北京交通大學

信号与系统

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利用MATLAB分析离散信号与系统

- $X(z)$部分分式展开的MATLAB实现
- $H(z)$零极点与系统特性的MATLAB计算

$X(z)$部分分式展开的MATLAB实现

离散信号的z变换$X(z)$通常可用有理分式表示：

$$X(z)=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\cdots+b_mz^{-m}}{a_0+a_1z^{-1}+a_2z^{-2}+\cdots+a_nz^{-n}}=\frac{\mathrm{num}(z)}{\mathrm{den}(z)}$$

$$=k+\frac{r_1}{1-p_1z^{-1}}+\frac{r_2}{1-p_2z^{-1}}+\cdots+\frac{r_n}{1-p_nz^{-1}}$$

$$[r,p,k]=\mathrm{residuez}(\mathrm{num},\mathrm{den})$$

num 为$X(z)$分子多项式的系数向量

den 为$X(z)$分母多项式的系数向量

$r=r_1, r_2, \ldots, r_n$ 为部分分式的系数，$p=p_1, p_2, \ldots, p_n$为极点。

[例] 利用MATLAB实现 $X(z)=\dfrac{18}{18+3z^{-1}-4z^{-2}-z^{-3}}$ 部分分式展开。

MATLAB程序为：

```
num = [18]; den = [18 3 –4 –1];
[r,p,k] = residuez(num, den)
```

运行结果为：

r =0.3600 , 0.2400 , 0.4000
p =0.5000 , – 0.3333 , – 0.3333
k =[]

故$X(z)$可展开为：$X(z)=\dfrac{0.36}{1-0.5z^{-1}}+\dfrac{0.24}{1+0.3333z^{-1}}+\dfrac{0.4}{(1+0.3333z^{-1})^2}$

$x[k]$可表示为：$x[k]=0.36(0.5)^k+0.24(-0.3333)^k+0.4(k+1)(-0.3333)^k$

$H(z)$零极点与系统特性的MATLAB计算

离散系统的系统函数$H(z)$可由有理分式表示：

$$H(z)=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\cdots+b_mz^{-m}}{a_0+a_1z^{-1}+a_2z^{-2}+\cdots+a_nz^{-n}}=\frac{\mathrm{num}(z)}{\mathrm{den}(z)}$$

利用impz函数计算$H(z)$的单位脉冲响应$h[k]$，调用形式为

h = impz(num, den, N)

num和den分别为$H(z)$分子多项式和分母多项式的系数向量。

N表示单位脉冲响应输出序列个数，返回值h是单位脉冲响应。

$H(z)$零极点分布图可用zplane函数画出，调用形式为：

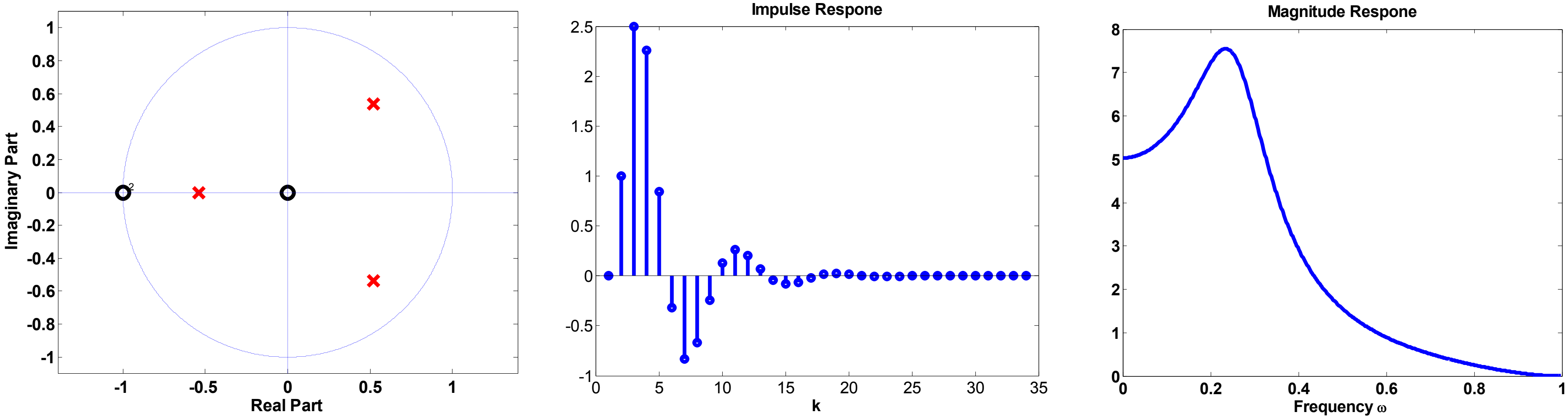
zplane(num, den)

[例] 试画出离散系统 $H(z)=\dfrac{z^{-1}+2z^{-2}+z^{-3}}{1-0.5z^{-1}-0.005z^{-2}+0.3z^{-3}}$ 零极点分布图，求其单位脉冲响应$h[k]$和频率响应$H(\mathrm{e}^{\mathrm{j}\Omega})$。

