问题与辨析__2020_08_31

1): 首先我们需要知道的是 RIS 引入之后，对整个信道衰落特性的影响，

通过分析，我们不难发现，当考虑 RIS 位置较高导致 BS-RIS 之间仅有 LoS 传输路径时，同时考虑 BS-Rx 之间的 NLoS 传输分量，那么可以证明此时整个 channel 的衰落特性为 Rice fading；

这个可以作为第一个点；

在此基础上，可以定义 virtual Rice factor，探索影响 Virtual Rice factor 的因素，

2): 我们可以 Rx 处于一个慢速的运动状态，此时 Rx 的运动引起的位移变化对角度参数的影响可以忽略；当然，也可以考虑一个时变的 Non-stationary 场景；

Non-stationary 还是 Stationary ？

不如考虑时变情况下，得到 Non-stationary 的信道模型，然后可以构建 time-varying 的 RIS 响应；

3): 当 BS-RIS 之间仅有 LoS 时，可以根据 Generalized law of reflection 得到 RIS 阵列的响应；

4): 考虑可以设置 RIS 的元素的相位响应是任意的 (不会得到最佳接收信号)，元素之间并不是独立的，那么求解 virtual LoS 的信道相关特性时就可以构建任意的图样的信道相关特性了；

5): 可以改变 RIS 阵列的摆放位置，让 RIS 阵列的摆放位置有一个偏移角度；

问题与辨析__2020_09_01 **(信道衰落特性的推导)**

1): 需要搞清楚，**Wideband channel** 的 **Rayleigh/Rice fading** 是**怎么回事**，**如何推导**呢?

首先看一下，课程 PPT 上是怎么讲的，

Central Limit Theorem

- Let μ_n be indepenedent RVs with mean $m_{\mu_n}$and variance $\sigma_{\mu_n}^2$, then

$$\mu = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{N} (\mu_n - m_{\mu_n}) \sim N(0, \sigma_\mu^2)$$

where $N(0, \sigma_\mu^2)$ denotes the normal (Gaussian) distribution with zero-mean and variance

$$\sigma_\mu^2 = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \sigma_{\mu_n}^2 .$$

Functions of One RV

- **Functions of one RV:**
  Suppose that x is a RV with PDF $p_x(x)$. Let $f(x)$ be a function of the RV x. Hence, the expression

$$y = f(x)$$

已知 x 的pdf，如何计算 y = f(x) 的pdf；

  defines a new RV.

- **Fundamental theorem (transformation of RVs):** The PDF $p_y(y)$ of the RV y can be expressed in terms of the PDF $p_x(x)$ of the RV x as follows

非常重要！！！

$$p_y(y) = \sum_{v=1}^{m} \frac{p_x(x_v)}{\left|\frac{d}{dx} f(x)\right|_{x=x_v}},$$

  where m is the number of real roots x_v $(v = 1, 2, \dots, m)$ defined by

$$y = f(x_1) = f(x_2) = \cdots = f(x_m).$$

Lab Simulation of Multipath Fading Channels

- **Aim:** To simulate the **complex lowpass impulse response** of the channel and approximate the statistical properties of the channel **reference model** as closely as possible. Meanwhile, keep the realization complexity of the **channel simulator** as low as possible.

- **Narrowband channel complex gain** $g(t)$=$g_I(t)$+$jg_Q(t)$
  - $\tilde{r}(t) = \tilde{s}(t) \cdot g(t).$

- **Wideband channel complex impulse response** $g(t,\tau)$=$g_I(t, \tau)$+$jg_Q(t, \tau)$
  - $\tilde{r}(t) = \tilde{s}(t) * g(t,\tau) = \int_0^{\infty} \tilde{s}(t-\tau) \cdot g(t,\tau) d\tau.$

Transmission of a Band-Pass Signal

- **Transmitted band-pass** signal: $s(t) = \mathrm{Re}[\tilde{s}(t) e^{j2\pi f_c t}]$
  - $\tilde{s}(t)$: complex (**low-pass**) envelope of the **transmitted** signal.
  - f_c : carrier frequency.
- Noiseless **received band-pass** signal: $r(t) = \mathrm{Re}[\sum_{n=1}^{N} C_n e^{j2\pi[(f_c+f_{D,n})(t-\tau_n)]} \tilde{s}(t-\tau_n)]$
  - C_n: amplitude of the nth propagation path.
  - τ_n : time delay of the nth propagation.

