Discussions toward   
**an integrative science**

Applied Category Theory 2018   
@ Leiden, the Netherlands

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Use this link to see the [ACT 2018 schedule](https://docs.google.com/spreadsheets/d/1Qnzv-zSNWQe4SX0SU-120X-PyENzOExFDeHd42cGv0c/edit#gid=0).

# Discussion Guide[[1]](#footnote-0)

Having a productive discussion is a hard job. In order to help you to get the most out of your discussions, we would like to provide some advice. We are completely aware that too much “structuring” can also negatively influence a creative process, so please don’t feel “parented” by the below suggestions but rather see them as just that - advice.

A general philosophy which we would like to emphasize is: *the discussion sessions are not there for you to learn a topic, but rather to discuss deep issues and to develop new ideas.*

At the beginning of discussion:

1. Pick a moderator and a minute taker. This is extremely useful.
2. Go around the discussion and ask everyone for his/her reason for attending the discussion, their ideas about the topic (if any), and what they would like to get out of the discussion. Nobody should hold back his/her ideas and intentions at this point, and everybody should be listened to. Try not to “criticize” at this stage.
3. Try not to go into details yet, but first gather all ideas.
4. If different directions emerge, don’t be afraid to split your group. It would of course be nice if you joined together again at the end to gather your individual outcomes.

To emphasize, **do not** spend a lot of time on clarifying (technical) background questions. Going into technical details usually takes a lot of time which you usually will not have at this point. We encourage you to minimize this to the absolutely necessary amount and rather be bold and try to develop and discuss ideas. The “experts” among you can always get together at a later point during the workshop to discuss technical details.

***Important: take minutes, and post the outcome of your minute-taking (a synthetic summary) to this document on the same day that the discussion took place.***

For minute takers:

* During the discussion session, take some preliminary notes about topics and points of relevance. Don’t try to be perfect or be afraid of not getting everything down. Just try to do it as well as possible.
* Immediately after the discussion, or on the same day that the discussion took place, the minute taker tries to create a synthetic summary of the discussion, highlighting the main points that were addressed and that are left open to investigate, and copies it into this document.
* In this way, other workshop participants can glimpse into your session and see if they want to take up a point or add something. Don’t worry if it is still preliminary and unpolished. You can always come back and change it later.
* The minutes posted this way will be shared only among participants and will not be made public, though we (the organizers) may use them to compose a final workshop report.
* Other people are encouraged to enrich the draft via suggestions and comments.

# Proposed Discussions

If you have an idea for a discussion, please put them in this list. We will sort out locations and exact times during the workshop.

1. Sentence as programs
2. Semantic categories that are richer than FVect
3. Tension between doing category theory and doing linguistics; using tricks, concrete examples, ability to implement
4. Is there a principled way of transforming Bart and Fabio’s category into a category of measures, akin to the Shannon entropy. + “Is entropy a functor?”
5. Probability of A \subset B as primitive. A “deconstructed measure space”. Is there a way of doing “relative probability” within a measure-theoretic framework; so I could work within a set of measure zero but still talk about subsets of that set? Are there existing axiomatizations (Popper)? Can we deal with this categorically? (+ relationship to control)
6. Is conditioning compositional? + Bayesian inversion as compactness + connection with open games.
7. How do objective priors (e.g. Jeffreys prior) fit in?
8. Categorical money theory = open games + petri nets + algebra of accounting [Viktor Winschel]
9. Category theory [towards an integrative science](http://www.appliedcategorytheory.org/workshops/)
10. Globular tutorial with Jamie Vicary (TBC)
11. Case studies of applied category theory in industry
12. Games, learning, and bidirectional transformations
13. The need for software (that isn’t a proof assistant)
14. The need for diagrammatic proof assistants
15. Open Petri nets
16. Directions, ideas and topics for a new regular workshop on compositional structures (Jamie)

# Schedule Day 1: Causality

#### 11:00 - 12:00 (Room: ALL) Talk to your officemates!

Attendees: everyone.

Summary: n/a.

