

CAMBRIDGE UNIVERSITY ENGINEERING DEPARTMENT



PART IB Paper 4: Thermodynamics

2023/2024

Lectures 1-5

Prof R. J. Miller

Contents of lecture course

<i>Lecture</i>	<i>Subject</i>
1	Analysis of steady flow processes (Revision)
2	2 nd law analysis
3	Gas turbines
4	Working fluids I
5	Working fluids II
6	Power generation I
7	Power generation II
8	Refrigeration cycles
9	Properties of mixtures
10	Combustion processes

Recommended books

The two books which are closest to this syllabus are:

'Thermodynamic: An engineering approach' 6th edition or more recent, by Yunus A. Cengel and Michael A. Boles, Mc GrawHill press.

'Fundamentals of Engineering Thermodynamics' 5th edition or more recent, by Michael J. Moran and Howard N. Shapiro, Wiley press.

A good short book which explain the laws of thermodynamics with as little maths as possible is:

'Four Laws that Drive the Universe' by Peter Atkins, Oxford press.

A good book on the physical meaning of the 2nd law of thermodynamics, which does not use Mathematics is:

'The 2nd law: Energy, Chaos and Form' by Peter Atkins, Scientific American Library press.

Copies of these books do not need to be the most recent editions and can be bought cheaply second hand.

Video lecture 1

<http://www.youtube.com/watch?v=2knWCuzcdJo>



PART IB Paper 4: Thermodynamics

2023/2024

Prof Rob Miller

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1.1 Rolls-Royce Trent XWB jet engine

First flight 15th July 2014



Built in Derby – powers the Airbus A350 family

Core thermal efficiency ~42%

1A cycle analysis, assuming all components reversible: **Efficiency ~60%**

2

1.2 By the end of the course you should

1. Understand how to undertake a thermodynamic analysis of a 'real', **irreversible**, heat engine.
2. Be able to quantify the impact of individual 'non-ideal', **irreversible**, components (e.g compressor, combustor or turbine) on **efficiency**.
3. Understand how to make design changes, which **reduce magnitude of the irreversibility**, and thus increase efficiency.

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1.3 How the heat engine made the modern world

'I see two paramount themes in the history of the past century: the growth of human control over inanimate forms of energy; and an ever increasing readiness to tinker with social institutions in the hope of attaining desired goals.'

- W. H. McNeill, The rise of the West(1963)

When McNeill talks of '*control over forms of energy*' he is describing the invention, from 1712, of engines which convert heat into shaft work



Kings College chapel -1515
(30 years – height 29m)



Burj Khalifa – Dubai -2009
(5 years – height 828m)

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1.3 How the heat engine made the modern world

1884 Steam turbine

(Charles Parsons – St Johns)



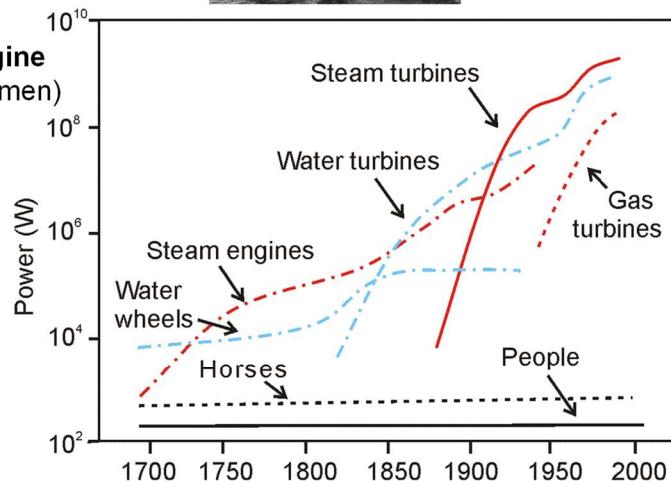
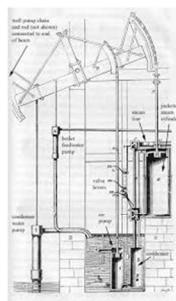
1930 Gas turbine

(Frank Whittle - Peterhouse)



1712 Steam engine

(Thomas Newcomen)

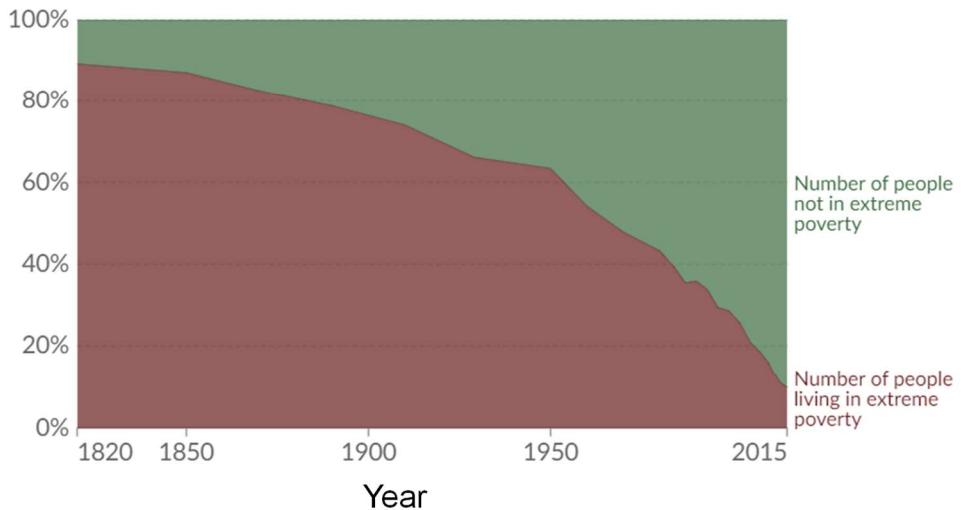


By 1870 British steam engines generated 3 GW (equivalent to 40 million men consuming 3 times the entire British wheat production)

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1.3 How the heat engine made the modern world

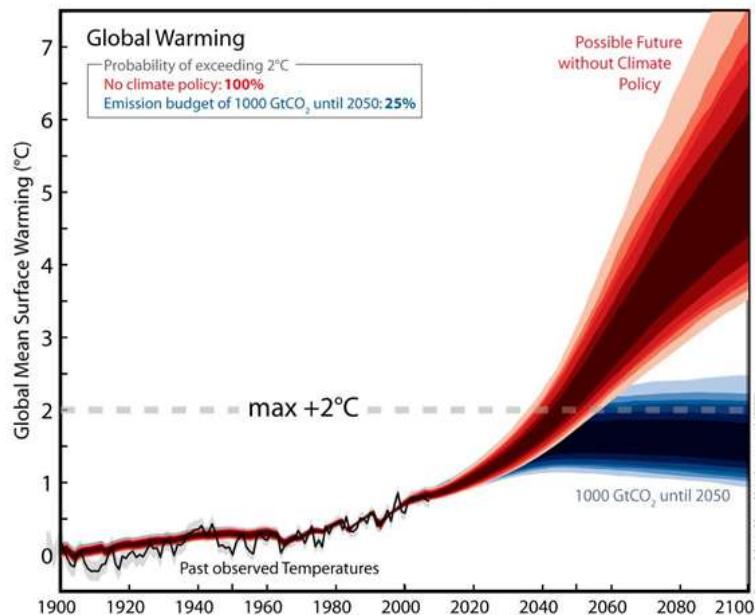
World population living in extreme poverty



"For the first time in history, the living standards of the masses of ordinary people had begun to undergo sustained growth ... Nothing remotely like this had happened before in history" - Nobel Prize winner R.E. Lucas, Jr.

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1.4 Impact of heat engines on the environment



Green house gas emissions and consequent global warming for a business-as-usual scenario (red) and for a mitigation scenario in which emissions are limited to 1000 GtCO₂ between 2000 and 2050 (blue). 7

1.5 Course

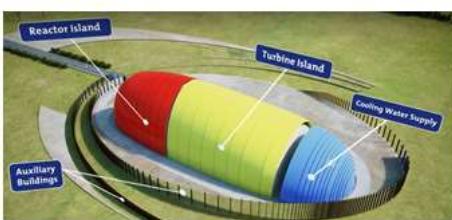
Lecture 3 – Jet engines



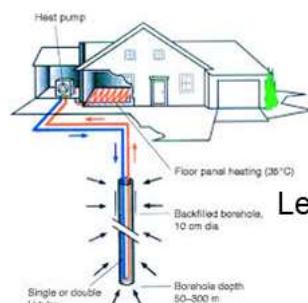
Lecture 4 & 5 – Working fluids



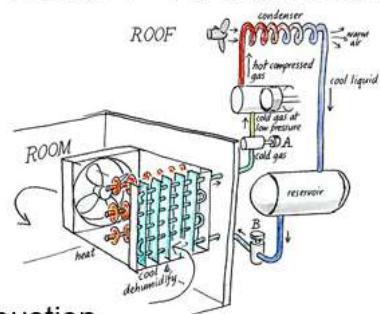
Lecture 6 & 7 – Power generation



Lecture 8 – Heating and cooling



Lecture 9 – Air conditioning

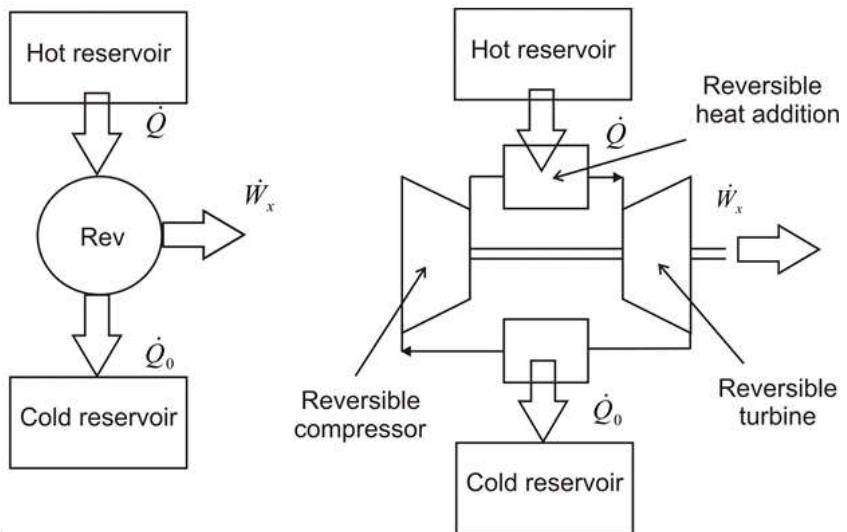


Lecture 10 Combustion



1.6 The ‘ideal’, reversible heat engine

Last year you considered the ‘ideal’, reversible heat engines



$$\eta = \frac{\dot{W}_x}{\dot{Q}} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}$$

‘Ideal’ Gas Turbine (Joule cycle)

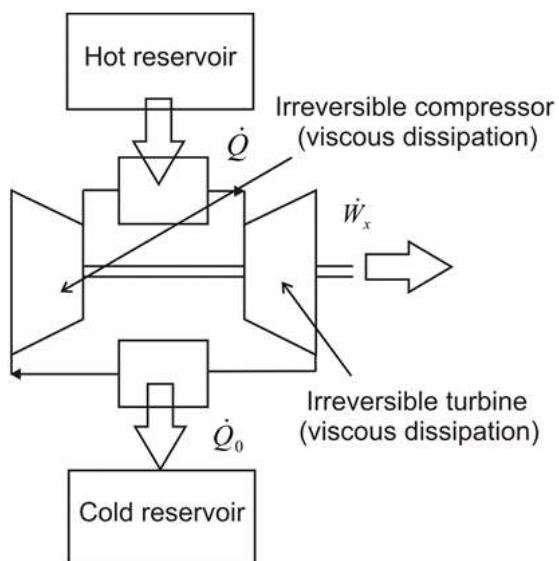
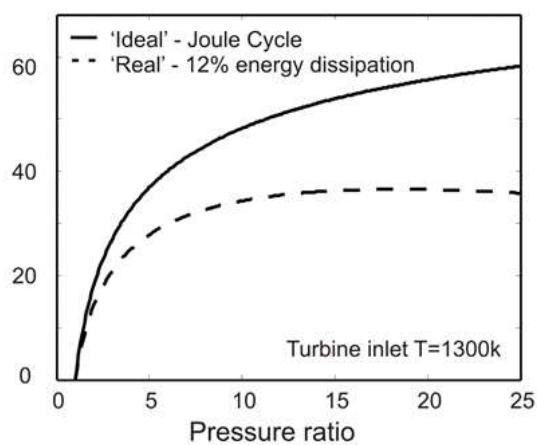
r_p is compressor pressure ratio

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1.7 The ‘real’, irreversible heat engine

This year we considered the ‘real’, irreversible heat engines

Efficiency %

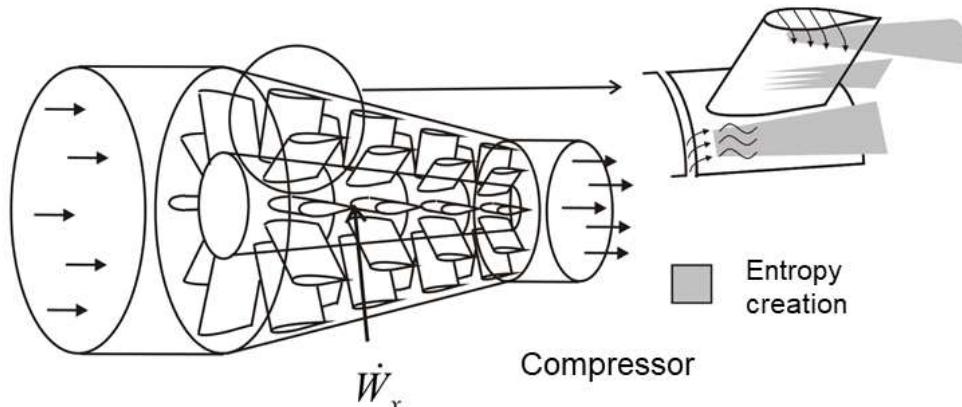


‘Real’ Gas Turbine

Before analysing the effect of **irreversibilities** on the whole heat engine we need to examine their effect on individual steady flow processes.

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1.8 Irreversible steady flow processes



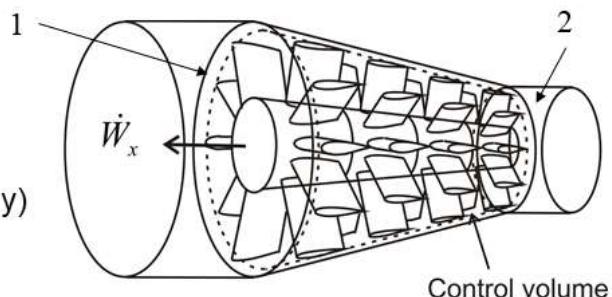
1. Reversible processes can be viewed as the theoretical ideal limit
2. Irreversible processes (e.g. viscous dissipation) create entropy.
3. Entropy convects like "smoke". It is being continually created wherever something deleterious to machine efficiency is taking place.
4. The difference between the 'ideal' power and the 'real' power is set by the increase in entropy from inlet to exit.
5. The aim of designer is to minimise entropy creation.

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1.9 Analysis of steady flow device

All devices in course:

- Steady flow.
- One exit and one inlet (mostly).
- KE is small at inlet and exit (mostly)



$$1^{\text{st}} \text{ law} \quad \dot{m} \left(\left(h_2 + \frac{v_e^2}{2} \right) - \left(h_1 + \frac{v_i^2}{2} \right) \right) = \dot{Q} - \dot{W}_x$$

Enthalpy balance equation - Determines the 'quantity' of energy.

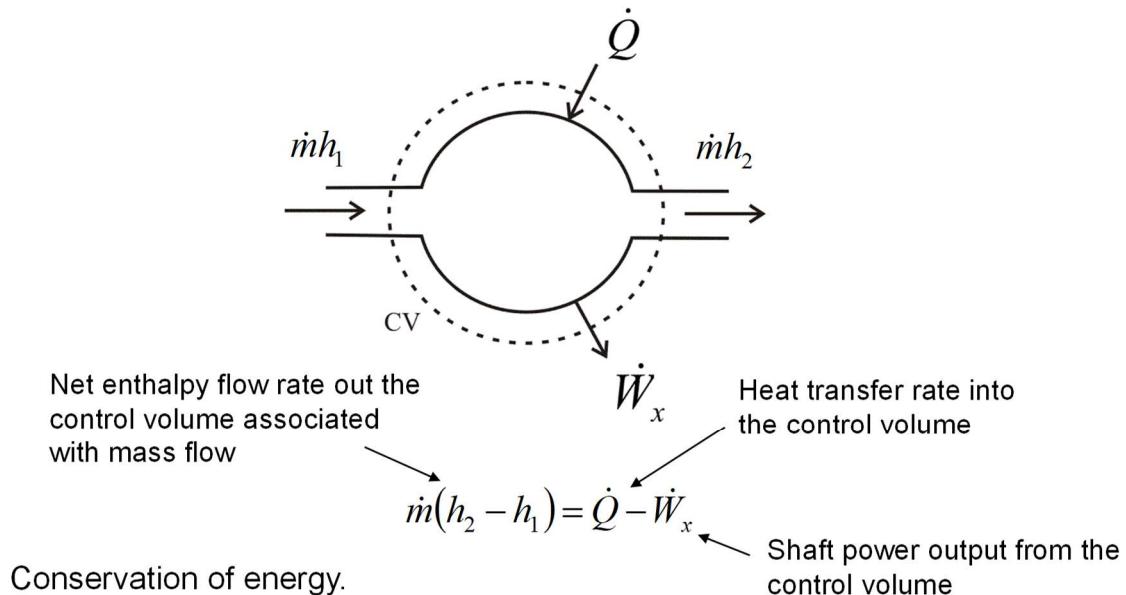
$$2^{\text{nd}} \text{ law} \quad \dot{m} (s_2 - s_1) = \int \frac{d\dot{Q}}{T} + \dot{S}_{irrev}$$

Entropy balance equation - Determines the 'quality' of energy.

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1st law – Energy ‘quantity’

Specific enthalpy, h , is a measure of energy ‘quantity’.



NB For ideal gases $h_2 - h_1 = c_p(T_2 - T_1)$

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The 1st law applied to steady flow processes

The 1st law can be written as

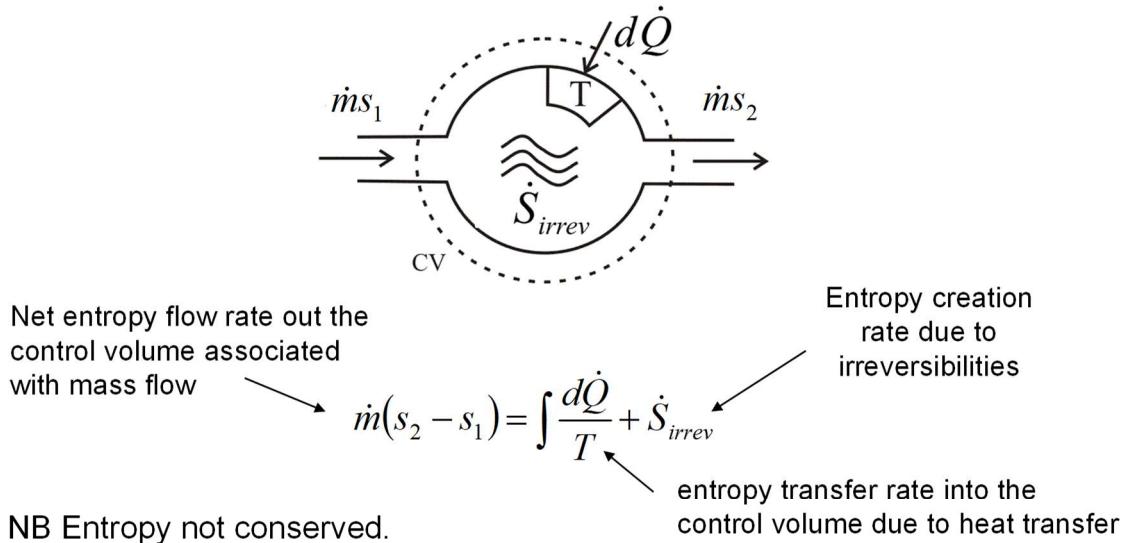
$$\dot{m} \left(\left(h_2 + \frac{v_2^2}{2} \right) - \left(h_1 + \frac{v_1^2}{2} \right) \right) = \dot{Q} - \dot{W}_x$$

The specific enthalpy of the flow is h . At inlet and exit of most of the steady flow device considered in this course the area of is large and the velocity of the flow is small. It is therefore a reasonable to assume, unless otherwise stated, that the velocity is zero.

