Copyright Notice

Convolutional Neural Networks

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Computer vision

Computer Vision Problems

Image Classification

64x64

Cat? (0/1)

Object detection

Neural Style Transfer

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Deep Learning on large images

64x64 × 3

Cat? (0/1)

12288

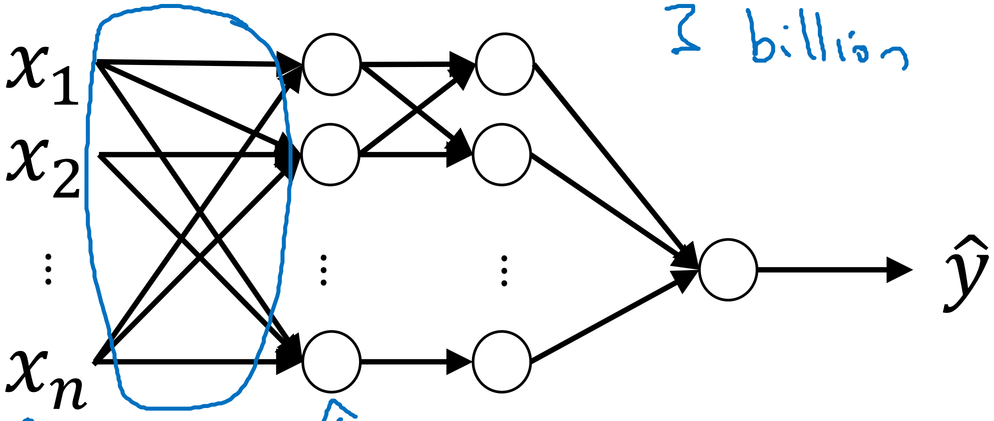
1000 × 1000 × 3

= 3 million

$x \in \mathbb{R}^{3M}$

$W^{[1]}$ (1000, 3M)

3 billion



3M

1000

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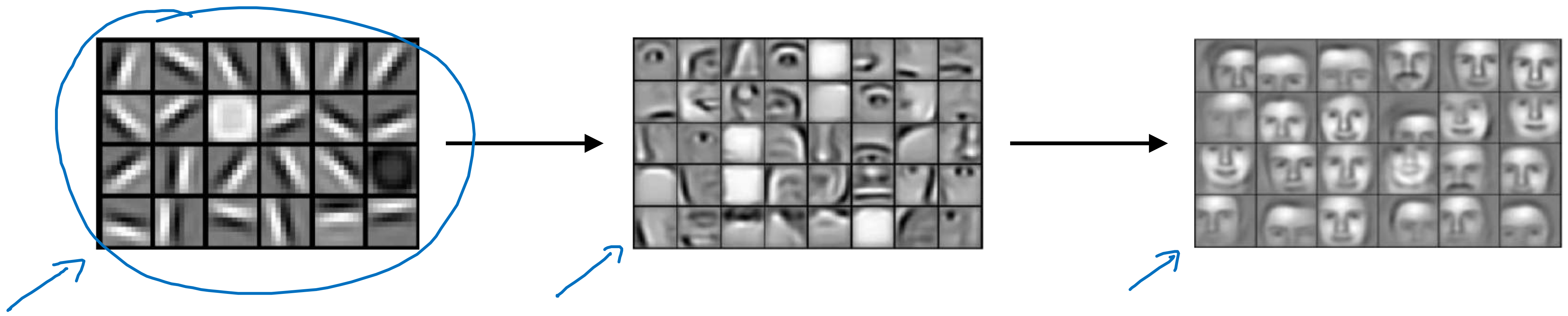
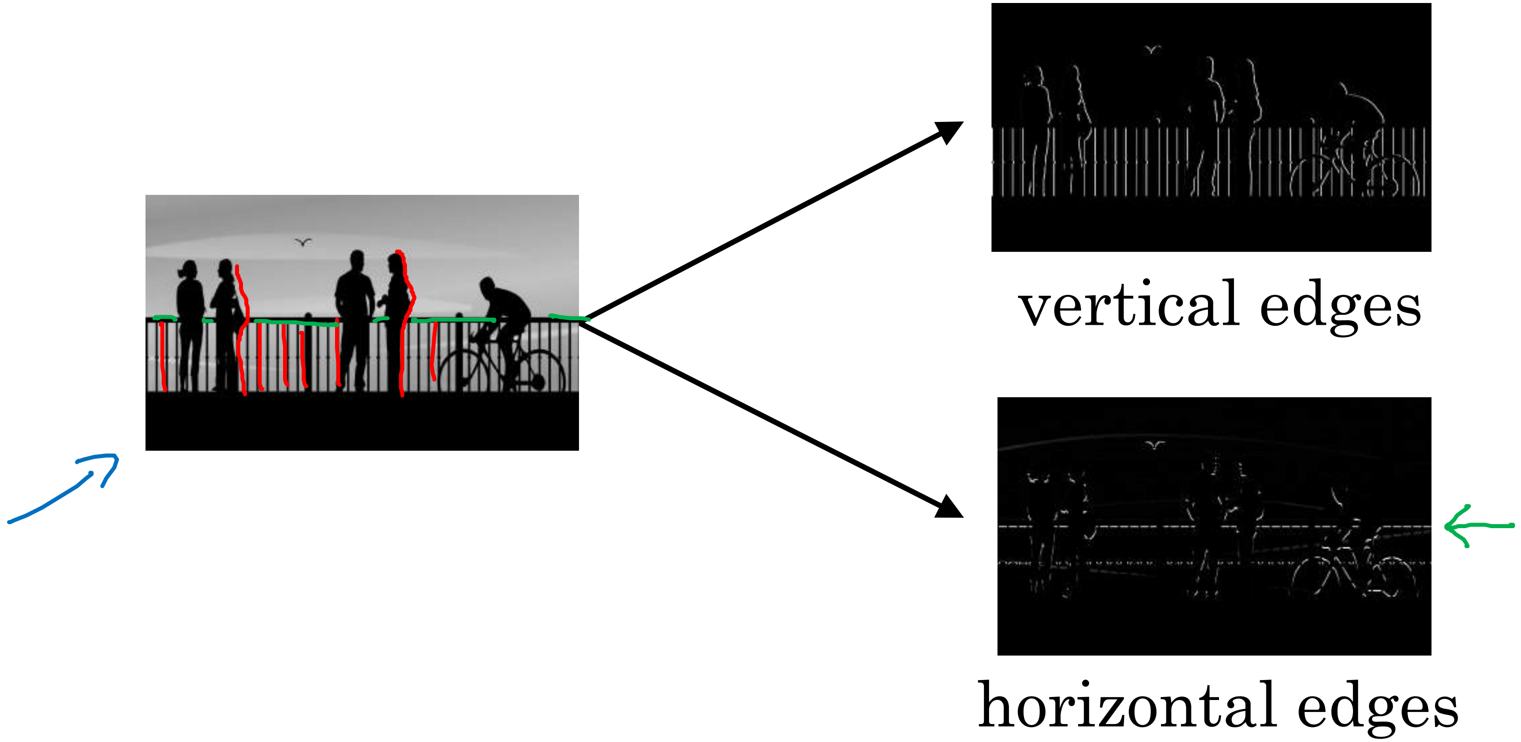
Convolutional Neural Networks

