

**Definition. Category**

A category consists of a class of **objects**

$$A, B, C, \dots$$

and a class of **morphisms**

$$f, g, h, \dots$$

for which the following axioms are fulfilled:

(1) To each morphism  $f$  there exists operators called **domain**, **codomain** and denoted by

$$\text{dom}(f), \text{cod}(f)$$

Each operator associates an object to  $f$ .

(2) If for two morphisms  $f$  and  $g$  we have  $\text{cod}(f) = \text{dom}(g)$ , then there exists a morphism named **composition** of  $f$  and  $g$  which is denoted by  $g \circ f$ . Moreover, we have

$$\text{dom}(g \circ f) = \text{dom}(f)$$

$$\text{cod}(g \circ f) = \text{cod}(g)$$

(3) For each object  $A$  there exists an morphism named **identity**, denoted by

$$\text{id}_A$$

and that fulfills  $\text{dom}(\text{id}_A) = A$  and  $\text{cod}(\text{id}_A) = A$ .

(4) The composition is required to be **associative**, that is, for any morphisms  $f, g, h$

$$(f \circ g) \circ h = f \circ (g \circ h)$$

given that the compositions are defined.

(5) Composing a morphism  $f$  that has  $\text{dom}(f) = A$  and  $\text{cod}(f) = B$  with corresponding identities, fulfills

$$\text{id}_B \circ f = f = f \circ \text{id}_A$$