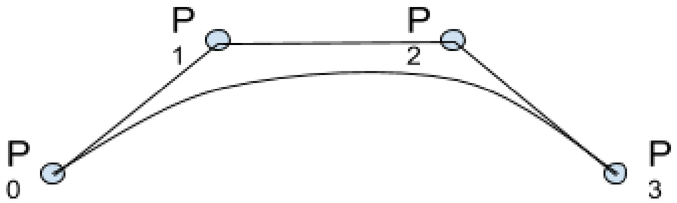
Shutter Curve

Bezier curve as pdf

We will use 3rd order bezier curve, which is:

$c(p) = P_0(1-p)^3 + P_1 3(1-p)^2 p + P_2 3(1-p)p^2 + P_3 p^3$

$P_i\ i \in 0..3$ - control points of the form $(t_i; f_i)$; $p \in [0; 1]$ - parameter;



Let's present $c(p) = (f(p), t(p))$, where $f(p)$ specifies probability density and $t(p)$ specifies the value of random variable. We assume $c(p)$ is valid pdf, to get cdf out of it, we'll need to get the curve in the form of $F(t)$ (we suppose it's possible to do so) and get this function integrated:

$cdf(t) = \int\limits_{t_{start}}^{t} F(t^*)dt^*$;

Let's substitute $t = t(p)$ here:

$cdf(p) = \int\limits_0^p F(t(p^*))t'(p^*)dp^*$;

From there note that $F(t(p)) = f(p)$ and, finally integral will look like:

$cdf(p) = \int\limits_0^p f(p^*)t'(p^*)dp^*$;

Now we recall that our curve can be not the valid pdf and have area under the curve not equal to 1. To get normalization factor, we will say that cdf(1) = 1, and hence

$K\int\limits_0^1 f(p^*)t'(p^*)dp^* = 1$;

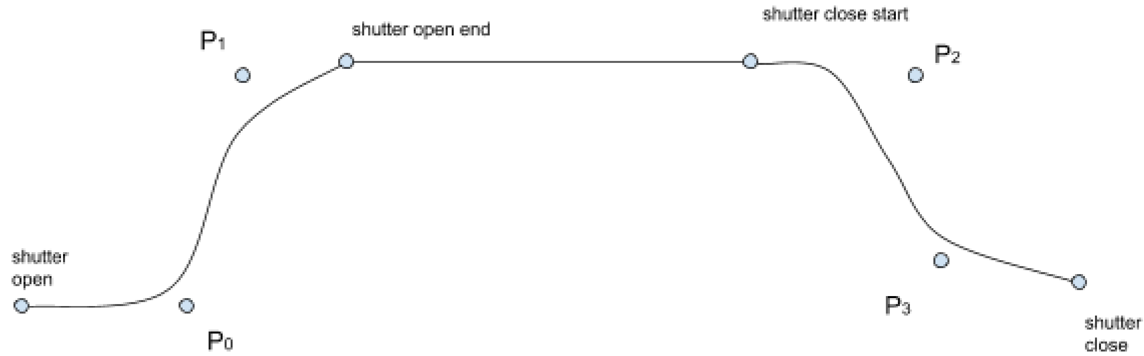
$K = \frac{1}{\int\limits_0^1 f(p^*)t'(p^*)dp^*}$ - our normalization factor;

We now want to inverse sample this curve; $cdf(p)$ and $t(p)$ form a new parametric curve, so the algorithm is:

1) For given value V find such parameter p^* that $cdf(p^*) = V$;
2) Evaluate $t(p^*)$ and return this new value

Since we have all derivatives, we'll use Newton method for our purpose. The numeric method is quite generic and there is no necessity to describe it here.

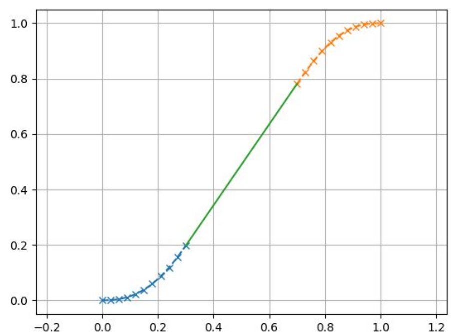
Combination of bezier curves and constant



In this case we have curve defined as an union of points:
$C = \{c_0(p) \mid c_1(p) \mid const(p),\ p \in [0; 1]\}$ where $c_0(p)$ and $c_1(p)$ are bezier curves and $const(p)$ is constant function (i.e. $t(p)$ runs from shutter_open_end to shutter_close_start and $f(p) = const$).
We'll deal with cdf for this curve as a separate 3 curves:

1) Bezier curve $cdf_0(p)$
2) Const $cdf_{const}(t)$
3) Bezier curve $cdf_1(p)$

Note that const cdf is NOT PARAMETRIC:
As the final point is to inverse sample the cdf, there is no point to present function in parametric form.
Note that further cdfs are not valid by themselves, but their union alongside normalization factor will make a valid cdf.



Const cdf

Actually, it's very simple:
$cdf_{const}(t) \ = const \cdot t + cdf_0(1)$
Getting into account that our $const = 1$: this section describes fully opened shutter, we get

$cdf_{const}(t) = t$
This cdf should be evaluated for $t \in [shutter\ open\ end;\ shutter\ close\ start]$

Bezier cdfs

As we have it earlier: $cdf_0(p)$, $cdf_1(p)$
Those are curves described in “bezier curve” section of this document.
$cdf_1(p)$ should include $cdf_0(1)$ and $shutter\ close\ start - shutter\ open\ end$ as addendum to make our final cdf continuous.
Note that we are skipping normalization stuff as we are going to handle this on the whole curve scope.

Normalization factor

As in bezier curve case we want our cdf to be 1 at the end of time domain, so
$K[cdf_0(1) + shutter\ close\ start - shutter\ open\ end + cdf_1(1)] = 1$
Hence,
$K = \frac{1}{cdf_0(1) + shutter\ close\ start - shutter\ open\ end + cdf_1(1)}$

Final inverse sampling algorithm:

1) With the given value V determine, what section do we need:
    a) if $V < K \cdot cdf_0(1)$, we are sampling the first bezier curve section as was described previously
    b) if $V \geq K \cdot cdf_0(1)$ and $V < K \cdot (cdf_0(1) + shutter\ close\ start - shutter\ open\ end)$, we are sampling const section ($t = \frac{V}{K}$)
    c) else we are sampling the second bezier curve