In the category of sets, given two set B and C, the set of all functions from C to B is often denoted by B^C. It can be constructed as a subset of $PP(C \times B)$, where P denotes the power-set operator. We now aim to generalize this concept in terms of category theory.

Definition. *In a category $\mathcal{C}$ for two objects $B, C \in \mathcal{C}$ we call an object exponential, denoted by*

$$\mathbf{C^B}$$

if the following is satisfied:
(1) $C^B \in \mathcal{C}$
(2) there is a morphism $\epsilon \in Hom(C^B \times B, C)$ such that:
For any $f \in Hom(A \times B, C)$ there exists a unique $\tilde{f} \in Hom(A, C^B)$ with $\epsilon \circ (\tilde{f} \times id_B) = f$.

At first glance this definition might look a little bewildering. First of all C^B is required to be just an object in $\mathcal{C}$. Don't let yourself confuse by its special notation! Secondly, the product $C^B \times B$ is required to exist and $\epsilon, \tilde{f}$ must be morphisms in $\mathcal{C}$.

To understand the meaning of ϵ consider in Set a function $f : A \times B \to C$. If we fix one element of A, say a, then we obtain a function $f_a : B \to C$. That is

$$f_a(b) = f(a, b)$$

This way

$$a \mapsto f_a$$

actually is a mapping

$$A \to C^B$$

This mapping corresponds exactly to the transpose $\tilde{f} \in Hom(A \in C^B)$. On the other hand, this transpose is connected to the function f by the property that

$$\tilde{f}(a)(b) = f(a, b)$$

The r.h.s we also can write as

$$\left(\tilde{f}(a), b\right) \mapsto \tilde{f}(a)(b)$$

or by regarding as product

$$(\tilde{f} \times id_B)(a, b) \mapsto \tilde{f}(a)(b)$$

From these expressions we see that this map operates by evaluating the the first component, which is in C^B, at the point given in the second component, which is in B.
This reveals the sense of ϵ, and it has to be considered like an abstract evaluation.

Like for products, the exponential must not necessarily exist. We say a category is **cartesian closed** if for any pair of objects its product and exponential exists.