Application Applied Category Theory 2019

J. Scott Carter

1. Relevant Background

My 1982, Yale University PhD dissertation represents stable homotopy classes by codimension 1 immersions without 1-dimensional multiple points when the Kervaire invariant vanishes.

Since then I have been a pioneer in developing diagrammatic techniques to depict higher dimensional phenomena. In particular, Masahico Saito and I established the movie move technique for representing knotted surfaces in 4-dimenisonal space. Daniel Jelsovsky, Seiichi Kamada, Laurel Langford, Masahico Saito, and I developed quandle cocycle invariants for knotted surfaces. The movie moves inspired a lot of subsequent foundational work in higher categories that included the formulation of the category of 2-tangles as a free braided monoidal 2-category [Baez-Langford]. Additional adjectives need to be inserted to make that statement precise. I think it is reasonable to say that the movie move results inspired the tangle hypothesis.

I have worked with categorical tools such as GLOBULAR.SCIENCE and I am looking forward to implementing example computations in HOMOTOPY.IO.

The order of my preferences for these mini-courses is as follows:

(1) Backens, *Simplifying quantum circuits using the ZX-calculus*;
(2) Fritz, *Partial evaluations, the bar construction, and second-order stochastic dominance*,
(3) Spivak, *Toward a mathematical foundation for autopoiesis.*

My plan is to be in the United Kingdom during the first week of July 2019. I hope to extend that trip through other personal and professional visits through the summer school. With housing support, I would be able to attend the school. I will seek other sources of support.

2. Statement on contributions of the school to my career

I hope to learn more about the language of diagrams and their metaphors.

In retirement, I plan to dedicate more of my time to learning and writing. My next project, in collaboration with Seiichi Kamada, is a book on "Diagrammatic Algebra." I have a provincial understanding of this subject, and I wish to broaden my horizons. Any examples that I learn from this course will be incorporated into our manuscript.

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Identities among morphisms give rise to higher morphisms. My purpose is to understand the relations among identities. At the most primitive levels of diagrammatics, there are trivalent junctions Y and ⅄, crossings X, cups and caps U and ∩, and identities I. Tensorial methods combine these to make grammatical strings, and when those strings represent isomorphic morphisms additional relations arise. Further higher categorical structures occur. Such ideas lead to the notion of knotted foams in 4-dimensions and higher that are the analogue of knotted trivalent graphs in 3-dimensions.

Among my research goals is the development of such a theory. The singular sets of foams give abstract tensor formulations for analogues of the Yang-Baxter relations and the Elliot-Biedenharn relations. Work on homology theory that is related to quandles (self-distributive structures) indicates that solutions to these equations are, at least in a formal setting, easy to come by.

I hope to learn and apply concepts from the workshop in different diagrammatic contexts. A colleague asked me once if I thought at all about quantum gravity. My answer was (paraphrase), "No, not in particular, but I expect that one day, I will look up from the Feynman-like diagrams that I draw and the relations among these, and hope to see that the theory coincides with a physically meaningful model." Just as the same twelve notes are used to create the world's music, the same few diagrams may give rise to metaphors among physical and linguistic theories.

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Curriculum Vita
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Education

B.S. March 1978, Summa Cum Laude, University of Georgia, Athens, Georgia, Mathematics Major.

Ph.D. in Mathematics, May 1982, Yale University, New Haven, Connecticut, Dissertation: "Surgery on Immersions: a Geometric Approach to Stable Homotopy," Advisor Prof. Ronnie Lee.

