We will use the following more or less standard notation:

$$X \backslash Y := \{z \in X : z \notin Y\}$$

For convenience let us restate the relation that holds between the sets:

$$A \times B \cup B \times A = C \times D \cup D \times C \tag{1}$$

We will proof the statement by contradiction and therefore assume it does not hold. Then either A or B is not equal to any of D or C. Without loss of generality we can assume this holds for A, that is, $A \neq C$ and $A \neq D$. In case B is the one, the same arguments as follow can be applied in analogy.

First, we are going to show that neither $A \subset C$ nor $A \subset D$. To the contrary we assume $A \subset C$. The case $A \subset D$ can be treated in full analogy. The entire chain of arguments is a repeatedly application of the equation (1).

By assumptions we have

$$C \backslash A \neq \emptyset$$

Now if we apply (1) from right to left we obtain

$$C \backslash A \times D \subset B \times A$$

This shows two things:

$$C \backslash A \subset B \tag{2}$$

$$D \subset A \tag{3}$$

Moreover, since $A \neq D$,

$$A \backslash D \neq \emptyset$$

Applying (1) from left to right on the subsets $A \backslash D$ and $C \backslash A$ gives

$$A \backslash D \times C \backslash A \subset C \times D$$

From this we infer

$$C \backslash A \subset D$$

Altogether we have

$$C\backslash A \times C\backslash A \subset C \times D$$

and this can be used to apply (1) from right to left:

$$C\backslash A \times C\backslash A \subset A \times B \cup B \times A$$

This gives an immediate contradiction since non of the factors on the left intersect with A. So we must refute the assumption $A \subset C$.

So far we have shown that under our assumption we have

$$A\backslash C \neq \emptyset \tag{4}$$

$$A\backslash D \neq \emptyset \tag{5}$$

By (1) applied from left to right we see

$$A \subset C \cup D$$

and with this using (4), (5) shows the existence of two distinct $a, a' \in A$ with

$$a \in D\backslash C \tag{6}$$

$$a' \in C\backslash D \tag{7}$$

This in particular shows $C\backslash D \neq \emptyset$ and $D\backslash C \neq \emptyset$.
Applying (1) from right to left shows

$$C\backslash D \times D\backslash C \subset A \times B \cup B \times A$$

that implies

$$C\backslash D \cap B \neq \emptyset \text{ or } D\backslash C \cap B \neq \emptyset$$

So either there is $b \in D\backslash C \cap B$ or $b' \in C\backslash D \cap B$. Then either

$$(a, b) \in D\backslash C \times D\backslash C$$

or

$$(a', b') \in C\backslash D \times C\backslash D$$

Especially this means

$$(a, b) \notin C \times D \cup D \times C$$

resp.

$$(a', b') \notin C \times D \cup D \times C$$

what contradicts (1) and shows we must drop our assumptions.

In summary we have shown either $A = C$ or $A = D$ resp. either $B = C$ or $B = D$. In order to finish the proof we have to consider the case $A = B$.
Without loss of generality we assume $A = C$. Then (1) becomes

$$C \times B = C \times D \cup D \times C$$

from what we infer $B = D$. A similar argument leads to $B = C$ if we assume $A = D$.
Finally, we have shown either $A = C$ and $B = D$ or $A = D$ and $B = C$.