The 1st law can therefore also be written as

$$\dot{m}(h_2 - h_1) = \dot{Q} - \dot{W}_x$$

2nd law – Energy ‘quality’



This course: Calculates entropy in terms of measurable **macroscopic** quantities.

However: ‘Qualitative’ understanding of the **microscopic** meaning of entropy may help interpretation (next 2 slides)

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Physical interpretation of the 2nd law of thermodynamics

The second law is based on human experience of spontaneous processes. Examples of such processes include a dropped ball coming to rest, a hot pan cooling or high pressure air leaking from a chamber into an evacuated space. What’s happening in each of these processes? Energy of some kind is changing from being ‘localised’, in the ball, pan or high pressure chamber, to becoming more ‘spread out’.

A commonly used analogy for the 2nd law is:

‘Energy spontaneously disperses from being localised to becoming spread out’.

Let’s look at the three examples stated earlier, in more detail:

1. The ball starts with potential energy. Dropping the ball results in PE changing into KE. Drag on the ball transfers the KE of the ball into KE of the air, which in turn is converted, by viscous dissipation, into a rise in internal energy of the air (a small increase in air temperature). When the ball hits the ground, and comes to rest, its remaining KE is converted into internal energy in the ground and ball. Heat transfer then occurs between the hot air, ball and ground and the surroundings. The first law states that energy is conserved and that the PE of the ball at the start is the same as the final rise in internal energy of the surroundings. The second law states that energy is dispersed, from being localised in the ball at the start, to being spread out in the surrounding air at the end.
2. A hot pan contains iron atoms which have a high vibrational kinetic energy. This vibrational kinetic energy is localised in the pan. This energy spontaneously disperses (as long as the pan

is not insulated). Whenever the less rapidly moving molecules in the cooler air of the room hit the hot pan, the fast-vibrating iron atoms transfer some of their energy to the air molecules. The second law states that energy is dispersed, from being localised in the pan at the start, to being spread out in the surrounding air at the end.

3. A high pressure gas flows out of a chamber, through a small hole, into a second evacuated chamber. This involves the spontaneous dispersal of energy from being localised in the high chamber into a combination of the two chambers. The first law states that energy is conserved and that the internal energy is the same both before and after the expansion (no work or heat is transferred). The second law states that energy is dispersed, from being localised in the high pressure chamber at the start, to being spread out in both chambers at end.

Let us now apply the 2nd law, in closed system form, to the three examples above.

$$S_2 - S_1 = \int \left(\frac{dQ}{T} \right)_{\text{boundary}} + \Delta S_{\text{irrev}}$$

This equation tells us that the entropy of a closed system (left hand side) is changed either by heat transfer on the boundary of the closed system (1st term right hand side) or by entropy creation due to an irreversible process within the closed system (2nd term right hand side).

Now consider the three examples previously discussed (ball, pan & high pressure chamber). If a single closed system is placed around the object and its surroundings, such that no heat is transferred to or from the boundary of the closed system, then the 2nd law becomes

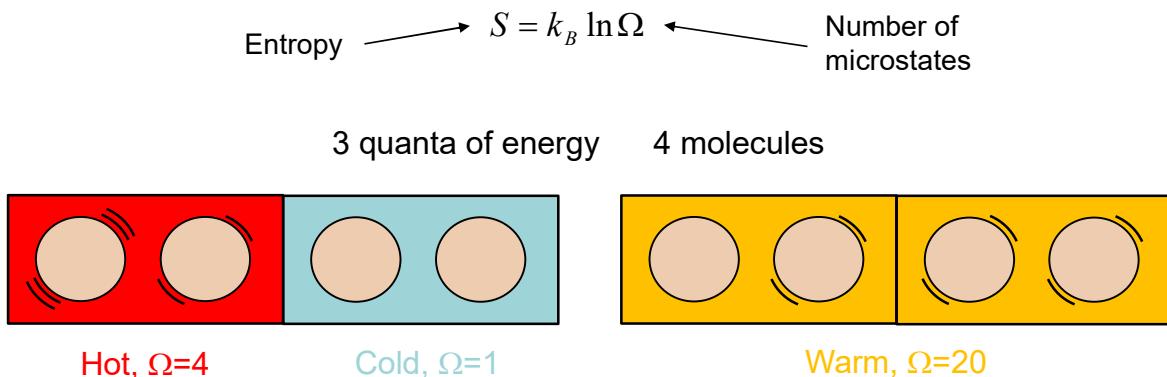
$$S_2 - S_1 = \Delta S_{\text{irrev}}$$

In all three examples the energy starts as localised in a small region of the control volume (within the ball, pan high pressure chamber) and ends up spread across the whole closed system as a small rise in temperature. Energy dispersion is irreversible and so $\Delta S_{\text{irrev}} > 0$. In other words, the energy spontaneously disperses from being localised within one part of the closed system to becoming spread out over the entire closed system and entropy creation is a measure of that irreversibility.

Another way of viewing this process is to consider the work that can be extracted from local ‘differences’ in flow properties. Differences in velocity can be removed reversibly by placing a miniature paddle wheel between the regions of high and low velocity, differences in temperature can be removed reversibly by placing a miniature Carnot cycle between the regions of high and low temperature and differences in pressure can be removed reversibly by placing a piston between regions of high and low pressure and extracting work as it moves. In all of these cases the ‘differences’ can be removed reversibly resulting in the maximum possible work extraction. If these ‘differences’ are instead allowed to disappear spontaneously, without work extraction, by energy dispersal, then the process is irreversible, and entropy is created.

'Qualitative' physical understanding of entropy

Entropy is a measure of the number of possible ways energy can be distributed in a system of molecules



Good movie explaining this in more detail

<https://ocw.mit.edu/courses/res-tll-004-stem-concept-videos-fall-2013/resources/entropy/>

But what is entropy? A molecular perspective.

Entropy is a measure of the number of ways energy can be distributed in a system of molecules.

To understand this definition, we need to be familiar with two points:

- 1) The idea that energy is quantised.
- 2) The idea that a microstate is a description of how the energy is distributed, at a specific instant in time, over every molecule in a system.

A high entropy therefore corresponds to there being a large number of possible microstates (possible ways in which the energy can be distributed in the system of molecules) and a low entropy corresponds to there being a small number of possible microstates.

Using this molecular definition of entropy, given above, we can now interpret the 2nd law. Writing the 2nd law for a control volume we have.

$$\dot{m}(s_2 - s_1) = \int \left(\frac{d\dot{Q}}{T} \right)_{\text{boundary}} + \dot{S}_{\text{irrev}}$$

The first term on the RHS represents the change in entropy due to heat addition at the boundary. If heat is added the number of possible ways of distributing the energy goes up, due simply to the fact that there is more energy, with the result that the entropy rises. The magnitude of this rise depends on the temperature at which the heat is added. If the heat is added at a high temperature then the increase in the number of possible ways of distributing the energy is smaller, and the entropy rise is small. If the heat is added at a low temperature, then the increase in the number of possible ways of distributing the energy is larger, and the entropy rise is large.

The link between the molecular world and macroscopic property entropy is given by Boltzmann's equation

$$S = k_B \ln \Omega$$

where $k_B = 1.381 \times 10^{-23}$ and Ω is the number of microstates. This equation directly links entropy to the number of ways energy can be distributed in a system of molecules. A video demonstrating this link, which I'd advise you to watch, can be found at:

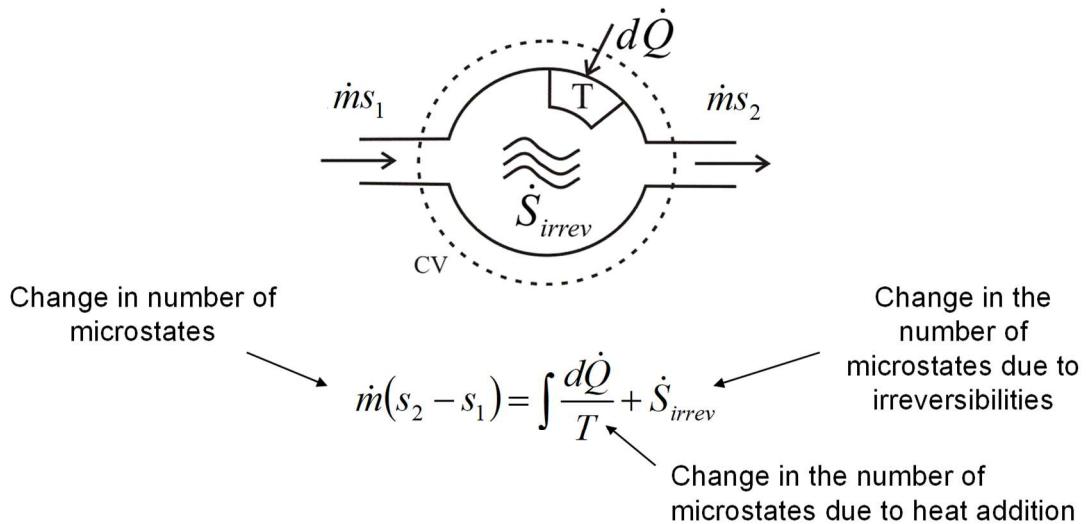
<https://ocw.mit.edu/courses/res-tll-004-stem-concept-videos-fall-2013/resources/entropy/>



A more detailed explanation of the link between the microscopic, molecular world, and macroscopic thermodynamic properties is beyond the scope of this course. For those of you interested, the subject is covered in depth in the 4th year Module 4A9 – ‘Molecular Thermodynamics’.

2nd law – Energy ‘quality’

Entropy is a measure of the number of ways energy can be distributed in a system of molecules



Adding \dot{W}_x does not change number of microstates

Adding \dot{Q} T_{low} - Large increase in number of microstates

T_{high} - Small increase in number of microstates

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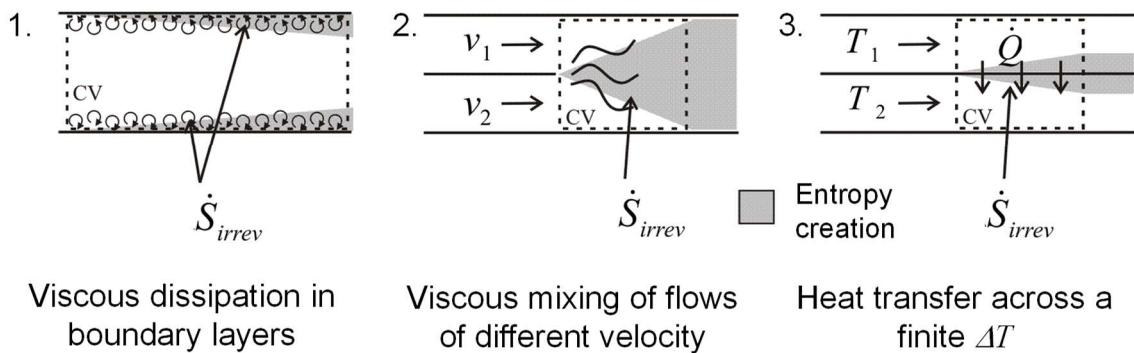
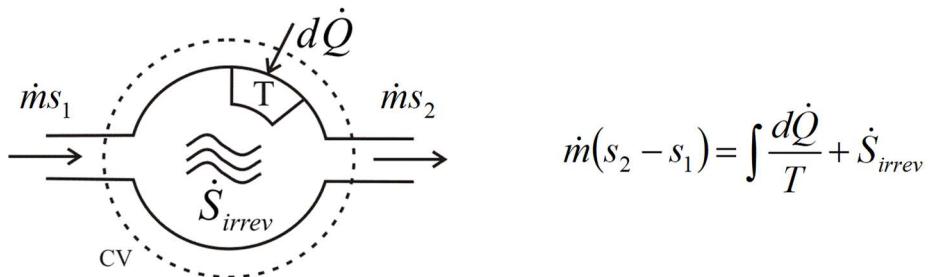
Common analogies for entropy

It is worth considering three common, ill-defined, but often useful, analogies for entropy, and their relationship to the concept of the number of microstates:

- 1) Entropy as a 'measure of disorder'. 'Disorder' is meant to refer to the number of microstates. High 'disorder' is therefore meant to refer to a large number of possible microstates, and thus a high entropy.
- 2) Entropy as 'dispersal of energy'. This is the analogy used above. As energy disperses the number of possible microstates (ways in which the energy can be distributed) rises and so energy dispersal is linked to entropy rise. This analogy works in many common cases but it should be remembered that it is only analogous to the concept of the number of microstates.
- 3) Entropy as 'energy quality'. This analogy derives from the fact that for a fluid with a fixed energy its entropy determines the maximum work which can be extracted in the presence of a fixed environment. The lower the entropy, the higher the potential to do work and therefore the higher the 'energy quality'. This can be seen from the example of a pan cooling. Initially the pan is hot, and the energy is localised in the pan. This is a low entropy condition and work could be extracted by operating a Carnot cycle between the pan and the environment. Once the pan has been cooled, to the temperature of the environment, the energy has dispersed. This is a high entropy condition and because the pan and environment are at the same temperature no work can be extracted. In this case the 'energy quality' is said to have reduced.

2.0 Entropy creation locally within the flow

In this course 3 flow processes which create entropy are considered.



NB generally differences ‘disappearing’ without doing work create entropy 18

Entropy creation locally within the flow

Up to this point we have discussed the change in entropy which occurs between the start and end of a process. In this section we will look at the irreversible flow phenomena responsible for entropy creation locally within the flow. In this course three sources of entropy creation will be considered, two viscous and one thermal:

- 1) Viscous dissipation in boundary layers.
- 2) Viscous mixing in the freestream of flows of different velocities.
- 3) Heat transfer across a finite temperature difference.

In each case ‘differences’, either in velocity or temperature, are allowed to disappear spontaneously and therefore are associated with entropy creation.

Finally, because great use will be made through this lecture of entropy creation it is worth introducing an extra analogy for entropy, proposed by Prof John Denton of Trinity Hall in 1992, and now used widely throughout industry and academia across the world.

‘Entropy may be considered to be like “smoke” that is created within the flow whenever something deleterious to a machine efficiency is taking place. For example, “smoke” is continually being created in the blade boundary layers and in the mixing of leakage flows with the mainstream. Once created the “smoke” can never be destroyed and it is convected downstream through the machine and diffuses into the surrounding flow. The concentration of “smoke” at the exit from the machine includes a contribution from every source within the machine and the loss of machine performance is proportional to the average concentration of “smoke” at its exit.’

The relationship between entropy creation due to irreversibility in a machine and its loss of performance will be presented in lecture 2.

The 'Tds' equations

The Tds relations were derived last year by considering a reversible process between two states separated by a delta change in thermodynamic properties. Even though they are derived for a reversible process, because entropy is a property of state, they can also be used for irreversible processes. The two Tds equations are

$$Tds = du + pdv$$

$$Tds = dh - vdp$$

Examples of delta processes

The second of the two 'Tds' equations is very useful for determining how properties change across delta steady flow processes. A number of examples follow:

1. Delta shaft work is added isentropically (adiabatically and reversibly). 1st law gives $dh = dw_x$

$$\therefore 0 = dw_x - vdp$$

Pressure must rise as isentropic work is done on the fluid.

2. Delta irreversible viscous process with no work and heat flux. 1st law gives $dh = 0$

$$\therefore Tds_{irrev} = -vdp$$

Pressure must fall as irreversible entropy rises.

3. Delta heat flow is added reversibly. 1st law gives $dh = dq$, 2nd law gives $dq = Tds$

$$\therefore dq = dq - vdp$$

Pressure is constant across a reversible heat addition.

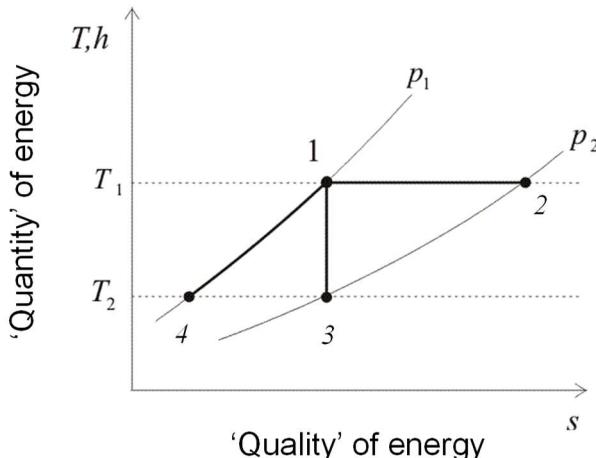
Example 1: Analysis of steady flow devices

Review of three steady flow devices commonly used in course. Aim is to show you how to approach the analysis of steady flow processes

1a - Reversible heat extraction

1b - Isentropic turbine

1c - Throttle

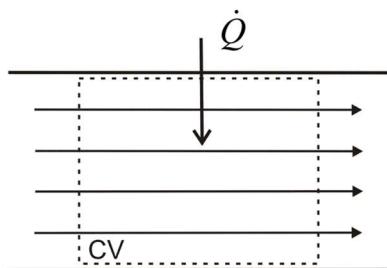


Use the 1st ('quantity' of energy) and 2nd ('quality' of energy) laws to analyse each.

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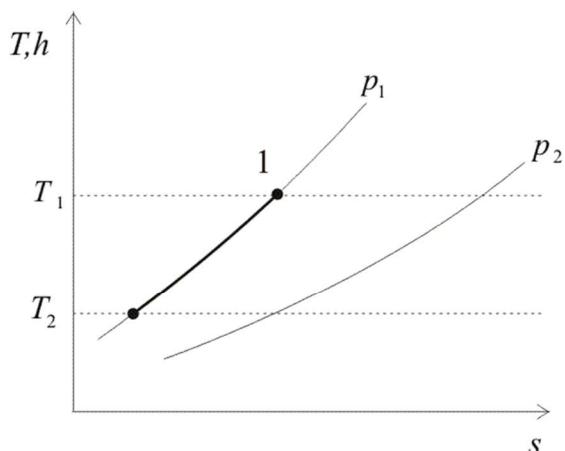
Example 1a: Reversible heat extraction.

Air flows steadily through a pipe. The flow is reversible. A heat flow rate is removed. $T_1 = 600\text{K}$ $T_2 = 300\text{K}$. Assume velocities are small and pressure is constant. Calculate the heat flow rate per kg flow, q , and the specific entropy change $s_2 - s_1$.

