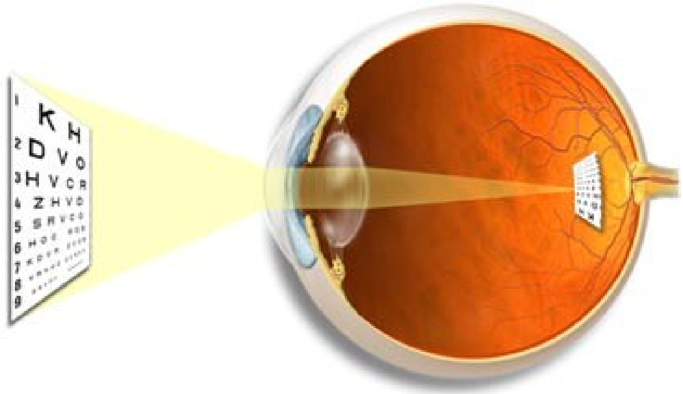
Digital Image Fundamentals

Image Quality

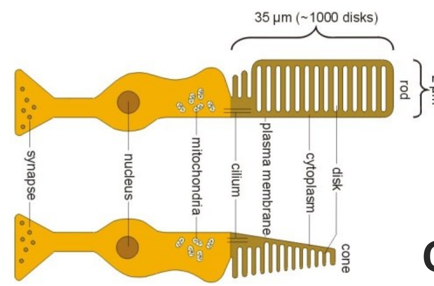
- Objective/ subjective
  - Machine/human beings
  - Mathematical and Probabilistic/ human intuition and perception

Structure of the Human Eye



photoreceptor cells

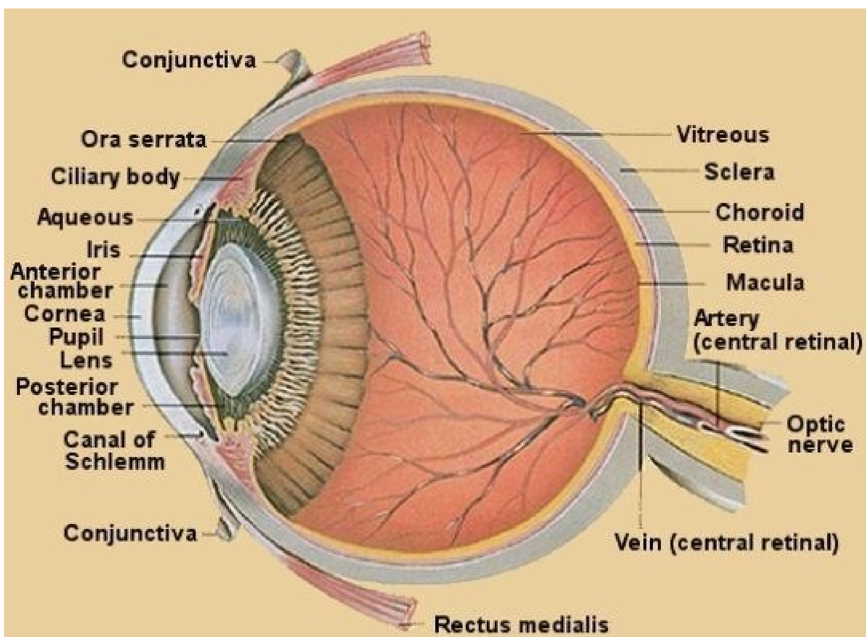
75~150 million

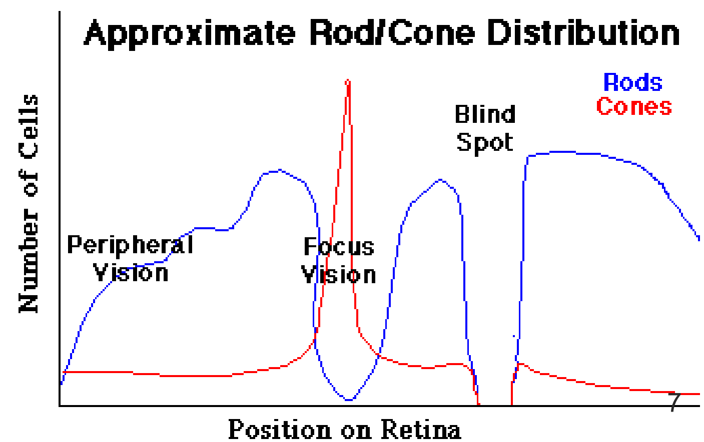


Rod cell

Cone cell

6~7 million

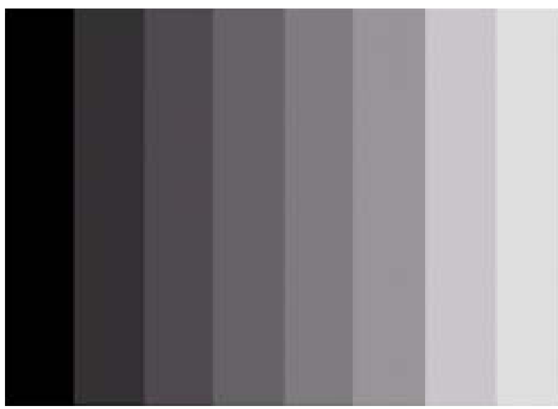


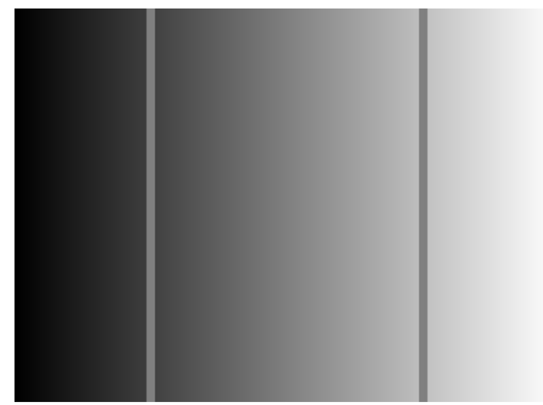


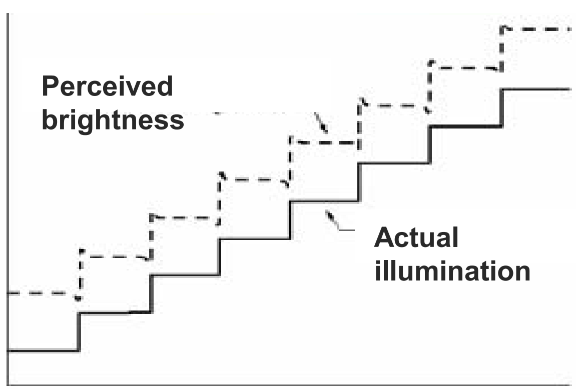
http://en.wikipedia.org/wiki/Blind_spot_(vision)

