1 Brief statement of interest

As a soon to be minted Ph.D. on the job market an opportunity like this, one that is both an opportunity to get involved in cutting edge applications and to make new professional and scholastic connections, is invaluable. I am a scholar with an interdisciplinary history and bent who has felt alone in believing that (higher) category theory can and should be applied and in particular that its most valuable applications may be far afield of the mathematicians usual redoubt, the applicability to physics. Even in the application process, this workshop has informed me of new possibilities. I expect that the Adjoint school will provide me with new projects, new directions, and new professional prospects.

2 Thesis description and other background

2.1 Background

I believe that I am well prepared for participating at the highest level in this workshop. I have organized and run:

- a one-semester seminar in Topos theory,
- a one-semester seminar in algebraic theories,
- a one-semester seminar in quasi-category theory,
- a one-semester seminar in higher algebra, and
- a one-semester seminar in linear logic with a view towards geometry.

I have participated and presented multiple times in a one-semester seminar on homotopy type theory that radically changed the trajectory of my doctorate. In summer schools I have presented talks:

- on simplicial sets,
- on the unstable $\mathbf{A}^1$-homotopy category,
- and on the theory of fibered categories in the context of ∞-cosmoi.

I am also familiar with Riehl and Shulman's type theory for ∞-categories. More, at my home university I have presented numerous times on the topics of (essentially) algebraic theories, test category theory, weakness and constructivism, and coherence in algebra and logic.

2.2 Thesis

My Ph.D. thesis entitled "Spectra as locally finite weak Z-groupoids", due to be defended in May of 2019, synthesizes:

- an elegant presentation of the suspension native to cellular sets and to HoTT,
- Kan's combinatorial spectra, and
- essentially algebraic theories of higher composition,

to provide a presentation of spectra as weak Z-groupoids satisfying a local finiteness condition.

2.3 The long arc

Alongside this purely mathematical work I have been engaged with the philosophies of mathematics and the natural and social sciences. I am interested in how epistemological and ethical concerns inform what sorts of mathematics and what sorts of inference from mathematics is appropriate for different domains. I am particularly interested in the abstraction of epistemology as opposed to metaphysics which is manifest in type theory and I have long been convinced that topological and geometric consideration of coherence might serve as powerful formal metaphor for many questions in the biological and social sciences.

Before I joined the PhD program in mathematics I completed a masters thesis, ostensibly in mathematics but really in behavioral economics, under the direction of Prof. Peter Dodd's at the University of Vermont. In that thesis I discover ways in which social structure, modeled by graphs, determines the effectiveness of many reasonable and cognitively accessible distribution strategies. In particular, we show that high Gini coefficient distributions are often stable even when agents are trying to redistribute resources equitably. In that time I also worked in computational and cognitive linguistics. The most indicative artifact of this work is the paper: http://www.nature.com/articles/srep12209, wherein we observe that Zipf's law holds for phrases, not words.

2.4 Commitment to being at Oxford

I have excellent reason to believe I will be able to secure funds for airfare, so it is highly likely, even in the absence of funding overall, that I will be able to attend.

3 Project preference order

Ranking these projects is a very difficult task. Each round of consideration I have given to them has yielded a different ordering. In the final analysis however, I have settled on:

1. Mehrnoosh Sadrzadeh - Formal and experimental methods to reason about dialogue and discourse using categorical models of vector spaces
2. David Spivak - Toward a mathematical foundation for autopoiesis

3. Miriam Backens - Simplifying quantum circuits using the ZX-calculus
4. Bartosz Milewski - Traversal optics and profunctors
5. Tobias Fritz - Partial evaluations, the bar construction, and second-order stochastic dominance
6. Pietr Hofstra - Complexity classes, computation, and Turing categories

PAUL LESSARD
Department of Mathematics
University of Colorado, Boulder
Boulder, CO 80309-0395 USA
617-407-0970
pale2772@colorado.edu
http://math.colorado.edu/~pale2772/

