UNIVERSITY OF CAMBRIDGE

# IIA SB4 – Fundamentals of Integrated Photonics

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**Centre for Photonic Systems   Engineering Department   Electrical Engineering Division**

# Outline

1. Background Introduction
2. Fundamentals of Integrated Photonics
3. Derivation of Optical Waveguide Modes
4. Coupling Between Waveguides
5. Mode Simulation and Case Analysis

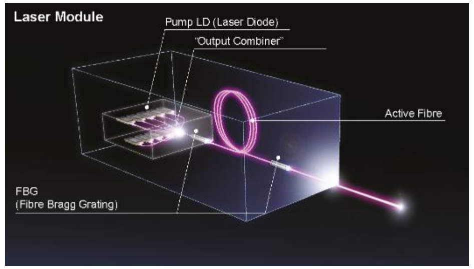
# Development of optical communication

## Theoretical Foundation

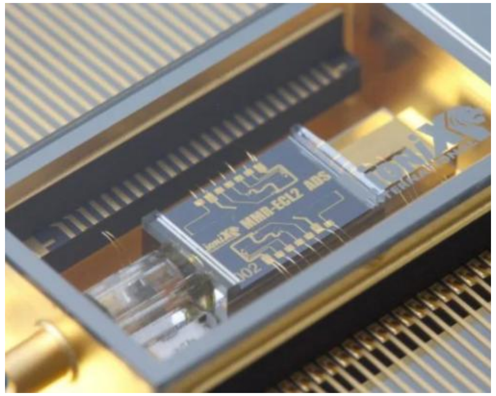
- In 1966, **Charles K. Kao** (Nobel Prize in Physics 2009) proposed the concept that **light could be guided in dielectric fibers** — paving the way for modern optical communication.

## Key Enabling Technologies

- **Lasers**: Invention of the III-V laser (first in 1960) and subsequent rapid development provided coherent light sources for optical links.
- **Low-loss optical fibers**: Key breakthroughs in reducing attenuation (1970s) made long-distance fiber transmission feasible.
- **Dense Wavelength Division Multiplexing (DWDM)**: Enables multiple channels to transmit simultaneously through a single fiber, allowing massive bandwidth scaling.
- **Optical Amplifiers (e.g., Erbium-Doped Fiber Amplifiers, EDFAs)**: Boost optical signals directly without electrical conversion, enabling long-haul transmission across global networks.




# From Optical Fibers to Integrated Photonics

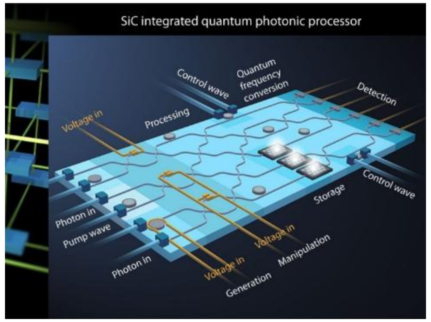


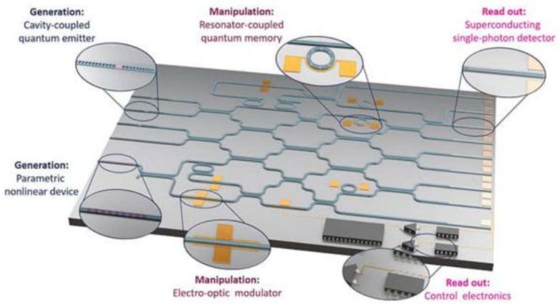
## Rise of Silicon Photonics:

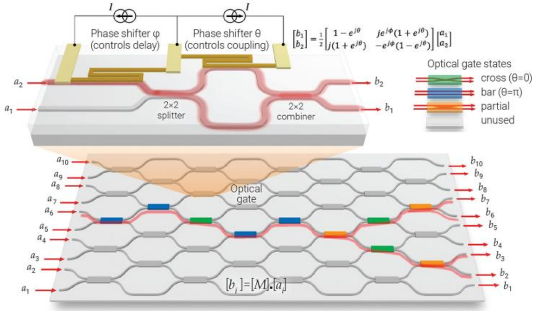
- With advances in nanofabrication technologies, the functionalities of traditional fiber-based systems can now be integrated onto a **single photonic chip**.
- This marks a paradigm shift from bulky **fiber-based optics** to **on-chip integration**, enabling scalable, stable, and manufacturable photonic systems.

## Key features:

- Integrated Photonic Circuits (PICs) manipulate light on a chip, similar to how electronic ICs control electrons.
- Fabricated via CMOS-compatible techniques (e.g., DUV lithography) on various material platforms, such as Si, SiN, and InP,
- Typical components include waveguides, couplers, resonators, lasers, modulators, detectors...




*Compact, low-cost, large-bandwidth, power-efficient, and etc.*

# Underlying Driving force — Rapid Growth of Data Traffic

Global internet traffic is **growing exponentially**, driven by:

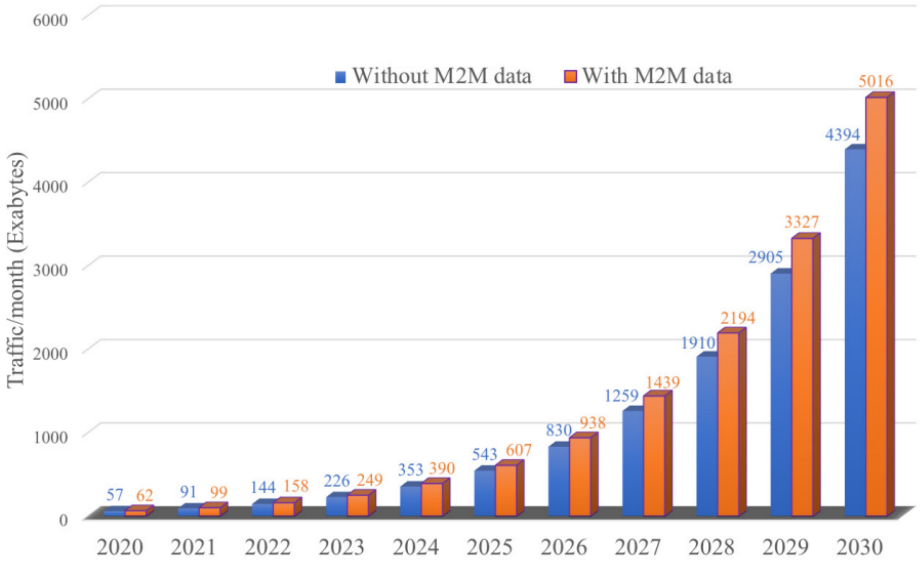
- Mobile internet, streaming video and real-time communications
- AI model training and edge computing
- Cloud services and IoT devices

This impose great performance pressure on data centers:

- Ultra-high bandwidth of >1 Tbps
- Low communication latency <1 µs
- Target energy consumption is <1 pJ/bit

**We Live in a World of Data**


**Data Needs to be Accessible Always and in Real-Time**



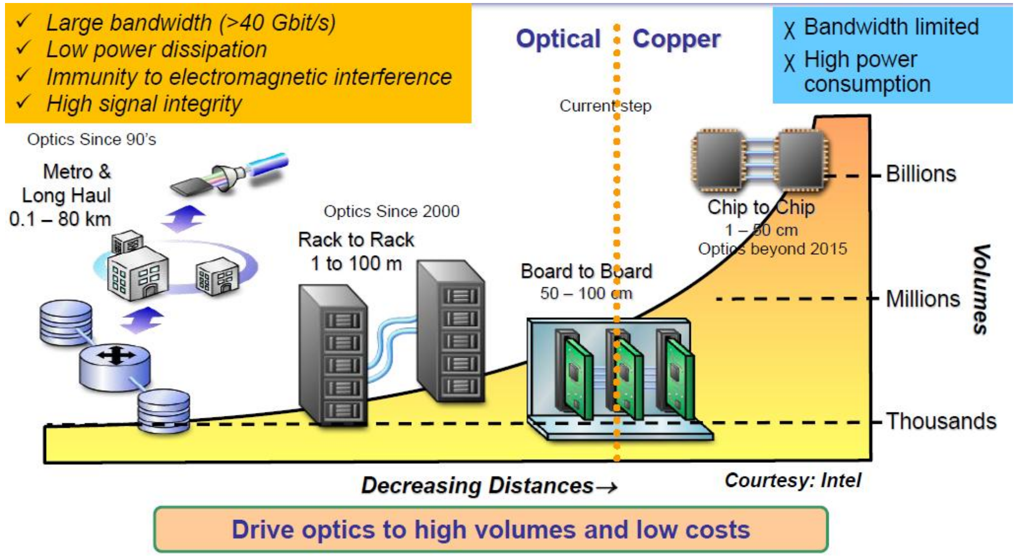
# Electrical vs Optical Interconnects in Data Centers