Edge detection example

Computer Vision Problem





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Vertical edge detection

$3\times1 + 1\times1 + 2\times1 + 0\times0 + 5\times0 + 7\times0 + 1\times-1 + 8\times-1 + 2\times-1 = -5$

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6×6

"convolution"

*

1	0	-1
1	0	-1
1	0	-1

3×3

→ filter

kernel

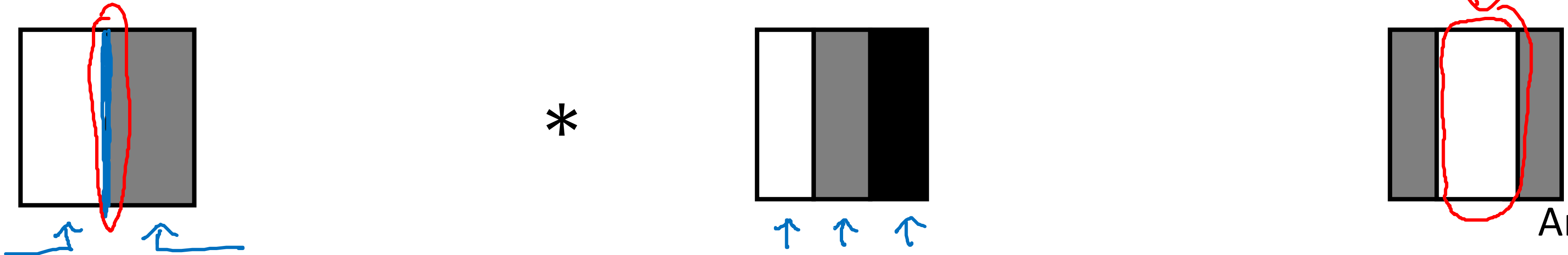
=

-5	-4	0	8
-10	-2	2	3
0	-2	-4	-7
-3	-2	-3	-16

4×4

Vertical edge detection

$$\underbrace{\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix}}_{6 \times 6} * \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}}_{3 \times 3} = \underbrace{\begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}}_{4 \times 4}$$



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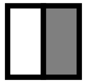
Convolutional Neural Networks

More edge detection

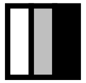
Vertical edge detection examples

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0



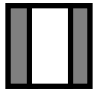
$*$

1	0	-1
1	0	-1
1	0	-1



$=$

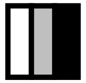
0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0



0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

$*$

1	0	-1
1	0	-1
1	0	-1



$=$

0	-30	-30	0
0	-30	-30	0
0	-30	-30	0
0	-30	-30	0

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Vertical and Horizontal Edge Detection

1	0	-1
1	0	-1
1	0	-1

Vertical

1	1	1
0	0	0
-1	-1	-1

Horizontal

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

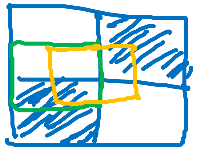
6x6

*

1	1	1
0	0	0
-1	-1	-1

=

0	0	0	0
30	10	-10	-30
30	10	-10	-30
0	0	0	0



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Learning to detect edges

1	0	-1
1	0	-1
1	0	-1

1	0	-1
2	0	-2
1	0	-1

Sobel filter

3	0	-3
10	0	-10
3	0	-3

Scharr filter

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

convolution

$*$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

3×3

$=$

45°
70°
73°

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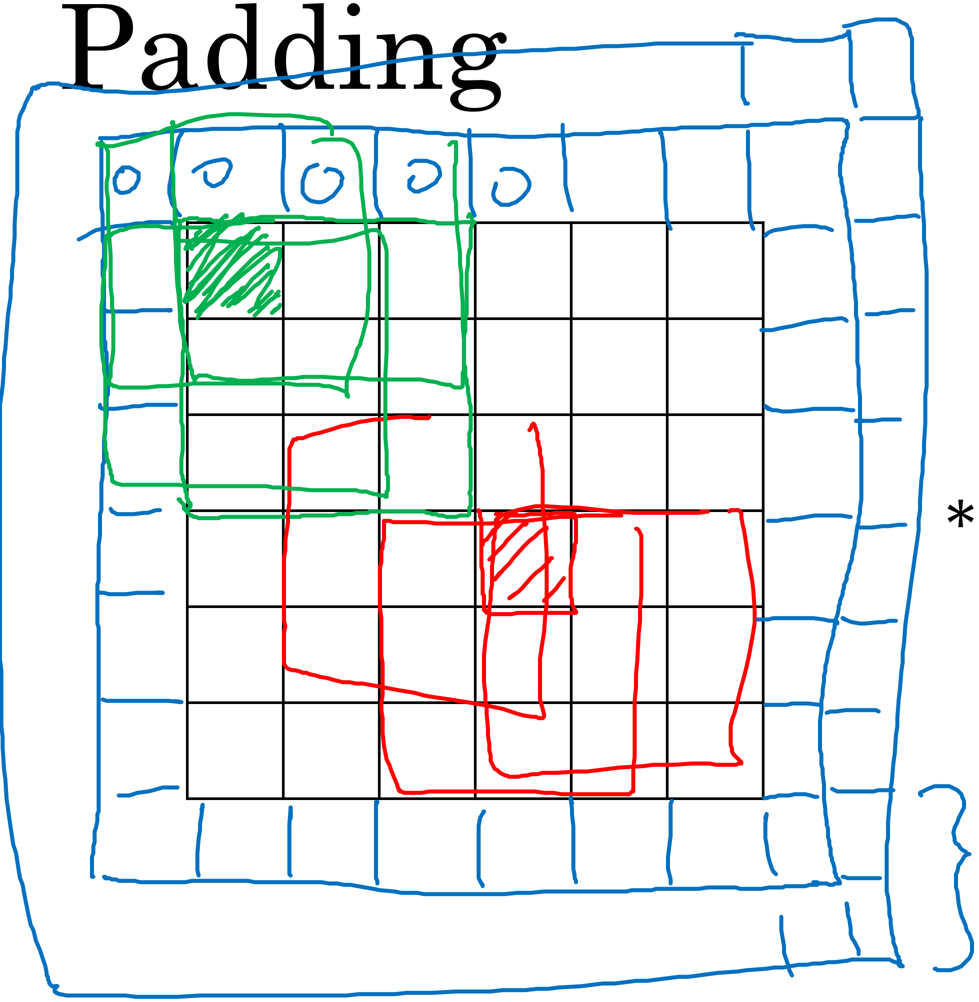
Convolutional Neural Networks

