Example 1. *Let us view the natural numbers N and all integers Z as objects in the category Mon.*

The inclusion $i : N \to Z$ provides a natural morphism within Mon of which we will show that it is monomorph.

For this let $g, h \in Hom(Z, N)$ and $i \circ g = i \circ h$. This implies for any $x \in Z$, $g(x) = h(x)$ and so $g = h$.

Next, we show i is epimorph as well.

For this, let us first make a simple observation:
For any morphism $f : Z \to N$ and positive $n \in Z$ we have by definition of morphisms in Mon,

$$0 = f(n - n) = f(n) + f(-n)$$

which implies

$$-f(n) = f(-n)$$

Now assume $f \circ i = g \circ i$ for $f, g \in Hom(Z, N)$. This trivially implies $f(z) = g(z)$ for $z \in Z \cap N$. For negative $z \in Z$, we have by the previous remark

$$f \circ i(-z) = f(-z) = -f(z)$$

and the same for g

$$g \circ i(-z) = -g(z)$$

Combined this yields,

$$-g(z) = -f(z)$$

Hence $g(z) = f(z)$.
Altogether this shows $f = g$ and thus i is epimorph.

Interestingly i is ***not isomorph!*** *Otherwise there would be a morphism $i^{-1} : Z \to N$ such that $i \circ i^{-1}(-1) = -1$, but this is impossible since $-1 \notin N$.*

For readers with knowledge in topology:

Example 2. *This example comes from topology and regards the category Top which objects are topological spaces and the morphisms are taken to be all continuous maps. Consider the identity map $id : (N, \tau) \to (N, \sigma)$, where N denotes the set of natural numbers one time equipped with the discrete topology τ and one times with the chaotic topology σ. The latter consists of the sets N and $\emptyset$ only. Trivially this map is an monomorphism and epimorphism, but it does not have an inverse. This inverse necessarily would be the map $id : (N, \sigma) \to (N, \tau)$, but this is not continuous.*