Applied category theory school application

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January 30, 2019

1 Category theory background and PhD information

Most of my exposure to category theory has been through its use as a language to concisely define things in algebraic geometry. This month, I audited a 3 week course on applied category theory, taught by David Spivak and Brendan Fong.

I completed my PhD in May 2018. My dissertation consisted of three projects in algebraic statistics. The one that I am most proud of was motivated by applications to low-rank matrix completion, and used tropical geometry to characterize the algebraic matroid underlying the Grassmannian of planes in affine n-space, and the determinantal variety of matrices of rank at most 2.

2 Project preference order

1. Toward a mathematical foundation for autopoiesis
2. Complexity classes, computation, and Turing categories
3. Simplifying quantum circuits using the ZX-calculus
4. Partial evaluations, the bar construction, and second-order stochastic dominance
5. Traversal optics and profunctors

6. Formal and experimental methods to reason about dialog and discourse using categorical models of vector spaces

3 To what extend I can commit to coming to Oxford

Whether or not I can come to Oxford for both weeks depends heavily on what my fiancée winds up doing for the summer, which is still up in the air. It is likely that I can come for at least the first week. Funding probably won't be an issue since I have some travel funds of my own, and I'm already planning on being in Europe for a conference from July 9-13.

4 Why I am interested

As a new postdoc, I am looking for new projects that are related to my current research area (algebraic statistics), but that take me in new directions. Applied category theory seems like a great candidate for this, especially since I happen to be at the same institution as Brendan Fong and David Spivak.

I am currently learning about causal inference (an area of expertise of my postdoc mentor, Caroline Uhler), and I'm wondering if any meaningful applications of category theory may arise here. In causal inference studies, one collects data from random variables $X_1, \ldots, X_n$ and wishes to construct a directed graph on vertex set $1, \ldots, n$ with a directed edge $i \to j$ when X_i directly influences X_j. Note that if $i \to j \to k$ is a directed path in this directed graph, then X_i influences X_k. Moreover, since each X_i tautologically determines itself, one could view such directed graphs as categories. I wonder if there is a meaningful way to view causal inference studies as functors. More generally, can viewing causal inference in category-theoretic terms lead to any insights that would elude us otherwise? One hope of mine is that this applied category school will give me the tools to ask good questions about connections between category theory and causal inference, and that such connections will lead to useful theorems and algorithms.

DANIEL IRVING BERNSTEIN

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Research interests: Algebraic statistics, combinatorics, tropical geometry, applied algebraic geometry, phylogenetics, contingency tables, convex geometry

EDUCATION

2018 Ph.D. North Carolina State University, Mathematics (advisor: Seth Sullivant)
2015 M.S. North Carolina State University, Mathematics
2013 B.S. Davidson College, Mathematics (departmental honors, *magna cum laude*)

APPOINTMENTS

Massachusetts Institute of Technology:

- NSF Mathematical Sciences Postdoctoral Research Fellow (Mentor: Caroline Uhler): 2018-2020

Brown University:

- ICERM Postdoc: Fall 2018

North Carolina State University (Raleigh NC):

- Research Assistant: Summer 2014, Spring 2015, Summer 2015, Fall 2015, Summer 2016
- Instructor: Spring 2016, Fall 2016, Spring 2017, Fall 2017
- Recitation Leader: Spring 2014 and Fall 2014
- Lecture Assistant: Fall 2013, Spring 2018

Max Planck Institute for Mathematics in the Sciences (Leipzig, Saxony, Germany) :

- Visiting PhD Student in Bernd Sturmfels's group on nonlinear algebra: Summer 2017

GRANTS, HONORS, AND AWARDS

2018 – 2020	NSF Mathematical Sciences Postdoctoral Research Fellowship ($150,000)
2018	Winton-Rose Award (NC State Math Department, $1,000)
2013 – 2014	North Carolina State University Graduate Fellowship ($4,000)
2013	William G. McGavock Mathematics Award (Davidson College Math Department)
2013	Patterson Prize for talk given at the MAA Southeastern Section Spring Meeting