MATLAB程序如下:

```
num = [0 1 2 1];
den = [1 –0.5 –0.005 0.3];
figure(1); zplane (num, den);
N = 34;
h = impz (num, den, N);
figure(2);  stem (h);
title ('Impulse Response');
w = linspace (0, pi, 1000);
H = freqz (num, den, w);
figure(3); plot (w/pi, abs(H));
xlabel ('Frequency \omega');
title ('Magnitude Response');
```

[例] 试画出离散系统 $H(z)=\dfrac{z^{-1}+2z^{-2}+z^{-3}}{1-0.5z^{-1}-0.005z^{-2}+0.3z^{-3}}$ 零极点分布图，求其单位冲激响应$h[k]$和频率响应$H(\mathrm{e}^{\mathrm{j}\Omega})$ 。

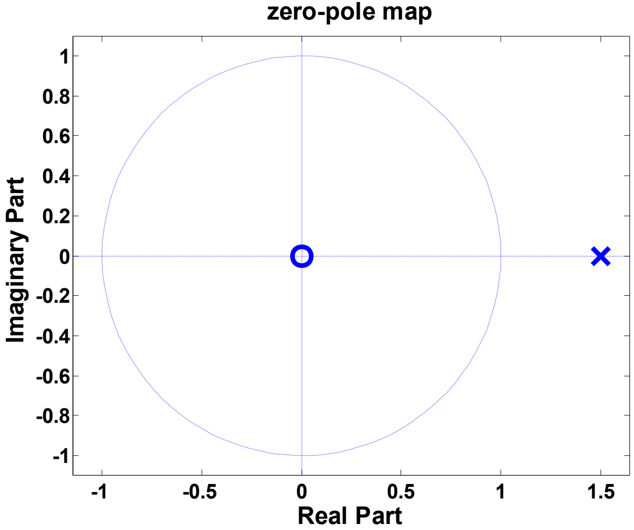


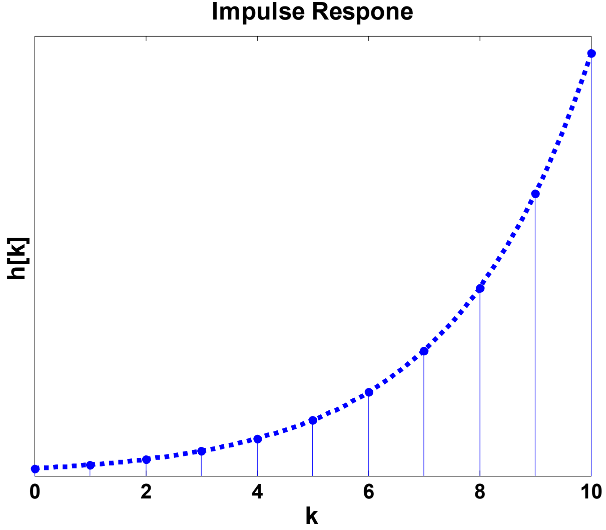
运行结果

极点在实轴上变化

```
num = [1];
z1 = 1.5;
den = [1, -z1];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z) = \frac{1}{1-1.5z^{-1}} = \frac{z}{z-1.5}$$

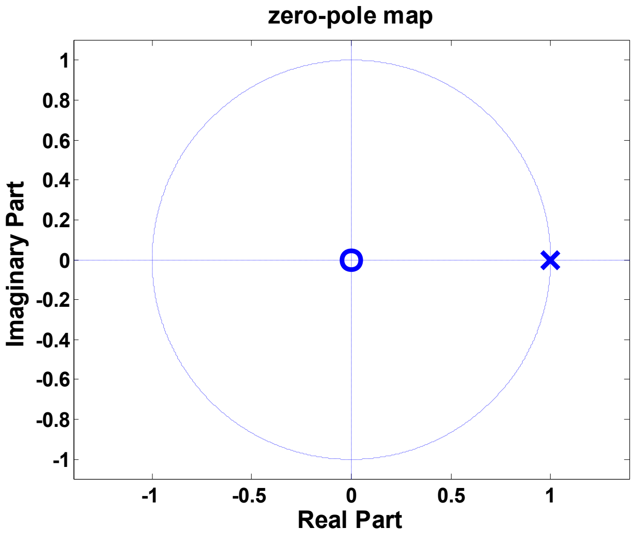


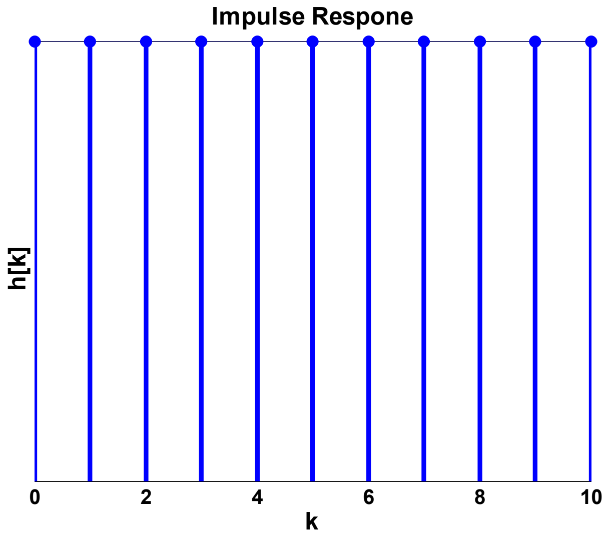


极点在实轴上变化

