Name: Toby Smithe

Expected PhD completion & topic: 2021.
I study the computational principles and mathematical structures underlying the brain's ability to construct and act upon a model of its environment (an interactive process broadly called *active inference*), and I build self-organising models of hippocampal circuits instantiating these principles.
(Please see my CV for a fuller description.)

Background: I am a mathematical neuroscientist with an interdisciplinary complex systems background (see CV). Whilst trying to understand how neural activity could exhibit compositionality [1, 2] and give rise to quantum-like behaviour in language and cognition [3], I realised that category theory will be the lingua franca of 21$^{\text{st}}$-century science, particularly of complex interacting systems. I attended the 'categorical quantum mechanics' course in Oxford, which introduced me to dagger compact categories for modelling quantum protocols, language and cognition (Abramsky, Coecke *et al.*; example references: [4–8]). I then performed an extensive survey of categorical topics relevant to my research: basic category theory (from Leinster's recent book [9]) and higher category theory [10]; effectus theory (from Jacobs *et al.*; [11–14]); differential linear logic (from Blute, Ehrhard, Murfet; [15–18]); coalgebraic descriptions of dynamics and computation (Jacobs, Pavlovic; [19, 20]); and operadic descriptions of open dynamical systems (Baez, Fong, Lerman, Sobocinski, Spivak; [21–25]). I also attended the 'Open Games' workshop at FLOC 2018[1]. I am now working on a synthesis of these ideas to elucidate the formal structure of computational neuroscience, particularly where it relates to my DPhil project, and which is fundamentally related to autopoiesis.

Project preference (high to low): Spivak (very major preference), Sadrzadeh (minor preference); rest equal.

Oxford availability: I live in Oxford and am available in mid-July.

Statement:
Computational neuroscience has a problem: no-one can define neural computation. This is central to a fundamental divide between two predominant modelling approaches. On one side, the "biophysical" approach aims for faithfulness with low-level electrophysiological data, but makes only qualitative claims about cognitive function. On the other, the "computational" approach derives from statistical models in cognitive science and machine learning – often to interpret high-level (*eg.*, fMRI or behavioural) data – typically making strong claims about the computations neural systems 'should' perform, without suggesting biologically plausible mechanisms. Neuroscience lacks the formal language necessary to bridge this divide; category theory – by emphasising composition, relationships, and interaction – provides the missing tools.

In the computational tradition, *active inference* ideas [27] are essential to both autopoiesis [28, 29] and my research. These models start from the good regulator theorem [30] that evolutionarily successful agents must embody accurate models of their environments[2]: they can update their internal expectations through perception, and, dually, modify external states through action. These ideas have a beautiful information-geometric interpretation in variational inference [31, 32], with the same form whether applied to single cells [33], agents with complex brains [34], or entire populations (such as species or organizations) [35].

Life being essentially autopoietic, corresponding mechanisms must obtain. Yet attempts to relate these models plausibly to neural quantities have failed, beholden to unidiomatic assumptions [31, 36] that ignore the 'operadic' structure of the brain [37]. My research takes a new approach by stripping back both "computational" and "biophysical" models, applying categorical open-systems and effectus-theoretic tools to clarify the fundamental structures and elucdiate the adjunctions that span the divide. I am applying to the School to learn from and share with the wider community, to ensure that my work coheres with the nascent study of autopoietic systems more generally.

Science – itself the expression of autopoiesis at our species' level – is moving away from specific objects of study, towards their common relationships and universal properties: a transition naturally framed in the austere language of category theory. Indeed, solving the problems above implies finding a "coalgebraic" Rosetta Stone [38] for biology, geometry, information and dynamics.

PS: Since 2017, I have even owned the `autopoies.is` domain name, and would be glad to host relevant resources there.

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[1] So I note that there is probably a structural connection between open games and autopoiesis (perhaps along the lines of Bolt et al. [26])...

[2] That is, agents obey an information-theoretic least-action principle to keep their expectations aligned with reality [31]; note that "environment" here also self-referentially includes hidden internal states, so *least-action entails homeostasis*.

Bibliography

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[2] Brenden M. Lake et al. “Building machines that learn and think like people”. In: *Behavioral and Brain Sciences* 40 (2017), e253.

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[31] Christopher L Buckley et al. “The free energy principle for action and perception: A mathematical review”. In: *Journal of Mathematical Psychology* 81 (2017), pp. 55–79.

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Biological systems are inherently autopoietic, so some mechanism must underlie this description. Yet attempts to relate these models to neural quantities have invariably failed, forced into implausible assumptions by ignoring the 'operadic' structure of the brain [36]. I aim to initiate a new approach by stripping back both kinds of model[3], applying categorical open-systems and effectus-theoretic tools to clarify the fundamental structures and elucdiate the adjunctions that span the divide. I am applying to the School to learn from and share with the wider community, to ensure that my work is commensurate with the nascent study of autopoietic systems more generally.

Science, itself the expression of autopoiesis at our species' level, is moving away from specific objects of study, towards their common relationships and universal properties: the austere language of category theory is perfect for framing this transition, forcing a distinction between necessary and contingent, thus unifying the present patchwork of paradigms.

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[3] Top-down "computational", and bottom-up "biophysical".