Yet, electrical links (e.g., copper wires, electronic chips) face:

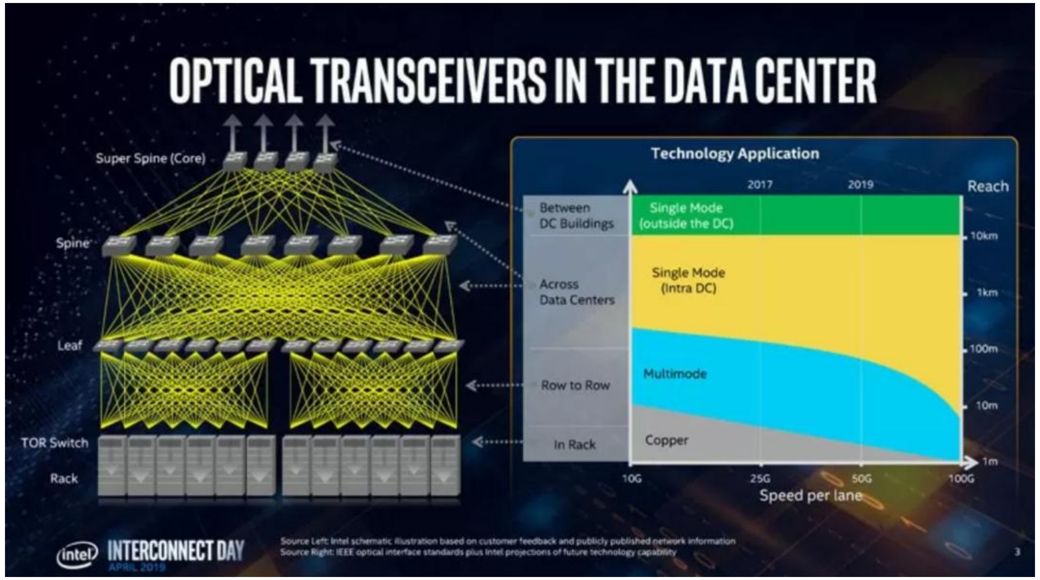
- Bandwidth limitations due to RC delay and skin effect
- High power consumption and complex heat management
- EMI (electromagnetic interference) sensitivity

In contrast, optical interconnects offer:

- Much larger bandwidth (via multiplexing technologies)
- Much lower power consumption
- Low latency, with immunity to EMI




Optics has progressively eliminated copper in the metro and long haul network in the last 20 years and will continue its migration all the way to chip to chip application in the next 10 years.

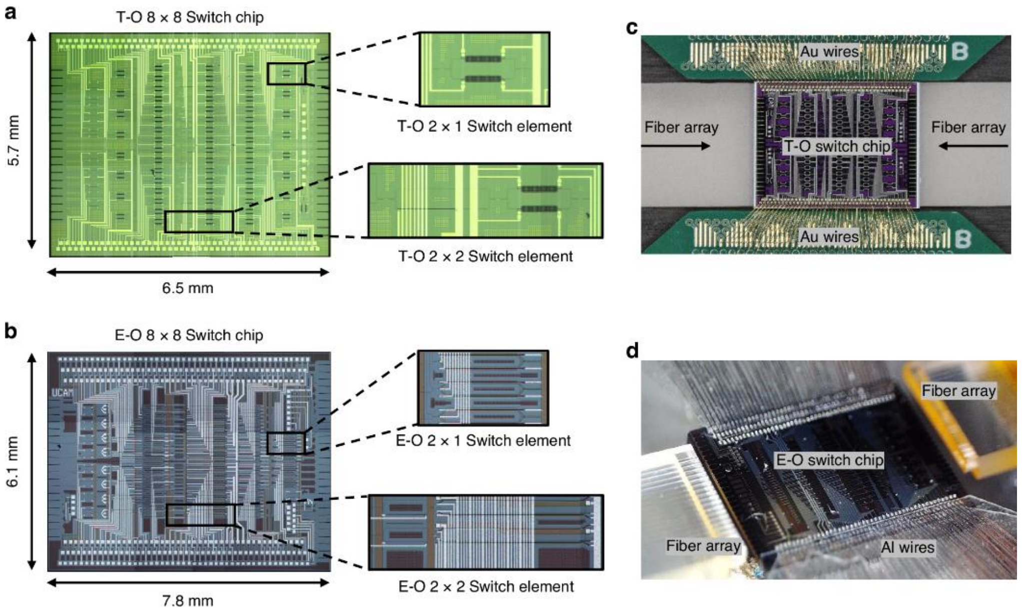


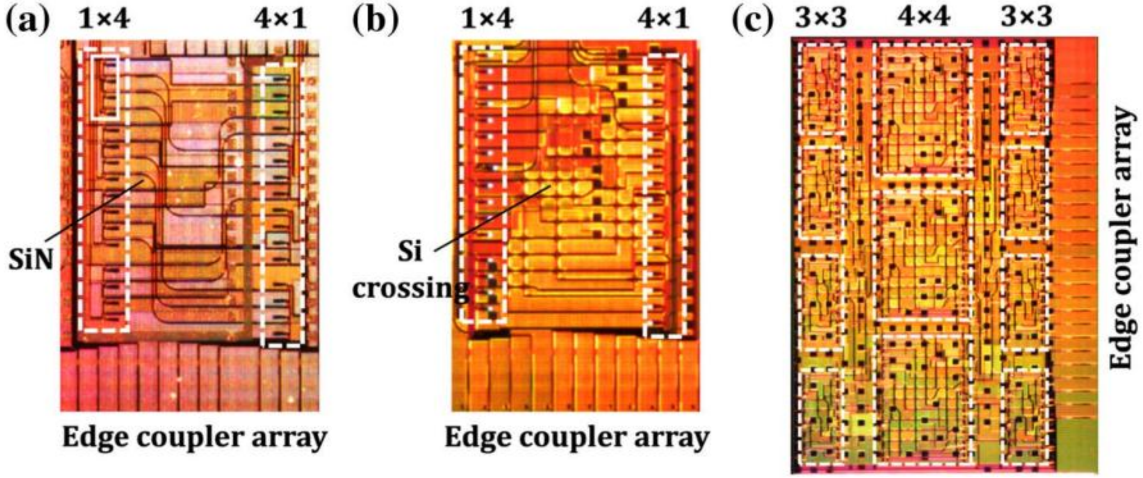

Photonic interconnects increasingly replacing copper wiring in interconnects

# Integrated Photonics for Optical Interconnect/Switching

Integrated optical switches empowering next-generation optical interconnection/switching:

- Enable compact, scalable optical interconnects and on-chip switching
- Supports high-throughput, low-latency communication for AI and HPC systems
- Largely adopted by leading companies, such as Google, Meta, Amazon, Alibaba Cloud…



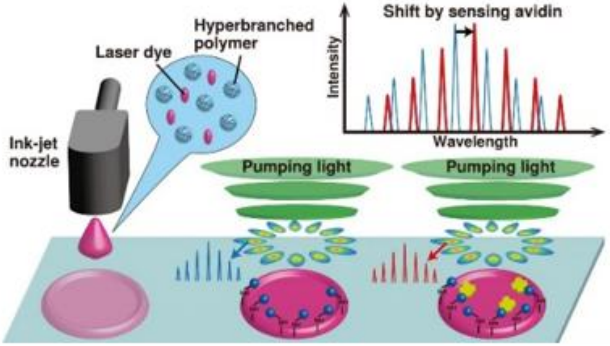



# Broad Application Perspectives

## Optical Interconnect

- Data centers and supercomputers
- Optical I/O and all-optical transmission
- Chip-to-chip optical communication
- 5G/6G fronthaul optical networks
- All-optical metro and access networks