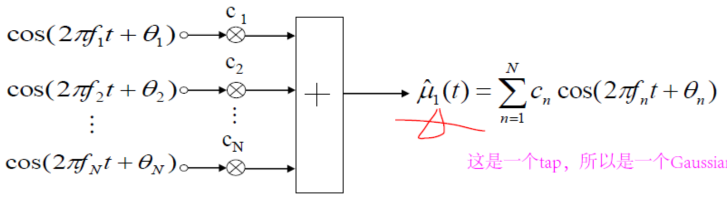
$\Rightarrow$ $r(t) = \mathrm{Re}[\tilde{r}(t) e^{j2\pi f_c t}]$

- $\tilde{r}(t) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)} \tilde{s}(t-\tau_n)$, complex (**low-pass**) envelope of the **received** signal.
- $\phi_n(t) = 2\pi[(f_c + f_{D,n})\tau_n - f_{D,n} t]$

  $\Rightarrow$ Since f_c is very large, very small changes in the path delay τ_n will cause large changes in the phases $\phi_n(t)$, due to the term $f_c \cdot \tau_n$.

4.3 Channel Model Parameter Computation Methods

- **Principle**: **Central Limit Theorem**.
  - ⇨ A Gaussian random process can be approximated by the superposition of a large number of properly weighted sinusoidal functions. 趋于无穷



这是一个tap，所以是一个Gaussian process

- **Application**: As long as the modeled fading envelope process involves one or more Gaussian random processes, the SoS channel modeling approach can be applied.
- **Classical channels**:
  - Rayleigh & Rice processes: 2 real Gaussian random processes
  - Suzuki process: 3 real Gaussian random processes 如: 对数正态分布
  - L-tap wideband channel (tapped delay line model): $2L$ real Gaussian random processes

Rice (Ricean) Fading (1/3)

- In some propagation environments, e.g., microcellular and mobile satellite environments, the received signal consists of scattered components plus a strong **specular** or **LoS** component.

⇒ $g_I(t)$ and $g_Q(t)$ are Gaussian random processes with **non-zero** means $m_I(t)$ and $m_Q(t)$, respectively.

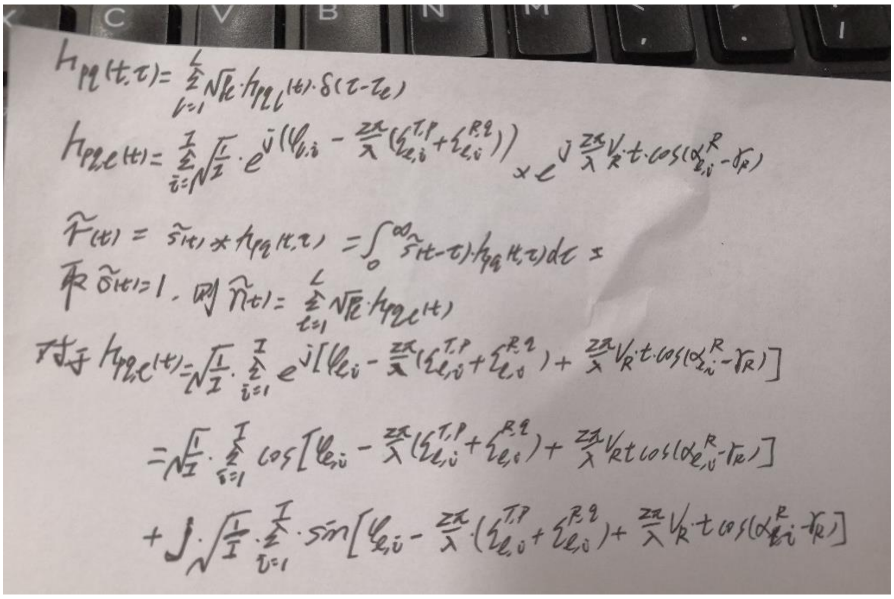
⇒ We again assume that $g_I(t)$ and $g_Q(t)$ are **uncorrelated** and have the same **variance** b_0.

⇒ The magnitude $\alpha(t)=|g(t)|=|g_I(t)+jg_Q(t)|$ has a **Ricean distribution**.

$$p_\alpha(x) = \frac{x}{b_0}\cdot\exp\left\{-\frac{x^2+s^2}{2b_0}\right\}\cdot I_0\left(\frac{xs}{b_0}\right)$$
$$= \frac{2x(K+1)}{\Omega_p}\cdot\exp\left\{-K-\frac{(K+1)x^2}{\Omega_p}\right\}\cdot I_0\left(2x\sqrt{\frac{K(K+1)}{\Omega_p}}\right),\quad x\ge 0.$$

- $s^2=m_I^2(t)+m_Q^2(t)$: is called the **non-centrality parameter**; the specular/LoS power.
- $2b_0$: scattered power.
- $\Omega_p=s^2+2b_0=E[\alpha^2]$: the average envelope power.
- $K=s^2/(2b_0)$: **Rice factor**.
  - ⇒ $s^2=\frac{K\Omega_p}{K+1}$, $2b_0=\frac{\Omega_p}{K+1}$. 直达路径与多径功率的比值
  - K=0: the channel exibits Rayleigh fading.
  - K→∞: the channel does not exhibit any fading at all.