Padding

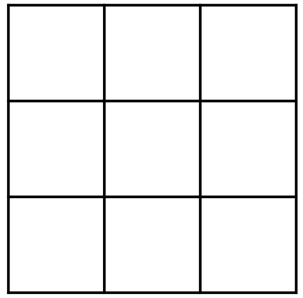
Padding

- shrinky output
- throw away info from edge



p=2

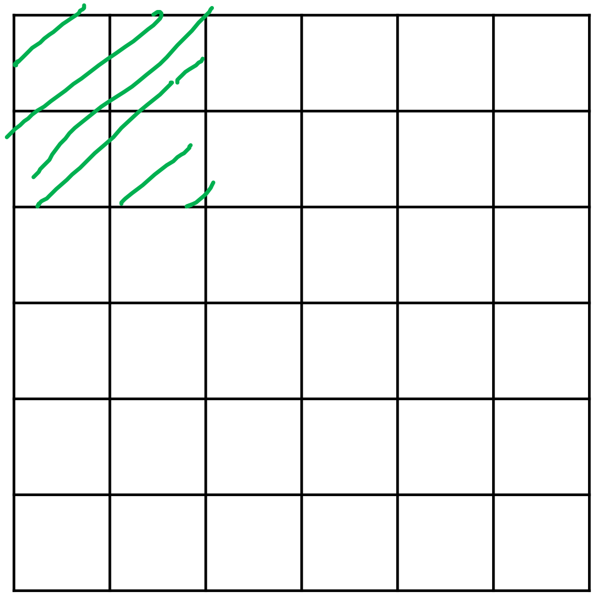
$*$



3x3

f x f

$=$



6x6

~~4x4~~

6x6 → 8x8

n x n

n-f+1 x n-f+1

6-3+1=4

p = padding = 1

n+2p-f+1 x n+2p-f+1

6+2-3+1 x ___ = 6x6

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Valid and Same convolutions

→ no padding

"Valid": $n \times n \ast f \times f \rightarrow n-f+1 \times n-f+1$

$6 \times 6 \ast 3 \times 3 \rightarrow 4 \times 4$

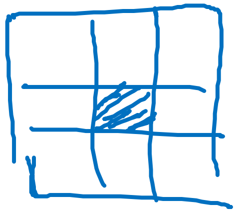
"Same": Pad so that output size is the same as the input size.

$n+2p-f+1 \times n+2p-f+1$

$\cancel{n}+2p-f+1 = \cancel{n} \Rightarrow \boxed{p = \frac{f-1}{2}}$

$3 \times 3 \quad p = \frac{3-1}{2} = 1$ | 5×5, $f=5$ $\quad p=2$

f is usually odd



1×1

3×3

5×5

7×7

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Convolutional Neural Networks

Strided convolutions

Strided convolution

2 3	3 4	7 3	4 4	6 3	2 4	9 4
6 1	6 0	9 1	8 0	7 1	4 0	3 2
3 -3	4 4	8 3	3 4	8 3	9 4	7 4
7 1	8 0	3 1	6 0	6 1	3 0	4 2
4 -3	2 4	1 3	8 4	3 3	4 4	6 4
3 1	2 0	4 1	1 0	9 1	8 0	3 2
0 -1	1 0	3 -1	9 0	2 -1	1 0	4 3

7×7

*

3	4	4
1	0	2
-1	0	3

3×3

=

91	100	83
69	91	127
44	72	74

3×3

stride = 2

$\lfloor z \rfloor = \text{floor}(z)$

$n \times n \quad * \quad f \times f$

padding p stride s

$s = 2$

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

$$\frac{7+0-3}{2} + 1 = \frac{4}{2} + 1 = 3$$

Summary of convolutions

$n \times n$ image $\qquad f \times f$ filter

padding $p \qquad$ stride s

Output size:

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \quad \times \quad \left\lfloor \underbrace{\frac{n+2p-f}{s}} + 1 \right\rfloor$$

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Technical note on cross-correlation vs. convolution

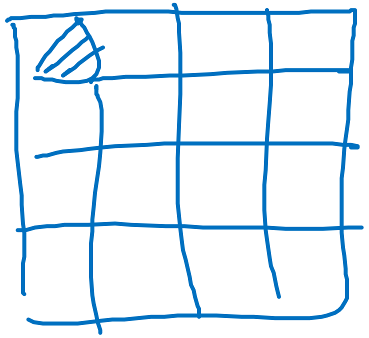
Convolution in math textbook:

2^{7}	3^{2}	7^{5}	4	6	2
6^{9}	6^{0}	9^{4}	8	7	4
3^{-1}	4^{1}	8^{3}	3	8	9
7	8	3	6	6	3
4	2	1	8	3	4
3	2	4	1	9	8

*

3	4	5
1	0	2
-1	9	7

=



The correct filter after flipping vertically and horizontally would be:

7	9	-1
2	0	1
5	4	3

$$(A * B) * C = A * (B * C)$$

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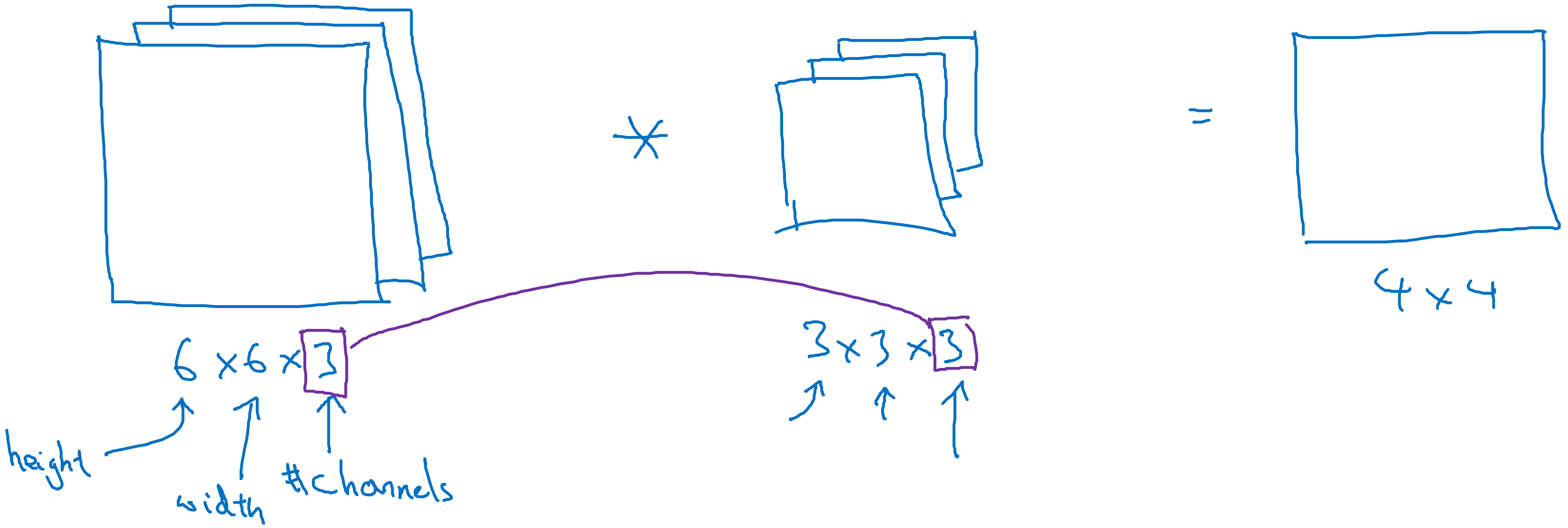
Convolutional Neural Networks