Positions Held

Dec. 7, 2018 - Present: Professor Emeritus, University of South Alabama, Department of Mathematics and Statistics, Mobile, Alabama.
Oct. 5, 2010 - June 1, 2018: Professor, University of South Alabama, Department of Mathematics and Statistics, Mobile, Alabama.
Jan. 2003 - Oct. 5, 2010: Chair, University of South Alabama, Department of Mathematics and Statistics, Mobile, Alabama.
May 2002 - Dec. 2002: Interim Chair, University of South Alabama, Department of Mathematics and Statistics, Mobile, Alabama.
Sept. 1996 - May 2002: Professor, University of South Alabama, Department of Mathematics and Statistics, Mobile, Alabama.
Aug. 1989 - May 1996: Assistant Professor or Associate Professor, University of South Alabama, Department of Mathematics and Statistics, Mobile, Alabama.
Aug. 1988 - June 1989: Lecturer, Wayne State University, Department of Mathematics, Detroit, Michigan.
Aug. 1987 - June 1988: Lecturer, University of Texas, Austin, Texas.
Aug. 1985 - June 1987: Assistant Professor, Department of Mathematics and Computer Studies, Lake Forest College, Lake Forest, Illinois.

Aug. 1982 - June 1985: Instructor, Mathematics Department, University of Texas, Austin, Texas.
1978 -1982: Teaching Assistant, Mathematics Department, Yale University, New Haven, Connecticut.

Publications

Journal Articles

1. *Surgery Theory of Immersions*, Proc. Northwestern Homotopy Theory Conf. (Miller and Priddy, eds.), AMS Contemp. Math Series 19 (1983), 23-37.

2. *Surgery on Codimension One Immersions in (n+1)-space: Removing n-tuple Points*, Trans. of the AMS 289, No. 1, (1986), 83-102.

3. *On Generalizing Boy's Surface: Constructing a Generator of the Third Stable Stem*, Trans. of the AMS 289, No. 1 (1986), 103-121.

4. *A Further Generalization of Boy's Surface*, Houston Journal of Mathematics 12, No. 1 (1986), 11-31.

5. *Simplifying the Self Intersection Sets of Codimension One Immersions in (n+1)-space*, Houston Journal of Mathematics 13, No. 3, 353-366.

6. *Surgery on the Equatorial Immersion I*, Illinois Journal of Mathematics 32, No. 4, Winter 1988, 703-715.

7. *Surgery on the Equatorial Immersion in Low Dimensions*, Differential Topology Proceedings, Siegen 1987, Springer LMN 1350, U. Koschorke, ed.

8. *Triple Points of Immersed Surfaces in Three Dimensional Manifolds*, (with Ki Hyoung Ko), Topology and Its Applications 32, (1989), 149-159.

9. *Immersed Codimension One Projective Spaces in Spherical Space Forms*, Proc. of the AMS 105, No. 1, January 1989, 254-257.

10. *Immersed Projective Planes in Lens Spaces*, Proc. of the AMS 106, No. 1 (May 1989), 251-260.

11. *Classifying Immersed Curves*, Proc. of the AMS 111, No.1 (Jan. 1991), 281-287.

12. *Extending Immersed Curves to Proper Immersions of Surface*, Topology and its Applications 40 (1991), 287-306.

13. *Closed Curves that Never Extend to Proper Maps of Disks*, Proc. of the AMS 113, No. 3 (Nov 1991), 879-888.

14. *Canceling Branch Points on Projections of Surfaces in 4-Space*, (with Masahico Saito), Proc. of the AMS. 116, No 1. (Sept 1992), 229-237.

15. *Extending Immersed Circles in the Sphere to Immersed Disks in the Ball*, Comm. Math. Helv. 67 (1992), 337-348.

16. *Syzygies among Elementary String Interactions in Dimension 2+1*, (with Masahico Saito), Letters in Mathematical Physics 23 (1991), 287-300.

17. *Planar Generalizations of the Yang Baxter Equation and Their Skeins*, (with Masahico Saito), Journal of Knot Theory and its Ramifications Vol 1, No. 2 (1992), 207-217.

18. *A Diagrammatic Theory of Knotted Surfaces*, (with Masahico Saito), in "Quantum Topology," ed. Randy Baadhio and Louis Kauffman, World Science Publishing (Singapore 1993), 91-115.

19. *Reidemeister Moves for Surface Isotopies and Their Interpretations As Moves to Movies*, (with Masahico Saito), Journal of Knot Theory and its Ramifications Vol 2, No 3 (1993), 251-284.

20. *Knotted Surfaces, Braid Movies, and Beyond*, (with Masahico Saito), in "Quantum Gravity," ed. John Baez, Oxford University Press (1994), 191-229.

