ECON 427 HW 1 Yihua Zhang (905231562)

1.

(a)

These are the datasets for the first question.

@data

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54.60031622,21.48897426,29.36021618,33.11134196,118.3433212,-1.471067262,Normal

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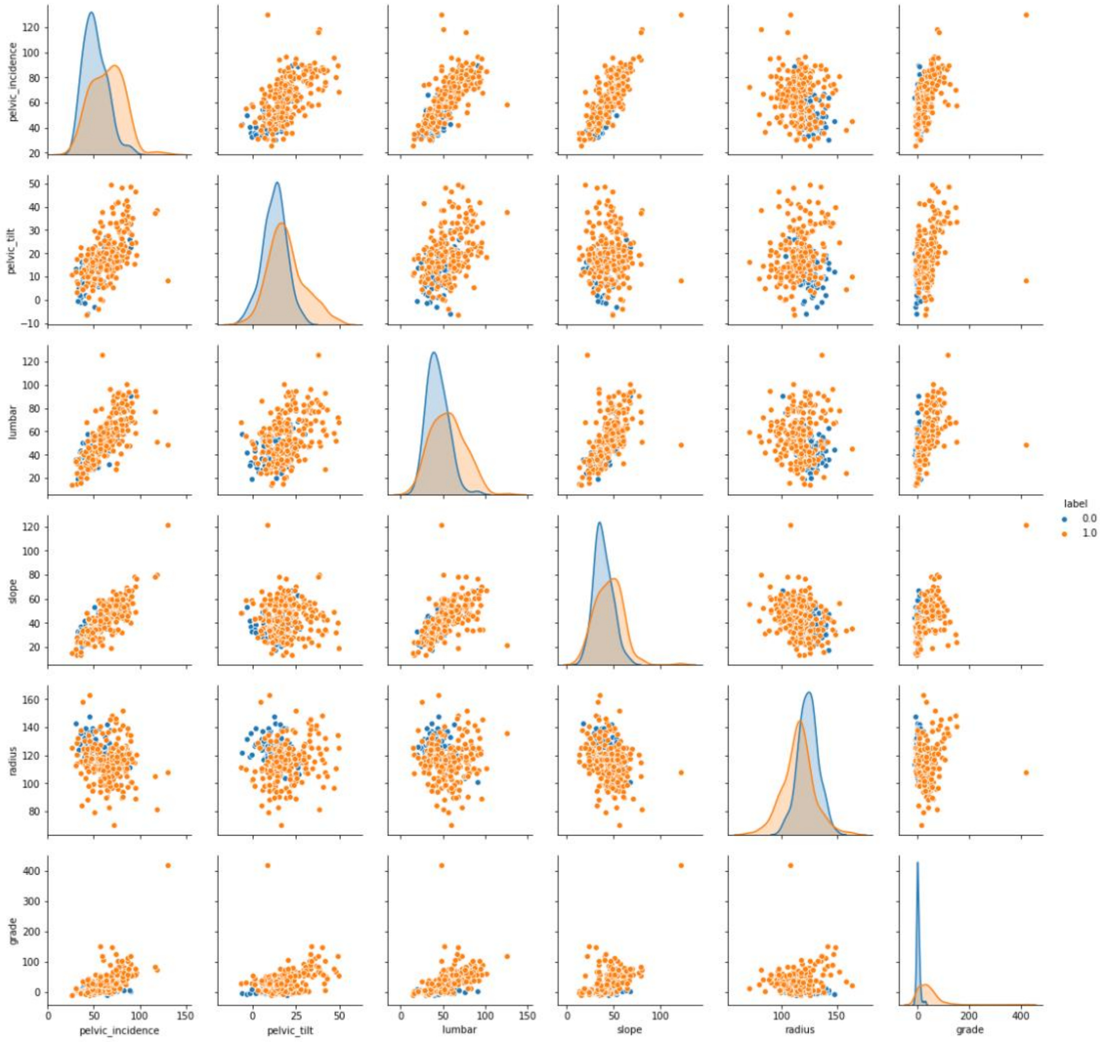
45.07545026,12.30695118,44.58317718,32.76849908,147.8946372,-8.941709421,Normal
47.90356517,13.61668819,36,34.28687698,117.4490622,-4.245395422,Normal
53.93674778,20.72149628,29.22053381,33.21525149,114.365845,-0.421010392,Normal
61.44659663,22.6949683,46.17034732,38.75162833,125.6707246,-2.707879517,Normal
45.25279209,8.693157364,41.5831264,36.55963472,118.5458418,0.214750167,Normal
33.84164075,5.073991409,36.64123294,28.76764934,123.9452436,-0.199249089,Normal

