Notation:
$|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^n$
$x \cdot y$ is the Euclidean inner product in $\mathbb{R}^n$
x^T denotes the transpose of $x \in \mathbb{R}^n$
∂_i is the partial derivative of the i'th coordinate

Let us fix $x \in \mathbb{R}^n$:
For any $h \in \mathbb{R}^n$ with $|h| = 1$ we define a function $g : \mathbb{R} \to \mathbb{R}$ by

$$t \mapsto f(x + t \cdot h)$$

That is, g follows f along the line $t \mapsto x + t \cdot h$.
As we know, the derivative of a function is determining locally its rate of change. Therefore, we easily can formulate our search for the direction of steepest decrease by

Find h which minimizes $g'(0)$

Since we are only interest in the direction, we require $|h| = 1$.
Using the chain rule we can compute g' by

$$g'(0) = \nabla f(x) \cdot h$$

The expression $\nabla f(x)$ is called gradient of f at x and is computed by

$$\begin{pmatrix} \partial_1 f(x) \\ \partial_2 f(x) \\ \vdots \\ \partial_n f(x) \end{pmatrix}$$