

If $f(c) < f(d)$, then f must take a local minimum within $[a, d]$, otherwise in $[c, b]$.

This is a consequence from f being continuous and as such it maps an interval onto an interval. This means somewhere it must take minimum value.