(b)
Here are the codes and graphs of (i), (ii) and (iii).
trykNN.py

```
##
import pandas as pd
import numpy as np
import math
import seaborn as sns
from sklearn.model_selection import train_test_split

users = pd.read_table('vertebral_column_data/column_2C.dat',sep = '::',header=None,engine='python')
users=users[0]
#loading the data and get into the list
datalist=np.zeros((len(users),7))
for idx in range (len(users)):
    Str=users[idx]
    Str=Str.split()

    for i in range(len(Str)):
        element=Str[i]
        if i<(len(Str)-1):
            datalist[idx][i]=float(element)
        else:
            if element=='AB':
                datalist[idx][i]=1
            else:
                datalist[idx][i]=0

newdf=pd.DataFrame({"label":datalist[:,6],"pelvic_incidence":datalist[:,0],"pelvic_tilt":datalist[:,1],"lumbar":datalist[:,2],"slope":datalist[:,3],"radius":datalist[:,4],"grade":datalist[:,5]})
##corresponding to b
sns.pairplot(data=newdf, hue='label', vars=['pelvic_incidence', 'pelvic_tilt', 'lumbar', 'slope','radius','grade'])
###
```

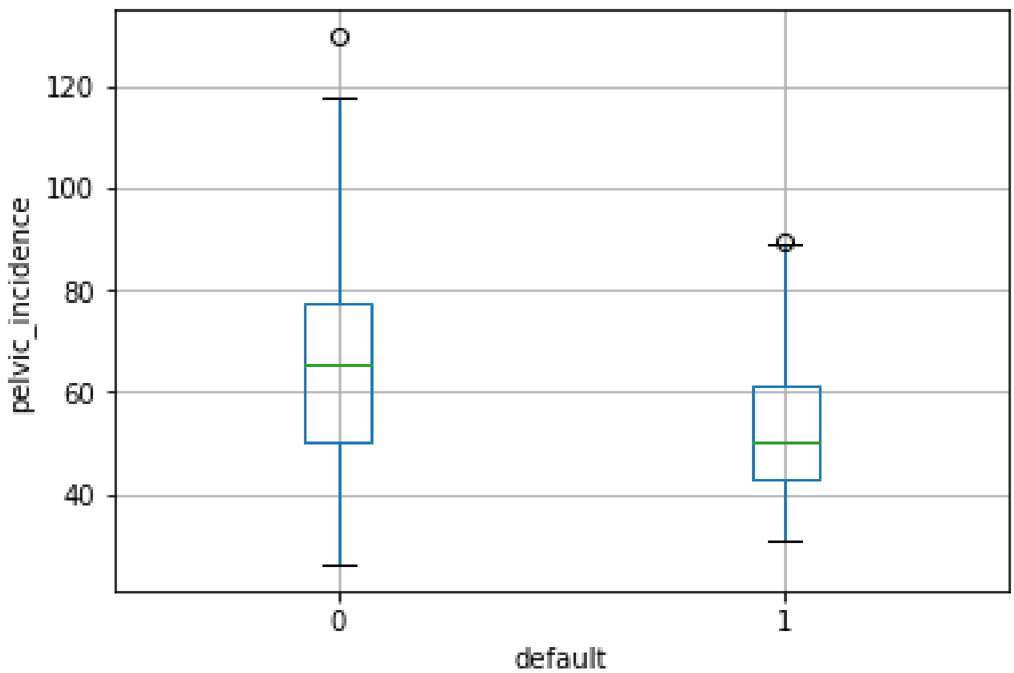
Here are the graphs for the questions:

