北京交通大学

信号与系统

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利用MATLAB分析连续信号与系统

- $X(s)$部分分式展开的MATLAB实现
- $H(s)$零极点与系统特性的MATLAB计算

$X(s)$部分分式展开的MATLAB实现

$$X(s)=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}$$

$$=k_1+k_2 s+\cdots+k_m s^m+\frac{r_1}{s-p_1}+\frac{r_2}{s-p_2}+\cdots+\frac{r_n}{s-p_n}$$

```
[r,p,k]=residue(num, den)
```

num,den分别为$X(s)$分子多项式和分母多项式的系数向量。

$r_1\ r_2,\ldots,\ r_n$为部分分式的系数，$p_1\ p_2,\ldots,\ p_n$为极点，

若$X(s)$为真分式， 则$k_1,\ k_2\ ,\ldots,\ k_m$为零。

[例] 利用部分分式展开法求$X(s)$的反变换。

$$X(s)=\frac{s+2}{s^3+4s^2+3s}$$

```
format rat  %将结果数据以分数的形式输出
num=[1 2];  den=[1 4 3 0];
[r,p,k]=residue(num, den)
```

运行结果为　r = –1/6, –1/2, 2/3

p = –3,　 –1,　0

k=[]

故$X(s)$可展开为 $X(s)=\frac{-1/6}{s+3}+\frac{-1/2}{s+1}+\frac{2/3}{s}$

$$x(t)=\mathscr{L}^{-1}[X(s)]=-\frac{1}{6}\mathrm{e}^{-3t}u(t)-\frac{1}{2}\mathrm{e}^{-t}u(t)+\frac{2}{3}u(t)$$

[例] 利用部分分式展开法求$X(s)$的反变换。

$$X(s)=\frac{2s^3+3s^2+5}{(s+1)(s^2+s+2)}$$

```
num=[2 3 0 5];
den=conv([1 1],[1 1 2]);
%将因子相乘的形式转换成多项式的形式
[r,p,k]=residue(num,den)
magr=abs(r)      %求r的模
angr=angle(r)   %求r的相角
```

[例] 利用部分分式展开法求$X(s)$的反变换。

$$X(s)=\frac{2s^3+3s^2+5}{(s+1)(s^2+s+2)}$$

运行结果为

- r = –2.0000 + 1.1339i, –2.0000 – 1.1339i, 3.0000
- p = –0.5000 + 1.3229i, –0.5000 – 1.3229i, –1.0000
- k =2
- magr =2.2991, 2.2991, 3.0000
- angr = 2.6258, –2.6258, 0

故$X(s)$可展开为

$$X(s)=2+\frac{2.2991\mathrm{e}^{\mathrm{j}2.6258}}{s+0.5-\mathrm{j}1.3229}+\frac{2.2991\mathrm{e}^{-\mathrm{j}2.6258}}{s+0.5+\mathrm{j}1.3229}+\frac{3}{s+1}$$

$$x(t)=2\delta(t)+1.1495\mathrm{e}^{-0.5t}\cos(1.3229t+2.6258)u(t)+3\mathrm{e}^{-t}u(t)$$

$H(s)$零极点与系统特性的MATLAB计算

$$H(s)=\frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$H(s)$零极点分布图可用pzmap函数画出，调用形式为

```
sys=tf(num, den)
pzmap(sys)
```

num,den分别为$H(s)$分子多项式和分母多项式的系数向量。

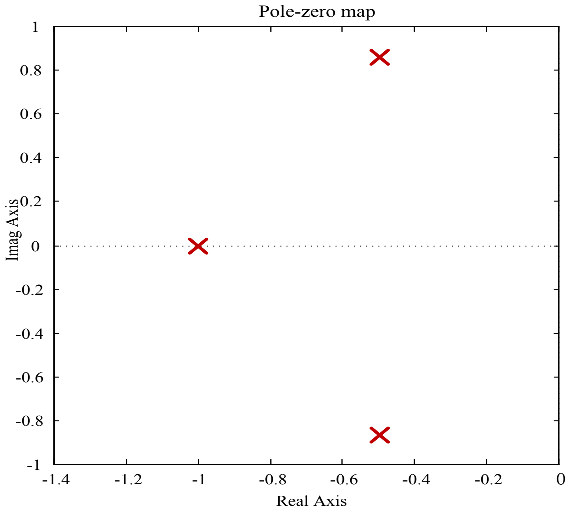
pzmap函数画出sys所描述系统的零极点图。

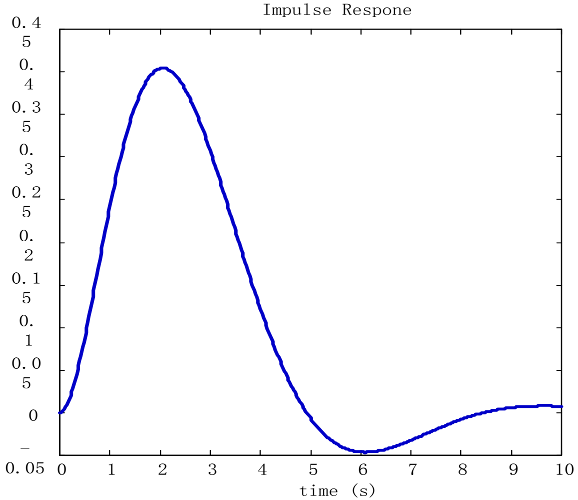
[例] 画出连续系统 $H(s)=\dfrac{1}{s^3+2s^2+2s+1}$ 的零极点分布图，求其冲激响应$h(t)$和频率响应$H(\mathrm{j}\omega)$，并判断系统是否稳定。

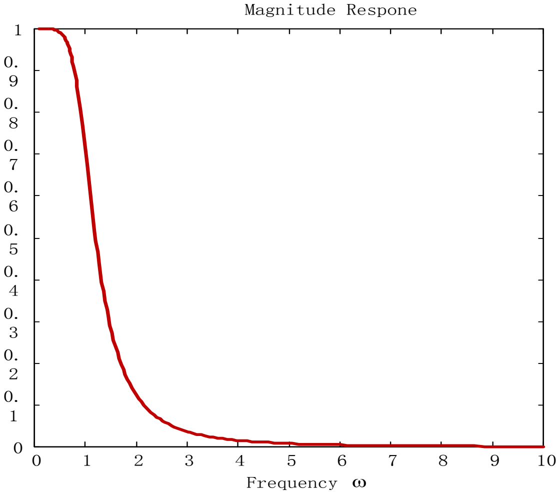
```
num=[1];  den=[1 2 2 1];
sys=tf(num,den);
figure(1); pzmap(sys);
t=0:0.02:10;
h=impulse(num,den,t);
figure(2);  plot(t,h)
title('Impulse Respone')
w=0:0.01:10;
H=freqs(num,den,w);
figure(3);  plot(w,abs(H))
xlabel('Frequency \omega')
title('Magnitude Respone')
```

[例] 画出连续系统 $H(s)=\dfrac{1}{s^3+2s^2+2s+1}$ 的零极点分布图，求其冲激响应$h(t)$和频率响应$H(\mathrm{j}\omega)$，并判断系统是否稳定。