Convolutions over volumes

Convolutions on RGB images



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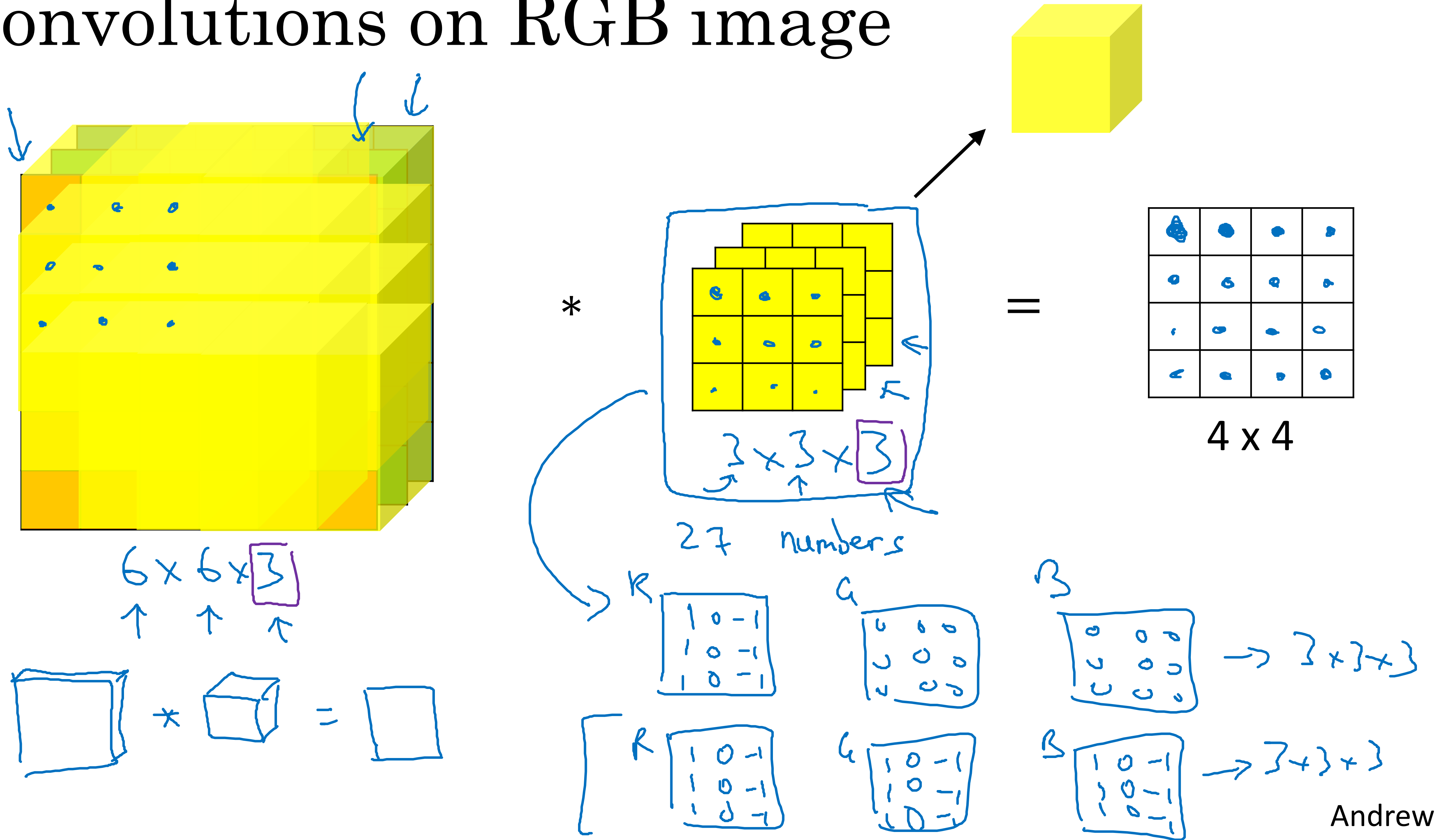
Convolutions on RGB image



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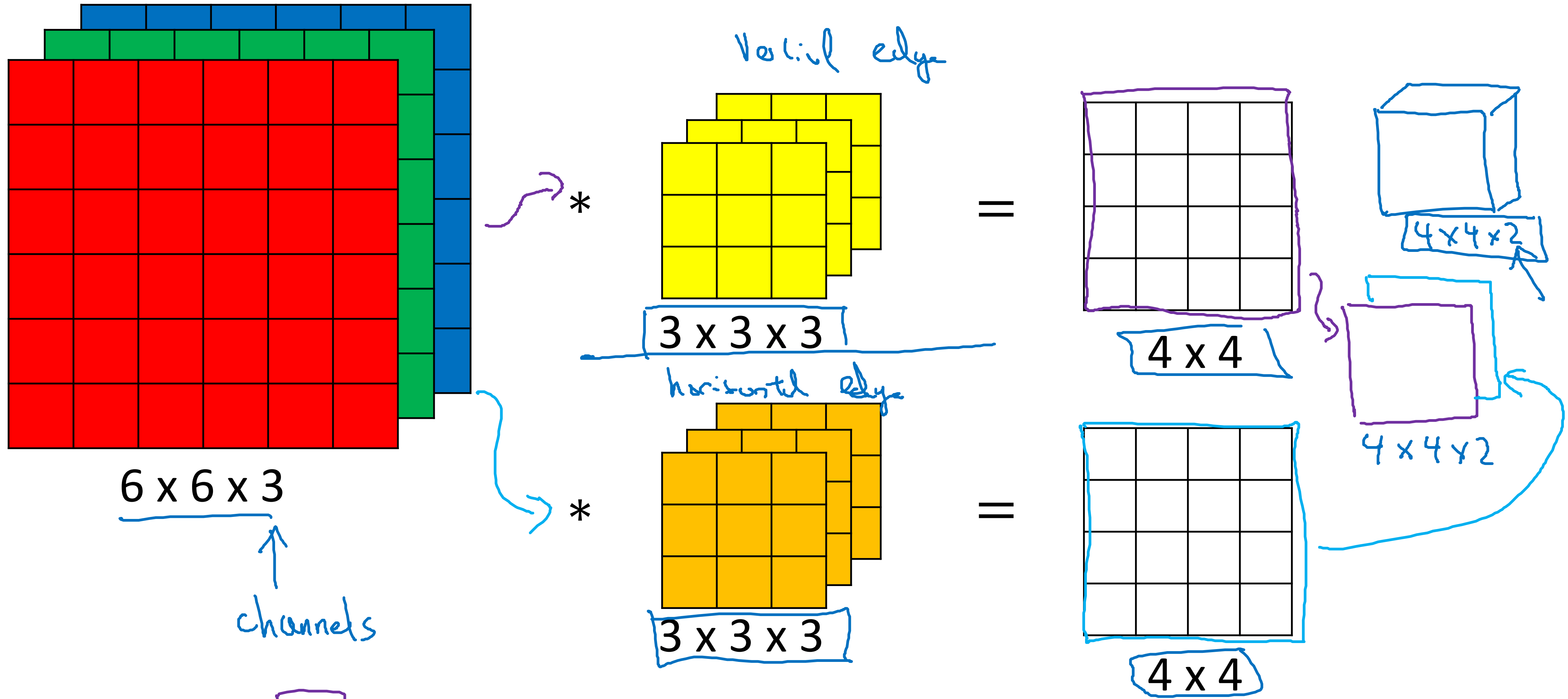
Multiple filters