#### 3:00 - 4:00 (Room: Main Hall) Compositionality, the journal: town hall

Attendees: Brendan, Nina, Josh, Bob Coecke, John Baez, Ross Duncan, Jamie Vicary, Helle Hvid Hansen, Fabrizio Genovese, Jelle Herold, Philipp Zahn, Mike Johnson, Jonathan Lorand, Tai Danae,

Summary: We talked about a number of subjects related to the forthcoming [journal](http://compositionality-journal.org), including

* The most important thing: have fast, efficient review processes! Guarantee a 30-60 day turnaround from reviewers!
* How can we exert pressure on reviewers? A solid deadline? Editorial wallet? Is that too corporate / not okay for nonprofits? That paper has been submitted for that issue, and the deadline is graded for issues. What about having a system to review the reviewers? A moderated post-publication forum for papers?
* We need a visual flow chart for the editorial review process of the journal.
* Where is the journal getting income (currently through a benefactor, but possibly eventually through a donation system),
* What can we do to make sure that people *outside of category theory* (sitting on, for example, a hiring board) take publications in this journal seriously?
* what does “compositionality” mean and the fact that many people think of themselves as working with compositionality even though they would not necessarily qualify under our scope. A manifesto linked on the front page that helps authors understand what is compositionality.
* the need for tools to track editors and their performance,
* blockchain implementation; Jelle would be interested in doing this,
* A suggestion: a series of reports about negative results: “anecdotes of failure”? For example, a series of links to 2-page reports on the arXiv. Ref. “daily wtf”.
* What lessons can we learn from TAC? Perhaps in managing editorial boards, or getting archived in real library collections.
* If the arXiv gets bought by Elvesier, what is our licensing agreement for articles?
* Could we have reviews for video lectures? Yes, if you want to do it yourself, the journal could help advertise this.
* Distill.pub interactive stuff: we’d like to offer a “pedagogy grant” that allows authors to write new things. Just a matter of cost.
* Short video abstracts option? E.g. this is at the NGP. Talk to Gowers about mini editorial introductions to the articles?
* Goal: emphasize clarity of exposition rather than formality.
* Post-publication reviews + open reviews? As a way of getting reviewers to invest?
* Encourage authors to include links to additional material + videos?

#### 3:00 - 4:00 (Room: 301) Relative measure theory

Attendees: David Spivak, Christina Vasilakopoulou, Samson Abramsky, Dan Cicala, Dan Marsden, Blake Pollard, Spencer Breiner, Brad Theilman

Summary: The discussion centered around finding axioms to capture a gadget where conditional probabilities are primitive. The suggested axioms begin with a distributive lattice C together with, the following conditions: for each subobject a of b,

1. P( a | b ) \* P( b | c ) = P( a | c )
2. P( 0 | a ) = 0 when a is not 0
3. P( a v a’ | b) + P( a ^ a’ | b ) = P( a | b ) + P( a’ | b )
4. For (a\_i)\_{i in I} with I a directed set, sup { P ( a\_i | b ) } = P ( V a\_i | b )
5. P ( a | a ) = 1.

A motivating example: given a system of non-deterministic dynamical systems we are interested in capturing a dynamical system with feedback loops. For instance, can we find P( a , c | a , b ) for the dynamical system

I.b --(n.id)--> a^\*.a.b --(id.f)--> a^\*.a.c

Where n is the unit, “.” is tensor, and a.b --(f)--> a.c is some dynamical system.

One difficulty of this is that composition of non-deterministic dynamical systems involves taking a weighted average of probabilities, but feedback does not play well with this.

As an exercise, we unsuccessfully tried to find a trace operator in the monoidal category STOCH.

#### 3:00 - 4:00 (Room: Organizer’s Room) Unifying the 3 (4?) notions of causality

Attendees: Aleks Kissinger, Fabio Gadducci, Fabio Zanasi, Jules Hedges, Sophie Raynor, Eliana Lorch, Pablo Andrés-Martínez, Bruno Gavranovic, Martha Lewis, Jade Master, Eswaran Subrahmanian, Dmitry Vagner.

Summary: We focused on the categorical formalisation of causal Bayesian Networks. The standard framework only allows us to consider causal structures defined by *acyclic graphs*. The main motivation of our categorification is to achieve a definition of these concepts where the extension to causal structures with cycles is natural. Three different results were discussed:

1. Given a DAG *G*, we can build a monoidal category *D(G)* where objects are the random variables in *G* (its vertices), each of them equipped with a *comonoid* structure, and morphisms are string diagrams built from comultiplication, counit and boxes of the form:



There is a monoidal functor *F: D(G)* ➡️ *Stoch* that also preserves comonoid structure. Such a functor precisely corresponds to ‘filling’ the boxes with stochastic matrices such that the probability distribution in the whole graph *G* is Markov relative to it. More info: [Brendan's Master's thesis](https://arxiv.org/abs/1301.6201).

We would define the Markov condition in a graph with cycles in a similar way, where instead of using simply comonoids, we have a Frobenius structure in every object (so we have caps and cups, and can draw loops).