系统稳定





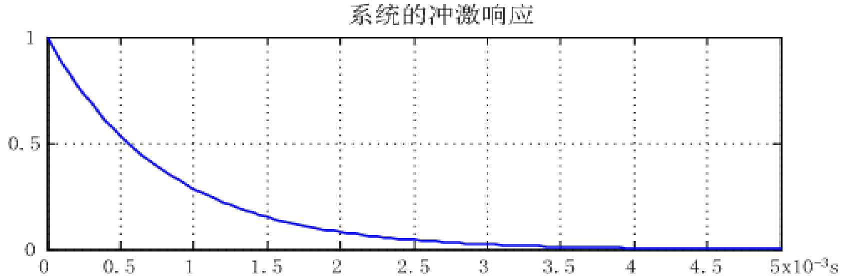


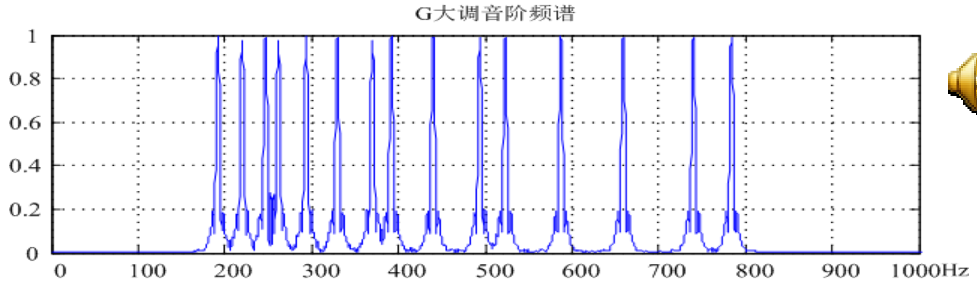
运行结果

[例] 讨论系统函数$H(s)$的零点和极点取不同值时，该系统的冲激响应$h(t)$和幅度响应$|H(j\omega)|$。

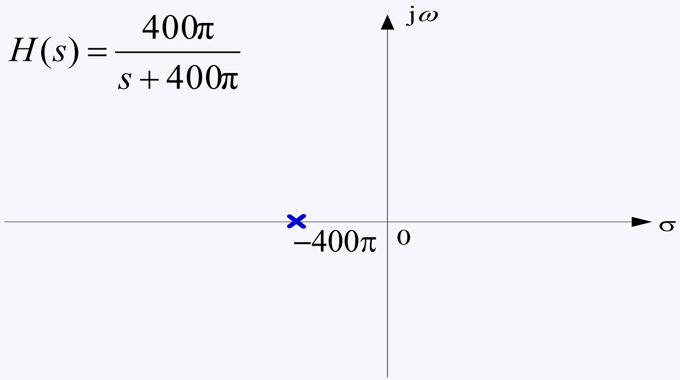
```
%以一阶系统为例
p1=input('p1=');
num=[1];
den=[1 –p1];
sys=tf(num,den);
figure(1); pzmap(sys);
t=0: 0.0001: 0.005;
h=impulse(num,den,t);
figure(2); plot(t,h)
w=0:0.1:1000*2*pi;
H=freqs(num,den, w);
figure(3); plot(w/2/pi, abs(H));
xlabel('Hz')
```

