Let ξ be an accumulation point of the sequence $(x_j^{(i)})$. Assume $f'(\xi) \neq 0$. By starting the iteration at ξ itself, must provide in the first cycle at least one coordinate axis j for which

$$\arg\min_{\tau} f(\xi + \tau e_i) < f(\xi)$$

This is since otherwise ξ would be a local minimum and thus $f'(\xi) = 0$. Let us denote this optimum by η. So,

$$f(\eta) < f(\xi)$$

From the iteration formula we obtain

$$|x_{j+1}^{(i)} - x_j^{(i)}| = |\tau_j^{(i)}|$$

which shows the sequence $(\tau_j^{(i)})$ being bounded as well. Thus by Bolzano-Weierstrass theorem for some sub-sequence we have $\tau_j^{(i)} \to \bar{\tau}$. By construction we have for each $\tau \in \mathbb{R}$

$$f(x_j^{(i)} + \tau_j^{(i)} e_j) \leq f(x_j^{(i)} + \tau e_j)$$

By taking the limit, this gives:

$$f(\xi + \bar{\tau} e_j) \leq f(\xi + \tau e_j)$$

In other words $\bar{\tau}$ solves

$$\arg\min_{\tau} f(\xi + \tau e_j)$$

Because of the required uniqueness of this optimum, we must have

$$\xi + \bar{\tau} e_j = \eta \tag{1}$$

By taking the limit of

$$x_{j+1}^{(i)} = x_j^{(i)} + \tau_j^{(i)} e_j$$

we obtain

$$\xi = \xi + \bar{\tau} e_j$$

which together with (1) would imply $\xi = \eta$ and contradicts $f(\eta) < f(\xi)$ as pointed out above.