北京交通大学

信号与系统

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离散时间基本信号

※ 实指数序列

※ 虚指数序列 & 正弦序列

※ 复指数序列

※ 单位脉冲序列

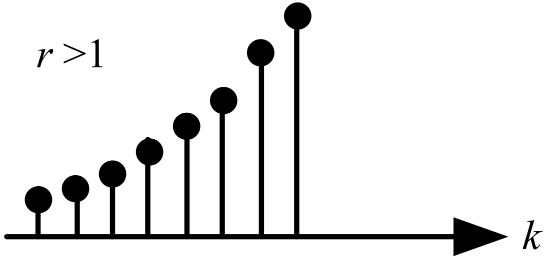
※ 单位阶跃序列

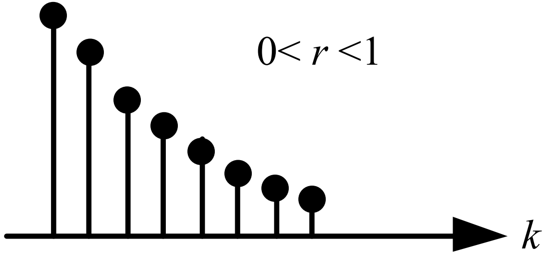
※ 矩形序列

※ 斜坡序列

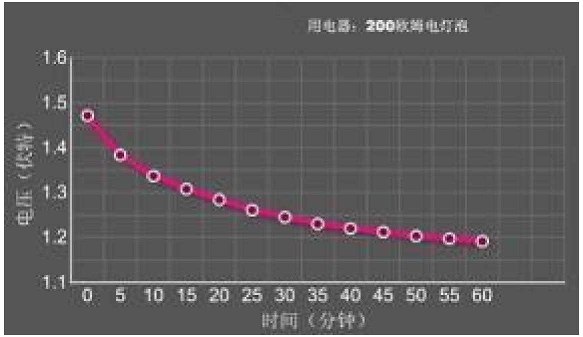
2.1 实指数序列

$$x[k] = Ar^k, \quad k \in Z$$





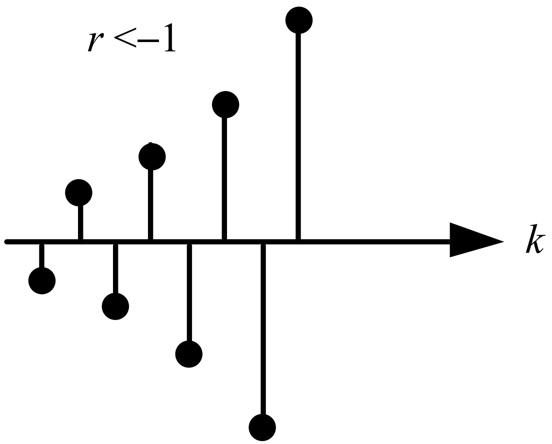
人口增长趋势图

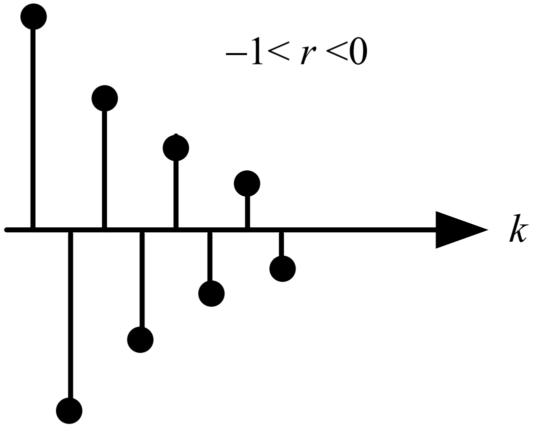


一次性电池放电测试图

2.1 实指数序列

$$x[k] = Ar^k, \quad k \in Z$$





2.2 虚指数序列 & 正弦序列

$x[k] = \mathrm{e}^{\mathrm{j}\Omega_0 k}$ $x[k] = A\cos(\Omega_0 k + \phi)$

利用Euler 公式可以将正弦序列和虚指数序列联系起来，即

$$\mathrm{e}^{\mathrm{j}\Omega_0 k} = \cos\Omega_0 k + \mathrm{j}\sin\Omega_0 k$$