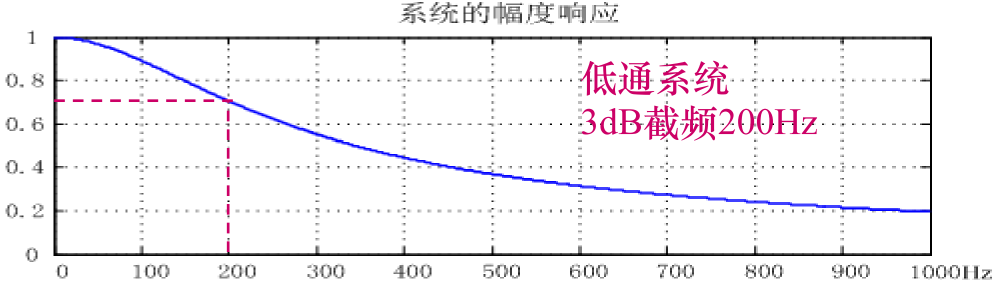
无零点；单极点p1= –400π

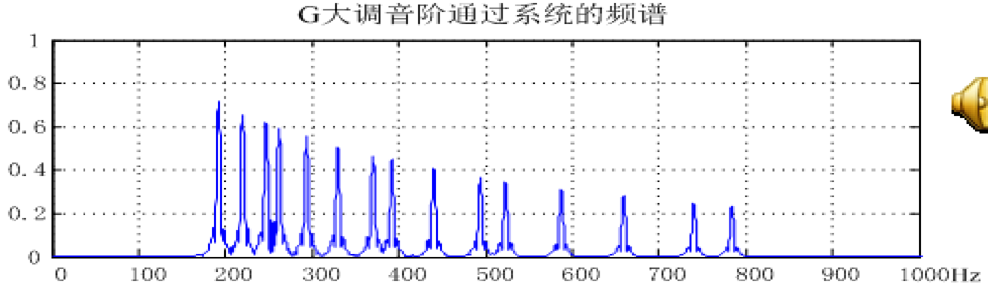




$$H(s)=\frac{400\pi}{s+400\pi}$$

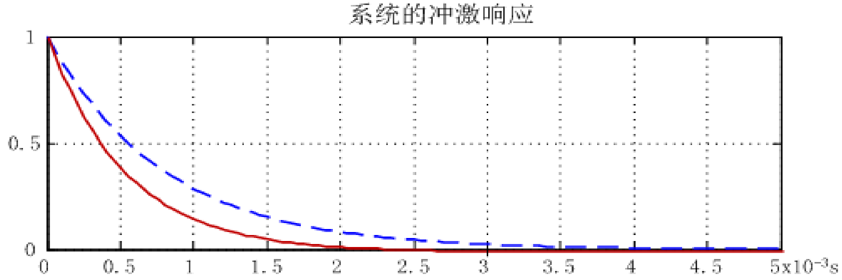


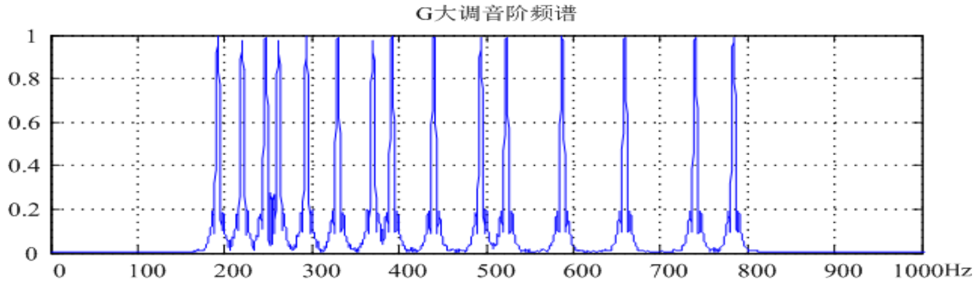




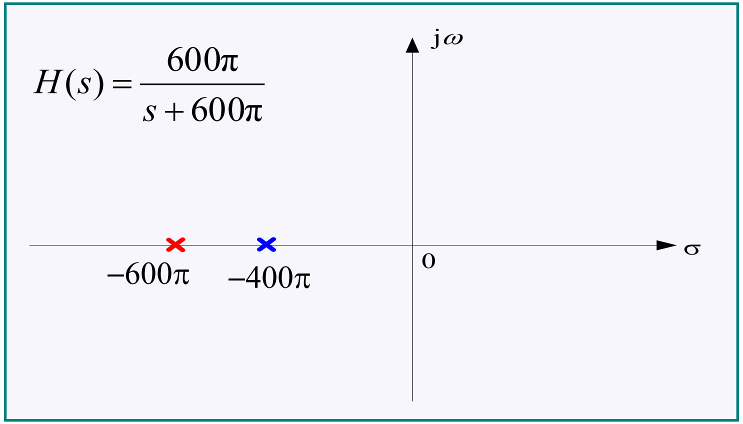
*S*左半平面的单实数极点

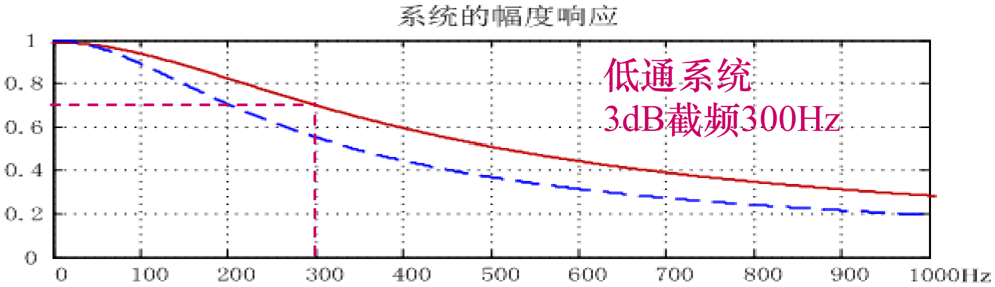
无零点；单极点p1= –600π



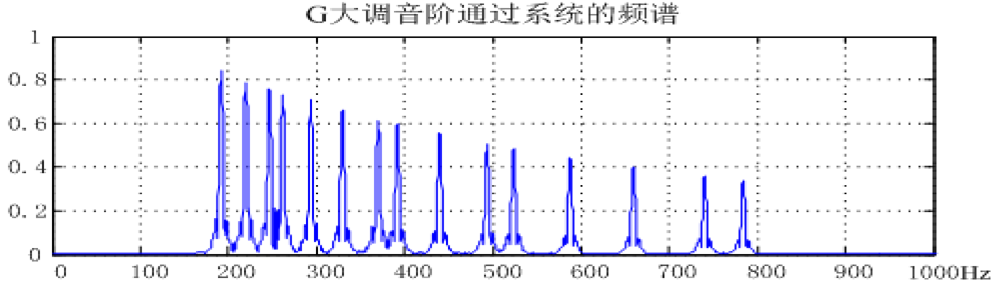


$$H(s)=\frac{600\pi}{s+600\pi}$$

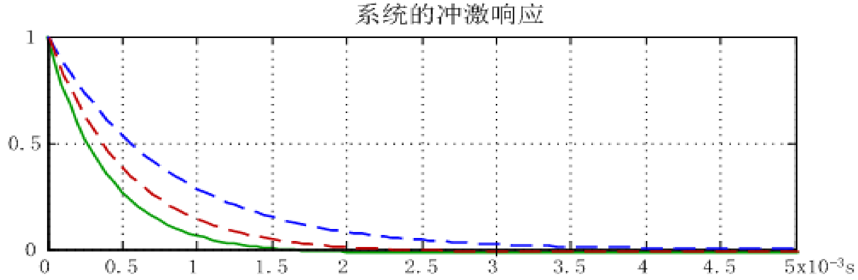


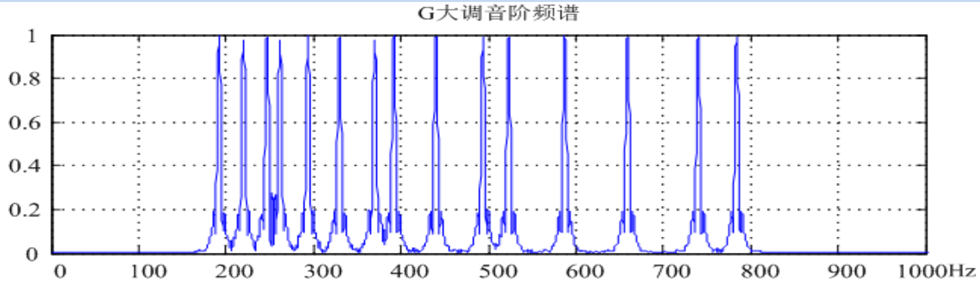


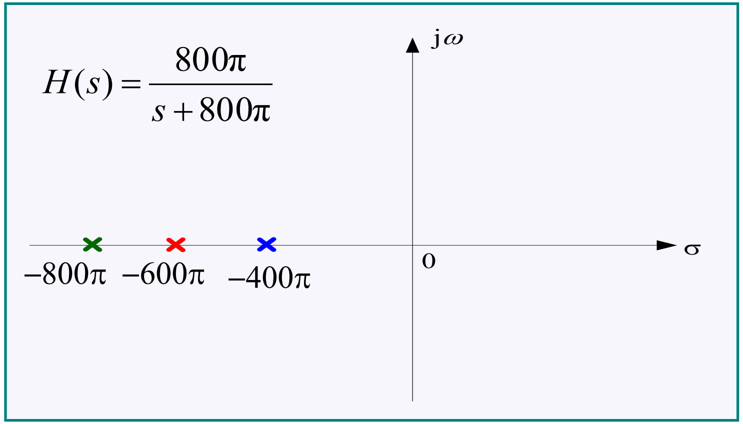
极点数值减小