PAPERS

Submitted papers and preprints

11. The tropical Cayley-Menger variety (with Robert Krone). Submitted. arXiv:1812.09370
10. The algebraic matroid of the funtf variety (with Cameron Farnsworth and Jose Israel Rodriguez). arXiv:1812.10353
9. Typical and Generic Ranks in Matrix Completion (with Greg Blekherman and Rainer Sinn). Submitted. arXiv:1802.09513
8. L-infinity optimization to Bergman fans of matroids with an application to phylogenetics. Submitted. arXiv:1702.05141

Papers in press and print

7. Unimodular hierarchical models and their Graver bases (with Christopher O'Neill). To appear in *Journal of Algebraic Statistics*. arXiv:1704.09018

6. Completion of tree metrics and rank 2 matrices. *Linear Algebra and its Applications.* **533** (2017), pp. 1-13. arXiv:1612.06797
5. L-infinity optimization to linear spaces and phylogenetic trees (with Colby Long). *SIAM Journal on Discrete Mathematics.* **31** (2017) no. 2, pp. 875-889. arXiv:1702.05127
4. Unimodular binary hierarchical models (with Seth Sullivant). *Journal of Combinatorial Theory, Series B.* **123** (2017), pp. 97-125. arXiv:1502.06131
3. Normal binary hierarchical models (with Seth Sullivant). *Experimental Mathematics.* **26** (2017) no. 2 pp. 153-164. arXiv:1508.05461
2. Bounds on the expected size of the maximum agreement subtree (with Lam Si Tung Ho, Colby Long, Mike Steel, Katherine St. John and Seth Sullivant). *SIAM Journal on Discrete Mathematics.* **29** (2015) no. 4, pp. 2065-2074. arXiv:1411.7338
1. On three sets with nondecreasing diameter (with Carl Yerger and David J. Grynkiewicz). *Discrete Mathematics.* **338** (2015) no. 8, pp. 1328-1344. arXiv:1407.5122

TALKS

Invited conference talks

16. *Nonlinear algebra and matrix completion.* November 16, 2018. Nonlinear algebra in applications workshop at Brown University, ICERM.
15. *Using tropical geometry to characterize the algebraic matroid for rank-2 matrix completion.* July 13, 2018. SIAM Annual Meeting, Minisymposium on Distance Geometry. Portland, OR.
14. *Typical and generic ranks in low-rank matrix completion.* April 21, 2018. AMS Spring Eastern Sectional Meeting at Northeastern University. Special Session on Algebraic Statistics.
13. *Tropical linear spaces in phylogenetics.* September 23, 2017. AMS Fall Southeastern Sectional Meeting at University of Central Florida. Special Session on Mathematics of Biomolecules: Discrete, Algebraic, and Topological.
12. *Tropical Geometry for Rigidity Theory and Matrix Completion.* August 1, 2017. SIAM Conference on Applied Algebraic Geometry, Minisymposium on Algebraic Methods in Rigidity Theory. Atlanta, Georgia
11. *Tropical linear spaces in phylogenetics.* May 27, 2017. Interactions between algebra and the sciences. Max Planck Institute for Mathematics in the Sciences. Leipzig, Germany
10. *Combinatorial properties of hierarchical models.* July 11, 2016. SIAM Annual Meeting, minisymposium on algebraic statistics
9. *Toric Varieties in Statistics.* April 9, 2016. Meeting on Algebraic Geometry for Applications. Clemson University. Clemson, SC.
8. *Hierarchical Models: Normality and Related Properties.* Oct 3, 2015. 2015 AMS fall central sectional meeting, special session on algebraic statistics and its interactions with combinatorics, computation, and network science. Loyola University. Chicago, IL.

Seminar talks

7. Applied algebra and geometry seminar at Massachusetts Institute of Technology (Nov 21, 2017)
6. Algebra, geometry, and combinatorics seminar at San Francisco State University (Oct 19, 2016)
5. Combinatorics, algebra, convexity, algorithms and optimization seminar at UC Davis (Oct 17, 2016)
4. Graduate student algebra seminar, about once each semester

Contributed conference talks

3. *Unimodular Binary Hierarchical Models.* March 2, 2015. Forty-Sixth Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Florida Atlantic University, Boca Raton, FL.

