北京交通大学

主讲人：陈后金

电子信息工程学院

系统频域分析举例

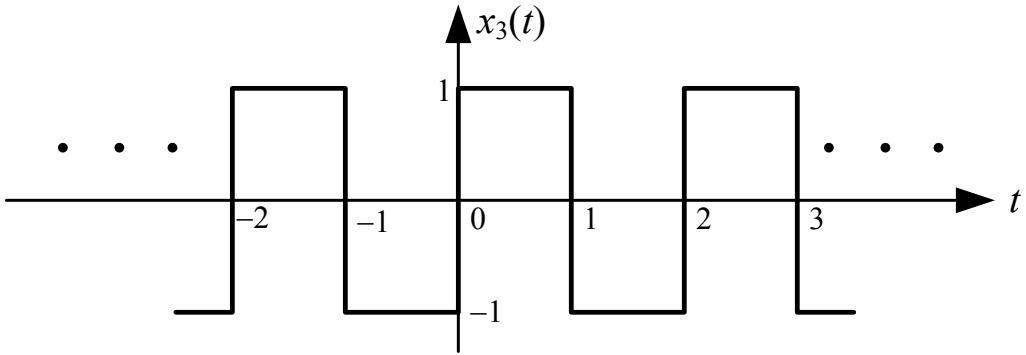
[例] 已知理想模拟低通滤波器的频率响应$H(\mathrm{j}\omega)$为

$$H(\mathrm{j}\omega)=\begin{cases}\mathrm{e}^{-\mathrm{j}2\omega} & |\omega|<2\pi\\ 0 & |\omega|>2\pi\end{cases}$$

(1) 若系统的输入信号$x_1(t)$=Sa(πt)，试求系统输出响应$y_1(t)$；

(2) 若系统的输入信号$x_2(t)$=Sa($3\pi t$)，试求系统输出响应$y_2(t)$；

(3) 若系统的输入信号$x_3(t)$是周期矩形波信号，试求系统输出响应$y_3(t)$。



系统频域分析举例

[例] 已知理想模拟低通滤波器的频率响应$H(\mathrm{j}\omega)$为

$$H(\mathrm{j}\omega)=\begin{cases}\mathrm{e}^{-\mathrm{j}2\omega} & |\omega|<2\pi\\ 0 & |\omega|>2\pi\end{cases}$$

解：(1) 信号$x_1(t)$=Sa(πt)通过系统的响应$y_1(t)$；

由基本信号的Fourier变换可知

$$\mathrm{Sa}(\pi t)\xrightarrow{\mathscr{F}}p_{2\pi}(\omega)$$

输出频谱为 $Y_1(\mathrm{j}\omega)=H(\mathrm{j}\omega)X_1(\mathrm{j}\omega)=p_{2\pi}(\omega)\mathrm{e}^{-\mathrm{j}2\omega}$

由Fourier反变换可得

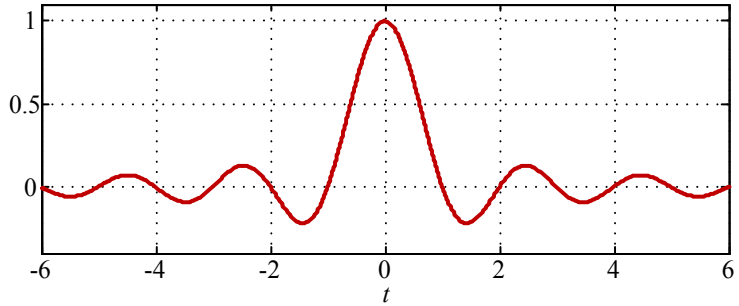
$$y_1(t)=\mathscr{F}^{-1}\{Y_1(\mathrm{j}\omega)\}=\mathrm{Sa}\left[\pi(t-2)\right]$$