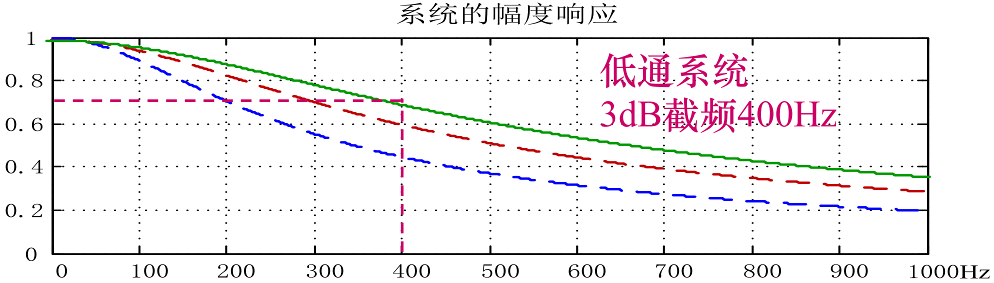


无零点；单极点p1= −800π

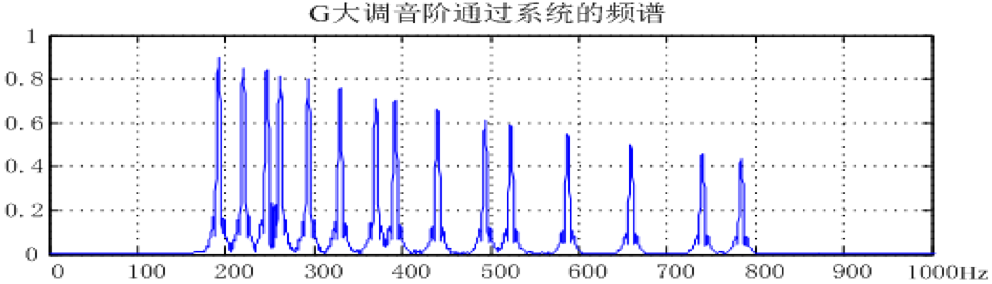




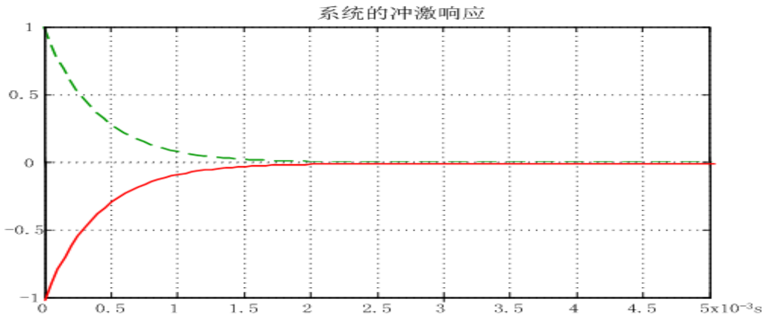


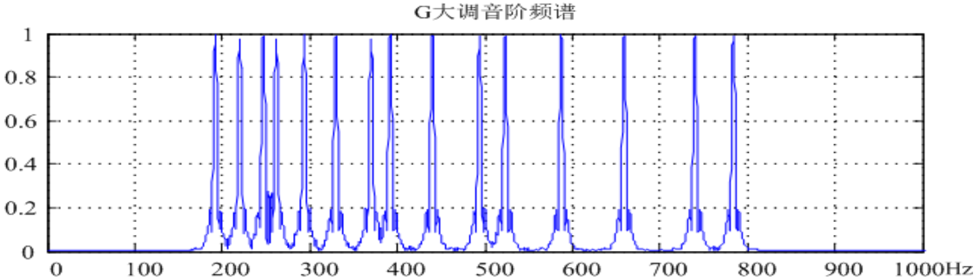


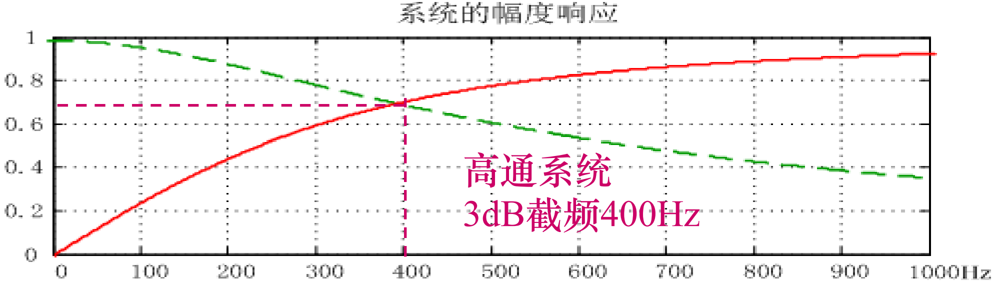
极点数值减小

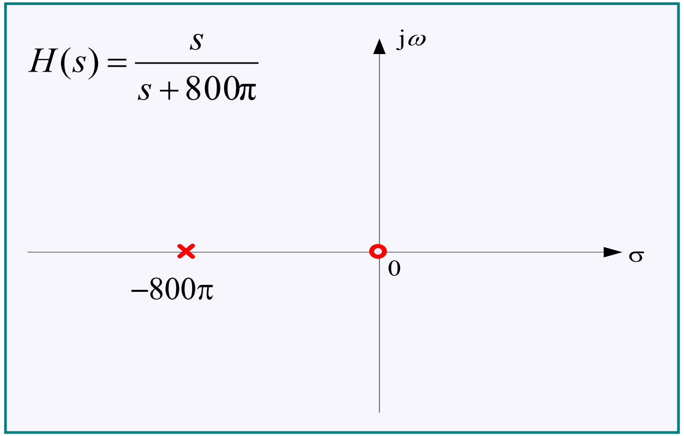


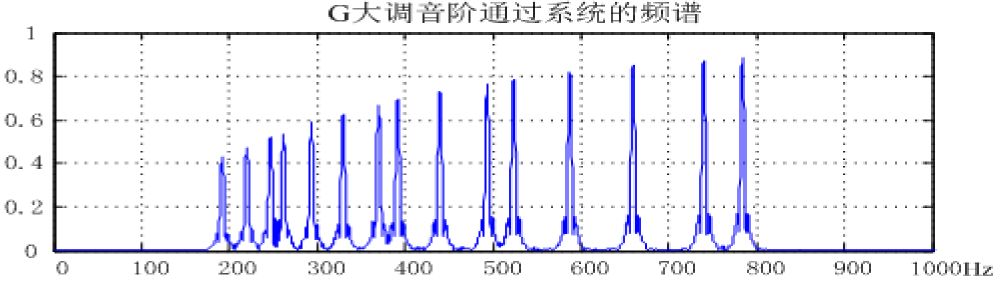
零点$z_1=0$；单极点$p_1=-800\pi$





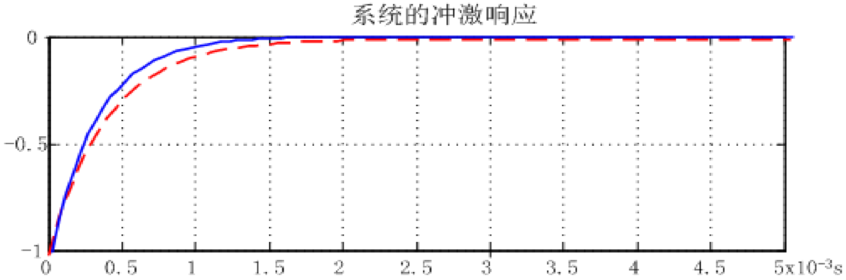


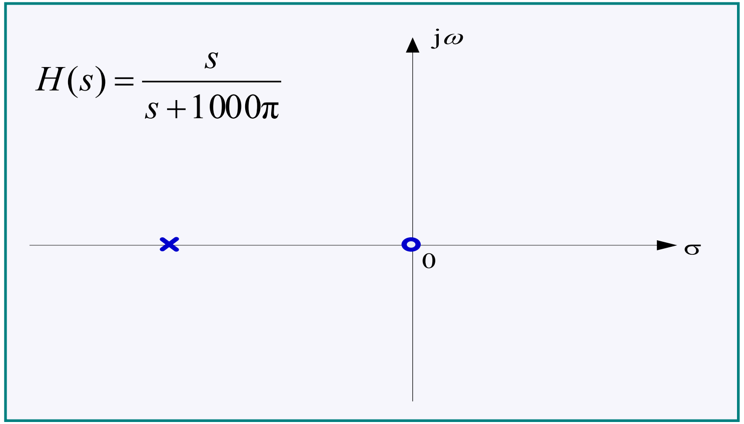




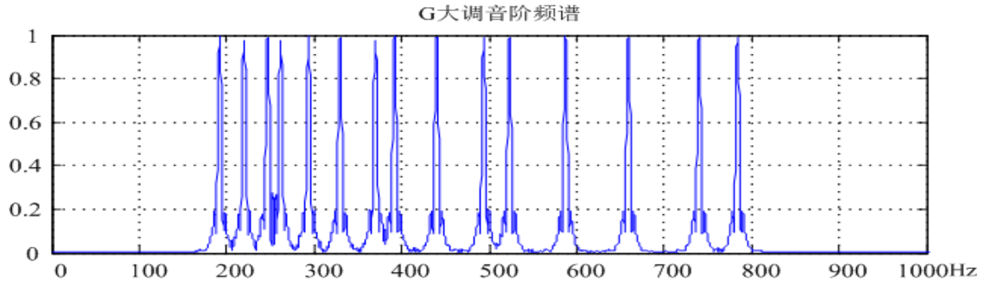
增加一个在原点的零点

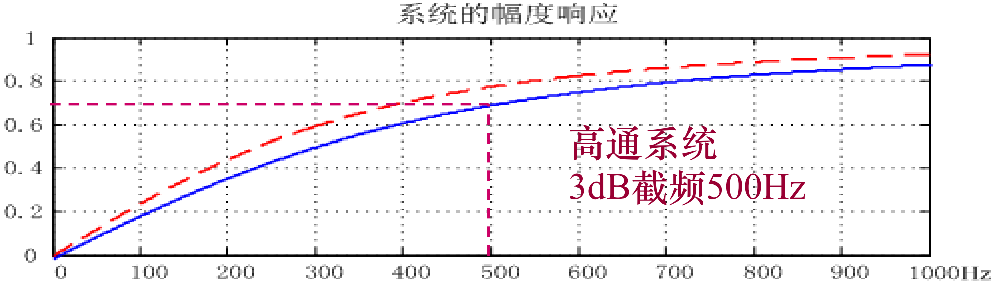
零点$z_1=0$；单极点$p_1=-1000\pi$

