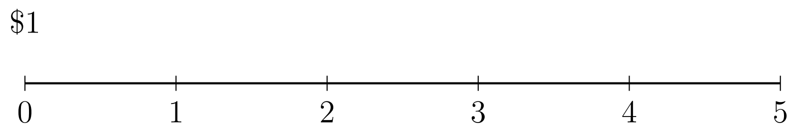
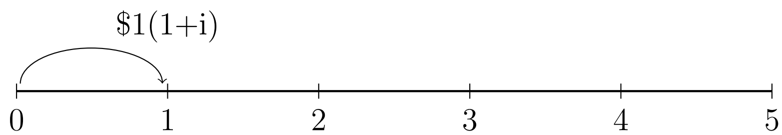
1 Moving money into the Future $(1+i)^n$

Imagine you have \$1 today, that is at $t=0$. And you have a bank account willing to pay $i\%$ per period. The period can be yearly, monthly, daily etc., you just have to be certain the interest rate corresponds to the period. For example, it would be wrong to compound a yearly interest rate at a daily-rate. (So, for this write-up let us assume all interest rates and periods are given on a yearly-basis to keep things straight forward.)

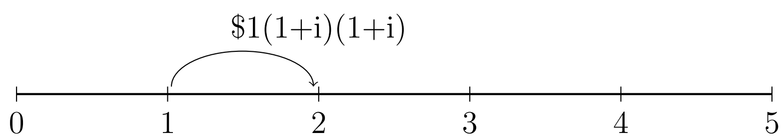


Question, if we invest our \$1 at $i\%$ how much will we have in one year? To computer this, we have to multiply our beginning value by $(1+i)$.

To move money 1-year into the future, multiply by $(1+i)$.



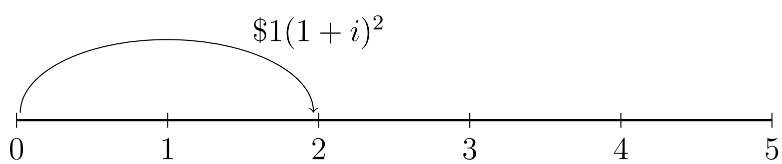
We are now at $t=1$. So, let's go forward one more year. How much will we have one year from now, or at $t=2$. Again, to move money one year into the future, multiply the beginning amount by (1+i).



Above, the beginning amount was $\$1(1+i)$, so one-step to the right is $\$1(1+i)(1+i)$.

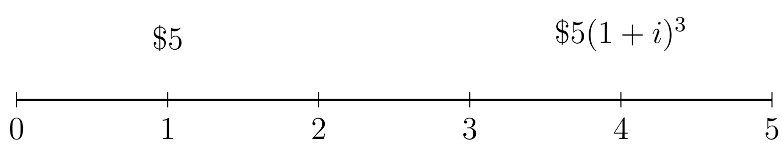
We are now two steps away from $t=0$. Notice, at first, we could have asked the question, how much money will we have in the account two years from now?.

The answer to this is to take two-steps to the right and compute two years worth of growth all at once. Above, we did it in two steps, but here we do it all at once.



Look at the exponent of the interest rate (1+i) and look at the t value (we took 2 steps to the right), and realize if we want to move money n periods into the future, we multiply the beginning value by $(1+i)^n$.

Question: I have \$5 at $t=1$ and want to know how much it will be worth at $t=4$, what do I multiply by?

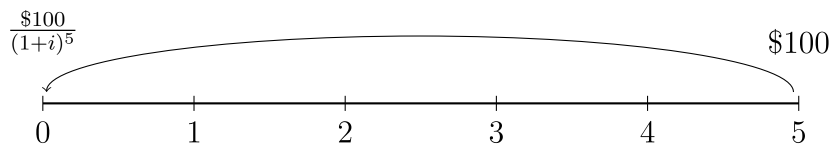


Three steps to the right.

2 Moving money back in Time $v \equiv \frac{1}{1+i}$

Often times payments are setup to be received in the future. We can move these payments back in time, to today. This is the opposite of what we did in Section 1. To pull payment back from the future and still maintain their correct values, we divide by powers of $(1+i)^n$. For each step to the left, we increment the value of n.

Example: Move \$100 to $t = 0$ from $t = 5$.



If you do this a lot, you begin to loathe writing all the fractions. So, a very neat notation was invented.

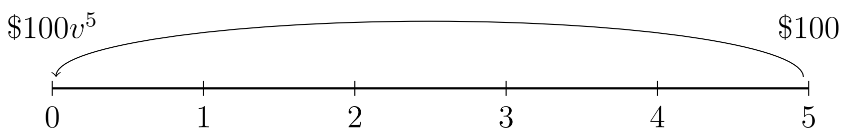
Define,

$$v^n \equiv \frac{1}{(1+i)^n}$$

.