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Toby St Clere Smithe

St Edmund Hall, Queen's Lane, Oxford, OX1 4AR

toby.smithe@psy.ox.ac.uk

Research and Education

9 Oct 2016 – **DPhil Theoretical Neuroscience**
University of Oxford

Thesis: **Neural circuits for navigating a structured environment**
Supervisors : Simon Stringer, Mark Buckley
Department of Experimental Psychology, University of Oxford

I build simple but biologically plausible models of neural circuits implicated in spatial navigation and memory, centred on the hippocampus, with a view to answering the question: how does the brain learn to represent the structure of the world, and act accordingly? The core of the project is a synthesis of classical Hebbian models with modern active inference methods, driven by the development of a new category-theoretic framework for computational neuroscience: just as our environment is hierarchical and compositional, so is the brain, and so should be our models of its circuits and representations. This part of the project frames active inference in open systems terms, and the neural circuit realisation thereof as maps in an associated effectus.

1 Sept 2015 – 31 July 2016 **MSc Complex Systems Science (Erasmus Mundus) (Year 2)**
École Polytechnique, Paris, France

Thesis: **Learning nonlinear dynamics with balanced spiking networks**
Supervisor : Sophie Denève, *Group for Neural Theory*
Laboratoire de Neurosciences Cognitives, École Normale Supérieure, Paris

Selected modules:
Mathematics, Vision and Learning (MVA) Masters, École Normale Supérieure, Cachan
- Probabilistic graphical models, including small project on nonlinear dynamical systems.

CogMaster Programme, École Normale Supérieure, Paris
- Theoretical neuroscience; neuroscience of perception, action & decision-making.

1 Sept 2014 – 31 Aug 2015 **MSc Complex Systems Science (Erasmus Mundus) (Year 1)**
Chalmers University, Gothenburg, Sweden

Thesis: **Investigations into nanochannel-confined DNA in two regimes**
Supervisor: Bernhard Mehlig, *Department of Physics, University of Gothenburg*

Selected modules:
- Computational and mathematical biology; bioinformatics.
- Neural networks, stochastic optimization; information theory for complex systems.

5 Oct 2013 – 11 Jul 2015 **BSc (Hons) Open (Mathematics) (2.i)**
Open University, UK

Selected modules:

- Mathematical statistics; optimization; calculus of variations; groups and geometry.

1 Oct 2010 – 30 Jun 2013 **BA (Hons) Psychology, Philosophy and Physiology (2.i)**
University of Oxford, UK

Publications and Talks

Journal Articles:

St Clere Smithe, T & Stringer, S.M. (2019). A common mechanism accounts for the differential development of place and head-direction cells. To be submitted to *Frontiers in Computational Neuroscience*.

St Clere Smithe, T. (2019). Chaos and computation in the cortex: how complexity makes structure seem random. To be submitted to *PLoS Computational Biology*.

Werner, E., Jain, A., Muralidhar, A., Frykholm, K., St Clere Smithe, T., Fritzsche, J., Westerlund, F., Dorfman, K. D., & Mehlig, B. (2018). Emergence of hairpins in the conformations of a confined polymer. *Biomicrofluidics 12, 024105*. arXiv:1611.05736.

St Clere Smithe, T., Iarko, V., Muralidhar, A., Werner, E., Dorfman, K. D., & Mehlig, B. (2015). Finite-size corrections for confined polymers in the extended de Gennes regime. *Physical Review E, 92(6), 062601.* arXiv:1510.03195.

Conference Talk:

[8 July 2014] T. St Clere Smithe: PyViennaCL – Very easy GPGPU linear algebra. SciPy 2014, Austin, Texas.

Conference Poster:

[13 October 2016] W. Lavrijsen, T. St Clere Smithe: Pythonization API for Cppyy. CHEP 2016, San Francisco, California.

Software Development

26 June 2017 – **GPGPU acceleration for biological neural networks**
Codeplay Software Ltd (part-time)

25 May 2015 – 21 Aug 2015 **ROOT and cppyy, Google "Summer of Code" sponsorship**
Software for Experiments Group, CERN, Switzerland

19 May 2014 – 18 Aug 2014 **PyViennaCL, "Google Summer of Code" sponsorships**
17 June 2013 – 23 Sept 2013 *Institute for Analysis and Scientific Computing*
Vienna University of Technology, Austria

This is a brief letter of recommendation for Toby St Clere Smithe.

I am an applied category theorist working within the Quantum Group at the University of Oxford, and have had several discussions with Toby regarding his work.

Toby is a DPhil. student in theoretical neuroscience at the University of Oxford. Toby identified a connection between the mathematical structures used in Categorical Quantum Mechanics, as developed in the Quantum Group of the department of Computer Science, and structures he anticipated would arise in mathematical models of the brain. Based on this observation, he took the initiative and approach members of our group, including myself, and we have been in regular discussion about his ideas. Toby has been actively exploring various categorical models of composition and there potential use in his work. He is very mathematically capable, and has a good understanding of categorical concepts, mainly via self study.

I believe that the ACT school would be highly relevant to Toby's work. He has a concrete application area in neuroscience, and has actively engaged with members of the applied category theory community to begin establishing a composition foundation for his work. As an applicant from the broader sciences, with an interesting multi-disciplinary background, I think he is exactly the type of student the school should be encouraging, and recommend him strongly for a place.

Regards

Dan Marsden