21. *New Solutions to the Permutohedron Equation*,(with Masahico Saito), in "Quantum Topology Kansas 1993," ed. David Yetter, World Science Publishing Company, (1994), 51-65,

22. *Knot Diagrams and Braid Theories in Dimension 4*, (with Masahico Saito), in "Real and Complex Singularities," ed. W. L Marar, Pitman Research Notes in Mathematics, Longman Publishing (1995).

23. *A Seifert Algorithm for Knotted Surfaces*, (with Masahico Saito), Topology, Vol 36, No. 1 (1996), 179-201.

24. *Braid and Movies*, (with Masahico Saito), Journal of Knot Theory and Its Ramifications, Vol 5, No. 5 (1996), 589-608.

25. *On Formulations and Solutions of Simplex Equations*, (with Masahico Saito), International Journal of Modern Physics A, Vol 11, No 24, (1996), 4453-4463.

26. *Normal Euler Classes of Knotted Surfaces and Triple Points on Their Projections*, (with Masahico Saito), Proc. Amer. Math. Soc., 125 (1997), no. 2, 617-623.

27. *A Combinatorial Description of Knotted Surfaces and Their Isotopies,* (with Joachim Rieger and Masahico Saito), Advances in Mathematics, 127, No. 1, April 15 (1997), 1-51.

28. *Surfaces in 3-Space that Do Not Lift to Embeddings in 4-Space,* (with Masahico Saito), Knot theory (Warsaw, 1995), 29–47, Banach Center Publ., 42, Polish Acad. Sci., Warsaw, 1998.

29. *Diagrammatics, Singularities, and Their Algebraic Interpretations,* (with Louis H. Kauffman and Masahico Saito), 10th Brazilian Topology Meeting (São Carlos, 1996). Mat. Contemp. 13 (1997), 21–115.

30. *Singularities of the Projections of Surfaces in 4-Space,* (with Vera Carrara and Masahico Saito), Singularities of the projections of surfaces in 4-space. Pacific J. Math. 199 (2001), no. 1, 21–40.

31. *Thin-G theory and Local Moves for Gems*, (with Sótenes Lins), Adv. Math. 143 (1999), no. 2, 251–283.

32. *Alexander Numbering of Knotted Surface diagrams,*(with Seiichi Kamada and Masahico Saito), Proc. Amer. Math. Soc., 128 (2000), 3761-3771.

33. *Structures and Diagrammatics of 4-Dimensional Topological Lattice Field Theories,* (with Louis Kauffman and Masahico Saito), Advances in Math., 146, 39-100 (1999).

34. *State-sum Invariants of Knotted Curves and Surfaces from Quandle Cohomology*, (with Daniel Jelsovsky, Seiichi Kamada, Laurel Langford, and Masahico Saito), Electron. Res. Announc. Amer. Math. Soc. 5 (1999), 146-156.

35. *Quandle Homology Groups, Their Betti Numbers, and Virtual Knots*, (with Daniel Jelsovsky, Seiichi Kamada, and Masahico Saito), J. Pure Appl. Algebra 157 (2001), no. 2-3, 135–155.

36. *Computations of Quandle Cocycle Invariants of Knotted Curves and Surfaces,* (with Daniel Jelsovsky, Seiichi Kamada, and Masahico Saito), Adv. Math. 157 (2001), no. 1, 36–94.

37. *Geometric Interpretations of Quandle Homology*, (with Seiichi Kamada, and Masahico Saito) J. Knot Theory Ramifications 10 (2001), no. 3, 345–386.

38. *Shifting Homomorphisms in Quandle Cohomology and Skeins of Cocycle Knot Invariants*, (with Daniel Jelsovsky, Seiichi Kamada, and Masahico Saito), Journal of Knot Theory and its Ramifications, Vol 10 (2001), no 4, 579-596.

39. *A Theorem of Sanderson on Link Bordisms in Dimension 4*, (with Seiichi Kamada, Shin Satoh, and Masahico Saito) Algebraic and Geometric Topology 1 (2001), paper no. 14, 299-310.

