To solve the system

$$
\begin{aligned}
v'(t) &= F(t, x(t), v(t)) \\
x'(t) &= v(t) \\
x(0) &= x_0 \\
v(0) &= v_0
\end{aligned}
$$

we use the Euler scheme to discretize in time. We could choose a more accurate scheme, but for showing the main idea it suffices. For each grid-point x_A we ask from where the volume arriving here has departed. This volume must exists since it solves the backward-in-time equivalent to above system. Formally this reads

$$
\tilde{v}(t^{k+1}, x_A) = \tilde{v}(t^k, x_D) + \Delta t \cdot F(t^k, x_D, \tilde{v}(t^k, x_D)) \tag{1}
$$

Note, by using $\tilde{v}(t, x)$ we make explicit that this is the approximate velocity of the volume currently located at x. Since we don't know x_D, equation (1) cannot be used alone in each step. The idea is to discretize $x'(t) = v(t)$ as a part of above system and to solve

$$
x_A = x_D + \Delta t \cdot \tilde{v}(t^k, x_D) \tag{2}
$$

for x_D. This is done by iterating the equation

$$
x_D = x_A - \Delta t \cdot \tilde{v}(t^k, x_D)
$$

which may be started with $x_D^0 := x_A$. For Δt being sufficiently small, the r.h.s is a contracting map and so Banach's fix-point theorem ensures a unique solution. After having found x_D, we would like to use it in (1) in order to approximate the velocity at x_A. But since x_D is certainly not matching a grid-point we have to interpolate $\tilde{v}(t^k, x_D)$ as well as $F(t^k, x_D, \tilde{v}(t^k, x_D)$ from known values.