## Optical sensing

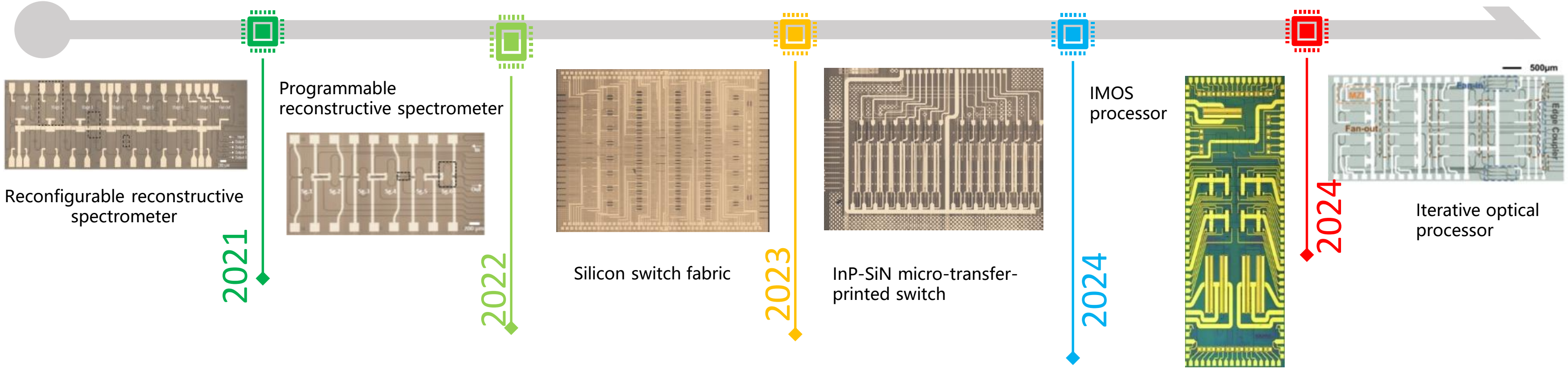


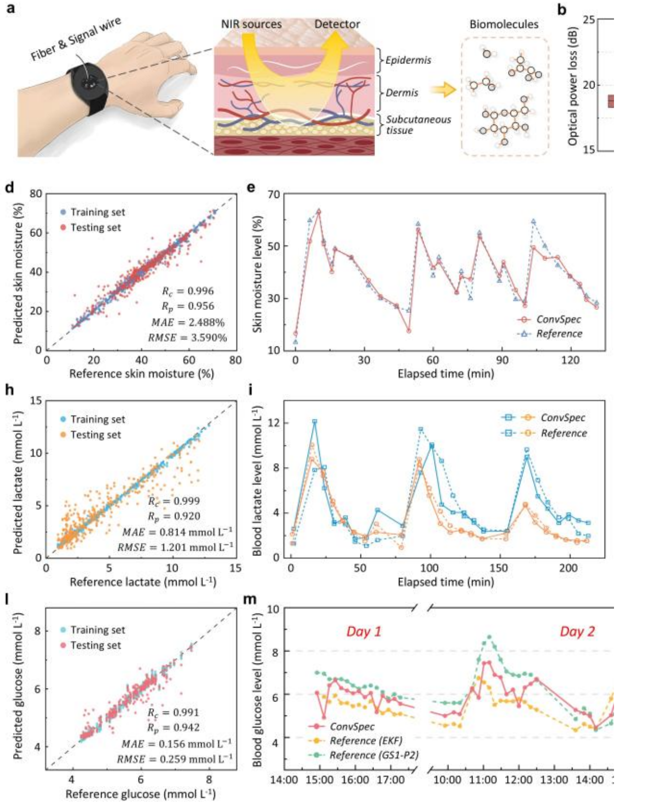

- Integrated spectroemter for biomedical, indusctral spectrosopic applications
- Portable/wearable sensing devices
- Hyperspectral imaging
- Optical coherence tomography (OCT)

## Optical computing

- AI acceleration photonic chips
- 5G/6G BBU signal processing
- Quantum computing processors
- Linear optical computing chips
- Reconfigurable photonic processors

# Recent Progresses




nature communications

Article

https://doi.org/10.1038/s41467-025-5

**Asymmetrical estimator for training encapsulated deep photonic neural networks**

nature photonics

Article

https://doi.org/10.1038/s41566-023-

**High-bandwidth perovskite photonic sources on silicon**

nature communications

Article

https://doi.org/10.1038/s41467-024-50302-3

**I/O-efficient iterative matrix inversion with photonic integrated circuits**

nature communications

Article

https://doi.org/10.1038/s41467-024-54708-x

**Chip-scale sensor for spectroscopic metrology**

nature photonics

Article

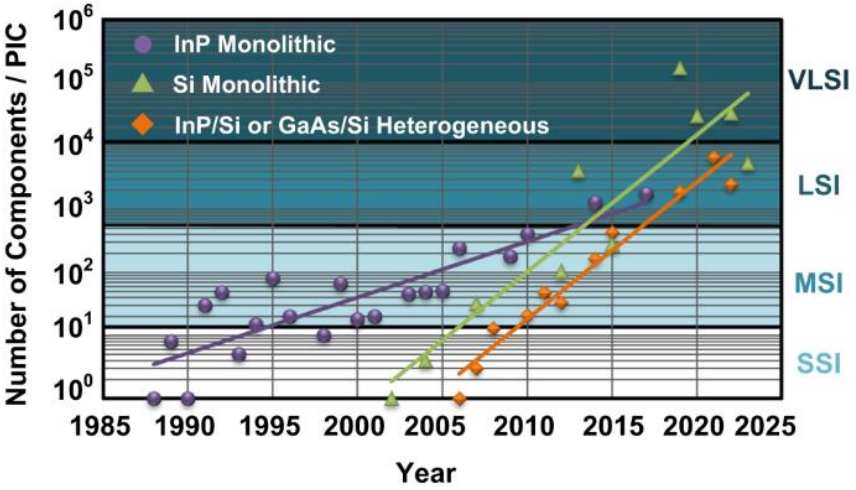
https://doi.org/10.1038/s41566-023-01244-7

**Massively scalable Kerr comb-driven silicon photonic link**

# Advances in Photonic Integration Technologies

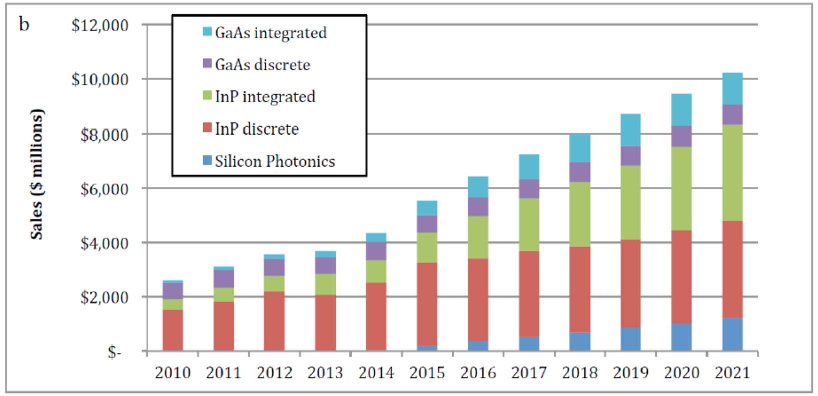
## Technology Trends

- **Rapid scaling in photonic integration**: From a few components per chip in the 1990s to **>$10^5$ components per PIC** today (approaching VLSI level)
- **Transition from monolithic to heterogeneous integration**: Combining **Si, SiN, and, InP** on a single chip platform via wafer-scale processes
- Leveraging **CMOS-compatible lithography (193nm DUV)** and **wafer-level packaging**




## Market Growth

- Strong year-over-year growth in the optical transceiver market → Dominated by **Silicon Photonics** and **InP integrated platforms**




*Source: LightCounting*

## Current Challenges

- Lack of industry-wide standardization for PIC design (no "Moore's Law" equivalent)
- Absence of low-cost on-chip light sources (especially integrated lasers on Si platforms)
- Difficulties in precise alignment between optical and electronic domains

# SB4: Weekly Plan

**Weekly Plan:**

**Week 1:** Introduce fundamental photonics theory and basic waveguide simulation with Lumerical software, focusing on understanding and setting up simple models.
**Week 2:** Dive into eigenmode analysis in silicon waveguides using both theoretical calculations and Lumerical FDTD. Design and simulate a single-mode waveguide, observe the mode propagation, and compare it with the calculated single-mode condition.
**Week 3:** Design and simulate waveguide photonic components — i.e., a directional coupler. Calculate their key structural parameters using theoretical models and simulate both components with Lumerical FDTD.
**Week 4:** Validate simulation results by comparing them with theoretical and numerical models, optimize their performance, ensuring accuracy and reliability of the simulations.