Education

2012-2019 UNIVERSITY OF COLORADO, Boulder CO
Doctor of Philosophy in Mathematics, May 2019 (expected)

2010-2013 UNIVERSITY OF VERMONT, Burlington VT
Master of Science in Mathematics, August 2013

2005-2008 THE EVERGREEN STATE COLLEGE, Olympia WA
Bachelor of Science, May 2008

Appointments

2012–2019 Graduate Teaching Fellow, University of Colorado Boulder

2011-2012 Graduate Teaching Fellow, University of Vermont

2011-2012 Graduate Research Fellow (NASA research grant #06811191), Vermont Advanced Computing Center & Complex Systems Center

2006-2008 Lead Tutor, QuaSR Center, The Evergreen State College

Research interests

I study the logic and algebra of higher categories. In practice this means I am interested in universal algebra, higher category theory, homotopy theory, categorical logic, and type theory. My current work aims to extend the program of synthetic homotopy theory begun with the advent of homotopy type theory to the domains of stable homotopy theory and motivic homotopy theory.

Beyond pure mathematics I am interested in the philosophy of mathematics and the natural and social sciences, cognitive science, economics, complex systems, and linguistics. As I move forward I very much hope to return to these topics again in interdisciplinary projects.

Preprints

2018 Joyal's Suspension on Θ and Kan's Combinatorial Spectra https://arxiv.org/abs/1812.00122

Publications

2015 Zipf's Law Holds for Phrases not Words with Jake Ryland Williams, Suma Desu, Eric M. Clark, James P. Bagrow, Christopher M. Danforth, and Peter Sheridan Dodds. Nature Scientific Reports 5, Art. 12209 2015

Invited talks, conference talks, and summer schools

2019 "Spectra are Locally Finite $\mathbf{Z}$-groupoids" Geometry in Modal Homotopy Type Theory, CMU

2018 "Fibrations and the Yoneda Lemma in Synthetic Higher Category Theory" Talbot, MIT

2015 "The Unstable $\mathbf{A}^1$-homotopy category", K-Theory Summer School, USC

2014 ""Semi-simplicial complexes and Postnikov systems" by Moore", Algebraic Topology Summer School, MSRI

Selected talks

2018 "A Cellular Suspension Functor for a New Presentation of Spectra" - Topology seminar - CU Boulder

2018 "Quillen's Theorem A and Grothendieck's Asphericite"' - Topology seminar - CU Boulder

2018 "Of Colimits, Sifted and Not, and the Naturalness of Algebraic Theories" - Slow Pitch seminar - CU Boulder

2017 "Symmetric Spectra Part II" - Topology seminar - CU Boulder

2017 "Symmetric Spectra Part I" - Topology seminar - CU Boulder

2016 "The etale topology for analysts" - Slow Pitch seminar - CU Boulder

Teaching

S-2019 Teaching assistant - Calculus III - 3 sections - CU Boulder

F-2018 Teaching assistant - Calculus II - 3 sections - CU Boulder

S-2018 Instructor - Calculus I - CU Boulder

F-2017 Instructor - Calculus III - CU Boulder

S-2017 Instructor - Calculus III - CU Boulder

F-2016 Teaching assistant - Calculus III - 4 sections - CU Boulder

S-2016 Instructor - Calculus II - CU Boulder

F-2015 Instructor - Calculus II - CU Boulder

S-2015 Instructor - Calculus I - CU Boulder

F-2014 Instructor - Calculus I - CU Boulder

S-2014 Instructor - Calculus I - CU Boulder

F-2013 Instructor - Calculus I - CU Boulder

S-2013 Teaching Assistant - Calculus I - 4 sections - CU Boulder

F-2012 Teaching Assistant -Pre-calculus - 4 sections - CU Boulder

F-2011 Instructor - Math 17: Mathematics in Nature and Society (Finite Mathematics) - UVM

Course development

I have developed course materials for inquiry based learning versions of the following courses:

- Calc II, Spring 2017: daily worksheets, activities, and assessments
- Calc III, Fall 2017: daily worksheets, activities, and assessments

Organization

F-2018 reading group on linear logic with a view towards geometry

F-2018 group independent study: topos theory

S-2017 group independent study: algebraic theories

F-2015 reading group in quasi-categories

2013-2015 Slow-Pitch seminar, CU Boulder

Affiliations

American Mathematical Society, Chapter Vice President 2017-2019

RESEARCH STATEMENT

PAUL LESSARD

Introduction

I study the logic and algebra of higher categories. In practice this means I am interested in universal algebra, higher category theory, homotopy theory, categorical logic, and type theory. My current work aims to extend the program of synthetic homotopy theory begun with the advent of homotopy type theory to the domains of (parameterized) stable homotopy theory and motivic homotopy theory.

Background

In a synthetic theory, entities are identified with how we may reason about them, whereas in an analytic theory all entities are built out of some more basic substance. The most familiar instance of this dichotomy is that of plane geometry where it is the distinction between the lines of Euclid, which may be taken to be atomic or primitive, and the lines of Descartes, which are solution sets to linear equations. While synthetic mathematics is broader than synthetic mathematics done by way of type theory, the synthetic mathematics I mean here is type theoretic.

First order logic and set theory abstract the manipulation of facts regarding mathematical entities i.e., the fact that $1+1-1=1$. Type theory on the other hand abstracts the notion of judgement and the construction of knowledge i.e., we know that $1+1-1=1$ because $1-1=0$ is the defining expression for -1 and we may add 1 to both sides.

Perhaps the best way to understand type theory is by way of the types-as-propositions interpretation: a type is a proposition and an inhabitant of a type is a proof of that proposition. For example, proofs that $1+1-1=1$ comprise a so called identity type of natural numbers, that is the type of proofs of equality of the two natural numbers $1+1-1$ and 1.

Now, since the 1980's it has been known[1] that dependent type theory comprises an internal language of locally cartesian closed categories. More recently it has been discovered that there is homotopical content to dependent type theories with an identity type. In fact, it has been found that dependent type theory with an identity type is an internal language for ∞-topoi, those higher categories which merit consideration as higher categories of spaces. Homotopy type theory is the topic spawned by that revelation.

The identity types at the heart of this new foundation for mathematics are governed by four rules. While the formalism in which these rules are expressed is specific to type theory the content is in fact familiar from topology. The formation rule, morally speaking, informs us that for any space A, there exists a space $A^{\mathbf{I}}$ of continuous functions from $\mathbf{I}$ the unit interval into the space A. The introduction rule embodies the existence of a section $\mathsf{refl}_{(\_)} : A \longrightarrow A^{\mathbf{I}}$ of $\mathsf{ev}_0 : A^{\mathbf{I}} \longrightarrow A$, the evaluation at 0 map. The elimination rule translates to the fact that, for a type/space A, and any fibration $\sum_{p:A^{\mathbf{I}}} Q(p) \longrightarrow A^{\mathbf{I}}$, where the fibration is presented

[1]See [Seely1].

University of Colorado
Boulder

Jonathan Wise
Assistant Professor
Department of Mathematics
University of Colorado
Campus Box 395
Boulder, CO 80309-0395

January 30, 2019

Letter of recommendation for Paul Lessard

Dear Colleagues:

It is my pleasure to recommend my student, Paul Lessard, for an postdoctoral fellowship at your institution. Paul is a unique individual and surely the most original and independent thinker I have met among graduate students here at the University of Colorado.

Officially, Paul is my advisee, but I learn far more from him in our meetings than he could possibly learn from me. In part, this is because Paul selected his thesis problem on his own, in an area where I can contribute interest but little expertise. Even where I have some intuition about higher category theory, Paul's intuition exceeds my own and takes him in different directions from the ones I can suggest. That is occasionally frustrating for me, but it works quite well for him. He has been able to find other sources within our department (Agnès Beaudry for homotopy theory, Keith Kearnes for foundations, Josh Grochow for computation) to aid his mathematical development, and he has been able to learn about and use cellular sets, test categories, homotopy type theory, etc. entirely on his own.