Summary: $n \times n \times \boxed{n_c} \ * \ f \times f \times \boxed{n_c} \longrightarrow n-f+1 \times n-f+1 \times n_c'$

$6 \times 6 \times 3 \qquad 3 \times 3 \times 3 \qquad 4 \times 4 \times 2$ ← # filters

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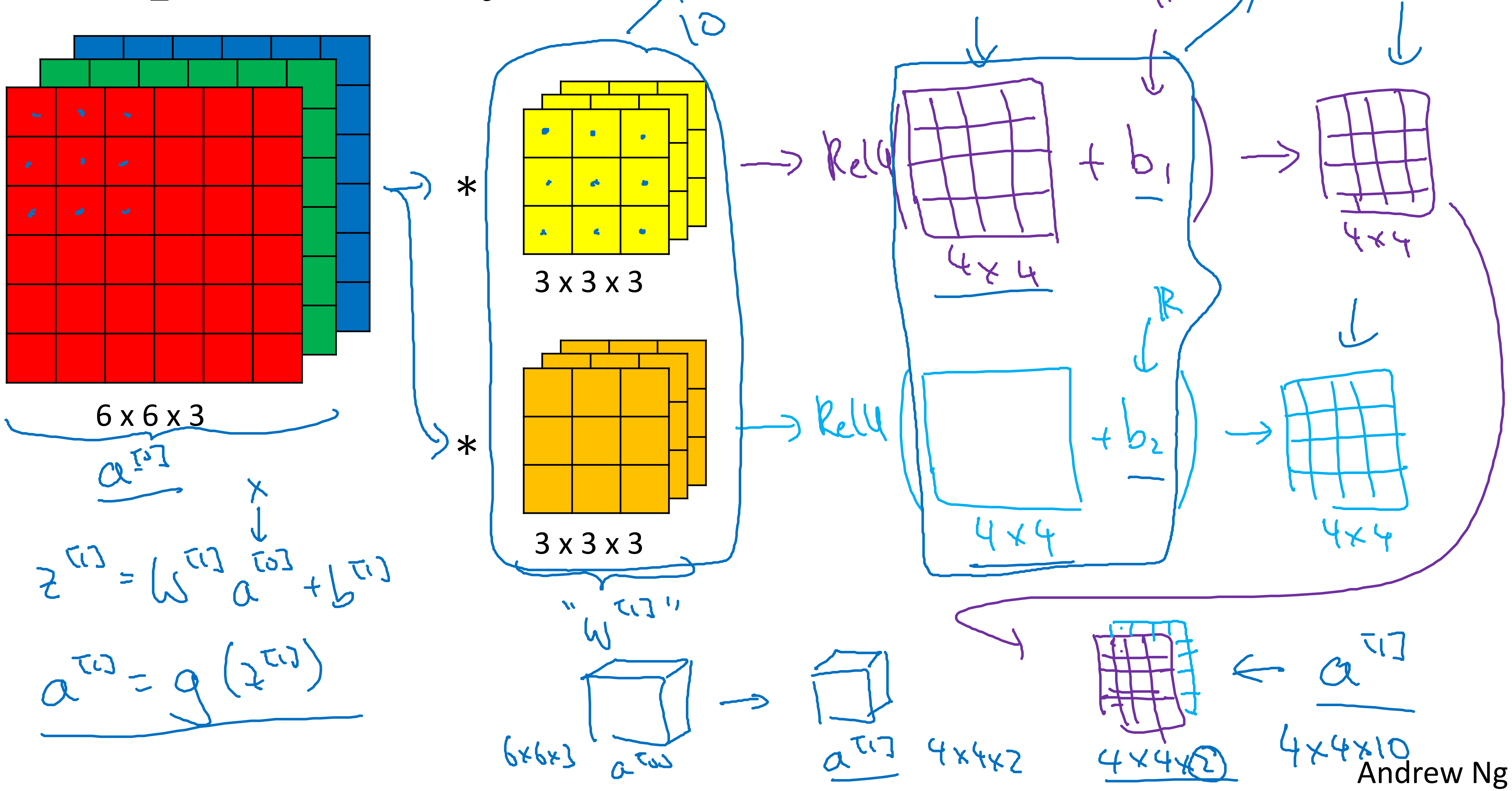
Convolutional Neural Networks

One layer of a convolutional network

Example of a layer

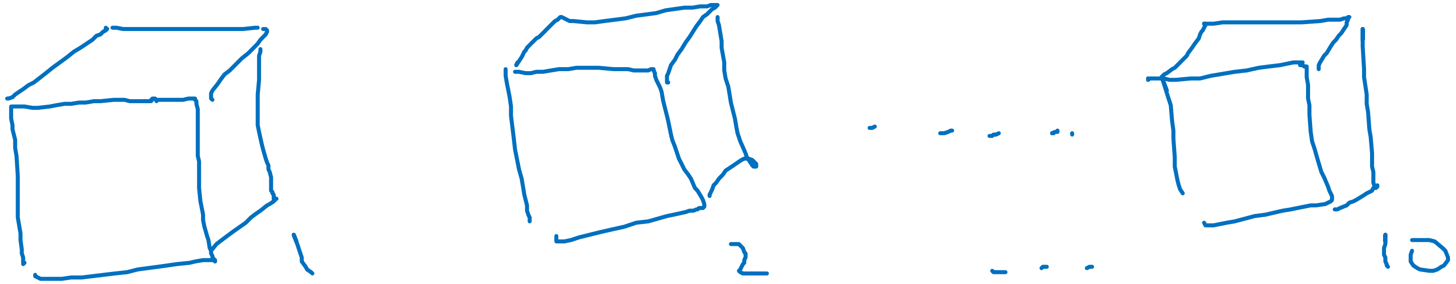


$$z^{[1]} = W^{[1]} a^{[0]} + b^{[1]}$$

$$a^{[1]} = g(z^{[1]})$$

Number of parameters in one layer

If you have 10 filters that are 3 x 3 x 3 in one layer of a neural network, how many parameters does that layer have?



3x3x3

27 parameters.

+bias

→ 28 parameters.

280 parameters.

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Summary of notation ★

If layer l is a convolution layer:

height, width, chanel

$f^{[l]}$ = filter size

$p^{[l]}$ = padding

$s^{[l]}$ = stride

$\boxed{n_c^{[l]}}$ = number of filters

→ Each filter is: $f^{[l]} \times f^{[l]} \times \boxed{n_c^{[l-1]}}$

Activations: $a^{[l]} \rightarrow n_H^{[l]} \times n_W^{[l]} \times n_c^{[l]}$

Weights: $f^{[l]} \times f^{[l]} \times n_c^{[l-1]} \times n_c^{[l]}$

bias: $n_c^{[l]}$ - $(1,1,1,n_c^{[l]})$ ← #filters in layer l.