```
num = [1];
z1 = 1.0;
den = [1, -z1];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1}{1-1\cdot z^{-1}}=\frac{z}{z-1}$$

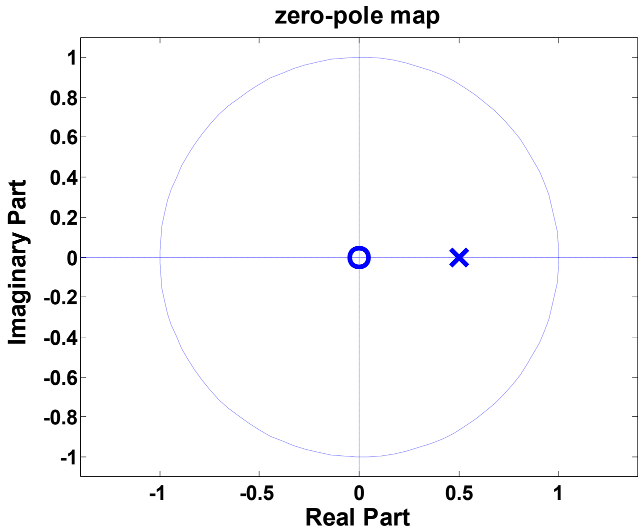


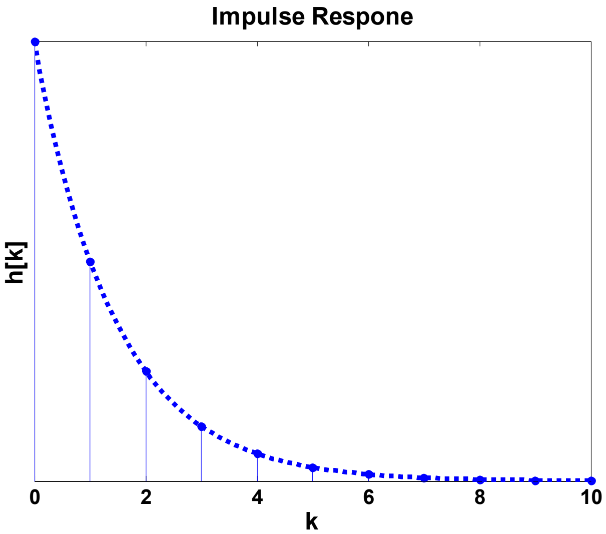


极点在实轴上变化

```
num = [1];
z1 = 0.5;
den = [1, –z1];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1}{1-0.5\cdot z^{-1}}=\frac{z}{z-0.5}$$

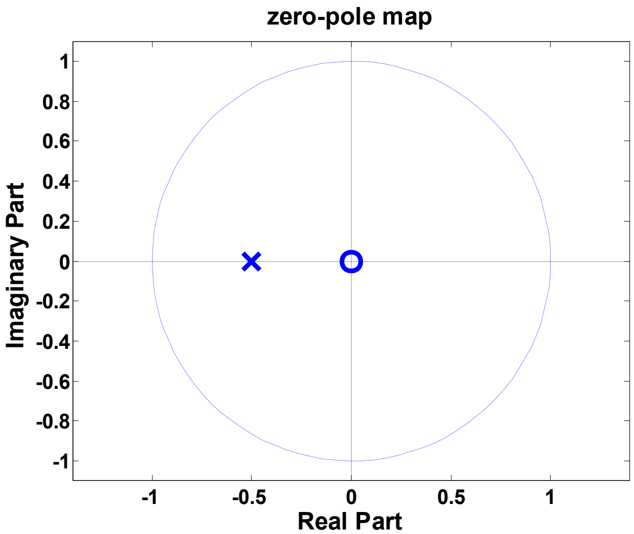


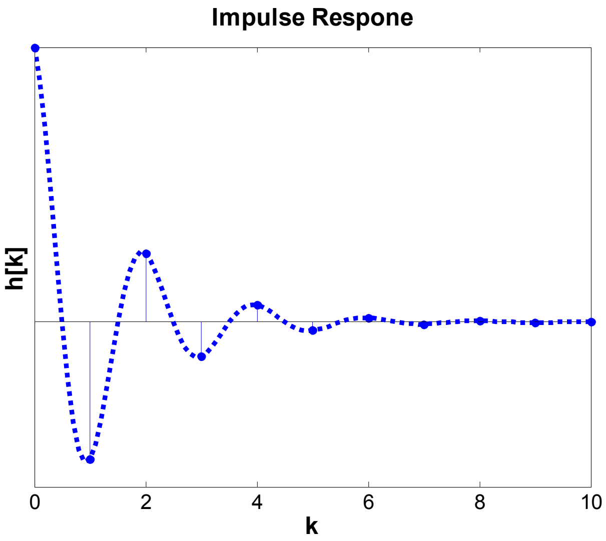


极点在实轴上变化