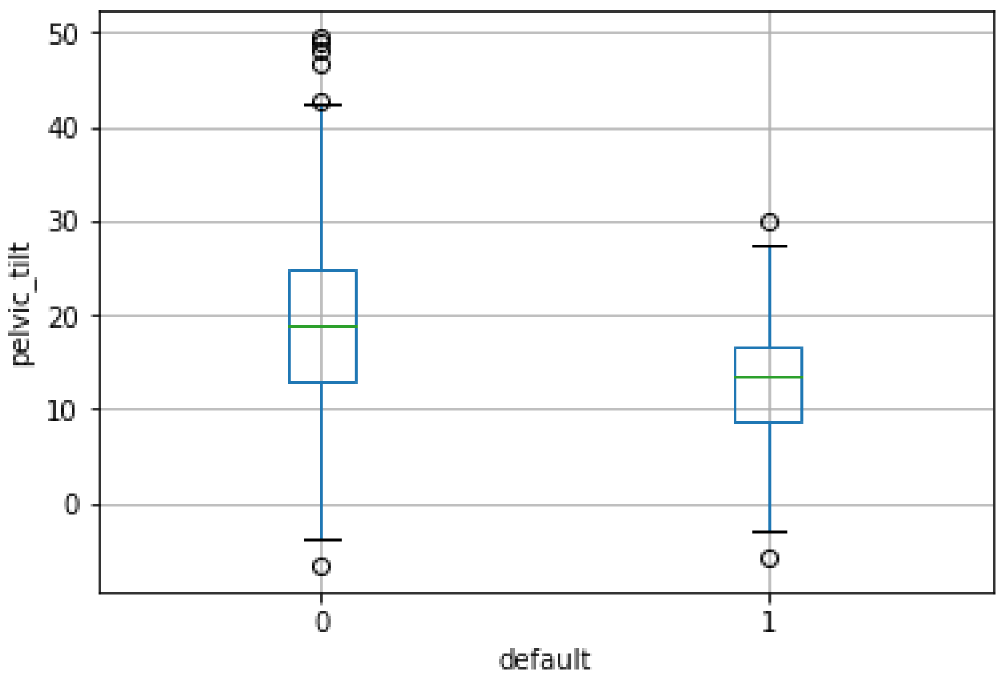


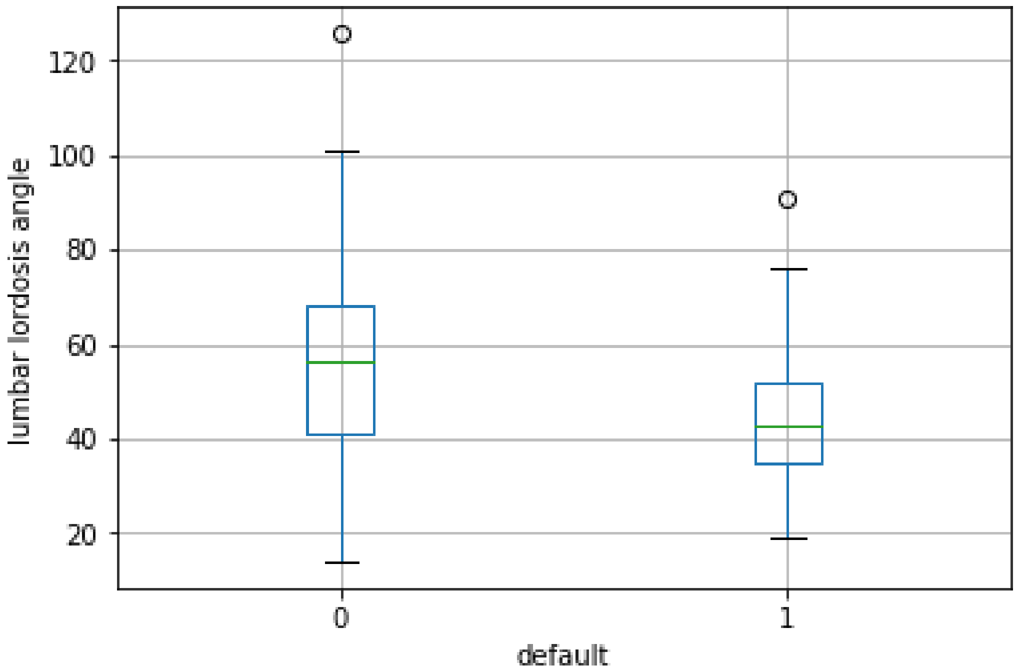
ii

Make boxplots: the following code is about pelvic incidence variables, the others boxplots are made in a similar way.

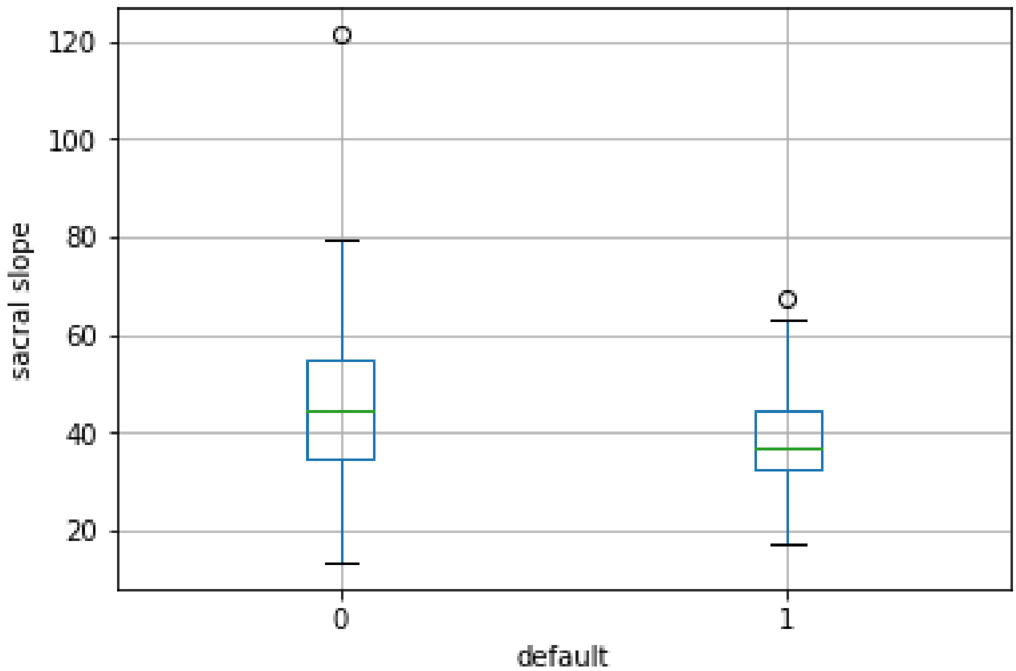
```
##
pel_1=newdf_1['pelvic_incidence']
pel_1=pel_1.rename(columns={'pelvic_incidence':'1'},inplace=True)
pel_0=newdf_0['pelvic_incidence']
pel_0=pel_0.rename(columns={'pelvic_incidence':'0'},inplace=True)

a=pd.concat([pel_1,pel_0],axis=1)
#plt.boxplot([pel_1,pel_0],patch_artist=True, boxprops={'color'})
a.boxplot()
plt.ylabel('pelvic_incidence')
plt.xlabel('default')
plt.show()
##
```