$$\cos\Omega_0 k = \frac{1}{2}(\mathrm{e}^{\mathrm{j}\Omega_0 k} + \mathrm{e}^{-\mathrm{j}\Omega_0 k})$$

$$\sin\Omega_0 k = \frac{1}{2\mathrm{j}}(\mathrm{e}^{\mathrm{j}\Omega_0 k} - \mathrm{e}^{-\mathrm{j}\Omega_0 k})$$

2.2 虚指数序列 & 正弦序列

$e^{j\Omega_0 k}$ 可由 $e^{j\omega_0 t}$ 抽样得到

$$e^{j\Omega_0 k} = e^{j\omega_0 t}\Big|_{t=kT}, \quad \Omega_0 = \omega_0 T$$

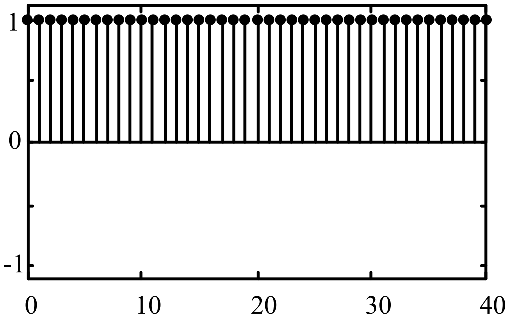
周期性:

若 $e^{j\Omega_0 N} = 1$，则 $e^{j\Omega_0 (k+N)} = e^{j\Omega_0 k} e^{j\Omega_0 N} = e^{j\Omega_0 k}$

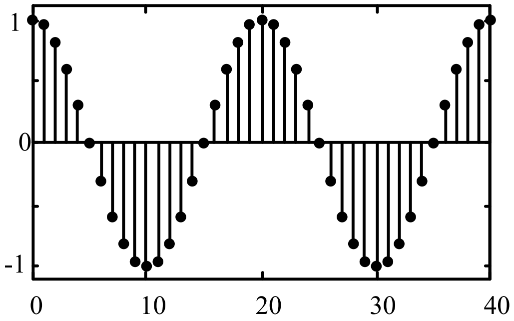
即$\Omega_0 N = m2\pi$ ，$m =$ 整数时，信号是周期信号。