系统频域分析举例

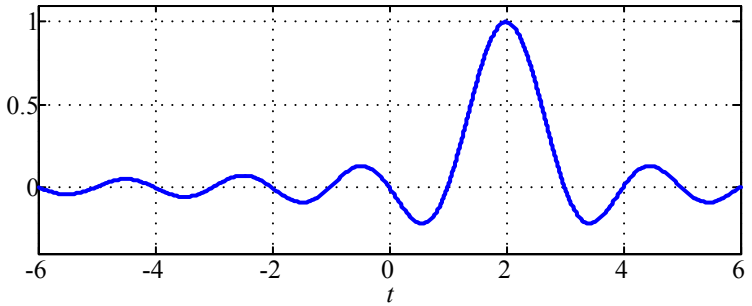
[例] 已知理想模拟低通滤波器的频率响应$H(\mathrm{j}\omega)$为

$$H(\mathrm{j}\omega)=\begin{cases}\mathrm{e}^{-\mathrm{j}2\omega} & |\omega|<2\pi\\ 0 & |\omega|>2\pi\end{cases}$$

解：(1) 信号$x_1(t)=\mathrm{Sa}(\pi t)$通过系统的响应$y_1(t)$；



系统输入$x_1(t)=\mathrm{Sa}(\pi t)$



系统输出$y_1(t)=\mathrm{Sa}[\pi(t-2)]$

系统频域分析举例

[例] 已知理想模拟低通滤波器的频率响应$H(\mathrm{j}\omega)$为

$$H(\mathrm{j}\omega)=\begin{cases}\mathrm{e}^{-\mathrm{j}2\omega} & |\omega|<2\pi\\ 0 & |\omega|>2\pi\end{cases}$$

解：(2) 信号$x_2(t)$=Sa($3\pi t$)通过系统的响应$y_2(t)$；

由基本信号的Fourier变换可知

$$\mathrm{Sa}(3\pi t)\xrightarrow{\mathscr{F}}(1/3)p_{6\pi}(\omega)$$

输出频谱为 $Y_2(\mathrm{j}\omega)=H(\mathrm{j}\omega)X_2(\mathrm{j}\omega)=(1/3)p_{4\pi}(\omega)\mathrm{e}^{-\mathrm{j}2\omega}$

由Fourier反变换可得

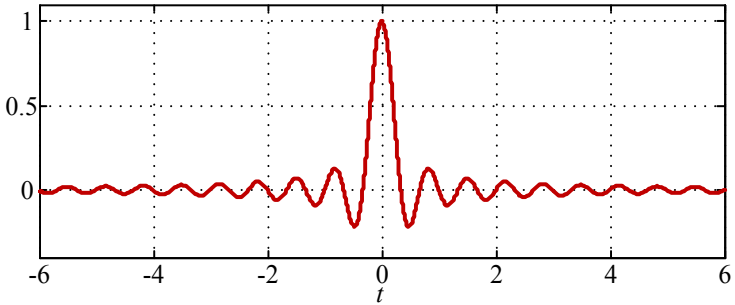
$$y_2(t)=\mathscr{F}^{-1}\{Y_2(\mathrm{j}\omega)\}=(2/3)\mathrm{Sa}\left[2\pi(t-2)\right]$$

系统频域分析举例

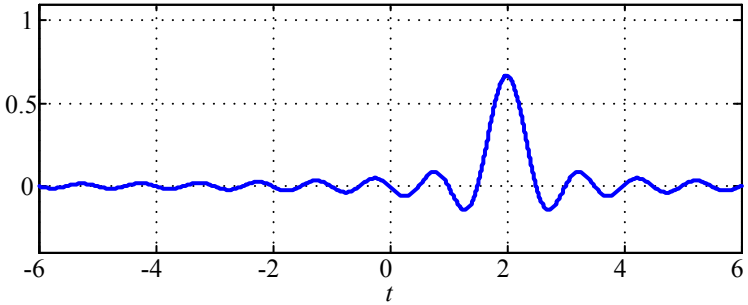
[例] 已知理想模拟低通滤波器的频率响应$H(\mathrm{j}\omega)$为

$$H(\mathrm{j}\omega)=\begin{cases}\mathrm{e}^{-\mathrm{j}2\omega} & |\omega|<2\pi\\ 0 & |\omega|>2\pi\end{cases}$$

解：(2) 信号$x_2(t)$=Sa(3πt)通过系统的响应$y_2(t)$；



系统输入$x_2(t)$=Sa(3πt)