**Coursework:**

| Coursework | Due date | Marks |
|---|---|---|
| Interim report 1 | 23rd of May | 20 |
| Interim report 2 | 3rd of June | 20 |
| Final summary report | 12th of June | 40 |

* 100% of the marks will be given based on individual performance.

# SB4: Schedule

**Compulsory sessions:**

| Week | Day | Date | Time | Location |
|---|---|---|---|---|
| 1 | Friday | 16th of May | 9:00 am – 11:00 am | CGC Seminar room |
| 1 | Tuesday | 20th of May | 11:00 am – 1:00 pm | CGC Seminar room |
| 2 | Friday | 23rd of May | 9:00 am – 11:00 am | **EED Seminar room** |
| 2 | Tuesday | 27th of May | 11:00 am – 1:00 pm | CGC Seminar room |
| 3 | Friday | 30th of May | 9:00 am – 11:00 am | **EED Seminar room** |
| 3 | Tuesday | 3rd of June | 11:00 am – 1:00 pm | CGC Seminar room |
| 4 | Friday | 6th of June | 9:00 am – 11:00 am | CGC Seminar room |
| 4 | Tuesday | 10th of June | 11:00 am – 1:00 pm | CGC Seminar room |

* Not having optional sessions as we have no need to stay in the lab and you can do your simulations anywhere

# Fabrication Processes of Integrated Photonic Chips

Using a passive **Silicon-on-Isolator (SOI) platform** as example:

**1. Typical SOI wafer stack:**

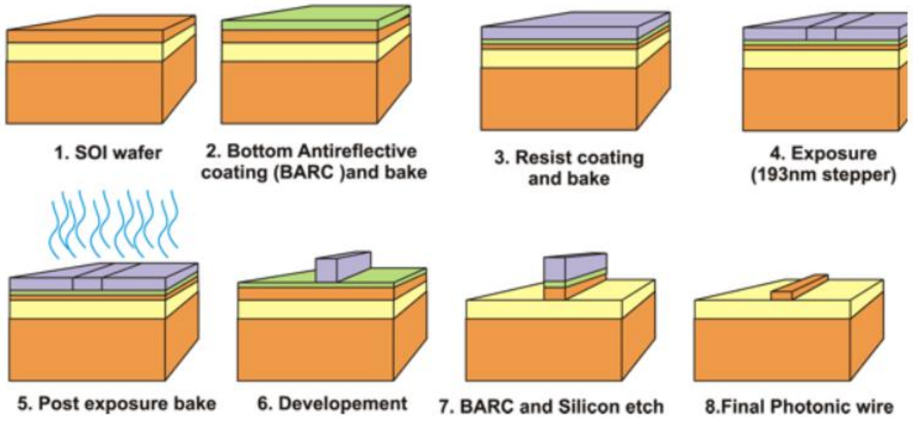
- Top: 220 nm **Silicon** layer (waveguide layer)
- Middle: 2 µm **Silicon dioxide** (buried oxide, BOX) layer
- Bottom: **Silicon** substrate

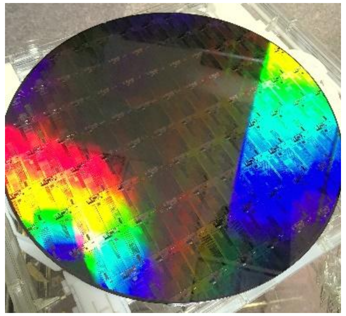
**2. Lithography & Etching:**

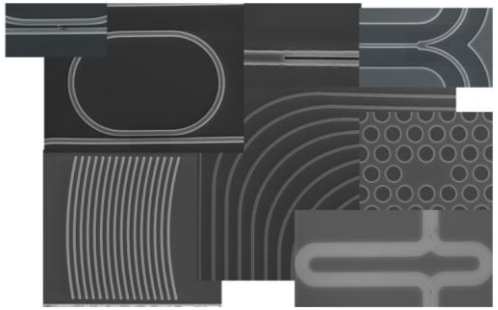
- Spin-coat photoresist and apply anti-reflective coating
- Use deep UV (DUV) exposure to transfer patterns
- Develop resist and perform reactive ion etching to define structures

**3. Post-processing & Packaging**

- Deposit upper cladding for protection (typically $SiO_2$ or polymer)
- Add metal pads or electrothermal heaters for electrical control
- Dice the chip, and polish facets or fabricate grating couplers for fiber interface

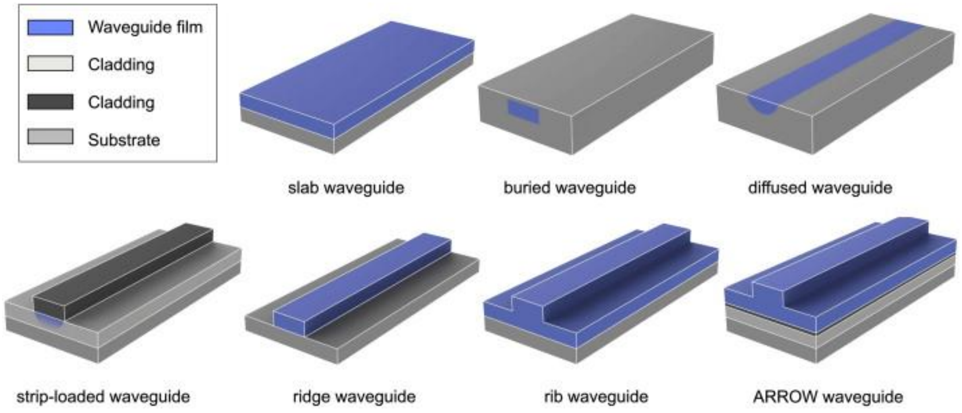




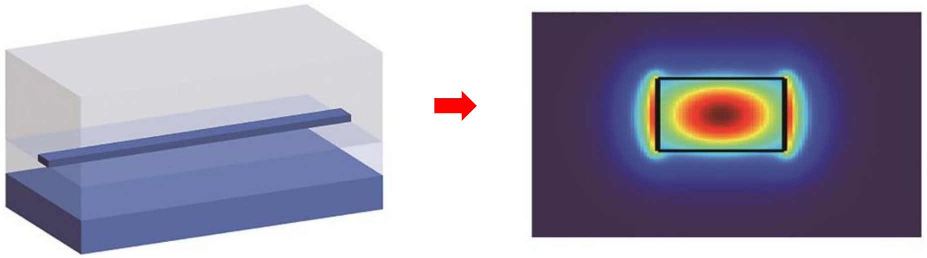
# Integrated Optical Waveguide

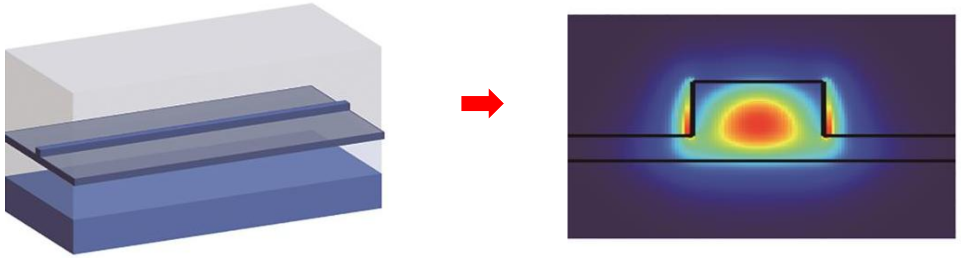
**Optical waveguide** – the most fundamental element in integrated photonics, guiding the propagation of light (like optical fibers)




- **Slab (Planar) Waveguide:** Index contrast (confinement) in one dimension
- **Strip/Ridge/Wire (rectangle) Waveguide :** Index contrast (confinement) in two dimensions
- **Rib waveguide, Slot waveguide and etc.**: more complex index confinement.