2): 通过多天查阅资料，进行对比推导分析，我们完成了 **RIS** 引入之后信道衰落特性由 **Rayleigh fading** 转换为 **Rice fading** 的证明：



$$h_{pq}(t,\tau)=\sum_{l=1}^{L}\sqrt{P_l}\cdot h_{pq,l}(t)\cdot\delta(\tau-\tau_l)$$

$$h_{pq,l}(t)=\sum_{i=1}^{I}\sqrt{\frac{1}{I}}\cdot e^{j(\varphi_{l,i}-\frac{2\pi}{\lambda}(\xi_{l,i}^{T,p}+\xi_{l,i}^{R,q}))}\times e^{j\frac{2\pi}{\lambda}V_R\cdot t\cdot\cos(\alpha_{l,i}^{R}-\gamma_R)}$$

$$\tilde{r}(t)=\tilde{s}(t)*h_{pq}(t,\tau)=\int_0^{\infty}\tilde{s}(t-\tau)\cdot h_{pq}(t,\tau)d\tau =$$

取 $\tilde{s}(t)=1$，则 $\tilde{r}(t)=\sum_{l=1}^{L}\sqrt{P_l}\cdot h_{pq,l}(t)$

对于 $h_{pq,l}(t)=\sqrt{\frac{1}{I}}\cdot\sum_{i=1}^{I}e^{j[\varphi_{l,i}-\frac{2\pi}{\lambda}(\xi_{l,i}^{T,p}+\xi_{l,i}^{R,q})+\frac{2\pi}{\lambda}V_R\cdot t\cdot\cos(\alpha_{l,i}^{R}-\gamma_R)]}$

$$=\sqrt{\frac{1}{I}}\cdot\sum_{i=1}^{I}\cos[\varphi_{l,i}-\frac{2\pi}{\lambda}(\xi_{l,i}^{T,p}+\xi_{l,i}^{R,q})+\frac{2\pi}{\lambda}V_R t\cos(\alpha_{l,i}^{R}-\gamma_R)]$$

$$+j\cdot\sqrt{\frac{1}{I}}\cdot\sum_{i=1}^{I}\cdot\sin[\varphi_{l,i}-\frac{2\pi}{\lambda}\cdot(\xi_{l,i}^{T,p}+\xi_{l,i}^{R,q})+\frac{2\pi}{\lambda}V_R\cdot t\cos(\alpha_{l,i}^{R}-\gamma_R)]$$

令 $m_i = \frac{1}{\sqrt{I}}\cos\left[\varphi_{\ell,i} - \frac{2\pi}{\lambda}\left(\xi_{\ell,i}^{T,p} + \xi_{\ell,i}^{R,q}\right) + \frac{2\pi}{\lambda} v_R \cdot t \cdot \cos\left(\alpha_{\ell,i}^{R} - \gamma_R\right)\right]$

$$E\{m_i\} = \int_{-\pi}^{\pi} \frac{1}{2\pi} \cdot m_i \, d\varphi_{\ell,i}$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(\varphi_{\ell,i} - a_i) \, d\varphi_{\ell,i}$$

$$= \frac{1}{2\pi} \sin(\varphi_{\ell,i} - a_i) \Big|_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi} \left[\sin(\pi - a_i) - \sin(-\pi - a_i)\right]$$

$$\sin a_i \quad - \quad \sin a_i$$

$$= 0$$

$$\sigma_{m_i}^2 = E\{(m_i - 0)^2\}$$

$$= \int_{-\pi}^{\pi} \frac{1}{2\pi} \cdot m_i^2 \, d\varphi_i$$

$$= \int_{-\pi}^{\pi} \frac{1}{2\pi} \cdot \frac{\cos(2\varphi_i - 2a) + 1}{2} \, d\varphi_i$$

$$= \frac{1}{4\pi I} \int_{-\pi}^{\pi} [\cos(2\varphi_i - 2a) + 1 \, d\varphi_i$$

$$= \frac{1}{4\pi I} \left[\frac{1}{2}\sin(2\varphi_i - 2a) + \varphi_i\right]\Big|_{-\pi}^{\pi}$$