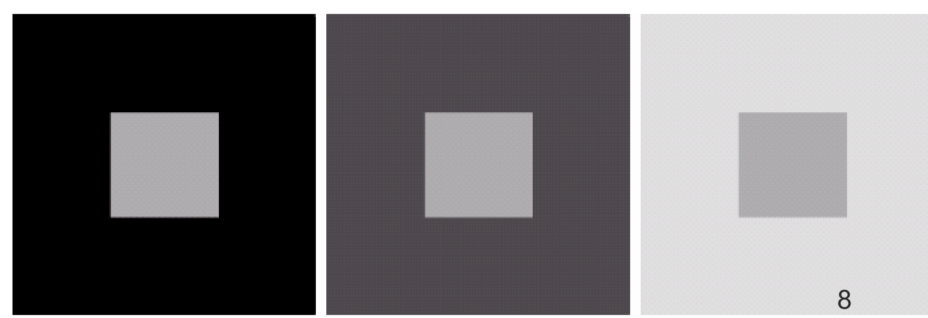
Human Visual Perception

- **Perceived brightness is NOT a simple function of intensity**



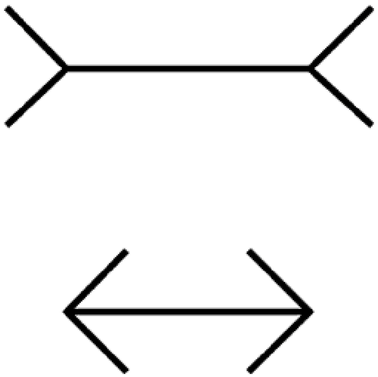


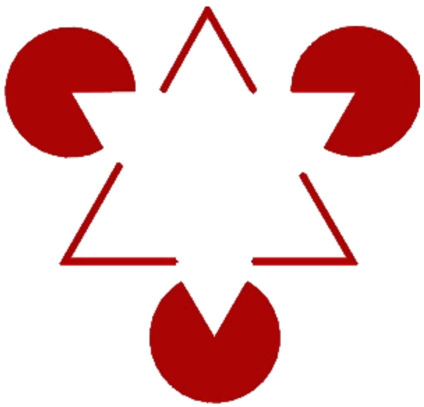


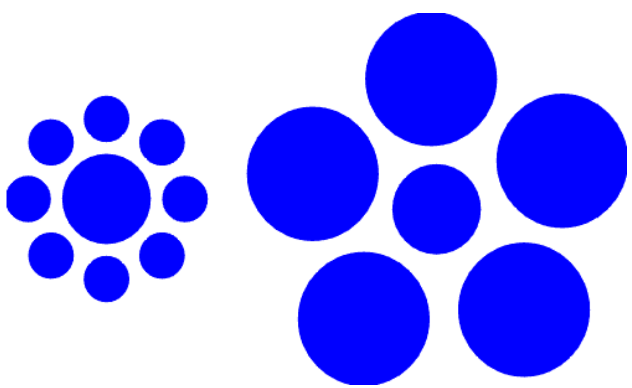


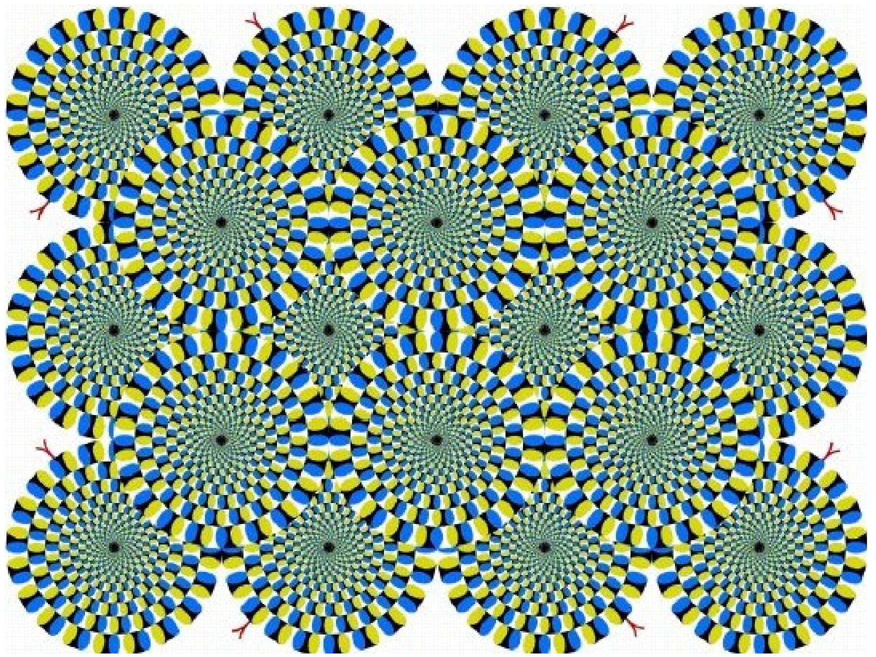
Human Visual Perception

- **Optical Illusion**







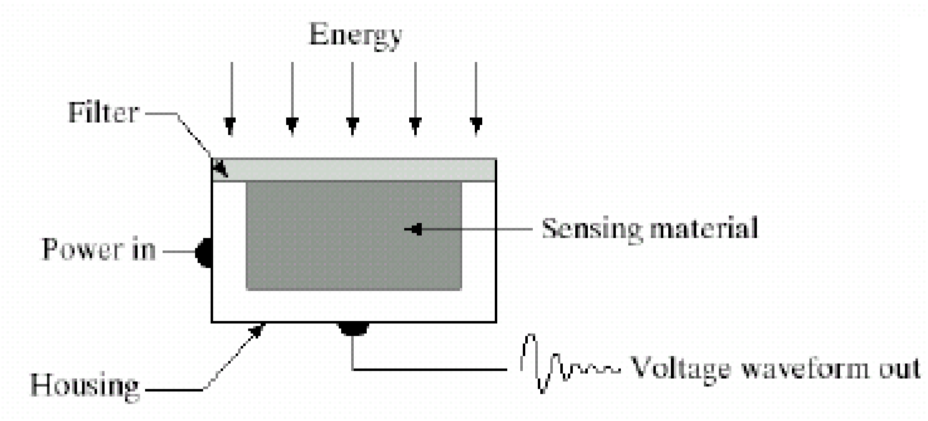


http://architecturehell.wordpress.com/

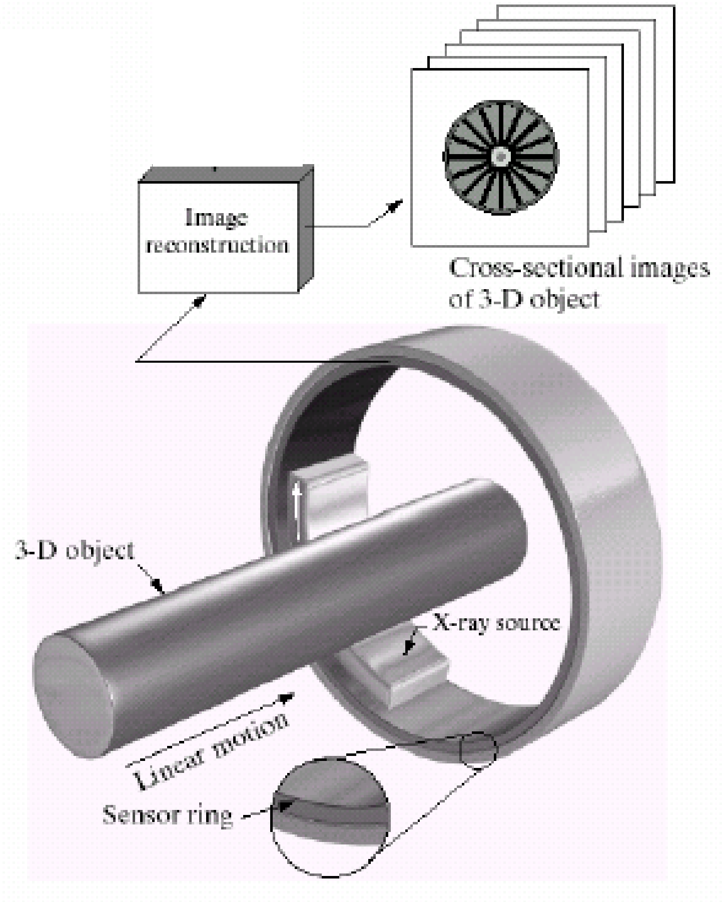
Image Sensing and Acquisition

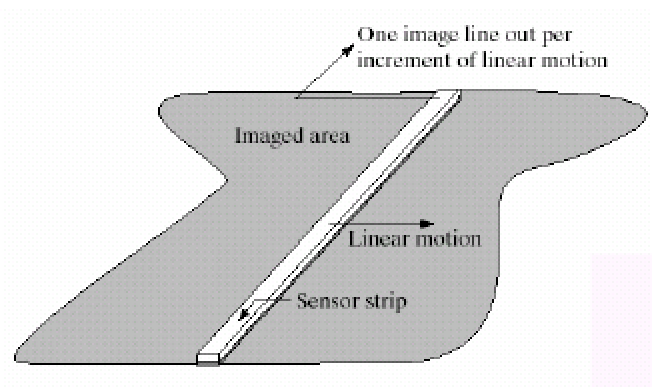
- Illumination Source
  - EM energy, ultrasound, synthesized, …
- Scene Element
  - Objects, human organs, buried mineral,…
- Sensing Material
  - Single sensor: photodiode
  - Sensor strips: require extensive processing
  - Sensor arrays: CCD & CMOS

Image Sensing and Acquisition



Single sensor





Sensor Strip

Circular Sensor Strip

Image Sensing and Acquisition

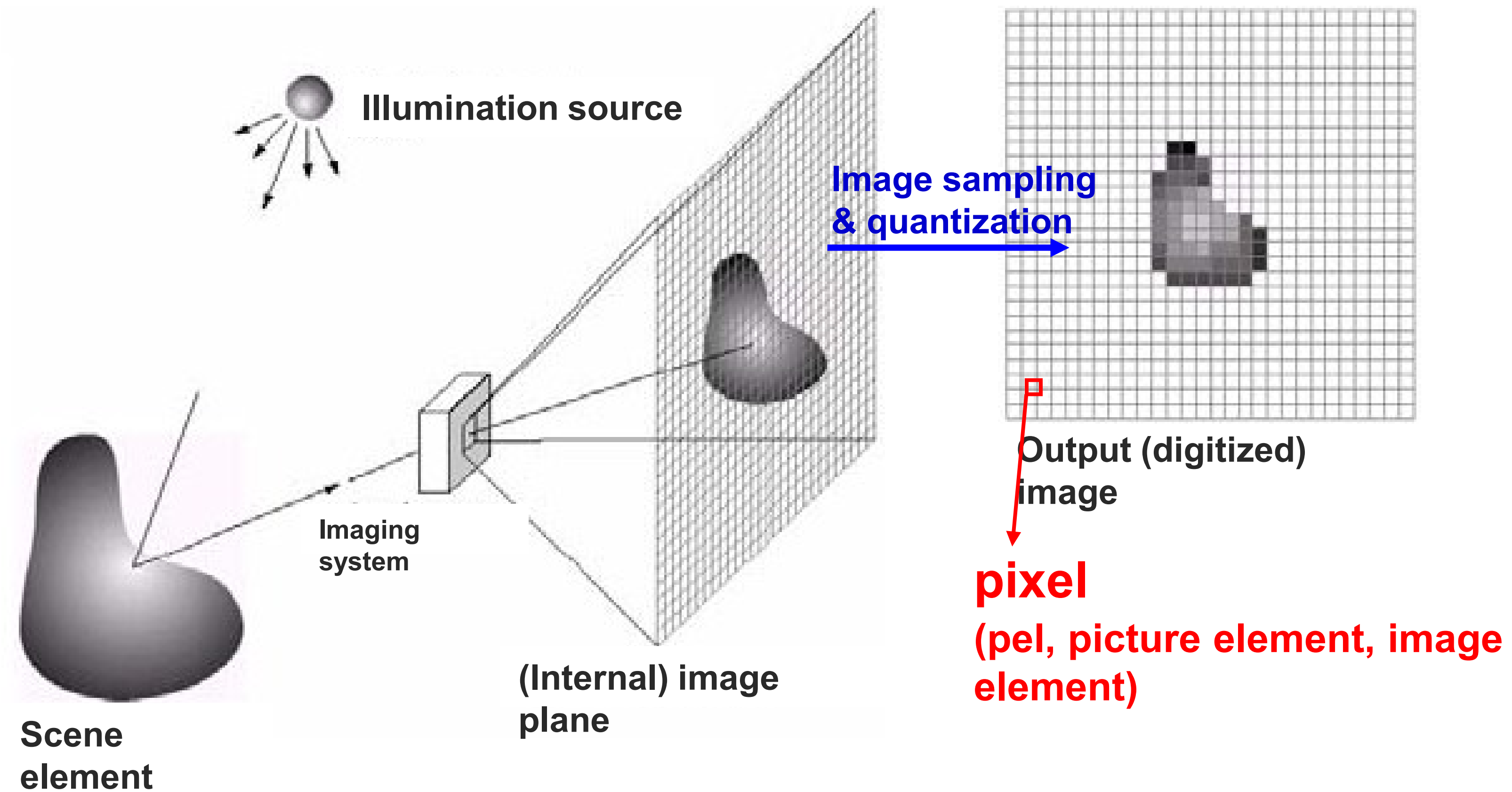
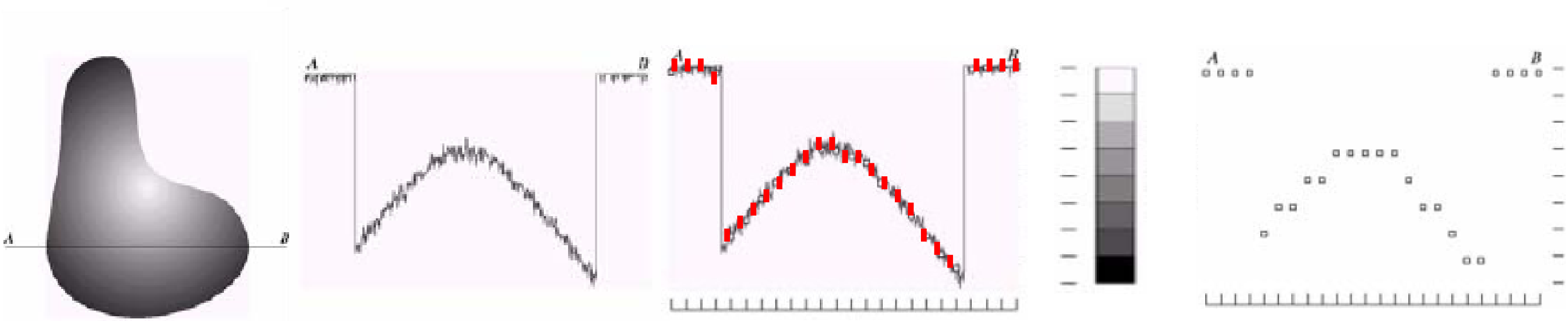


