北京交通大学

信号与系统

主讲人：陈后金

电子信息工程学院

连续非周期信号的频域分析

- 连续非周期信号的频域表示
- 典型连续非周期信号的频谱
- 连续时间傅里叶变换的性质

典型连续非周期信号的频谱

常见连续时间信号

单位冲激信号	直流信号	符号函数信号
单位阶跃信号	虚指数信号	余弦信号
一般周期信号	单位冲激串信号	…

典型连续非周期信号的频谱

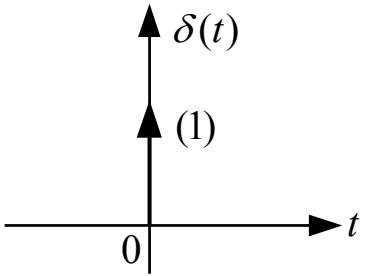
1. 单位冲激信号$\delta(t)$

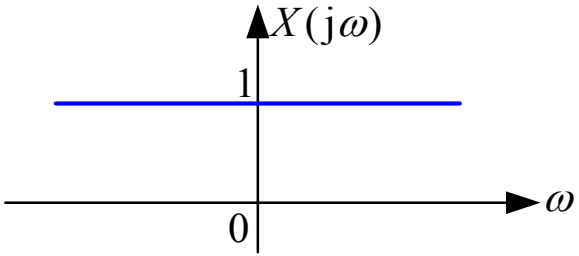
$$\mathscr{F}[\delta(t)]=\int_{-\infty}^{\infty}x(t)\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t$$

$$=\int_{-\infty}^{\infty}\delta(t)\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t$$

$$=1$$

$$\mathscr{F}[\delta(t)]=1$$





典型连续非周期信号的频谱

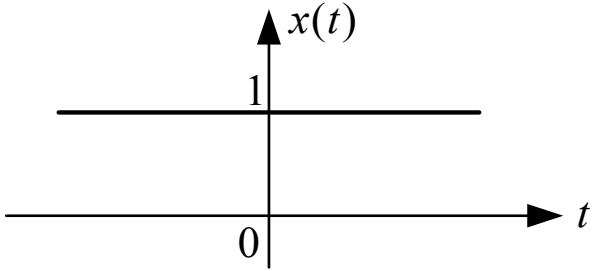
2. 直流信号 $x(t)=1,-\infty<t<\infty$

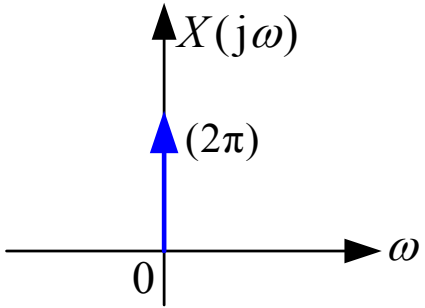
$$\mathscr{F}[x(t)]=\int_{-\infty}^{\infty}x(t)\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t$$

$$=\int_{-\infty}^{\infty}\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t \quad =2\pi\delta(\omega)$$

$$\delta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}1\cdot\mathrm{e}^{\mathrm{j}\omega t}\mathrm{d}\omega$$

$$\mathscr{F}[1]=2\pi\delta(\omega)$$

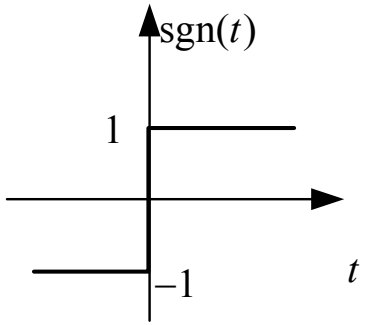


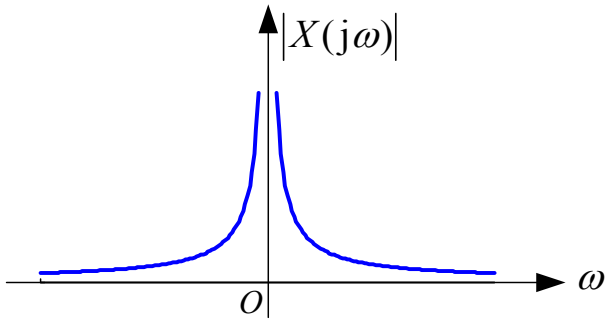


典型连续非周期信号的频谱

3. 符号函数信号$\mathrm{sgn}(t)$

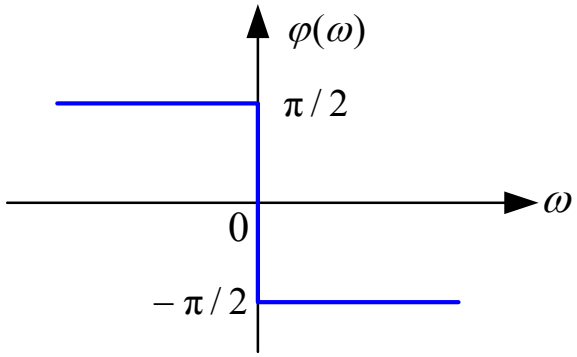
$$\mathrm{sgn}(t)=\begin{cases}-1 & t<0\\ 0 & t=0\\ 1 & t>0\end{cases}$$





