Example. *Let X be a partial ordered set. That is there exists a binary relation $\leq$ on X that fulfills:*
(1) if $a \leq b$ and $b \leq c$ then $a \leq c$
(2) if $a \leq b$ and $b \leq a$ then $a = b$
(3) $a \leq a$

We can define a category by taking as objects all elements from X and as morphisms all relations of the form $a \leq b$ for any $a, b \in X$. That means, the set $Hom(a,b)$ is either empty or consists of the only morphism $a \leq b$.
By (1) associativity is trivially satisfied for the composition. Moreover, (3) ensures the existence of the identity map: $id_a = a \leq a$.

For a product $a \times b$ the following necessarily would have to be fulfilled: For any x with $x \leq a$ and $x \leq b$

$$x \leq a \times b$$

Projections must fulfill

$$a \times b \leq a$$

and

$$a \times b \leq b$$

Altogether, these statements are equivalent of $a \times b$ being the largest lower bound of a and b.
This is an interesting example, where the product directly presents a known concept of partial orders.