```
num = [1];
z1 = – 0.5;
den = [1, – z1];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1}{1-(-0.5)\cdot z^{-1}}=\frac{z}{z-(-0.5)}=\frac{z}{z+0.5}$$

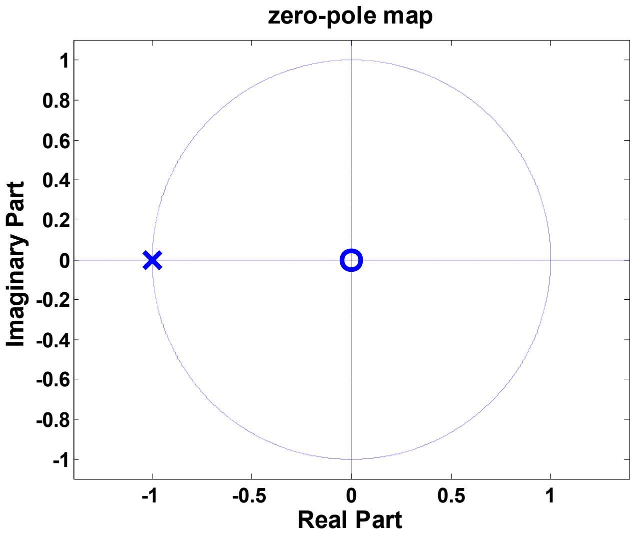


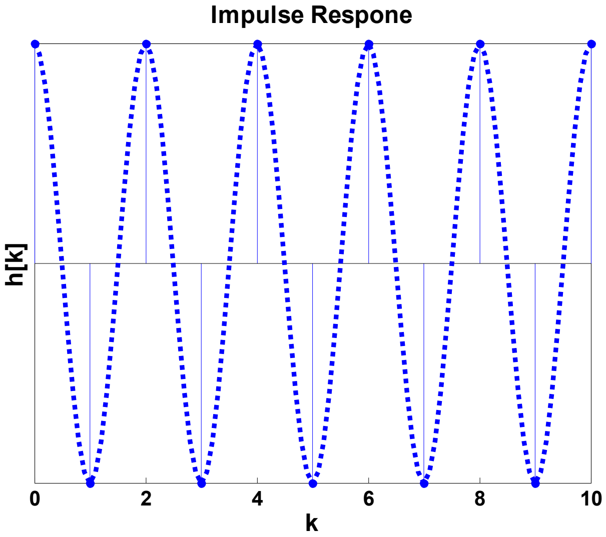


极点在实轴上变化

```
num = [1];
z1 = – 1;
den = [1, – z1];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1}{1-(-1)\cdot z^{-1}}=\frac{z}{z-(-1)}=\frac{z}{z+1}$$

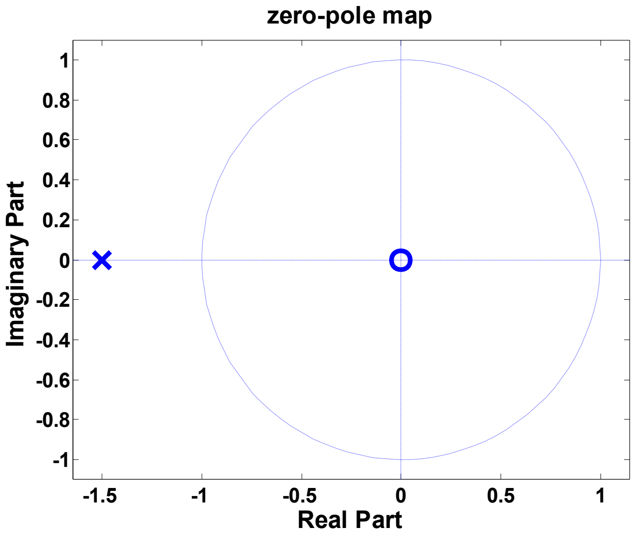


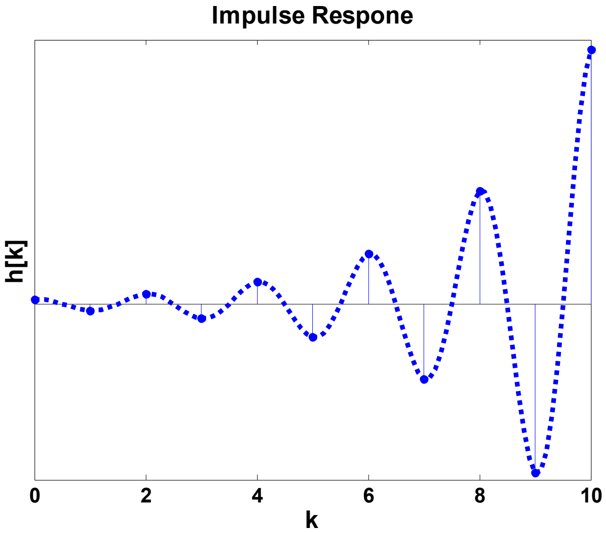


极点在实轴上变化