$$\mathscr{F}[\mathrm{sgn}(t)\mathrm{e}^{-\sigma|t|}]=\int_{-\infty}^{0}(-1)\mathrm{e}^{\sigma t}\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t+\int_{0}^{\infty}\mathrm{e}^{-\sigma t}\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t$$

$$=\frac{-1}{\sigma-\mathrm{j}\omega}+\frac{1}{\sigma+\mathrm{j}\omega}$$

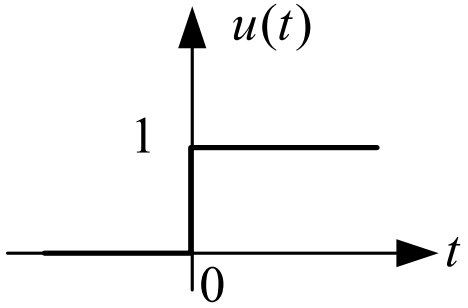


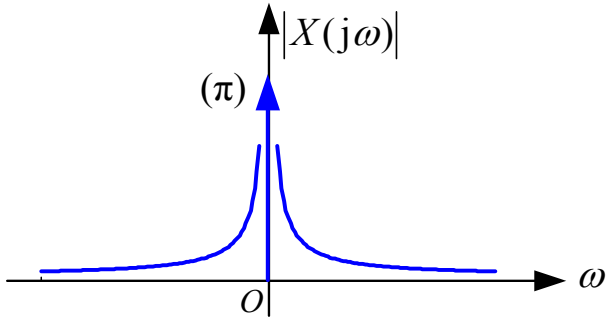
$$\mathrm{sgn}(t)=\lim_{\sigma\to 0}\mathrm{sgn}(t)\mathrm{e}^{-\sigma|t|}$$

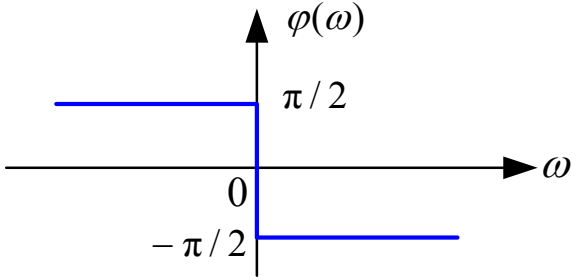
$$\mathscr{F}[\mathrm{sgn}(t)]=\frac{2}{\mathrm{j}\omega}$$

典型连续非周期信号的频谱

4. 单位阶跃信号$u(t)$







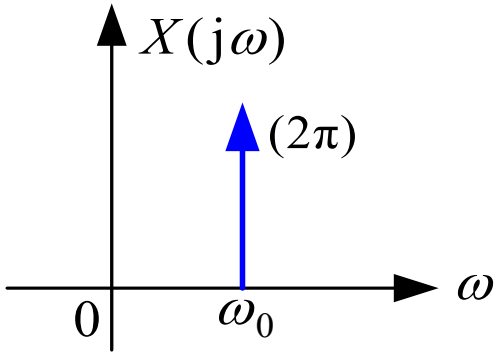
$$\mathscr{F}[u(t)]=\pi\delta(\omega)+\frac{1}{\mathrm{j}\omega}$$

典型连续非周期信号的频谱

5. 虚指数信号 $e^{j\omega_0 t}\ (-\infty < t < \infty)$

$$\mathscr{F}[e^{j\omega_0 t}] = \int_{-\infty}^{\infty} e^{-j(\omega-\omega_0)t}\,dt$$