**Waveguide Mode** – the spatial distribution of optical energy within the waveguide





# Fundamentals of Electromagnetic waves

**Maxwell's Equations —** *describe how **electric fields (E/D)** and **magnetic fields (B/H)** are generated and interact*

| Name | Differential form | Integral form |
|---|---|---|
| Gauss's law | $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$ | $\oiint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \dfrac{Q(V)}{\varepsilon_0}$ |
| Gauss's law for magnetism | $\nabla \cdot \mathbf{B} = 0$ | $\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$ |
| Maxwell–Faraday equation (Faraday's law of induction) | $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ | $\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\dfrac{\partial \Phi_{B,S}}{\partial t}$ |
| Ampère's circuital law (with Maxwell's correction) | $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$ | $\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \dfrac{\partial \Phi_{E,S}}{\partial t}$ |

$$\mathrm{D} = \varepsilon \mathrm{E} \quad \mathrm{B} = \mu \mathrm{H} \quad \mathrm{J} = \sigma \mathrm{E}$$

*Form the foundation for deriving **wave propagation**, **light behavior**, and **mode formation** in optical waveguides*

**Monochromatic Assumption (i.e. for a specific frequency $\omega$)**

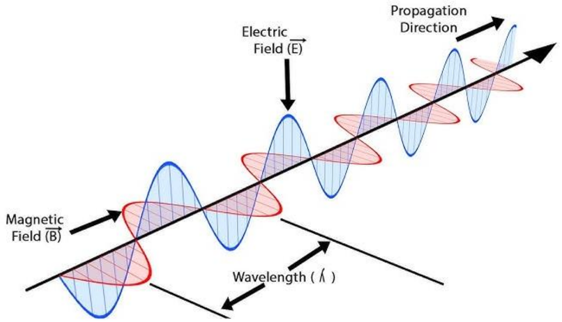
$$\mathbf{E}(r,t) = \mathbf{E}(r)e^{i\omega t}, \quad \mathbf{H}(r,t) = \mathbf{H}(r)e^{i\omega t}$$

**Maxwell's wave equation (Helmholtz equation):**

$$\nabla^2 \mathbf{E}(r) + k^2 n^2(r)\mathbf{E}(r) = 0$$

$$\nabla^2 \mathbf{H}(r) + k^2 n^2(r)\mathbf{H}(r) = 0$$

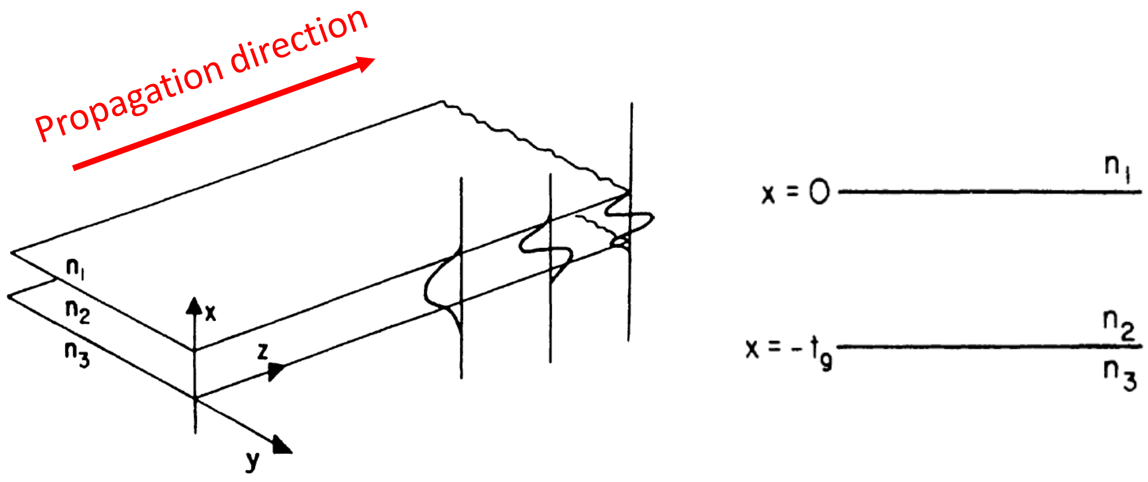
In free space (uniform media), electromagnetic waves are perfectly transverse: i.e. the electric field (***E***) and magnetic field (***H***) oscillate **perpendicularly** to each other and to the direction of propagation.




*TEM (Transverse Electric and Magnetic) wave*

# Waveguide modes in a Planar Waveguide

**Three-Layer Planar Waveguide** (index contrast in x direction ) **:**




Maxwell's wave equation (Helmholtz equation):

$$\nabla^2 \boldsymbol{E}(\boldsymbol{r}) + k^2 n^2(\boldsymbol{r}) \boldsymbol{E}(\boldsymbol{r}) = 0 \qquad (1.1)$$

where $\boldsymbol{E}$ : the electric field vector
$\boldsymbol{r}$ : the position vector, i.e. $\boldsymbol{r}(x, y, z)$
$\boldsymbol{n}(\boldsymbol{r})$ : the index of refraction
$\boldsymbol{k} \equiv \boldsymbol{\omega}/\boldsymbol{c}$ : the free-space wavenumber

Uniform plane wave propagating in the z direction; however, the index contrast in the x direction prevents maintaining a pure TEM mode, leading to **TE (Transverse Electric)** and **TM (Transverse Magnetic)** modes.

As for TE modes, where $E(z) = 0$, the monochromatic assumption becomes:

***Effective refractive index*** *experienced by the guided mode*

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}(\mathrm{x, y}) \exp(-i\beta \mathrm{z})$$ where $\beta$ is the propagation constant, $\beta = kn_{eff} = \omega n_{eff}/c$ (1.2)

# Waveguide modes in a Planar Waveguide

Substituting (1.2) into (1.1), the wave equation becomes:

$$\partial^2 \boldsymbol{E}(x,y)/\partial x^2 + \cancel{\partial^2 \boldsymbol{E}(x,y)/\partial y^2} + \left[k^2 n^2(\boldsymbol{r}) - \beta^2\right]\boldsymbol{E}(x,y) = 0 \qquad (1.3)$$

Since the waveguide is assumed infinite in the *y* direction, (1.3) can be rewritten separately for the three regions in *x*:

Region 1 $\quad \partial^2 E(x,y)/\partial x^2 + \left[k^2 n_1^2 - \beta^2\right]E(x,y) = 0$

Region 2 $\quad \partial^2 E(x,y)/\partial x^2 + \left[k^2 n_2^2 - \beta^2\right]E(x,y) = 0 \qquad (1.4)$

Region 3 $\quad \partial^2 E(x,y)/\partial x^2 + \left[k^2 n_3^2 - \beta^2\right]E(x,y) = 0$

The solutions of (1.4) are either **sinusoidal or exponential** functions of *x* in each of the regions, depending on whether $(k^2 {n^2}_i - \beta^2), i = 1, 2, 3$ **is greater or less than zero**.