```
num = [1];
z1 = – 1.5;
den = [1, – z1];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1}{1-(-1.5)\cdot z^{-1}}=\frac{z}{z-(-1.5)}=\frac{z}{z+1.5}$$



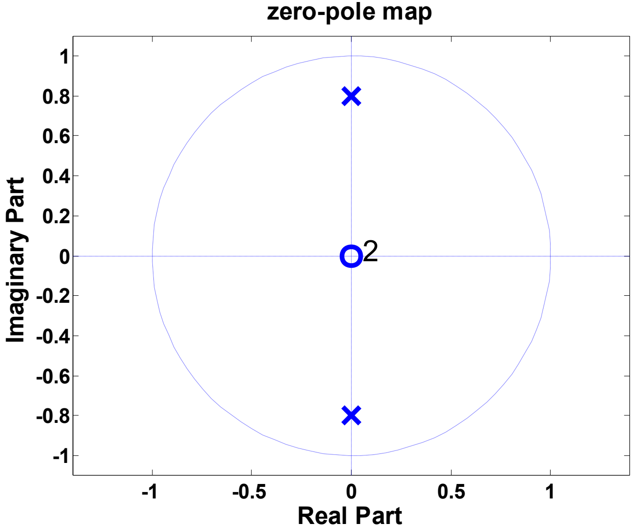


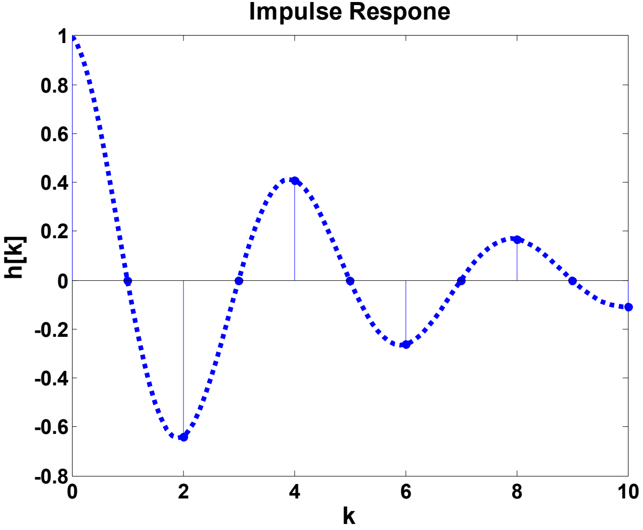
极点在虚轴上变化

```
r = 0.8;
num = [1];
den = [1, 0, r*r];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1}{1+0.8^2 z^{-2}}=\frac{1}{(1-0.8jz^{-1})(1+0.8jz^{-1})}$$

$$=\frac{z^2}{(z-0.8j)(z+0.8j)}=\frac{z^2}{z^2+0.8*0.8}$$



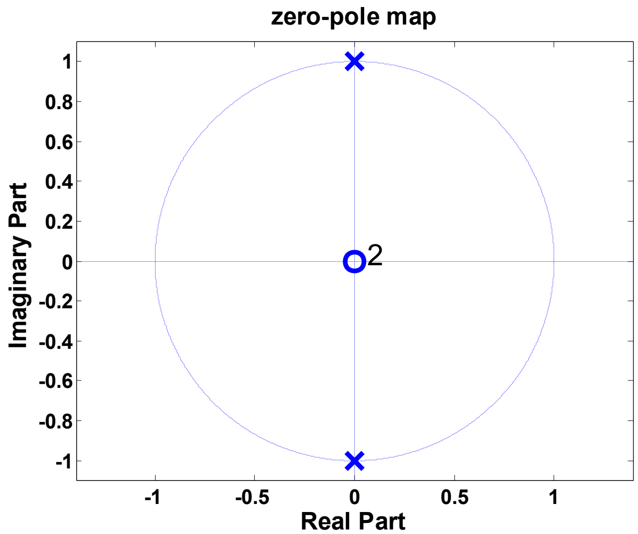


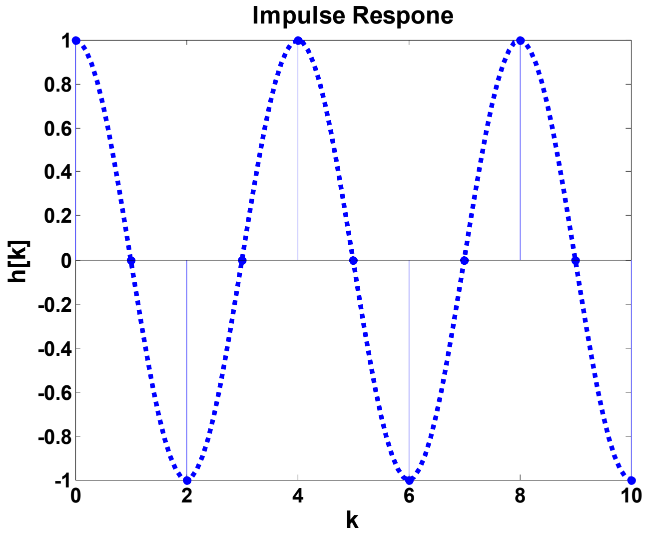
极点在虚轴上变化