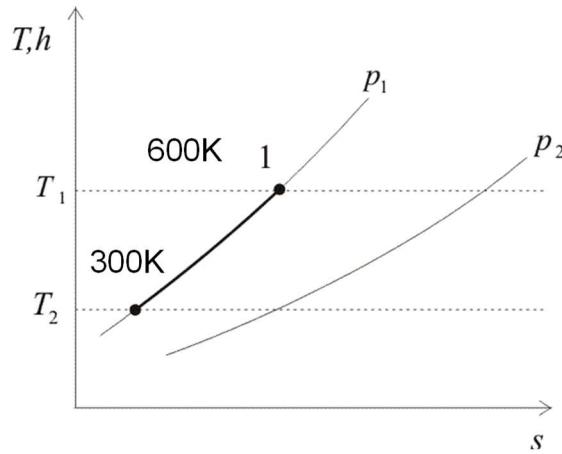


$$1^{\text{st}} \text{ law} \quad \dot{m}(h_2 - h_1) = \dot{Q} - \dot{W}_x$$

$$2^{\text{nd}} \text{ law} \quad \dot{m}(s_2 - s_1) = \int \frac{d\dot{Q}}{T} + \dot{S}_{\text{irrev}}$$



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Fill in

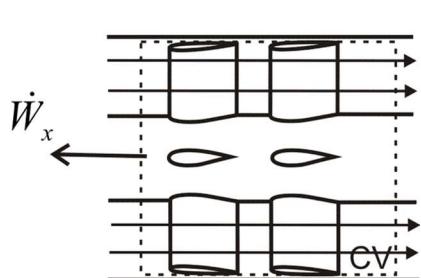
$$1^{\text{st}} \text{ law} \quad \dot{m}(h_2 - h_1) = \dot{Q} - \dot{W}_x$$

$$2^{\text{nd}} \text{ law} \quad \dot{m}(s_2 - s_1) = \int \frac{d\dot{Q}}{T} + \dot{S}_{irrev}$$

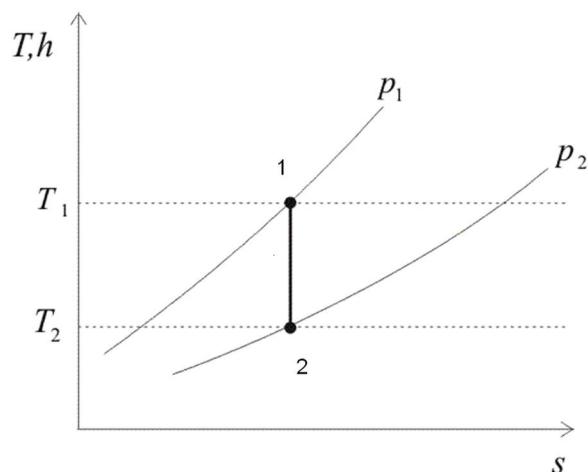
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Example 1b: Isentropic turbine.

Air flows steadily through a reversible adiabatic turbine. $T_1 = 600\text{k}$ $T_2 = 300\text{k}$, calculate the power output per kg flow, w_x , and the turbine pressure ratio.

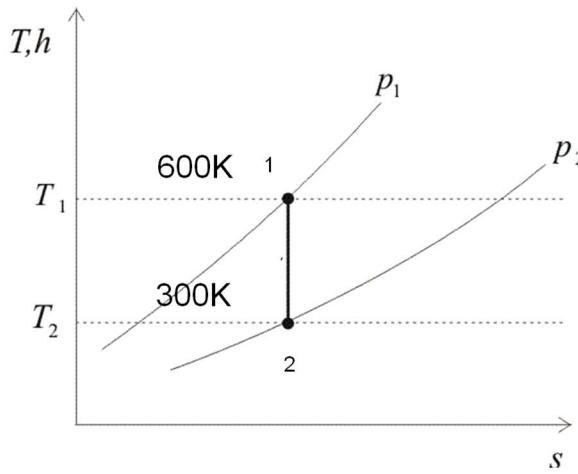


$$1^{\text{st}} \text{ law} \quad \dot{m}(h_2 - h_1) = \dot{Q} - \dot{W}_x$$



$$2^{\text{nd}} \text{ law} \quad \dot{m}(s_2 - s_1) = \int \frac{d\dot{Q}}{T} + \dot{S}_{irrev}$$

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$$1^{\text{st}} \text{ law} \quad \dot{m}(h_2 - h_1) = \dot{Q} - \dot{W}_x \quad \dot{Q} = 0$$

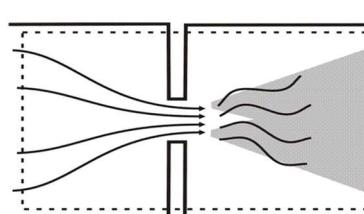
$$\dot{W}_x = \frac{\dot{W}_x}{\dot{m}} = (h_2 - h_1) = c_p(T_2 - T_1) = 1.005 \times 300 = 301.5 \text{ kJ/kg}$$

Isentropic expansion

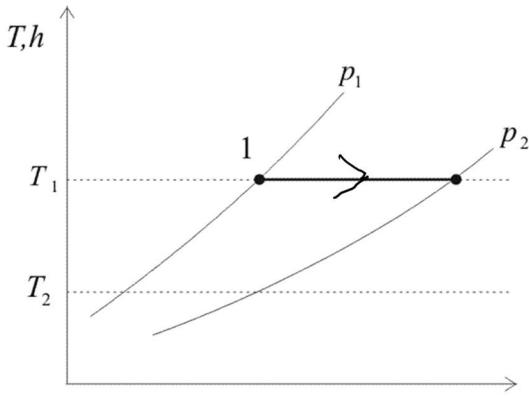
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \quad 0.5 = \left(\frac{p_2}{p_1} \right)^{\frac{0.4}{1.4}} \quad \frac{p_1}{p_2} = 11.3 \quad 23$$

Example 1c: Throttle.

Air flows steadily through a throttle. The rate of heat transfer and rate of shaft work are zero. $T_1 = 600\text{K}$, $P_1/P_2 = 11.3$. Calculate the specific entropy rise.



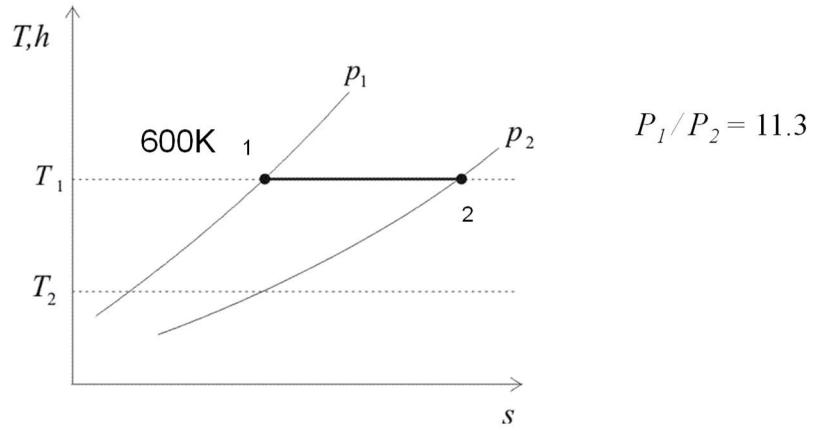
Irreversible entropy



$$1^{\text{st}} \text{ law} \quad \dot{m}(h_2 - h_1) = \dot{Q} - \dot{W}_x$$

$$2^{\text{nd}} \text{ law} \quad \dot{m}(s_2 - s_1) = \int \frac{d\dot{Q}}{T} + \dot{S}_{irrev}$$

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$$1^{\text{st}} \text{ law} \quad \dot{m}(h_2 - h_1) = \dot{Q} - \dot{W}_x = 0$$

The control volume is adiabatic with zero work

$$2^{\text{nd}} \text{ law} \quad \dot{m}(s_2 - s_1) = \int \frac{d\dot{Q}}{T} + \dot{S}_{irrev} = \dot{S}_{irrev}$$

$$(s_2 - s_1) = \frac{\dot{S}_{irrev}}{\dot{m}} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \overset{0}{\leftarrow} = -0.287 \times \ln \frac{1}{11.3} = 0.696 \text{ kJ/kgK}$$

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Entropy change of a perfect gas

For an ideal gas the Tds equations can be written as

$$ds = c_v \frac{dT}{T} + \frac{p}{T} dv \quad \text{and} \quad ds = c_p \frac{dT}{T} - \frac{v}{T} dp$$

For a perfect gas the equations can be integrated to give

$\Delta s = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$	$\text{and} \quad \Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$
--	---

Because entropy is a property of state these equations can be used to find its change between any two points in a flow. Both are given in the CUED data book.

Lecture 2: 2nd law analysis

2. 2nd law analysis: Max available power

2.1 The different value of work and heat

The 1st law allows you to account energy but \dot{W}_x and \dot{Q} appear to have the same value.

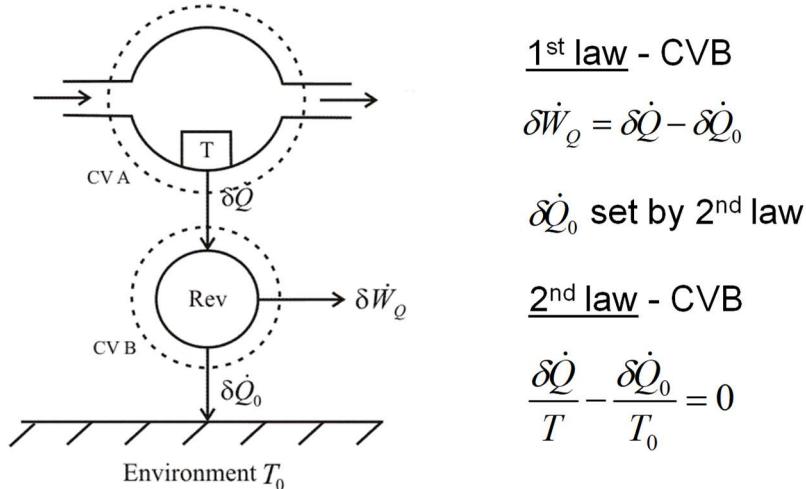
$$\dot{m}(h_2 - h_1) = \dot{Q} - \dot{W}_x$$

The 2nd law tells you that work has more value than heat.

$$\dot{m}(s_2 - s_1) = \int \frac{d\dot{Q}}{T} + \dot{S}_{irrev}$$

But how much work is available from a unit of heat?

$d\dot{Q}$ is removed from CV A. What is maximum available power $d\dot{W}_Q$?



The 2nd law sets the magnitude of the heat flow rate of $\delta\dot{Q}_0$ that must be transferred to the environment.

$$d\dot{W}_Q = \left(1 - \frac{T_0}{T}\right) d\dot{Q}$$

2

$$d\dot{W}_x = \left(1 - \frac{T_0}{T}\right) d\dot{Q}$$

$T \rightarrow \infty$ \dot{Q} is as good as \dot{W}_x

$T \rightarrow T_0$ \dot{Q} is useless.

So the relative value of heat compared to work is dependant on the temperature from which it comes, T , and the temperature of the environment T_0 .

NB The magnitude of the heat flow rate to the environment $\delta\dot{Q}_0$ is set by the requirement that heat transfer to the environment is reversible.

3

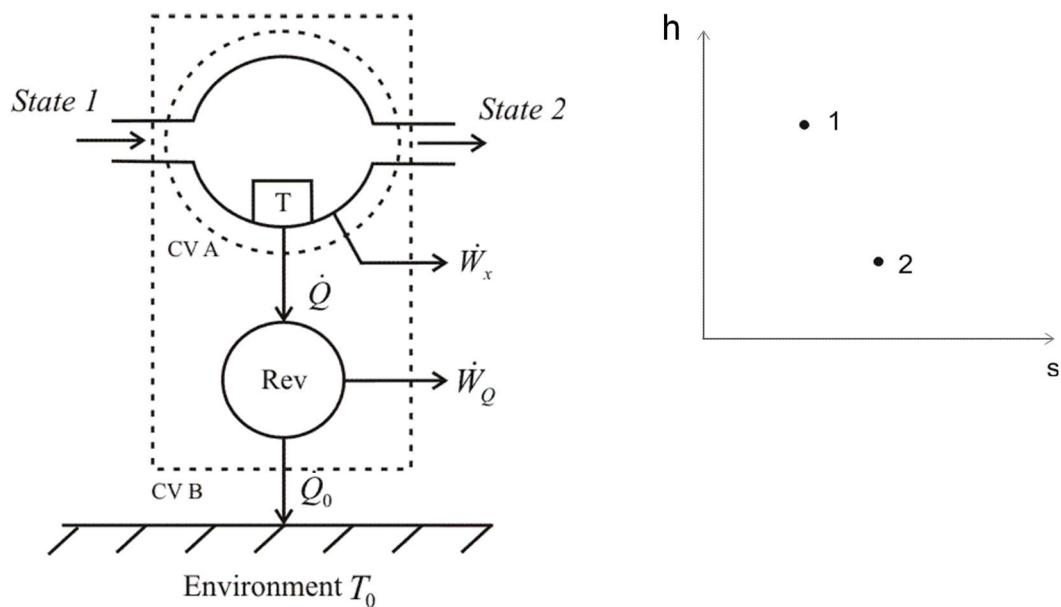
$$d\dot{W}_x = \left(1 - \frac{T_0}{T}\right) d\dot{Q}$$

Question: A guy in the pub offers you three energy sources which should you buy? The room is at 300K.

1. 100 kW hours of electricity.
2. 300 kW hours of heat from a solar thermal plant at 600K.
3. 200 kW hours of heat from a gas turbine combustor at 1500K.

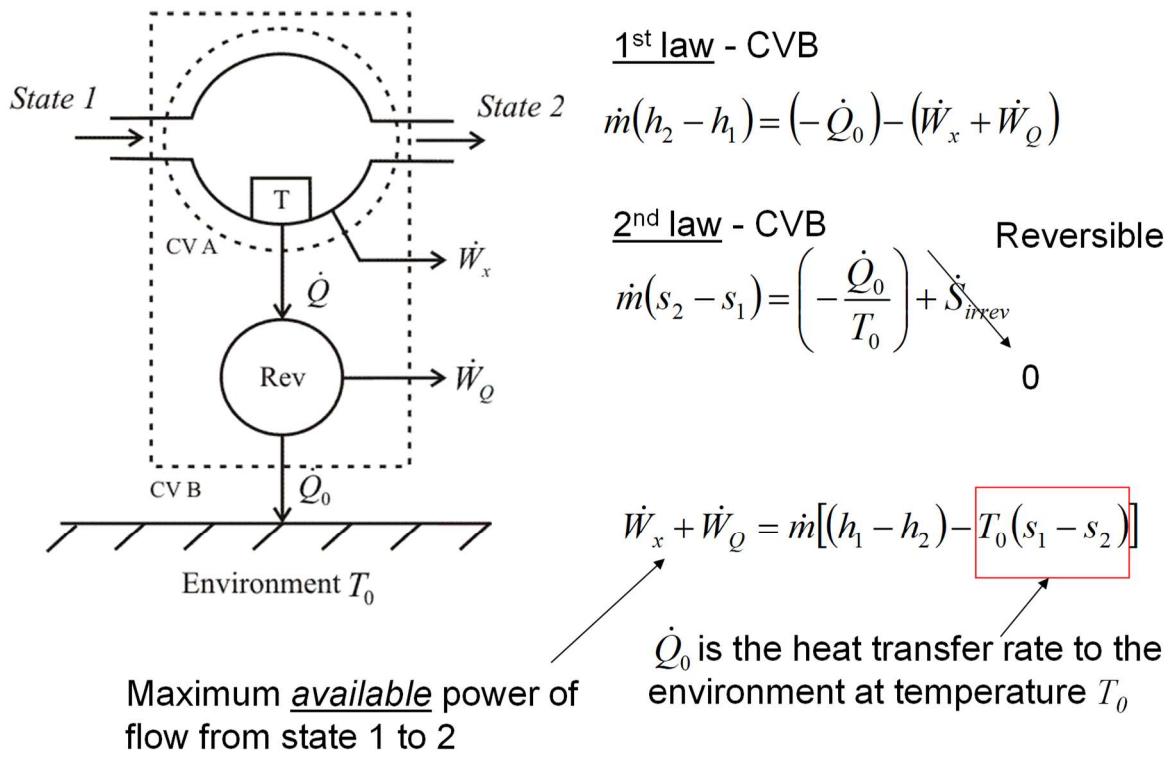
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2.2 Maximum *available* power in a steady flow device



The **maximum available power** can be achieved when the process is **reversible**.

5



6

Rewriting gives

$$\dot{W}_{max12} = \dot{m}(b_1 - b_2) = \dot{m}[(h_1 - T_0s_1) - (h_2 - T_0s_2)]$$

Where b is defined as the specific steady flow availability function

So the maximum available power which can be extracted between states 1 & 2, per kg mass flow rate is the decrease in b.

$$b = h - T_0s$$

NB Although h and s are thermodynamic properties, T_0 is the reservoir temperature and so b is not a thermodynamic property.

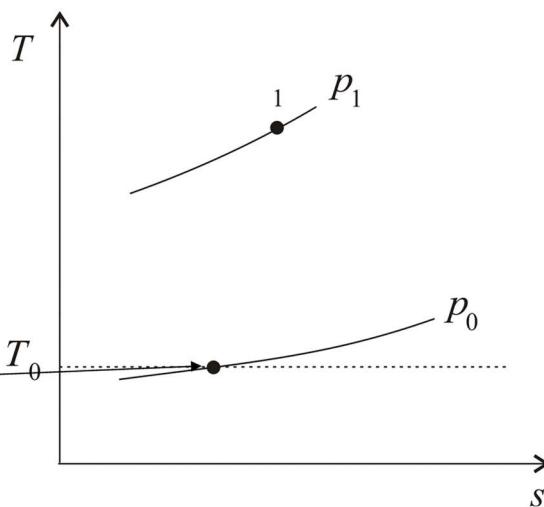
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2.3 The dead state

When a fluid is in equilibrium with the environment p_0, T_0 it is said to be at the dead state.

At the dead state the fluid has no available power.

Dead state in equilibrium with environment



The max available power that can be extracted from a fluid at state 1 is

$$\frac{\dot{W}_{\max}}{\dot{m}} = b_1 - b_0 = (h_1 - T_0 s_1) - (h_0 - T_0 s_0) = e_1$$

The specific steady-flow exergy e is the difference between b and b_0 8

Exergy and the dead state

The datum levels of h and s are arbitrary and therefore the absolute level of b is meaningless. Only changes in b make sense. When a fluid is at equilibrium with the environment, it has zero potential for doing work. It would therefore be physically meaningful if b were zero at this condition. This will not be so unless h and s also happen to be zero. Rather than change the h and s datum levels in all the different thermodynamic tables, it is common to use a new quantity called *exergy*.

When the fluid is at equilibrium with the environment it is said to be in its *dead state* (p_0, T_0). The dead state is usually taken as $p_0 = 1$ bar and $T_0 = 25$ °C. The *specific steady-flow exergy* e is then defined as the difference between the actual value of b and its value b_0 at the dead state,

$$e = (b - b_0) = (h - T_0 s) - (h_0 - T_0 s_0)$$

where h_0 and s_0 are the fluid specific enthalpy and entropy at (p_0, T_0) . Hence, $e_0 = 0$.

Exergy has gained popularity in thermodynamics and has appeared in popular publications such as New Scientist. There are chapters on it in each of the course text books. It should, however, be noted that $\Delta b = \Delta e$ and so analysis can often be done with either steady flow availability or steady flow exergy. In this course we will stick to steady flow availability.

2.4 Energy unavailable for work

'Entropy is the measure of a system's energy that is unavailable for work'

Encyclopaedia Britannica – Joseph Keenan

$$\dot{W}_{max} = (\dot{H}_1 - \dot{H}_0) - \dot{Q}_0$$

Maximum available power Difference in enthalpy flow rate from the dead state Energy flow rate unavailable for work (Heat transferred to environment)

Entropy flow rate fixes the energy flow rate unavailable for work

$$\dot{Q}_0 = T_0 (\dot{S}_1 - \dot{S}_0)$$

Energy flow rate unavailable for work Entropy flow rate Set by equilibrium with the environment

9

Energy unavailable for work

In his entry for entropy in the 1976 edition of the Encyclopaedia Britannica Prof Joseph Keenan of MIT wrote

'Entropy is the measure of a system's energy that is unavailable for work'

To understand this definition of entropy you must consider the system isolated except for interaction with an environment of constant pressure and temperature. In bringing the system reversibly into equilibrium with the environment the entropy of the system must be reduced by $S_1 - S_0$, where S_1 is the entropy of the system, and S_0 is the entropy of the system when it is at the pressure and temperature of the environment. The entropy transferred to the environment is achieved by a heat transfer \dot{Q}_0 , where

$$\dot{Q}_0 = T_0 (S_1 - S_0)$$

where T_0 is the temperature of the environment. \dot{Q}_0 is therefore considered to be the system's energy which is unavailable for work. Because S_0 and T_0 is fixed by the environment, \dot{Q}_0 is fixed by the entropy of the system S_1 alone.

