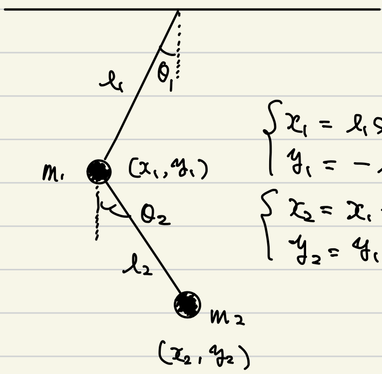

$$\begin{cases} x_1 = l_1 \sin\theta_1 \\ y_1 = -l_1 \cos\theta_1 \end{cases}$$

$$\begin{cases} x_2 = x_1 + l_2 \sin\theta_2 \\ y_2 = y_1 - l_2 \cos\theta_2 \end{cases}$$

$$L = \frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2 - mg\, y_1 - mg\, y_2$$

$$= \frac{m_1}{2}\left[l_1^2 \cos^2\theta_1 \cdot \dot{\theta}_1^2 + l_1^2 \sin^2\theta_1 \cdot \dot{\theta}_1^2\right] + \frac{m_2}{2}\left[(l_1 \cos\theta_1 \cdot \dot{\theta}_1 + l_2 \cos\theta_2 \cdot \dot{\theta}_2)^2 + (l_1 \sin\theta_1 \cdot \dot{\theta}_1 + l_2 \sin\theta_2 \cdot \dot{\theta}_2)^2\right]$$

$$+ mg\, l_1 \cos\theta_1 + m_2 g\, (l_1 \cos\theta_1 + l_2 \cos\theta_2)$$

$$= \frac{m_1}{2} l_1^2 \dot{\theta}_1^2 + \frac{m_2}{2}\left(l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 (\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2)\right)$$

$$+ mg\, l_1 \cos\theta_1 + m_2 g\, (l_1 \cos\theta_1 + l_2 \cos\theta_2)$$

$$= \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\theta}_1^2 + \frac{m_2}{2}\left(l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)\right)$$

$$+ mg\, l_1 \cos\theta_1 + m_2 g\, (l_1 \cos\theta_1 + l_2 \cos\theta_2)$$

① w.r.t. θ_1

$$\frac{\partial L}{\partial \dot{\theta}_1} = (m_1 + m_2) l_1^2 \dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial L}{\partial \theta_1} = -m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_1 g l_1 \sin\theta_1 - m_2 g\, l_1 \sin\theta_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0$$

$$\Leftrightarrow (m_1 + m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_2 (\cancel{\dot{\theta}_1} - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$

$$\cancel{+ m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2)} + (m_1 + m_2) g l_1 \sin\theta_1 = 0$$

2 w.r.t. θ_2

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial L}{\partial \theta_2} = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_2 g l_2 \sin\theta_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0$$

$$\Leftrightarrow m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 - \cancel{\dot{\theta}_2}) \sin(\theta_1 - \theta_2)$$

$$\cancel{- m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2)} + m_2 g l_2 \sin\theta_2 = 0$$

$$\begin{pmatrix} (m_1 + m_2) l_1^2 & m_2 l_1 l_2 \cos(\theta_1 - \theta_2) \\ m_2 l_1 l_2 \cos(\theta_1 - \theta_2) & m_2 l_2^2 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix}$$

$$= \begin{pmatrix} - m_2 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1 + m_2) g l_1 \sin\theta_1 \\ m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - m_2 g l_2 \sin\theta_2 \end{pmatrix}$$