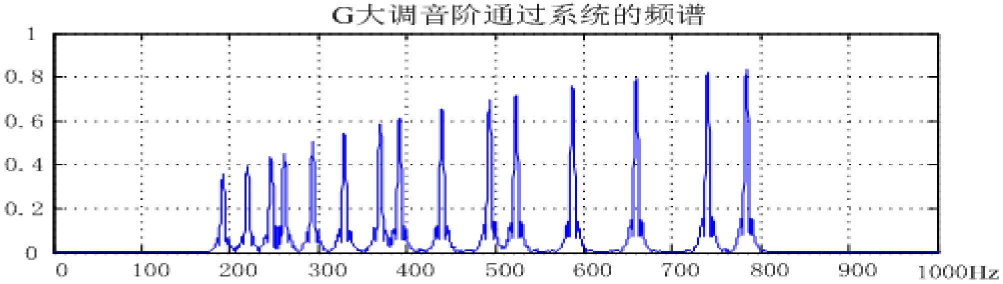




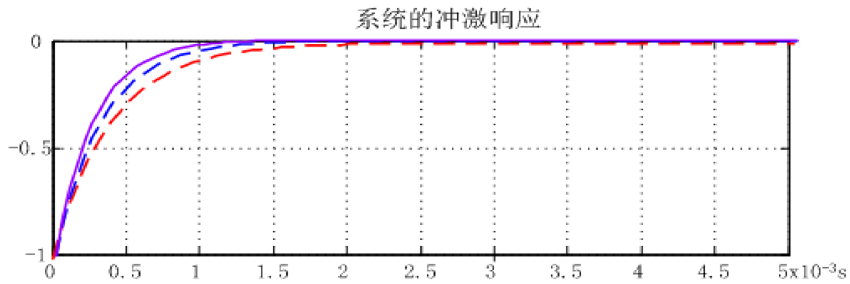
极点数值减小

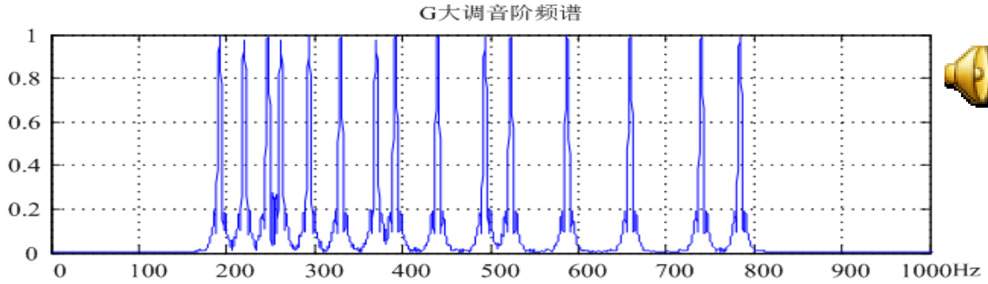


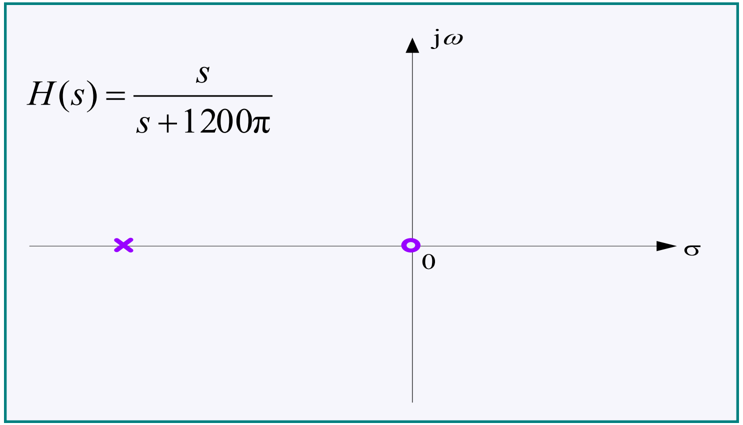


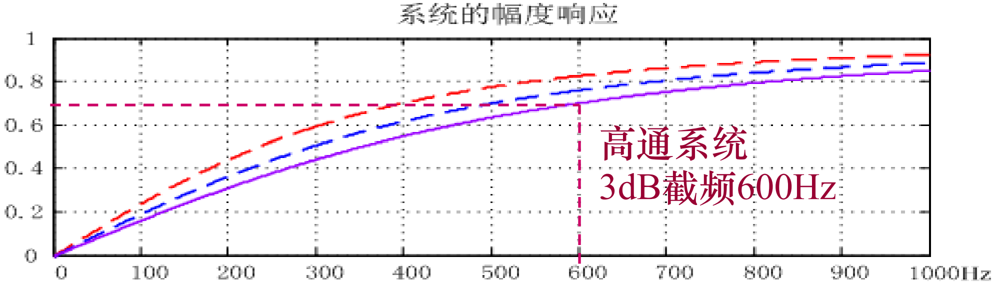


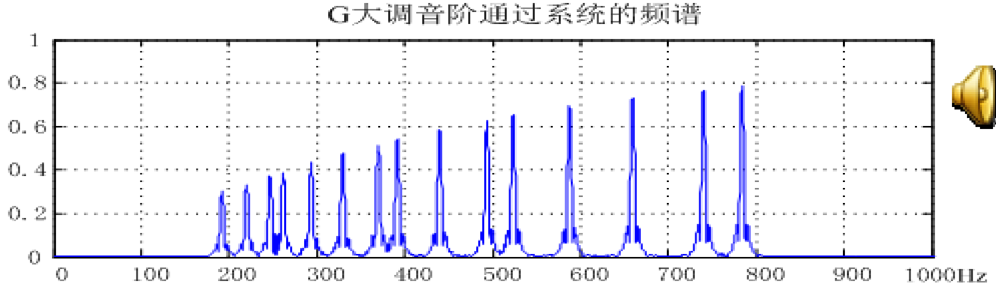
零点z_1=0；单极点p_1=−1200π





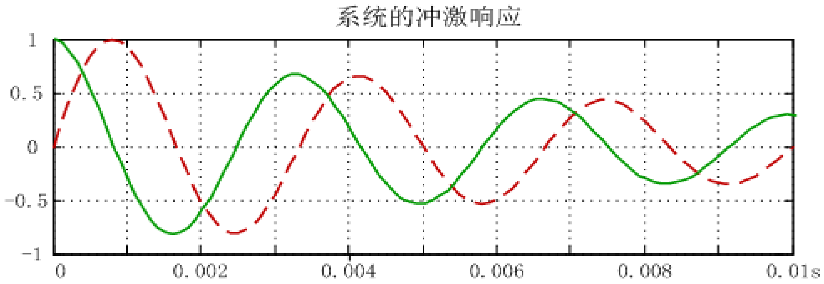


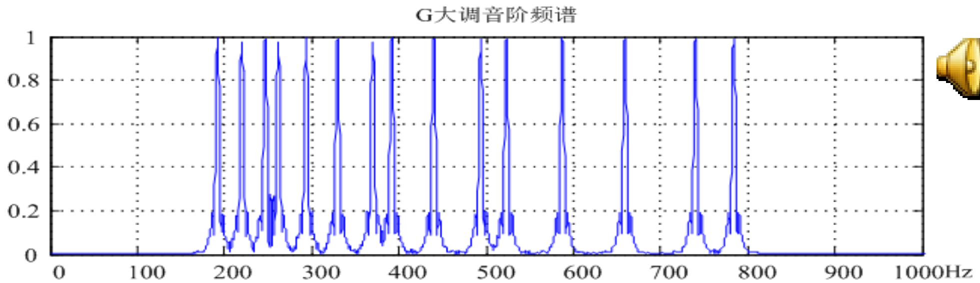




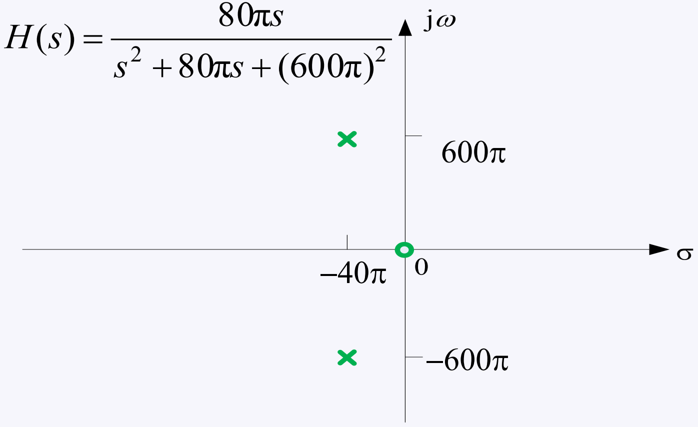
极点数值减小

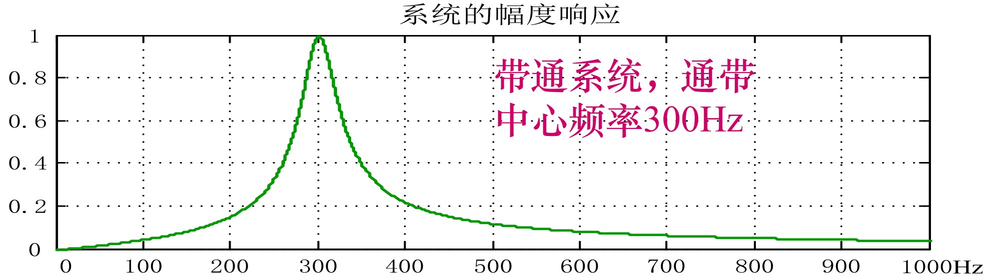
零点$z_1=0$；共轭极点$p_1\approx -40\pi+j600\pi$, $p_2 \approx -40\pi-j600\pi$