Input: $n_H^{[l-1]} \times n_W^{[l-1]} \times \boxed{n_c^{[l-1]}}$ ←

Output: $n_H^{[l]} \times n_W^{[l]} \times \boxed{n_c^{[l]}}$ ←

$$n_{\cancel{H}W}^{[l]} = \left\lfloor \frac{n_{\cancel{H}W}^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

floor

$$A^{[l]} \rightarrow m \times \underbrace{n_H^{[l]} \times n_W^{[l]} \times n_c^{[l]}}$$

$n_c \times n_H \times n_W$

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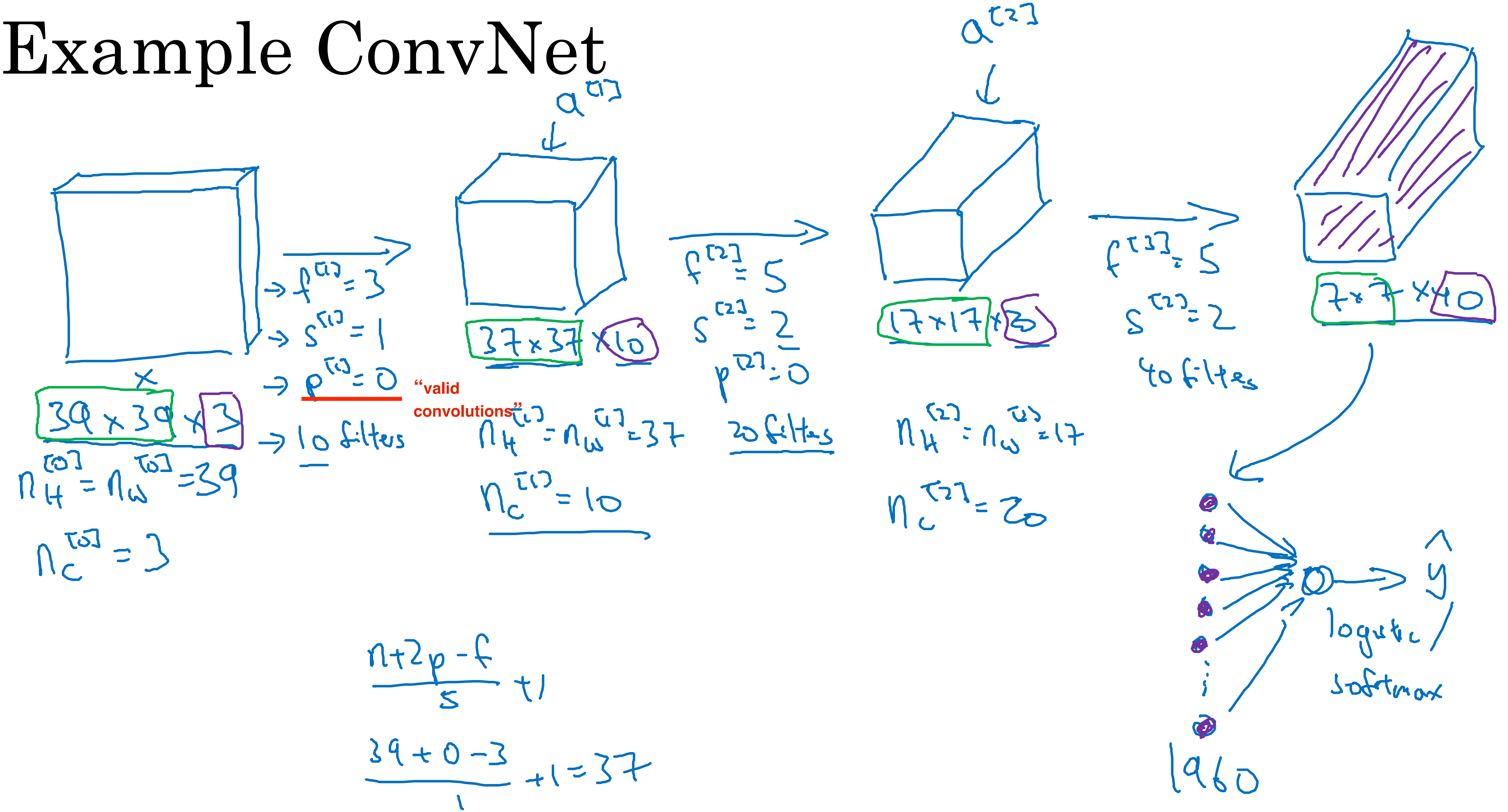
Convolutional Neural Networks

A simple convolution network example

Example ConvNet



Types of layer in a convolutional network:

- Convolution (CONV) ←
- Pooling (POOL) ←
- Fully connected (FC) ←

previous slides

Fortunately pooling layers and fully connected layers are a bit simpler than convolutional layers to define.

Convolutional Neural Networks

Pooling layers

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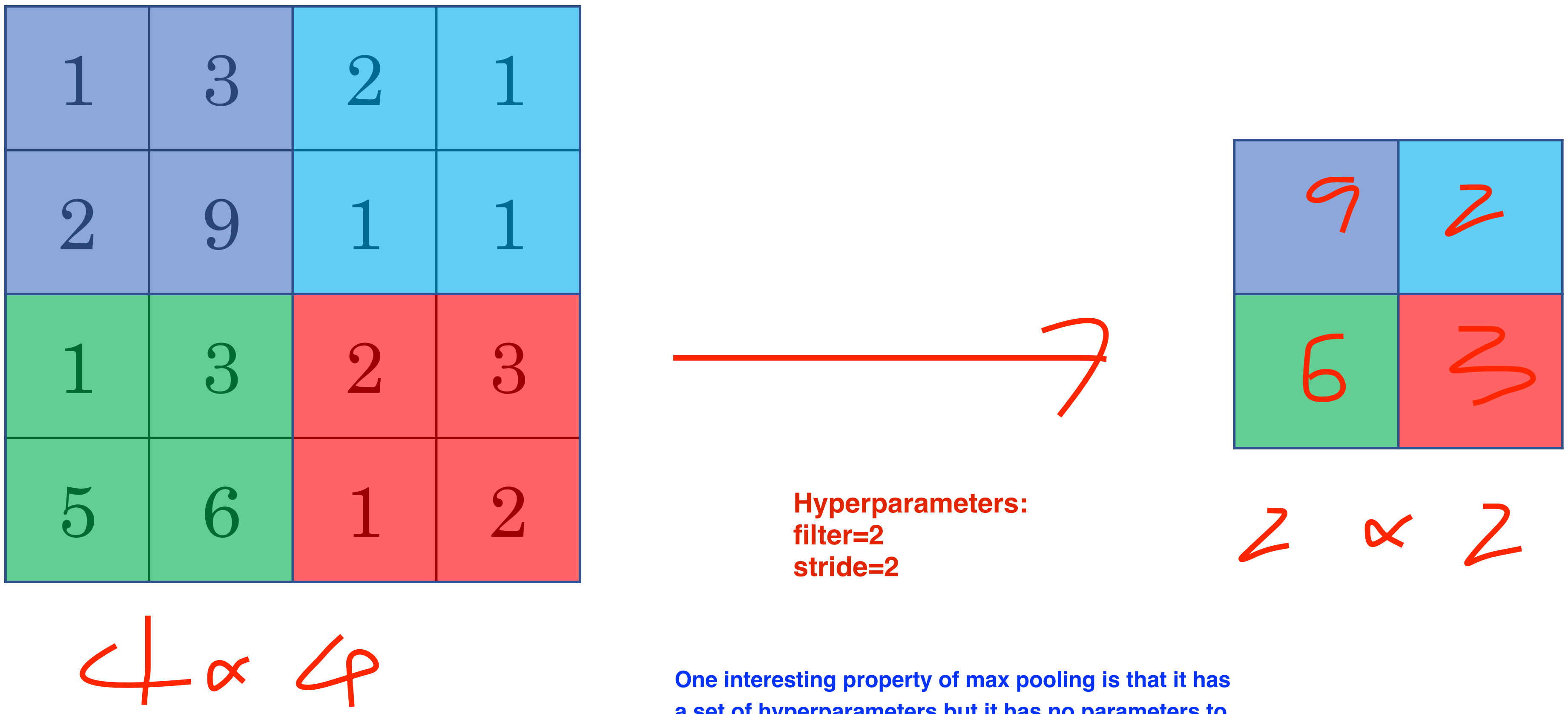
Pooling layer: Max pooling