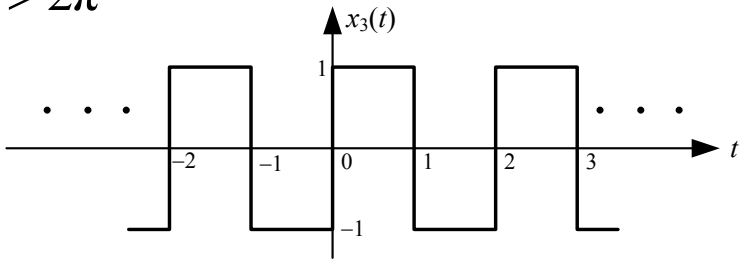


系统输出$y_2(t)$=(2/3)Sa[2π(t–2)]

系统频域分析举例

[例] 已知理想模拟低通滤波器的频率响应$H(\mathrm{j}\omega)$为

$$H(\mathrm{j}\omega)=\begin{cases}\mathrm{e}^{-\mathrm{j}2\omega} & |\omega|<2\pi\\ 0 & |\omega|>2\pi\end{cases}$$



解： (3) 信号$x_3(t)$ 通过系统的响应$y_3(t)$

信号的基频 $\omega_0=2\pi/T_0=2\pi/2=\pi$

由于理想低通滤波器的截止频率$\omega_c=2\pi$，所以只有直流和基波能通过该理想低通滤波器。由于$x_3(t)$实奇对称，$x_3(t)$中无直流分量。

$$C_1=\frac{1}{2}\int_0^2 x_3(t)\cdot\mathrm{e}^{-\mathrm{j}\pi t}\mathrm{d}\ =\frac{2}{\mathrm{j}\pi}$$

$$C_{-1}=C_1^*=-\frac{2}{\mathrm{j}\pi}$$

系统频域分析举例

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$$H(\mathrm{j}\omega)=\begin{cases}\mathrm{e}^{-\mathrm{j}2\omega} & |\omega|<2\pi\\ 0 & |\omega|>2\pi\end{cases}$$

解：(3) 信号$x_3(t)$ 通过系统的响应$y_3(t)$

$$x_3(t)=\sum_{n=-\infty}^{\infty}C_n\ \mathrm{e}^{\mathrm{j}n\pi t}$$

$$C_{-1}=-\frac{2}{\mathrm{j}\pi}\qquad C_0=0\qquad C_1=\frac{2}{\mathrm{j}\pi}$$

$$y_3(t)=C_{-1}H(-\mathrm{j}\pi)\mathrm{e}^{-\mathrm{j}\pi(t-2)}+C_1H(\mathrm{j}\pi)\mathrm{e}^{\mathrm{j}\pi(t-2}=\frac{4}{\pi}\sin(\pi t)$$

系统频域分析举例

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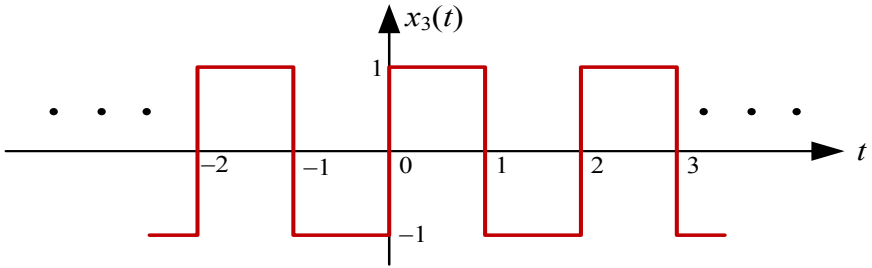
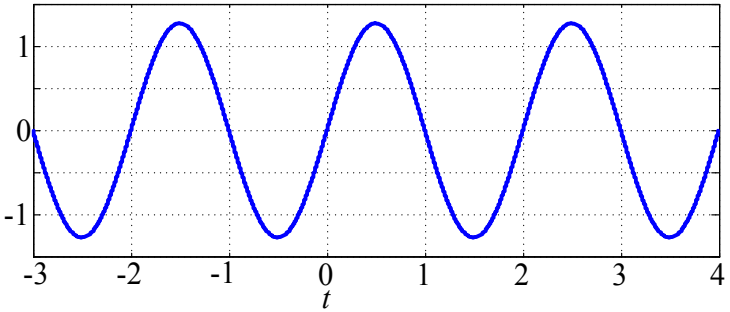
$$H(\mathrm{j}\omega)=\begin{cases}\mathrm{e}^{-\mathrm{j}2\omega} & |\omega|<2\pi\\ 0 & |\omega|>2\pi\end{cases}$$

