Definition. ***Monomorphism***
A morphism $f : A \to B$ of some category is called monomorphism if the following condition holds:
For any two morphisms $g, h : B \to A$, from $f \circ g = f \circ h$ follows $g = h$.

Definition. ***Epimorphism***
A morphism $f : A \to B$ of some category is called epimorphism if the following condition holds:
For any two morphisms $g, h : B \to A$, from $g \circ f = h \circ f$ follows $g = h$.

These two definitions resemble the alternative way of describing an injective resp. surjective function.

A function f which is injective and surjective has an inverse f^{-1} which fulfills $f \circ f^{-1} = id$ and $f^{-1} \circ f = id$. This can be obtained from $(y, x) \in f^{-1} \;:\leftrightarrow\; (x, y) \in f$. We can ask if the analogous statement holds in general categories. First let us clarify what is meant by inverse in terms of morphisms:

Definition. ***Isomorphism***
A morphism $f : A \to B$ is called isomorphism if the exists a morphism $f^{-1} : B \to A$ such that

$$f^{-1} \circ f = id_A$$

and

$$f \circ f^{-1} = id_B$$

The morphism f^{-1} is called **inverse** and moreover uniquely determined by its definition.
To this, assume there is a further inverse g. Then $g \circ f = id_A$. By composing with f^{-1} we obtain $(g \circ f) \circ f^{-1} = id_A \circ f^{-1}$. Associativity yields, $g \circ id_B = id_A \circ f^{-1}$ and by using properties of the identity, $g = f^{-1}$.

In category theory isomorphic objects are usually considered the same.

Theorem. *If f is an isomorphism then it is an epimorphism and monomorphism.*

Proof. Assume g and h are morphisms that fulfill $f \circ g = f \circ h$. By composing with f^{-1} from the left on both sides we obtain $g = h$. This shows f is a monomorphism. In analogous manner one can show that f is epimorphism. $\square$