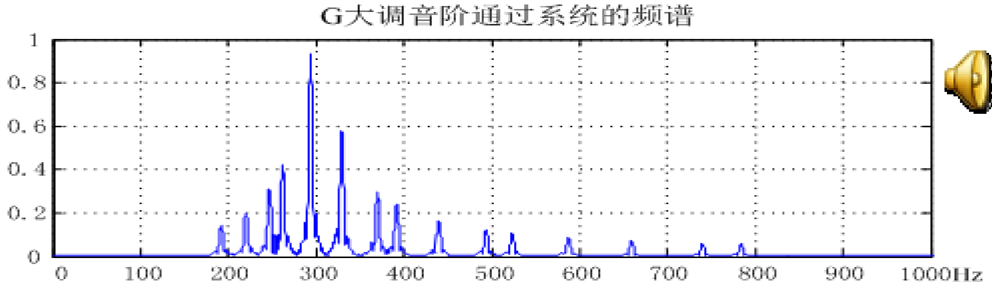


$$H(s)=\frac{80\pi s}{s^2+80\pi s+(600\pi)^2}$$

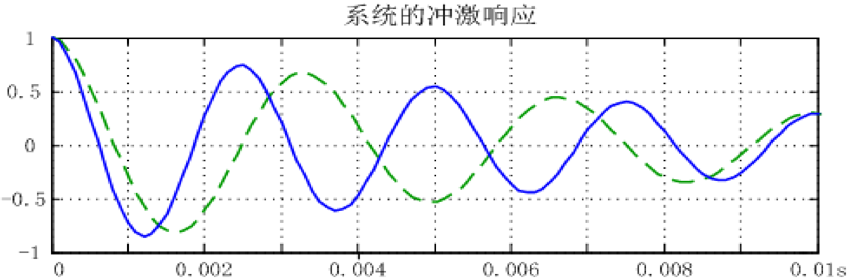


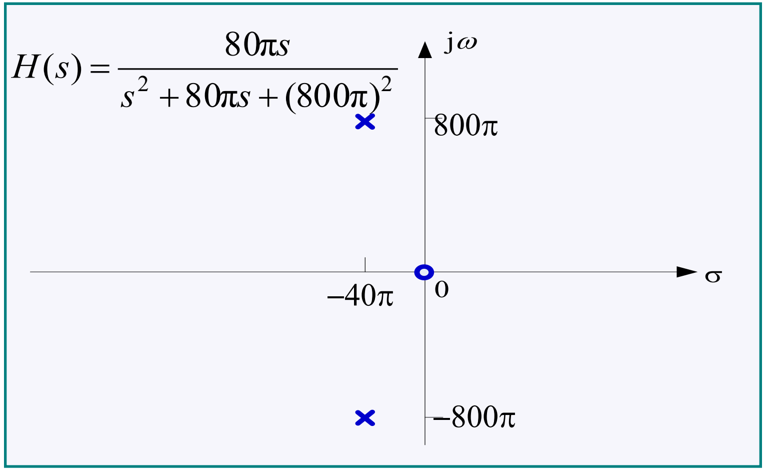


增加一个在原点的零点

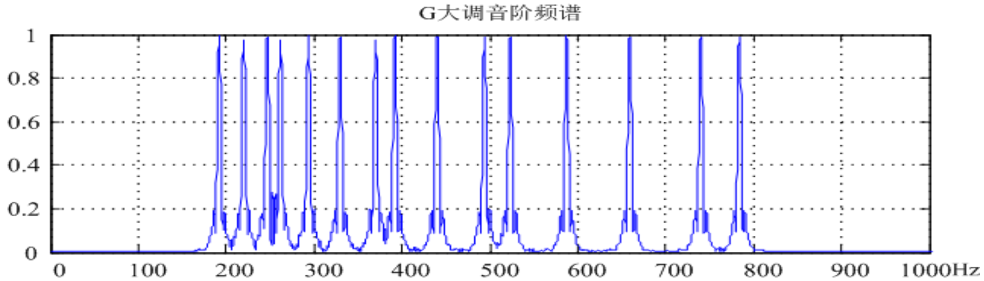


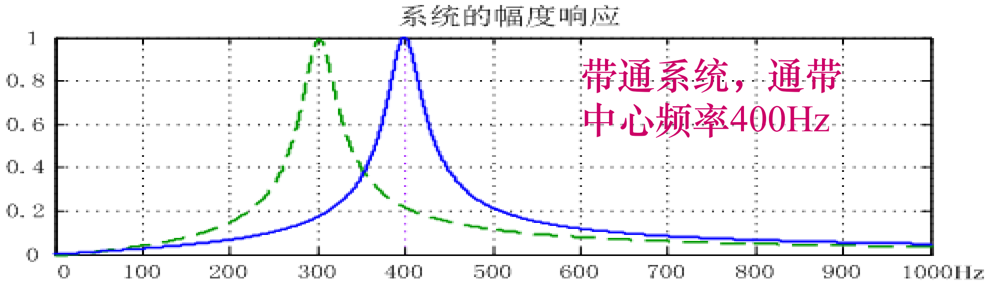
零点$z_1=0$；共轭极点$p_1 \approx -40\pi+j800\pi$, $p_2 \approx -40\pi-j800\pi$

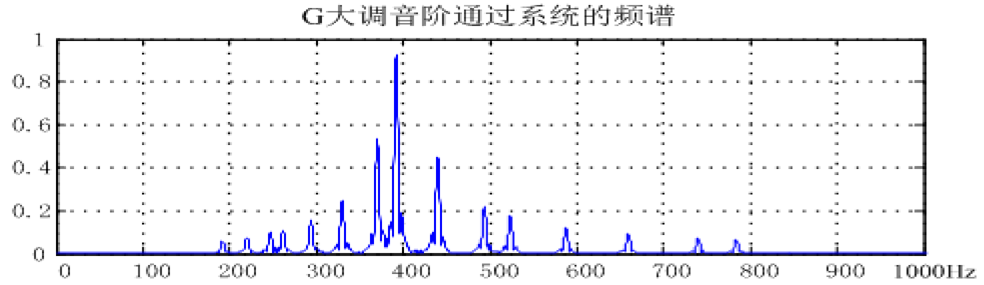




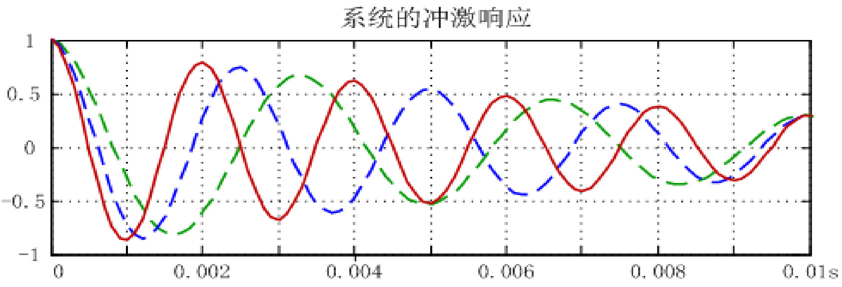
极点实部不变，虚部增大

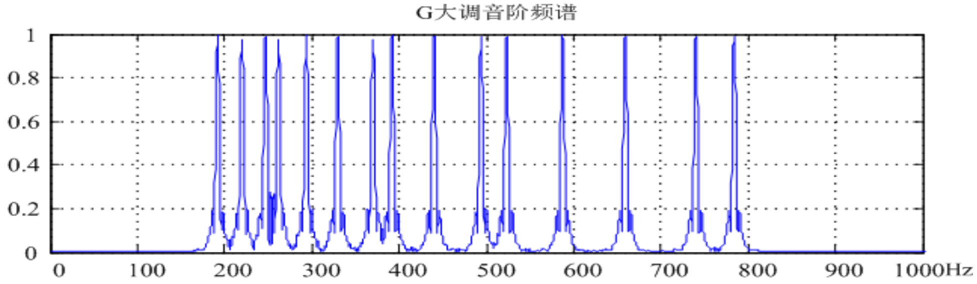




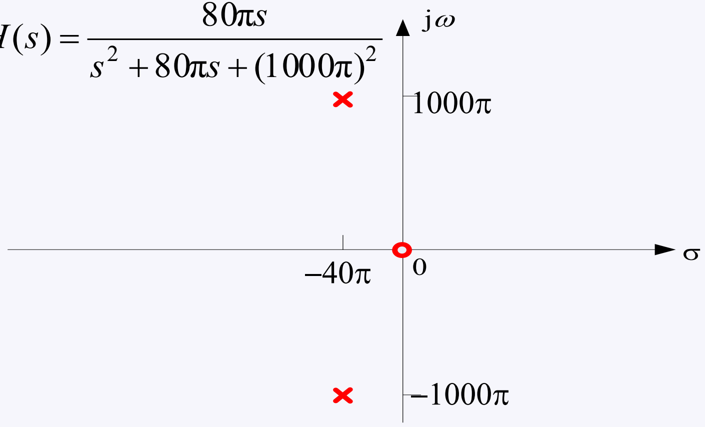


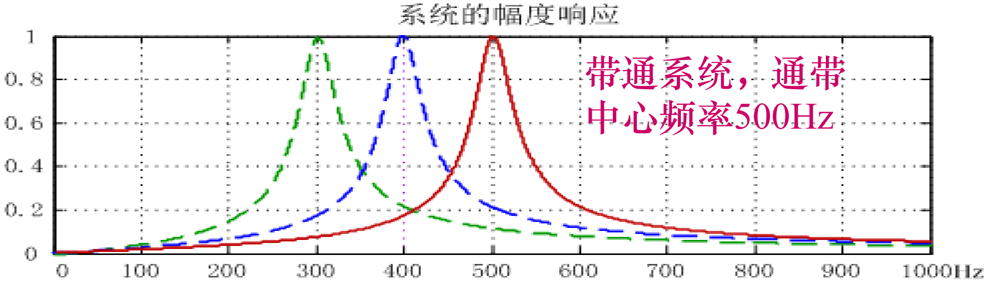
零点$z_1=0$；共轭极点$p_1\approx-40\pi+j1000\pi$, $p_2\approx-40\pi-j1000\pi$