# Waveguide modes in a Planar Waveguide

Therefore, the transverse function $E_y(x)$ has the general form:

Region 1 $\partial^2 E(x,y)/\partial x^2 + \left[k^2 n_1^2 - \beta^2\right] E(x,y) = 0$

Region 2 $\partial^2 E(x,y)/\partial x^2 + \left[k^2 n_2^2 - \beta^2\right] E(x,y) = 0$

Region 3 $\partial^2 E(x,y)/\partial x^2 + \left[k^2 n_3^2 - \beta^2\right] E(x,y) = 0$

$$E_y(x) = \begin{cases} A\exp(-qx) & 0 \le x \le \infty \\ B\cos(hx) + C\sin(hx) & -t_g \le x \le 0 \\ D\exp\left[p\left(x+t_g\right)\right] & -\infty \le x \le -t_g \end{cases} \qquad (1.5)$$

where *A, B, C, D, q, h,* and *p* are all constants that can be determined by matching **the boundary conditions.**

**Boundary conditions:**

At refractive index interfaces, the **tangential components** of both electric field and magnetic field (corresponding to the derivative of electric field) **must be continuous**, in order to avoid unphysical discontinuities (e.g. infinite surface current).

i.e. $E_y(x)$ and $\partial E_y(x)/\partial x$ must be continuous at the interfaces ($x = 0$, and $x = -t_g$)

# Waveguide modes in a Planar Waveguide

Then, the transverse function $E_y(x)$ can be expressed in terms of a single constant C', as:

$$E_y(x) = \begin{cases} C' \exp(-qx) & 0 \le x \le \infty \\ C'\left[\cos(hx) - (q/h)\sin(hx)\right] & -t_g \le x \le 0 \\ C'\left[\cos(ht_g) + (q/h)\sin(ht_g)\right]\exp\left[p(x+t_g)\right] & -\infty \le x \le -t_g \end{cases} \quad (1.6)$$

Substituting (1.6) into (1.3), we get:

$$q = \left(\beta^2 - n_1^2 k^2\right)^{1/2}, \qquad h = \left(n_2^2 k^2 - \beta^2\right)^{1/2},$$
$$p = \left(\beta^2 - n_3^2 k^2\right)^{1/2}, \qquad k \equiv \omega / \mathrm{c}. \quad (1.7)$$

As $\partial E_y(x)/\partial x$ continuous at $n_2$-$n_3$ boundary (i.e. $x = -t_g$), then:

$$-h\sin(-ht_g) - h(q/h)\cos(-ht_g) = p\left[\cos(ht_g) + (q/h)\sin(ht_g)\right] \quad \text{or} \quad \tan(ht_g) = \frac{p+q}{h\left(1 - pq/h^2\right)} \quad (1.8)$$

Leads to discrete values of $\beta \Rightarrow \beta_m$ ($q_m$, $h_m$ and $p_m$ )

- Solutions for $\beta_m$ can be obtained graphically or computed numerically using software tools.

# Waveguide modes in a Planar Waveguide

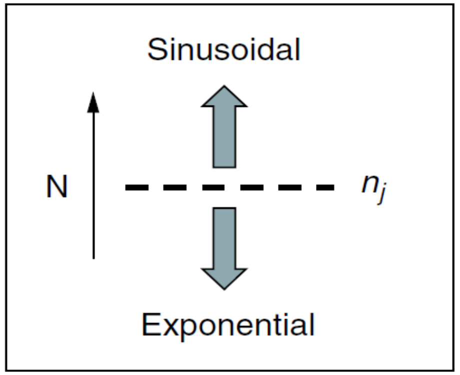
**TE Mode Profiles**:

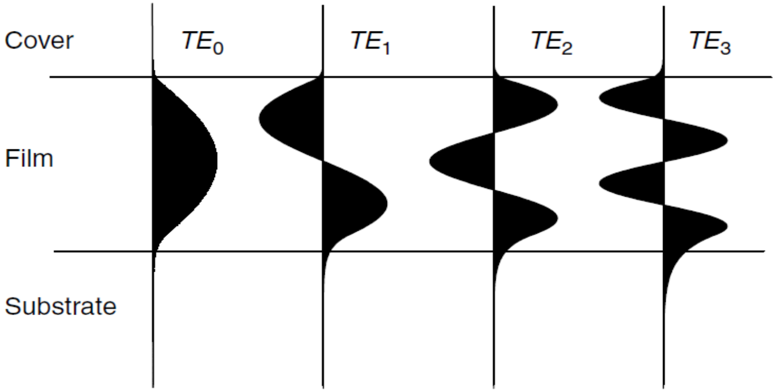
- In the core region (higher refractive index), the optical field oscillates sinusoidally.

  The number of field nodes defines the **mode order**:

  — The $TE_0$ mode, with no internal nodes, is called the **fundamental mode**.

  — Higher-order modes (e.g., $TE_1$, $TE_2$, and $TE_3$) have one, two, three, or more nodes, respectively.
- In other regions (lower refractive index), the field decays exponentially, called the evanescent field/tail .



# Waveguide modes in a Planar Waveguide

In analog, for **TM Modes** (i.e., $H(z) = 0$), we have:

$$H_y(x,z,t) = H_y(x)e^{i(\omega t - \beta z)} \qquad E_x(x,z,t) = \frac{i}{\omega\varepsilon}\frac{\partial H_y}{\partial z} = \frac{\beta}{\omega\varepsilon}H_y(x)e^{i(\omega t-\beta z)} \qquad E_z(x,z,t) = -\frac{i}{\omega\varepsilon}\frac{\partial H_y}{\partial x}$$

These equations yield:

$$H_y(x) = \begin{cases} -C'\left[\frac{h}{\bar{q}}\cos(ht_g) + \sin(ht_g)\right]\exp\left[p(x+t_g)\right] & -\infty \le x \le -t_g \\ C'\left[-\frac{h}{\bar{q}}\cos(hx) + \sin(hx)\right] & -t_g \le x \le 0 \\ C' - \frac{h}{\bar{q}}\exp(-qx) & 0 \le x \le \infty \end{cases}$$

**Boundary condition:** $\partial H_y/\partial x$ is continuous at the $n_2$-$n_3$ boundary

$$\tan(ht_g) = \frac{h(\bar{p}+\bar{q})}{h^2 - \bar{p}\bar{q}}$$

To summarize, the **non-zero field components**:

**TE Modes:** $\boldsymbol{E_y}$, $H_x$ , $H_z$ *(minor)*; **TM Modes:** $\boldsymbol{H_y}$, $E_x$ , $E_z$ *(minor)*;

# Cutoff Condition

The **cutoff condition** represents the point at which the field becomes oscillatory in *Regions 1* and *3*, i.e., when the mode can no longer be well confined within the waveguide.

For symmetrical planar waveguide (i.e. $n_1 = n_3$) at cutoff condition, the magnitude of β is given by (see Eq. 1.7):

$$\beta = kn_1 = kn_3 \qquad p = q = 0$$

$$k = \omega / c = 2\pi / \lambda_0 \;\rightarrow\; h = k\left(n_2^2 - n_1^2\right)^{1/2} = k\left(n_2^2 - n_3^2\right)^{1/2}$$

For waveguiding of a given mode to occur, one must have (see Eq. 1.8):

$$tan(ht_g) = \frac{p+q}{h(1-pq/h^2)} = 0 \;\rightarrow\; ht_g = m_s\pi, \quad m_s = 0,1,2,3,\ldots$$

$$\rightarrow \quad \Delta n = (n_2 - n_1) > \frac{m_s^2\lambda_0^2}{4t_g^2(n_2+n_1)}, \quad m_s = 0,1,2,3,\ldots \qquad (1.9)$$

The lowest mode $m_s = 0$

- $\Delta n>0 \Rightarrow$ any wavelength could be guided in this mode
- small $\Delta n$ and/or large $\lambda_0/t_g \Rightarrow$ poor confinement $\Rightarrow$ large evanescent tails