如果$\Omega_0 /2\pi = m/N$，该离散信号为周期序列;
当N、m是不可约的整数，信号的周期为N。

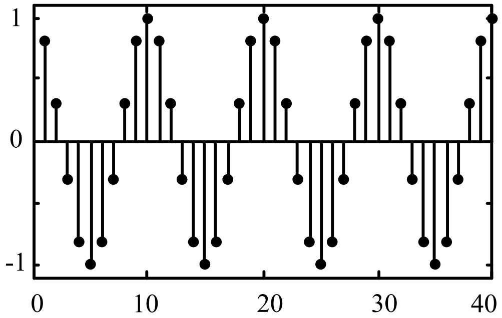
$\cos[\Omega_0 k]$当角频率Ω_0从0增加到π时的波形



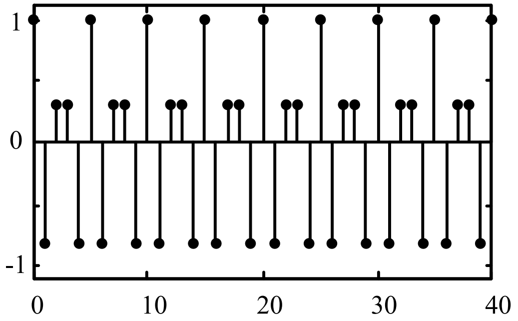
$\Omega_0=0$



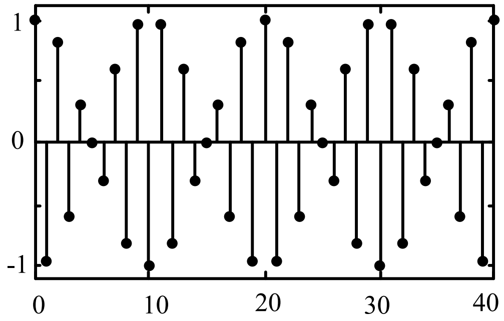
$\Omega_0=0.1\pi$



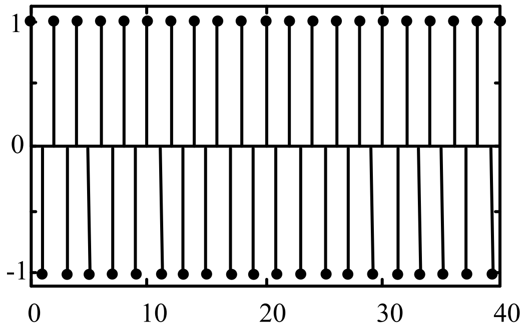
$\Omega_0=0.2\pi$



$\Omega_0=0.8\pi$



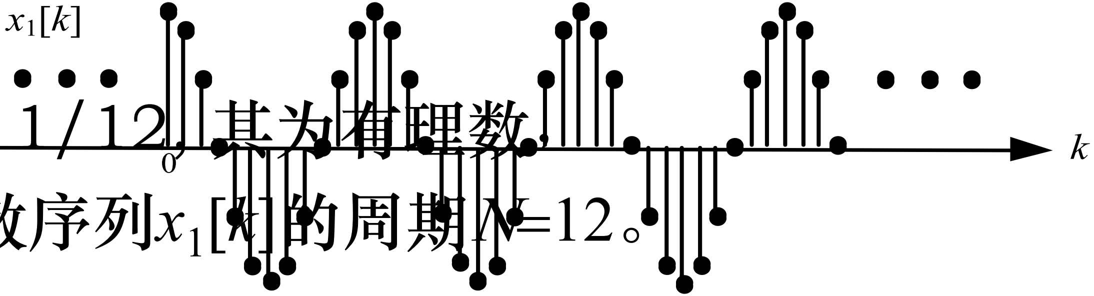
$\Omega_0=0.9\pi$



$\Omega_0=\pi$

[例]判断下列离散序列是否为周期信号

(1) $x_1[k]=\cos(k\pi/6)$



$\Omega_0=\pi/6$

$\Omega_0/2\pi = 1/12$，其为有理数，

所以离散序列$x_1[k]$的周期N=12。

(2) $x_2[k]=\cos(k/6)$

$\Omega_0=1/6$

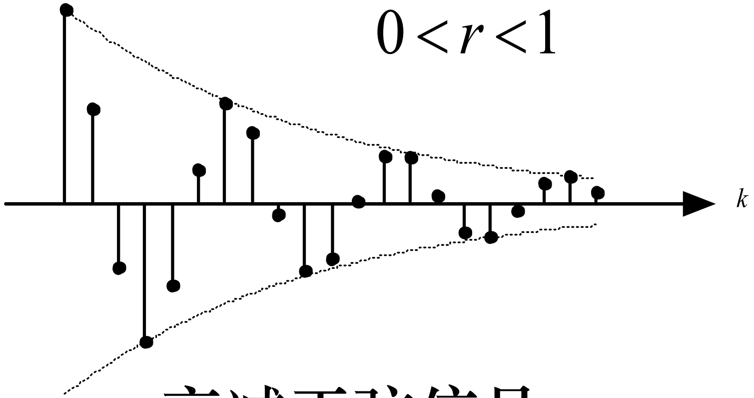
$\Omega_0/2\pi = 1/12\pi$，$1/12\pi$是非有理数，

