Stirling Engine

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1 Introduction

The Stirling engine is a heat engine that works by a cycle of compression and expansion of a gas between 2 different temperatures: T_+(the higher temperature) and T_-(the lower temperature) and transforms heat energy into mechanical work. The cycle comprises 2 isothermal at temperatures T_+ and T_- and two isovolumetrics from T_+ to T_- and vice versa.
To maximise the work produced, the engine has to operate most reversibly, this is done via a regenerator, a fixed matrix of material where heat from the hot fluid is intermittently stored in a thermal storage medium before it is transferred to the cold fluid.

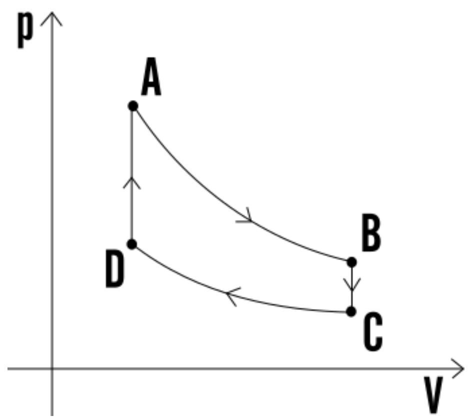


Figura 1: Stirling cycle

The main goal of the first part of the experience is to find the efficiency of the heat engine, given by:

$$\eta = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} \tag{1}$$

In the second part of the experience the cycle is inverted to obtain a refrigerator, which is characterized by the coefficient of performance:

$$COP = \frac{Q_c}{W} \tag{2}$$

Where Q_c is the heat removed from the cold reservoir.

2 Set Up and Measurement Procedure

The single-piston Stirling engine consists of a cylindrical chamber with a coaxial piston and a displacer inside, which has a 90° offset, connected to a hot and cold source. The difference in temperature between the upper (connected to the hot source) and lower (connected to the cold source) parts of the cylindrical chamber produces a pressure gradient which makes the gas and as a consequence the piston move. This is done using, also, a regenerator made of copper mesh.
The hot source is provided by a resistor connected to a multi-meter in such a way as to control the voltage and current, while the cold source is provided by a constant flux of cold water pumped by a container containing a mix of water and ice. Knowing the flux and the difference in temperature of the water before and after entering into the cycle makes it possible to estimate the exiting heating (Q_{out}). Moreover, the piston is connected to a string wrapped around a pulley connected to a variable resistor. This setup allows us to determine changes in volume by monitoring changes in voltage. Finally, pressure is estimated using a piezoelectric manometer. The measurements were conducted by altering the voltage applied to the dissipative resistor, thus controlling the heat input into the engine. For each voltage value, three repetitions were collected to improve statistical reliability.
In the second part of the experience, the cycle is inverted connecting one of the two multimeters to the wheel in such a way as to provide work to the refrigerator. The other multimeter is linked to the dissipative resistor to make the temperature constant and ensure that the temperature does not drop below 0°C and so avoid condensation. By knowing the power supplied to the resistor to stabilize the temperature we can know the amount of heat removed from the cold source. The measurements were obtained by adjusting the voltage of the generator powering the wheel. Three repetitions were performed for each voltage setting to improve statistical robustness.

3 Data Analysis

3.1 Pressure and volume measurements

Since we read values of pressure and volume in voltage we have to convert them into the right units. For the pressure we use the following formula:

$$P[KPa] = \frac{1}{0.004}\left(\frac{V_{out}}{V_{in} - 0.04}\right) \tag{3}$$

where $V_{in} = 5.21V$ and V_{out} is obtained by the software. For the volume, we know that:

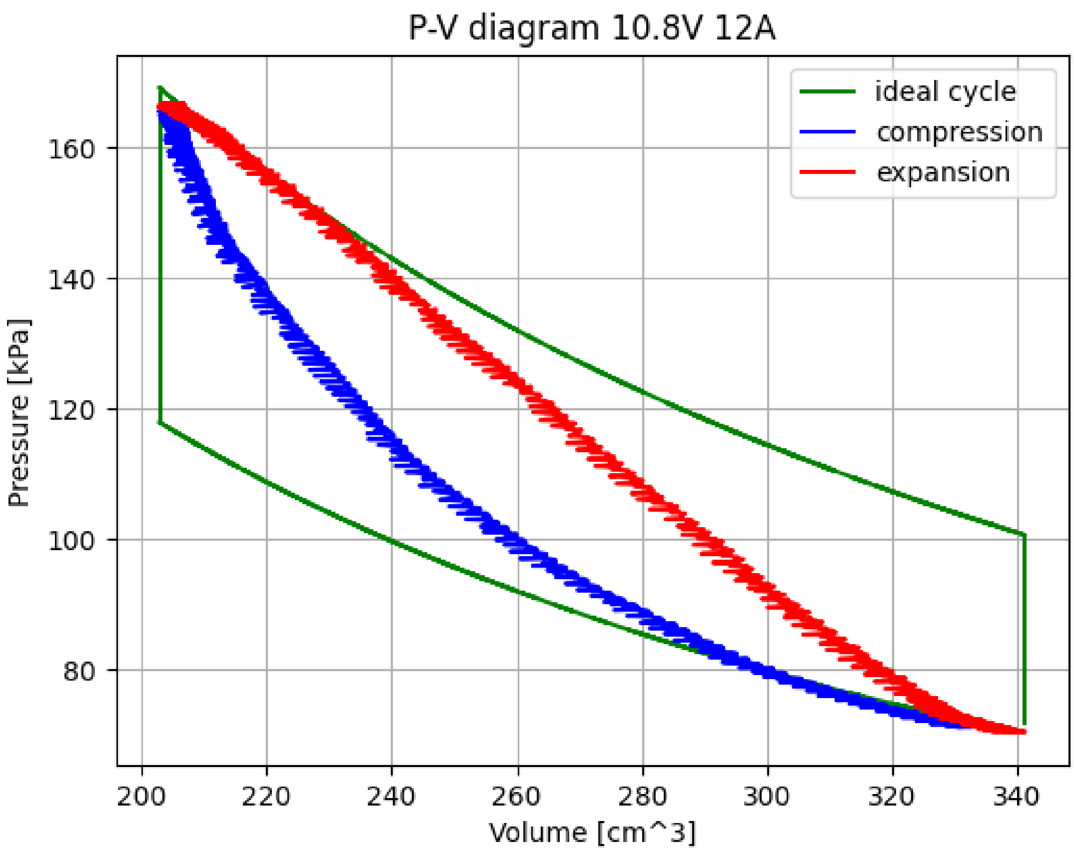
$$\frac{V_{out}}{V_{in}} = \frac{R_1 + xR2}{2R_1 + R_2} \tag{4}$$

where $R_1 = 1.2K\Omega$, $R_2 = 20K\Omega$, and x is the increment of R_2 knowing that R_2 increases of $2K\Omega$ each time that the pulley makes one round. To compute the displacement and thus the volume, we have to know the difference in resistance ($\Delta R = x(t_2) \cdot R_2 - x(t_1) \cdot R_2$) at two different times:

$$\Delta R = \frac{\Delta V}{V_{in}}(2R_1 + R_2) \tag{5}$$

Where ΔV is provided by the software, then we can find the fraction of a turn: $\Delta_{turn} = \frac{\Delta R}{2000}$ and finally compute the displacement $\Delta z = \Delta_{turn} \cdot \pi d$, where $d = 23.45 \pm 0.01mm$ is the diameter of the pulley. Finally, knowing the initial volume: $V_i = 200cm^3$, we can find the volume at each time $V(t) = V_i + \Delta z \cdot \pi R_p$, where $R_p = 30mm$ is the piston radius.

3.2 Stirling engine



The work W in eq. 1 can be calculated in 4 different ways:

- fitting the isothermals just in the first part of the cycle: $W_1 = nR(T_+ - T_-)log\left(\frac{V_{max}}{V_{min}}\right)$
- calculating the area inside the cycle: $W_2 = \oint PdV$
- $W_3 = Q_{in} - Q_{out}$ where Q_{in} is calculated as $Q_{in} = IV\Delta t$, where I and V are the current and the voltage provided by the generator to the resistance while Δt is the period of a cycle. $Q_{out} = \dot{m}c_v\Delta T\Delta t$, where $c_v = 4186J/kg \cdot K$ is the specific heat for water, ΔT is the difference in temperature between cycle incoming and outcoming water and $\dot{m}$ is the flux of water which is measured using a chronometer and a graduated cylinder: $\dot{m} = \frac{0.201 \pm 0.01kg}{61 \pm 1s} = 3.3 \pm 0.1g/s$.

