Application for Applied Category Theory 2019

Applicant Details

Name	Thomas C. Fraser
Supervisor	Robert W. Spekkens
PhD Degree	Quantum Information & Foundations [Sept. 2018 – Aug. 2020]
Location	Perimeter Institute, Waterloo, Ontario, Canada
Financial Commitment	If funding is unavailable, I can fully fund my participation using alternative sources.

Project Preference Order

1. Simplifying Quantum Circuits Using the ZX-Calculus with M. Backens
2. Partial Evaluataions, the Bar Construction, and Second-Order Stochastic Dominance with T. Fritz
3. Toward a Mathematical Foundation for Autopoiesis with D. Spivak
4. Formal and Experimental Methods to Reason About Dialogue and Discourse Using Categorical Models of Vector Spaces with M. Sadrzadeh
5. Complexity Classes, Computation, and Turing Categories with P. Hofstra
6. Traversal Optics and Profunctors with B. Milewski

My Participation in the ACT2019 School

Although, as I hope to illustrate below, I have a strong background and research interest in the project proposed by Miriam Backens, the primary reason why I wish to attend this research school is to understand the challenges faced by applied category theorists in other disciplines and to study the problem solving strategies they employ. I strongly believe category theory provides a framework in which concepts and solutions from one field can be translated into solutions for another, and more importantly, that these translations are discovered during research programs like ACT2019.

Personally, I view this research school as an opportunity to foster collaborations and to develop ongoing correspondences with the greater applied category theory community. Provided I am presently at the early stages of my researcher career, I can foresee that ACT2019 will have a significant and positive impact on my incipient motivations and perspectives.

Finally, if funding is unavailable for me to attend the conference at Oxford University in July, I have already confirmed the financial support of my supervisor and home institution.

General Background

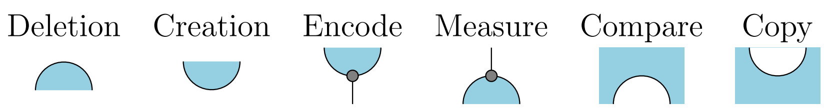
My first introduction to and appreciation for category theory came from reading *Physics, Topology, Logic and Computation: A Rosetta Stone* [BS10] by Baez and Stay. As a student of mathematical physics, the Rosetta Stone eloquently organized and cataloged my understanding of physics; a group G became a category on one object with invertible morphisms, representations of G became functors $F, F' : G \to \mathrm{Hilb}$ and interwiners made sense as a natural transformation $\alpha : F \Rightarrow F'$ between representations. More importantly, the Rosetta Stone revealed to me analogous structures in functional programming and topology which once seemed disconnected. My desire to understand Lane's proof [Lan63] that all monoidal categories are equivalent to one wherein the associator and units are taken as identities naturally lead to my interest in weak n-categories wherein coherence laws of order m are held by morphisms of order $m+1$, and in particular, their application to theoretical physics [BL09]. Although most of my research is concerned with monoidal categories that admit graphical calculi [Sel10], specifically dagger compact closed categories for which finite dimensional Hilbert spaces are complete [Sel12], I have a basic understanding of applications to linguistics [Lam58] and have begun reading about intuitionistic logic as an internal logic of topos theory [MM12].

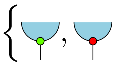
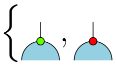
In the summer of 2018, I began a PhD in Quantum Information and Quantum Foundations under the supervision of Rob Spekkens at the Perimeter Institute in Canada. In broad strokes, my research objective is to understand and clarify the differences (and similarities) between classical and quantum physics in terms of their computational resources and causal structure.