Image Sampling & Quantization

$$f(x,y) \rightarrow \boxed{\text{A/D}} \rightarrow F(j,k)$$

sampling **quantization**

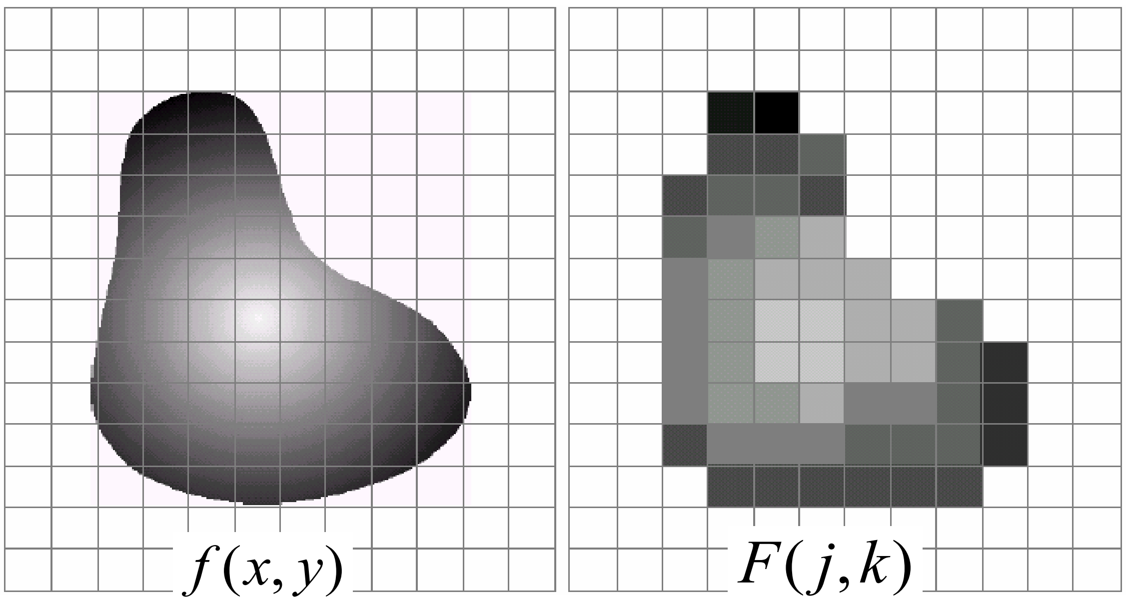


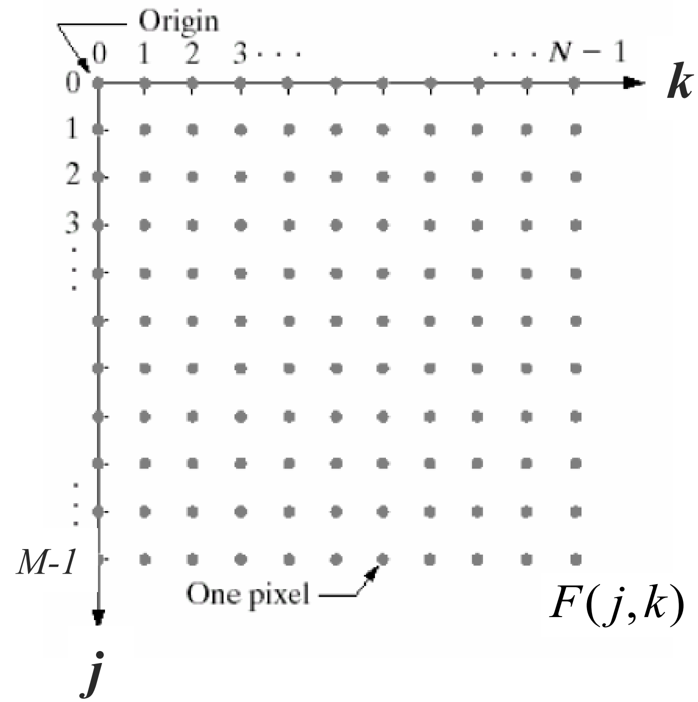
Original image | $f(x,y)$ | sampling | quantization | $F(j,k)$

sampling $\downarrow$ coordinates

quantization $\downarrow$ amplitude

Image Sampling & Quantization



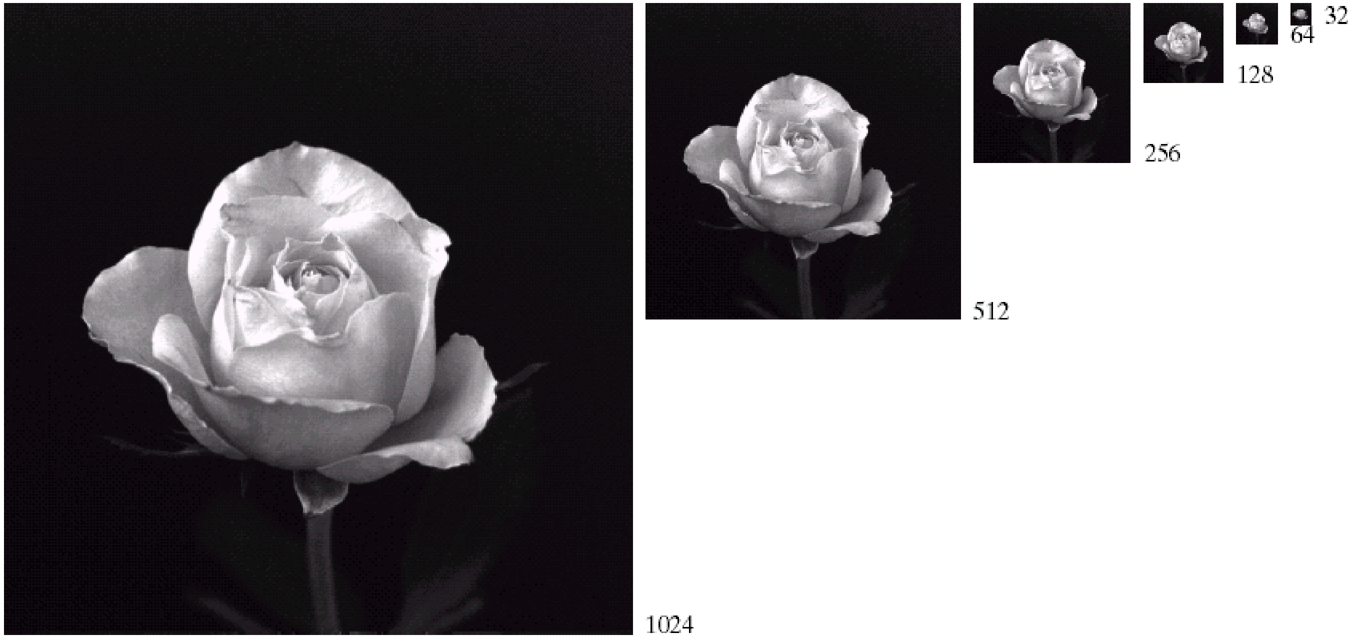


$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

$$F(j,k) = \begin{bmatrix} F(0,0) & F(0,1) & \cdots & F(0,N-1) \\ F(1,0) & F(1,1) & \cdots & F(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ F(M-1,0) & F(M-1,1) & \cdots & F(M-1,N-1) \end{bmatrix}$$

