Example 1. ***Monoid***
This is a very often appearing structure in category theory. A monoid is a set S together with a binary relation $\cdot : S \times S \to S$ fulfilling the following:
(1) it is associative, that is $(x \cdot y) \cdot z = x \cdot (y \cdot z)$
(2) it has a neutral element e, which satisfies $e \cdot x = x = x \cdot e$ for all $x \in S$.

A moinid can be seen as a category with only one object and morphisms taken to be all elements in S. Moreover, composition is defined to be the binary relation $\cdot$. One easily verifies that all axioms are fulfilled. Note, since the category has only one element, say A, any morphism x has dom$(x) = A =$ cod(x). Therefore, composition works among all morphisms, that is in analogy with the operation $\cdot$ between elements of S.
Examples for monoids are N, Q, R, the natural numbers, the rationals and the reals. The binary relation can be taken to be the addition or multiplication.

Example 2. ***The category*** Mon
From monoids we can build a category Mon which objects are all monoids and morphism functions which preserve the monoid structure in the following sense.

If $f : X \to Y$ is a morphism then
(1) for any $x, y \in X$

$$f(x \cdot y) = f(x) \cdot f(y)$$

(2) $f(e_X) = e_Y$ where e_X, e_Y are the corresponding neutral elements of X resp. Y

One easily checks that this still holds when composing two morphisms $f \circ g$. Therefore the composition of functions provides a composition *between morphisms which fulfills all axioms of a category.*