40. *Diagrammatic Computations for Quandles and Cocycle Knot Invariants*, (with Seiichi Kamada and Masahico Saito), AMS Contemporary Math Series, ed. Lou Kauffman, David Radford, and Fernando Sousa.

41. *Stable Equivalence of Knots on Surfaces and Virtual Knot Cobordisms*, (with Seiichi Kamada and Masahico Saito), Journal of Knot Theory and its Ramifications, Vol 11, No 3 (May 2002), 311-322.

42. *Twisted Quandle Homology Theory and Cocycle Knot Invariants*,(with Mohammed Elhamdadi, and Masahico Saito), Algebr. Geom. Topol. 2 (2002) 95-135.

43. *Bordism of Unoriented Surfaces in 4-Space*, (with Seiichi Kamada, Shin Satoh, and Masahico Saito) Michigan Math. J. 50 (2002), no. 3, 575–591.

44. *Quandle Cohomology and State-sum Invariants of Knotted Curves and Curfaces,* (with Daniel Jelsovsky, Seiichi Kamada, Laurel Langford, and Masahico Saito), Trans. Amer. Math. Soc. 355 (2003), no. 10, 3947–3989.

45. *Quandle Homology Theory and Cocycle Knot Invariants*, (with Masahico Saito), Proceedings of Symposia in Pure Mathematics Vol 71 (2003), 249-268, ed. Mattic, et al.

46. *Cocycle Knot Invariants, Quandle Extensions, and Alexander Matrices*, (with Angela Harris, Marina Nikiforou, and Masahico Saito), in Low Dimensional Topology of the 21st Century, ed. Hitoshi Murakami, RIMS Kokyuroku 1272 (Kyoto 2002), also available at math.GT/0204113

47. *Extensions of Quandles and Cocycle Knot Invariants* (with Mohamed Elhamdadi, Marina Appiou Nikiforou, and Masahico Saito), J. Knot Theory Ramifications 12 (2003), no. 6, 725–738, also available at math.GT/0107021

48. *Generalizations of Quandle Cocycle Invariants and Alexander Modules from Quandle Modules* (with Masahico Saito) Intellegence of Low Dimensional Topology, Shodo-Shima, JAPAN (Decemeber 2003), 77-90.

49. *Homology Theory for the Set-theoretic Yang-Baxter Equation and Knot Invariants from Generalizations of Quandles,* (with Mohamed Elhamdadi and Masahico Saito), Fund. Math. 184 (2004), 31–54.

50. *Cocycle Knot Invariants from Quandle Modules and Generalized Quandle Homology,* (with Matias Graña, Mohamed Elhamdadi and Masahico Saito) Osaka J. Math. 42 (2005), no. 3, 499–541.

51. *Ribbon Concordance of Surface-knots via Quandle Cocycle invariants,* (with Masahico Saito and Shin Satoh) J. Aust. Math. Soc. 80 (2006), no. 1, 131–147.

52. *Ribbon-moves for 2-knots with 1-handles Attached and Khovanov-Jacobsson numbers*, Proc. Amer. Math. Soc. 134 (2006), no. 9, 2779–2783.

53. *A Lower Bound for the Number of Reidemeister Moves of Type III*, (with Mohamed Elhamdadi, Masahico Saito, and Shin Satoh) Topology and its Applications, 153 (15), 2788-2794.

54. *Categories for Knotted Curves and Surfaces and Quandles,* In Sica, Giandomenico, ed. "What is Category Theory? Advanced Studies in Mathematics and Logic." Polimetrica, Publisher, Italy, 17-44.

55. *Set Theoretic Yang-Baxter Solutions via Fox Calculus,* (with Masahico Saito), J. Knot Theory Ramifications 15 (2006), no. 8, 949–956, math.GT/0503166.

56. *Cohomology of the Adjoint of Hopf algebras*, Journal of Generalized Lie Theory and Applications, Vol 2, No. 1, March 2008, 19-34.