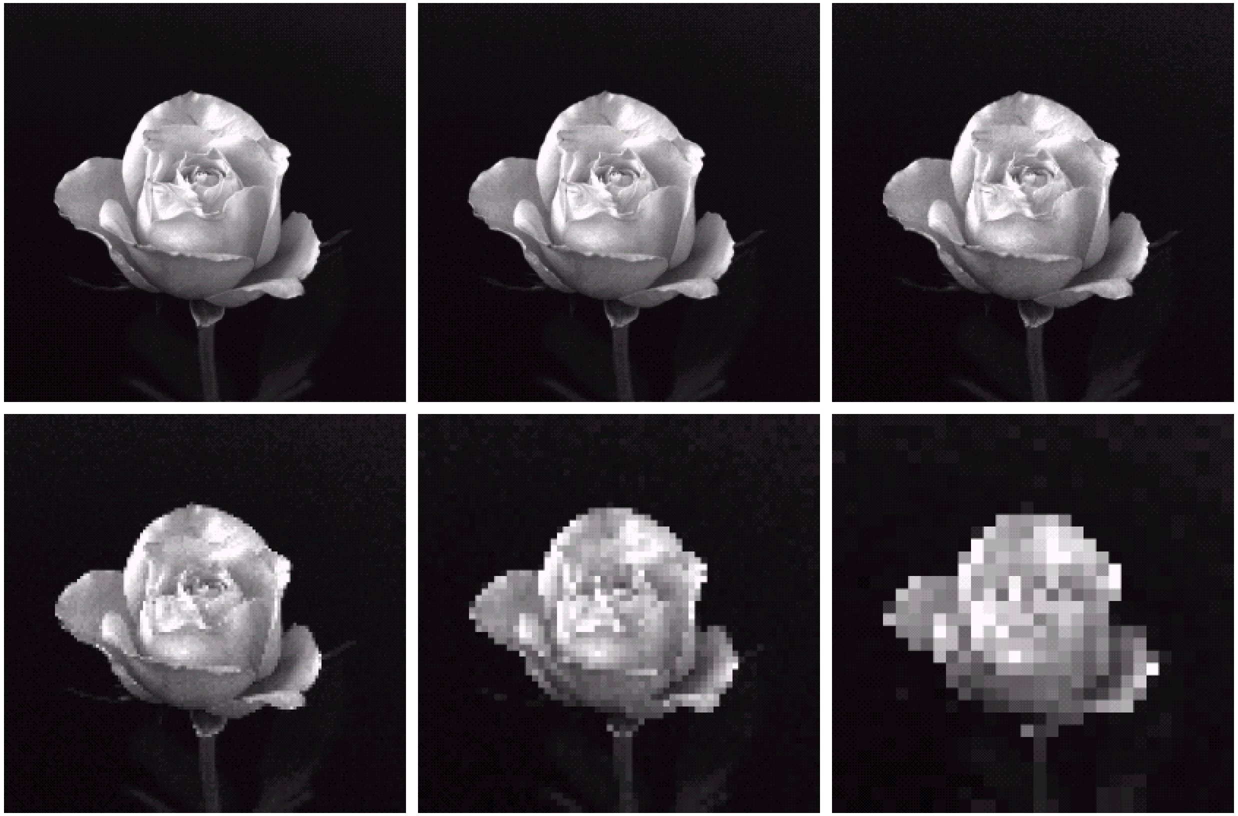
Downsampling

- **1024x1024 → 32x32**
  - Downsampled by a factor of 2

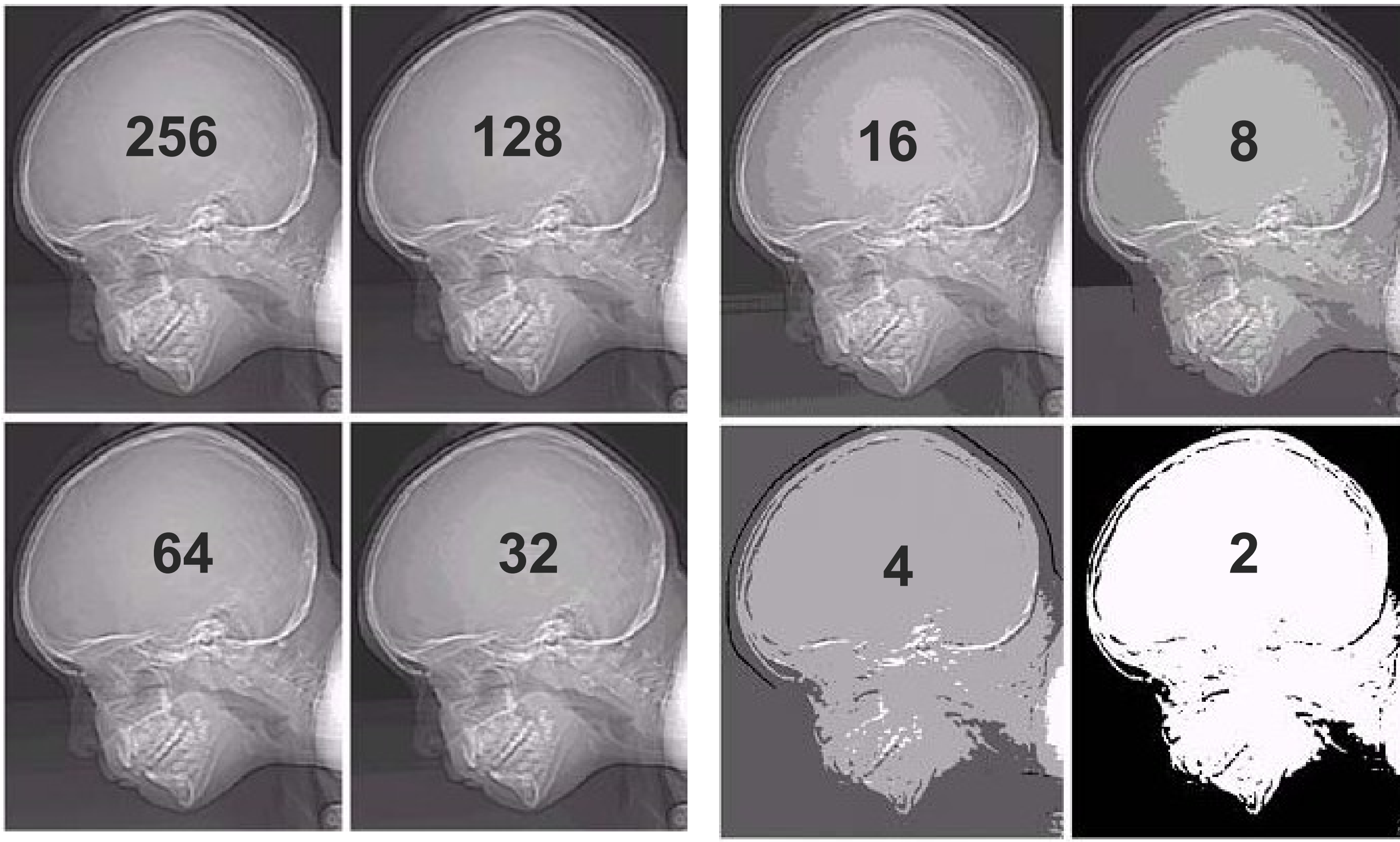


Re-Sampling

- **Zero-Order-Hold Method (ZOH)**
  - **Row and column duplication**

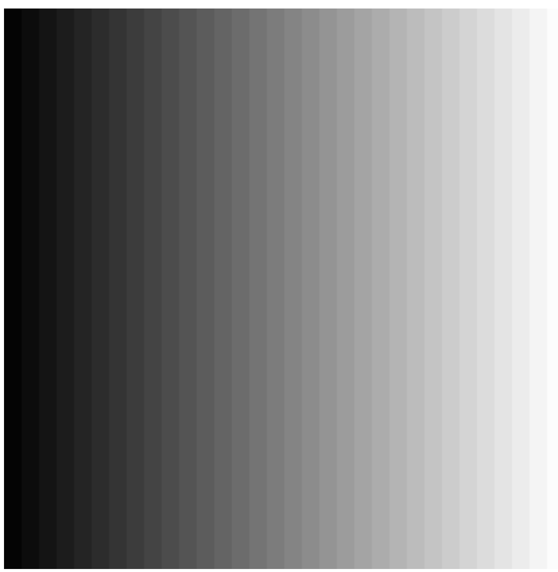


L=256,128,64,32,16,8,4,2

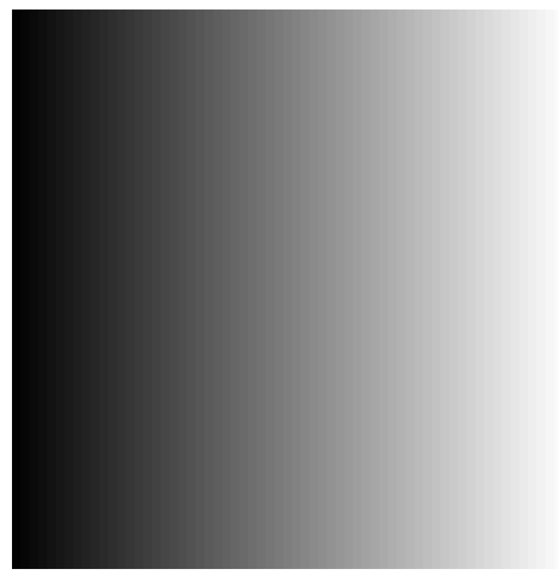


Digital Image Representation

- 8-bit image is commonly used
  - Storage
  - Human perception



32 steps (5 bits) in gray level



64 steps (6 bits) in gray level

Image Enhancement

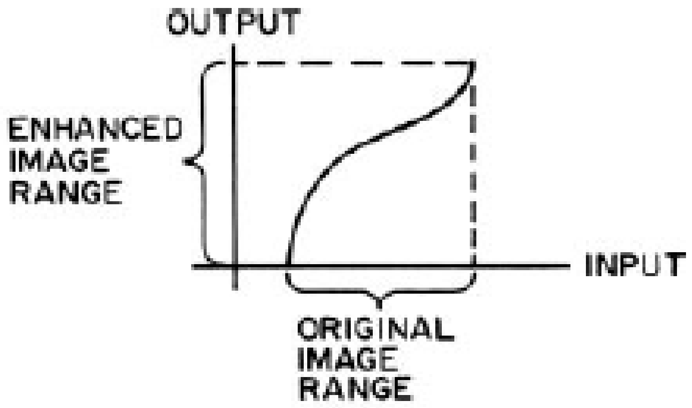
Image Enhancement

- Goal of Image Enhancement
  - make images more appealing
  - no theory, ad-hoc rules, derived with insights

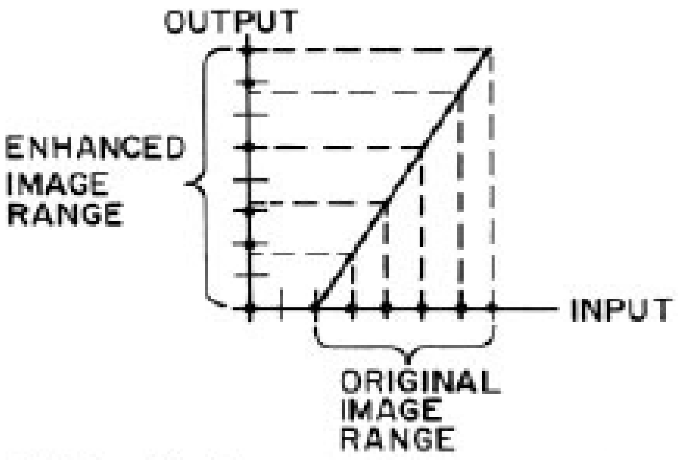
- Two Approaches
  - Contrast Manipulation
  - Histogram Modification

Contrast Manipulation

- **Transfer Function**
  - Linear
  - Nonlinear
  - Piecewise



Continuous Image

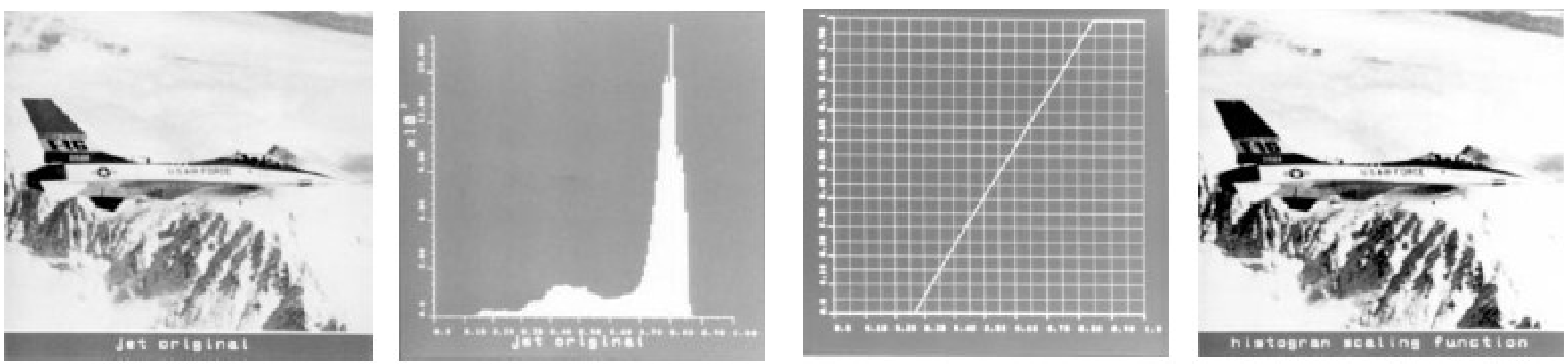


Quantized Image

Contrast Manipulation

- **Linear scaling and clipping**

$$G(j,k) = T\left[F(j,k)\right] \quad 0 \le F(j,k) \le 1$$

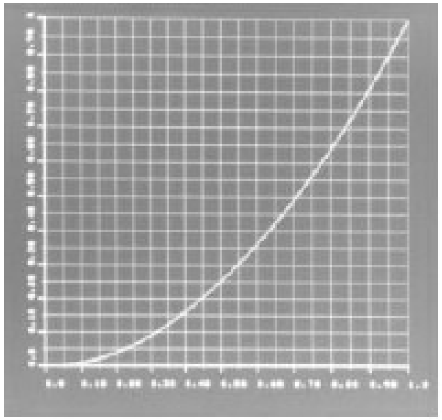


(*a*) Original (*b*) Original histogram (*c*) Transfer function (*d*) Contrast stretched

