Wind Power

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1 Introduction

The objective of the initial phase of this experiment is to comprehend and confirm the operation of a three-phase generator utilising electromagnetic induction. Within a hollow cylinder known as the stator, a permanent magnet (rotor) rotates. The stator is equipped with coils arranged around magnets. As the rotor turns, it induces an oscillating electromotive force within the coils, generating alternating current (AC) with a frequency determined by the rotor's angular velocity:

$$\nu = \frac{N_{rpm} \cdot P}{120} \tag{1}$$

Where P is the number of magnetic poles, N_{rpm} is the number of rotations per minute. The 3 currents of three-phase have a phase displacement of 120 degrees.
In the second part of the experience, we try to measure the efficiency of the engine to convert the wind power into an electric one. In order to do this, firstly we verify the cubic relation between power and velocity:

$$P = \frac{1}{2}\rho A v^3 \tag{2}$$

Where is the turbine surface, ρ is the air density and v is the wind speed.
Then we verify the Betzel limit:

$$c_P = \frac{P}{P_0} \leq 59\% \tag{3}$$

Where P is the wind power on the rotor and P_0 is the wind input power.

2 Measurement Procedure

Firstly we want to measure the number of poles of the stator. In order to do this we use a photodiode connected to an oscilloscope to measure the rotational frequency of the wind turbine. We measure the difference of phase between the currents which has to be $120/2 = 60$ degrees and the rotational frequency.
Then we want to find the load that maximises the delivered power. To do this we connect the turbine to a rheostat and two multimeters to measure the current and voltage. We vary the resistance and we take measures of I and V.
Finally, we want to measure the efficiency of the turbine so we measure the diameter of the wind

tunnel: $d = 54.0 \pm 0.5cm$, so the area will be: $A = (\frac{d}{2})^2\pi = 0.23 \pm 0.01m^2$ and we take six measurements of different wind speeds with a hot wire anemometer at five different positions and three different depths($8.5 \pm 0.5cm, 23.5 \pm 0.5cm$ and $28.5 \pm 0.5cm$) in the wind tunnel namely: at $240.0 \pm 0.5cm$ before the turbine, $90.0 \pm 0.5cm$ before the turbine, before the turbine at two different angles: 0 degrees and 45 degrees, behind the turbine (only two measures in depth: $8.5 \pm 0.5cm, 23.5 \pm 0.5cm$) and $110.0 \pm 0.5cm$ behind the turbine.
We measure the values of I and V for each wind speed using the multimeters.
We had to take care to insert the anemometer straight because the speed that is measured is very sensible to angle variations.