$$= \frac{1}{4\pi I} \cdot \left[\frac{1}{2} \sin(2\varphi_i - 2a)\Big|_{-\pi}^{\pi} + 2\pi\right]$$

$$= \frac{1}{2I}$$

$\therefore E\{m_i\} = 0, \ Var[m_i] = \sigma_{m_i}^2 = \frac{1}{2I}$

$h_{pq,\ell}(t) = x + j \cdot y$

则 $x = \sum_{i=1}^{I} \frac{1}{\sqrt{I}} \cdot \cos\left[\varphi_{\ell,i} - \frac{2\pi}{\lambda}\left(\xi_{\ell,i}^{T,p} + \xi_{\ell,i}^{R,q}\right) + \frac{2\pi}{\lambda} v_R \cdot t \cdot \cos\left(\alpha_{\ell,i}^{R} - \gamma_R\right)\right]$, $m_x = E\{x\} = 0$

$\sigma_x^2 = Var\{x\} = \frac{1}{2I} \times I = \frac{1}{2}$

$y = \sum_{i=1}^{I} \frac{1}{\sqrt{I}} \sin\left[\varphi_{\ell,i} - \frac{2\pi}{\lambda}\left(\xi_{\ell,i}^{T,p} + \xi_{\ell,i}^{R,q}\right) + \frac{2\pi}{\lambda} v_R \cdot t \cdot \cos\left(\alpha_{\ell,i}^{R} - \gamma_R\right)\right]$

$m_y = E\{y\} = 0$

$\sigma_y^2 = Var\{y\} = \frac{1}{2I} \times I = \frac{1}{2}$

$n_i = \frac{1}{\sqrt{I}} \sin\left[\varphi_{\ell,i} - \frac{2\pi}{\lambda}\left(\xi_{\ell,i}^{T,p} + \xi_{\ell,i}^{R,q}\right) + \frac{2\pi}{\lambda} v_R \cdot t \cdot \cos\left(\alpha_{\ell,i}^{R} - \gamma_R\right)\right]$

$$E\{n_i\} = \int_{-\pi}^{\pi} \frac{1}{2\pi} \cdot n_i \, d\varphi_{\ell,i}$$

$$= \int_{-\pi}^{\pi} \frac{1}{2\pi\sqrt{I}} \cdot \sin\left[\varphi_{\ell,i} - \frac{2\pi}{\lambda}\left(\xi_{\ell,i}^{T,p} + \xi_{\ell,i}^{R,q}\right) + \frac{2\pi}{\lambda} v_R t \cos\left(\alpha_{\ell,i}^{R} - \gamma_R\right)\right] d\varphi_{\ell,i}$$

$$= \int_{-\pi}^{\pi} \frac{1}{2\pi\sqrt{I}} \sin(\varphi_{\ell,i} - a_i) \, d\varphi_{\ell,i}$$

$$E\{(n_i - 0)^2\} = \int_{-\pi}^{\pi} \frac{1}{2\pi} \cdot \frac{1}{I} \cdot \sin^2(\varphi_{\ell,i} - a_i) \, d\varphi_{\ell,i}$$

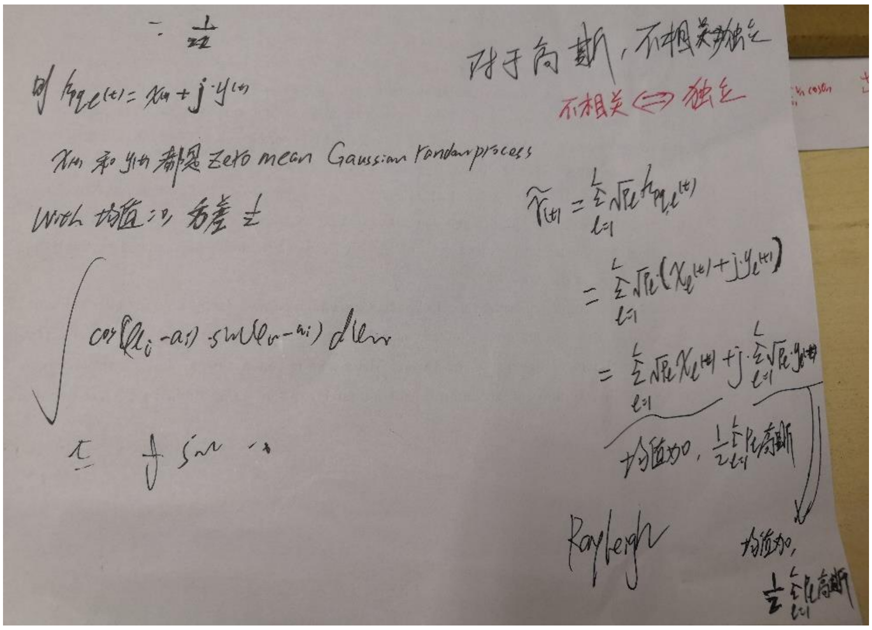
$$= \int_{-\pi}^{\pi} \frac{1}{2\pi I} \frac{1 - \cos(2\varphi_{\ell,i} - 2a)}{2} \, d\varphi_i$$