120
100
80
60
40
pelvic_incidence
0
1
default

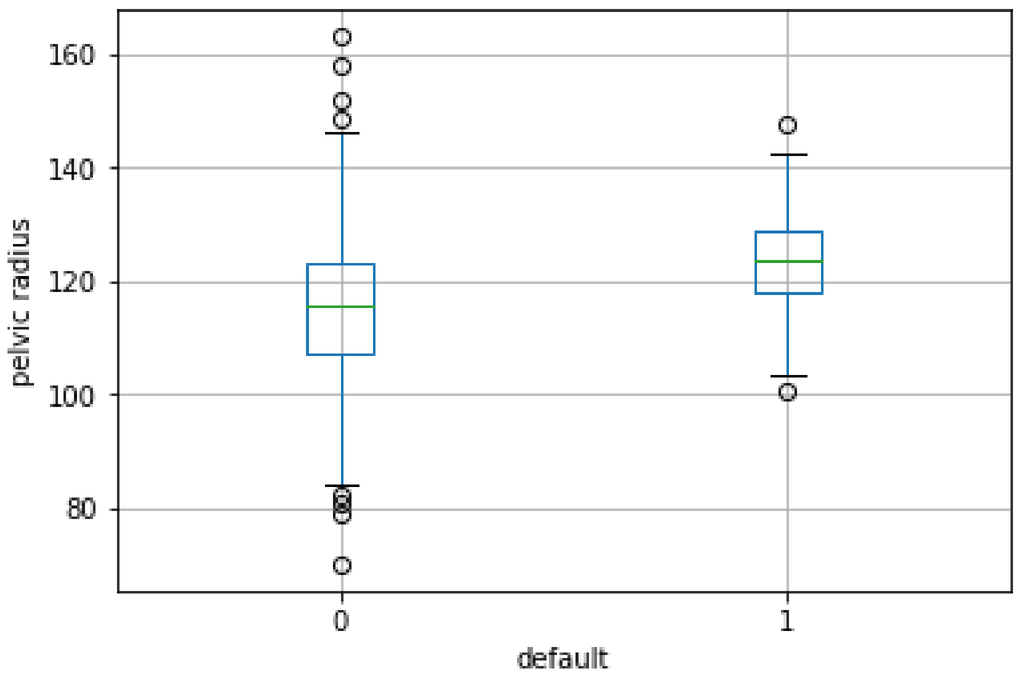

50
40
30
20
10
0
pelvic_tilt
0
1
default

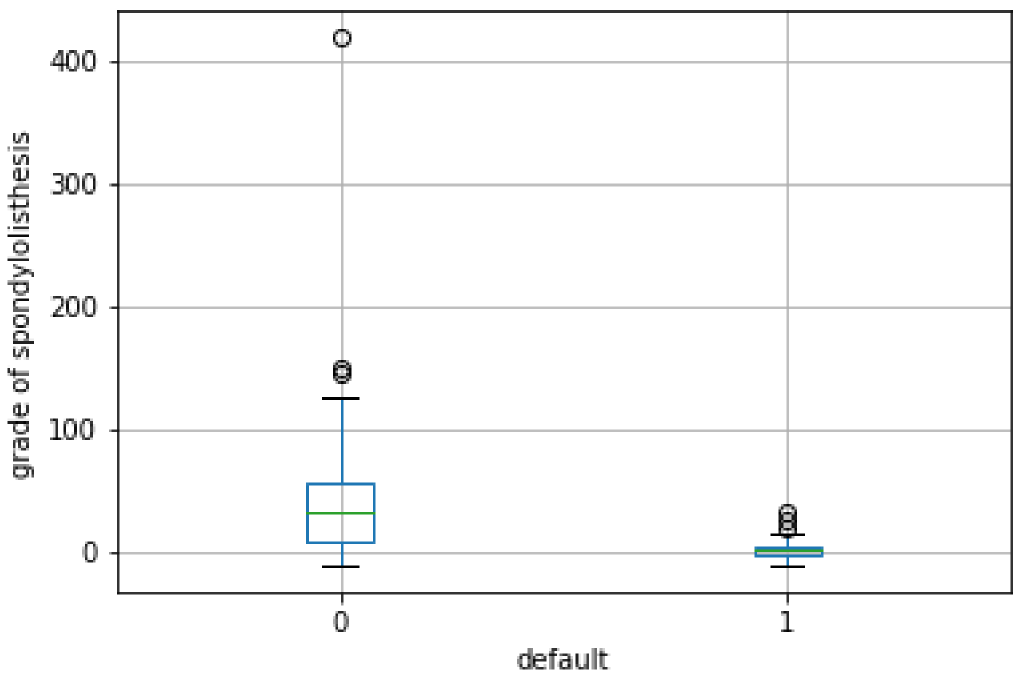


120
100
80
60
40
20
lumbar lordosis angle
0
1
default



120
100
80
60
40
20
sacral slope
0
1
default





iii

```
##
X=newdf[['pelvic_incidence','pelvic_tilt','lumbar','slope','radius','grade']]
y=newdf['label']

x0=X[y==0]
```

```
x1=X[y==1]

x0_train=x0.head(70)
y0_train=y[y==0].head(70)

x0_test=x0[70:len(x0)]
y0_test=y[y==0].head(x0_test.shape[0])

x1_train=x1.head(140)
y1_train=y[y==1].head(140)

x1_test=x1[140:len(x1)]
y1_test=y[y==1].head(x1_test.shape[0])