Notes:
1. This is just notation.
2. Moving a payment from the future to time $t = 0$ is called finding the payment's *Present Value.*

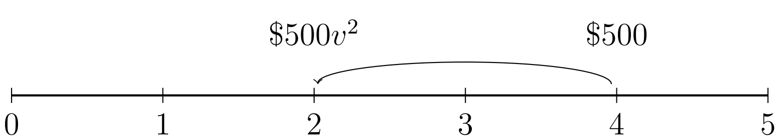
So, PV(\$100 to be received at $t = 5$) = \$100 v^5.



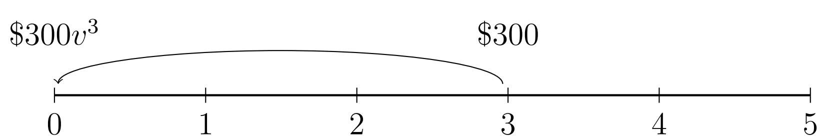
The above two number lines say the same thing. For the rest of this paper, we will always use the v notation; unless context makes writing the details of the division as a fraction more clear.

Examples:

1. Move \$500 to $t = 2$ from $t = 4$ (two steps *Left*).

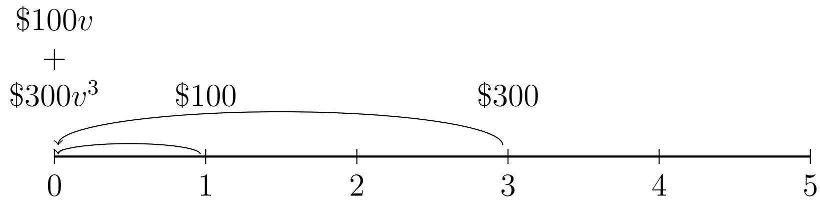


2. Move \$300 to $t = 0$ from $t = 3$, that is find its Present Value (PV).



$$PV = \$300v^3$$

3. What is the Present Value (PV) of two payments, \$100 received at $t = 1$ and \$300 at $t = 3$?.



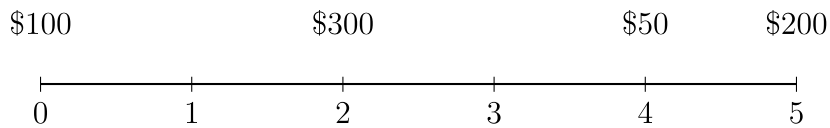
$$PV = \$100v + \$300v^3$$

3 Combos: $(1+i)^n$ and v^n

For illustration, let's move some payments around for practice.

Also, I'm going to add another piece of notation. Let PV_n be the value of all the payments at $t = n$. So, the present value of something at $t = 0$ would be PV_0 and the present value at $t = 2$ would be written PV_2, and so on.

Assume an interest rate i, and find the PV of these four payments at each point of the number line. (I don't think anybody would ever do anything like this except as a practice problem, like this one.)



Present Values at time 0:

$$PV_0 = \$100 + \$300v^2 + \$50v^4 + \$200v^5$$

Present Value at time 1:

$$PV_1 = \$100(1+i) + \$300v + \$50v^3 + \$200v^4$$

Present Value at time 2:

$$PV_2 = \$100(1+i)^2 + \$300 + \$50v^2 + \$200v^3$$

Present Value at time 3:

$$PV_3 = \$100(1+i)^3 + \$300(1+i) + \$50v + \$200v^2$$

Present value at time 4:

$$PV_4 = \$100(1+i)^4 + \$300(1+i)^2 + \$50 + \$200v$$

Present value at time 5:

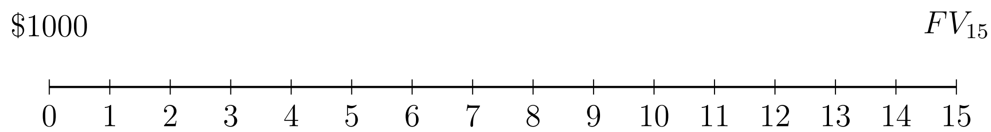
$$PV_5 = \$100(1+i)^5 + \$300(1+i)^3 + \$50(1+i) + \$200$$

Note: if the timing of a payment occurs at the same time as the valuation point, then it is not adjusted by anything.

4 Example Problems

1.
The amount an investor will have in 15 years if $1000 is invested today at an annual interest rate of 9% will be closest to:

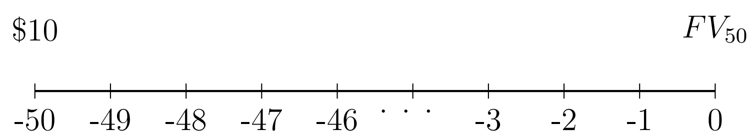
A $1,350., **B** $3,518., **C** $3,642., **D** $9,000.



$$FV_{15} = \$1000(i+i)^{15} = \$1000(1.09)^{15} = \$3,642.48$$

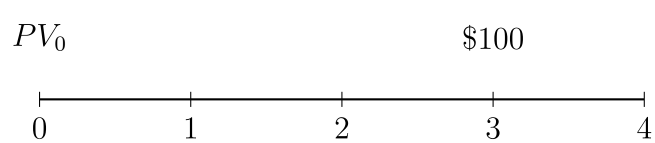
Note: FV is for Future Value.