$$= \frac{1}{4\pi I} \int_{-\pi}^{\pi} (1 - \cos(2\varphi_i - 2a)) \, d\varphi_i$$

$$= \frac{1}{4\pi I} (2\pi$$

$$= \frac{1}{2I}$$

对于



上述式对 **Wideband NLoS components** 的 channel fading 进行推导，我们发现推导的结果**确实是 Rayleigh fading** channel。那么类似的，我们可以对 **Rice fading channel 进行推导**；

问题与辨析__2020_09_03

1): 若要根据 Generalized Law of Reflection 得出结论，那么感觉应该先考虑单天线的场景，进行推导；

此时，由于只有一根天线，因而传输距离与天线编号无关；

倘若考虑 MIMO 多天线，那根据表达式，要想实现最佳反射波束指向，那么 RIS 元素的相位响应应该会和天线的 index 有关。那么不同的天线就需要不同的响应，这样是不太好的。所以，我们暂时**先考虑仅有单天线**的情况；

2): 现在在考虑信道特性时，我们不应该再考虑 RIS 元素之间是相互独立的，而是应该**认为存在交叉相乘项**；

3): 有一个问题就是，在传统的信道模型中，对于 Rice 衰落信道，一般是已知了 Rice factor，因而在表述 LoS 和 NLoS 的传输功率时，可以比较好的表述；

但是，此处是我们通过证明，发现信道是 Rice fading，然后又要去定义 Rice factor，因而相对来说就显得更加困难了；

4): 还有一个问题就是，根据 Generalized law of reflection 计算 RIS 的响应时，在计算时是通过 source 和 destination 的位置坐标 (距离)进行计算的，而距离变量似乎不会考虑 Doppler frequency，也就是说优化得到的 RIS 的相位响应并不能消除 Doppler frequency 的影响？？？；

在计算 Doppler frequency 时，那个 AoA 角度是如何选取的呢？是否可以选取主径的角度进行代替呢？

问题与辨析__2020_09_04

1): 在最近几天，我们通过推导分析证明了在引入 RIS 之后，Tx 和 Rx 之间的信道衰落特性由 Rayleigh fading 转变为 Rice fading。

那么接下来顺理成章的我们想要去**求解**对应 **Rice fading 情况**下的 **virtual Rice factor** $K_{\mathrm{Rice}}^{\mathrm{vir}}$，此处$K_{\mathrm{Rice}}^{\mathrm{vir}}$之所以称为是 **virtual Rice factor**，是因为此处并没有 LoS 信道，而是由 RIS 的引入提供的虚拟直达径 (virtual LoS component)，本质上是一个 physical NLoS component。因此，与传统 LoS 引起的 Rice fading 是不同的，所以此处我们将其称为 virtual Rice factor；

2): 如果需要求解 **virtual Rice factor** $K_{\mathrm{Rice}}^{\mathrm{vir}}$，那么就需要求解 RIS 提供的 virtual LoS component 的功率，以及其他 NLoS components 的功率；

这就涉及到一个功率的分配问题。在传统的信道模型中，对于 Rice fading 信道的建模一般都是直接给出 Rice factor K_{Rice}，然后对于其他 NLoS components 的功率都是进行归一化了。这样，在已知天线对的有效接收功率的情况下，就可以根据比例系数计算 LoS，NLoS 的功率分批情况，同时再根据 NLoS 内部的不同主径的功率因子系数就可以进一步计算得到不同的主径的功率；

但是，但是。**现在是需要反过来**，根据 virtual LoS component 和 NLoS components 提供的功率来反过来计算 **virtual Rice factor** $K_{\mathrm{Rice}}^{\mathrm{vir}}$，那可就完全不一样了。

首先，virtual LoS component 和 NLoS components 之间的功率如何分配，就是一个难点；

再者，NLoS components 之间不同的主径的功率如何定义、求解和分配，又是一个难点；

最后，RIS 提供的 virtual LoS component 的功率如何计算又是一个难点；

综上，要想给出 **virtual Rice factor** $K_{\mathrm{Rice}}^{\mathrm{vir}}$的定义，我们需要**解决**上述三个大难题；