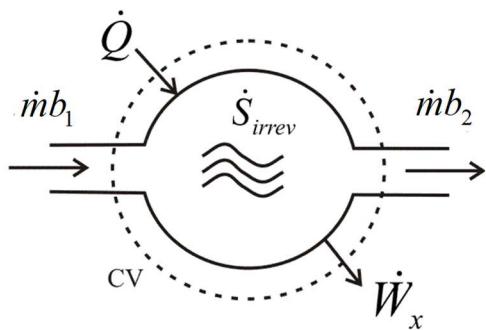
Fill in

Example 1:

There is a 1kg/s mass flow rate of air at a temperature 500K and a pressure 5 bar. The environment is at 1bar and 300K. Calculate the maximum available power which can be extracted from the air.

10

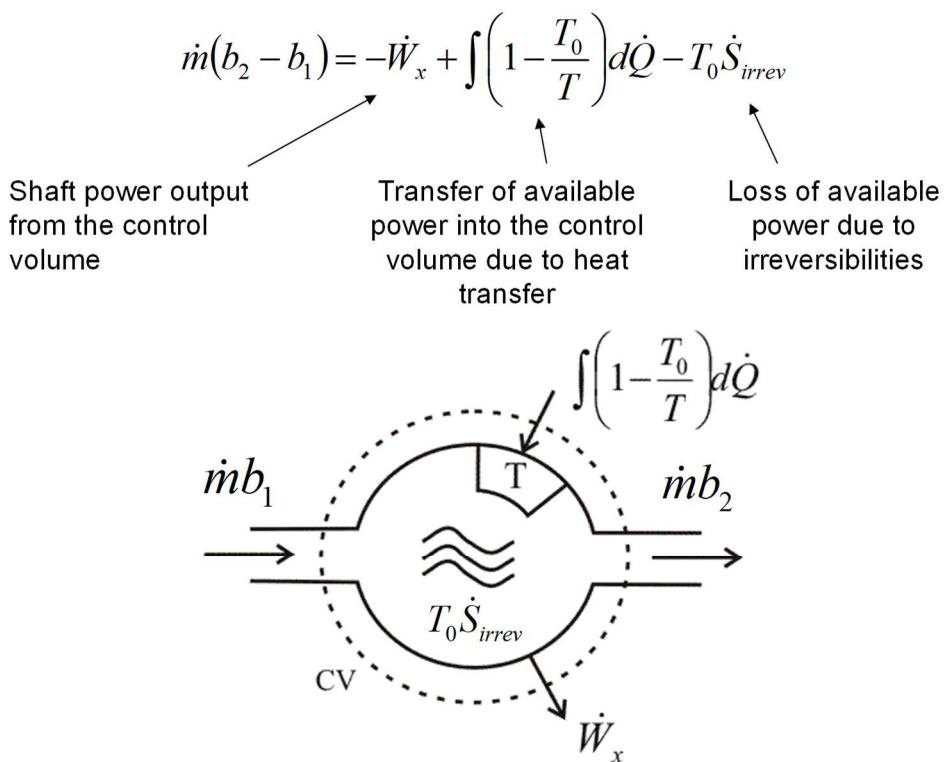
2.5 Availability balance equation - Steady flow device



NB balance equation
state 2 - state 1

$$\begin{aligned}
 \dot{m}(b_2 - b_1) &= \dot{m}[(h_2 - h_1) - T_0(s_2 - s_1)] \\
 \text{1st law} \quad \dot{m}(h_2 - h_1) &= \dot{Q} - \dot{W}_x \qquad \text{2nd law} \quad \dot{m}(s_2 - s_1) = \int \frac{d\dot{Q}}{T} + \dot{S}_{irrev} \\
 \dot{m}(b_2 - b_1) &= -\dot{W}_x + \dot{Q} - \int \frac{T_0}{T} d\dot{Q} - T_0 \dot{S}_{irrev} \\
 \dot{m}(b_2 - b_1) &= -\dot{W}_x + \int \left(1 - \frac{T_0}{T}\right) d\dot{Q} - T_0 \dot{S}_{irrev}
 \end{aligned}$$

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NB notice form of heat transfer term from section 2.1

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Transfer of available power due to heat transfer

As long as the heat is supplied at a temperature above that of the environment it transfers available power into the control volume. If the temperature is constant, then the transfer of available power due to heat transfer is

$$\left(1 - \frac{T_0}{T}\right) \dot{Q}$$

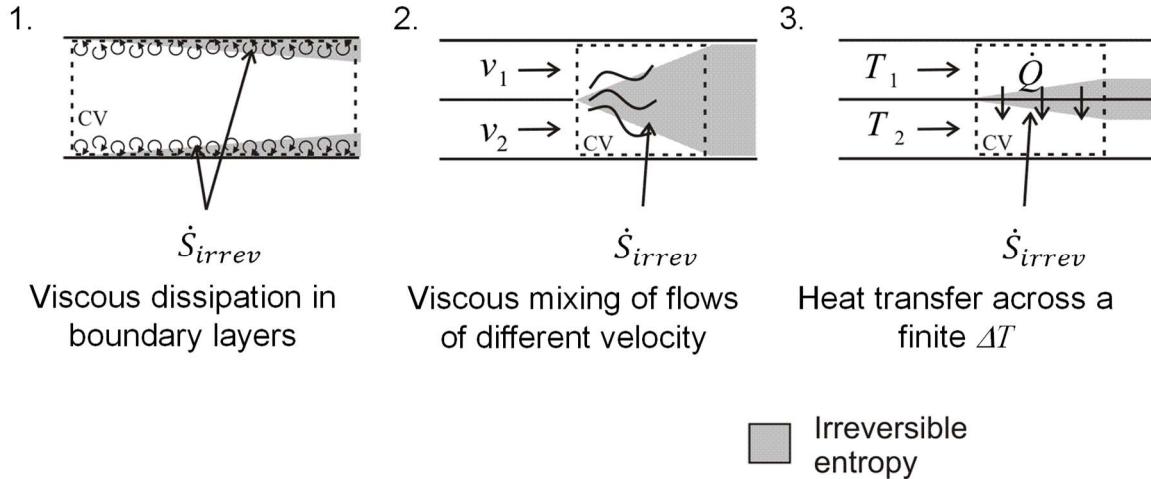
If the temperature varies it becomes

$$\int \left(1 - \frac{T_0}{T}\right) \delta Q$$

2.6 Loss of available power due to irreversibilities

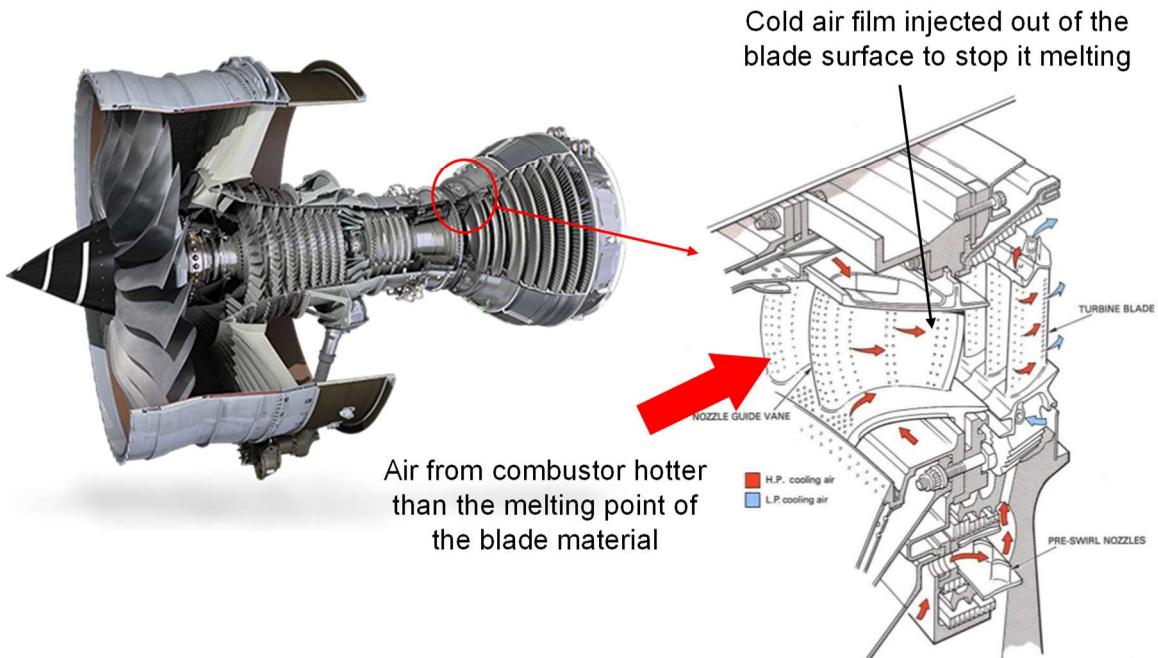
$$\dot{m}(b_2 - b_1) = -\dot{W}_x + \int_0^T \left(1 - \frac{T_0}{T}\right) d\dot{Q} - T_0 \dot{S}_{irrev}$$

Irreversibilities cause a reduction in the available power which can be extracted from a flow



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2.6 Loss of available power due to irreversibilities



Often the sources of irreversibility are complex. The designer should aim to avoid velocity or temperature difference mixing out without doing work

Loss of available power due to irreversibilities

Any process which is irreversible involves a loss of the available power. As discussed in the last lecture the three irreversible flow processes considered in this course are the ***viscous dissipation in boundary layers***, ***viscous mixing of flows of different velocity***, and ***heat-transfer across a finite temperature difference***.

The loss of available power due to the irreversibility is given by

$$T_0 \dot{S}_{irrev}$$

This is one of the most important concepts in thermofluids.

It is useful at this point to remember the analogy between entropy and “smoke”, introduced in lecture 1.

“Smoke” is continually being created in the blade boundary layers and in the mixing of leakage flows with the mainstream. Once created the “smoke” can never be destroyed and it is convected downstream through the machine and diffuses into the surrounding flow. The concentration of “smoke” at the exit from the machine includes a contribution from every source within the machine and the loss of machine performance is proportional to the average concentration of “smoke” at its exit.

2.7 How to approach a question

1. Sketch schematic of problem and define Control Volume.
2. Sketch process on a h-s or T-s diagram.
3. Determine whether the question requires a 1st law analysis or a 2nd law analysis. NB A 1st law analysis is usually quicker.

Calculate the shaft work output?

Calculate the heat addition?

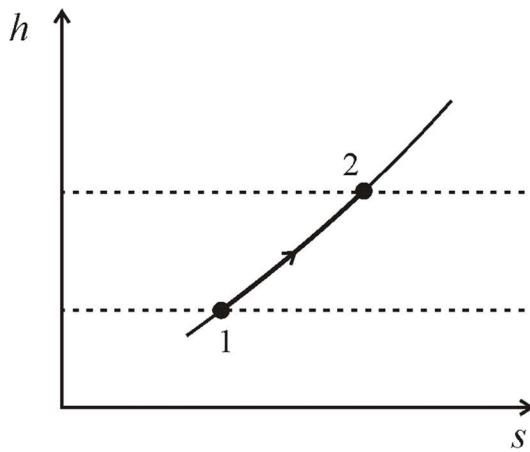
Calculate the loss of available power in a device?

Calculate the change in availability between two states?

Calculate the rate of transfer of available power due to heat-transfer?

Example 2: Heat addition

The temperature at the inlet and exit of a jet engine combustor is $T_1=900\text{K}$, $T_2=1800\text{K}$. There is no pressure drop in the combustor. The mass flow is 10kg/s. Use the properties of air throughout. The environment is at 300K. The combustor can be modelled as a reversible heat addition. Calculate the rate of transfer of available power due to heat addition into combustor.



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$$\dot{m}(b_2 - b_1) = -\dot{W}_x + \int \left(1 - \frac{T_0}{T} \right) d\dot{Q} - T_0 \dot{S}_{irrev}$$

$$\dot{m}(b_2 - b_1) = \dot{m}[(h_2 - h_1) - T_0(s_2 - s_1)]$$

$$\dot{m}(b_2 - b_1) = \dot{m} \left[c_p(T_2 - T_1) - T_0 \left(c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{p_2}{p_1} \right) \right) \right]$$

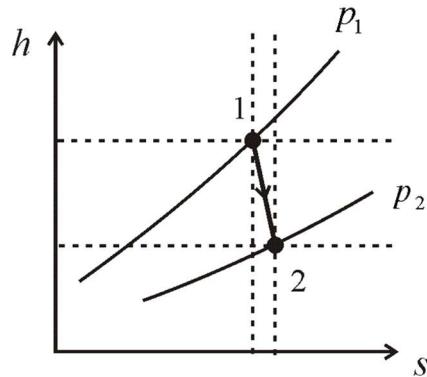
$$\dot{m}(b_2 - b_1) = 10 \left[1.005 \times 10^3 (1800 - 900) - 300 \left(1.005 \times 10^3 \ln \left(\frac{1800}{900} \right) \right) \right]$$

Rate of transfer of available power due to heat addition = 6.96 MW

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Example 3: Irreversible turbine

The adiabatic turbine of a jet engine has an inlet and exit pressure and temperature of 15 bar, 1800K, and 1bar, 920K. The mass flow is 10kg/s. Use the properties of air throughout. The environment is at 300K. Calculate (a) the power output and (b) the loss of available power due to irreversibilities in the turbine.



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Ex 3a – 1st Law analysis

$$1^{\text{st}} \text{ law} \quad \dot{m}(h_2 - h_1) = \dot{Q} - \dot{W}_x$$

$$\dot{W}_x = 10 \times 1.005 \times 10^3 (1800 - 920)$$

$$\boxed{\text{Shaft power output} = 8.84 \text{MW}}$$

Ex 3b – 2nd Law analysis

$$\text{Lost power potential} = T_0 \dot{S}_{irrev}$$

$$= T_0 \times \dot{m} \times \left(c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right) \right)$$

$$= 300 \times 10 \times \left(1005 \times \ln\left(\frac{920}{1800}\right) - 287 \times \ln\left(\frac{1}{15}\right) \right)$$

$$\boxed{\text{Loss of available power} = 0.31 \text{MW}}$$

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Splitting energy into available energy and unavailable energy

It is often insightful to consider the energy of a system as split into two parts, the part which is available for work and the part which is unavailable for work. The change in the energy across a control volume is therefore made up of the sum of the changes in the part which is available for work and a change in the part which is unavailable for work.

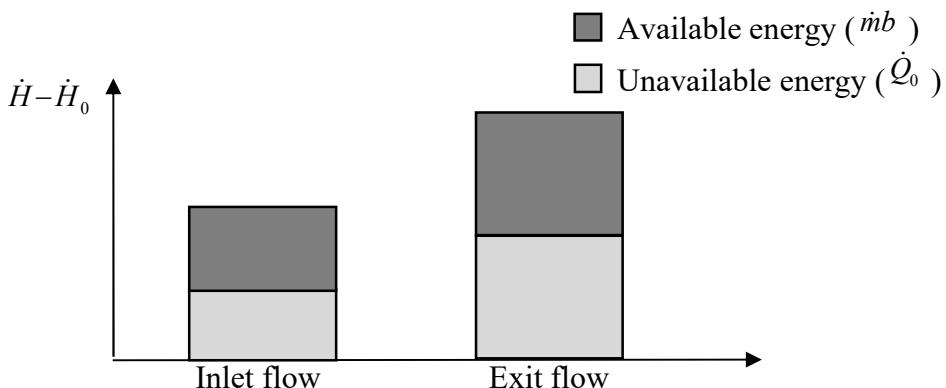
$$\Delta\text{energy} = \Delta\text{available energy} + \Delta\text{unavailable energy}$$

$$\Delta\dot{H}_{12} = \dot{m}\Delta b_{12} + \Delta\dot{Q}_{0_2}$$

where 1 is the inlet and 2 is the exit of a control volume and where the unavailable energy of the flow at inlet \dot{Q}_{0_1} is

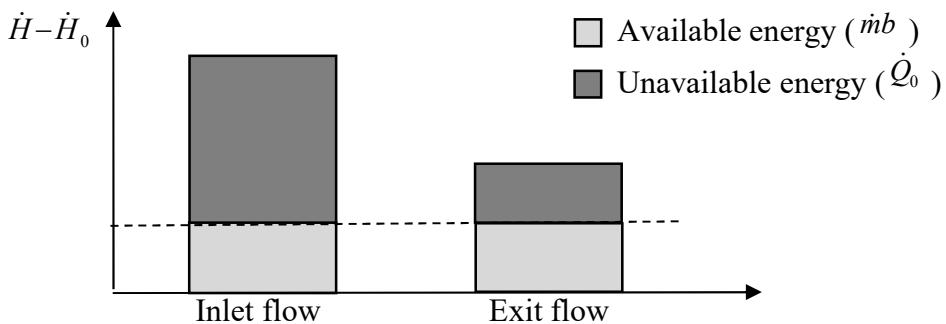
$$\dot{Q}_{0_1} = T_0(\dot{S}_1 - \dot{S}_0)$$

Consider again example 1 from lecture 1 but this time from the perspective of the energy of the system being split into the two parts. In part (a) a reversible heat transfer to the flow increases both the enthalpy and the entropy of the flow. The effect of this on the available energy and unavailable energy of the flow is shown below.



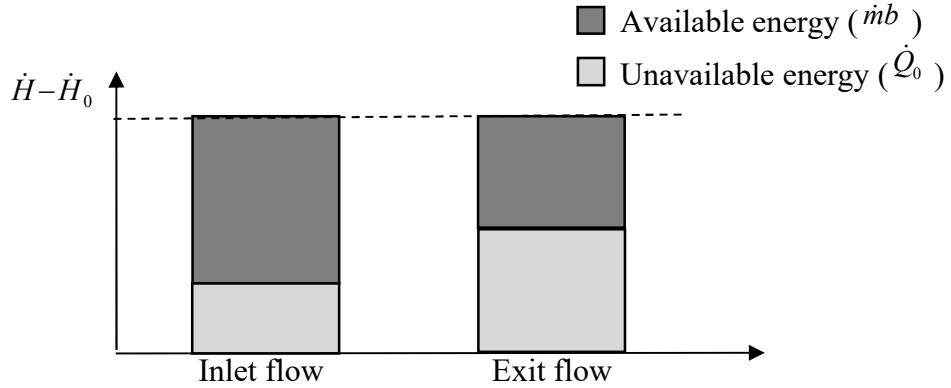
Because the entropy of the flow has risen the energy in that flow which is unavailable for work also rises. This means that the increase in the available energy of the flow must be smaller than the increase in the energy of the flow. This shows that the ‘value’ of heat, in terms of the work that can be extracted from it, is always less than one, set by the Carnot efficiency between the temperature of the flow and the environment.

In part (b) of example 1 an isentropic turbine reduces the enthalpy of the flow, but the entropy of the flow remains unchanged.



Because the entropy of the flow remains unchanged the energy which is unavailable for work remains unchanged. This means that the drop in energy of the flow is equal to the drop in available energy of the flow.

Finally, in part (c) of example 1 the throttle increases the entropy of the flow but leaves the energy of the flow unchanged.