3 Data Analysis

3.1 Number of poles

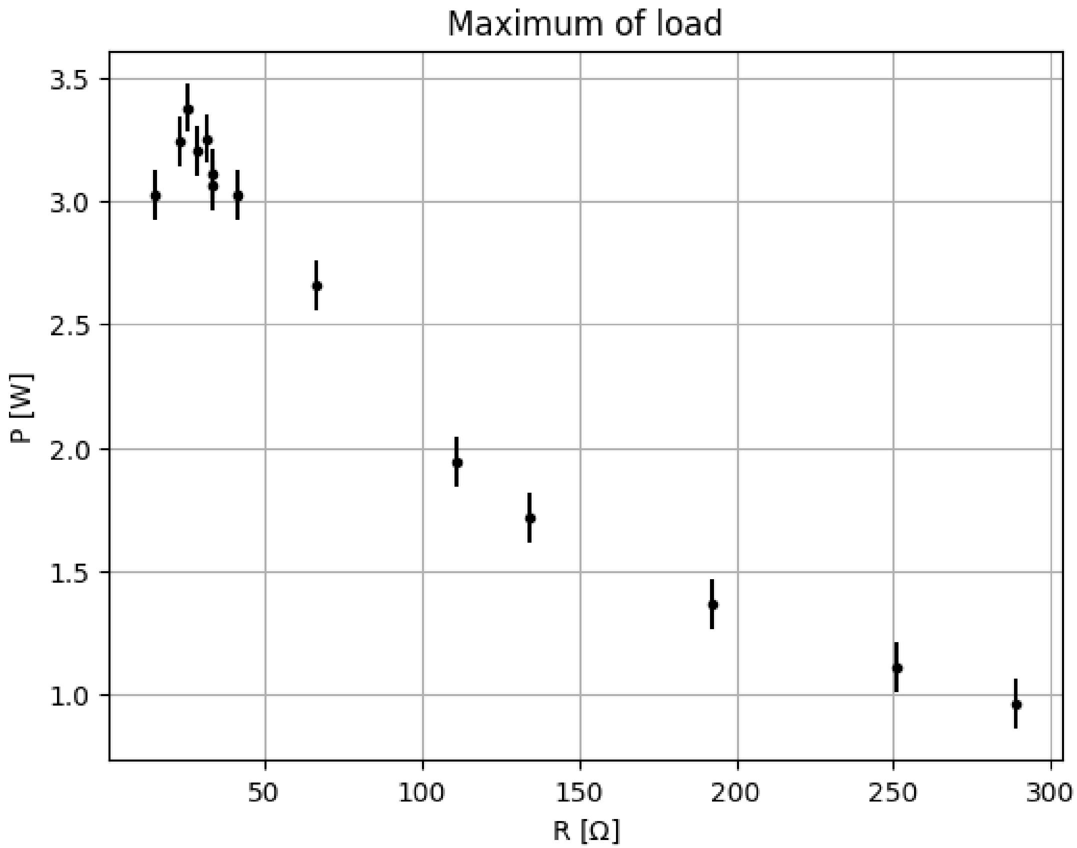
Firstly we want to find the number of poles of the three-phase generator. We measure the period of the turbine: $\Delta T = 68 \pm 1ms$ so the number of revolutions per minute will be: $N_{rpm} = \frac{1}{\Delta T} \cdot 60 = 875 \pm 1rpm$, the current frequency is: $\nu = 58 \pm 1Hz$. The number of poles will be:

$$P = \frac{120 \cdot \nu}{N_{rpm}} = 8.04 = 8 \; poles \tag{4}$$

Which means that the rotor is composed of 4 magnets.

3.2 Maximum load

Now we find the load that maximises the power generated.