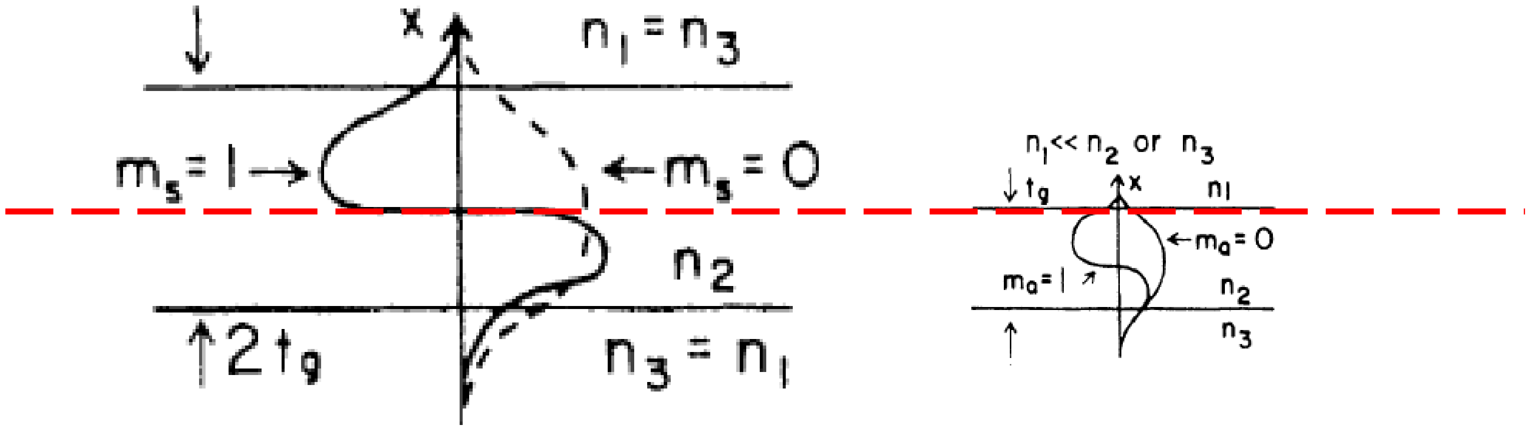
The cutoff condition given in (1.9) determines which modes can be supported by a waveguide with a given $\Delta n$ and ratio of $\lambda_0/t_g$.

# Cutoff Condition

For asymmetrical planar waveguide (e.g., $n_2 > n_3 >> n_1$) , the cutoff condition becomes:

$$\Delta n = (n_2 - n_3) > \frac{(2m_a+1)\lambda_0^2}{16(n_2+n_3)t_g^2} = \frac{m_s^2\lambda_0^2}{4(n_2+n_3)(2t_g)^2}, \quad m_s = (2m_a+1), \quad m_a = 0,1,2,3,\ldots \qquad (1.10)$$

By comparing Eq. 1.9 & 1.10, it can be seen that the asymmetric waveguide supports only modes are similar/correspond to the odd modes of a symmetric waveguide of twice its thickness.




| | Sym. $2t_g$ | Asym. $t_g$ |
|---|---|---|
| $m_{s,a}$ = | 1 | 0 |
| | 3 | 1 |
| | 5 | 2 |
| | 7 | 3 |

# Modes in a Strip (rectangular) Waveguide

To simplify the analysis, we follow Marcatili's method (an approximate solution), assuming that all modes are well-guided, i.e., above the cutoff condition.

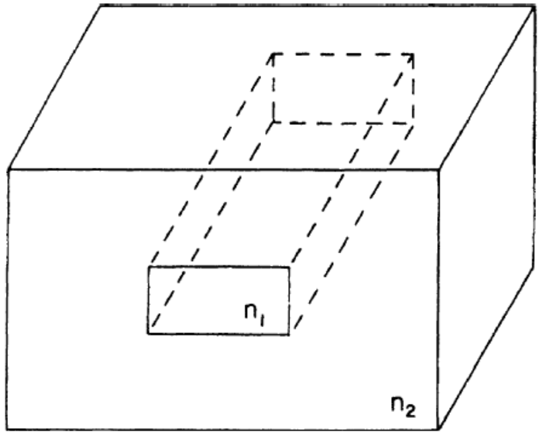
- The field magnitudes in the shaded corner regions are small enough to be neglected
- Field decays exponentially in Regions 2-5 with most of the power being confined to Region 1.

Since the index contrast exists in both the *x* and *y* directions, the electric field can be denoted as:

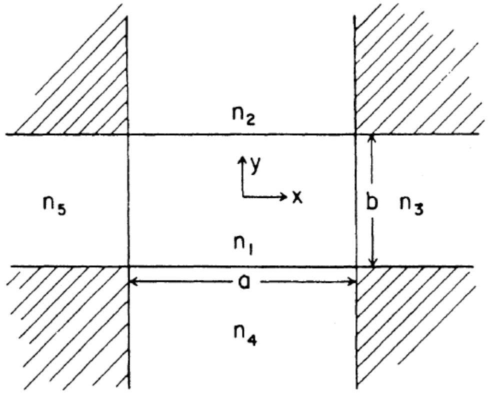
$$E^x_{pq} \text{ and } E^y_{pq} ,$$

i.e. the electric field distributing along either x or y direction

where *p* and *q* represent the number of field nodes in the *x* and *y* directions, respectively.




$n_1 > n_2$

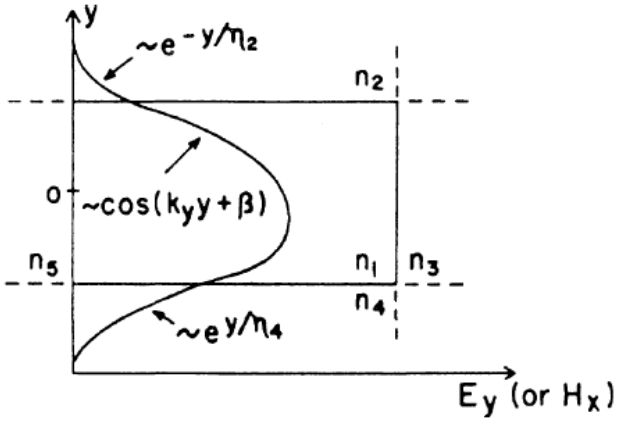


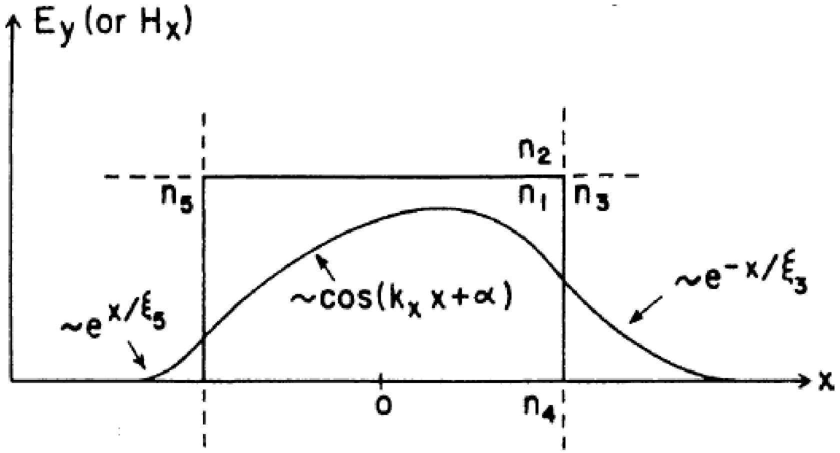

$n_1 > n_2, n_3, n_4, n_5$

# Modes in a Strip (rectangular) Waveguide

- For $E^{x}_{pq}$ modes, dominate components are: $E_x$ , $H_y$
- For $E^{y}_{pq}$ modes, dominate components are: $E_y$ , $H_x$

*For example, the mode profile of* $E^{y}_{00}$ as:




Note that the shape of the mode is characterized by **extinction coefficients** $\eta_2$, $\xi_3$, $\eta_4$, and $\xi_5$ in the regions where it is exponential, and by **propagation constants** $k_x$ and $k_y$ in *Region 1*