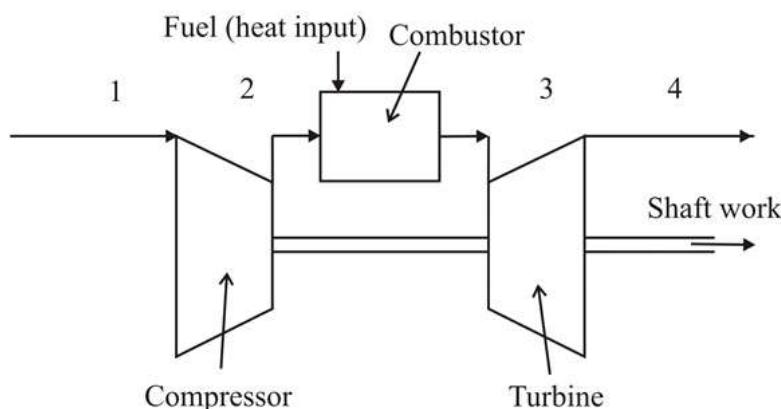
Because the entropy of the flow rises the energy which is unavailable for work rises. Because the energy of the flow remains unchanged across the throttle the increase in unavailable energy is equal to the decrease in available energy.

Lecture 3: Gas turbines

3. Gas turbines

3.1 Introduction

1st year ideal air-standard Joule cycle. 2nd year real gas turbines.



1. 2-4% pressure drop in combustor.
2. Combustion changes gas composition.
3. The compressor and turbine contain irreversibilities.

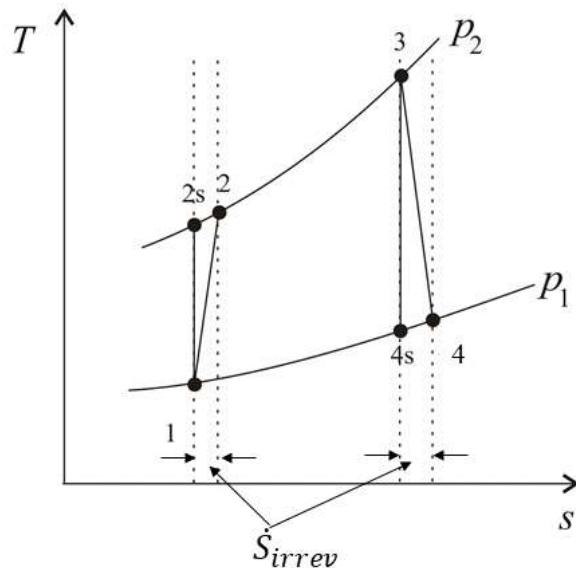
Real gas turbines

The ideal reversible cycle which most closely models a simple gas turbine is a Joule cycle. In the US the same cycle is known as the Brayton cycle. In a simple gas turbine the compressor inlet air is taken from atmosphere and the turbine exit is exhausted to atmosphere. The air standard Joule cycle is limited in a number of ways. In a real gas turbine:

- Across the combustor there is normally a drop of 2-4% in pressure.
- The combustion processes changes the gas composition and so the thermodynamic properties of the working fluid in the turbine are different from in the compressor.
- Within the compressor and turbines there is viscous dissipation in the boundary layer and mixing of flows or different velocity and temperature. These processes are irreversible and therefore creation entropy. Each of these irreversible events causes a loss in the available power of the flow.

In the analysis in this lecture only the final one of these real effects will be considered. This is the main effect in real gas turbines. In examination and supervision problems you may be asked to include compressor and turbine irreversibilities, use combustion product gas products in the turbine and exit nozzle and include a combustor pressure drop. The final two are easy to include.

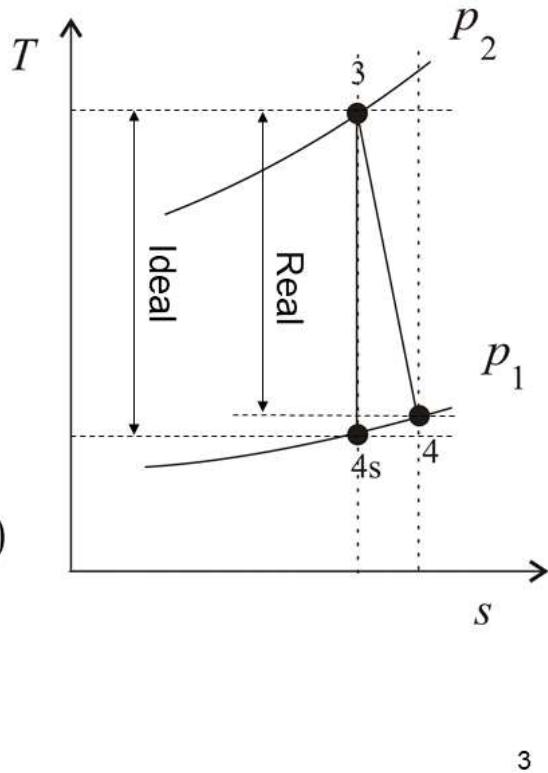
3.2 Compressor and turbine irreversibilities



Compressor and turbine blades have large surface area, contain many leakage jets and involve the mixing of streams of different temperature. These all result in irreversible entropy generation.

3.3 Isentropic turbine efficiency

$$\eta_T = \frac{(\dot{W}_T)_{real}}{(\dot{W}_T)_{ideal}} =$$



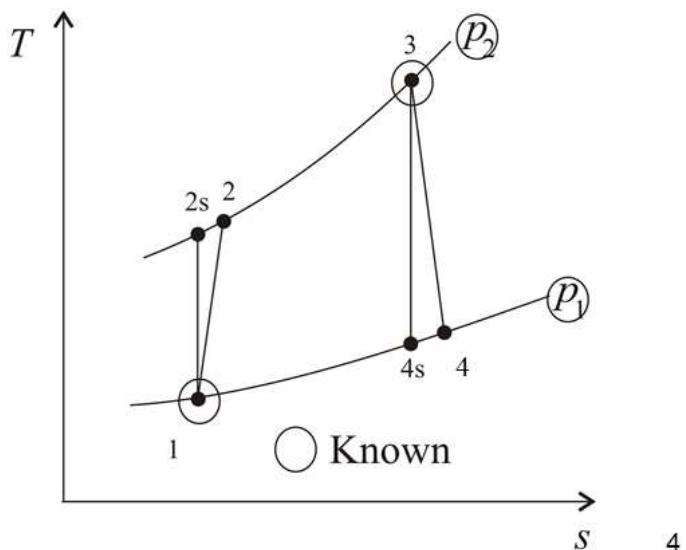
Calculate T_{4s}

$$\frac{T_{4s}}{T_3} = \left(\frac{P_1}{P_2} \right)^{\left(\frac{\gamma-1}{\gamma} \right)}$$

3

Example 1 – 1st law analysis of gas turbine

$T_1 = 300K$, $T_3 = 1500K$, $\eta_C = 90\%$, $\eta_T = 90\%$, $p_2/p_1 = r_p = 25$. The mass flow rate of air is 120kg/s. The mass flow rate of fuel and the pressure loss in the combustion chamber may both be neglected. Calculate the thermal efficiency of the cycle.



Fill in

Compressor

$$\left. \begin{aligned} \frac{T_{2s}}{T_1} &= \left(\frac{p_2}{p_1} \right)^{\frac{(\gamma-1)}{\gamma}} = r_p^{\frac{(\gamma-1)}{\gamma}} = 25^{0.2857} = 2.508 \\ T_{2s} &= 2.508 T_1 = 752.4K \end{aligned} \right\} \text{Isentropic 1st year analysis}$$

$$\eta_c = \frac{(h_{2s} - h_1)}{(h_2 - h_1)} = \frac{T_{2s} - T_1}{T_2 - T_1} \quad T_2 = T_1 + \frac{(T_{2s} - T_1)}{\eta_c} =$$

$$\dot{W}_C = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1) =$$

Combustor

$$\dot{Q}_{2-3} = \dot{m}(h_3 - h_2) = \dot{m}c_p(T_3 - T_2) =$$

5

Fill in

Turbine

$$\left. \begin{aligned} \frac{T_3}{T_{4s}} &= \left(\frac{p_3}{p_4} \right)^{\frac{(\gamma-1)}{\gamma}} = r_p^{\frac{(\gamma-1)}{\gamma}} = 25^{0.2857} = 2.508 \\ T_{4s} &= T_3 / 2.508 = 598.1K \end{aligned} \right\} \text{Isentropic 1st year analysis}$$

$$\eta_T = \frac{(h_3 - h_4)}{(h_3 - h_{4s})} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

$$T_4 = T_3 + \eta_T(T_3 - T_{4s}) =$$

$$\dot{W}_T = \dot{m}(h_3 - h_4) = \dot{m}c_p(T_3 - T_4) =$$

Cycle efficiency

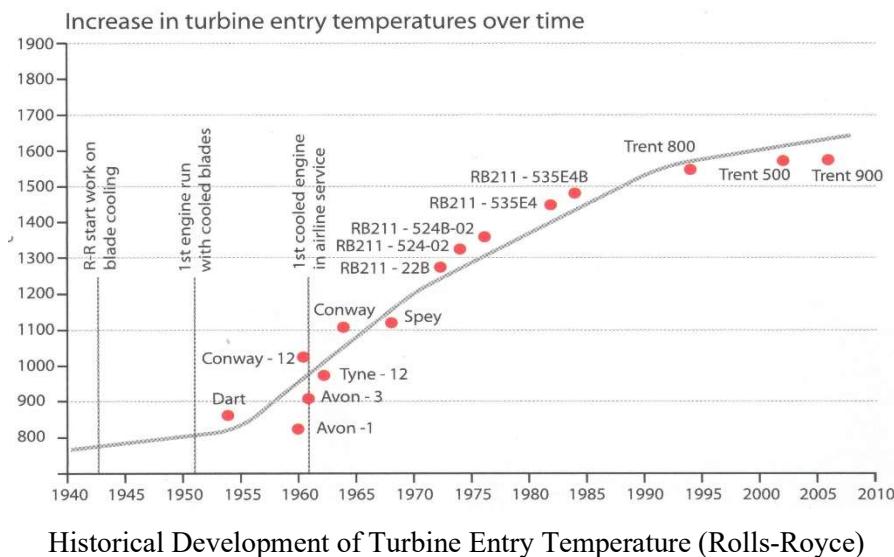
$$\eta_{IdealJouleCycle} = 60.1\%$$

$$\eta_{1stLaw} = \eta_{Thermal} = \frac{\dot{W}_T - \dot{W}_C}{\dot{Q}_{2-3}} =$$

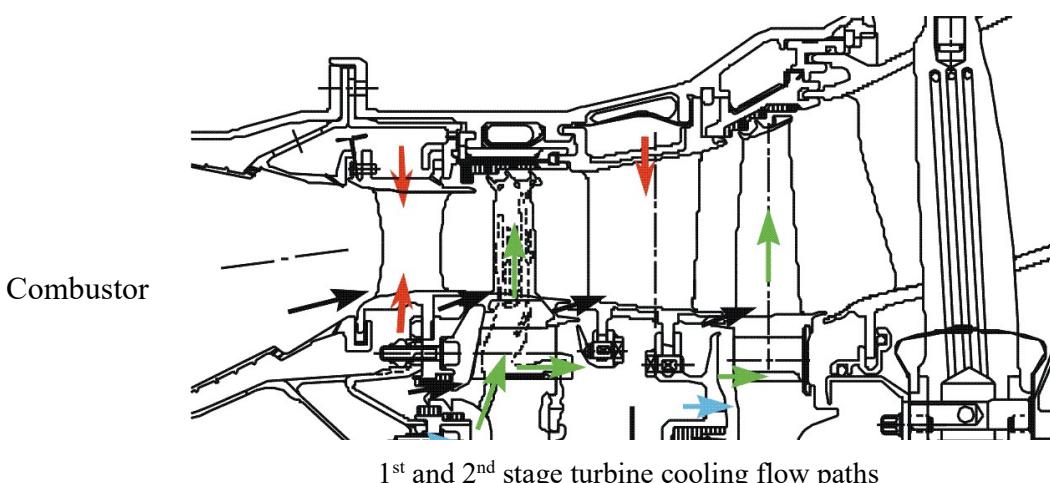
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Historical rise in gas turbine efficiency

The efficiency of gas turbines has risen dramatically over the last 50 years. The early gas turbines built in the 1940s and 1950s had cycle efficiencies of about 17%. This was for two main reasons, firstly the efficiency of the compressors and turbines were low (high irreversible entropy generation), and secondly the temperature of the combustor (the temperature at which the heat is added to the cycle) was low. Remember that low temperature of heat addition means that the power potential of the added heat is low. The combustor exit temperature was limited by the 1st stage turbine metallurgical limits of the time. Over the last 50 years irreversibility entropy generation in the compressor and turbine has been reduced and combustor exit temperatures have been raised. The historical variation in turbine entry temperature of Rolls Royce engines is shown in the figure below.

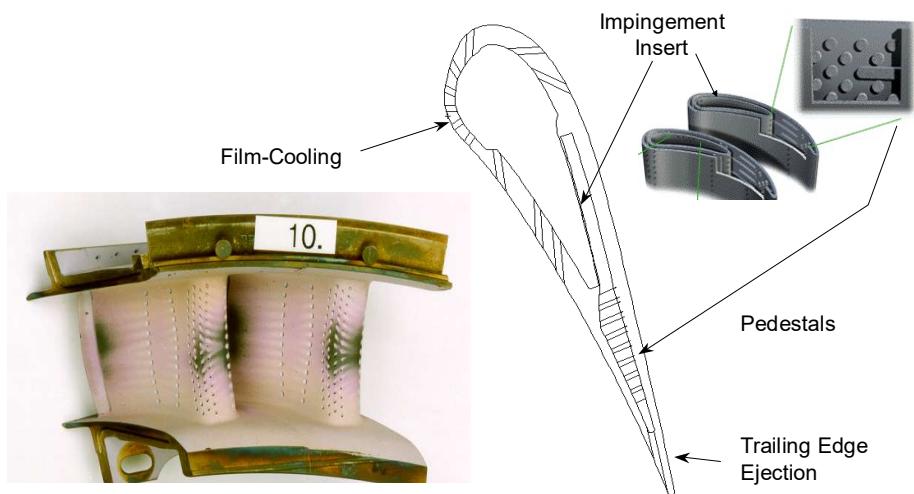


Currently, the turbine entry temperature (TET) is 1870 K for an aero-engine on take-off, 1780 K for an aero-derivative gas turbine, 1700 K for a heavy-duty industrial machine which have to have longer working life. For comparison the melting temperature of the blade metal is about 1800 K. Creep limit for aero-engines and aero-derivatives is about 1370 K. Creep limit for heavy-duty industrial machines is about 1250 K. If the TET is higher than about 1200 K, it is necessary to cool the turbine blades.



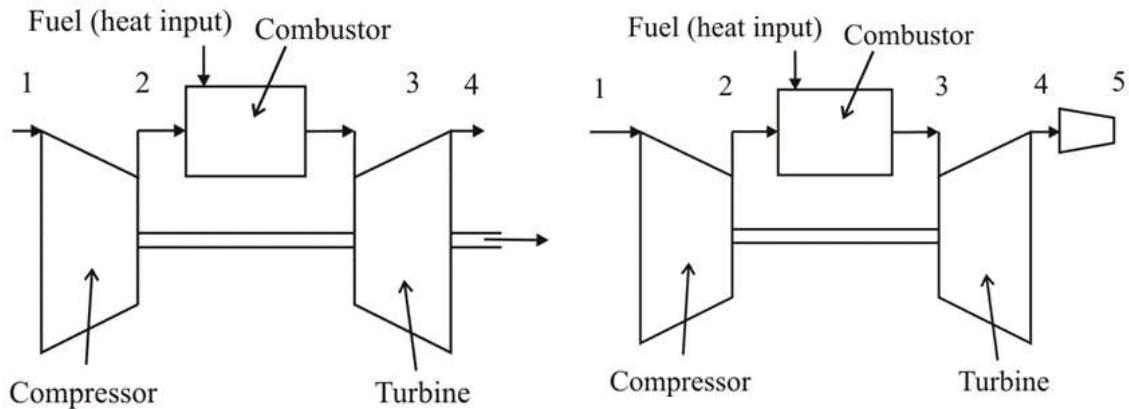
Turbine blade cooling is crucial to the successful development of high performance gas turbines. Around 15-25 % of the compressor air bypasses the combustion chamber and is used in cooling the turbine. The black, green, red and blue arrows in the figure above show different cooling

flows in the 1st and 2nd stage turbine. If turbine designers are to maximise efficiency, then they need to inject the cooling flows with as little irreversible entropy generation as possible.



A turbine cooling system which uses internal convective and external film cooling

3.4 Gas turbine & Jet engine



Gas turbine (Land & Marine)

$$P_4 = P_{atmos}$$

$$\dot{W}_x = \dot{W}_T - \dot{W}_C$$

Jet engine (aero)

$$P_5 = P_{atmos}$$

$$\begin{aligned} \dot{W}_T &= \dot{W}_C \\ h_4 + \frac{0^2}{2} &= h_5 + \frac{v_5^2}{2} \end{aligned}$$

7

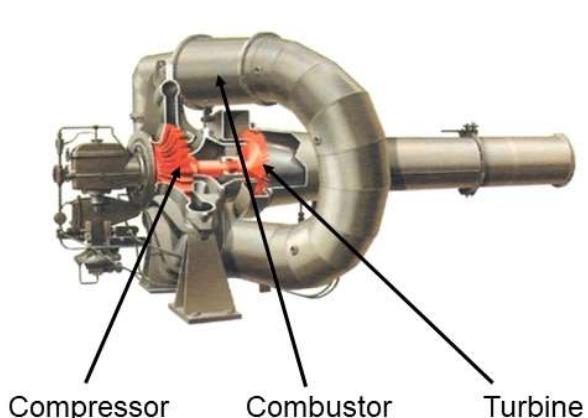
Use of gas turbines

The majority of the western world's naval fleets use gas turbines for propulsion and electrical power generation. The General Electric LM2500 gas turbine used to power ships have a simple-cycle thermal efficiency of 37%. Compared to a steam turbine and diesel-propulsion systems, the gas turbine offers greater power for a given size and weight, high reliability and long life. Another advantage is that where a steam turbine would typically take 4 hours to start up a gas turbine takes less than 2 minutes. Many modern marine propulsion systems combine diesel and gas turbines to provide efficient low-power propulsion from the diesel engine at cruise and high-power from the gas turbine at higher speed.

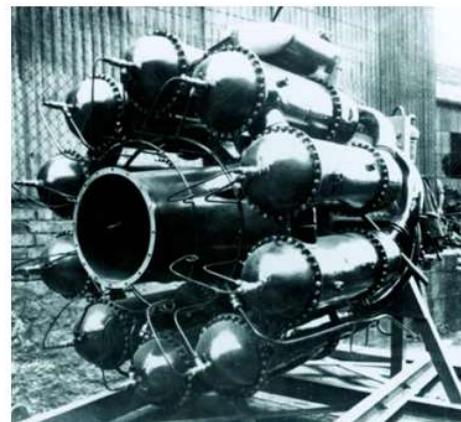
Gas turbines are also used for a wide range of land based power generation from remote power generation and micro turbines for local distributed power to large power stations. Their power output can range from 50kW for a micro turbine to 250MW for a heavy duty power station. The efficiency of the gas turbine tends to rise with their size. The Rolls-Royce Industrial Trent (40:1 pressure ratio) has an overall thermal efficiency of 42%. This is an aero-derivative gas turbine which means it is an aero-gas turbine converted to operate on the ground. Heavy duty industrial gas turbines, specifically for land based use, are manufactured by companies such as Alstom, Mitsubishi and Siemens. They typically have an efficiency of 38%.

All civil aircraft are powered by gas turbines manufactured by one of four companies General Electric, Pratt & Whitney, Rolls-Royce and SNECMA. The jet engine under the wing has a gas turbine at its core. Part of the turbine power is used to power a fan at the front which in the Trent 700 provides 75,000lbs of thrust.