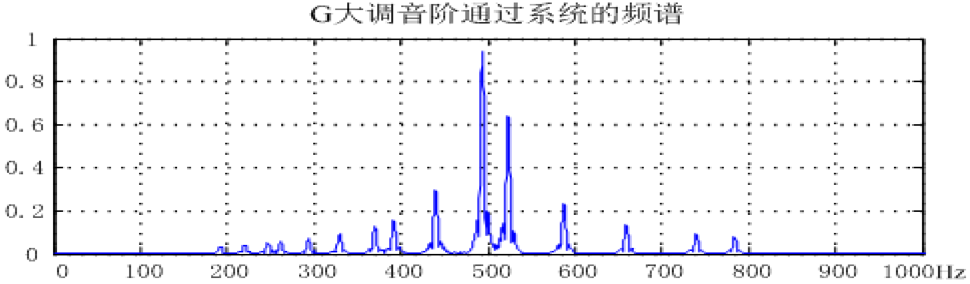




$$H(s)=\frac{80\pi s}{s^2+80\pi s+(1000\pi)^2}$$



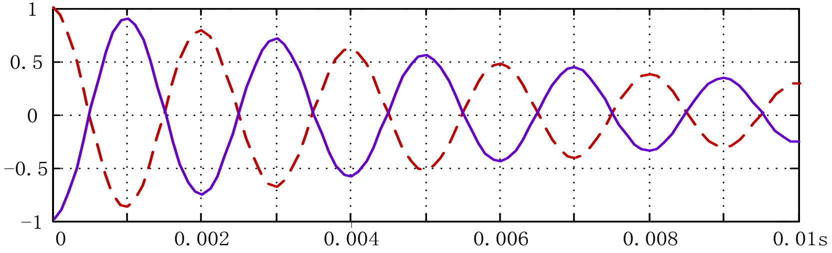




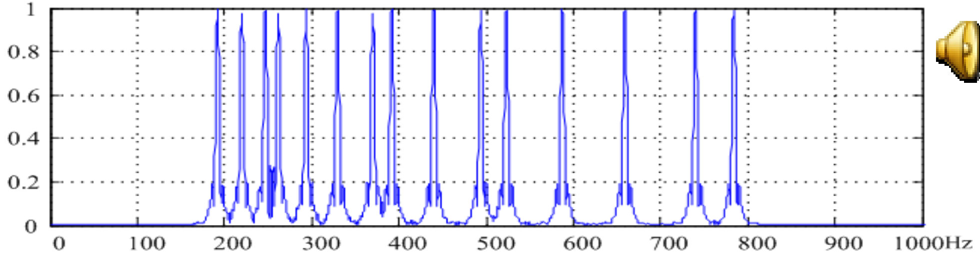
极点实部不变，虚部增大

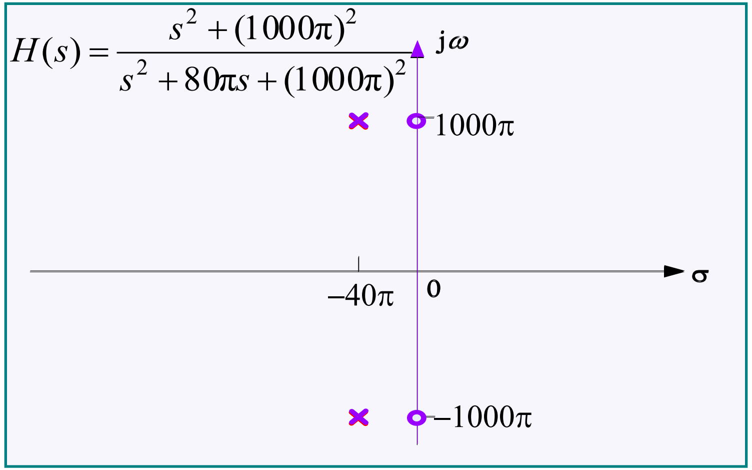
零点$z_1=j1000\pi$, $z_2=-j1000\pi$；极点$p_1\approx-40\pi+j1000\pi$, $p_2\approx-40\pi-j1000\pi$