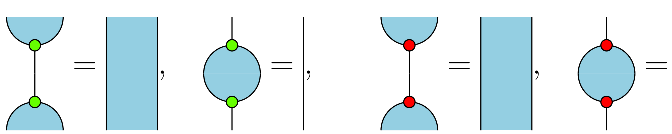
Specific Background for The ZX-Calculus Project

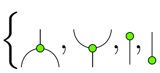
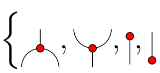
Recently, I began working on a research project which aims to translate the ZX-calculus of Coecke and Duncan [CD11] into the language of 2-categories. Unlike the work of Cicala [Cic17] wherein the utility of 2-categories is leveraged to treat the rewrite rules of the ZX-calculus internally, my approach is analogous to that of Vicary [Vic12] wherein classical information, quantum systems and their interactions are represented using 0-,1-, and 2-cells respectively. If successful, it is my intention to study the normal forms the graphical calculi admits and to develop a complete graphical calculi for pure qubit quantum theory.

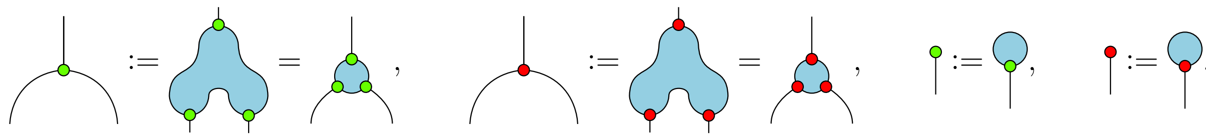
As already articulated by Vicary [Vic12], there are conceptual and algebraic advantages to a 2-categorical approach. For example, in this language the elementary 2-cells admit an operational interpretation in terms of the classical information "stored" in the shaded regions:



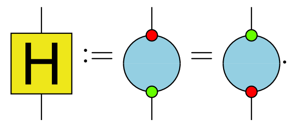
Given a pair of colored 2-cells $\Big\{$$\Big\}$ and their respective adjoints $\Big\{$$\Big\}$ which are vertical unitaries,



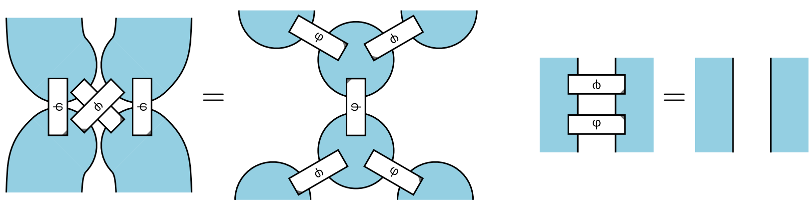
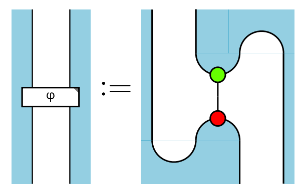
it is known that $= \Big\{$$\Big\}$ and  $= \Big\{$$\Big\}$ each form commutative †-Frobenius algebras[1] where,



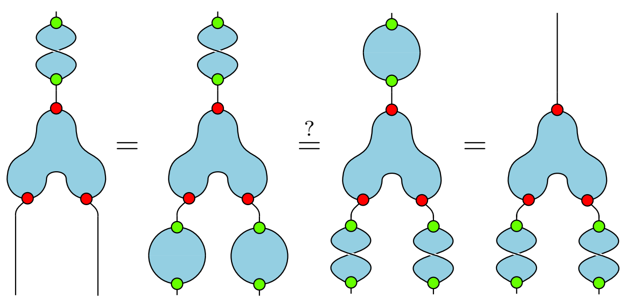
Additionally, all of the rules of the ZX-calculus involving the Hadamard follow from the following,



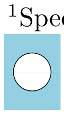
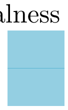
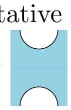
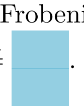
In the ZX-calculus, $\{{\color{green}\bullet},{\color{red}\bullet}\}$ also form a bialgebra. This is accomplished if and only if the following (and its adjoint) holds,

 where 

Translating the remaining rules of the ZX calculus, which involve decorating $\{{\color{green}\bullet},{\color{red}\bullet}\}$ with angular values $\alpha \in [0, 2\pi)$, is an ongoing research effort of mine.



