Example 1. *We take as objects all sets and as morphisms all binary relations between two sets. That is, if A, B are sets and $f : A \to B$ is a morphism, then f can be viewed as subset of $A \times B$. In general these morphisms are not functions, it could be that $(a, b) \in f$ and $(a, c) \in f$ for $b, c \in B$ with $b \neq c$.*
For two morphisms $f : A \to B$ and $g : B \to C$ we define composition as follows:

$$(a, c) \in g \circ f \;\; :\leftrightarrow \;\; \exists b \in B \; (a, b) \in f \wedge (b, c) \in g$$

In words, the composition relates a to c if and only if there is a 'bridge' b such that f relates a to b and g relates b to c.
Before showing that the axioms of category are fulfilled note how this category is abstracting the notion of functions. Although, from a point-wise view, these morphisms are not functions, they still behave like functions w.r.t an appropriate defined composition.
For any object A we define

$$id_A = \{(a, a) \mid a \in A\}$$

One immediately sees that this serves correctly as identity morphism. It remains to proof associativity. Assume we are given $f : A \to B$, $g : B \to C$ and $h : C \to D$.

$$(a, d) \in (h \circ g) \circ f$$

means, there exists $b \in B$ such that

$$(a, b) \in f \tag{1}$$

and

$$(b, d) \in h \circ g$$

Moreover, the latter implies there exists $c \in C$ such that

$$(b, c) \in g \tag{2}$$

and

$$(c, d) \in h \tag{3}$$

All this follows directly from the definition of $\circ$*. By putting equations (1), (2) and (3) together, lets us infer by doing similar considerations that* $(a, d) \in h \circ (g \circ f)$*.*
This shows $(h \circ g) \circ f = h \circ (g \circ f)$*.*