所以离散序列$x_2[k]$是非周期序列。

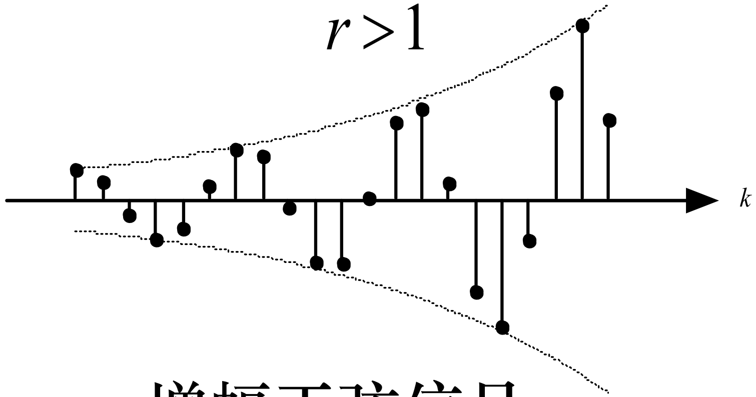
2.3 复指数序列

$$x[k] = Ar^k e^{j\Omega_0 k} = Az^k$$

$$Ar^k e^{j\Omega_0 k} = Ar^k \cos(\Omega_0 k) + jAr^k \sin(\Omega_0 k)$$



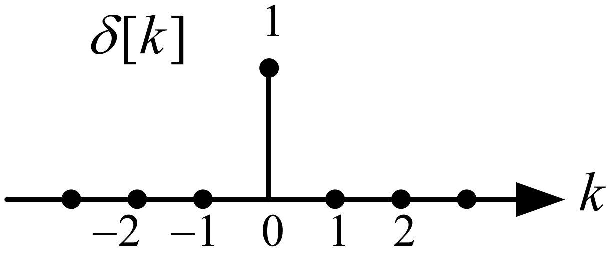
衰减正弦信号



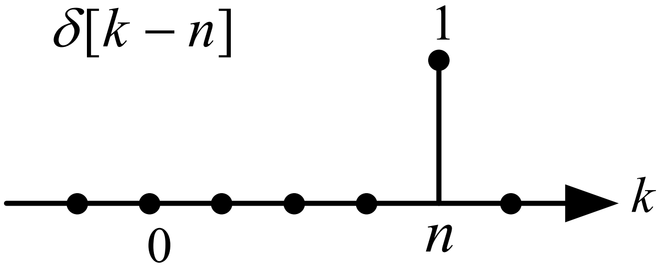
增幅正弦信号

2.4 单位脉冲序列

$$\delta[k]=\begin{cases}1 & k=0\\ 0 & k\neq 0\end{cases}$$

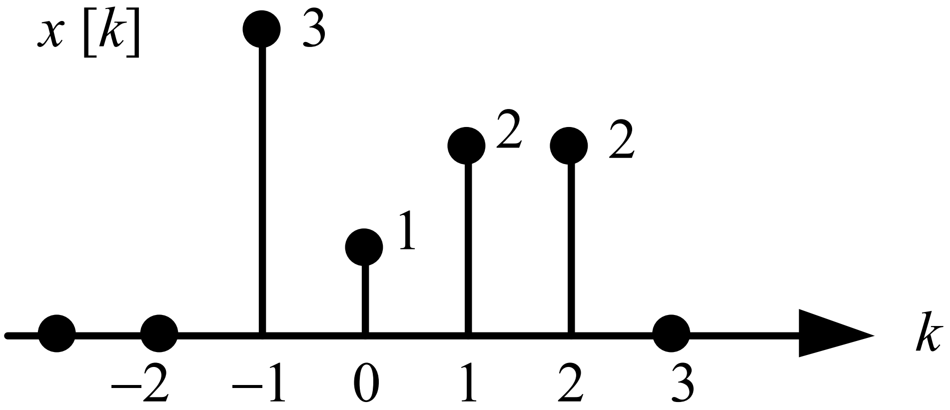


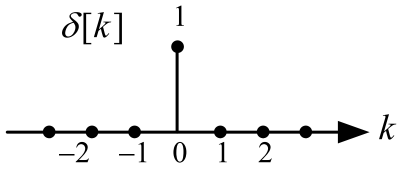
$$\delta[k-n]=\begin{cases}1 & k=n\\ 0 & k\neq n\end{cases}$$