解：(3) 信号$x_3(t)$ 通过系统的响应$y_3(t)$



系统输入$x_3(t)$



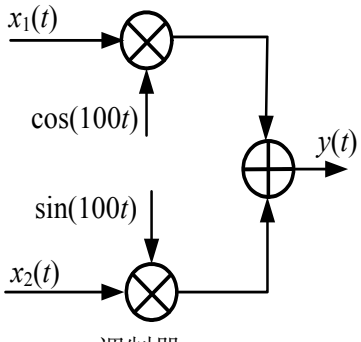
系统输出$y_3(t)=4\sin(\pi t)/\pi$

系统频域分析举例

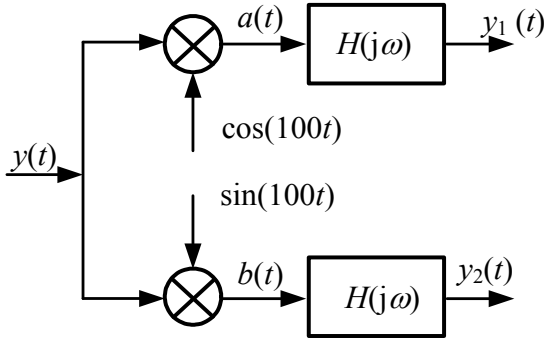
[例] 正交幅度调制系统如图所示

(1) 试求$y(t)$、$a(t)$和$y_1(t)$的频谱并画出频谱图；

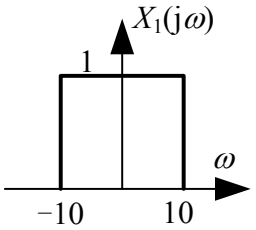
(2) 试求$b(t)$和$y_2(t)$的频谱并画出频谱图。

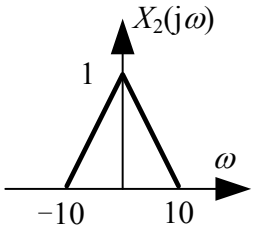


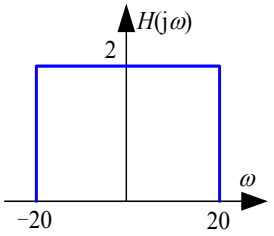
调制器



解调器



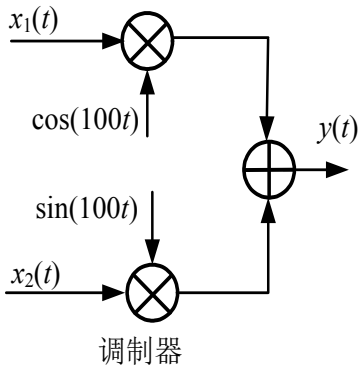


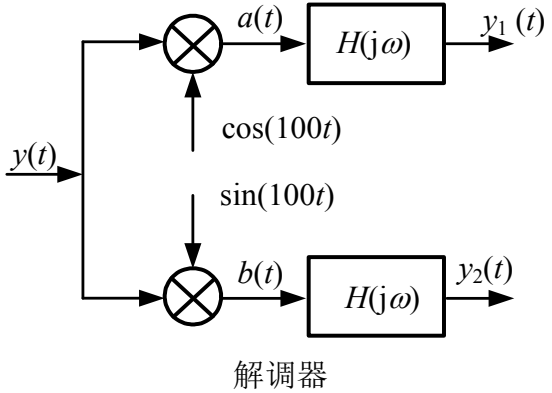


系统频域分析举例

[例] 正交幅度调制系统

解：(1) $y(t)$、 $a(t)$和$y_1(t)$的频谱





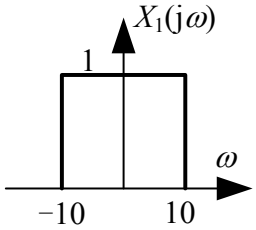
$$Y(j\omega)=[X_1(j(\omega-100))+X_1(j(\omega+100))]/2+[X_2(j(\omega-100))-X_2(j(\omega+100))]/2j$$