x_train=x0_train.append(x1_train)
y_train=y0_train.append(y1_train)

x_test=x0_test.append(x1_test)
y_test=y0_test.append(y1_test)

##
```

(c)

(i)

Here are the codes for the k-nearest neighbors with the Euclidean metric.

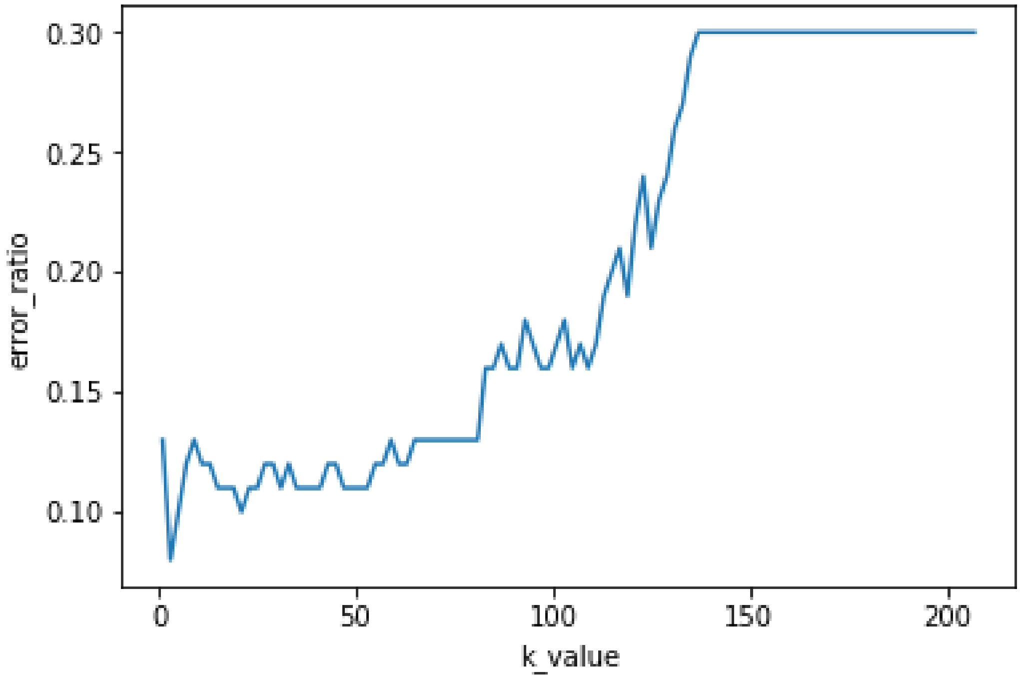
```
##
def Euclidean(single_data,norm_x_train):
    diffmat=np.tile(single_data,(norm_x_train.shape[0],1))-norm_x_train
    sq_diff=diffmat**2
    distance=(sq_diff.sum(axis=1))**0.5

    return distance    #Euclidean metric
##
```

(ii)

In this question…

trykNN3.py

k values：{1， 3， 5， ……209}



When K=3, the minimum error rate is 0.08.