Hyperparameters:
filter=2
stride=2

One interesting property of max pooling is that it has a set of hyperparameters but it has no parameters to learn. There's actually nothing for gradient descent to learn. Once you fix f and s, it's just a fixed computation and gradient descent doesn't change anything.

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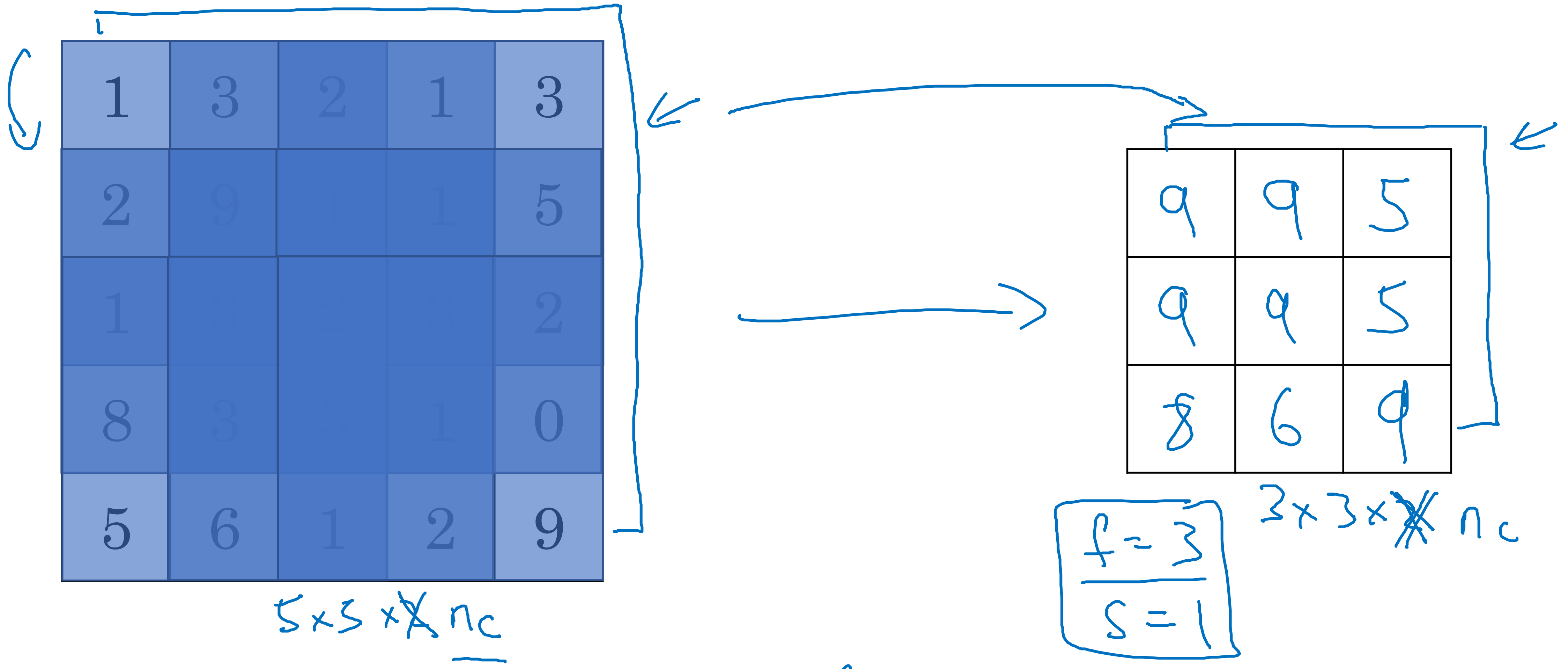
Pooling layer: Max pooling



$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

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Pooling layer: Average pooling

1	3	2	1
2	9	1	1
1	4	2	3
5	6	1	2

$\longrightarrow$

3.75	1.25
4	2

$f = 2$

$s = 2$

$\underline{7 \times 7} \times 1000 \longrightarrow 1 \times 1 \times 1000$

max pooling is used much more often than average pooling with one exception, which is sometimes very deep in a neural network.

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Summary of pooling

Hyperparameters:

f : filter size

s : stride

Max or average pooling

$f=2, s=2$

$f=3, s=2$

When you do max pooling, usually, you do not use any padding, although there is one exception that we'll see next week as well.

→ ~~p: padding.~~

No parameters to learn!