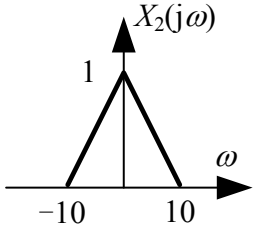
$$\mathrm{Re}\{Y(j\omega)\}=[X_1(j(\omega-100))+X_1(j(\omega+100))]/2$$

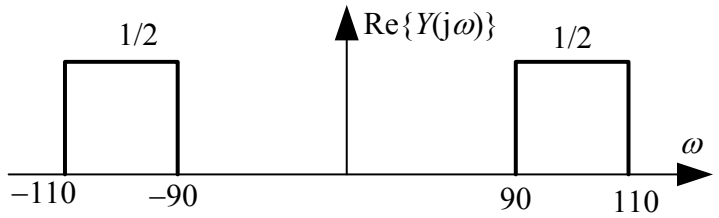
$$\mathrm{Im}\{Y(j\omega)\}=[-X_2(j(\omega-100))+X_2(j(\omega+100))]/2$$

系统频域分析举例

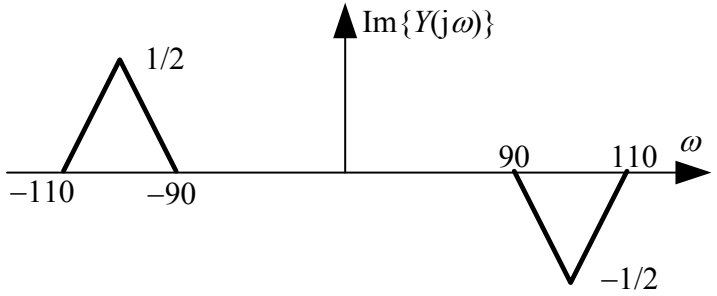
解：(1) $y(t)$、 $a(t)$和$y_1(t)$的频谱







$$\mathrm{Re}\{Y(j\omega)\}=[X_1(j(\omega-100))+X_1(j(\omega+100))]/2$$

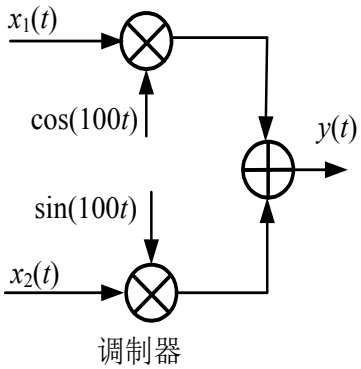


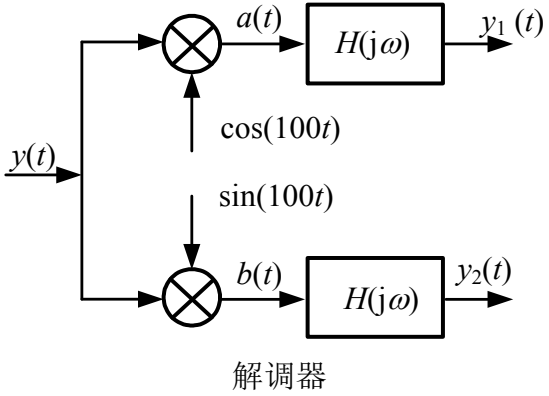
$$\mathrm{Im}\{Y(j\omega)\}=[-X_2(j(\omega-100))+X_2(j(\omega+100))]/2$$

系统频域分析举例

[例] 正交幅度调制系统

解：(1) $y(t)$、 $a(t)$和$y_1(t)$的频谱





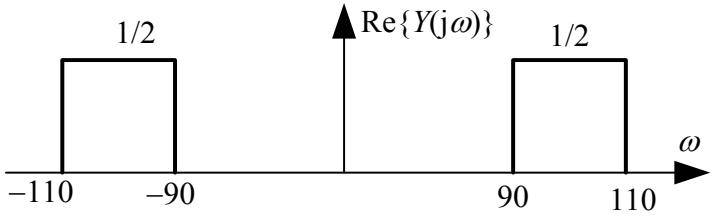
$$A(j\omega)=[Y(j(\omega-100))+Y(j(\omega+100))]/2$$