Calculating the confusion matrix

```
##
tp=0
fn=0
fp=0
tn=0
for row in range (test_num):
    compare=best_compare[row,:]
    y_real=compare[0]
    y_pre=compare[1]
    if y_real==1 and y_pre==1:
        tp+=1
    else:
        if y_real==1 and y_pre==0:
            fn+=1
        else:
            if y_real==0 and y_pre==1:
                fp+=1
            else:
                tn+=1


confusion_matrix=[[tp,fp,tp+fp],[fn,tn,fn+tn],[tp+fn,fp+tn,tp+fn+fp+tn]]
##
```

Now, we will get the results like this…

Confusion matrix

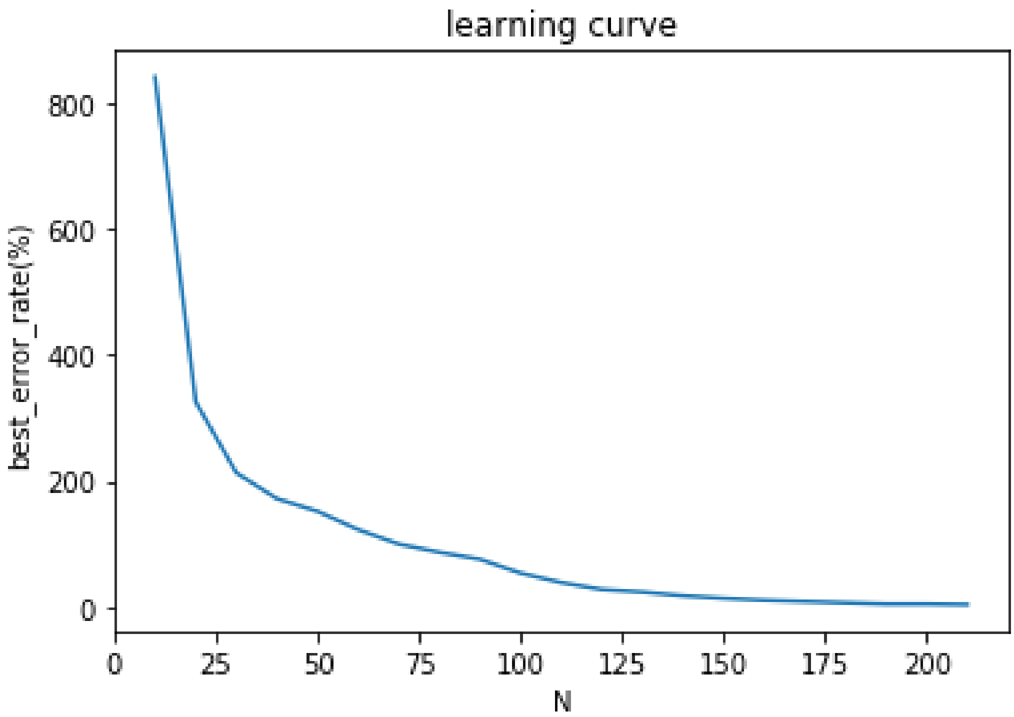
		True status		
		1	0	total
Predicted status	1	69	7	76
	0	1	23	24
	total	70	30	100