```
r = 1;
num = [1];
den = [1, 0, r*r];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1}{1+z^{-2}}=\frac{1}{(1-\mathrm{j}z^{-1})(1+\mathrm{j}z^{-1})}$$

$$=\frac{z^2}{(z-\mathrm{j})(z+\mathrm{j})}=\frac{z^2}{z^2+1}$$



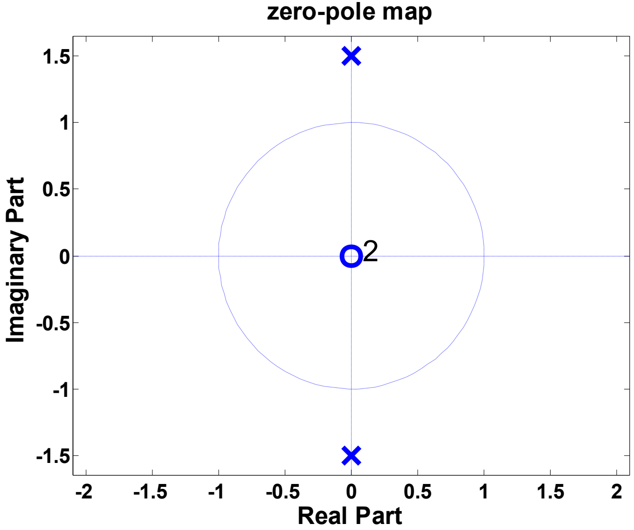


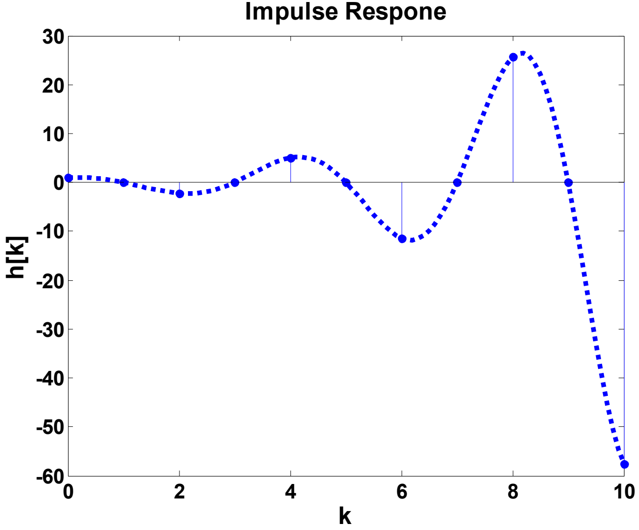
极点在虚轴上变化

```
r = 1.5;
num = [1];
den = [1, 0, r*r];
zplane (num, den);
h = impz(num, den, 11);
```

$$H(z)=\frac{1}{1+1.5^2z^{-2}}=\frac{1}{(1-1.5\mathrm{j}z^{-1})(1+1.5\mathrm{j}z^{-1})}$$