3): 通过查阅文献资料，我们找到了在传统信道模型中如何去定义 NLoS components 不同的主径的功率，具体可以见文档“**Path_power 怎么弄.docx**”

indoor office scenario [12]). The mean powers $\tilde{P}'_n$ of clusters are generated as [12]

$$\tilde{P}'_n = \exp\left(-\tilde{\tau}_n \frac{r_\tau - 1}{r_\tau \sigma_\tau}\right) 10^{-\frac{Z_n}{10}} \quad (17)$$

where Z_n follows a Gaussian distribution $\mathcal{N}(0, 3)$ [12]. Unlike the virtual delay and mean power of clusters, which are generated as in the WINNER II channel model, generations of angular parameters, relative delays of rays, and mean power of rays are not following the WINNER II channel model.

当然，也可以从几项标准中找到类似的定义如“**IMT-2020**”，“**WINNER II D.1.1.2 ver 1.1**”，“**3GPP TR 38.901 v16.1.0 (2019.12)**”，具体的说明和参数取值可以参见对应的标准。

不过，比较大的一个问题还是，RIS 提供的 virtual LoS component 得信号功率应该如何计算；

4): 考虑到此处我们需要构建的是一个 Non-stationary 的信道模型，因而距离和角度参数都是时变的，因而传输路径就会改变，所以上面**第 3)点**的**公式(17)**所给出的 cluster 的功率就应该进行时变的更新；

同样的，RIS 提供的 virtual LoS component 的传输距离也会改变，因而传输功率也需要更新；幸运的是，在参考**文献[1]**中给出了一种 **cluster 功率更新**的方法，

[1] S. B. Wu, *et. al*, “**A general 3-D non-stationary 5G wireless channel model**,” IEEE TCom, 2018.

indoor office scenario [12]). The mean powers $\tilde{P}'_n$ of clusters are generated as [12]

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Another important aspect is the evolution of cluster mean power. Constant cluster mean powers were assumed in [28] and [29], which were not sufficient to characterize time evolution of the channel. Therefore, in this paper, with the assumption that the cluster mean powers satisfy the inverse square law, the time evolution of cluster mean power can be expressed as (derivations given in Appendix B)

功率更新

$$\tilde{P}_{n,m_n}(t + \Delta t) = \tilde{P}_{n,m_n}(t)\frac{3\tau_n(t) - 2\tau_n(t + \Delta t) + \tau_{m_n}}{\tau_n(t) + \tau_{m_n}}. \quad (27)$$

The mean power terms $\tilde{P}_{n,m_n}$ in the mean power evolution in (27) are not normalized. They need to be normalized such that $P_{n,m_n} = \tilde{P}_{n,m_n} / \sum_{n,m_n} \tilde{P}_{n,m_n}$ before being plugged (代入) into (10). To guarantee smooth power transitions when clusters

当然，此处我们在对**功率进行更新**时，可以直接使用上述**公式(17)**，代入**时变的参数**即可！

另外需要说明的是，虽然这样子可以较好的**解决 NLoS components** 的 **cluster 功率**问题；但是 **RIS 提供的 virtual LoS component** 的**功率问题**却**没有**很好的**解决**！！！

5): 关于 **RIS 提供的 virtual LoS component** 的**功率**问题，目前还没有比较好的解决办法

i): 一种思路是采用路径损耗的方式进行计算，

目前已有较多的研究成果讨论路径损耗相关的建模和计算工作。但是采用路径损耗的方法建模的一个不足之处在于路径损耗计算所得的结果是 pathloss，并不能表示 **RIS 的功率系数**；

ii): 另一种思路是采用类似于 NLoS components 的功率系数的方法，

也就是说采用传统的 NLoS components 的 cluster 的功率的建模方法。但是由于已有的研究成果及结论表明 RIS 信道的功率衰减是两段距离相乘的平方$(d_1 d_2)^2$，而不是距离之和的平方$(d_1 + d_2)^2$。因此，在采用传统的 NLoS components 的 cluster 的功率的建模方法时，我们将 RIS 提供的功率表示为两段功率的乘积；同时，将单个 RIS 元素的功率建模为一个 cluster 的功率。

从直观上来理解，**RIS** 提供的 virtual LoS component 路径的**功率系数**会和 **RIS 的元素数目**，**RIS 的尺寸**，**RIS 的位置(传输距离)** 有关，