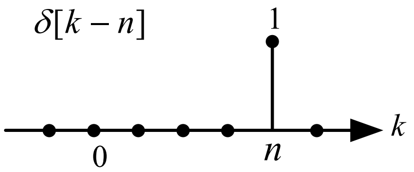
2.4 单位脉冲序列

➢ 作用：表示任意离散时间序列





单位脉冲序列

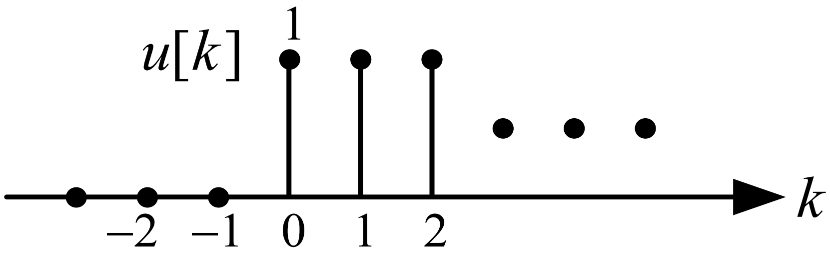


有位移的单位脉冲序列

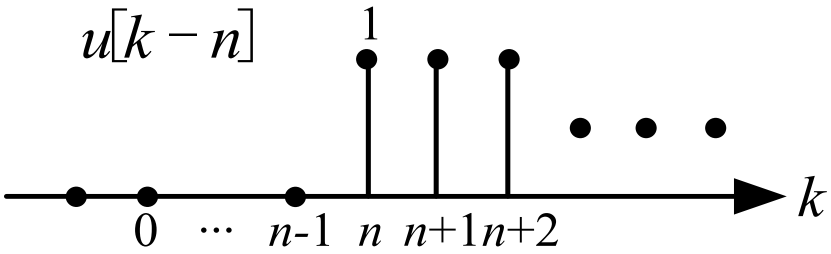
$$x[k] = 3\delta[k+1] + \delta[k] + 2\delta[k-1] + 2\delta[k-2]$$