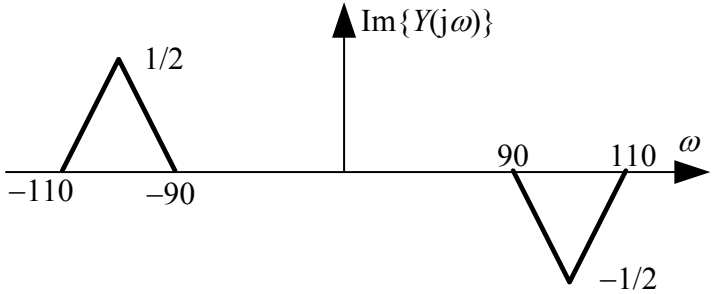
$$\mathrm{Re}\{A(j\omega)\}=[\mathrm{Re}\{Y(j(\omega-100))\}+\mathrm{Re}\{Y(j(\omega+100))\}]/2$$

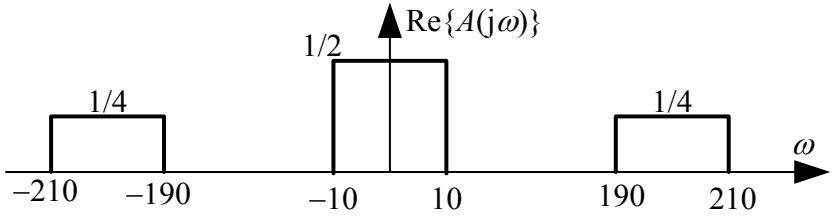
$$\mathrm{Im}\{A(j\omega)\}=[\mathrm{Im}\{Y(j(\omega-100))\}+\mathrm{Im}\{Y(j(\omega+100))\}]/2$$

系统频域分析举例

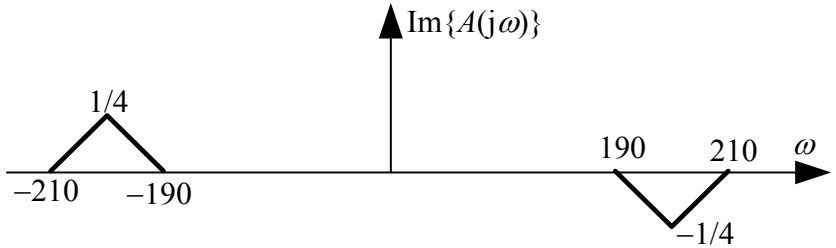
解：(1) $y(t)$、 $a(t)$和$y_1(t)$的频谱







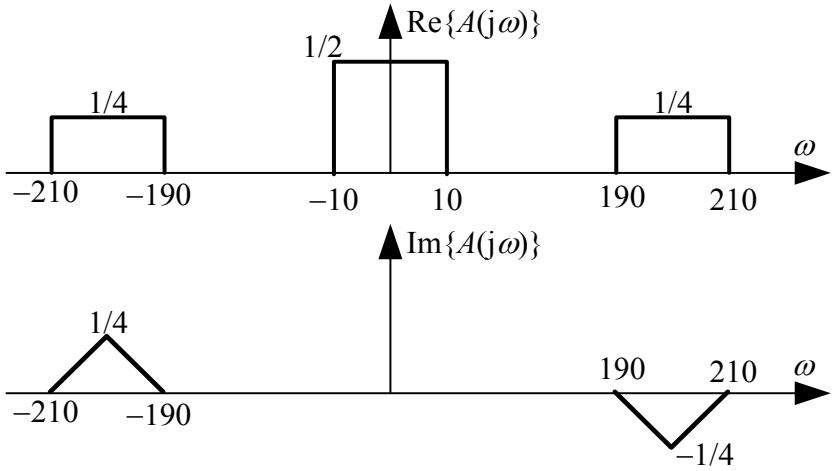
$$\mathrm{Re}\{A(\mathrm{j}\omega)\}=[\mathrm{Re}\{Y(\mathrm{j}(\omega-100))\}+\mathrm{Re}\{Y(\mathrm{j}(\omega+100))\}]/2$$