1. Such *D(G)* can be defined as the free hypergraph category over the set of boxes defined above. [Here](https://arxiv.org/pdf/1602.06771.pdf), it is shown that such a construction can be also achieved by taking the (discrete) cospans of the category *Hyp/ΣG* (the category of hypergraphs and hypergraph homomorphisms sliced by *G*, seen as a hypergraph). One interesting feature of this construction is that the object corresponding to *ΣG* (our original graph) is terminal.
2. On another note, given the following two categories:

* *C,* where objects are pairs of a DAG and a probability distribution over its vertices, and morphisms are graph homomorphisms that respect conditional independencies.
* *D*, category of string diagrams built from comonoids and the boxes described above.

There is a pair of adjunctions *F* ⊣ *G*, where *F:* *C* ➡️ *D* corresponds to ‘factoring’ the probability distribution (disintegration) and *G: D* ➡️ *C* corresponds to ‘flattening’ back to the joint distribution. This result is proven by a student of Fabio Zanasi at UCL.

# Schedule Day 2: AI & Cognition

#### 11:30 - 12:30 (Room: Main Hall) Learners, Lenses, and Open Games

Attendees: Mike Johnson, David Spivak, Jules Hedges, Brendan Fong, Joshua Tan, John Baez, Christina V, Eliana L, Harald van Mil, Viktor, Philipp, Joe Moeller, Helle Hvid Hansen, Bruno, Dan Cicala, Jamie Vicary, Pablo A, Blake P, Kathryn H, Eswaran S, John Foley, Ross Duncan

Summary: We covered a conjectural correspondence between three fields: databases (in particular, a category of asymmetric lenses), machine learning (Spivak’s Learner category), and economics (Hedges’ category of open games).

Databases and lenses

1. Mike Johnson: bidirectional transformations. In database theory, a query (or view) is a one directional transformation. The opposite direction is the “view-update” problem: what is the right transformation on the BIG database when something changes in the view? In general, there could be lots of things required to get a consistent database instance, outside of the changes immediately given in the view.
2. Bidirectional transformations are much more general than the way they show up in databases. In all sorts of change management: codebase + documentation, architectural diagrams.
3. Lenses capture such bidirectional transformations. “Good” lenses or c-lenses (c for category \subset vs. delta-lenses \superset lenses which are Set-based) are *(op-)fibrations*: a functor with Cartesian lifts. The data of a lens is defined by:
   1. g: S \to V “Get the view”.
   2. p : S x V \to S “Put”. (where V represents a new database state)

Subject to three axioms: the putget axiom. Putting then getting gets back the new state. Getput axiom: getting then putting gets back the original state. The putput axiom: if I do two changes of V, they can be commuted so long as … (?)

1. A symmetric lens is where the two things both know things that the other does not. This is not the case for the view-update problem. This is an asymmetric lens.
2. There’s a category of (asymmetric) lenses, where asymmetric lenses are the morphisms. There’s also a category of (symmetric) lenses, where the pullbacks of two asymmetric lenses are morphisms.

*Learners*

From Fong, Spivak, and Tuyeras: an (A,B)-learner is

1. a set P (parameter)
2. I : P x A -> B (implementation)
3. U : P x A x B -> P (update)
4. r : P x A x B -> A (request)

*Games*

1. First, a diversion to Haskell lenses:
   1. (S,S’) -> (V, V’), to deal with polymorphic type system in Haskell
   2. a map of this form is composed of two maps:
   3. get: S to V
   4. put: S x V’ to S’
2. *Jules’ claim*:open games are to Haskell lenses like learners are to regular lenses
3. an open game is a map G: (X,S) -> (Y, R) determined by
   1. a set \Sigma, strategy profiles
   2. a function \Sigma x X \to Y, play function
   3. \Sigma x X X R \to S, “coplay”
   4. X \times (Y \to R)\to (\Sigma \to P\Sigma), called best response

Discussion

* Claim: there’s a monoidal functor from Learner to Games.
* Intuition: neurons are game-playing agents whose utility space S, R is…? Neurons change the strategy via the update function.
* Intuition: you can think of the neural network as a bidirectional transformation, with fixed states, where the implementation is the “get”, while the request is the “put”. In databases, you enforce these compatibility conditions… but that’s too restrictive to capture semantics of ML, e.g. in gradient descent.
* Intuition: if (PxA) = S —> combines conditions 2-4 in learner into one thing: S x V -> S. 2 is the get, 3-4 is the put.
* So P x A \to A is a classical asymmetric lens, where the update is just changing A leads to changing the copy of A in P x A. P x A -> B?? gets you a symmetric lens! Then the pullback is the composition for learners! So there is a fully faithful functor from Learner to symmetric lens?
* Question: Forward propagation in the symmetric lens is the implementation or “running the hypothesis”, so what does request or backprop look like?
* Connection from Chenchang Zhu? In geometry, often you want to define a map from A to B, but A is not flexible enough, so you divide A finer via covers, you call it the fiber replacement P x A.
* Question: can we model GANs using open games ~ learners translation?

