**ACT school 2018 plans**

Here we discuss the next steps in preparation for our meeting in Leiden. (Jules?) suggested the slogan of developing a Categorical Systems Theory.

We discussed the general idea that if you throw away the idea of inputs and outputs (*as powerfully argued by Willems who rails against giving inputs and outputs definition status*) open systems typically result in a symmetric monoidal category **Beh** which has a relational feel (so probably a forgetful functor to **Rel\_**\times - the \times means that we take cartesian product as monoidal product).

We are interested in discovering nice (string diagrammatic) syntax for these things. One hypothesis is that the algebra of cartesian bicategories will give us suitable algebraic structure for diagrammatic reasoning about behaviours. Syntax is related with semantics by a symmetric monoidal functor

**Syntax** -> **Semantics**

There’s a whole bunch of examples of this kind of situation, e.g.

* Petri nets with boundaries (here the semantics is the concept of Span(Graph) as an algebra of transition systems, with a bunch of papers by Walters et al)
* Signal flow graphs (I could include a more detailed bibliography if someone is interested!)
* Calculus of conjunctive queries, a.k.a. the fragment of FOL that one can think about string-diagrammatically (see below for a link to my paper with Bonchi and Seeber)
* Passive linear circuits <https://arxiv.org/pdf/1504.05625.pdf> - with an explanation of the relationship with Graphical Linear Algebra (see Proposition 7.5) - Q. can we find a cartesian bicategory structure here?
* ...

One idea for our week together would be to discover some more, perhaps based on the examples in Willems’ paper, e.g. the hydraulics example.

Here’s some more reading:

* The idea of cartesian-bicategories-as-syntax is explored in my arXiv paper with Bonchi and Pavlovic (<https://arxiv.org/abs/1711.08699>). Lawvere theories are certain kinds of props, and come with the classic functorial semantics (models as cartesian functors to **Set**). Frops (“Frobenius props”) are also kinds of props, that come with a functorial semantics (models as cartesian bifunctors to **Rel**). You’ll notice that the paper is a definition and a bunch of examples: an open problem is to find some nice characterisations for categories of models. I would argue that this is something close to a “proof theory of open systems” coined by Jules.
* With Bonchi and Seeber we have proven a completeness result for free Frops - those frops induced from a signature with generators and no equations. We framed this as a problem of database theory: here’s a link to the [paper](http://users.ecs.soton.ac.uk/ps/papers/cocq.pdf).

More things to think about

* Since we are throwing away inputs and outputs Joshua suggested that it may be interesting to think about causality. Aleks Kissinger has been doing very interesting work on this: perhaps we could interact with Aleks’s group?
* There are some deeper questions about “why are monoidal categories the right thing anyway?”. Eliana asked about this, and Jules followed up by asking about theorems that relate monoidal categories and surface diagrams.
* Given that many examples in Willems concern the solution space of systems of differential equations, perhaps we could look at compositional techniques? Jules was mentioning some work in this area.
* Another area which is interesting to me is open algebras of account systems. A possible semantic category is described in this [paper](https://arxiv.org/abs/0803.2429). We could try to give an open Petri-net like syntax here and define a monoidal functor.
* We could discuss more about tool support. From general purpose string diagram drawing via diagrammatic reasoning assistants through to iPhone games :)

Notes from seminar discussion

* Jonathan asked about causality too, including the idea of a sliding scale of “when a measurement becomes a cause”. Generally, we agreed that dynamical systems, in Willems sense, must also be interactive. Sophie raised the idea of compact symmetric multicategories as a syntax for decorated graphs with some edges oriented and others not; could this be a good setting to formalize how to induce an IO division from the “higher-order” behavioral data?
* Eliana and Cory raised possible applications of the behavioral approach to control to reinforcement learning. How would that work?

“Height of the water” = a parameterized family of morphisms? Each morphism representing a water tank system. Plugging in values / internal states = “synchronization with the environment” = objects in the category. Jules: “endogenization” of an exogenous variable, thus allowing it to interact with other endogenous stuff.

What about composing tanks / flows of different materials? Maybe we can do something with viscoscity or other variables under the behavior of “mixing” different materials? Is there some algebraic relation we can model?

To read: cartesian bicategories (again), passive linear networks, Categories in Control