3.5 First jet engine



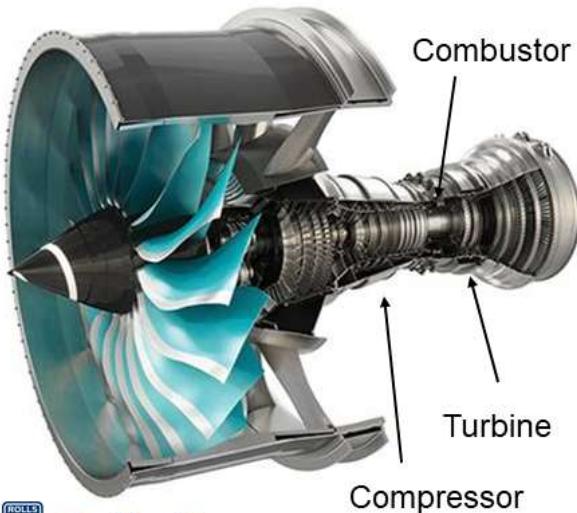
WU (Whittle Unit)



W1 engine

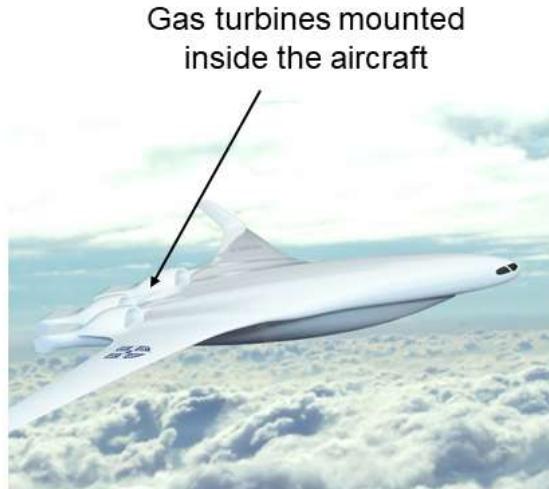
First jet engine tested by Frank Whittle in 1937

3.6 Reducing engine emissions



Rolls-Royce

25% reduction in fuel burn



Cambridge-MIT Silent Aircraft Initiative

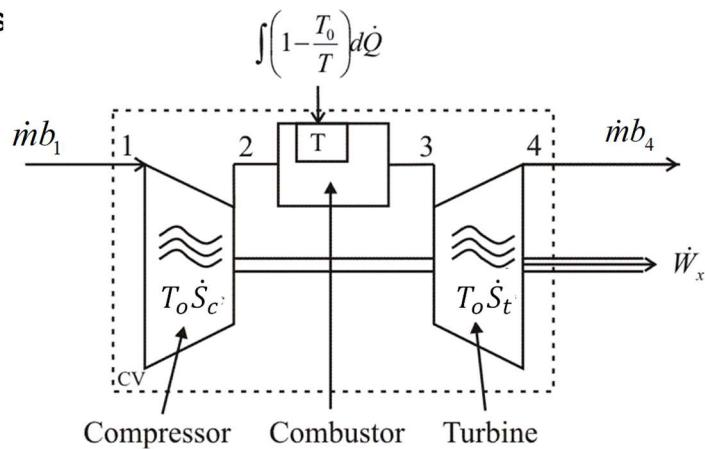
NASA N+3 up to 70% reduction in fuel burn

<https://www.youtube.com/watch?v=JxkJ-FwFeVI> (Time in video 0:27 – 1:26)

9

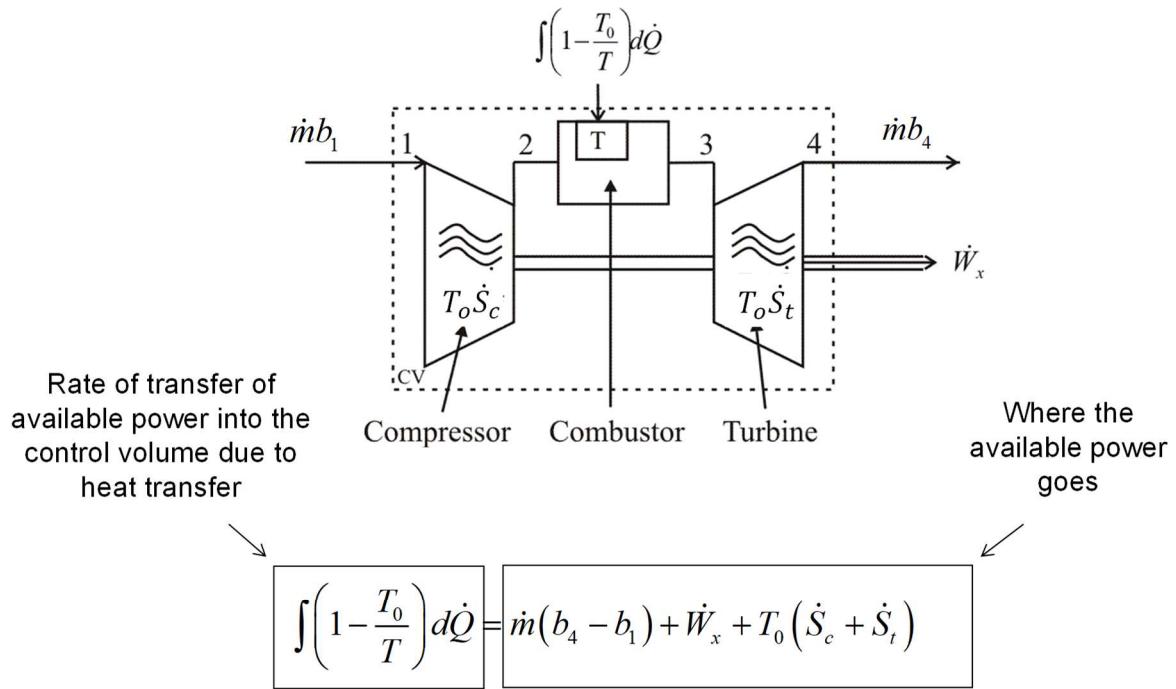
Example 2: 2nd Law analysis of gas turbine in Ex1.

Applying an availability analysis to a cycle allows us to examine where the available power transferred in the engine as heat goes. This is known as 2nd law analysis



$$\dot{m}(b_4 - b_1) = -\dot{W}_x + \int \left(1 - \frac{T_0}{T}\right) d\dot{Q} - T_0(\dot{S}_c + \dot{S}_t)$$

10



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Transfer of available power into the combustor

$$\frac{1}{\dot{m}} \int \left(1 - \frac{T_0}{T}\right) d\dot{Q} = (b_3 - b_2) = (h_3 - h_2) - T_0(s_3 - s_2)$$

$$(s_3 - s_2) = c_p \ln\left(\frac{T_3}{T_2}\right) - R \ln\left(\frac{P_3}{P_2}\right)$$

$$\frac{1}{\dot{m}} \int \left(1 - \frac{T_0}{T}\right) d\dot{Q} = 1.01(1500 - 802.7) - 300 \left(1.01 \times \ln \frac{1500}{802.7}\right) = 515.0 \text{ kJ/kg}$$

12

Shaft power extraction

$$\frac{\dot{W}_x}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_c}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1)$$

$$\frac{\dot{W}_x}{\dot{m}} = 1.01 \times (1500 - 688.3) - 1.01 \times (802.7 - 300) = 312.1 \text{ kJ/kg}$$

Available power in exhaust flow

$$(b_e - b_i) = (h_4 - h_i) - T_0(s_4 - s_i)$$

$$(b_e - b_i) = 1.01 \times (688.3 - 300) - 300 \left(1.01 \times \ln \left(\frac{688.3}{300} \right) \right)$$

$$(b_e - b_i) = 140.6 \text{ kJ/kg}$$

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Loss of available power due to irreversibilities in compressor

$$T_0 \Delta s_c$$

$$(s_2 - s_1) = c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

$$T_0 \Delta s_c = 300 \times \left(1.01 \times \ln \frac{802.7}{300} - 0.287 \times \ln \frac{25}{1} \right) = 21.1 \text{ kJ/kg}$$

Loss of available power due to irreversibilities in turbine

$$T_0 \Delta s_t$$

$$(s_4 - s_3) = c_p \ln \left(\frac{T_4}{T_3} \right) - R \ln \left(\frac{P_4}{P_3} \right)$$

$$T_0 \Delta s_t = 300 \times \left(1.01 \times \ln \frac{688.3}{1500} - 0.287 \times \ln \frac{1}{25} \right) = 41.1 \text{ kJ/kg}$$

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$$\int \left(1 - \frac{T_0}{T} \right) d\dot{Q} = \dot{m}(b_e - b_i) + \dot{W}_x + T_0(\dot{S}_c + \dot{S}_t)$$

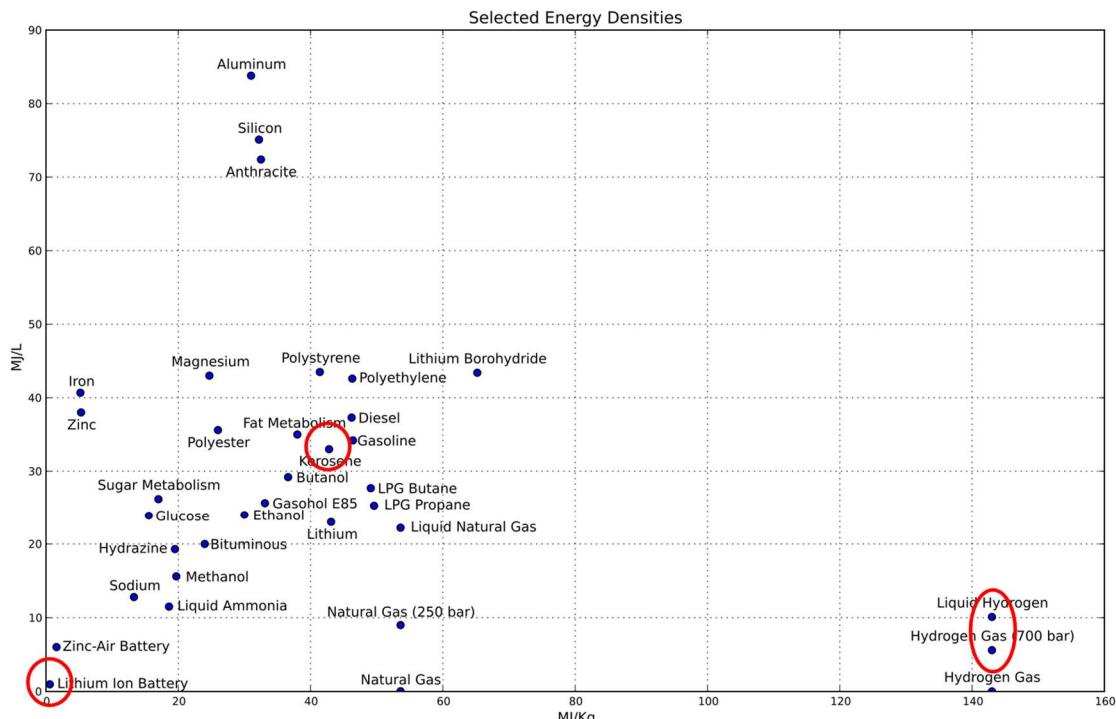
Where does the available power transferred into the control volume as heat go?

Available power transferred into the combustor	515.0	100%
Shaft power extracted	312.1	60.6%
Loss of available power in the compressor	21.1	4.1%
Lost of available power in the turbine	41.1	8%
Available power in exhaust flow	140.6	27.3%
Total RHS	515.0	100%

$$\eta_{Rational} = \eta_{2ndLaw} = \frac{312.1}{515.0} = 60.6\% \quad \eta_{Thermal} = \eta_{1stLaw} = 44.3\%$$

15

3.7 Zero carbon flight



Hydrogen produced by a massive expansion of offshore wind

16

3.7 Zero carbon flight



In September 2020 Airbus announced that they would have a hydrogen aircraft on the market by 2035

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Lecture 4: Working fluids I

4. Working fluids I

4.1 Introduction

Every thermodynamic device (e.g. refrigerators, heat pumps, engines) relies on a *working substance*.

The working substance is usually a fluid (gas or a liquid) whose function is to provide a medium for the transfer of heat and work.

Common working fluids:

1. **Air** - reciprocating engines, gas turbines.
2. **Water** - steam engines, steam turbine power plant.
3. **HFC s** - heat pumps, refrigeration plant.

Benefit of real fluids

Air-standard cycles (i.e. Joule cycle) using an ideal gas do not approach Carnot because Q not at constant T . Other working fluids will permit practical cycles which are closer to the ideal.

2

4.2 Reminder

Recall that:

- **Properties** (e.g. p , v , T) at a particular state have one value.
- A **state** refers to the condition of a system as described by its properties.
- A **pure substance** has a homogeneous and invariable composition.
- **Phases** are solid, liquid vapour.

In this course each working fluid will be considered to be:

1. A pure substance.
2. One, two or more different phases.

Example of a pure substance:

1. Water H_2O , present as liquid, vapour (steam) or as a mixture of liquid and vapour.
2. Air is not strictly a pure substance, since its composition varies during liquefaction due to the different boiling points of nitrogen (-195.8°C) and oxygen (-183.0°C). Nevertheless a substance that is present in only a single phase may usually be considered to be pure. Gaseous air (79% nitrogen and 21% oxygen by volume) is pure to the extent that its composition is constant.

3

4.3 How many properties fixes a state?

We know that not all properties are independent.

Example: For an ideal gas 2 properties set the state.

$$p = \frac{RT}{v}$$

Two-property rule:

In the absence of external effects (motion, gravity, electromagnetism or surface tension) the state of a pure substance is fixed by the values of 2 independent properties.

This is known as the **state principle** and allows us to say $p=f(T,v)$ etc.

For the case of a ideal gas $p=f(T,v)$ can be written as the simple **equation of state** above.

4

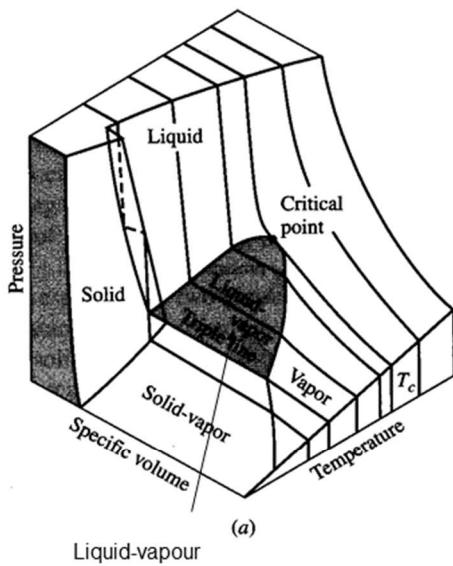
4.4 Some remarks about the state principle

1. Equations of state depend on the detailed atomic and molecular structure of a substance, and cannot be deduced from the Laws of Thermodynamics.
2. They must either be obtained by experimental measurement or deduced from statistical mechanics.
3. It is not normally possible to write an equation of state in closed form, except for a very limited range of properties. The major exception is the ideal gas equation of state, $pv=RT$.

So if $p=f(T,v)$, we should be able to experimentally determine $p-v-T$ surfaces.

5

4.5 Experimentally measured p - v - T surfaces



Surface shows different phases.

1. At high T becomes vapour/gas.
2. As T is dropped becomes liquid.
3. Finally at low T becomes solid.
4. In between two-phase regions.

6

p - v - T surface

From experiments it is known that the temperature and specific volume can be regarded as independent and pressure determined as a function of these two

$$p=p(T,v)$$

The graph of such a function is a surface, the **p - v - T surface**.

Terminology:

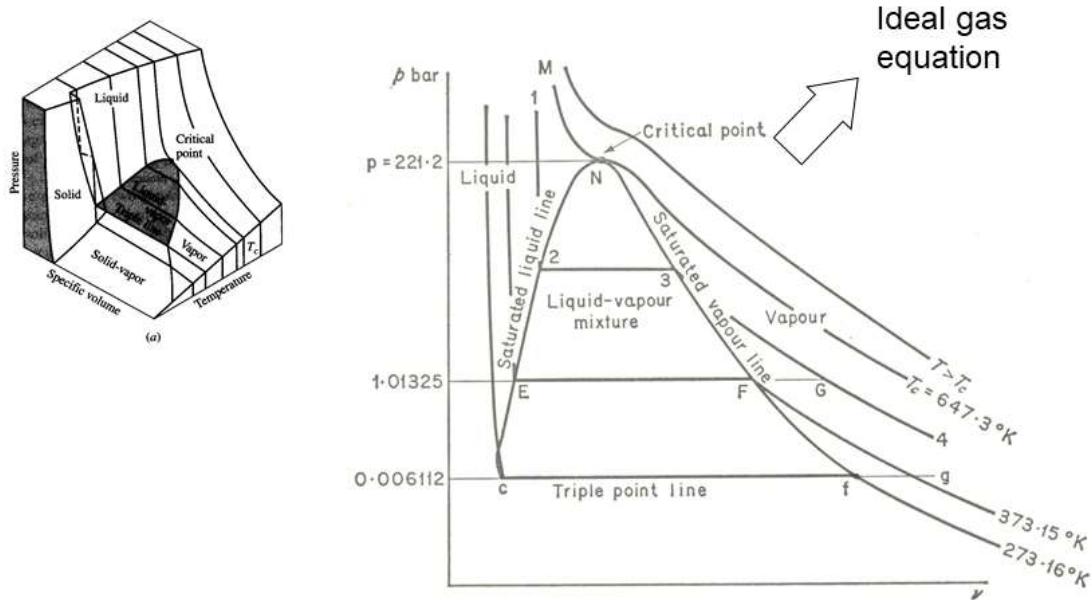
The **saturation state** is a state at which phase change starts and ends.

The **vapour dome** is the region composed of the two-phase liquid-vapour states. The saturated liquid and vapour lines are the lines bordering the vapour dome.

The **critical point** is the point where the saturated liquid and vapour lines meet. The critical temperature of a pure substance is the maximum temperature at which liquid and vapour phases can coexist.

4.6 p-v diagram

Projecting the $p\text{-}v\text{-}T$ surface onto the $p\text{-}v$ plane

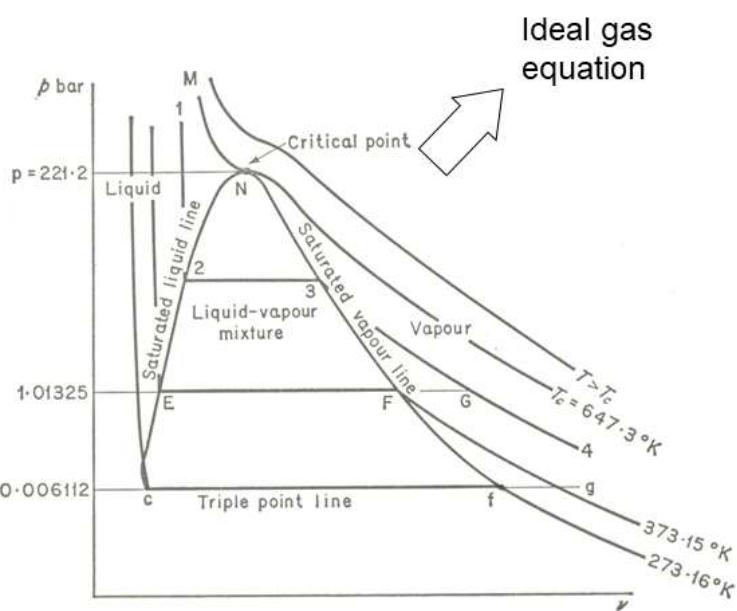


7

$p\text{-}v$ diagrams normally omit the solid phase, as this has little practical relevance.

1. The **saturated liquid line** (where the fluid is sometimes described as being **wet saturated**).
2. The **saturated vapour line** (where the fluid is sometimes described as being **dry saturated**).
3. The **liquid**, region, (where the fluid is sometimes described as being **subcooled**).
4. The **liquid-vapour** region.
5. The **vapour region**, (where the fluid is said to be **superheated**).
6. The **critical point** is the point with the maximum temperature at which liquid and vapour phases can exist.
7. Above the critical point no distinction can be made between liquid and vapour.
8. The **triple line** is the line along which three phases can exist together.