问题与辨析__2020_09_10

1): 在求解 **RIS** 提供的 virtual LoS component 的**功率系数**时，我们采用**路径损耗**的方法进行定义

and $n \in \left[1 - \frac{N}{2}, \frac{N}{2}\right]$, assuming that both of N and M are even numbers. We use the symbols d_1, d_2, θ_t, φ_t, θ_r and φ_r to represent the distance between the transmitter and the center of the RIS, the distance between the receiver and the center of the RIS, the elevation angle and the azimuth angle from the center of the RIS to the transmitter, the elevation angle and the azimuth angle from the center of the RIS to the receiver, respectively. Let $r^t_{n,m}$, $r^r_{n,m}$, $\theta^t_{n,m}$, $\varphi^t_{n,m}$, $\theta^r_{n,m}$ and $\varphi^r_{n,m}$ represent the distance between the transmitter and $U_{n,m}$, the distance between the receiver and $U_{n,m}$, the elevation angle and the azimuth angle from $U_{n,m}$ to the transmitter, the elevation angle and the azimuth angle from $U_{n,m}$ to the receiver, respectively.

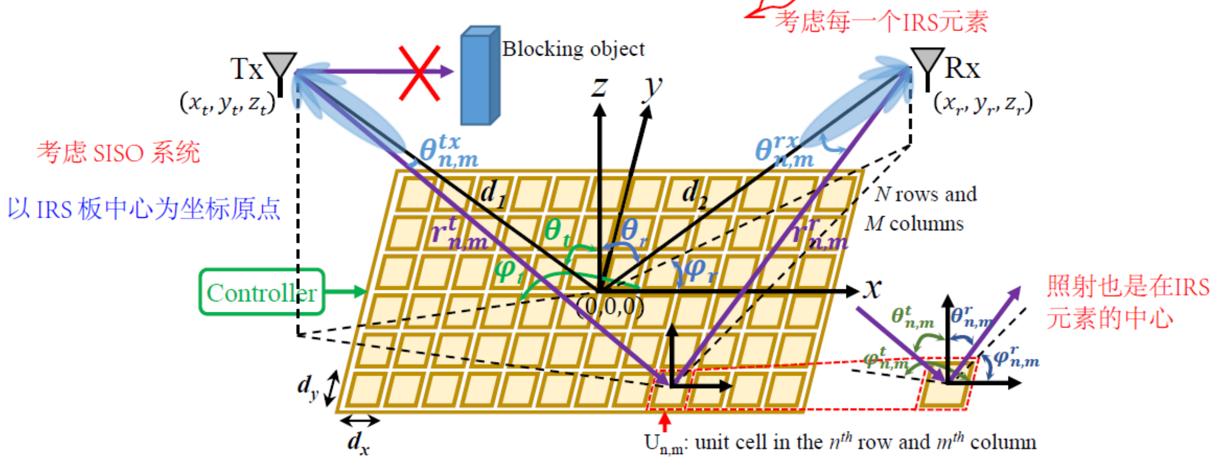

Fig. 2. RIS-assisted wireless communication without the direct path between the transmitter and the receiver.

As shown in Fig. 2, the transmitter emits a signal to the RIS with power P_t through an antenna with normalized power radiation pattern $F^{tx}(\theta, \varphi)$ and antenna gain G_t. The signal is reflected by the RIS and received by the receiver with normalized power radiation pattern $F^{rx}(\theta, \varphi)$ and antenna gain G_r. Let $\theta^{tx}_{n,m}$, $\varphi^{tx}_{n,m}$, $\theta^{rx}_{n,m}$ and $\varphi^{rx}_{n,m}$ represent the elevation angle and the azimuth angle from the transmitting antenna to the unit cell $U_{n,m}$, and the elevation angle and the azimuth angle from the receiving antenna to the unit cell $U_{n,m}$, respectively.

The following key result presents the connection between the received signal power of the receiver and the various parameters described above in RIS-assisted wireless communication systems.

Theorem 1. The received signal power in RIS-assisted wireless communications is as follows

自己推导的General的公式

$$P_r = P_t \frac{G_t G_r G d_x d_y \lambda^2}{64\pi^3} \left| \sum_{m=1-\frac{M}{2}}^{\frac{M}{2}} \sum_{n=1-\frac{N}{2}}^{\frac{N}{2}} \frac{\sqrt{F_{combine}}\,\Gamma_{n,m}}{r^t_{n,m} r^r_{n,m}} e^{\frac{-j2\pi(r^t_{n,m}+r^r_{n,m})}{\lambda}} \right|^2, \tag{3}$$

距离相乘

四个联乘的效果

where $F_{combine}=F^{tx}\left(\theta^{tx}_{n,m},\varphi^{tx}_{n,m}\right)F\left(\theta^{t}_{n,m},\varphi^{t}_{n,m}\right)F\left(\theta^{r}_{n,m},\varphi^{r}_{n,m}\right)F^{rx}\left(\theta^{rx}_{n,m},\varphi^{rx}_{n,m}\right)$, which accounts for the effect of the normalized power radiation patterns on the received signal power.