Precision=0.908

Fscore=0.945

(iii)

learning_curve.py



The codes are here：

```
##
N_range=np.arange(10,220,10)
N_best_error_ratio=np.zeros_like(N_range)

for N in N_range:
    x_train,y_train,x_test,y_test=prepare_data(x0,x1,y,N)
    norm_x_train=norm2(x_train)
    norm_x_test=norm2(x_test)

    test_num=norm_x_test.shape[0]
    y_train_dist=np.zeros((norm_x_train.shape[0],2))

    compare=np.zeros((test_num,2))
```

```
    k_value=np.arange(1,min(N,196+5),5)
    error_ratio=[]
    for k in k_value:
        error_sum=0

compare_result,error_sum_result=kNN_compare(k,norm_x_test,norm_x_train,y_test,test_num)
        error=error_sum_result/test_num
        error_ratio.append(error)

    min_index=np.array(error_ratio).argsort()
    k_best=k_value[min_index[0]]

best_compare,best_error=kNN_compare(k_best,norm_x_test,norm_x_train,y_test,test_num)
    idx_e=int(N/10-1)
    N_best_error_ratio[idx_e]=best_error/N*100



plt.figure(1)
plt.xlabel('N')
plt.ylabel('best_error_rate(%)')
plt.title('learning curve')
plt.plot(N_range,N_best_error_ratio)
##
```

(d)

(I)

The definition of Minkovski distance:

```
#
def Minkow(single_data,norm_x_train,pvalue):
    distance=np.zeros((norm_x_train.shape[0]))
    x1=single_data
    for row in range(norm_x_train.shape[0]):
        x2=norm_x_train[row,:]
        dist=DistanceMetric.get_metric('minkowski',p=pvalue)
        xmat=[x1,x2]
        distance[row]=dist.pairwise(xmat)[0][1]
    return distance
#
```

A. When p=1, it will be the Manhattan Distance.

The file is knn_d.py

When using Minkow function, make the p-value to 1.

In the kNN_compare function, the minimum value of the error rate in distance=Minkow(single_data,norm_x_train,1)
Is 0.11， the k is 6

B． For the（list）q_value，with log10(p)
The corresponding file is knn_d2.py

```
# q_value=np.arange(0.1,1.1,0.1)
##
for q in q_value:
    pvalue=10**q
    error_sum=0

compare_result,error_sum_result=kNN_compare_p(k,norm_x_test,norm_x_train,y_test,
test_num,pvalue)
    error=error_sum_result/test_num
    error_ratio.append(error)
##
distance=Minkow(single_data,norm_x_train,pvalue)
```

After the calculation,
The lowest error rate is 0.1, and for the corresponding log10(p) is 0.6.

C．

```
# distance=chebdist(single_data,norm_x_train)
##
def chebdist(single_data,norm_x_train):
    distance=np.zeros((norm_x_train.shape[0]))
    x1=single_data
    for row in range(norm_x_train.shape[0]):
        x2=norm_x_train[row,:]
        dist=DistanceMetric.get_metric('chebyshev')
        xmat=[x1,x2]
        distance[row]=dist.pairwise(xmat)[0][1]
    return distance
##
```

As the result, the smallest error rate is 0.1 and the k will be 16.

(ii)
Mahalanobis distance

```
#distance=Minkow(single_data,norm_x_train,pvalue)
def mahadist(single_data,norm_x_train):
    distance=np.zeros((norm_x_train.shape[0]))
    x1=single_data
    for row in range(norm_x_train.shape[0]):
        x2=norm_x_train[row,:]
```

```
X=np.vstack([x1,x2])
XT=X.T
S=np.cov(X)
SI=np.linalg.inv(S)
n=XT.shape[0]

for i in range(0,n):
    for j in range(i+1,n):
        delta=XT[i]-XT[j]
        d=np.sqrt(np.dot(np.dot(delta,SI),delta.T))
        distance[row]=d
```

For the return distance, the smallest error rate is 0.1 and the k will be 21.

(e)

```
##
        count_class={}
        for ik in range (k):
            vote_y=y_train.iloc[sort_dist[ik]]

            count_class[vote_y]=count_class.get(vote_y,0)+float(1/distance[sort_dist[ik]]) #

##
```

	Euclidean	Manhattan	Chebyshev
Majority vote	K=16 Error rate=0.1	K=6 Error rate=0.11	K=16 Error rate=0.07
Weight	K=6 Error rate=0.1	K=26 Error rate=0.1	K=16 Error rate=0.1