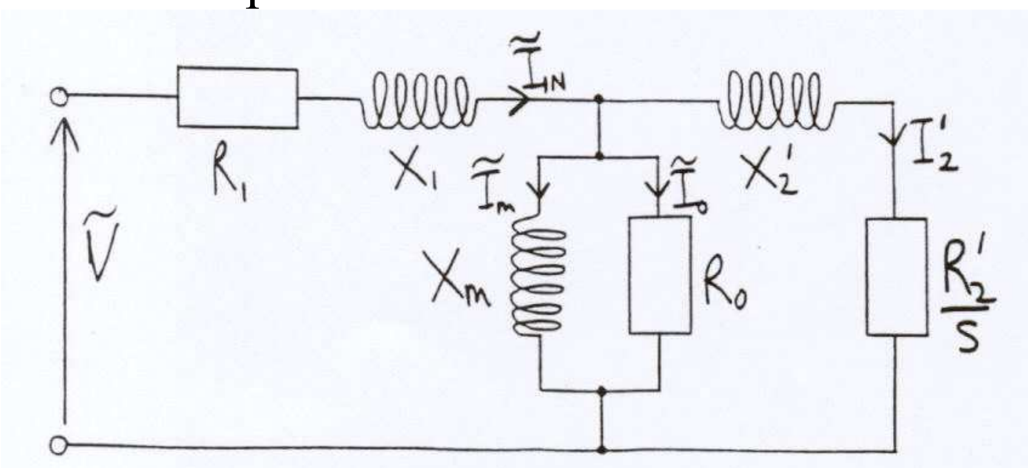

Fig. 9.4

In lecture 3 it was shown that the emf induced in a coil by a rotating magnetic field is proportional to the magnitude of the field, and the speed of the field w.r.t. the coil. If the rotor starts to rotate such that the slip is s, then the speed of the field w.r.t. the rotor winding is $s\omega_s$. Therefore, the magnitude of the emf induced in the rotor is s × the magnitude that would be induced by the same field, if the rotor were stationary i.e. sE_2. Furthermore, X_2 is the rotor leakage reactance calculated at the supply frequency (say, ωL_2). When the rotor rotates, the frequency of the rotor current becomes $s\omega$, and so the leakage reactance becomes $s\omega L_2 = sX_2$. Therefore, E_2 is equation 9.8 is replaced by sE_2, and X_2 by sX_2 to give equation 9.9. This can then be divided through by s to give 9.10, in which E_2 and X_2 are the induced emf in the rotor and the rotor leakage reactance, calculated as if the rotor were stationary. This results in the equivalent circuit of fig. 9.3. In this equivalent circuit:

1. All currents and voltages are at supply frequency.
2. All reactances are calculated at supply frequency.
3. The magnetising flux has to cross the air-gap, resulting in a far smaller magnetising reactance than that of a transformer. Therefore, unlike the transformer, the parallel branch comprising X_m and R_0 cannot be moved to the terminals.

Finally, as for the transformer, the rotor parameters can be referred to the stator by multiplying them by k^2. This results in the final equivalent circuit shown in fig. 9.4, which is also given in the Electrical and Information Science Data book.

9.3 Determination of equivalent circuit parameters.

No-load test - Example 9.1

A no-load test is carried out on a 415 V, 50 Hz, 4-pole, three-phase star-connected induction motor. Find R_0 and X_m:

V_{line}	I_{line}	P_{in}(total)	Speed (rpm)
415 V	2.8 A	705 W	1500

Motor is supplied at rated voltage, but with no mechanical load.

$$\therefore\ \omega_r \to \omega_s \quad \therefore\ s \to 0 \quad \therefore\ R_2'/s \to \infty.$$

Approximate equivalent circuit is shown below.

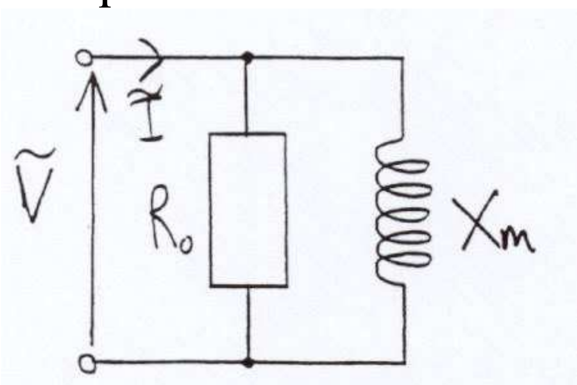

Fig 9.5

Star-connected $\Rightarrow$ $V_{ph}=V_l/\sqrt{3}=415/\sqrt{3}=240V$

$$P = 3\frac{V_{ph}^2}{R_0} \quad \Rightarrow \quad 705 = 3\times\frac{240^2}{R_0} \quad R_0 = 245.1\Omega$$

In order to be able to predict induction motor performance, the equivalent parameters must be found. Two tests are required to achieve this. Given the similarity between the operation of the induction motor and the transformer, and their equivalent circuits, we can borrow the ideas seen last year on finding the transformer equivalent circuit parameters.