Finally, in 2018 I attended the Perimeter Institute's winter school. During this research school, myself and three other graduate students studied the connection between models of quantum computing, their efficient simulation, quantum contextuality, and the stabilizer subtheory [How+14; KWB18; Got98; JM08]. Perhaps the failure of the ZX-calculus to be complete for the entirety of pure qubit quantum theory is related to the conextuality thereof, and that category theory can illuminate this connection.

[1] Specialness of the commutative †-Frobenius algebras requires that the *copy* and *compare* 2-cells compose as  =  but note that  $\neq$ .

References

[BL09] John C Baez and Aaron Lauda. "A prehistory of n-categorical physics". In: *arXiv preprint arXiv:0908.2469* (2009).

[BS10] John Baez and Mike Stay. "Physics, topology, logic and computation: a Rosetta Stone". In: *New structures for physics*. Springer, 2010, pp. 95–172.

[CD11] Bob Coecke and Ross Duncan. "Interacting quantum observables: categorical algebra and diagrammatics". In: *New Journal of Physics* 13.4 (2011), p. 043016.

[Cic17] Daniel Cicala. "Categorifying the ZX-calculus". In: *arXiv preprint arXiv:1704.07034* (2017).

[Got98] Daniel Gottesman. "The Heisenberg representation of quantum computers". In: *arXiv preprint quant-ph/9807006* (1998).

[How+14] Mark Howard et al. "Contextuality supplies the 'magic'for quantum computation". In: *Nature* 510.7505 (2014), p. 351.

[JM08] Richard Jozsa and Akimasa Miyake. "Matchgates and classical simulation of quantum circuits". In: *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*. Vol. 464. 2100. The Royal Society. 2008, pp. 3089–3106.

[KWB18] Angela Karanjai, Joel J Wallman, and Stephen D Bartlett. "Contextuality bounds the efficiency of classical simulation of quantum processes". In: *arXiv preprint arXiv:1802.07744* (2018).

[Lam58] Joachim Lambek. "The mathematics of sentence structure". In: *The American Mathematical Monthly* 65.3 (1958), pp. 154–170.

[Lan63] Saunders Mac Lane. "Natural associativity and commutativity". In: *Rice University Studies* (1963).

[MM12] Saunders MacLane and Ieke Moerdijk. *Sheaves in geometry and logic: A first introduction to topos theory*. Springer Science & Business Media, 2012.

[Sel10] Peter Selinger. "A survey of graphical languages for monoidal categories". In: *New structures for physics*. Springer, 2010, pp. 289–355.

[Sel12] Peter Selinger. "Finite dimensional Hilbert spaces are complete for dagger compact closed categories". In: *arXiv preprint arXiv:1207.6972* (2012).

[Vic12] Jamie Vicary. "Higher quantum theory". In: *arXiv preprint arXiv:1207.4563* (2012).

Thomas C. Fraser

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Education

2018 – Present **Ph.D., Quantum Information**
Perimeter Institute For Theoretical Physics

2017 – 2018 **M.Sc., Theoretical Physics, Quantum Information**
Thesis: A Combinatorial Approach To Causal Inference
Perimeter Scholars International Program, Perimeter Institute For Theoretical Physics

2012 – 2017 **B.Sc., Mathematical Physics, Astrophysics Specialization**
Cumulative Average: **97.79%**
University of Waterloo, Waterloo, ON

2008 – 2012 **High School Diploma**
Specialist High Skills Major (SHSM) in Energy
Renfrew Collegiate Institute, Renfrew, ON

Awards & Scholarships

2018 **UW Graduate Award & Marie Curie Award**
Sufficiently High Entrance Average, Department Selected

2018 **The Joanne Cuthbertson and Charlie Fischer Graduate Student Award**
Support for an exceptional Masters student from the 2017/2018 PSI class

2017 **Governor General's Silver Academic Medallion**
Highest Graduating Average in Graduating Class

