$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = F(t, x, u)$$

where u is a function $[0, T] \times \mathbb{R}^n \to \mathbb{R}^n$ and F an operator which maps the function u to another function and may depend explicitly on time t and space x. The expression $u \cdot \nabla u$ reads in coordinates as:

$$\sum_i^n u_i \frac{\partial u_j}{\partial x_i}$$

This is the 'problematic' non-linearity.