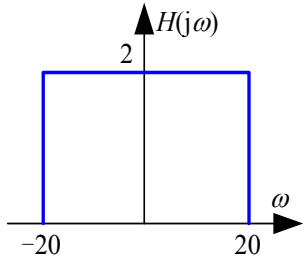


$$\mathrm{Im}\{A(\mathrm{j}\omega)\}=[\mathrm{Im}\{Y(\mathrm{j}(\omega-100))\}+\mathrm{Im}\{Y(\mathrm{j}(\omega+100))\}]/2$$

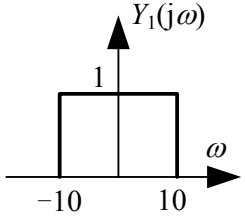
系统频域分析举例

解：(1) $y(t)$、 $a(t)$和$y_1(t)$的频谱





$$Y_1(\mathrm{j}\omega) = H(\mathrm{j}\omega)A(\mathrm{j}\omega)$$

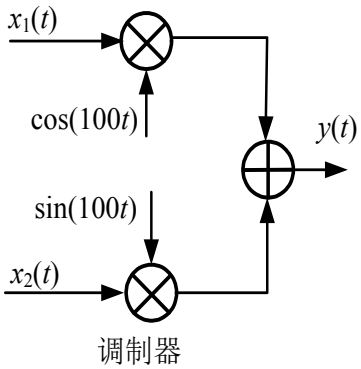


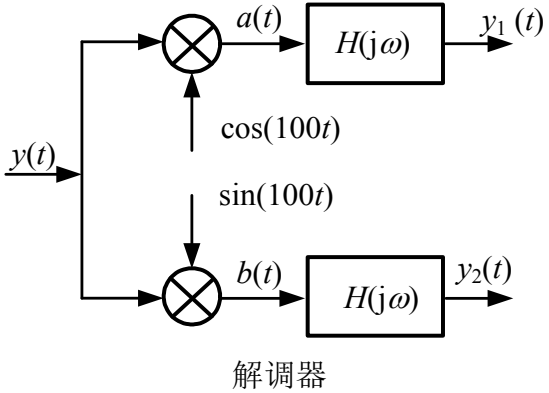
由图可见 $Y_1(\mathrm{j}\omega) = X_1(\mathrm{j}\omega)$

系统频域分析举例

[例] 正交幅度调制系统

解：(2) $b(t)$和$y_2(t)$的频谱





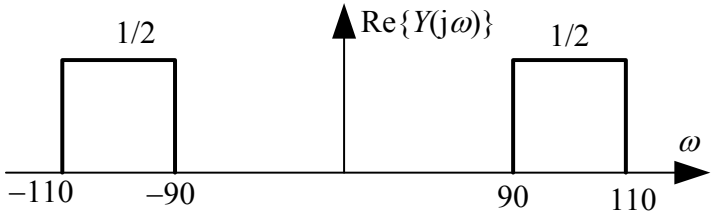
$$B(j\omega)=[Y(j(\omega-100))-Y(j(\omega+100))]/2j$$

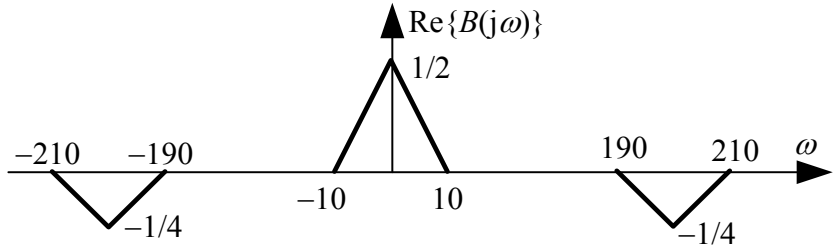
$$\mathrm{Re}\{B(j\omega)\}=[\mathrm{Im}\{Y(j(\omega-100))\}-\mathrm{Im}\{Y(j(\omega+100))\}]/2$$

$$\mathrm{Im}\{B(j\omega)\}=[-\mathrm{Re}\{Y(j(\omega-100))\}+\mathrm{Re}\{Y(j(\omega+100))\}]/2$$