$$=\frac{z^2}{(z-1.5\mathrm{j})(z+1.5\mathrm{j})}=\frac{z^2}{z^2+1.5*1.5}$$



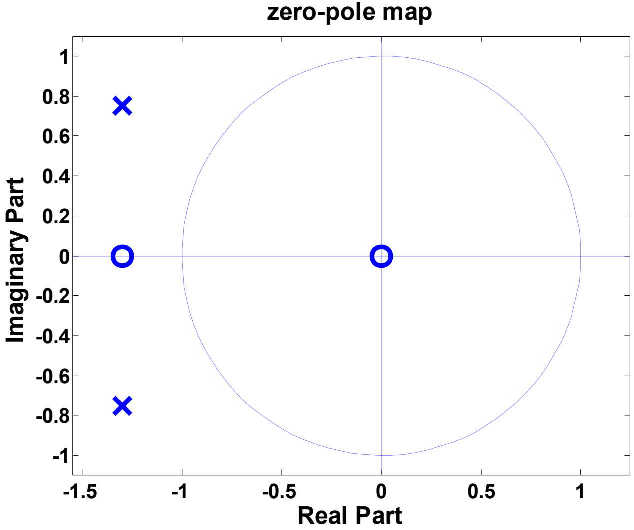


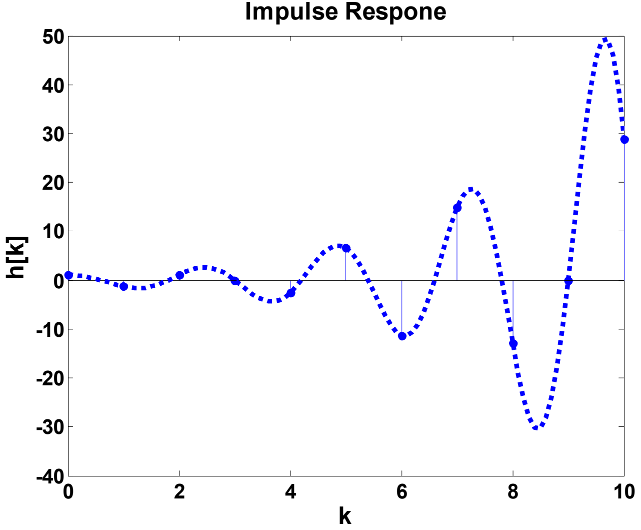
零极点在左半平面变化

r = 1.5;   theta = 5*pi/6;
num = [1, – r*cos(theta)];
den = [1, – 2*r*cos(theta), r*r];
zplane (num, den);
h = impz (num, den, 11);

$$H(z)=\frac{1-r\cos(\theta)z^{-1}}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}=\frac{1-r\cos(\theta)z^{-1}}{(1-r\,\mathrm{e}^{\mathrm{j}\theta}z^{-1})(1-r\,\mathrm{e}^{-\mathrm{j}\theta}z^{-1})}$$

$$=\frac{z\left[z-1.5\cdot\cos(5\pi/6)\right]}{z^2-2\cdot1.5\cdot\cos(5\pi/6)+1.5*1.5}$$



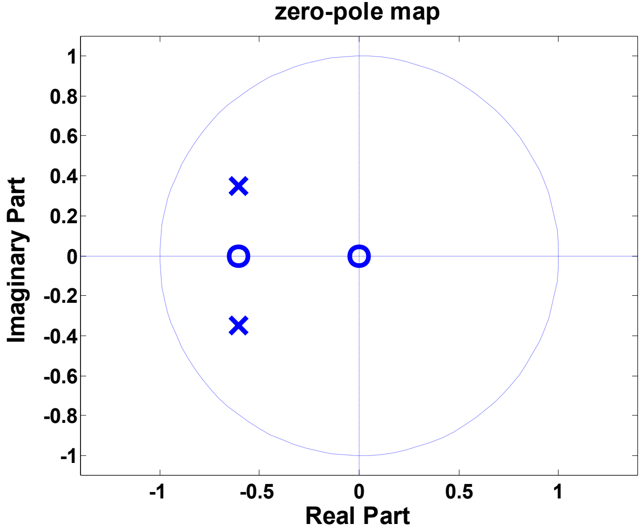


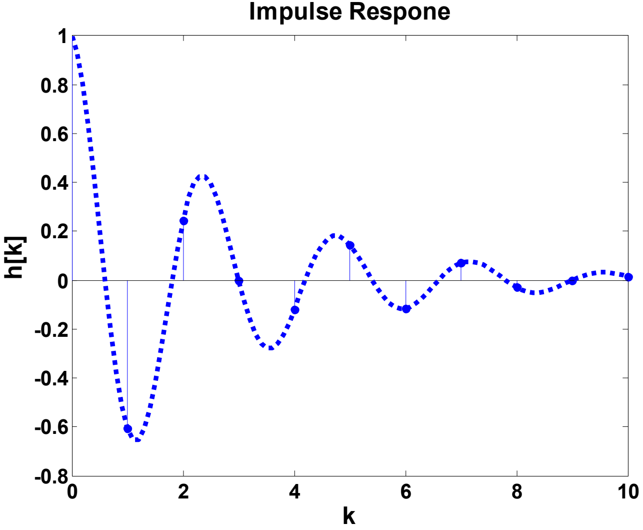
零极点在左半平面变化

```
r = 0.7;   theta = 5*pi/6;
num = [1, – r*cos(theta)];
den = [1, – 2*r*cos(theta), r*r];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1-r\cos(\theta)z^{-1}}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}=\frac{1-r\cos(\theta)z^{-1}}{(1-r\mathrm{e}^{\mathrm{j}\theta}z^{-1})(1-r\mathrm{e}^{-\mathrm{j}\theta}z^{-1})}$$

$$=\frac{z[z-0.7\cdot\cos(5\pi/6)]}{z^2-2\cdot 0.7\cdot\cos(5\pi/6)+0.7*0.7}$$



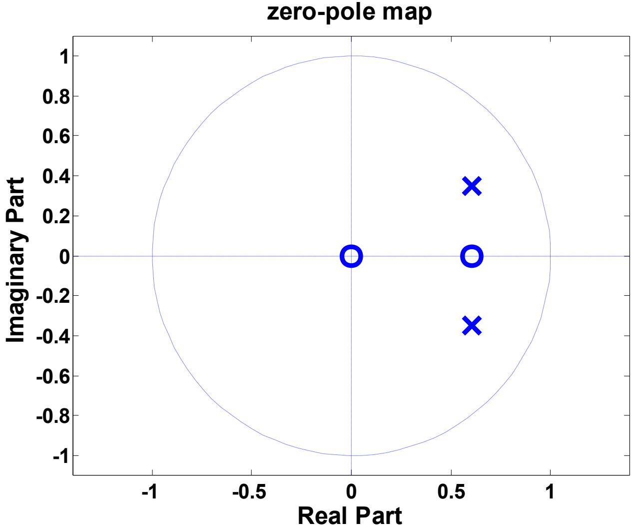


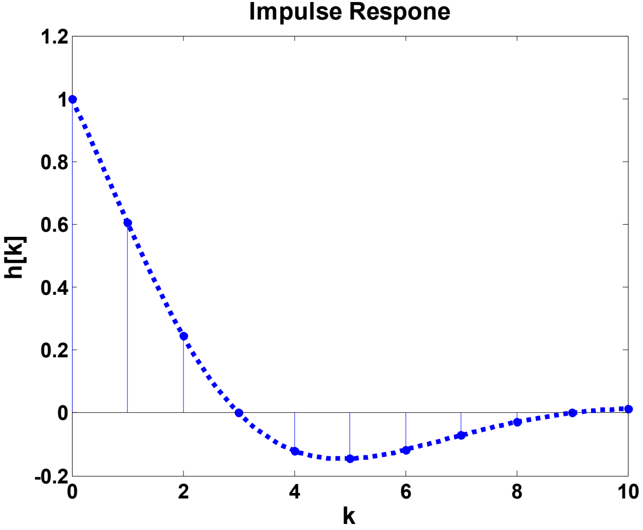
零极点在右半平面变化