Contrast Manipulation

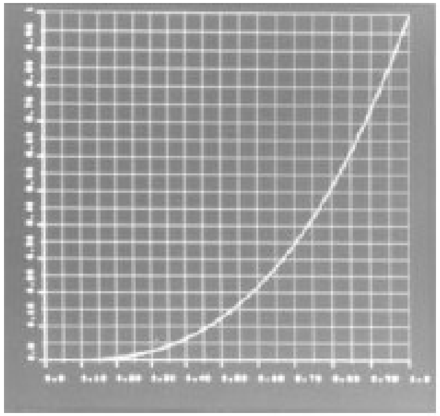
- **Power-Law**

$$G(j,k) = [F(j,k)]^p \qquad 0 \le F(j,k) \le 1$$



(a) Square function

(b) Square output



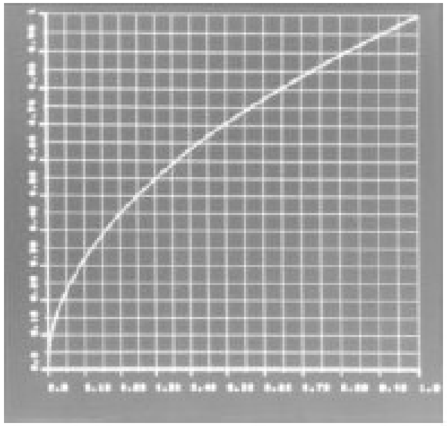
(c) Cube function

(d) Cube output

Contrast Manipulation

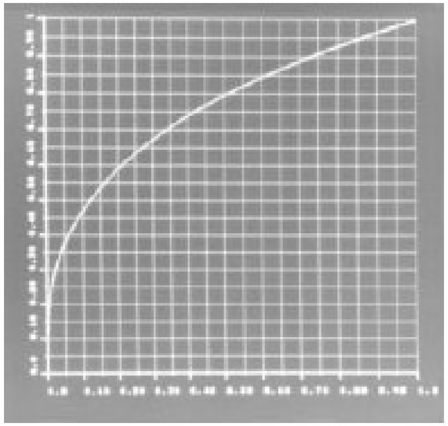
- **Power-Law**

$$G(j,k) = \left[F(j,k)\right]^p \quad 0 \le F(j,k) \le 1$$



(*a*) Square root function

(*b*) Square root output

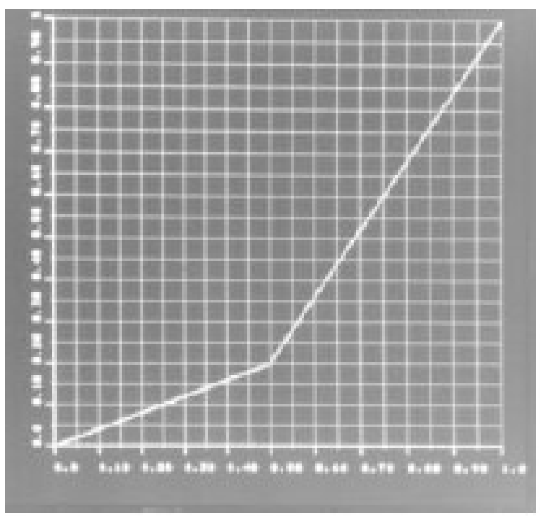


(*c*) Cube root function

(*d*) Cube root output

Contrast Manipulation

- **Rubber Band Transfer Function**
  - Piecewise linear transformation
  - Inflection point (control point)



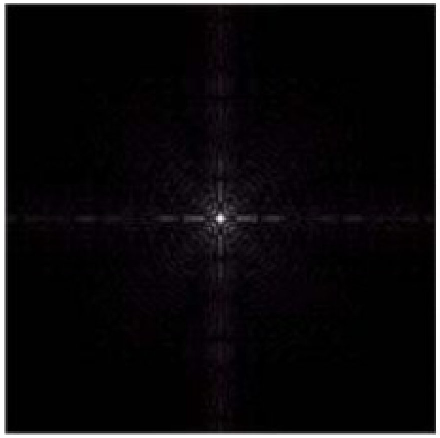
Can choose the area where we want to stretch or reduce the contrast

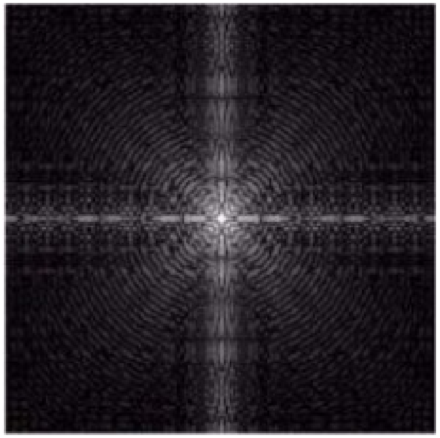
Contrast Manipulation

- **Logarithmic Point Transformation**

$$G(j,k) = \frac{\log_e\{1 + aF(j,k)\}}{\log_e\{2.0\}} \qquad 0 \le F(j,k) \le 1$$

Fourier Spectrum





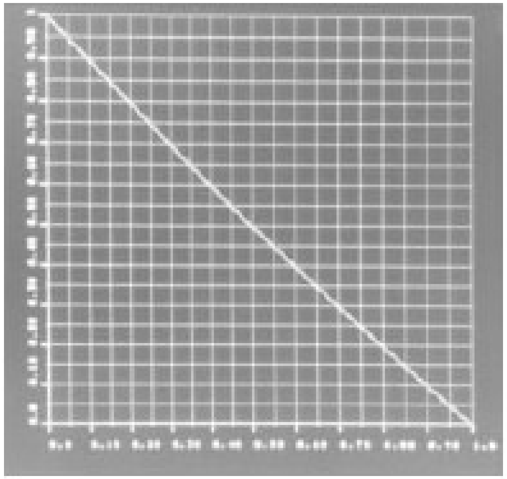
$0 \sim 1.5 \times 10^6 \longrightarrow$ 0~6.2

Useful for scaling image arrays with a very wide dynamic range

Contrast Manipulation

- **Reverse Function**

$$G(j,k) = 1 - F(j,k) \quad 0 \leq F(j,k) \leq 1$$



(a) Reverse function

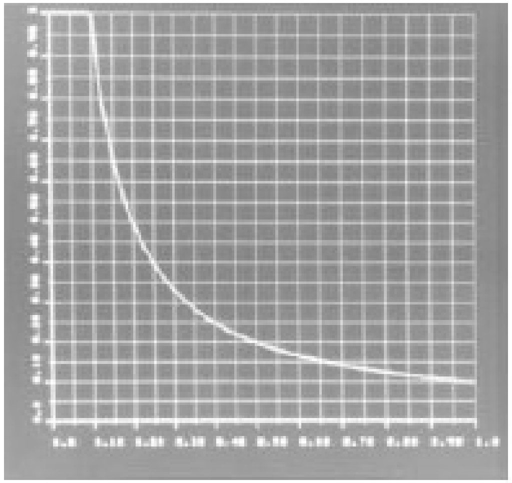
(b) Reverse function output

Able to see more details in dark areas of an image

Contrast Manipulation

- **Inverse Function**

$$G(j,k)=\begin{cases}1 & 0\le F(j,k)\le 0.1\\ \dfrac{0.1}{F(j,k)} & 0.1\le F(j,k)\le 1\end{cases}$$