#### 11:30 - 12:30 (Room: 301) Translation + Richer Semantics than FVect for

Attendees: Giuseppe Greco, Jade Master, Brad Theilman, Tai-Danae Bradley, Michael Moortgat, Mehrnoosh Sadrzadeh, Bob Coecke, Martha Lewis

Summary:

* We summarized the translation model that we worked on during the School week
* Suggestion to use Abstract Categorial Grammar as a way of mediating translations between grammars of other languages
* Discussion of concept representation in the Poincare disc as a means of encoding concepts with more structure than standard vector spaces

Afternoon meeting:

Giuseppe Greco, Jade Master, Michael Moortgat, Mehrnoosh Sadrzadeh, Bob Coecke, Martha Lewis, Daniel Cicala, Samson Abramsky

* Michael summarised need for non-associative tensor
* Description of box and diamond modalities for structural control

#### 11:30 - 12:30 (Room: Organizer’s Room) How to apply CT to X?

Attendees: Spencer Breiner, Jonathan Lorand, Fabio Gaducci,

Summary:

#### 15:30 - 17:00 (Main Hall) Software for ACT (ft. Jamie Vicary)

Attendees: many

Summary: see tinyurl.com/ACT2018CCT or [in table format](http://www.joshuatan.com/a-comparison-of-category-theory-software/)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Software package** | **Does it exist?** | **Author** | **Goal** | **User interaction** | **Automation** | **How to define categories** | **Categorical setting** | **Structures it can compute** | **Architecture** | **Directed? (i.e. categories vs. groupoids)** |
| HoTT libraries in Cog or Agda | Yes | Various | Research | Code | Proof assistant | Via presentations | Higher (weak) | Limits, colimits, functors | Library | No |
| Opetopic | Yes | Eric Finster | Research | Geometrically | None | Via presentations | Higher (weak) |  | Library | Yes |
| [Quantomatic](https://quantomatic.github.io/) | Yes | Aleks Kissinger et al. | Research, scale, interoperability (?) | Geometrically | Proof assistant | Via presentations | Strict, symmetric monoidal |  | Standalone | No |
| Algebraic Query Langauge (AQL) | Yes | Ryan Wisnesky, David Spivak | Commercial, teaching | Code | Query optimizer, data migration | Explicitly, and via presentations | Monoidal | Limits, colimits, functors | Standalone + library | Yes |
| Globular | Yes | Jamie Vicary et al. | Research, publishing | Geometrically | None | Via presentations | Higher (semi-strict) |  | Standalone | Yes |
| TikZit | Yes | Aleks Kissinger et al. | Publishing | LaTeX | None |  |  |  | Library |  |
| Typedefs | Under development | Jelle Herold et al. | Commercial | Code |  |  |  |  |  |  |
| [Proto-Quipper-M](https://arxiv.org/abs/1706.02630) | Yes | Francisco Rios, Peter Selinger | Research |  |  |  |  |  |  |  |
| Rholang | Under development | Mike Stay et al. | Commercial |  |  |  |  |  |  |  |
| EASIK | Yes | Bob Rosebrugh et al. | Research | Geometrically |  |  |  |  |  |  |
| [Cateno](https://github.com/jasonmorton/Cateno) | Yes | Jason Morton | Research |  |  |  |  |  |  |  |
| [PySheaf](https://github.com/kb1dds/pysheaf) | Yes | Michael Robinson | Research | Code |  |  |  |  |  |  |
| Specware | Yes | Kestrel Institute | Commercial |  |  |  |  |  |  |  |
| Catlab | Under development | Evan Patterson |  |  |  |  |  |  |  |  |
| OICOS | Under development | Viktor Winschel, Philipp Zahn | Commercial |  |  |  |  |  |  |  |
| Statebox | Under development | Jelle Herold et al. | Commercial |  |  |  |  |  |  |  |
| [TikzWD](https://github.com/appliedcategorytheory/TikZWD) | Yes | Patrick Schulz, David Spivak | Publishing | LaTeX |  |  |  |  |  |  |
| DSL for Operads | No | TBD | Simulation | Geometrically | None | Via presentations | Operads |  | Standalone |  |
| Common File Format for Categorical Constructions | No | TBD | Infrastructure | Code | Type-checking | Explicitly, or via presentations | All |  | File format |  |