- Knowing the moment of inertia of the flywheel: $\tau = 0.043 kg \cdot m^2$, $W_4 = \frac{1}{2}\tau\omega^2$ where ω is the angluar frequency of the flywheel.

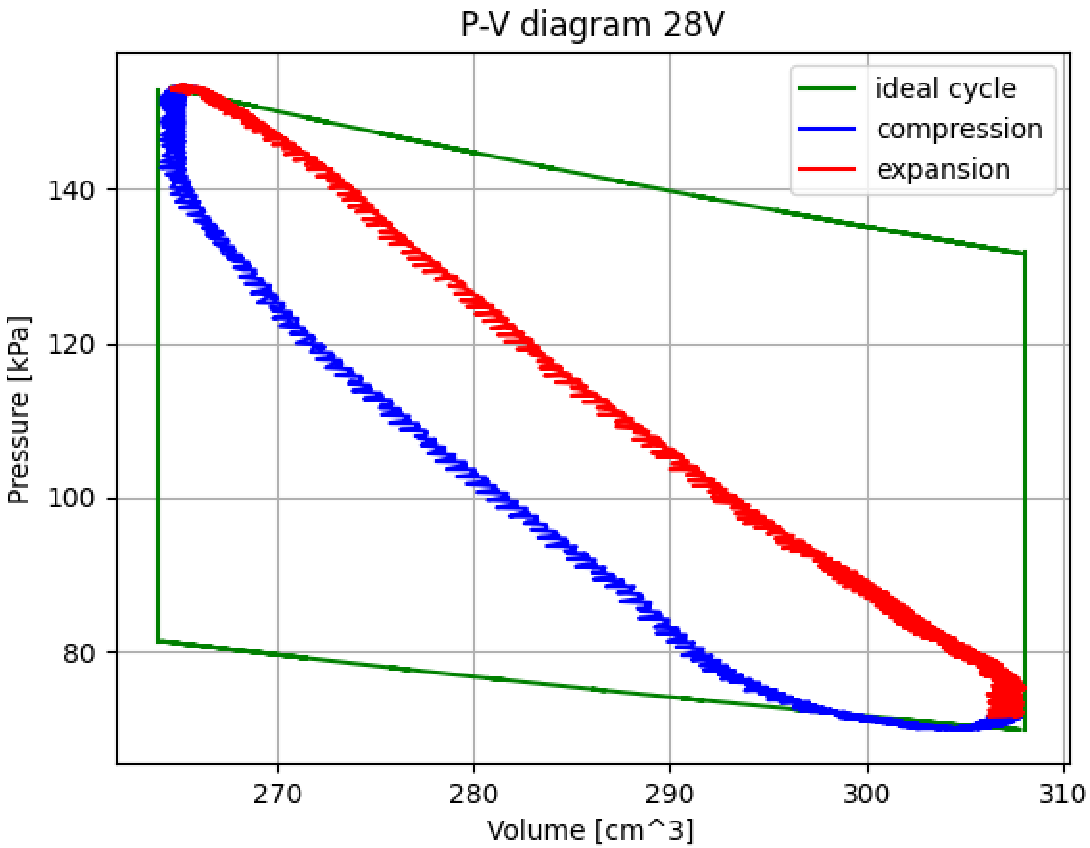
Q_{in} of eq. 1 is calculated as $Q_{in} = IV\Delta t$.
In the following table, to calculate η, the 4 methods of finding W are used and the values of η are in the same order as the works in the list before.
The ideal efficiency of the Stirling engine is found as an ideal Carnot efficiency by fitting the isothermal lines to find T_+ and T_-: $\eta_0 = 1 - \frac{T_-}{T_+}$.
The three values of voltage and current on the resistance are:

1. $I_1 = 14.0 \pm 0.1A$, $V_1 = 12.7 \pm 0.1V$
2. $I_2 = 12.0 \pm 0.1A$, $V_2 = 10.8 \pm 0.1V$
3. $I_3 = 10.0 \pm 0.1A$, $V_3 = 9.0 \pm 0.1V$

	I_1, V_1	I_2, V_2	I_3, V_3
η_0	0.43 ± 0.06	0.37 ± 0.04	0.31 ± 0.03
η_1	0.16 ± 0.02	0.18 ± 0.02	0.18 ± 0.02
η_2	0.05 ± 0.01	0.07 ± 0.01	0.07 ± 0.01
η_3	0.71 ± 0.15	0.54 ± 0.11	0.36 ± 0.07
η_4	0.15 ± 0.02	0.24 ± 0.03	0.17 ± 0.03

The errors are calculated as standard deviations between the three cycles with the same I and V.

3.3 Stirling Refrigerator



The removed heat by the cold reservoir, Q_C in eq. 2, is given by:

$$Q_c = IV\Delta t \tag{6}$$

Where I and V are the current and the voltage provided to the resistance to keep the temperature constant. The same ways as before are used to calculate the work, with the only difference that W_4 is not used because the flywheel was detached.
The coefficients of performance, in the table, are in the same order as before.
The four values of voltage and current are:

1. $\Delta V_1 = 26.0 \pm 0.1V$, $I_{s1} = 1.12 \pm 0.01A$, $I_1 = 3.3 \pm 0.1A$, $V_1 = 2.88 \pm 0.1V$
2. $\Delta V_2 = 28.0 \pm 0.1V$, $I_{s2} = 1.10 \pm 0.01A$, $I_2 = 3.3 \pm 0.1A$, $V_2 = 2.88 \pm 0.1V$
3. $\Delta V_3 = 30.0 \pm 0.1V$, $I_{s3} = 1.06 \pm 0.01A$, $I_3 = 3.3 \pm 0.1A$, $V_3 = 2.88 \pm 0.1V$

where ΔV e I_s are the current and the potential difference on the engine.

The efficiency of the engine is calculated as:

$$\eta_{eng} = \frac{W}{W_{eng}} \tag{7}$$

where $W_{eng} = I_s \Delta V \Delta t$ and W is calculated using the methods as before.

	$30V$	$28V$	$26V$
η_0	0.46 ± 0.05	0.47 ± 0.05	0.47 ± 0.05
η_1	0.27 ± 0.04	0.27 ± 0.04	0.29 ± 0.04
η_2	0.10 ± 0.02	0.07 ± 0.02	0.09 ± 0.02
η_3	0.70 ± 0.10	0.69 ± 0.10	0.72 ± 0.10

	$30V$	$28V$	$26V$
cop_0	2.15 ± 0.04	2.13 ± 0.04	2.11 ± 0.04
cop_1	1.10 ± 0.09	1.16 ± 0.09	1.23 ± 0.09
cop_2	2.90 ± 0.20	4.60 ± 0.60	4.10 ± 0.60
cop_3	0.42 ± 0.06	0.44 ± 0.06	0.49 ± 0.06
cop_5	0.29 ± 0.02	0.31 ± 0.02	0.32 ± 0.02

Where cop_5 is the value of the coefficient of performance using as work $W_{eng} = I_s \Delta V \Delta t$ and cop_0 is the ideal one: $cop_0 = \frac{T_+}{T_+ - T_-}$

4 Conclusions

The values of η_3 in the Stirling engine are too high compared to the other values, this is probably because the cycle is not isolated to the environment so the room could heat the water inside the cycle, furthermore also the friction of the piston could heat the water as well.
The values of η_2 of the real cycle are less than the ideal Carnot cycle (η_0) and less than the fitted cycle (η_1) as expected.
Regarding the Stirling refrigerator, the values of η_3 are too high for the same reason as before so the values of *cop* are too low compared to the other values.
The values of cop_5 are very low because the efficiency is as if it were 1.
The efficiency of the real cycle (η_2) is lower than the ideal efficiency (η_0) and the fitted one (η_1) so the real cycle values of *cop* (cop_2) are greater than cop_1 and cop_0 (less work to have the same Q_c).