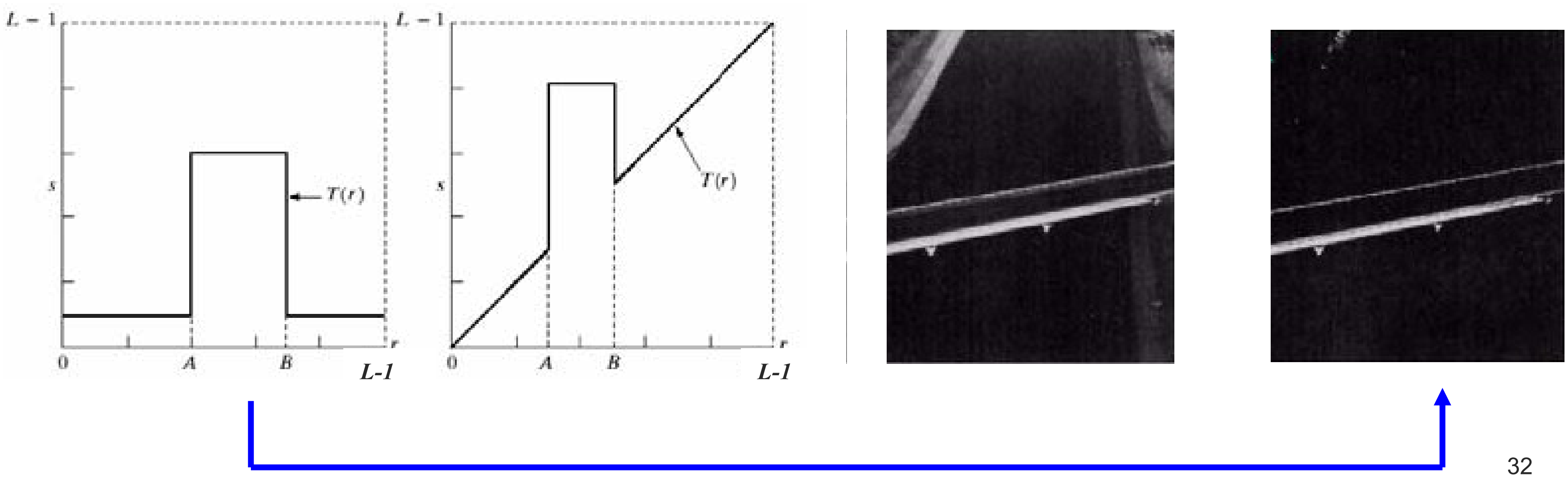


(c) Inverse function

(d) Inverse function output

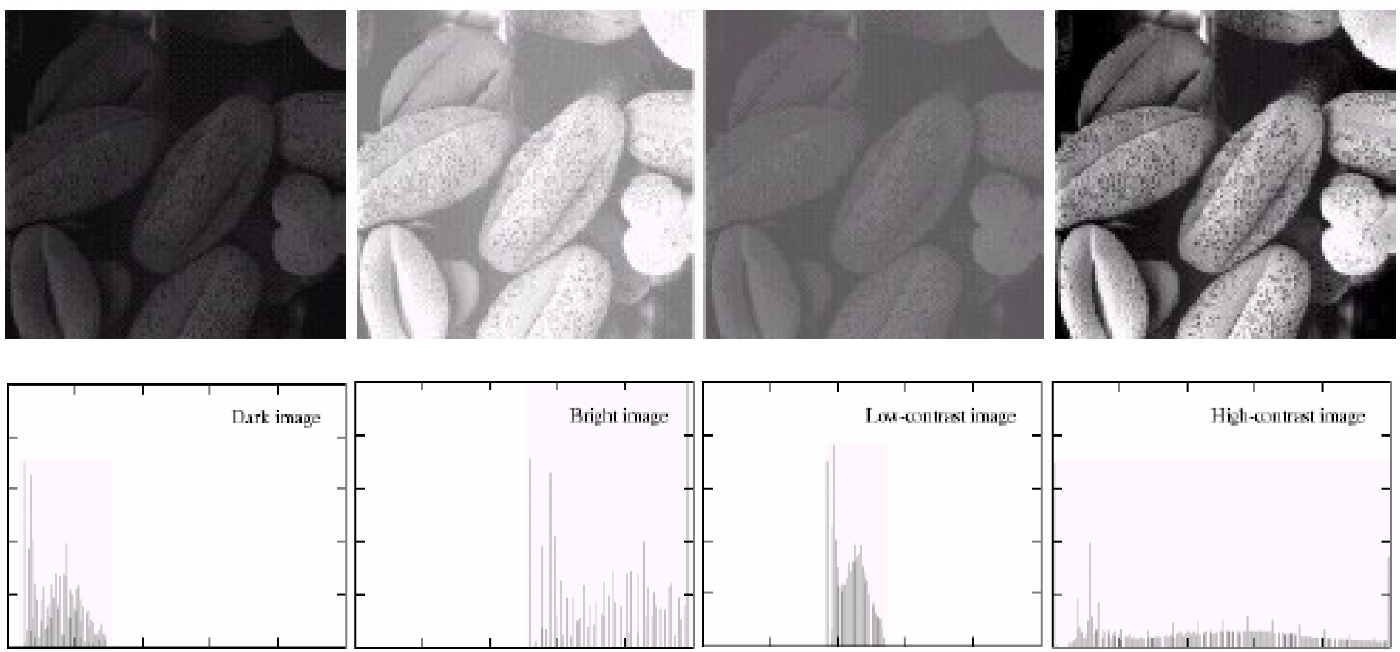
Contrast Manipulation

- **Amplitude-Level Slicing (Gray-Level Slicing)**



Histogram Modification

- **Goal**
  - **Rescale the original image so that the histogram of the enhanced image follows some desired form**

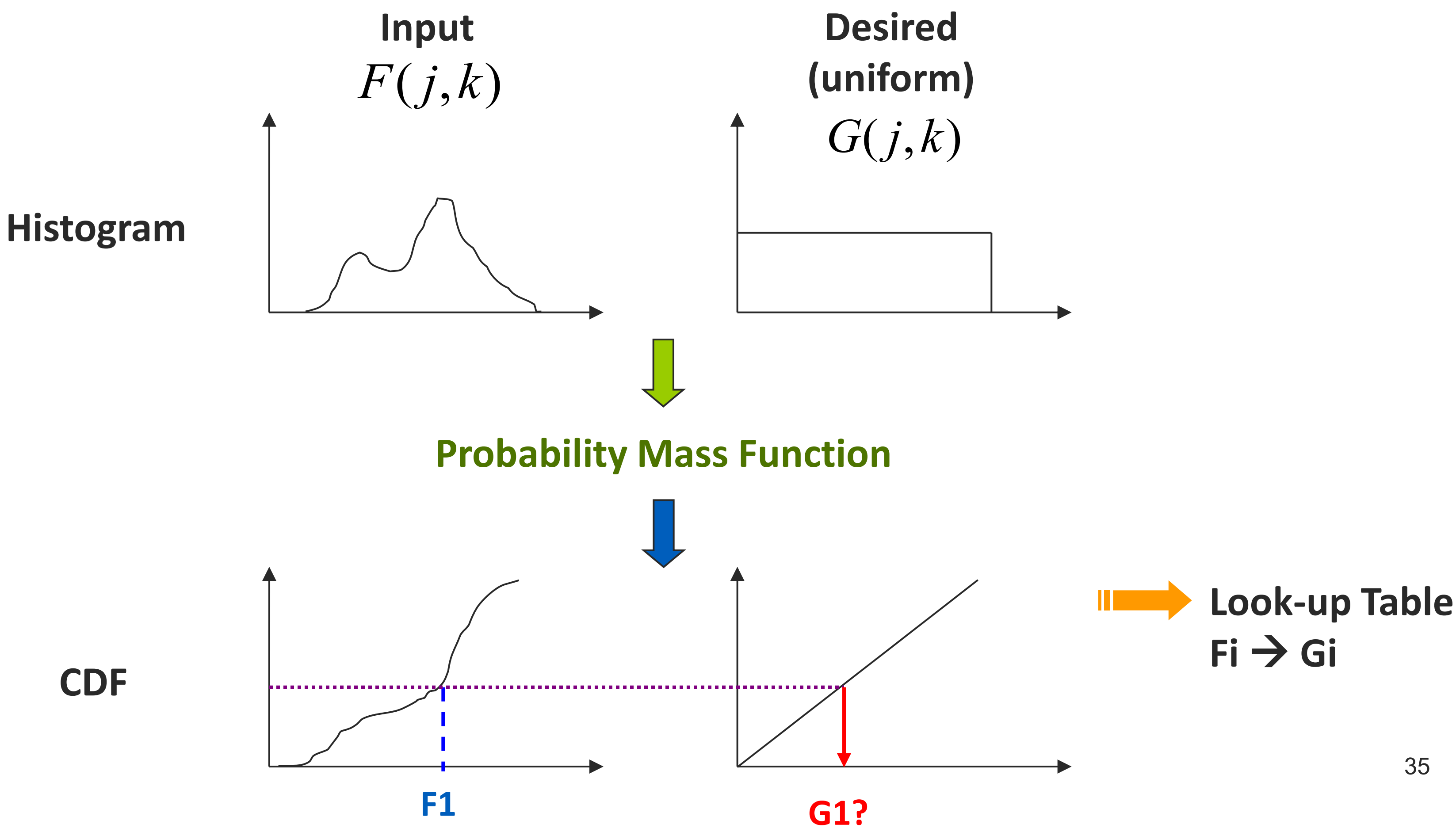


Histogram Modification

- Histogram Equalization
  - make the output histogram to be uniformly distributed
    - Transfer function
    - Bucket filling

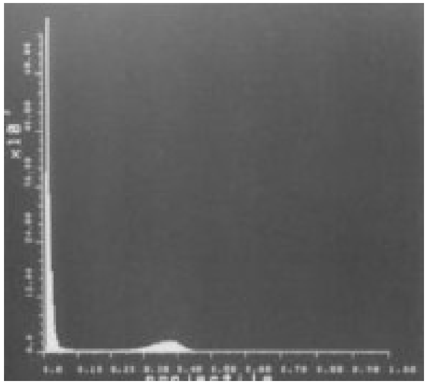
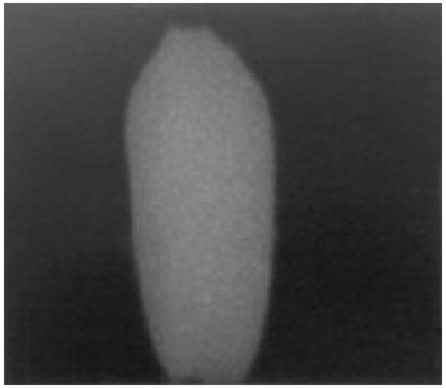
Histogram Equalization

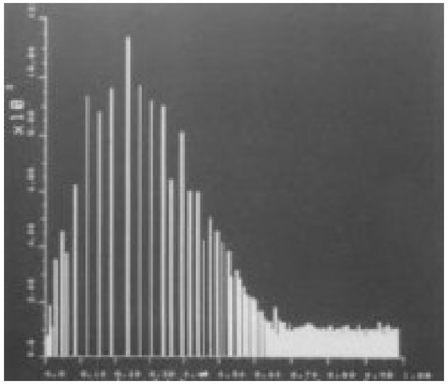
- Transfer Function



Histogram Equalization

- **Transfer Function**
  - **Output histogram not really uniformly distributed**
  - **Still keep the shape**
  - **More flat than the original histogram**



Histogram Equalization

- Bucket Filling

arbitrary

F(j,k)	# of pixels
0	1
1	2
2	5
⋮	⋮
255	3

uniform

G(j,k)	# of pixels
0	N/256
1	N/256
2	N/256
⋮	⋮
255	N/256

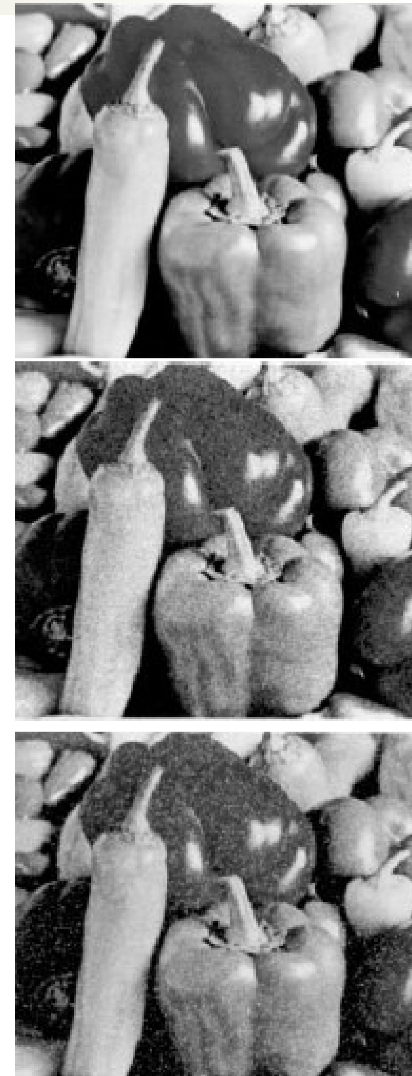
N: # of total pixels

  - Not 1-1 mapping
  - Accumulated probability may not end exactly at the boundary of a bin → split it out

Noise Cleaning

Noise Cleaning

- **Noise**
  - electrical sensor noise
  - photographic grain noise
  - channel error
  - etc.
- **Characteristics of the noise**
  - discrete
  - not spatially correlated
  - higher spatial frequency



Noise Cleaning

- Two types of noise
  - Uniform Noise
    - Additive uniform noise, Gaussian noise
  - Impulse Noise
    - Salt and pepper noise

- Solutions
  - Uniform Noise → low-pass filtering
  - Impulse Noise → non-linear filtering

Basics of Spatial Filtering

- **Mask**
  - filter, kernel, template
  - m x n
    - m=2a+1, n=2b+1,
      where a and b are nonnegative integers
    - e.g. 3x3 mask