57. *Cohomology of Categorical Self-Distributivity,* (with Alissa Crans, Mohamed Elhamdadi, Masahico Saito), Journal of Homotopy and Related Structures, Vol 3, No. 1, 13-63, math.GT060717.

58. *Cohomology of Frobenius Algebras and the Yang-Baxter Equation,* (with Alissa Crans, Mohamed Elhamdadi, Enver Karadayi, and Masahico Saito), in Communications of Contemporary Mathematics (Lin Memorial Issue ed. Birman and Tian, eds.) 10 (2008), suppl. 1, 791814. arxiv.0705.3231.

59. *Virtual Knot Invariants from Group Biquandles and Their Cocycles,* (with Mohamed Elhamadadi, Masahico Saito, Daniel Silver, and Susan Williams), J. Knot Theory and Its Ramifications Vol 18(7) (July 2009) 957-972, arxiv.0206255.

60. *Symmetric Extensions of Dihedral Quandles and Triple Points of non-orientable surfaces,* (with Kanako Oshiro and Masahico Saito), Topology Appl. 157(5) (2010), 857–869.

61. *Algebraic Structures Derived from Foams*, (with Masahico Saito), to appear Journal of Lie Algebras and Related Structures, vol. 5 (2011), arxiv.1001.0775

62. *Heron's Formula from a* 4*-dimensional Perspective*, (with David Mullens), Visual Mathematics, 13, No. 1, (2011), on line at http://www.mi.sanu.ac.rs/vismath/

63. *A Survey of Quandle Ideas*, in "Introductory lectures on knot theory," 22-53, Series on Knots and Everything, 46, World Science Publ. (Hackensack, 2012).

64. *Classical Knot Theory,* Symmetry 4 (2012), No. 1, 225-250.

65. *A Knotted 2-dimensional Foam with Non-trivial Cocycle Invariant*, (with Atsushi Ishii), in "Intelligence of Low Dimensional Topology, (Oct. 2012), RIMS, Kyoto, 43-56.

66. *Braids and Branched Coverings of Dimension Three*, (with Seiichi Kamada), in "Intelligence of Low Dimensional Topology, (Oct. 2012), RIMS, Kyoto, 64–81.

67. *Reidemeister/Roseman-type Moves to Embedded Foams in 4-dimensional Space*, to appear in L. H. Kauffman and V. O. Manturov (Eds.) New Ideas in Low-Dimensional Topology. arxiv.1210.3608

68. *How to Fold a Manifold*, (with Seiichi Kamada), to appear in L. H. Kauffman and V. O. Manturov (Eds.) New Ideas in Low-Dimensional Topology. arxiv.1301.4259

69. *Three Dimensions of Knot Coloring*, (with Dan Silver and Susan Williams) Amer. Math. Monthly 121 (2014), no. 6, 506–514. arxiv.1301.5378

70. *Invariants of Links in Thickened Surfaces*, (with Dan Silver and Susan Williams), Algebr. Geom. Topol. 14 (2014), no. 3, 1377–1394, arxiv.1304.4655

71. *Non-orientable surfaces in 4-dimensional space,* J. Knot Theory Ramifications 23 (2014), no. 11, 1430002, 52 pp.

72. *Some elementary aspects of 4-dimensional geometry,* (with David Mullens), Symmetry 7 (2015), no. 2, 515?545.

73. *Reidemeister/Roseman-type moves to embedded foams in 4-dimensional space,* in "New ideas in low dimensional topology," 1– 30, Ser. Knots Everything, 56, World Sci. Publ., Hackensack, NJ, 2015.

74. *How to fold a manifold,* (with Seiichi Kamada) in "New ideas in low dimensional topology," 31–77, Ser. Knots Everything, 56, World Sci. Publ., Hackensack, NJ, 2015.

75. *Three-dimensional braids and their descriptions* (with Seiichi Kamada) Topology Appl. 196 (2015), part B, 510–521.

76. *Geometric and homological considerations of local crossings of n-foams*, J. Knot Theory Ramifications 24 (2015), no. 13, 1541007, 50 pp.

77. *Twist spinning knotted trivalent graphs,* (with Seung Yeop Yang) Proc. Amer. Math. Soc. 144 (2016), no. 3, 1371?1382.