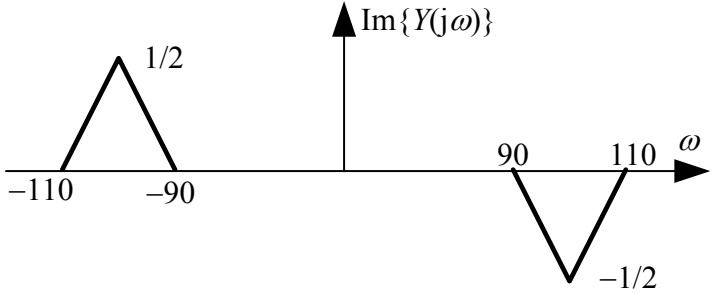
系统频域分析举例

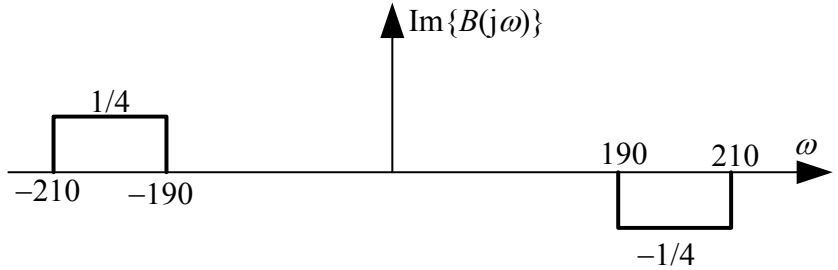
解：(2) $b(t)$和$y_2(t)$的频谱





$$\mathrm{Re}\{B(\mathrm{j}\omega)\}=[\mathrm{Im}\{Y(\mathrm{j}(\omega-100))\}-\mathrm{Im}\{Y(\mathrm{j}(\omega+100))\}]/2$$

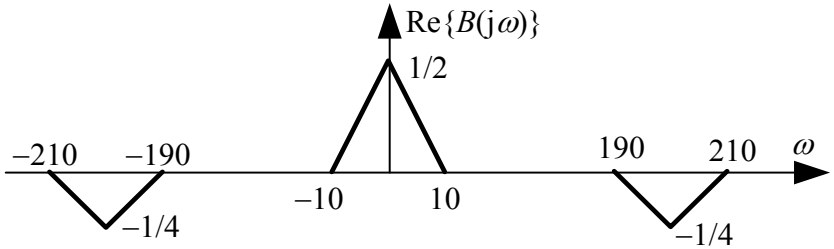


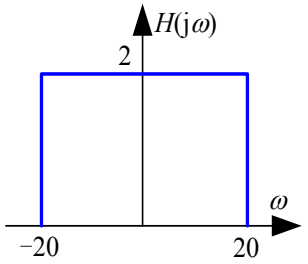


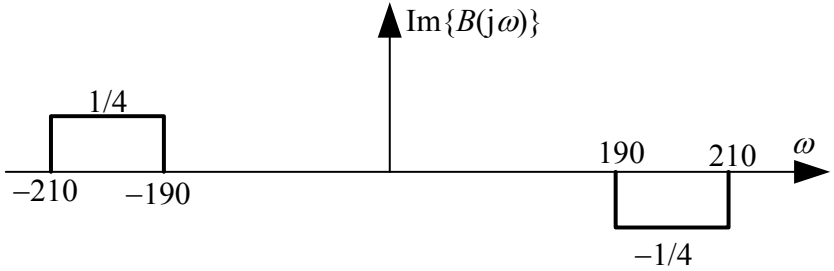
$$\mathrm{Im}\{B(\mathrm{j}\omega)\}=[-\mathrm{Re}\{Y(\mathrm{j}(\omega-100))\}+\mathrm{Re}\{Y(\mathrm{j}(\omega+100))\}]/2$$

系统频域分析举例

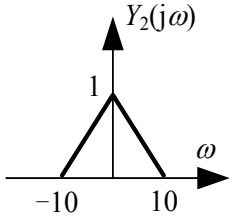
解：(2) $b(t)$和$y_2(t)$的频谱







$$Y_2(\mathrm{j}\omega)\} = H(\mathrm{j}\omega)B(\mathrm{j}\omega)$$



由图可见　$Y_2(\mathrm{j}\omega) = X_2(\mathrm{j}\omega)$

谢　谢

本课程所引用的一些素材为主讲老师多年的教学积累，来源于多种媒体及同事、同行、朋友的交流，难以一一注明出处，特此说明并表示感谢！