2. *A Strictly Increasing Function with Derivative Zero Almost Everywhere.* Spring, 2013. MAA Southeastern Section Spring Meeting, Winthrop University, Rock Hill, SC.
1. *Data Clustering and Movie Recommendations.* Spring, 2012. MAA Southeastern Section Spring Meeting, Clayton State University, Morrow, GA.

CONFERENCE ORGANIZING

3. *AMS Special Session on Algebraic Statistics (a Mathematics Research Communities Session)* at the Joint Mathematics Meetings. Atlanta GA, January 5, 2017
2. *Second Triangle Area Math Graduate Conference.* Raleigh NC, October 24, 2015.
1. *First Triangle Area Math Graduate Conference.* Raleigh NC, March 21, 2015.

CONFERENCES AND WORKSHOPS ATTENDED

30. *Bi-annual algebraic and tropical meetings of Brown and Yale* at Yale (November 29, 2018)
29. *Applied Algebra Day* at Massachusetts Institute of Technology (November 17, 2018)
28. *Nonlinear Algebra in Applications* at Brown University, ICERM (November 12-16, 2018)
27. *Real Algebraic Geometry and Optimization* at Brown University, ICERM (October 15-19, 2018)
26. *Algebraic Geometry Northeastern Series* at Brown University (September 21-23, 2018)
25. *Core Computational Methods* at Brown University, ICERM (September 17-21, 2018)
24. *Nonlinear Algebra Bootcamp* at Brown University, ICERM (September 5-12, 2018)
23. *SIAM Annual Meeting* in Portland, OR (July 9-13, 2018)
22. *AMS Spring Eastern Sectional Meeting* at Northeastern University (April 21-22, 2018)
21. *AMS Fall Southeastern Sectional Meeting* at University of Central Florida (September 23-24, 2017)
20. *SIAM Conference on Applied Algebraic Geometry* at Georgia Tech (July 31-August 4, 2017)
19. *NSF/CBMS Conference on Tensors and Their Uses in Approximation Theory, Quantum Information Theory and Geometry* at Auburn University (July 24-28, 2017)
18. *Algebraic and Combinatorial Phylogenetics* at Barcelona Graduate School of Mathematics (June 26-30, 2017)
17. *Interactions Between Algebra and the Sciences* at Max Planck Institute for Mathematics in the Sciences, Leipzig, Saxony, Germany (June 27, 2017)
16. *Computing in Tropical Geometry* at Zuse Institute Berlin (May 11-12, 2017)
15. *Joint Mathematics Meetings* (January 4 - 7, 2017)
14. *MSRI Summer School on Tropical Curves and Chip Firing* (July 25 - August 5, 2016).
13. *SIAM Annual Meeting* in Boston, MA (July 11 - 15, 2016).
12. *Summer School on Algebra, Statistics, and Combinatorics* at Aalto University, Helsinki, Finland (June 27 - July 2, 2016).
11. AMS *Mathematical Research Communities in Algebraic Statistics*, Snowbird UT (June 12-18, 2016).
10. *Meeting on Algebraic Geometry for Applications* at Clemson University, (April 9, 2016).
9. *AMS Fall Central Sectional Meeting* at Loyola University Chicago. (October 3-4, 2015).
8. Macaulay2 Workshop at Boise State University (May 27-30,2015)
7. *Forty-Sixth Southeastern International Conference on Combinatorics, Graph Theory, and Computing* at Florida Atlantic University (March 2-6, 2015)
6. *NSF/CBMS Conference on Mathematical Phylogeny* at Winthrop University (June 28 - July 22, 2014)
5. *Algebraic Statistics* at Illinois Institute of Technology (May 19-22, 2014)
4. *Joint Mathematics Meetings* (Jan 15-18, 2014)
3. *MAA Southeastern Section Spring Meeting* at Winthrop University (Spring 2013)
2. *MAA Southeastern Section Spring Meeting* at Clayton State University (Spring 2012)
1. Workshop in Epidemic Models at SAMSI and NCSU (May 16 - 20, 2011)