It's just a fixed function.

$$n_H \times n_W \times n_C$$

$$\downarrow$$

$$\left\lfloor \frac{n_H - f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n_W - f}{s} + 1 \right\rfloor \times n_C$$

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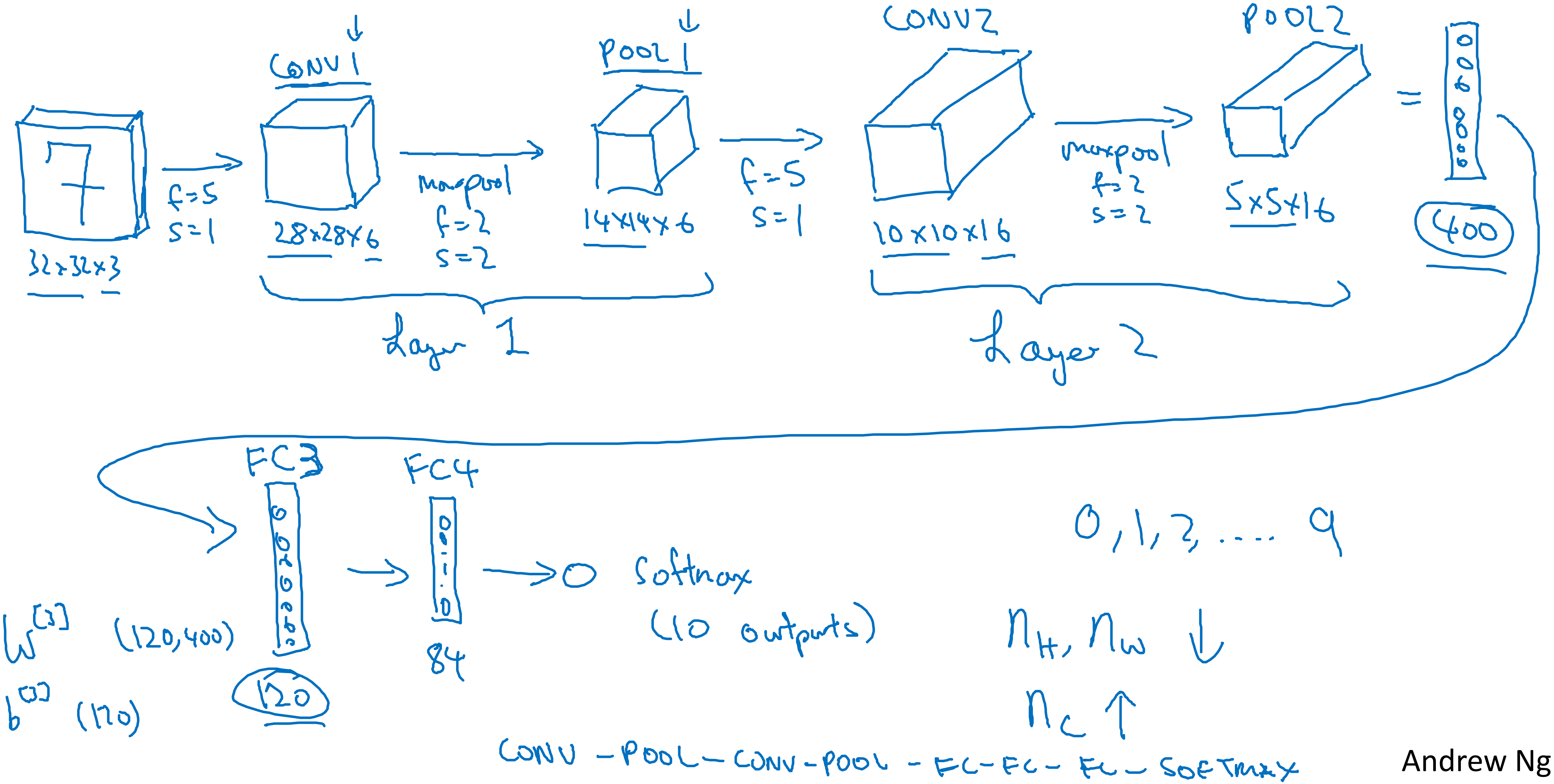
Convolutional Neural Networks

Convolutional neural network example

Neural network example (LeNet-5)



Neural network example

	Activation shape	Activation Size	# parameters
Input:	(32,32,3)	3,072 $a^{[0]}$	0
CONV1 (f=5, s=1)	(28,28,8)	6,272	608 (5*5*3 + 1) * 8 = 608
POOL1	(14,14,8)	1,568	0
CONV2 (f=5, s=1)	(10,10,16)	1,600	3216
POOL2	(5,5,16)	400	0
FC3	(120,1)	120	48120
FC4	(84,1)	84	10164
Softmax	(10,1)	10	850

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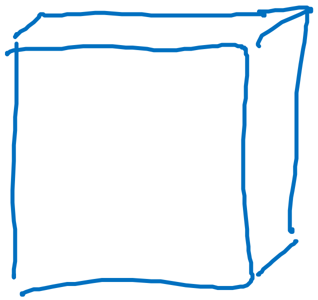
Convolutional Neural Networks

Why convolutions?

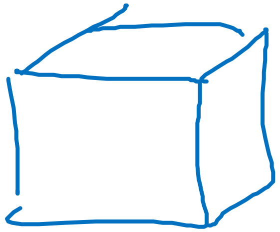
Why convolutions



32×32×3

$\xrightarrow{f=5}$

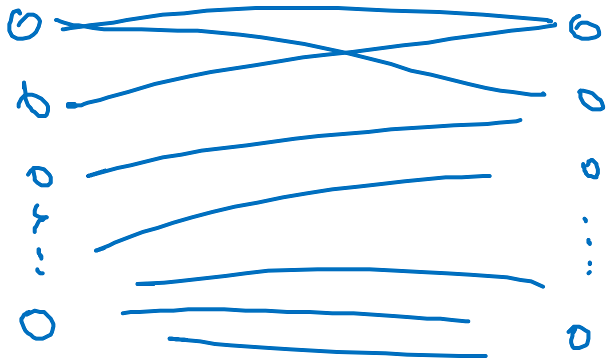
6 filters



28×28×6

3,072

4,704



3072

4704

5×5 − 25

26

~~6 × 26 = 156~~ parameters

(5*5*3+1)*6 = 456

3,072 × 4,704 ≈ 14M

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Why convolutions

translation invariance

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}}_{3 \times 3} = \begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}$$

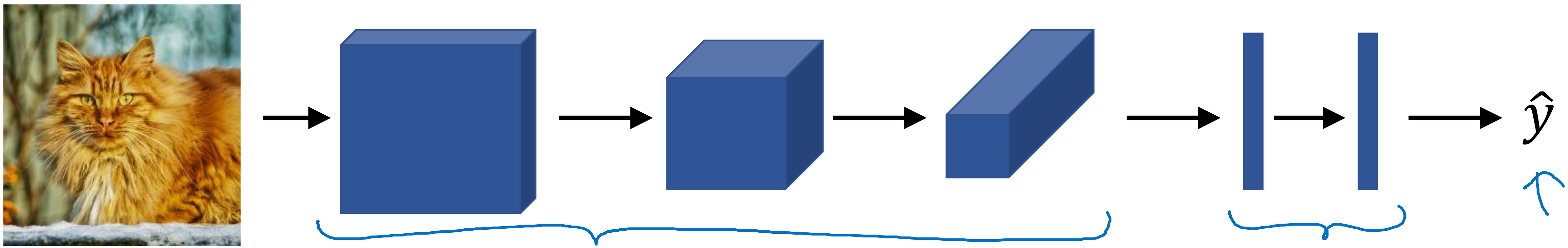
Parameter sharing: A feature detector (such as a vertical edge detector) that's useful in one part of the image is probably useful in another part of the image.

Sparsity of connections: In each layer, each output value depends only on a small number of inputs.

Putting it together

Training set $(x^{(1)}, y^{(1)}) \dots (x^{(m)}, y^{(m)})$.

$\boxed{w, b}$



Cost $J = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

Use gradient descent to optimize parameters to reduce J