8



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p-v diagram

For engineering purposes this diagram is probably the most useful. The properties of a substance are shown by plotting the boundaries between the phases together with lines of constant temperature (isotherms). The diagram divides into regions of solid, liquid and vapour together with two-phase regions containing solid-vapour or liquid-vapour mixtures in equilibrium. It is evident that the temperature and pressure are not independent in two-phase regions.

The temperature at which a phase change takes place for a given pressure is called the **saturation temperature**, and the pressure at which a phase change takes place for a given temperature is called the **saturation pressure**. The solid-vapour and liquid-vapour regions are separated by the triple point line. They are also separated from the single-phase regions by the **saturation lines**. The liquid-vapour region is of greatest engineering interest since it involves a phase change between fluids.

On the boundary between the liquid-vapour region and the liquid region the substance is said to be a **saturated liquid**. On the boundary between the liquid-vapour region and the vapour region the substance is said to be a **saturated vapour**. If the pressure is lowered at constant temperature or the temperature is raised at constant pressure the vapour is said to be an unsaturated vapour or a **superheated vapour**. The difference between the temperature of a superheated vapour and the saturation temperature at the same pressure is called the **degree of superheat**. The saturated liquid line meets the saturated vapour line at the critical point. The isotherm running through this point is called the **critical isotherm**. A substance in a state above the critical isotherm cannot be liquefied by isothermal compression alone and must first be cooled to below the critical temperature. Thus the critical isotherm marks a boundary between vapour and gas. If a substance at normal temperatures and pressures lies above its own critical isotherm it is called a gas, otherwise it is a vapour. This is not a rigid distinction and no phase change is involved.

4.7 Phase change

Examples of phase changes are:

1. Liquid to vapour is called **vaporisation**.
2. Solid to liquid is called **melting**.
3. Solid to vapour is called **sublimation**.

1. During a phase change both phases can coexist.
2. A change of phase is accompanied by absorption or release of energy resulting from the breaking or reformation of intermolecular bonds.
3. During phase change p and T not independent properties.
4. When a phase change occurs the heat transferred is called **latent heat**.
5. Latent heat is a function of the p or T .
6. Melting or freezing is known as the **latent heat of fusion**.
7. Boiling or condensation is known as the **latent heat of vaporisation**.

Best to see example of phase change.

10

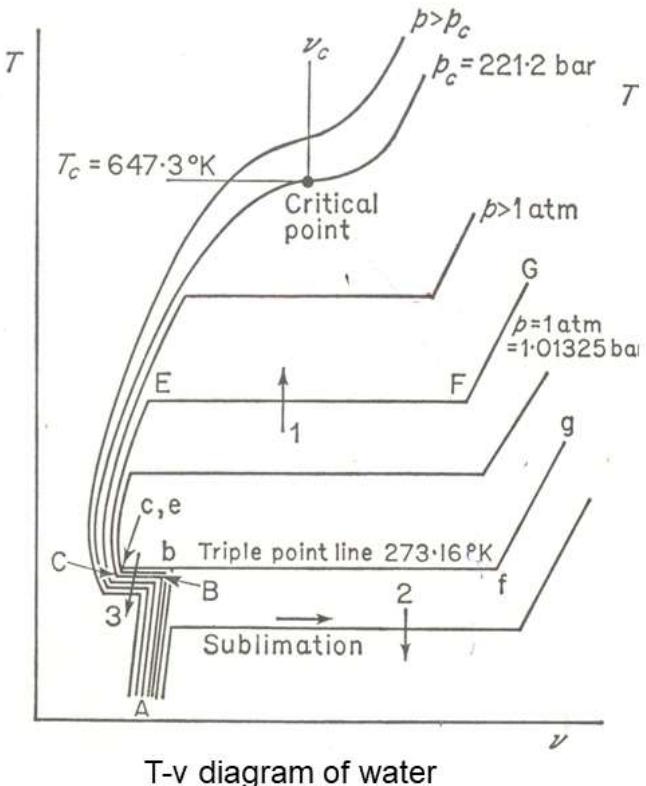
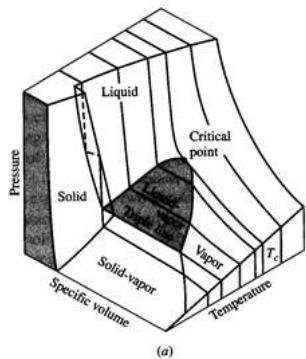
Example 1: Phase change on a $T\text{-}v$ diagram

Water (H_2O): Consider a lump of ice at atmospheric pressure and 270K, i.e. just below the normal freezing point. The sequence of events during heating at constant pressure can be traced out on a $T\text{-}v$ diagram.

1. Ice at 270K. Heating causes a rise in temperature according to the specific heat capacity of ice, and there is an increase in specific volume due to thermal expansion.
2. Ice at 273.15K. Further heating causes no rise in temperature, but instead a phase change from ice to water occurs at constant temperature. This temperature is the *melting point* at atmospheric pressure. For ice, melting is accompanied by a reduction in specific volume. This is anomalous, since most substances expand on melting. The heat supplied per unit mass in going from ice to water is the latent heat of fusion for water at atmospheric pressure. During the phase change ice and water coexist in equilibrium.

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Fill in



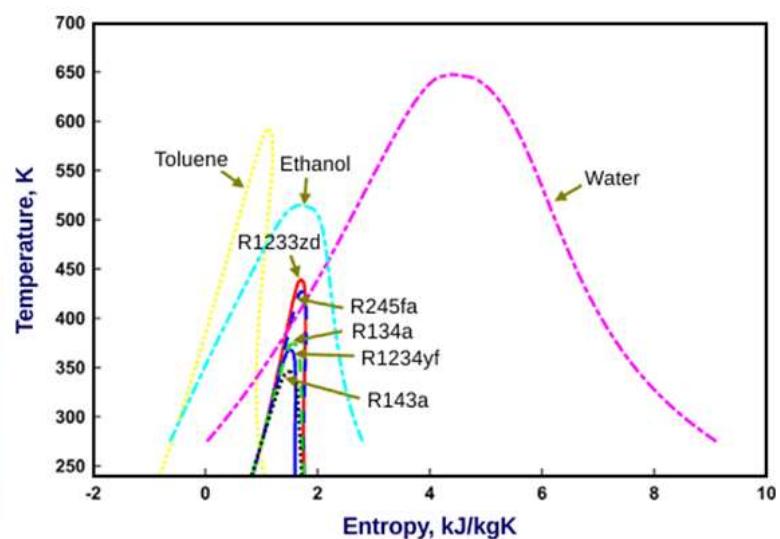
T-v diagram of water

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3. Water at 273.15K. The phase change is complete. Further heating causes the temperature to rise according to the specific heat capacity of water. The specific volume falls slightly up to 277K (for water), then rises due to thermal expansion.
4. Water at 373.15K. Further heating causes no further rise in temperature, but instead a phase change from water to steam occurs at constant temperature. This temperature is the *boiling point* at atmospheric pressure. Boiling (vaporation) is accompanied by a large increase in specific volume. The heat supplied per unit mass in going from water to steam is the latent heat of vaporisation for water at atmospheric pressure. During the phase change water and steam coexist in equilibrium.
5. Steam at 373.15K. The phase change is complete. Further heating causes the temperature to rise according to the specific heat capacity for steam at constant pressure. The steam is said to be superheated.
6. Steam at 400K. Further heating causes no further phase changes and the equation of state approximates to the ideal gas equation of state.

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4.8 Extracting power from low grade heat



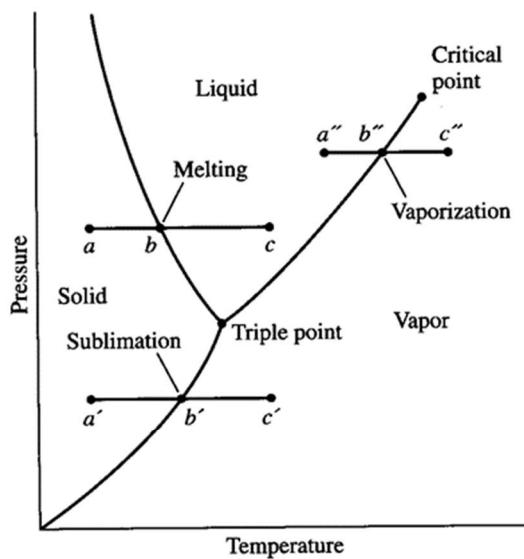
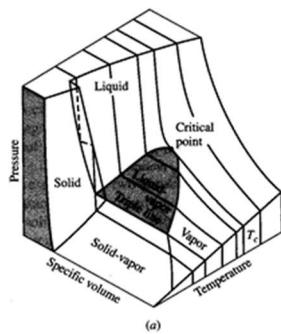
Tailoring the properties of the working fluid so that the liquid vapour region is at the temperature at which 'low grade' is provided.

(Organic Rankine Cycle – Lecture 6&7)

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4.9 p-T diagram

p - T diagram is known as a **phase diagram** since all three phases are separated by lines.



p and T linked and constant during phase change.

The vapourisation line ends at the critical point as no distinction can be made between vapour or liquid.

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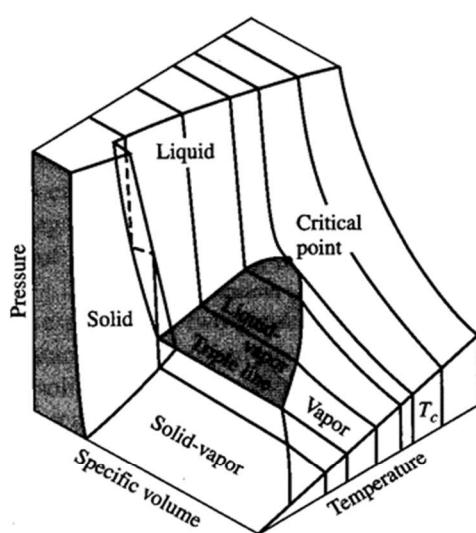
p-T diagram

The same information can be displayed on a p - T diagram by plotting the pressure against temperature during a change of phase. Curves of sublimation, fusion and vaporisation appear, separating regions of constant phase. The triple point appears at the junction of the three curves. The vaporisation curve ends abruptly at the critical point.

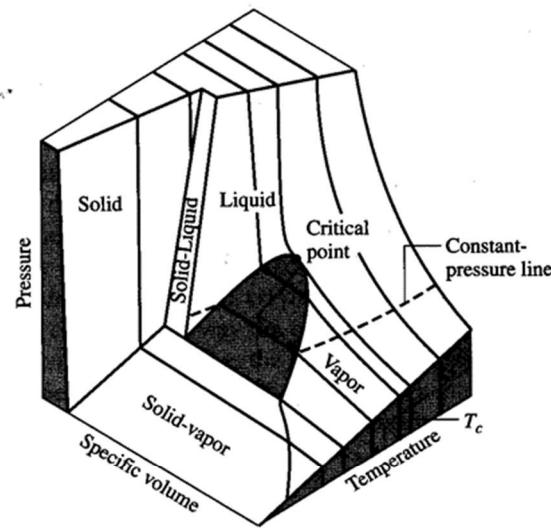
Water (H_2O) is a good example: Consider a system consisting of a unit mass of ice at a temperature below the triple point temperature. Let us begin where the system is at state a shown in the figure below. This point has a pressure greater than the triple point pressure. Suppose the system is slowly heated while maintaining the pressure constant and uniform throughout. The temperature increases with heating until point b . At this point the ice is a saturated solid. Additional heat at a fixed pressure results in the formation of liquid without a change in temperature. For most substance the specific volume increases during melting, but with water the opposite occurs.

Next consider a system initially at a' where the pressure is less than the triple point pressure. In this case as it is heated it passes through the two-phase solid-vapour region into the vapour region along $a'-b'-c'$. The case of vaporisation by heating is shown by line $a''-b''-c''$.

4.10 Expansion and contraction on freezing



Water
(expands on freezing)



Most other substances
(contract on freezing)

4.11 Triple point

<https://www.youtube.com/watch?v=BLRqpJN9zeA>

(0:23 - boiling, 1:05 – Triple point)

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Non-ideal gases

At the pressures and temperatures that usually occur in engineering applications many substances are found to lie in a state which is to the right of the critical isotherm (the constant temperature line which runs through the critical point). In this section we will restrict ourselves to two regions in this area: (1) the region where the temperature is in excess of twice absolute critical temperature; and (2) the region of low pressure where the pressure is one atmosphere or less. The so-called ‘permanent’ gases oxygen, hydrogen, nitrogen etc., all lay in the first region at ordinary working temperatures (the critical temperature of the specified gases being 154.8k, 33.99k and 126.2k). Water vapour at atmosphere pressure lies in the second and is in fact super-heated steam at low pressure. The behaviour of substances in these two regions approximate very closely to those of a *ideal gas*. An ideal gas is a particularly useful concept as its properties are related in a very simple way.

From kinetic theory, the conditions for a gas to be ideal are

- The molecules are rigid, and no momentum or time is lost during collisions.
- The volume occupied by the molecules is negligible compared to the total volume.
- The forces between adjacent molecules are negligible.

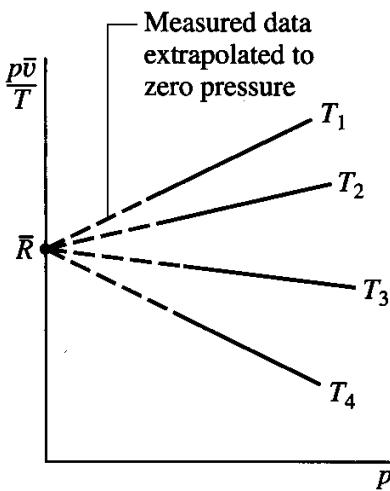
Condition (a) is satisfied for all gases, or else the pressure of a gas contained in a constant volume closed vessel would fall with time.

Condition (b) is satisfied at low pressures, since the molecules are far apart. It is less accurate at high pressures when the molecules are closer together.

Condition (c) is also satisfied at low pressures and also at high temperatures, when the molecules are moving faster and so spend less time in close proximity to each other. It is less accurate at high pressures and/or low temperatures.

As pressure is raised and/or temperature is dropped, conditions (b) and (c) are not strictly true, and the behaviour of the real gas differs from that of an ideal gas.

Let a gas be confined in a piston and the entire assembly be maintained at a constant temperature. The piston can be moved to various positions so that a series of equilibrium states at constant temperature can be obtained. The ratio $p\bar{v}/T$, where \bar{v} is the volume per mole, of the gas in the cylinder is plotted against pressure below.



The lines for several temperatures are plotted. When the ratios are extrapolated to zero pressure, precisely the same limiting value is obtained for each curve.

$$\lim_{p \rightarrow 0} \frac{p\bar{v}}{T} = \bar{R}$$

Where \bar{R} denotes the common limit for all temperatures. If this was repeated for all gases at all temperatures the limit would be \bar{R} in all cases. \bar{R} is called the ***universal gas constant***. Its value as determined experimentally is

$$\bar{R} = 8.314 \text{ kJ/kmol} \cdot \text{K}$$

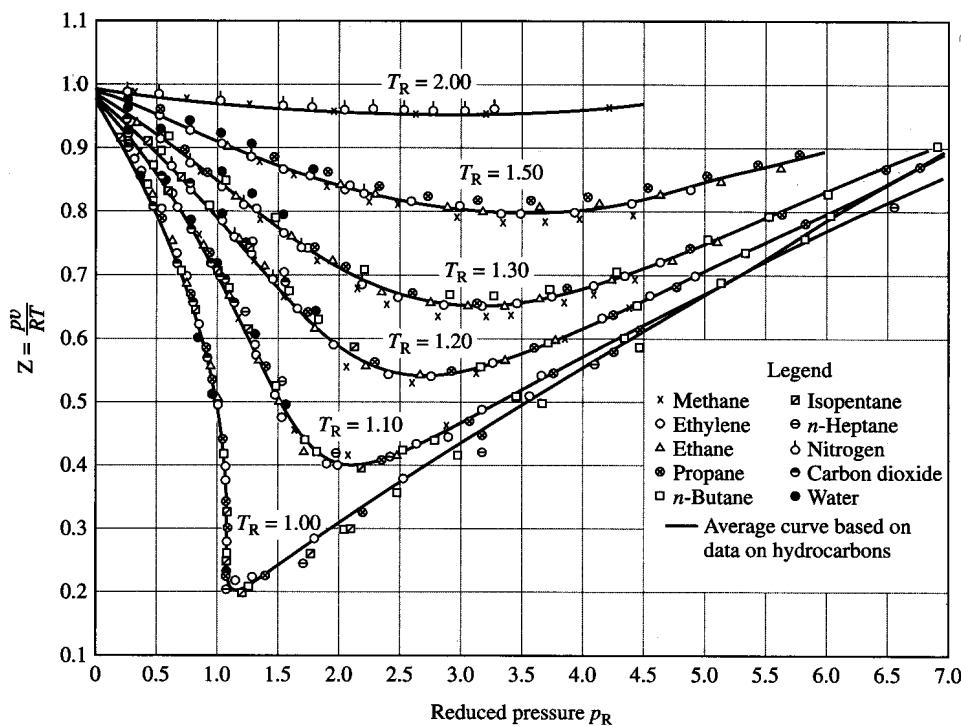
The ***compressibility factor***, Z , of a gas is denoted as

$$Z = \frac{\bar{p}\bar{v}}{\bar{R}\bar{T}} \text{ or } Z = \frac{p\bar{v}}{RT}$$

As pressure tends to zero Z tends to 1 and the ideal gas equation is obtained. As the pressure is raised two molecule and then three molecule interaction becomes more significant and Z diverges from 1. The behaviour of all gases are qualitatively similar. The figure below shows Z plotted against ***reduced pressure***, P_R , for a number of ***reduced temperatures***, T_R , for a range of substances. Where

$$P_R = \frac{P}{P_c} \text{ and } T_R = \frac{T}{T_c}$$

And T_c is the critical temperature and P_c is the critical pressure.



Generalised compressibility chart for various gases

The graph can be used to show that the behaviour of all real gases are the same but in practice measured p - v - T surfaces should be used for accuracy.

Another equation for describing the behaviour of a real gas is van der Waals equation of state.

$$\left(p + \frac{a}{v^2} \right) (v - b) = RT$$

where a and b are constants. This equations is determined by allowing for the attractive force between molecules, a/v^2 term, and the volume occupied by them, b .

Lecture 5: Working fluids II

5. Working fluids II

5.1 Introduction

1. Thermodynamic property data can be retrieved in various ways, including tables, graphs, equations and computer software.
2. The emphasis of the present lecture is the use of tables and graphs.
3. These are commonly available for substances of engineering interest.
4. The lecture explains how the tables and graphs in the CUED Thermofluids Data Book should be used.

5.2 Dryness fraction

1. A substance in the two-phase liquid-vapour region consists of both liquid and vapour coexisting in equilibrium.
2. x is called the *dryness fraction* of the mixture.
3. Each 1 kg of mixture contains x kg of vapour and $(1-x)$ kg of liquid.

(f =fluid,liquid, g =gas)

$$m = m_f + m_g$$

The volume of the mixture is then

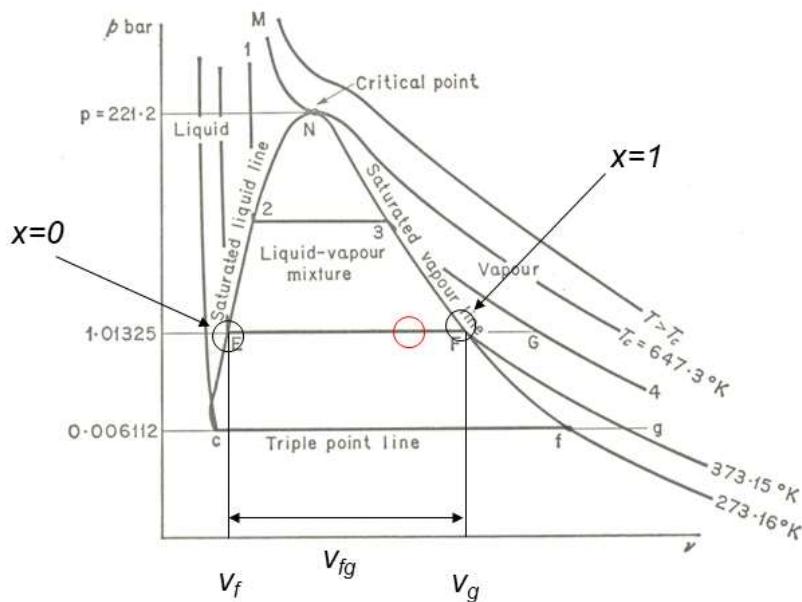
$$V = m_f v_f + m_g v_g$$

Hence the specific volume of the mixture is

$$v = \frac{V}{m} = \frac{m_f v_f + m_g v_g}{m_f + m_g} = (1-x)v_f + x v_g = v_f + x(v_g - v_f) = v_f + x v_{fg}$$

Thus the dryness fraction provides a measure of how far the phase change has proceeded.