2016 **Mike Lazaridis Scholarship**
Theoretical Physics Fellowship at Perimeter Institute

2015 **Xerox Research Centre of Canada Limited Award**
Best Work-term Report "Acoustic Modelling Using Mel-Frequency Cepstral Coefficients"

2015 **C. C. Lim Physics Prize**
Top Marks in Undergraduate Thermodynamics

2013 **Don E. Brodie Scholarship**
Highest Experimental Physics Lab Performance

2012 **A. Donald Maynes Scholarship**
Outstanding Academic Record

2012 **BMO Undergraduate Entrance Scholarship**
Outstanding Academic Average

2012 – 2017 **Dean's Honour List**
Academic Performance

2012 **President's Distinction Scholarship**
Entrance Average

2012 **Governor General's Bronze Academic Medallion**
Highest Graduating Average in Graduating Class

Academic Works

Published

April 2018	*Machine Learning Peeling and Loss Modelling of Time-Domain Reflectometry* J. R. Rinehart, J. H. Béjanin, T. C. Fraser, M. Mariantoni
September 2017	*Causal Compatibility Inequalities Admitting of Quantum Violations in the Triangle Scenario* T. Fraser, Elie Wolfe

Invited Talks At Conferences

November 2016	Quantum Networks Conference at International Institute for Physics, Natal, Brazil *Causal Compatibility Inequalities Admitting of Quantum Violations in the Triangle Scenario*

Course Notes

Winter 2016	*General Relativity*
Winter 2016	*Statistical Mechanics*
Fall 2016	*Applied Probability*
Fall 2016	*Quantum Physics 3*
Fall 2016	*Electricity & Magnetism 3*
Fall 2016	*Cosmology*

Project Papers

April 2016	*Variations in Stellar Metallicity* T. Fraser
January 2016	*Acoustic Modelling Using Mel-Frequency Cepstral Coefficients* T. Fraser

Computational Skills

Languages C, C++, Python, Mathematica, Matlab, Java, Actionscript, JavaScript

Research & Work Experiences

Mike Lazaridis Fellow

Perimeter Institute For Theoretical Physics. Waterloo, ON — May 2016 – September 2016

Research in quantum foundations studying quantum non-locality from the perspective of causal inference. Discovered new causal compatibility inequalities leading to a better understanding of quantum information resources. Computationally simulated six-entangled qubits and associated measurements to find new entanglement resources. Invented new computational techniques for solving the marginal satisfiability problem capable of out-performing existing methods when large computational networks are required.

Research & Development Data Scientist

Sysomos. Toronto, ON — September 2015 – January 2016

Industry application of varied machine learning methods. Designed algorithms to perform automatic speech recognition on digital video extracted from Twitter. Implemented advanced signal processing techniques to perform acoustic modelling. Worked with a massive parallel computing architecture to process billions of data sources. Designed and built native Android & iOS apps from scratch. Culminated in award winning paper.

Video Game Developer

Lunarch Studios. Waterloo, ON — September 2014 – May 2015

Built an highly-compatible graphics engine that supports dynamic assets loaded asynchronously. Acted as project manager to complete large-scale, internal projects. Developed a highly scalable server platform with integration between multiple software languages. Researched and implemented numerous bin-packing algorithms in order to optimize texture loading performance.

Mathematics Tutor

Humber College. Toronto, ON — January 2014 – May 2014

Tutored thousands of students one-on-one in fields such as statistics, technical math, engineering, biomechanics, and business. Lead an initiative to write and produce high quality educational videos to help students with their studies. Developed a multi-platform, browser-based student sign-in system in order to collect meaningful statistics to improve effectiveness of math centre. Designed and produced graphic art to promote and develop a mathematics community.

Solar Panel Technician

OVG Solar, Inc. Renfrew, ON — June 2011 – August 2011

Industry level experience engineering, assembling and maintaining numerous solar panel arrays. Worked in a team of carpenters, electricians and skilled engineers under flexible hours across all of eastern Ontario.