系统的冲激响应



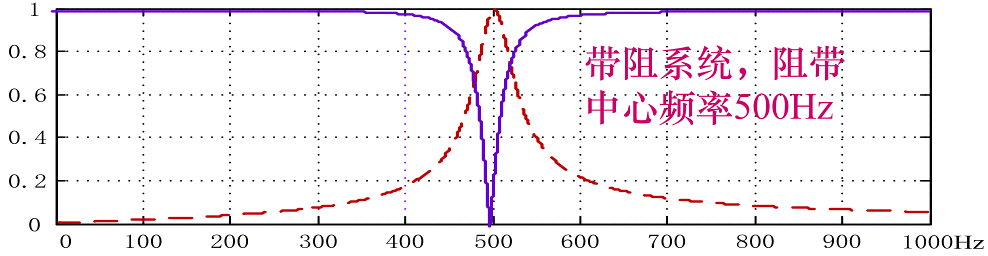
G大调音阶频谱



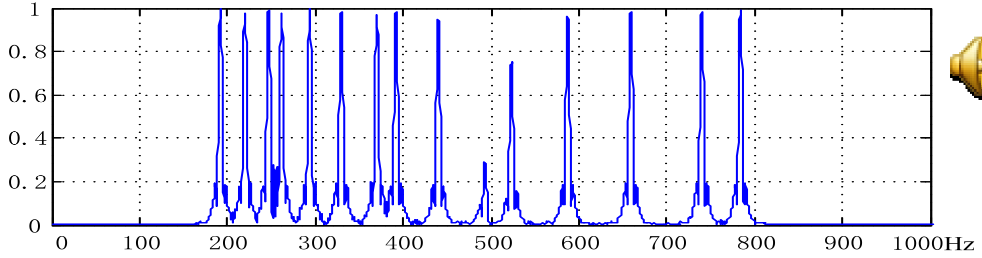


$$H(s)=\frac{s^2+(1000\pi)^2}{s^2+80\pi s+(1000\pi)^2}$$

系统的幅度响应

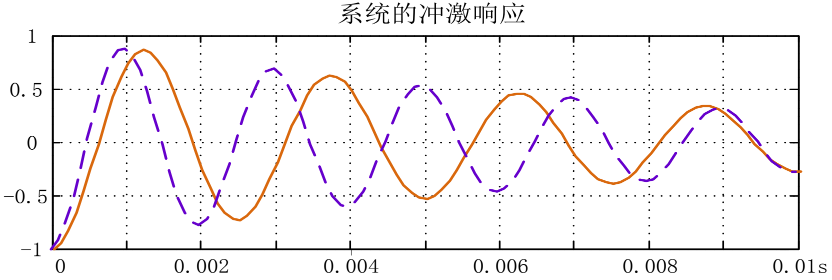


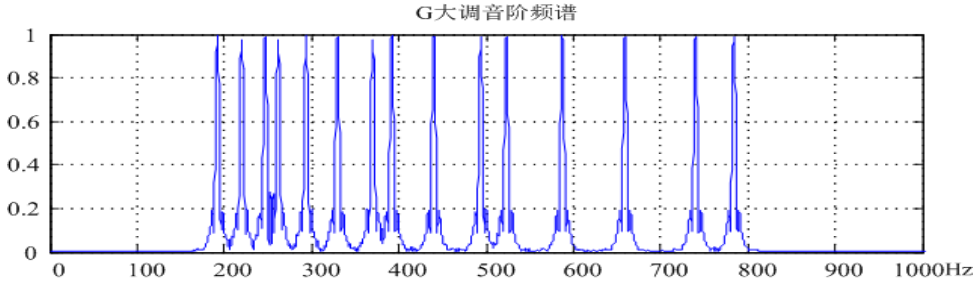
G大调音阶通过系统的频谱



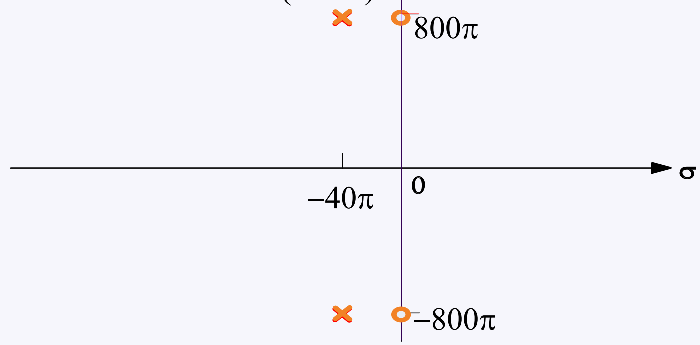
虚轴上的共轭零点

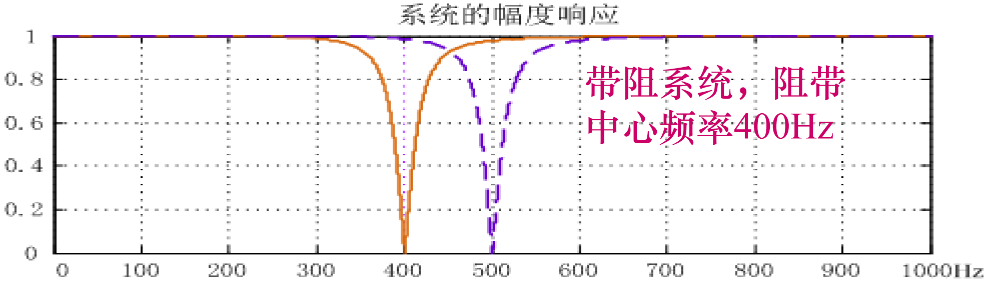
零点$z_1=j800\pi$, $z_2=-j800\pi$；　极点$p_1\approx-40\pi+j800\pi$, $p_2\approx-40\pi-j800\pi$

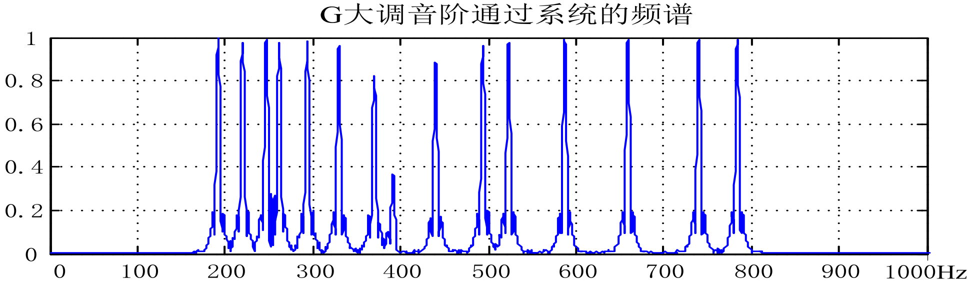




$$H(s)=\frac{s^2+(800\pi)^2}{s^2+80\pi s+(800\pi)^2}$$

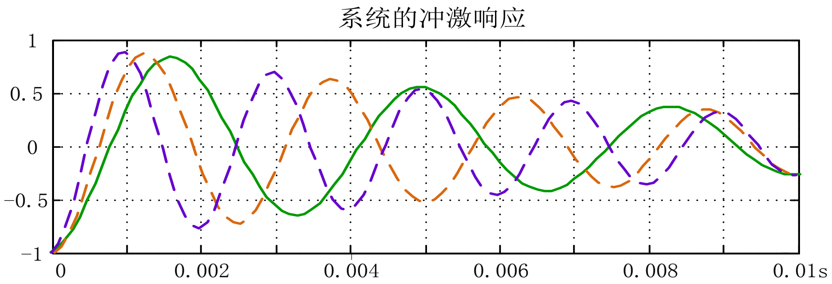


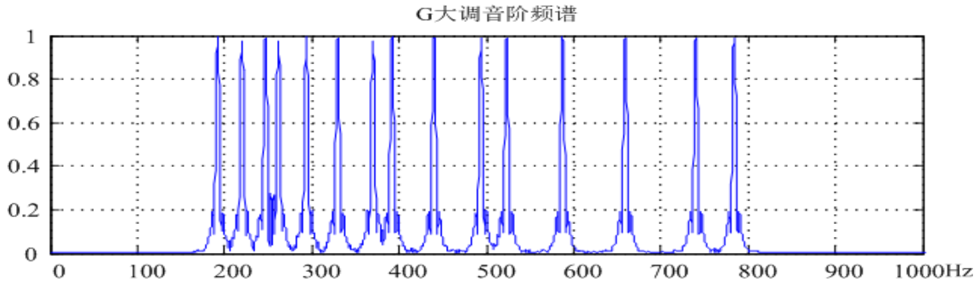




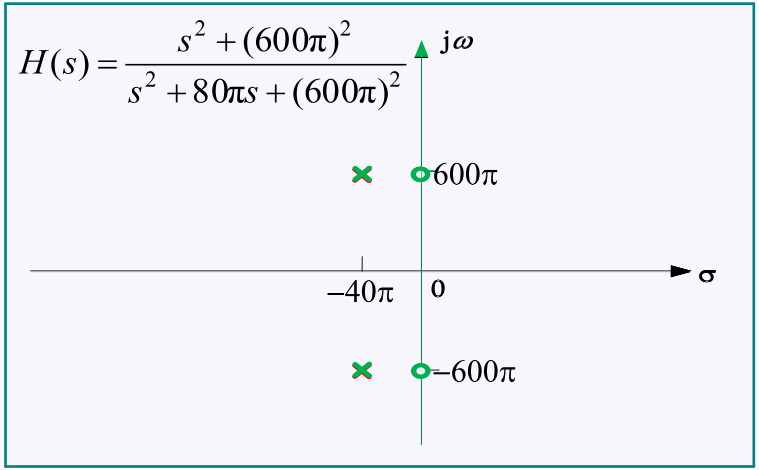
零点和极点虚部减小

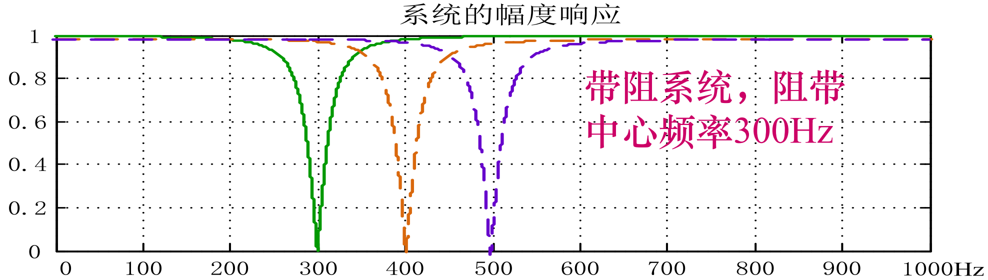
零点z_1=j600π, z_2=–j600π；极点$p_1 \approx -40\pi+j600\pi$, $p_2 \approx -40\pi-j600\pi$

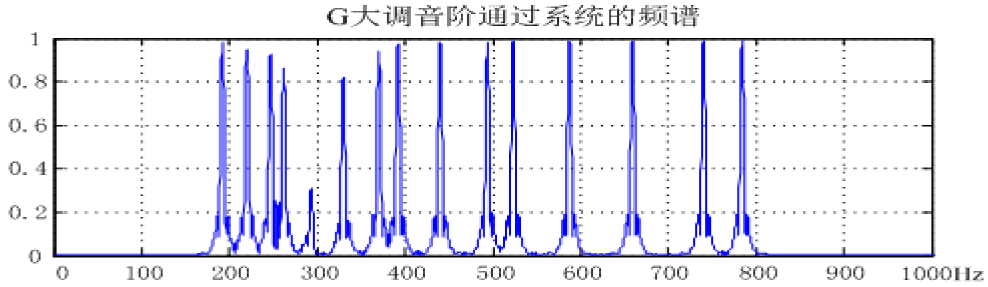




$$H(s)=\frac{s^2+(600\pi)^2}{s^2+80\pi s+(600\pi)^2}$$







零点和极点虚部减小

谢 谢

本课程所引用的一些素材为主讲老师多年的教学积累，来源于多种媒体及同事、同行、朋友的交流，难以一一注明出处，特此说明并表示感谢！