2



$$v = \frac{V}{m} = (1-x)v_f + x v_g = v_f + x v_{fg}$$

The above 'lever rule' applies to all intensive properties (v , u , h , s , etc.).

3

5.3 Tabulated properties

1. There are many ways in which to present thermodynamic data for substances (**tables**, **graphs**, computer programs).
2. CUED Thermofluid's data book provides a good (and useful!) example.
3. The main tables for this course are concerned with water and steam, since H_2O remains a major working fluid for many engineering applications.
4. The Thermofluids Data Book 2008 Edition was issued to you this year and this is the version which will be used.

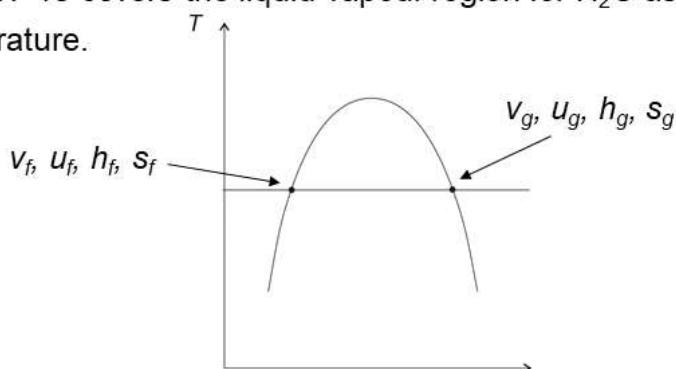
5.3.1 Tables for perfect and semi-perfect gases

1. Page 12 provides basic data for perfect gases.
2. Page 13 deals with semi-perfect gases. At low p and over the range of T listed (200K to 3000K) the gases listed are semi-perfect. This means c_p and c_v varies and the h can be looked up.

4

5.3.2 Tables for water and steam

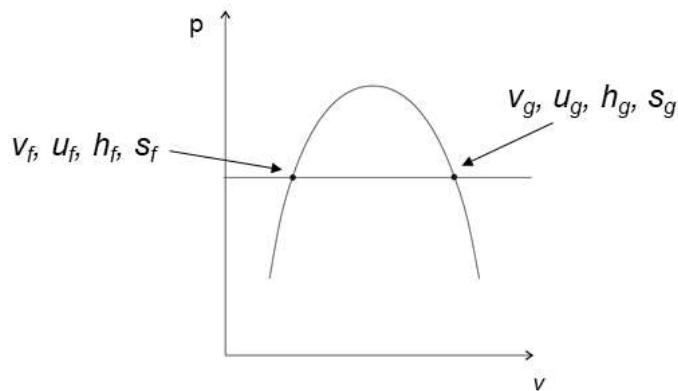
1. Page 16 contains data for the triple point and the critical point for
2. Page 17-18 covers the liquid-vapour region for H_2O as function of temperature.



Temp. °C <i>T</i>	Pressure bar <i>p</i>	Specific volume m^3/kg		Spec. int. energy kJ/kg		Specific enthalpy kJ/kg		Specific entropy kJ/kg K		Temp. °C <i>T</i>
		<i>v_f</i>	<i>v_g</i>	<i>u_f</i>	<i>u_g</i>	<i>h_f</i>	<i>h_{fg}</i>	<i>h_g</i>	<i>s_f</i>	
0.01	0.00611	0.001000	206.005	0.0	2375.0	0.0	2500.9	2500.9	0.000	9.156
2	0.00706	0.001000	179.776	8.4	2377.6	8.4	2496.2	2504.6	0.031	9.103
4	0.00814	0.001000	157.135	16.8	2380.4	16.8	2491.4	2508.2	0.061	9.051
6	0.00935	0.001000	137.652	25.2	2383.1	25.2	2486.7	2511.9	0.091	8.999
8	0.01073	0.001000	120.846	33.6	2385.9	33.6	2481.9	2515.6	0.121	8.949

5

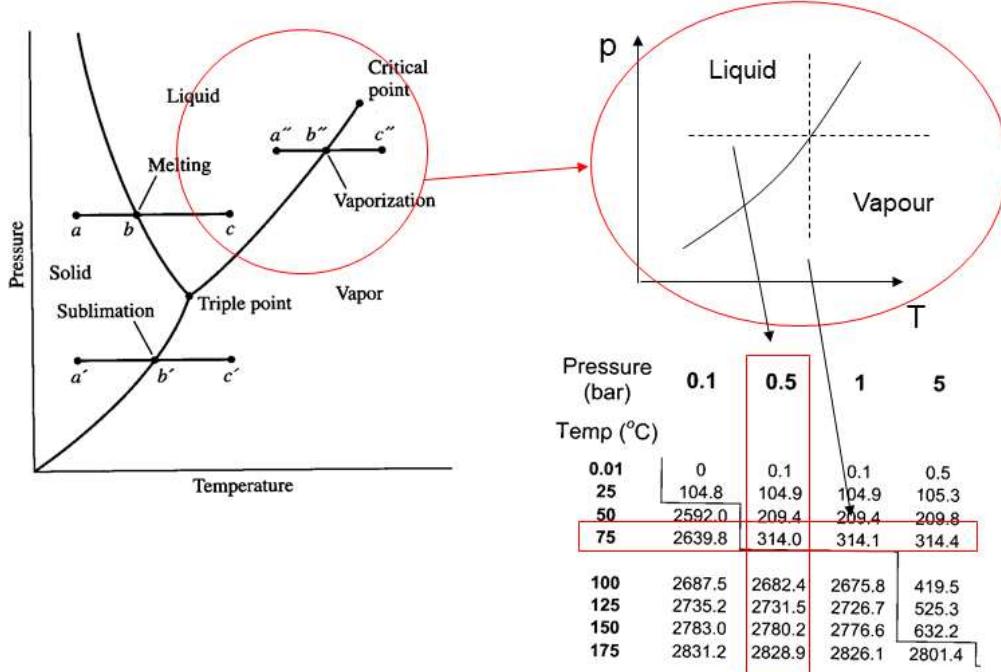
3. Page 19-22 covers the liquid-vapour region for H_2O as function of pressure.



Pressure bar <i>p</i>	Temp. °C <i>T</i>	Specific volume m³/kg <i>v_f</i>	Spec. int. energy kJ/kg <i>u_f</i>	Spec. int. energy kJ/kg <i>u_g</i>	Specific enthalpy kJ/kg <i>h_f</i>	Specific enthalpy kJ/kg <i>h_{fg}</i>	Specific enthalpy kJ/kg <i>h_g</i>	Specific entropy kJ/kg K <i>s_f</i>	Specific entropy kJ/kg K <i>s_g</i>	Pressure bar <i>p</i>
1.0	99.61	0.001043	1.6941	417.4	2505.5	417.5	2257.4	2674.9	1.303	7.359
1.5	111.35	0.001053	1.1594	467.0	2519.2	467.1	2226.0	2693.1	1.434	7.223
2.0	120.21	0.001061	0.8858	504.5	2529.1	504.7	2201.5	2706.2	1.530	7.127
2.5	127.41	0.001067	0.7187	535.1	2536.8	535.3	2181.1	2716.5	1.607	7.053
3.0	133.52	0.001073	0.6058	561.1	2543.1	561.4	2163.5	2724.9	1.672	6.992

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4. Page 23-26 covers date outside the the liquid-vapour region for H_2O as function of temperature (horizontally) and pressure (vertically).



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5.3.3 Transport properties

Why they are called transport properties is not clear.

The tables tend to contain properties useful in flow problems (such as pipe flow).

Page 27 contains transport properties for H₂O from the triple point to the critical point, v_f , v_g , c_{pf} , c_{pg} , thermal conductivity, dynamic viscosity and Prandtl number.

Page 28 contains (c_p , thermal conductivity, dynamic viscosity and Prandtl number) data for steam at atmospheric pressure.

Page 28 also contains corresponding data for air at atmospheric pressure.

Page 29 contains corresponding data for carbon dioxide at atmospheric pressure.

Page 29 also contains corresponding data for hydrogen at atmospheric pressure.

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5.3.4 Properties of gases and liquids at sea level and conditions

Page 31 contains data on air, carbon dioxide, hydrogen and helium under sea level conditions. Density, dynamic viscosity, kinematic viscosity, speed of sound and thermal conductivity are tabulated.

Page 31 also contains data on mercury and water at sea level.

Density, dynamic viscosity, kinematic viscosity and thermal conductivity are tabulated.

5.3.5 The international Standard Atmosphere

Page 31 gives data for the international standard atmosphere for sea level conditions (mostly repeat of air in table above).

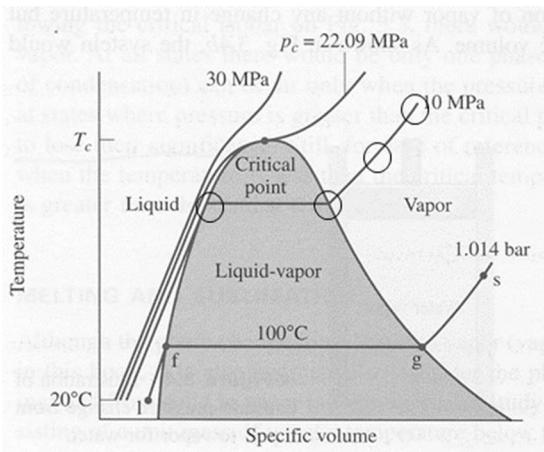
Page 32 for conditions at altitude. Temperature, pressure, density and kinematic viscosity are tabulated.

e.g. how p , T , ρ vary with altitude up to 30,000m.

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5.3.6 Data for Refrigerant HFC-134a (CH₂FCF₃)

Page 37 contains a table of saturation properties for refrigerant HFC-134a. The format is similar to the saturation tables for H₂O, except that data *h* and *s* at superheated conditions (at 20K and at 40K above the saturation temperature) is included in the table.



PROPERTIES TABLE FOR REFRIGERANT R-134a (CH₂FCF₃)

Saturation Temp. °C	Saturation Pressure bar	Saturated						Superheated by				Saturation Temp. °C	
		Specific volume m ³ /kg		Specific enthalpy kJ/kg		Specific entropy kJ/kg K		20K		40K			
		v _f	v _g	h _f	h _g	s _f	s _g	h	s	h	s		
T _{sat}	P _{sat}											T _{sat}	
-45	0.39	0.00070	0.46458	141.9	370.8	0.7687	1.7722	385.8	1.8348	401.3	1.8949	-45	
-40	0.51	0.00071	0.36094	148.1	374.0	0.7956	1.7643	389.2	1.8270	405.0	1.8869	-40	
-35	0.66	0.00071	0.28390	154.4	377.2	0.8221	1.7574	392.7	1.8201	408.6	1.8797	-35	
-30	0.84	0.00072	0.22585	160.8	380.3	0.8483	1.7512	396.1	1.8139	412.3	1.8734	-30	
-25	1.06	0.00073	0.18155	167.2	383.4	0.8743	1.7458	399.5	1.8085	416.0	1.8678	-25	

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Example 1:

Find the *h* of H₂O at 17.5MPa and 455°C?

The phase is not known.

First check the saturation tables for H₂O.

A pressure of 17.5MPa = 175 bar appears on page 22.

175	354.67	0.001804	0.00793	1679.4	2390.7	1711.0	818.5	2529.5	3.840	5.143	175
180	356.99	0.001840	0.00750	1699.1	2374.9	1732.2	777.8	2510.0	3.872	5.106	180
185	359.26	0.001881	0.00709	1719.3	2357.9	1754.1	734.9	2489.0	3.905	5.067	185
190	361.47	0.001926	0.00668	1740.3	2339.1	1776.9	689.2	2466.0	3.940	5.026	190
195	363.63	0.001977	0.00627	1762.3	2318.4	1800.9	639.8	2440.7	3.976	4.981	195

$$T_{saturation} = 354.67^\circ\text{C}.$$

The given *T* is higher, therefore the H₂O is in the vapour phase.

Now go to page 23 for the enthalpy of H₂O.

400	3279.9	3279.3	3278.6	3272.3	3264.5	3248.3	3214.5	3178.2	3139.4	3097.4	2975.7	2816.9
425	3331.8	3331.2	3330.5	3324.9	3317.8	3303.3	3273.2	3241.4	3207.7	3172.0	3072.3	2953.0
450	3384.0	3383.5	3382.8	3377.7	3371.3	3358.2	3331.2	3302.9	3273.3	3242.3	3157.9	3061.7
475	3436.6	3436.2	3435.6	3430.9	3425.1	3413.2	3388.7	3363.4	3337.1	3309.7	3236.6	3155.8

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The answer lies between tabulated values so interpolation is necessary.

The general formula for linear interpolation is

$$y = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

Choosing to interpolate in pressure first (*is this choice important?*) gives

$$h(450) = 3157.9 + \left(\frac{3061.7 - 3157.9}{200 - 150} \right) (175 - 150) = 3109.8 \text{ kJ/kg}$$

$$h(475) = 3236.6 + \left(\frac{3155.8 - 3236.6}{200 - 150} \right) (175 - 150) = 3196.2 \text{ kJ/kg}$$

then interpolating again in temperature gives the required answer

$$h = 3109.8 + \left(\frac{3196.2 - 3109.8}{475 - 450} \right) (455 - 450) = \boxed{3127.1 \text{ kJ/kg}}$$

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Example 2:

A rigid vessel of volume 1m^3 contains 0.7kg of H_2O at a pressure of 1 bar. Calculate the fraction by mass of vapour and the specific enthalpy of the mixture.

The total volume

$$V = m_f v_f + m_g v_g$$

and the total mass is

$$m = m_f + m_g$$

Thus the specific volume of the mixture is

$$v = \frac{V}{m} = \frac{m_f v_f + m_g v_g}{m_f + m_g} = (1 - x)v_f + xv_g$$

so that the dryness fraction x is

$$x = \frac{v - v_f}{v_g - v_f} = \frac{1/0.7 - 0.001043}{1.6941 - 0.001043} = 0.843$$

The specific enthalpy is then

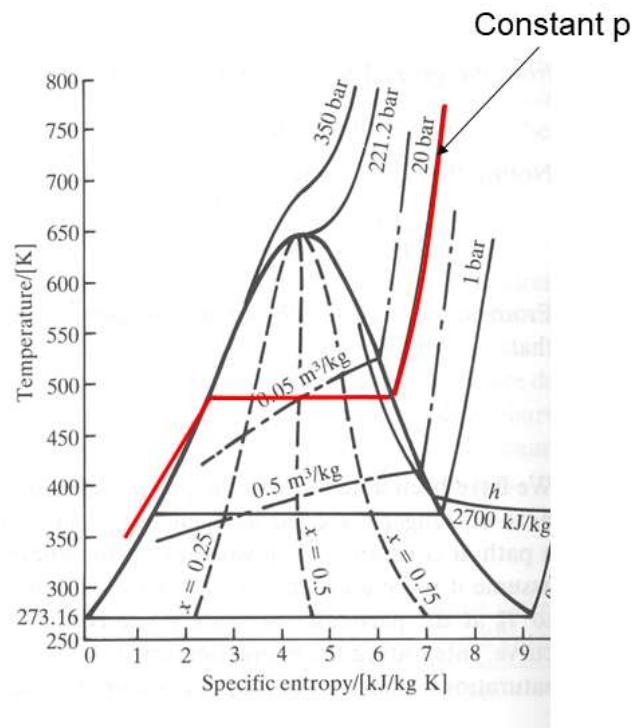
$$h = (1 - x)h_f + xh_g = h_f + xh_{fg} = 417.5 + 0.843 \times 2257.4 = \boxed{2320 \text{ kJ/kg.}}$$

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5.4 Graphical representation

5.4.1 T-s diagram

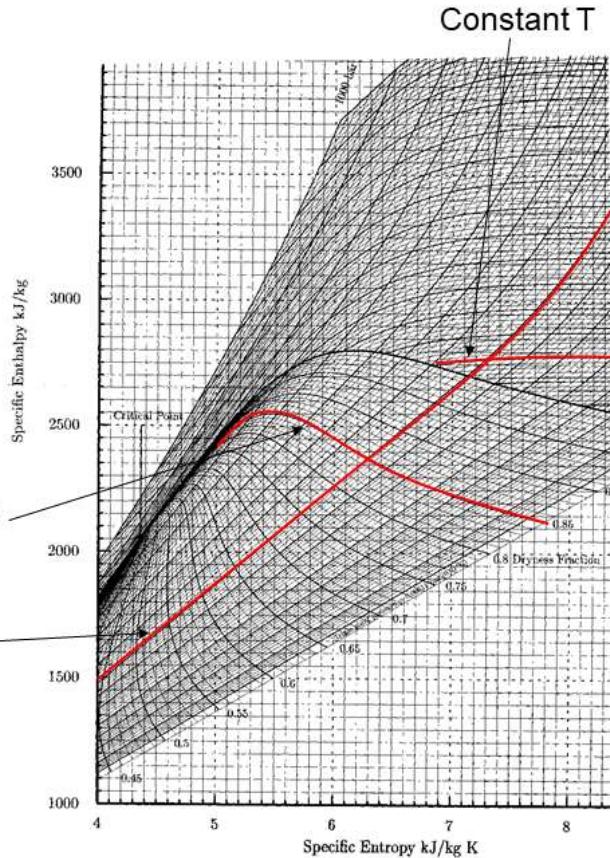
1. Most commonly sketched graph for cycles.
2. Good for comparing to Carnot cycle.



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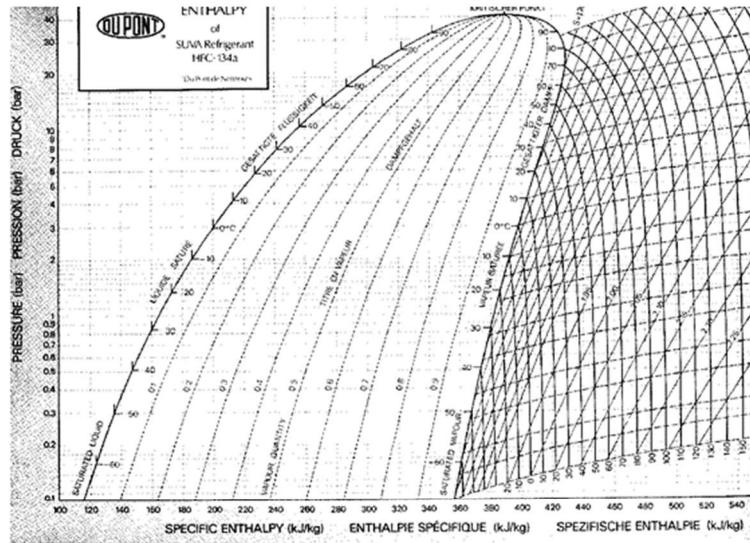
5.4.2 h-s diagram

1. Easiest graph to solve cycle problems on.
2. Much quicker than using tables but you need to be able to use both.



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5.4.3 p-h diagram



1. Used for refrigerant calculations.
2. Not as easy to use. Sometimes have to used tables.

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