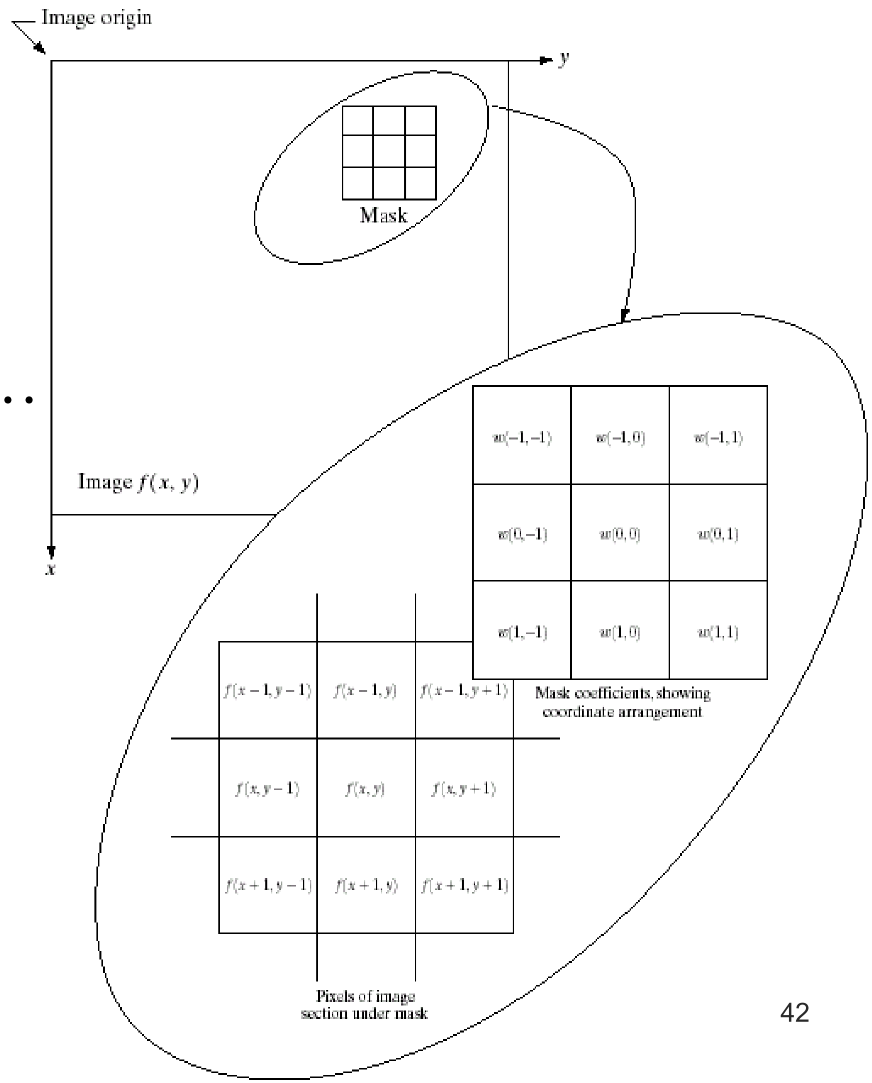
$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	W(0,0)	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

- **Spatial Filtering/Convolution**

$$
\begin{aligned}
G(j,k) \quad = \quad & w(-1,-1)F(j-1,k-1) + w(-1,0)F(j-1,k) + \cdots \\
& + w(0,0)F(j,k) + \cdots \\
& + w(1,0)F(j+1,k) + w(1,1)F(j+1,k+1)
\end{aligned}
$$

Basics of Spatial Filtering

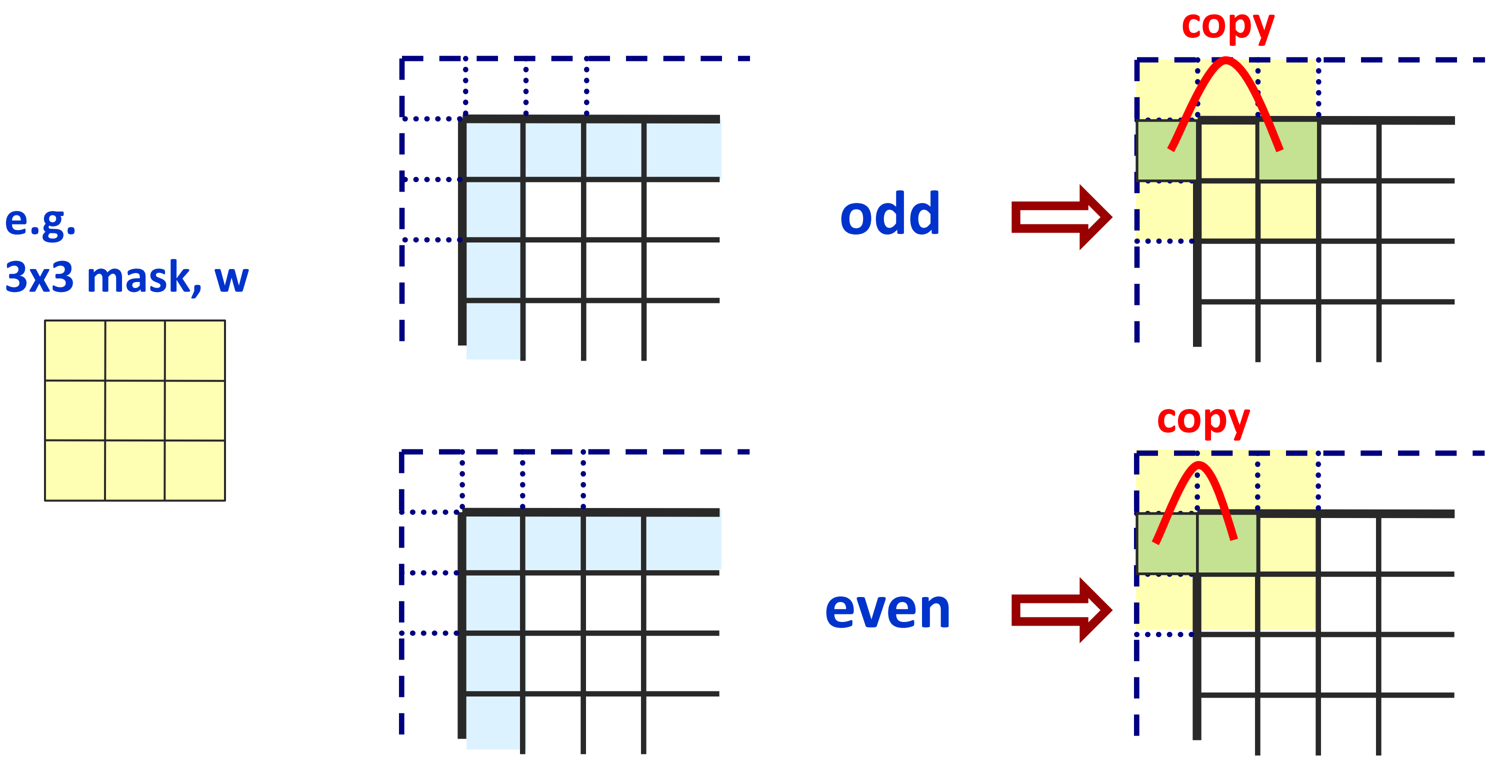
$$
\begin{aligned}
G(j,k) = \; & w(-1,-1)F(j-1,k-1) \\
& + w(-1,0)F(j-1,k) + \cdots \\
& + w(0,0)F(j,k) + \cdots \\
& + w(1,0)F(j+1,k) \\
& + w(1,1)F(j+1,k+1)
\end{aligned}
$$



Q: Boundary pixels?

Basics of Spatial Filtering

- Boundary Extension (3x3 mask)



Q: 5x5 mask?

Noise Cleaning

- **Uniform noise**
  - **Perform low-pass filtering**

$$H = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad H = \frac{1}{10}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad H = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

  - **General form**

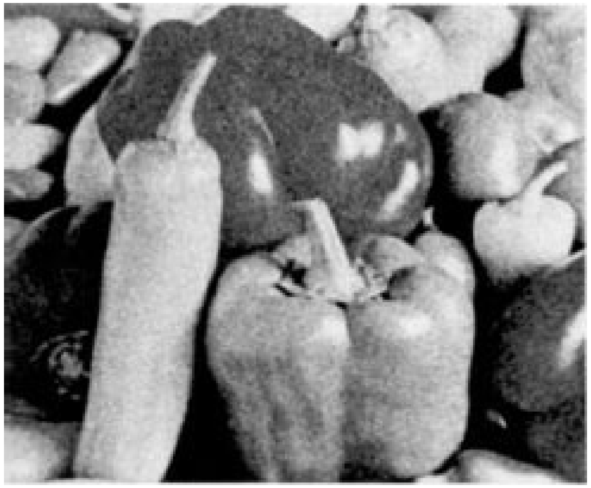
$$H = \frac{1}{(b+2)^2}\begin{bmatrix} 1 & b & 1 \\ b & b^2 & b \\ 1 & b & 1 \end{bmatrix}$$

e.g.

$$F = \begin{bmatrix} 0 & 0 & 180 & 180 \\ 0 & 0 & 180 & 180 \\ 0 & 0 & 180 & 180 \\ 0 & 0 & 180 & 180 \end{bmatrix}$$

High Frequency Noise Removal

- Low-pass filtering
  - Normalized to unit weighting
  - Averaging
  - Smaller/Larger filter size ?



3x3

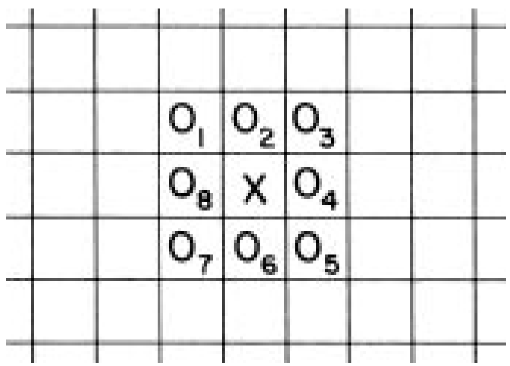
7x7

Noise Cleaning

- Impulse noise
  - black: pixel value =0 → dead sensor
  - white: pixel value=255 → saturated sensor
- Solutions
  - Outlier detection
  - Median filtering
  - Pseudo-median filtering (PMED)

Impulse Noise Removal

- **Outlier detection**



$$if \quad \left| x - \frac{1}{8}\sum_{i=1}^{8} O_i \right| > \varepsilon \qquad then \quad x = \frac{1}{8}\sum_{i=1}^{8} O_i$$

How to choose ε ?

Larger window?