JOURNALS REFEREED

2. Forum of Mathematics, Sigma
1. Journal of Combinatorial Theory, Series B

OTHER PROFESSIONAL SERVICE AND OUTREACH

Brown University, Institute for Computational and Experimental Research in Mathematics

1. Co-organizer (with Greg Blekherman and Rainer Sinn) of a working group in matrix completion (Fall 2018)

North Carolina State University Math Department

5. Judge at the MAA poster session for undergraduate research at the 2017 JMM (January 6, 2017)
4. Co-organizer (with Emily Barnard) of NCSU's Graduate Student Algebra and Combinatorics Seminar (Fall 2014 - Spring 2017)
3. *Math Circle* (assistant), May 3, 2014
2. *AMS Graduate Student Chapter*, Fall 2013 - Spring 2016
   -Organize professional development and networking events for graduate students including a semesterly conference (see conference organizing)
   -President: Fall 2014 - Spring 2016, Treasurer: Fall 2013 - Fall 2014
1. *Math Doesn't Bug Me Team* (volunteer), Fall 2013 - Present
   -Outreach events for children in elementary and middle school

Davidson College Math Department

1. *Bernard Society* (officer), Fall 2012 - Spring 2013
   -Organize departmental events

TEACHING EXPERIENCE

North Carolina State University Mathematics Department

3. *Calculus III* (Main Instructor): Spring 2016, Fall 2016, Spring 2017, Fall 2017
2. *Calculus for Life and Management Sciences* (Recitation Leader): Spring 2014 and Fall 2014
1. *Applied Differential Equations* (Lecture Assistant): Fall 2013

Davidson College

2. *Center for Teaching and Learning* (math and computer science tutor), Fall 2011 - Spring 2013
1. *Programming and Problem Solving* (homework grader), Fall 2010

PROFESSIONAL DEVELOPMENT FOR TEACHING

2. NCSU graduate student teaching assistant spring workshop (May 7, 2014)
1. NCSU graduate student teaching assistant fall workshop (August 14-18, 2014)

MEMBERSHIPS

2. American Mathematical Society (AMS)
1. Society for Industrial and Applied Mathematics (SIAM)

PROGRAMMING LANGUAGES

Proficient in Java, C, C++, R, Python, Sage, Macaulay2, Mathematica

REFERENCES

Seth Sullivant (Ph.D. Advisor)
Professor of Mathematics
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Jesús De Loera
Professor of Mathematics
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North Carolina State University is a land-grant university and a constituent institution of the University of North Carolina

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September 15, 2017

To the Hiring Committee,

It is a pleasure to write this letter of recommendation for **Daniel Bernstein** who has applied for a position in your department. In short, I give Daniel my highest recommendation: he has an extensive research record in algebraic statistics, has been incredibly independent throughout his Ph.D., has a commitment to service in the mathematical community, and is a passionate teacher.

Daniel is my Ph.D. student; he is by far the strongest student at NCSU I have interacting with in the 9 years I have been here. Daniel is producing research in applied algebraic geometry comparable to the best students and postdocs in this area. Daniel's research area is algebraic statistics, which uses tools from algebraic geometry, commutative algebra, and combinatorics to solve problems in statistics and its applications. Since coming to NCSU he has worked in three subareas of algebraic statistics leading to seven publications with more papers on the way. While motivated by algebraic statistics, the mathematical tools that Daniel employs comes from a wide range of areas including matroid theory, tropical geometry, optimization, and real algebraic geometry.