# Schedule Day 3: Dynamical Systems

#### 11:30 - 12:30 (Room Main Hall) Relational vs Input-State-Output in dynamical systems

Attendees:

Summary:

#### 11:00 - 12:00 (Room 301) Approximation and Uncertainty in Category Theory

Attendees: Joshua Tan, Sophie Raynor, Spencer Breiner, Jelle Herold, Jules Hedges, Dan Marsden, Tai Danae-Bradley, Brad Theilman, John Foley, Martha Lewis

Summary: We covered three main ideas.

1. “epsilon category theory” Ross is interested in quantum circuits; differential duration. Every box in my string diagram has some known noise property, and I want to search through or optimize the noise parameters. So as I execute through the string diagram, I keep track of a probability distribution through all of it. What about approximation between diagrams? Prakash’s stuff: dealing with equality up to epsilon. Quantitative homotopy theory in string diagrams; can the path length in string diagrams represent things like “execution time”? Have geometric topologists thought about this sort of stuff?
2. “categorical numerical analysis” Jules is interested in compositional numerical approximation, especially in economic models: how errors propagate across different kinds of programs (represented by different diagrams). Topologically equivalent manipulations of string diagrams lead to different errors due to this fact. “Non-faithful computation”. Errors are structured by the diagrams, but they somehow don’t respect the structure of the diagrams!
   1. Dan Marsden: maybe a useful way of thinking about these kinds of behaviors? TX -> X -> BX, want to force a “niceness” condition between TX and BX via a distributive law. So diagrams live in TX, X has diagrams, behaviors on diagrams in BX. Ref. bialgebraic semantics by Bartek Klin.
3. “scientific category theory” or “moral category theory” Josh wants approximate or partial or “weak” composition for scientific modeling: how do we get a more “combinatorial” working language that allows easier presentation and experimentation, rather than strict verification of categorical properties? Close to the idea of inferring categorical structure from data. “Process mining” as a non-categorical way of inferring models from data? But it seems categorical. Dan Ghica worked on something, a ML program, that learns categorical structure? Relation to coarsening, or Jurgen Jost’s idea for different description levels. Is a 2-cat way the right way. Quite closely related to epsilon CT. **We will talk about this tomorrow.**
   1. Related to question of approximate translation in DisCoCat?

#### 11:00 - 12:00 (Room 364) Modular Proof Systems

Attendees:

Summary:

#### 11:00 - 12:00 (Room 365) Learning the ACT Toolkit

Attendees:

Summary:

# Schedule Day 4: Systems Biology

#### 11:30 - 12:30 (Room 364) Causality & Petri Nets

Attendees: Dan Ghica, Fabio G, Aleks K, Samson S, Ross D, Pablo M, Spencer B, Sub S, Jean K, Josh T

Summary: Roughly, the problem is that we are trying integrate causality with event tracking as in Petri nets: production / consumption. A causal process has to be related to the notion of executing something. We can denominate some of these semantics in terms of events; an event occurrence is something that happens once.

Problem: disjunctive causality? Variables a, b can both generate c, but either conjunctively or disjunctively. Is it possible to formulate this sort of question within concurrency models?

Another approach is that of *partial* events; events that have incomplete descriptions. But we should expect that completing the events with more context does not allow you to break the specified causal order.

#### 3:30 - 5:00 (Room 364) Scientific category theory

Attendees: Bob C, Josh S, Jules H, Brad T, Pablo M, Aleks K, Jade M

Summary: <http://www.joshuatan.com/the-failures-of-category-theory/>

#### Evening (Common Room) Category theory as an integrative science

Attendees: Joe, Jade, Jules, Christina, Dan C, Josh

Summary: Can we write a research program, “something that we can paste into funding applications”, about how to sell CT as an integrative science.

* “What after compositionality? So you can write functors!”
* Really follows many people’s ideas and intuitions about category theory
* Want not only more of the connecting stuff, but also to understand what the important applications are, “what’s the point”, to science or to industry. Why do you want functors?

# Schedule Day 5: Closing

#### 11:30 - 12:30 (Room ?)

Attendees:

Summary:

1. Thanks to the organizers of the [Rethinking Workshop](http://rethinking-workshop.org) for sharing their discussion guide. [↑](#footnote-ref-0)