From functions we consider the properties of being injective and surjective. Let us given a function $f : X \to Y$ between two sets.

Injectivity states that for a given y there is at most one element x such that $f(x) = y$.

Surjectivity states that for each y there is at least one x such that $f(x) = y$.

Lemma. *A function $f : X \to Y$ is injective if and only if for any two function $g, h : Y \to X$, from $f \circ g = f \circ h$ follows $g = h$.*

Proof. If f is injective then this is clear since otherwise one would have a $y \in Y$ with $g(y) \neq h(y)$ but $f(g(y)) = f(h(y))$ which contradicts the injectivity. On the other hand, if above statement would hold for each g, h but f would not be injective, then there is $y \in Y$ and $x_1 \neq x_2$ with $f(x_1) = f(x_2) = y$. We construct two different functions $g, h : Y \to X$, where g only takes the constant value x_1 and h the constant value x_2. It follows $f \circ g = f \circ h$ and by assumption this implies $g = h$ in contradiction to $g \neq h$. $\square$

And similar for surjectivity:

Lemma. *A function $f : X \to Y$ is surjective if and only if for any two function $g, h : Y \to X$, from $g \circ f = h \circ f$ follows $g = h$.*

Proof. If f is surjective than the entire Y is its image. So $g = h$ follows immediately. The other direction follows from noting that if the image of f would be not the entire Y, then two functions with $g \circ f = h \circ f$ always could be made different on the part of Y which is not in the image of f. We silently excluding the trivial cases of either $Y = \emptyset$ or X having only one element. $\square$