Definition. ***Retraction***

A morphism $f \in Hom(A, B)$ is called a ***retraction*** *if there exists a right inverse that is, a $h \in Hom(B, A)$ with*

$$f \circ h = id_B$$

Definition. ***Section***

A morphism $f \in Hom(A, B)$ is called a ***section*** *if there exists a left inverse that is, a $h \in Hom(B, A)$ with*

$$h \circ f = id_B$$

With this we can formulate the following theorem:

Theorem. *A monomorphism f is an isomorphism if and only if it is a retraction.*

Proof. Assume $f \in Hom(A, B)$ is monomorph. Then there is some $g \in Hom(B, A)$ such that

$$f \circ g = id_B$$

Using this, we get

$$f \circ (g \circ f) = (f \circ g) \circ f = id_B \circ f = f \circ id_B$$

Since f is monomorph this implies

$$g \circ f = id_B$$

and this altogether $g = f^{-1}$. □

In analogy we have

Theorem. *A epimorphism f is an isomorphism if and only if it is a section.*

Proof. Assume $f \in Hom(A, B)$ is an epimorphism. Then there exists $g \in Hom(B, A)$ such that

$$g \circ f = id_B$$

Using this, we get

$$(f \circ g) \circ f = f \circ (g \circ f) = f \circ id_B = id_B \circ f$$

Since f is epimorph this implies

$$f \circ g = id_B$$

and thus $g = f^{-1}$. $\square$