```
r = 0.7;   theta = 1*pi/6;
num = [1, – r*cos(theta)];
den = [1, – 2*r*cos(theta), r*r];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1-r\cos(\theta)z^{-1}}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}=\frac{1-r\cos(\theta)z^{-1}}{(1-r\mathrm{e}^{\mathrm{j}\theta}z^{-1})(1-r\mathrm{e}^{-\mathrm{j}\theta}z^{-1})}$$

$$=\frac{z\left[z-0.7\cdot\cos(\pi/6)\right]}{z^2-2\cdot0.7\cdot\cos(\pi/6)+0.7*0.7}$$



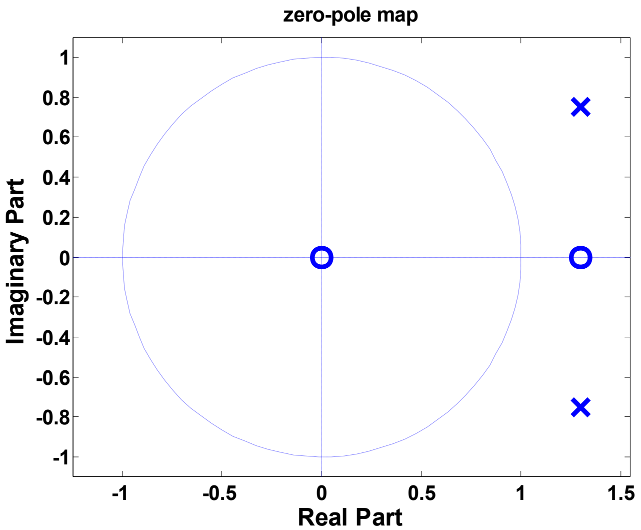


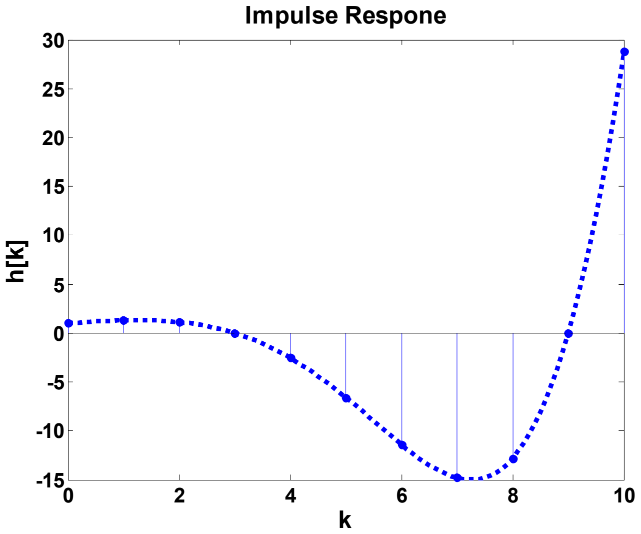
零极点在右半平面变化

```
r = 1.5;   theta = 1*pi/6;
num = [1, – r*cos(theta)];
den = [1, – 2*r*cos(theta), r*r];
zplane (num, den);
h = impz (num, den, 11);
```

$$H(z)=\frac{1-r\cos(\theta)z^{-1}}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}=\frac{1-r\cos(\theta)z^{-1}}{(1-r\mathrm{e}^{\mathrm{j}\theta}z^{-1})(1-r\mathrm{e}^{-\mathrm{j}\theta}z^{-1})}$$

$$=\frac{z[z-1.5\cdot\cos(\pi/6)]}{z^2-2\cdot1.5\cdot\cos(\pi/6)+1.5*1.5}$$





谢 谢

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