

Definition. *Product of morphisms*

In a category \mathcal{C} consider two morphisms $f \in \text{Hom}(A, B)$ and $g \in \text{Hom}(C, D)$. Moreover, assume the products $A \times C$ and $B \times D$ exist with the corresponding projection maps p_A, p_C resp. q_B, q_D .

We consider the following compositions:

$$f \circ p_A \in \text{Hom}(A \times C, B)$$

$$g \circ p_C \in \text{Hom}(A \times C, D)$$

By the definition of products applied on $B \times D$ and both compositions, we conclude there exists

$$\langle f \circ p_A, g \circ p_C \rangle \in \text{Hom}(A \times C, B \times D)$$

We denote this morphism by $f \times g$, that is:

$$f \times g := \langle f \circ p_A, g \circ p_C \rangle$$

Remember the last example where object were elements of a partial ordered set and morphisms all ordered pairs. The product of two objects a, b turned out to be the largest lower bound of a and b .

Now, if we have the morphisms $a \leq b$, $c \leq d$ and the products $a \times c$, $b \times d$ exist, then the product of these morphisms is simply

$$a \times c \leq b \times d$$

Translating this back to the notation of partial ordered sets, this gives a well-known result:

If $a \leq b$ and $c \leq d$, then the largest lower bound of a, c is lower or equal the largest lower bound of b, d .