2.5 单位阶跃序列

$$u[k]=\begin{cases}1 & k\ge 0\\ 0 & k<0\end{cases}$$



$$u[k-n]=\begin{cases}1 & k\ge n\\ 0 & k<n\end{cases}$$

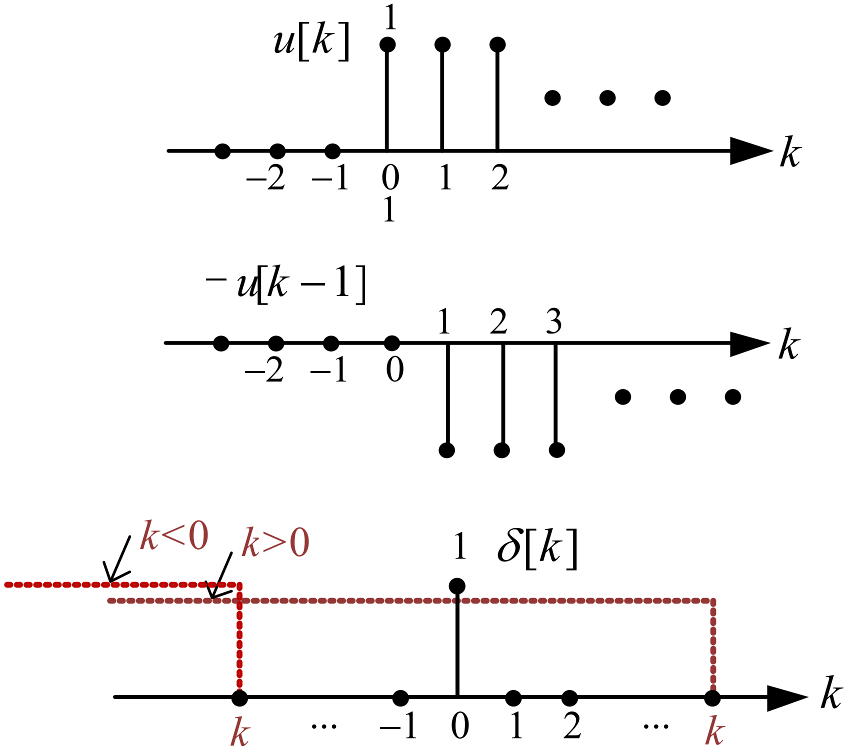


2.5 单位阶跃序列

➢ $\delta[k]$与$u[k]$的关系

$$\delta[k]=u[k]-u[k-1]$$

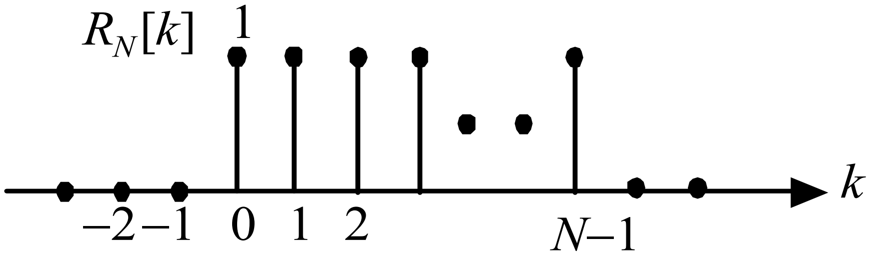
$$u[k]=\sum_{n=-\infty}^{k}\delta[n]$$



2.6 矩形序列

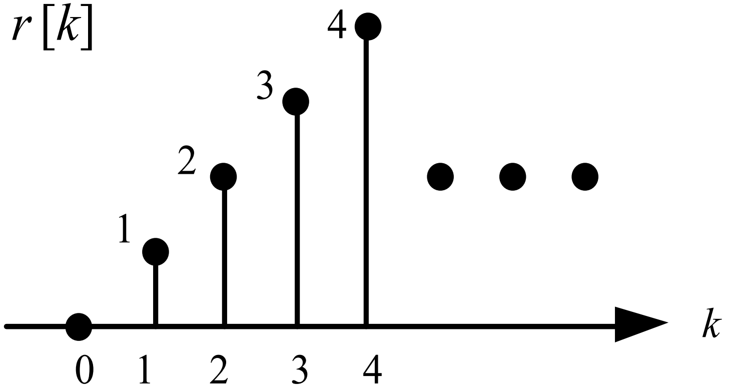
$$R_N[k] = \begin{cases} 1 & 0 \le k \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