Paul's thesis project is about the suspension functor in cellular sets, and more broadly, about the connection between cellular sets and homotopy type theory. In his manuscript, "Pursuing Stacks", Grothendieck proposed globular sets as a combinatorial model of homotopy theory. Like simplicial sets, which model homotopy types as simplices glued along faces, globular sets aim to model homotopy types as discs glued along hemispheres. They have the unusual feature that, in contrast to the situation for simplices, the set of maps from the n-globe to the $(n+1)$-globe is independent of $n \geq 0$. One therefore has a canonical extension of the category of globes to include globes of negative dimension. I once remarked to Paul — rather naively — that presheaves on this enlarged category of globes should model spectra. Paul later informed me that there is a precedent for this in a simplicial model for spectra constructed by Kan that allows negative dimensional simplices.

Paul took this idea and ran with it, quickly discovering just how naive it was. The first obstacle he discovered was that globular sets do not model homotopy types until they are equipped with additional structure to encode the concatenation of paths. An appropriate structure, called cellular sets, was identified by Joyal, in which cells are indexed by elaborate systems of planar trees. Paul must have explained this structure to me a half dozen times, but I never could grasp it intuitively. Eventually, Paul found an alternate construction that I could understand in the work of Berger, relying on iterated wreath products.

Paul realized that my conjecture makes sense when it is formulated about cellular sets in place of globular sets, and he set about to prove it. His plan to prove it is entirely his own, and is currently half way to fruition. The first step is to recognize that the globular shift operation corresponds, under topological realization, to suspension. Paul has done this in a paper already available on the preprint archive.

The second step is to describe a category of "negative dimensional cells" and recognize presheaves on that category as the result of formally inverting the suspension operation. Here again my intuition about negative dimensional cells was incorrect, as Paul discovered, for a technical reason involving the specification of a basepoint. Paul has also identified the right way to fix this, and I'm quite confident that what he has will work. He has already done what should be the hard work of verifying that the pointed presheaves on this category do model spectra in the first paper.

This result is only the beginning a much larger program that Paul has laid out in his proposal. I will do my best to comment on it, although I must admit that it goes well beyond my ability to comment in detail. Homotopy type theory proposes a connection between homotopy theory and logic by which a theorem is identified with the collection (the *type*) of all proofs of that theorem. Two proofs may or may not be equivalent, and there is a type of equivalences between them, a type of equivalences between equivalences, and so on. One can see a globular structure emerging from this description, although Voevodsky's original homotopical interpretation of type theory went through simplicial sets.

Homotopy type theory has a number of applications, both known and anticipated. Voevodsky was motivated by the potential for proof verification. It also gives a synthetic approach to the homotopy theory of spaces, meaning that homotopy theory emerges from the rules for logical reasoning, and doesn't require a construction within them.

The natural question that Paul considers is whether there is a parallel synthetic approach to *stable* homotopy theory. As all constructions of the ∞-category of spectra and its symmetric monoidal structure are quite complicated, a synthetic approach would be quite valuable. Paul proposes that this should come from Girard's *linear* type theory. As I understand it, linear type theory is modeled by vector spaces, whereas type theory could be modeled by sets. It is quite plausible that there is intrinsic homotopical information in linear type theory, just as there is in type theory, and that it should be modeled by the linear analogues of homotopy types, namely spectra. It is natural also to expect that a globular structure should emerge from a linear type theory, and therefore that Paul's category is the natural one in which to produce this model.

To conclude, Paul has been a strong and highly motivated student, and would be both a valuable collaborator and eager apprentice to a postdoctoral advisor who shares his interest in the combinatorial and computational aspects of homotopy theory. I recommend him very highly.

Sincerely,

Jonathan Wise