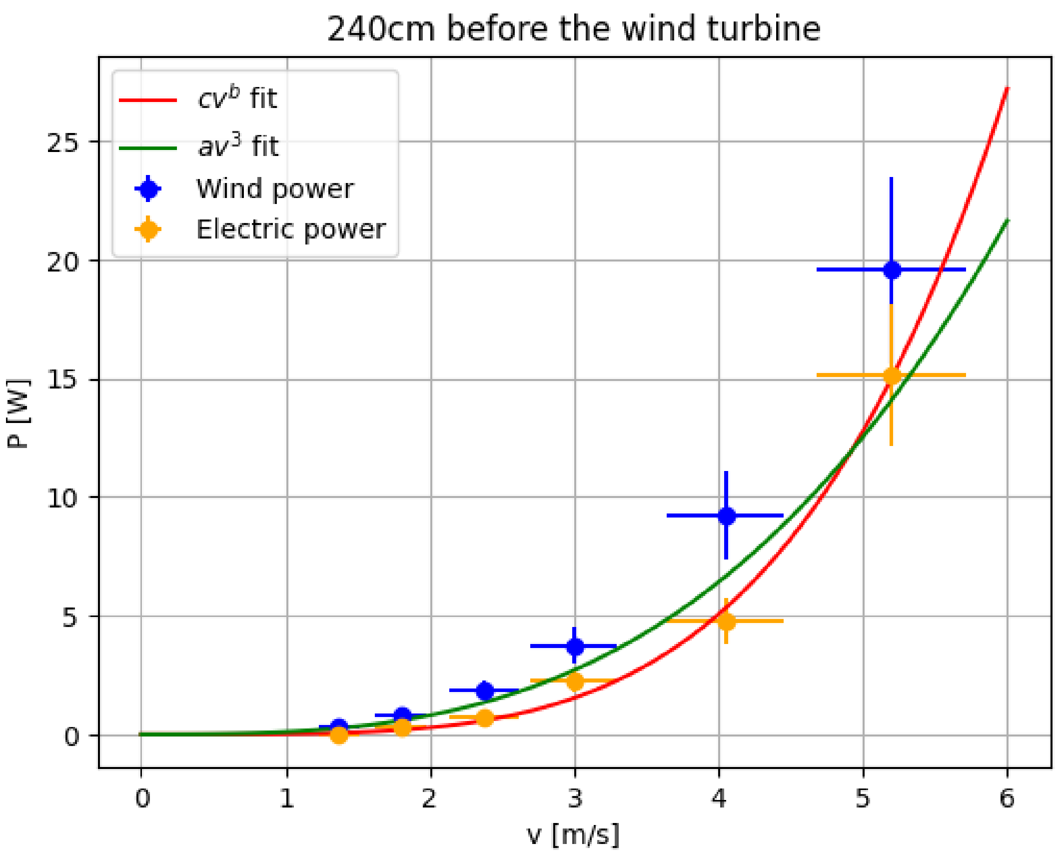
We find that $R_{max} = 26 \pm 1\Omega$.

3.3 Validation of cubic law

We try to verify the cubic law between the speed of the wind and the power generated:

$$P_{wind} = \frac{1}{2}\rho A v^3 \tag{5}$$

where the air density $\rho = 1.2 \pm 0.1 kg/m^3$.
We fit the measurements of electric power found through I and V measurements: $P_{electric} = I \cdot V$ using both a law: $P = av^3$ and $P = cv^b$. We use as wind speed the values measured in the asymptotic behaviour: more distant from the turbine; furthermore we take just the maximum value because we suppose that the anemometer does not overestimate the wind speed, just a little movement of the anemometer angle can vary a lot the value of wind speed therefore above all the deeper ones.
We plot the values of the so-called "Electric power" and "Wind power" versus wind speeds and the 2 fits:



We find that:

$$a = 0.10 \pm 0.05 \frac{W \cdot s^3}{m^3}, \qquad b = 4.1 \pm 0.5, \quad c = 0.02 \pm 0.01 \tag{6}$$

So the efficiency is calculated as:

$$\eta = \frac{av^3}{\frac{1}{2}\rho A v^3} = \frac{2a}{\rho A} = 0.12 \pm 0.05 \tag{7}$$

3.4 Validation of Betz limit

Finally, we try to verify the Betz limit namely the outgoing wind power has to be $\leq 59\%$ of the incoming wind power. In order to do that we write the power on the turbine as:

$$W_B = v_R T = v_R A \Delta P = \frac{1}{2}(v_0 + v_1) A \frac{1}{2} \rho (v_0^2 - v_1^2) \tag{8}$$

Where v_R is the average wind speed before and after the turbine, T is the force of the wind acting on the turbine, ΔP is the pressure of the wind on the turbine, v_0 and v_1 are the wind speeds before