$$= 2\pi\delta(\omega-\omega_0)$$



$$\mathscr{F}[e^{j\omega_0 t}] = 2\pi\delta(\omega-\omega_0)$$

典型连续非周期信号的频谱

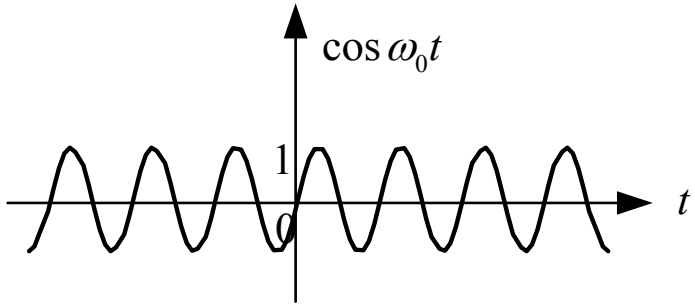
6. 余弦信号 $\cos(\omega_0 t)$

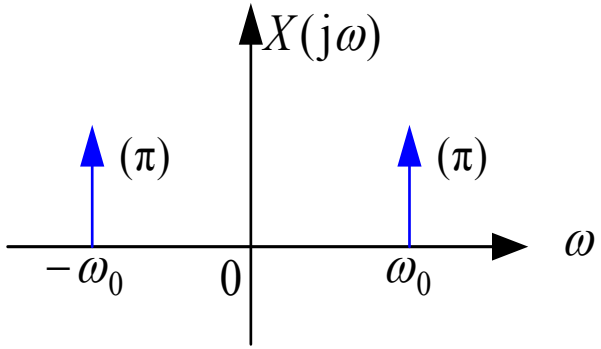
$$\cos(\omega_0 t)=\frac{1}{2}(\mathrm{e}^{\mathrm{j}\omega_0 t}+\mathrm{e}^{-\mathrm{j}\omega_0 t})$$

$$\mathscr{F}[\mathrm{e}^{\mathrm{j}\omega_0 t}]=2\pi\delta(\omega-\omega_0)$$

$$\mathscr{F}[\cos\omega_0 t]=\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$$

$$\mathscr{F}[\sin\omega_0 t]=\mathrm{j}\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$$





典型连续非周期信号的频谱

7. 一般周期信号

$$\tilde{x}(t)=\sum_{n=-\infty}^{+\infty}C_n\mathrm{e}^{\mathrm{j}n\omega_0 t}\qquad(\omega_0=\frac{2\pi}{T_0})$$

$$\mathscr{F}[\tilde{x}(t)]=X(\mathrm{j}\omega)=\mathscr{F}[\sum_{n=-\infty}^{+\infty}C_n\mathrm{e}^{\mathrm{j}n\omega_0 t}]=\sum_{n=-\infty}^{+\infty}C_n\cdot\mathscr{F}[\mathrm{e}^{\mathrm{j}n\omega_0 t}]$$

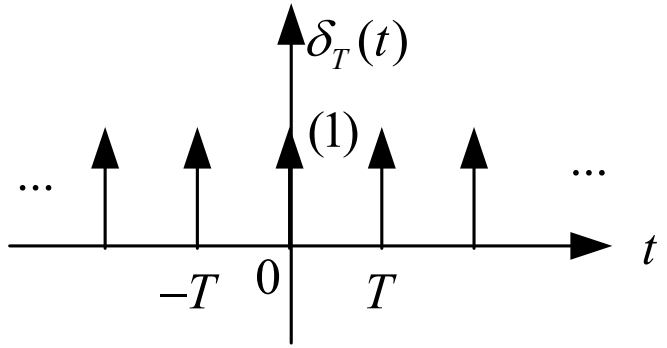
$$\mathscr{F}[\tilde{x}(t)]=2\pi\sum_{n=-\infty}^{+\infty}C_n\delta(\omega-n\omega_0)$$

典型连续非周期信号的频谱

8. 单位冲激串 $\delta_T(t)$

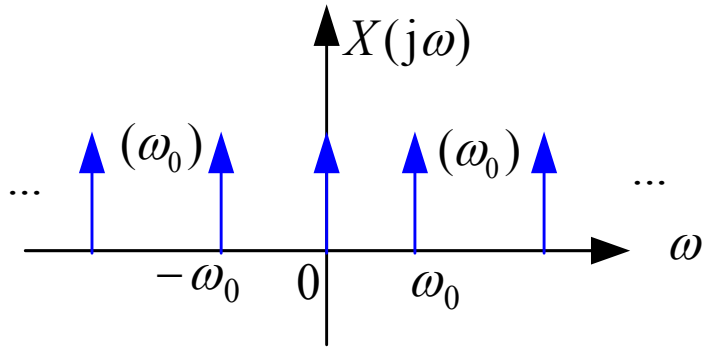
$$\delta_T(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)=\sum_{n=-\infty}^{+\infty}C_n\mathrm{e}^{\mathrm{j}n\omega_0 t}$$

$$C_n=\frac{1}{T}\int_{-T/2}^{T/2}\delta(t)\mathrm{e}^{-\mathrm{j}n\omega_0 t}\mathrm{d}t=\frac{1}{T}\int_{-T/2}^{T/2}\delta(t)\mathrm{d}t=\frac{1}{T}$$



$$\mathscr{F}[\delta_T(t)]=2\pi\sum_{n=-\infty}^{+\infty}\frac{1}{T}\delta(\omega-n\omega_0)$$

$$\mathscr{F}[\delta_T(t)]=\omega_0\sum_{n=-\infty}^{+\infty}\delta(\omega-n\omega_0)$$



谢　谢

本课程所引用的一些素材为主讲老师多年的教学积累，来源于多种媒体及同事、同行、朋友的交流，难以一一注明出处，特此说明并表示感谢！