2.
Fifty years ago, an investor bought a share of stock for $10. That stock has paid no dividends during this period, yet it has returned 20%, compounded annually, over the past 50 years. If this is true, the share price is now closest to: **A** $1,000., **B** $4,550., **C** $45,502., **D** $91,004.



$$FV_{50} = \$10(i+i)^{50} = \$10(1.20)^{50} = \$91,004.38$$

3.
How much must be invested today at 0% to have $100 in three years?
[**A**] $77.75, [**B**] $100.00, [**C**] $126.30, [**D**] $87.50

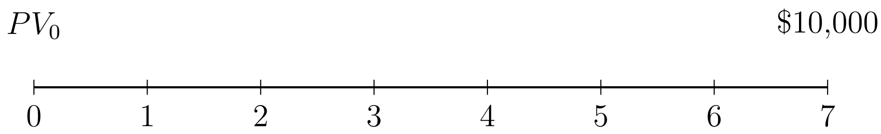


$$PV_0 = \$100v^3 = \frac{\$100}{(1+i)^3} = \frac{\$100}{(1.00)^3} = \$100$$

4.

How much must be invested today, at 8% interest, to accumulate enough to retire a $10,000 debt due seven years from today?

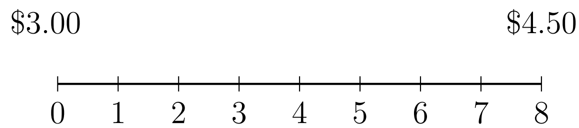
A. $3,265., **B.** $5,835., **C.** $6,123., **D.** $8,794.



$$PV_0 = \$10,000v^7 = \frac{\$10,000}{(1+i)^7} = \frac{\$10,000}{(1.08)^7} = \$5,834.90$$

5.

An analyst estimates that XYZ's earnings will grow from $3.00 a share to $4.50 per share over the next eight years. The rate of growth in XYZ's earnings is closest to: [**A**] 4.9%, [**B**] 5.2%, [**C**] 6.7%, [**D**] 7.0%



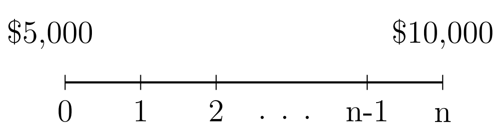
Solve for i:

$$\begin{aligned} \$3.00(1+i)^8 &= \$4.50 \\ (1+i)^8 &= \frac{\$4.50}{\$3.00} \\ 1+i &= \left(\frac{\$4.50}{\$3.00}\right)^{\frac{1}{8}} \\ i &= \left(\frac{\$4.50}{\$3.00}\right)^{\frac{1}{8}} - 1 \approx 5.2\% \end{aligned}$$

6.

If $5,000 is invested in a fund offering a rate of return of 12% per year, approximately how many years will it take for the investment to reach $10,000?

[**A**] 4.33 years, [**B**] 5.75 years, [**C**] 6.12 years, [**D**] 7.50 years

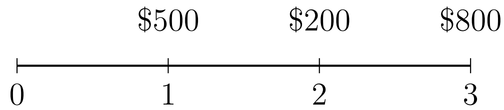


Solve for n:

$$\begin{aligned} \$5,000(1+i)^n &= \$10,000 \\ \$5,000(1.12)^n &= \$10,000 \\ (1.12)^n &= \frac{\$10,000}{\$5,000} \\ n \;\; ln(1.12) &= ln(2) \\ n &= \frac{ln(2)}{ln(1.12)} \approx 6.116 \end{aligned}$$

7.

An investment is expected to produce the cash flows of \$500, \$200, and \$800 at the end of the next three years. If the required rate of return is 12%, the PV is closest to:
[**A**] \$835., [**B**] \$1,175., [**C**] \$1,235., [**D**] \$1,500.

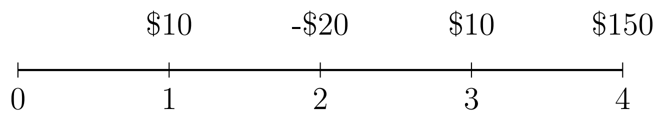


$$\begin{aligned} PV_0 &= \$500v + \$200v^2 + \$800v^3 \\ &= \$1,175.23 \end{aligned}$$

Note: I typically compute v first and store it in memory 'Key 7'.

8.

Given an 8.5% discount rate, an asset that generates cash flows of \$10 in year 1, -\$20 in year 2, \$10 in year 3, and then sold for \$150 at the end of year 4 has a present value of:
[**A**] \$163.42, [**B**] \$150.00, [**C**] \$135.58, textbf[D] \$108.29



$$\begin{aligned} PV_0 &= \$10v - \$20v^2 + \$10v^3 + \$150v^4 \\ &= \$108.29 \end{aligned}$$

5 Annuities

Here we go ...