Modelling Green Energy Conversion Networks for Generating Hydrogen Electric Vehicle Fuel

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What are we modelling?

The goal is to turn solar energy into hydrogen fuel to be used in HEVs. Here is a simple linear system for this:

$$\underset{\bullet}{\text{Sun}} \xrightarrow[\text{PV cells}]{} \underset{\bullet}{\text{Battery}} \xrightarrow[\text{Electrolyser, Compressor}]{} \underset{\bullet}{H_2 \text{ fuelling station}}$$

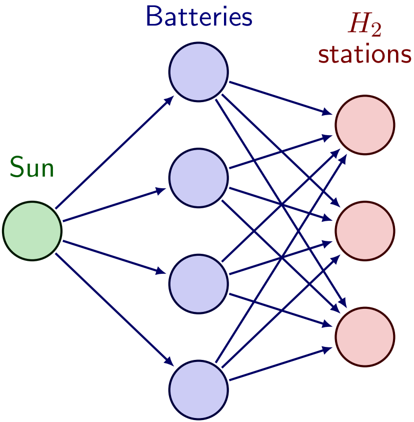
Observe:

- The Sun, battery and H_2 fuelling station are energy stores.
- The PV cells and electrolyser are energy converters.

We can abstract this into a graph with:

- **Vertices** representing energy stores.
- **Edges** representing energy converters.

A more complex example



If this sort of generalised, extensible and extendable approach is consistently taken, it is more useful for future investigation.

Theory

Begin with some definitions:

- $\mathcal{V}$ is the set of vertices mapped to energy stores
- $\mathcal{E}$ is the set of edges mapped to energy converters
- $G \triangleq (\mathcal{V}, \mathcal{E}) \ni \mathcal{E} \subseteq \left\{(x, y) : (x, y) \in \mathcal{V}^2 \text{ and } x \neq y\right\}$
- N is the cardinality of $\mathcal{V}$.
- $\underline{\underline{A_w}} \ni \forall v_i \in \mathcal{V}, \nexists j : \left[\sum_{k=1}^{N} \left(\underline{\underline{A_w}} + \underline{\underline{A_w}}^\top\right)^k\right]_{ij} = 0$ is G's weighted adjacency matrix.

Hence the graph G is defined to be simple, directed and weakly connected. Normalised edge weights $\in [0, 1]$ are used to proportion power transfer ratios.

Non-simple networks can be modelled by edge subdivisions or adjusting edge weights.

Ideal behaviour

Consider a single vertex with energy value u_i.

$$\xrightarrow[\text{Net power in}]{\qquad\qquad} \overset{u_i}{\bullet} \xrightarrow[\text{Net power out}]{\qquad\qquad}$$

Ignoring all constraints causes immediate energy propagation to and from all neighbours.

During time-step Δt:

$$\Delta u_i = -\overbrace{\sum_{j=1}^{N} \underline{\underline{A_w}}_{ij} u_i}^{\text{energy out}} + \underbrace{\sum_{j=1}^{N} \underline{\underline{A_w}}_{ji} u_j}_{\text{energy in}}$$

$\forall \underline{\underline{A_w}}, \forall \underline{u} : \sum_{i=1}^{N} \Delta u_i = 0 \therefore$ this operator conserves energy.

Non-ideal behaviour

We have the following non-idealities:

- **Vertex maximum capacity**: $u_{i_{max}} \ni 0 \leq u_i \leq u_{i_{max}}$.
- **Edge maximum power transfer**: $\underline{\underline{P}}_{ij} \ni \underline{\underline{A_w}}_{ij} u_i \leq \underline{\underline{P}}_{ij} \Delta t$ where $\underline{\underline{P}}_{ij}$ is the maximum power transfer from i to j.
- **Vertex self-discharge**: loss of energy stored over time.
- **Edge process inefficiency**: losses during power conversion.

Ignore the last two for now. We can then apply the remaining constraints to the previous equation:

$$\Delta u_i = \min \left(u_{i_{\max}} - u_i, \begin{array}{c} -\sum_{j=1}^{N} \min(\underline{\underline{A_w}}_{ij} u_i, \underline{\underline{P}}_{ij} \Delta t) \\ +\sum_{j=1}^{N} \min(\underline{\underline{A_w}}_{ji} u_j, \underline{\underline{P}}_{ji} \Delta t) \end{array} \right)$$

What about power losses?

