Definition 1. ***Functor***
A functor F between to categories C and D consists of two functions. One which maps objects from C to objects in D and one which maps morphisms from C to morphisms in D. Moreover, the following must hold:
(1) for $f \in Hom(A, B)$, $F(f) \in Hom(F(A), F(B))$
(2) $F(id_A) = id_{F(A)}$
(3) $F(g \circ f) = F(g) \circ F(f)$

Functors defined this way are structure preserving mappings between categories. Let us consider an example for a functor.

Example 1. *Remember the category N and Z as monoids of natural numbers and integers. Both categories consist of only one object. So we could let a functor map the object of N map to the object of Z. The morphisms in both categories are the elements and the composition, in case the monoids are written additive, is the $+$-operator. By (3) we have the requirement*

$$F(n + m) = F(n) + F(m)$$

And by (2)

$$F(0) = 0$$

Therefore the set of all possible functors between both categories are exactly the set of all linear mappings. For instance $F : n \mapsto 2n$.

For readers with knowledge in topology the following example will be a interest.

Example 2. *Given a topological space X, we define a category C by taking as objects all its elements. The morphisms are defined as follows. For two objects $x, y \in X$, the morphism $x \to y$ is included in $Hom(C)$ if and only if*

$$\forall U \ U \in \mathcal{B}(x) \to y \in U$$

That is, if for any open set U that contains x it also contains y. The composition is defined by

$$(x \to y) \circ (y \to z) = x \to z$$

We still have to verify that all axioms for this set of morphisms are fufilled.
Clearly we have $x \to x \in Hom(C)$. Hence for any object x, $id_x \in Hom(C)$. Moreover, is $x \to y$ and $y \to z$ are in $Hom(C)$, then each open set U containing x must contain y. But U is also an open set for y, so it must contain z as well. This shows, the composition is well-defined. Now, it is trivial to verify the composition is associative.
Let X, Y be topological spaces and C_X, C_Y the corresponding categories like above. If $f : X \to Y$ is a continuous mapping, then f is a functor between these categories. We verify the axioms for a functor:
(1) requires that f must operate on morphisms like so:

$$f : (x \to y) = f(x) \to f(y)$$

Then trivially (2) is fulfilled, since $f(x \to x) = f(x) \to f(x)$. By the definition of the composition, (3) is seen immediately as well. It remains to show that from $(x \to y) \in Hom(C_X)$ it follows $(f(x) \to f(y)) \in Hom(C_Y)$.
Let V be an open set containing $f(x)$. Then by continuity $f^{-1}(V)$ is open as well. Moreover, $x \in f^{-1}(V)$ and thus $y \in f^{-1}(V)$. But this implies $f(y) \in V$ that shows $(f(x) \to f(y)) \in Hom(C_Y)$.
The question arises if a functor between these categories necessarily is a continuous function. The answer is no. Just consider $f : [0,1] \to \{0,1\}$, with $f|[0,1/2[= 0$ and $f|[1/2,1] = 1$. Let $\{0,1\}$ carry the discrete topology and $[0,1]$ having a topology generated by the following basis: all open sets for points $\xi \in]0,1]$ and an open set for 0 which is $0, 1/4$. This implies $0 \to 1/4$ and trivially $f(0) \to f(1/4)$. By construction, f is not continuous at $1/2$.