Research Summary: When Daniel first started working with me, I gave him the problem of classifying which hierarchical models are normal. This normality condition is important for constructing Markov bases for random walks on contingency tables, but is also driven by motivating problems in optimization and statistical disclosure limitation. For each simplicial complex Γ and positive integer vector $d = (d_1, \ldots, d_n)$ is a matrix/vector configuration/toric variety, whose combinatorial properties are intimately connected to statistical properties of the model. We undertook a computational study of the normality property, which was in our joint paper "Normal binary hierarchical models", with appeared in *Experimental Mathematics*. This first foray led us to realize that to really understand the normality property we would first need to understand when the associated matrix is unimodular. (Unimodularity means that all associated integer programs can be solved efficiently with linear program.) This turned out to be a majorly challenging problem: through a difficult 28 page proof we managed to give a complete characterization of the simplicial complexes for which there is a d that yields a unimodular matrix. This appears in "Unimodular binary hierarchical models" in *J. Combinatorial Theory: Series B* and is a major work of structural combinatorics. Daniel really impressed me with his ability to master extremely technical complex arguments: it was he who produced the ultimate complex induction on which the whole proof rests. He also discovered the key idea that unimodularity should be preserved under taking the vertex link of a simplicial complex. Daniel in joint work the Chris O'Neill ("Unimodular hierarchical models and their Graver bases") extended these results to give a complete characterization of all pairs (Γ, d)

that give a unimodular configuration. While these results are technical and somewhat specialized, they really show Daniel's abilities to dig deep and persevere on a hard problem.

The next area Daniel has been involved in concerns combinatorial problems in mathematical phylogenetics. This area of mathematical biology is concern with reconstructing evolutionary histories of collections of species, which are typically represented by trees. Daniel was involved in a joint project with 5 coauthors on the shapes of random trees ("Bounds on the expected size of the maximum agreement subtree") that is leading to a wealth of new research. He also did some beautiful work with Colby Long on L^∞ optimization for phylogenetic tree reconstruction. Among all the different optimization criteria that can be used to reconstruct a phylogenetic tree, this is the only instance where there is a polynomial time algorithm. However, the geometry of the problem is complex, as there usually is not a unique optimum: the set of optima forms a tropical polytope. Daniel study of the geometry of these polytopes, and how to compute them efficiently ("L-infinity optimization to linear spaces and phylogenetic trees"). Along the way, he also gave a detailed study of the L^∞ optimization to a linear space. One beautiful result that Daniel showed is that the geometry of the set of closest points to a linear space is completely governed by the matroid of that linear space. Daniel also extended these ideas from phylogenetic tree space to arbitrary tropical linear spaces ("L-Infinity optimization to Bergman fans of matroids with an application to phylogenetics").

I think that Daniel's best result to date is in his beautiful solo-authored paper "Completion of tree metrics and rank 2 matrices". Matrix completion problems play an important role in the sciences, where one takes a partially observed matrix and tries to complete it to a low rank matrix. The problem has a potentially rich combinatorial side, but Daniel has been the only researcher to crack this combinatorial approach with his major result that gives a combinatorial description of the bases of the rank 2 completion matroid. The result is a masterful combination of tools from tropical geometry, combinatorial phylogenetics, and matroid theory. This result plays an analogous role in matrix completion as Laman's Theorem does in rigidity, giving a complete characterization in the 2 dimensional case. What's even more impressive is how Daniel's result suggests a range of new problems to explore in the area, developing and extending the techniques he created for the rank 2 case to higher rank. He has proposed a range of problems to pursue in this area, including connections to rigidity theory, and I am excited to see where these techniques will take him.

Teaching and Service: Daniel also has a significant commitment to teaching and to service to the mathematical community. He already organized an AMS Special Session, he organized a graduate student seminar at NCSU, he served as an officer in the graduate student AMS group at NCSU, and he helped create and organize two graduate student conferences in the Raleigh/Durham area. He has been involved in various mathematical outreach activities in the community. He is passionate about teaching and does a great job at it. Daniel is an exceptionally well-rounded mathematician.

Summary: Daniel is an exceptional mathematician, well above any student I've interacted with at NCSU. Starting in his third year of graduate study, Daniel has been pursuing his own research agenda: he is the most independent graduate student I have supervised and more so than many postdocs I have known. He is at the top among current graduating students in applied algebraic

geometry or applied discrete math. I would put him in the same class as Elina Robeva (current NSF Postdoc at MIT) or Luke Oeding (former NSF Postdoc, now faculty at Auburn U.) at a similar career stage. I give Daniel my strongest recommendation for a position in your department.

Sincerely,

Seth Sullivant
Professor of Mathematics
North Carolina State University
Fellow of the AMS