Proof of Theorem 1 其实就是球面波辐射，然后计算RIS元素的面积，然后得到单个的辐射功率

The power of the incident signal into unit cell $U_{n,m}$ can be expressed as

单个IRS的面积

自由空间公式!

$$P^{in}_{n,m} = \frac{G_t P_t}{4\pi {r^t_{n,m}}^2} F^{tx}\left(\theta^{tx}_{n,m},\varphi^{tx}_{n,m}\right)F\left(\theta^{t}_{n,m},\varphi^{t}_{n,m}\right) d_x d_y, \tag{19}$$

计算了单个IRS单元的距离和角度

and the electric field of the incident signal into $U_{n,m}$ is given by

IRS接收功率与电场的关系

为什么电场公式是这样的呢?

$$E^{in}_{n,m} = \sqrt{\frac{2Z_0 P^{in}_{n,m}}{d_x d_y}}\, e^{\frac{-j2\pi r^t_{n,m}}{\lambda}}, \tag{20}$$

where Z_0 is the characteristic impedance of the air, and $r^t_{n,m}$ can be written as

Tx到单个IRS的距离

仅有LoS路径!

$$r^t_{n,m} = \sqrt{\left(x_t - \left(m-\frac{1}{2}\right)d_x\right)^2 + \left(y_t - \left(n-\frac{1}{2}\right)d_y\right)^2 + {z_t}^2}. \tag{21}$$

According to the law of energy conservation, for the unit cell $U_{n,m}$, the power of the incident signal times the square of the reflection coefficient is equal to the total power of the reflected signal, thus we have

$$P^{in}_{n,m}\Gamma^2_{n,m} = P^{reflect}_{n,m}, \tag{22}$$

where $P^{reflect}_{n,m}$ is the total reflected signal power of the unit cell $U_{n,m}$, and the reflection coefficient can be written as

$$\Gamma_{n,m} = A_{n,m} e^{j\phi_{n,m}}, \tag{23}$$

where $A_{n,m}$ and $\phi_{n,m}$ represent the controllable amplitude and phase shift of $U_{n,m}$, respectively.

The power of the reflected signal received by the receiver from $U_{n,m}$ can be expressed as

自由空间公式！！！

$$P^r_{n,m} = \frac{G P^{reflect}_{n,m}}{4\pi {r^r_{n,m}}^2} F\left(\theta^{r}_{n,m},\varphi^{r}_{n,m}\right)F^{rx}\left(\theta^{rx}_{n,m},\varphi^{rx}_{n,m}\right) A_r, \tag{24}$$

where A_r represent the aperture of the receiving antenna, and $r^r_{n,m}$ can be written as

Rx到单个IRS的距离

仅有LoS路径!

$$r^r_{n,m} = \sqrt{\left(x_r - \left(m-\frac{1}{2}\right)d_x\right)^2 + \left(y_r - \left(n-\frac{1}{2}\right)d_y\right)^2 + {z_r}^2}. \tag{25}$$

By combining (19), (22) and (24), the electric field of the reflected signal received by the receiver from $U_{n,m}$ is obtained as

接收的来自(m,n)元素的功率

Rx接收的功率与电场关系

$$E^r_{n,m} = \sqrt{\frac{2Z_0 P^r_{n,m}}{A_r}} e^{\frac{-j2\pi r^t_{n,m}}{\lambda}} e^{\frac{-j2\pi r^r_{n,m}}{\lambda}}$$
$$= \sqrt{\frac{Z_0 P_t G_t G d_x d_y F_{combine}}{8\pi^2}} \frac{\Gamma_{n,m}}{r^t_{n,m} r^r_{n,m}} e^{\frac{-j2\pi(r^t_{n,m}+r^r_{n,m})}{\lambda}}. \tag{26}$$

The total electric field of the received signal is the superposition of the electric fields reflected by all unit cells toward the receiver, which can be written as

$$E^r = \sum_{m=1-\frac{M}{2}}^{\frac{M}{2}} \sum_{n=1-\frac{N}{2}}^{\frac{N}{2}} E^r_{n,m}. \tag{27}$$

接收的孔径，对于RIS元素，其孔径为 d_1 乘以 d_2

The received signal power of the receiver is

$$P_r = \frac{|E^r|^2}{2Z_0} A_r, \tag{28}$$

where the aperture of the receiving antenna can be written as

$$A_r = \frac{G_r \lambda^2}{4\pi}. \tag{29}$$

We obtain Theorem 1 by substituting (26), (27), and (29) into (28).