Impulse Noise Removal

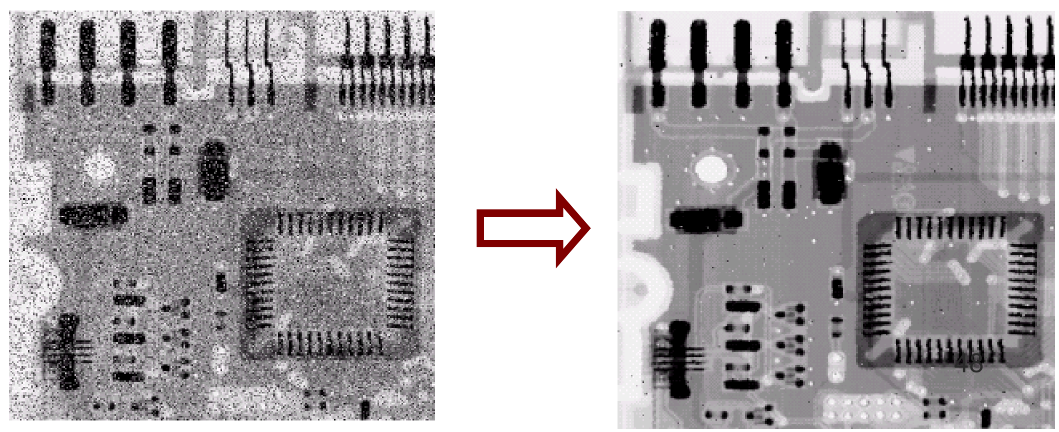
- **Median filtering**

  $$a_1, \ldots, a_N \quad where \quad N \quad is \quad odd$$

  - **sort those values in order**
  - **pick the middle one in the sorted list**
  - **e.g. 3x3 mask:**

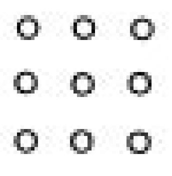
$$I = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 8 & 7 \\ 1 & 5 & 6 \end{bmatrix}$$

→ Median is 3

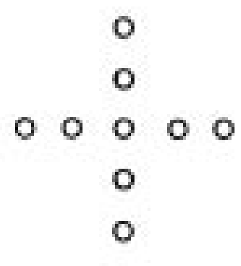


Impulse Noise Removal

- Median filtering
  - Preserve sharp edges
  - Effective in removing impulse noise
  - 1D/2D (directional)
    - e.g. 2D



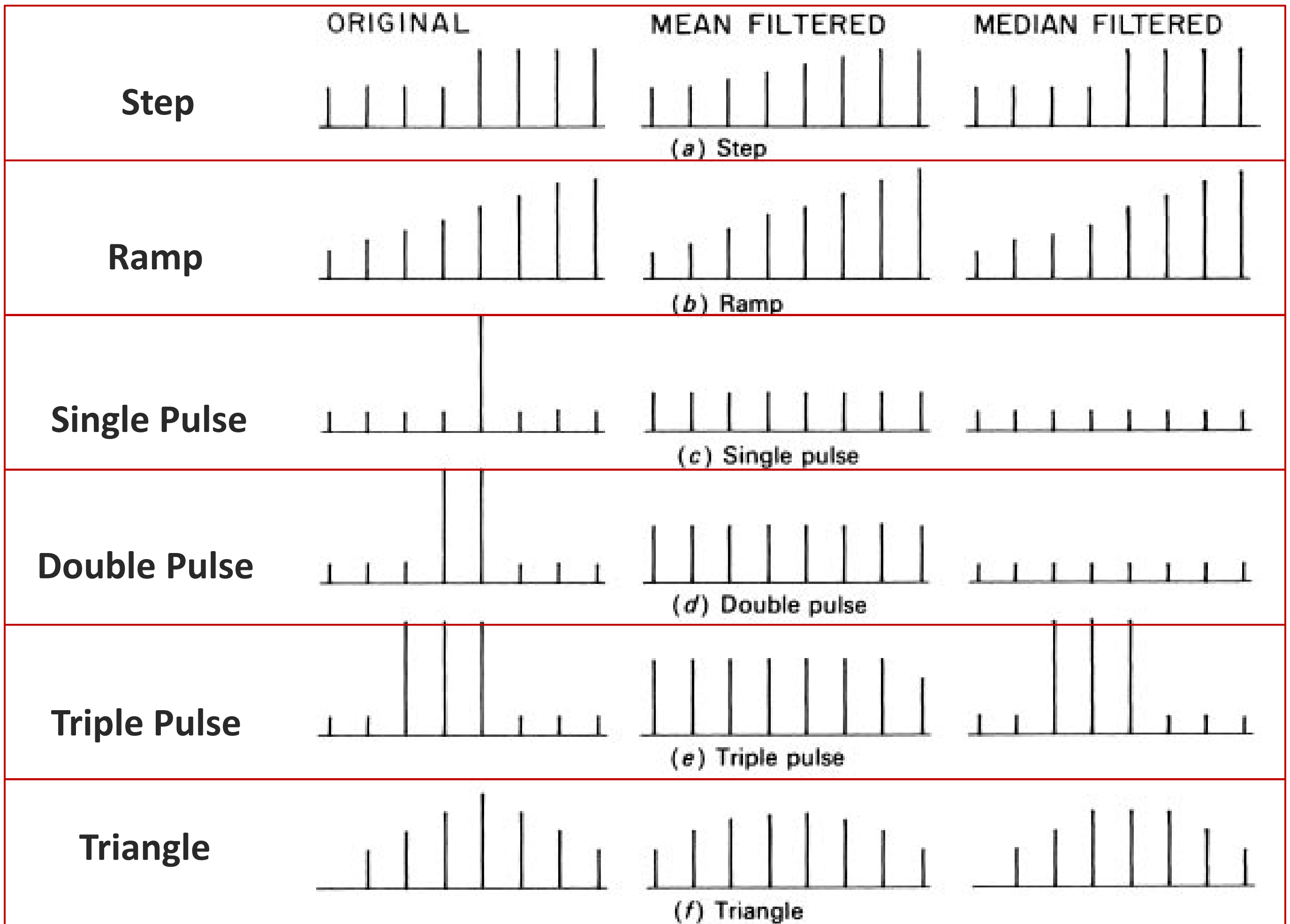
square



cross

Impulse Noise Removal

- e.g. 1D (window size = 5)



Impulse Noise Removal

- Median filtering
  - Fast computation
    - Approximation of median
      - e.g. 5-element filter

        a, b, c, d, e

        → MED(a, b, c, d, e)

        =max(min(a,b,c) , min(a,b,d), …)

        =min(max(a,b,c) , max(a,b,d), …)

        → there are 10 possible choices

        → could be narrowed down

Impulse Noise Removal

- **Pseudomedian filtering (PMED)**
  - e.g. 5-element filter

    a, b, c, d, e → spatially ordered

    MAXMIN = A (under estimated)

    = max(min(a,b,c) , min(b,c,d) , min(c,d,e))

    MINMAX = B (over estimated)

    = min(max(a,b,c) , max(b,c,d) , max(c,d,e))

    → PMED(a, b, c, d, e)

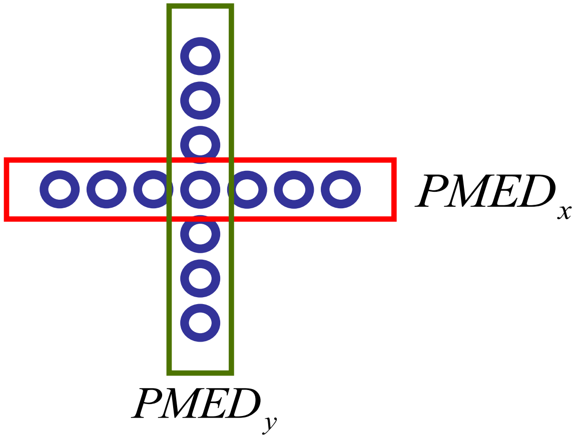
    = 0.5 * (A + B) = 0.5 * (MAXMIN + MINMAX)

    ~ MED(a, b, c, d, e)

Impulse Noise Removal

- **Pseudomedian filtering (PMED)**
  - **2D case**

$$PMED = \frac{1}{2}\left(PMED_x + PMED_y\right)$$



$$\begin{aligned} PMED &= \frac{1}{2}\max\left(MAXMIN(x_c), MAXMIN(y_R)\right) \\ &+ \frac{1}{2}\min\left(MINMAX(x_c), MINMAX(y_R)\right) \end{aligned}$$

Impulse Noise Removal

- Pseudomedian filtering (PMED)
  - MAXMIN
    - Remove salt noise
  - MINMAX
    - Remove pepper noise
  - May cascade two operations
    - Remove salt and pepper noise

Impulse Noise Removal

Original noisy image

MAXMIN

MINMAX of MAXMIN

MINMAX

MAXMIN of MINMAX

Q: same results?

Quality Measurement

- **Peak signal-to-noise ratio (PSNR)**
  - **Mean squared error (MSE)**

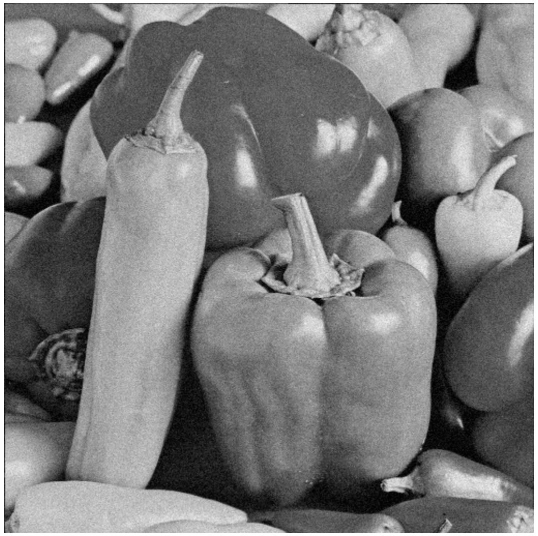
$$MSE = \frac{1}{w*h}\sum_{j}\sum_{k}\left[F(j,k) - F'(j,k)\right]^2$$

  - **The PSNR is defined as**

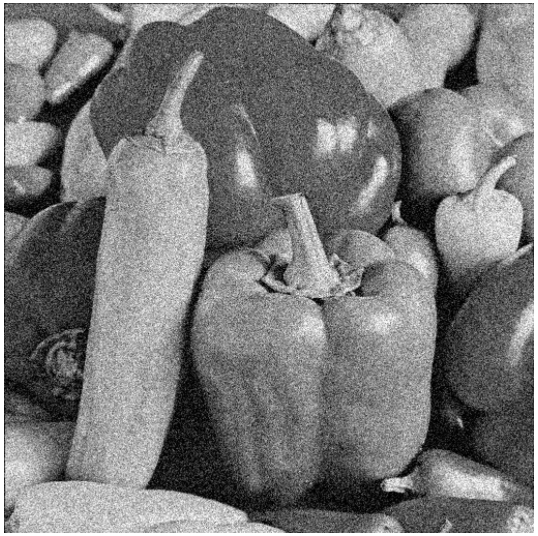
$$PSNR = 10 \times \log_{10}\left(\frac{255^2}{MSE}\right)$$

Example

Original image



Gaussian noise (σ=10)
PSNR : 28.18dB



Gaussian noise (σ=30)
PSNR : 18.81dB

Q: Represent perceived visual quality?