⑥ w.r.t. θ_2

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1 - \theta_2)$$

$$\frac{\partial L}{\partial \theta_2} = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_2 g l_2 \sin\theta_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0$$

$$\Leftrightarrow m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 - \cancel{\dot{\theta}_2}) \sin(\theta_1 - \theta_2)$$

$$\cancel{- m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2)} + m_2 g l_2 \sin\theta_2 = 0$$

$$\begin{pmatrix} (m_1+m_2) l_1^2 & m_2 l_1 l_2 \cos(\theta_1 - \theta_2) \\ m_2 l_1 l_2 \cos(\theta_1 - \theta_2) & m_2 l_2^2 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix}$$

$$= \begin{pmatrix} - m_2 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1+m_2) g l_1 \sin\theta_1 \\ m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - m_2 g l_2 \sin\theta_2 \end{pmatrix}$$