Firstly, if the motor is supplied with its rated voltage, but has no mechanical load coupled to it, it will accelerate up to almost synchronous speed (there will always be a very small difference between the motor speed and synchronous speed owing to friction torques). Since the slip s is very close to zero, the impedance of the branch containing $R_2'/s+jX_2'$ will become infinitely large i.e. open-circuit. Thus it may be ignored. Furthermore, the impedance of the parallel branch containing R_0 and X_m is far larger than the series components R_1 and X_1, and so R_1 and X_1 may be ignored also. This results in the equivalent circuit shown in fig. 9.5. This is identical to the transformer equivalent circuit valid for open-circuit conditions. The no-load test is always performed at rated voltage, since the iron losses (and hence the value for R_0) and the level of saturation (and hence the value for X_m) depend on the supply voltage.

In this example, the motor is star-connected, but its quoted rated voltage means line-line, hence the need to divide by √3 to obtain the phase voltage.

$$S = \sqrt{3}V_l I_l = \sqrt{3} \times 415 \times 2.8 = 2013 \text{ VA}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{2013^2 - 705^2} = 1885 \text{ VAR}$$

$$Q = 3\frac{V_{ph}^2}{X_m} \quad \Rightarrow \quad 1885 = 3 \times \frac{240^2}{X_m} \quad X_m = 91.7\,\Omega$$

Locked rotor test - Example 9.2

Locked rotor test is carried out on the same 415 V, 50 Hz, 4-pole, three-phase star-connected induction motor. R_1 is 0.6 Ω, and the stator:rotor leakage reactance is in the ratio 5:8. Find R_2', X_2' and X_1.

V_{line}	I_{line}	P_{in}(total)	Speed (rpm)
200 V	38.6 A	4920 W	0

Rotor held stationary $\Rightarrow$ s = 1 $\quad \therefore \; R_2'/s$ is small $\Rightarrow$ parallel components may be neglected in comparison to series components, giving the equivalent circuit shown below.

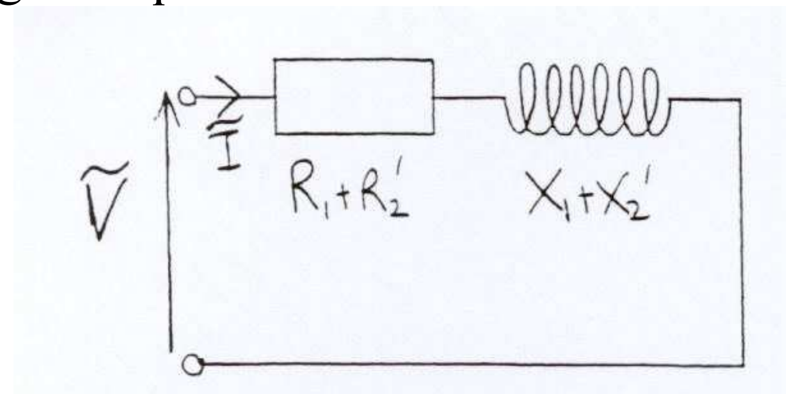

Fig. 9.6

If the supply voltage, V, input current, I and the power per phase P are all measured, then R_0 and X_m may be found as shown opposite. The measured input power is consumed by R_0, giving the equation opposite. The reactive power per phase may be found from the power triangle and equated with the reactive power consumed by X_m, as shown opposite.

In the locked rotor test, it is usual to excite the motor such that the input current is equal to the rated current, and to measure the applied voltage, the input current and the power consumed (per phase).

The locked rotor test , as the name implies, is carried out with the rotor held stationary, so that the speed is zero, and slip is 1. In that case, R_2'/s becomes R_2', which is small. The series parameters ($R_2'+jX_2'$) are much smaller than the magnetising branch with which they are in parallel. Consequently, the magnetising branch may be ignored, giving the equivalent circuit shown in fig. 9.6. This is analogous to the short-circuit test which is carried out on transformers.

$$P = 3I_{ph}^2\left(R_1 + R_2'\right) \qquad 4920 = 3\times 38.6^2\left(R_1 + R_2'\right)$$

$$\therefore \quad R_1 + R_2' = 1.1\,\Omega \quad R_1 = 0.6\,\Omega \text{ so } R_2' = 0.5\,\Omega$$

$$S = \sqrt{3}V_l I_l = \sqrt{3}\times 200\times 38.6 = 13.37 \text{ kVA}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{13.37^2 - 4.92^2} = 12.43 \text{ kVAR}$$

$$Q = 3I_{ph}^2\left(X_1 + X_2'\right) \quad 12430 = 3\times 38.6^2\left(X_1 + X_2'\right)$$

$$\therefore \quad X_1 + X_2' = 2.78\,\Omega \quad X_1 = \frac{5}{13}\times 2.78 = 1.07\ \Omega$$

$$X_2' = \frac{8}{13}\times 2.78 = 1.71\ \Omega$$

All of the power consumed is dissipated in the stator and rotor windings as heat, from which R_1+R_2' may be found as shown opposite. With a separate measurement of the stator resistance, using an ohm-meter, the referred rotor resistance, R_2' may then be determined.

The apparent power per phase, S, may be found since V and I are known, and then the reactive power consumed per phase may be calculated from the power triangle. In turn, this can be equated with the reactive power consumed by X_1+X_2', to find the total leakage reactance referred to the stator.

The separation of X_1 and X_2' is difficult, and requires additional tests to be carried out. It is often necessary to assume that they are equal. However, in this example we are told their ratio, and so they are found as shown opposite.