Let us consider a velocity field given as a function

$$u : [0, T] \times \mathbb{R}^n \to \mathbb{R}^n$$

and a particle which location is tracked by $x : [0, T] \to \mathbb{R}^n$. So if we want to know the velocity of the particle at time t we must consider $u(t, x(t))$. This actually we can regard as a function $v : [0, T] \to \mathbb{R}^n$ defined by $v(t) = u(t, x(t))$.
By Newton's law, the change of velocity is proportional to forces acting on a particle. Thus,

$$\frac{dv(t)}{dt} = F(t, x(t)) \tag{1}$$

Here $F(t, x(t))$ are all forces acting at time t on the particle at $x(t)$. This equation is already very expressive, since it tells exactly how each particle in a fluid (for instance), would move forward.
If we replace v by its definition, that is $v(t) = u(t, x(t))$ and apply the chain rule, the r.h.s calculates to

$$\frac{dv(t)}{dt} = \frac{du(t, x(t))}{dt} = \frac{\partial u}{\partial t} + \left(\frac{dx(t)}{dt} \cdot \nabla \right) u \tag{2}$$

Since velocity of the particle at $x(t)$ is given by

$$v(t) = \frac{dx(t)}{dt}$$

we find by inserting this into (2)

$$\begin{aligned} &\frac{\partial u}{\partial t} + \left(\frac{dx(t)}{dt} \cdot \nabla \right) u = \\ &\frac{\partial u}{\partial t} + (v(t) \cdot \nabla) u = \frac{\partial u}{\partial t} + (u \cdot \nabla) u \end{aligned}$$

Thus the problem (1) is transformed into one involving

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = F(t, x)$$