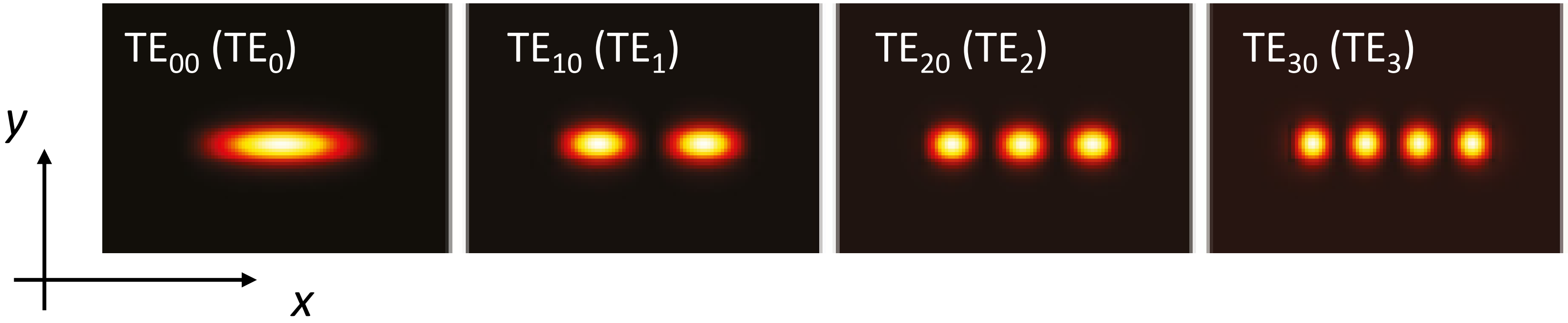
# Modes in a Strip (rectangular) Waveguide

In practice, the width ($x$-direction) of a rectangular waveguide is usually larger than its height ($y$-direction), for example, a **500 nm** $\times$ **220 nm** SOI waveguide.

In such cases:

- The refractive index contrast along **y-direction** is stronger, leading to **tighter confinement** and typically a **single-peaked field profile** without nodes (i.e., no oscillations along $y$).
- Along **x-direction**, the confinement is weaker, making it easier to form **higher-order modes** with multiple nodes/peaks.

Thus, the dominant field component is $E^y_{pq}$ ($p$=0,1,2,3,... and $q$=0), and the supported modes are known as **quasi-TE modes.**

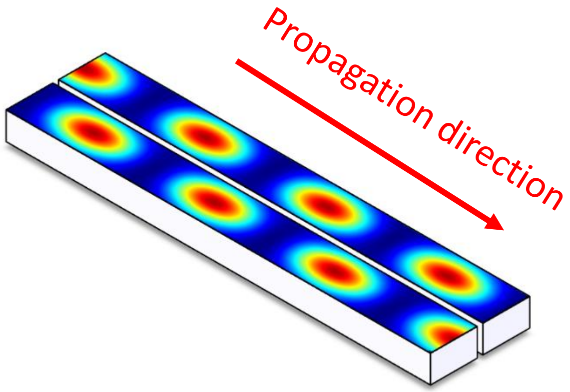


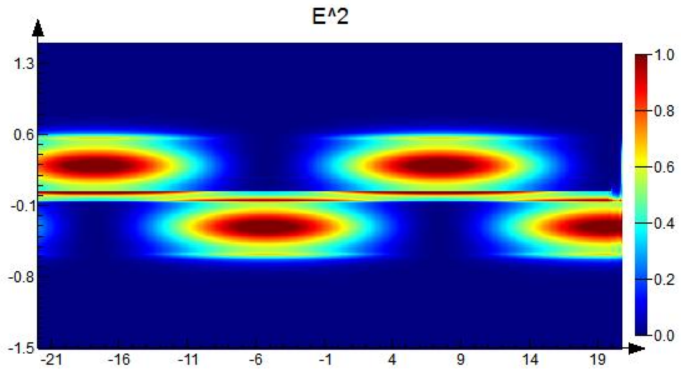
# Basics of Mode Coupling

When two waveguides are brought close to each other, a phenomenon called **mode coupling** occurs, which means that optical energy periodically transfers between the two waveguides.

This happens because:

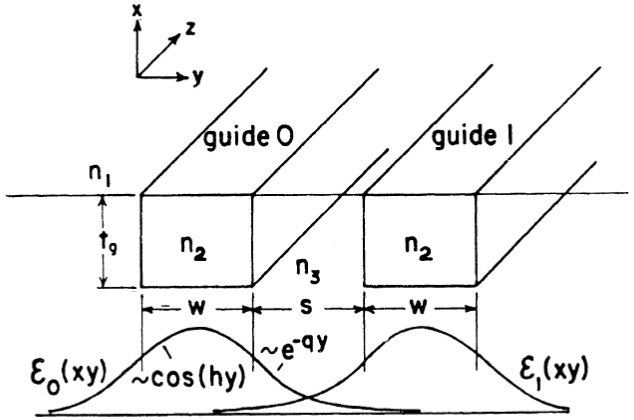
- The **evanescent fields** of the two waveguides extend into the surrounding space, and when placed close enough, **they overlap and interact**.
- This interaction enables **periodic energy transfer** between the two waveguides, where energy oscillates back and forth between the waveguides (like beating)



# Basics of Mode Coupling

## Coupled-Mode Theory of Synchronous Coupling




Coupling between two identical lossless waveguides ($\beta = \beta_0 = \beta_1$), and the field of the propagating mode is denoted, as:

$$\bar{E}(x,y,z) = A(z)\bar{E}(x,y)$$

where $A(z)$ is the complex amplitude of the field, such that the mode power equals

$$P(z) = |A(z)|^2 = A(z)A^*(z)$$

The mode coupling can be described by the general **coupled mode equations** for the amplitudes of the two modes:

$$\begin{cases} \dfrac{dA_0(z)}{dz} = -i\beta A_0(z) - i\kappa A_1(z) \\ \dfrac{dA_1(z)}{dz} = -i\beta A_1(z) - i\kappa A_0(z) \end{cases} \quad (1.11)$$

where $\beta_0$ and $\beta_1$ are the mode propagation constants, and $\kappa$ is the **coupling coefficient** between modes, which quantifies the strength of interaction between the two waveguides: the larger $\kappa$, the faster the energy transfer between them.

# Basics of Mode Coupling

Assume that the light is all within Waveguide *0* at the position *z=0*, we have:

$$A_0(0) = 1 \qquad \text{and} \qquad A_1(0) = 0$$

Thus, the solutions of the coupled mode equations are:

*power flow in waveguides*

$$\begin{cases} \dfrac{dA_0(z)}{dz} = -i\beta A_0(z) - i\kappa A_1(z) \\ \dfrac{dA_1(z)}{dz} = -i\beta A_1(z) - i\kappa A_0(z) \end{cases} \Rightarrow \begin{cases} A_0(z) = \cos(\kappa z) e^{-i\beta z} \\ A_1(z) = -i \sin(\kappa z) e^{-i\beta z} \end{cases} \Rightarrow \begin{cases} P_0(z) = A_0(z) A_0^*(z) = \cos^2(\kappa z) \\ P_1(z) = A_1(z) A_1^*(z) = \sin^2(\kappa z) \end{cases}$$

It can be seen that the **coupling length** *L* necessary for complete transfer of power from one guide to the other is given by:

$$L = \frac{\pi}{2\kappa} + \frac{m\pi}{\kappa} \qquad \text{where } m = 0,1,2,3 \ldots$$

# Simulation tools

Due to the complexity of mathematical models for optical waveguides, many computer programs and simulation tools have been developed to facilitate waveguide design and analysis.

Commercial software for waveguide mode analysis:

Ansys Lumerical FDTD/MODE

COMSOL Multiphysics (FEM)

In the following sessions, we will use **Lumerical** simulation tools (also **Matlab**-based numerical models) to perform calculations and simulations for optical waveguides and related photonic devices.

# Report 1

1. Download and install *MATLAB* (please follow the instructions at https://help.eng.cam.ac.uk/software/matlab/).

2. Write a *MATLAB* Script to simulate the modes supported in a symmetric **Silicon-on-Isolator (SOI)** planar waveguide ($\lambda$ = 1550 nm) with varying core thicknesses using the cutoff condition (see Eq. 1.9).

3. Try different material systems, such as **$Si_3N_4$** or **$LiNbO_3$** and repeat the analysis. Compare material influence on mode support.

4. Also, change the operating wavelengths, e.g., repeat analysis at **1310 nm**, **1600 nm**, to see the dispersion effect. Note, do consider the **material dispersion,** i.e. the refractive index of materials themselves also dependence on wavelength

**Summarize your findings in Report 1**, including key figures (e.g., mode count vs. thickness) and critical discussion.