78. *Fractal simplices*, J. Knot Theory Ramifications 25 (2016), no. 9, 1641003, 15 pp.

79. *Homology for quandles with partial group operations,* (with Atsushi Ishii, Masahico Saito, and Kokoro Tanaka, Pacific J. Math. 287 (2017), no. 1, 19?48.

80. *A prismatic classifying space*, (with Victoria Lebed and Seung Yeop Yang) to appear in "Non-Associative Mathematics and Its Applications," ed. Vojtechovsky et. al. Contem. Math, AMS (Providence 2018) preprint at arxiv.org/abs/1711.06215.

81. *Polytopes, Tensors, Graphs, Foams, and Homology*, to appear J. Knot Theory Ramifications.

Books

"How Surfaces Intersect in Space: an Introduction to Topology," World Scientific Publishing (1st edition Feb. 1993), (2nd edition 1995).
"The Classical and Quantum $6j$-symbols," Princeton University Press (1995) (with Daniel E. Flath and Masahico Saito).
"Knotted Surfaces and Their Diagrams," American Mathematical Society Surveys and Monographs Series, Vol 55, (1997) (with Masahico Saito).
"Knotted Surfaces in 4-dimensional Spaces," (with Seiichi Kamada and Masahico Saito), Encyclopaedia of Mathematics, 142, series in Low Dimensional Topology 111, Springer (Berlin 2004).
"Intelligence of Low Dimensional Topology, 2006," Ed. Carter, Kamada, Kauffman, Kawauchi, and Kohno, World Scienctific publishing (Singapore, 2007).
"An Excursion in Diagrammatic Algebra. Turning a Sphere from Red to Blue," Series on Knots and Everything, 48. World Scientific Publishing (Hackensack, 2012). manuscript at

`http://www.southalabama.edu/mathstat/personal_pages/carter/talks.html`

Unpublished

A Geometric Method to Compute Some Elementary Integrals, (with Abhijit Champanerkar) preprint available at arxiv.org/pdf/math/0608722
Amusing Permutation Representations of Group Extensions, (with Yongju Bae and Byeorhi Kim), preprint available at https://arxiv.org/abs/1812.08475

Professional Memberships

Member American Mathematical Society
Managing Editor Journal of Knot Theory and Its Ramifications

Recent Honor

$\Phi K \Phi$ scholar of the year, 2006.

Grants Awarded

1996, National Security Agency, "Generalizations of the Temperley-Lieb Algebra and Applications," # MDA904-96-10071.

2000, National Science Foundation, "Cohomology State-Sum Invariants in Dimensions 3 and 4." # DMS-9988107.

2003, National Science Foundation, "Collaborative Research: Cocycle Invariants of Low-Dimensional Knots and Manifolds," # DMS-0301095.

2006, National Science Foundation, "Collaborative Research: Algebraic Structures and Cohomology Theories Associated to Knottings," # DMS-0603926.

2012, Brain-Pool Trust, Ministry of Education Science and Technology and the Korean Federation of Science and Technology Societies, grant support for visiting Kyungpook National University, Daegu, Korea, Dec. 2011-Aug. 2012.

2018, Japanese Society for the Promotion of Science, Ten month grant to study and collaborate on topics in higher dimensional algebra, # JSPS 18511.

To whom it may concern,

This is a reference to support Professor Scott Carter's application to participate in the ACT school.

Scott is mathematician, recently retired as a professor in Mathematics from the University of Alabama. He is well known internationally as an expert in knotted surfaces, and has written many books and papers on this topic. Although Scott and I have not published together, we have had many interesting discussions about mathematics, and I know him quite well.

Scott uses category theory for his work, and is intensely interested in other applications of this technology. He is a creative, personable and open-minded individual, who I think would put a lot of energy into the course, and who would get a lot out of it. He is also, of course, a highly skilled and experienced mathematician. I support his application to participate in the School in the strongest possible way. Scott is of course older than the typical applicant for the ACT school; I hope that this fact will not disadvantage him.

Best wishes,
Jamie Vicary