Consider an arbitrary energy conversion p_{xy} with efficiency η_p.

$$\begin{tikzcd} x \arrow[r, "p_{xy},\ \eta_p=1"] & y \end{tikzcd}$$

If p is not, in fact, ideal, then we can model this by partitioning some power off into an energy wastage sink. Let's suppose it is 70% efficient:

$$\begin{tikzcd} x \arrow[r, "p_{xy},\ \eta_p=70\%"] \arrow[d, "30\%"] & y \\ \text{energy sink} & \end{tikzcd}$$

To handle self-discharge losses, increase the weight of the edges connecting vertices to the sink.

Demo: a simple linear system

Studying the simple linear system from earlier to demonstrate the model:

$$\text{Sun} \xrightarrow[\text{PV cells}]{} \text{Battery} \xrightarrow[\text{Electrolyser, Compressor}]{} H_2 \text{ fuelling station}$$

Reasonable assumptions about component behaviour and economics and solar power input over the course of a representative year were used.

Geography-dependent data was sampled for Cyprus.

Results: minimum budget to support HEVs

L-BFGS-B optimisation was used to minimise the total budget required to sustain the system:

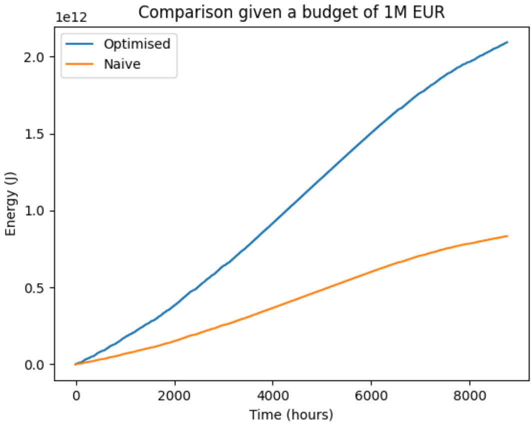
# HEVs	Min. budget (€)	Solution η	PV capacity (kW)
1	70.8k	0.55	4.6
10	715k	0.55	46
100	6800k	0.55	460

It was not possible to simulate 1000 HEVs due to constraints from the PV data API.

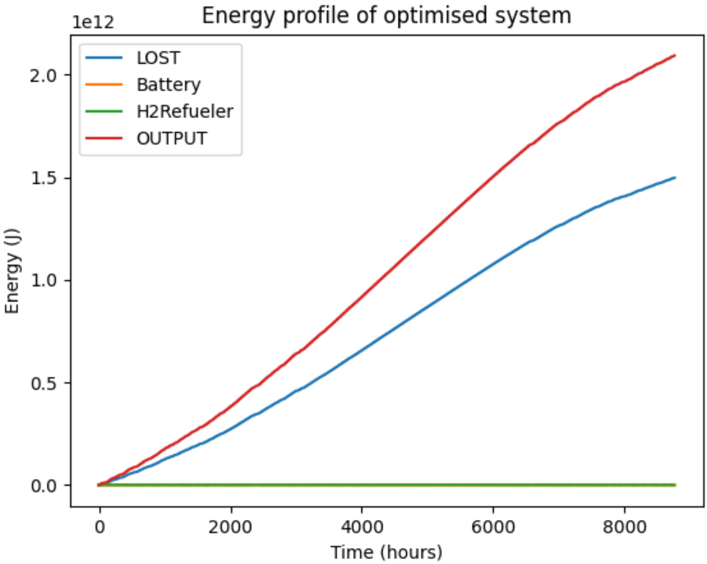
Results: optimising given budget

For €1M, use **L-BFGS-B optimisation** to find the optimal budget allocation that maximises the number of HEVs supported:

PV (€)	Battery (€)	Electrolyser (€)	H_2 station (€)
645k	244k	107k	3.90k



Results: optimising given budget (cont.)



Future potential

- **More accurate modelling** of component behaviour, e.g. using characterisations dependent on more parameters.
- **Expand components into sub-networks** to give a more detailed analysis.
- **Investigate larger networks**, e.g. a potential national PV-HEV grid.