$$R_N[k] = u[k] - u[k-N] = \sum_{m=0}^{N-1} \delta[k-m]$$

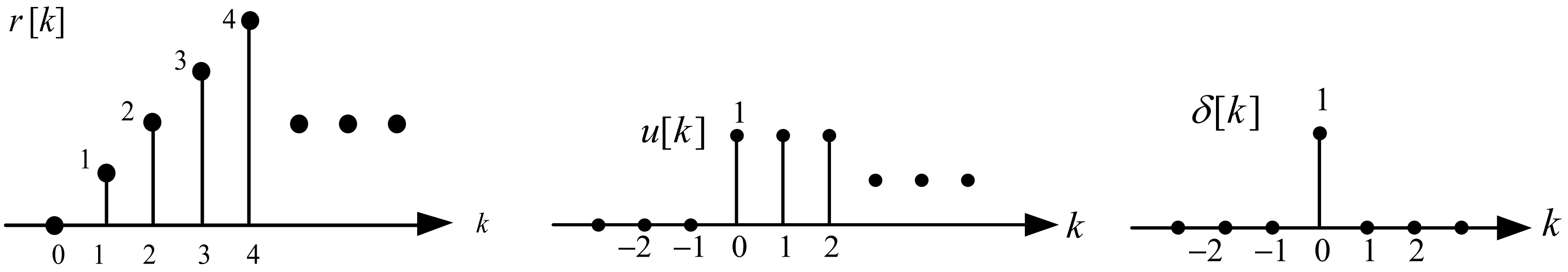


2.7 斜坡序列

$$r[k]=ku[k]$$



总结

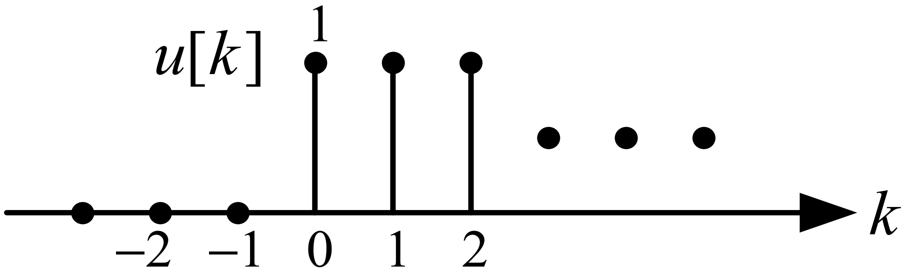


差分关系 $\delta[k]=u[k]-u[k-1]$ $u[k]=r[k+1]-r[k]$

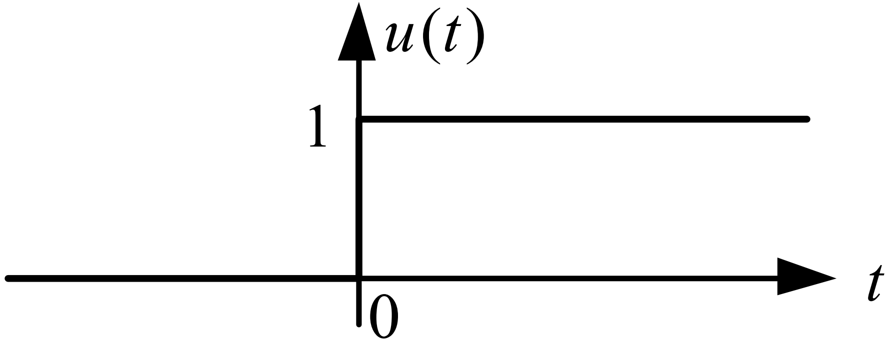
求和关系 $u[k]=\sum_{n=-\infty}^{k}\delta[n]$ $r[k+1]=\sum_{n=-\infty}^{k}u[n]$

$u(t)$与$u[k]$的区别

$$u[k]=\begin{cases}1 & k\geq 0\\ 0 & k<0\end{cases}$$

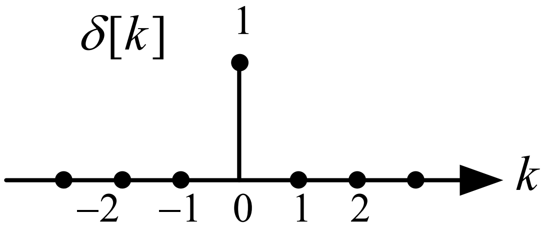


$$u(t)=\begin{cases}1 & t>0\\ 0 & t<0\end{cases}$$



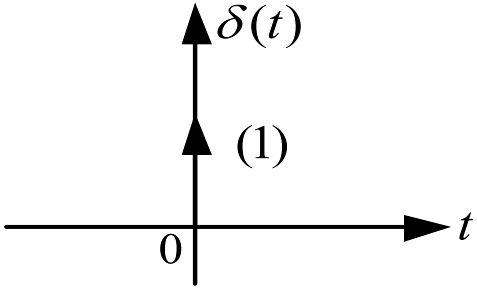
$\delta(t)$与$\delta[k]$的区别

$$\delta[k]=\begin{cases}0 & k\neq 0\\ 1 & k=0\end{cases}$$



$$\delta(t)=0\,,\quad t\neq 0$$

$$\int_{-\infty}^{+\infty}\delta(t)\,\mathrm{d}t=1$$



谢 谢

本课程所引用的一些素材为主讲老师多年的教学积累，来源于多种媒体及同事、同行、朋友的交流，难以一一注明出处，特此说明并表示感谢！