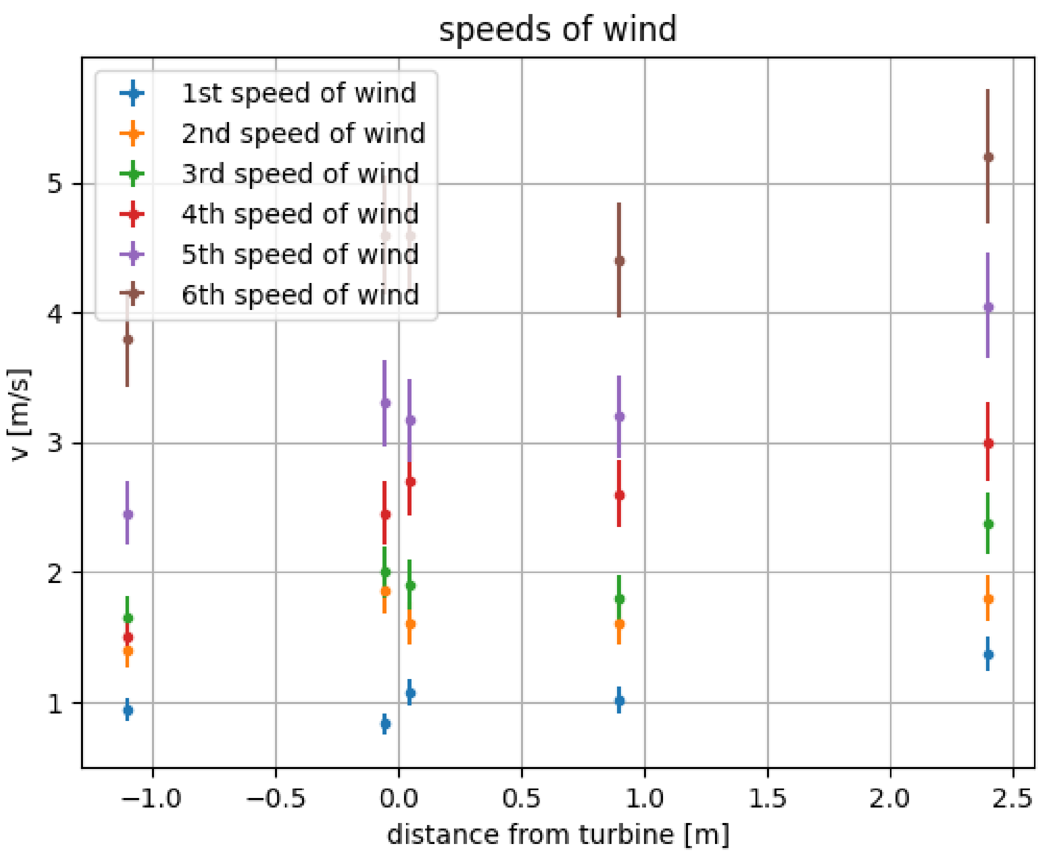
and after the turbine respectively.
So c_P will be:

$$c_P = \frac{W_B}{\frac{1}{2}\rho A v_0^3} = \frac{\frac{1}{2}(v_0+v_1)(v_0^2-v_1^2)}{v_0^3} \tag{9}$$

$v_0[m/s]$	1.4 ± 0.2	1.8 ± 0.2	2.4 ± 0.3	3.0 ± 0.4	4.1 ± 0.5	5.2 ± 0.6
$v_1[m/s]$	0.9 ± 0.3	1.4 ± 0.3	1.7 ± 0.4	1.5 ± 0.5	2.5 ± 0.6	3.8 ± 0.8
c_P	0.45 ± 0.06	0.35 ± 0.03	0.44 ± 0.06	0.56 ± 0.09	0.51 ± 0.08	0.40 ± 0.05

4 Conclusions

We saw that in general the wind speed decreases as approaching the turbine because of turbulence and decreases behind it since part of wind speed energy is absorbed by the turbine:



In the plot the positive values of distance mean that the wind speed values are taken before the turbine, instead negative values mean behind the turbine.

Regarding the validation of the cubic law, we found that the measurements do not follow the cubic law but are better fitted by a fourth power law. This behaviour could be due to the big uncertainty of the wind speed measurements: just a little movement of the hot wire anemometer angle can change the value measured.
The efficiency calculated can be reasonable for a wind turbine.
We found that the Betz limit is respected for each value of wind speed.