In many application the r.h.s $F(t,x)$ in addition depends on $u(t)$ itself. Since $u(t)$ is a function $\mathbb{R}^n \to \mathbb{R}^n$ this makes $F$ an operator on some appropriate Banach-space $B$:

$$F : [0,T] \times B \to B$$

And example of this could be $F(t,u) = \nabla \cdot \nabla u$ which appears as friction force in the Navier-Stokes equation. Then $u$ would be required to be two times differentiable and as a Banach-space one could consider the two times continuous differentiable functions.
If $F$ fulfills a Lipschitz-condition of the sort

$$|F(t,u) - F(t,w)| \leq L|u - w|$$

for some $L > 0$, then a general form of Picard-Lindeloef theorem states that the following problem locally has a unique solution:

$$u'(t) = F(t,u) \tag{1}$$

with $u(0) = u_0$. This is an initial value problem for a vector-field having at time 0 the value $u_0$. With this solution $u$ we can pose another initial value problem:

$$x'(t) = u(t, x(t)) \tag{2}$$

with $x(0) = x_0$. The solution $x$ we search is a function $[0,T] \to \mathbb{R}^n$. Under suitable assumptions on $B$, for instance if it is the two times continuous differentiable function space, this system, by again applying Picard-Lindeloef, has a unique solution $x$.
But then the function $v : t \mapsto u(t, x(t))$ fulfills