High Pass and Low Pass Filters in Frequency Domain

To apply a filter in the frequency domain, the steps are:

1. Convert the image to the frequency domain using **Fourier Transform**.
2. Multiply the transformed image with the filter mask.
3. Apply **Inverse Fourier Transform** to convert back to the spatial domain.

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1. Low Pass Filter (LPF)

- Purpose → Retain low frequencies (smooth parts), remove high frequencies (sharp details and noise).

- Formula:

$$H(u,v) = \begin{cases} 1, & D(u,v) \leq D_0 \\ 0, & D(u,v) > D_0 \end{cases}$$

Where:

- $H(u,v)$ → Filter function in frequency domain
- $D(u,v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}$ → Distance from the center
- D_0 → Cutoff frequency

Example of Low Pass Filter

➡ If D0=50 (cutoff distance), only frequency values within a circle of radius 50 around the center will be retained → Results in a **blurred image**.

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2. High Pass Filter (HPF)

- Purpose → Retain high frequencies (sharp details and edges), remove low frequencies (smooth areas).

- Formula:

$$H(u,v) = \begin{cases} 0, & D(u,v) \leq D_0 \\ 1, & D(u,v) > D_0 \end{cases}$$

where:

- $D(u,v)$ → Distance from the center
- D_0 → Cutoff frequency

Example of High Pass Filter

➡️ If D0=50 (cutoff distance), only frequency values **outside a circle** of radius 50 around the center will be retained → Results in a **sharper image** (edge-enhanced).

Types of Filters

1. Ideal Filter (Sharp Cutoff)

- Formula (Low Pass):

$$H(u,v) = \begin{cases} 1, & D(u,v) \leq D_0 \\ 0, & D(u,v) > D_0 \end{cases}$$

➡️ Retains low-frequency values and completely removes high frequencies → Causes ringing effect (Gibbs effect).

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2. Butterworth Filter (Smooth Cutoff)

- Formula (Low Pass):

$$H(u,v) = \frac{1}{1 + \left(\frac{D(u,v)}{D_0}\right)^{2n}}$$

- Formula (High Pass):

$$H(u,v) = \frac{1}{1 + \left(\frac{D_0}{D(u,v)}\right)^{2n}}$$

where:

- n→ Order of the filter (higher = sharper cutoff).
  Produces smoother transitions without ringing.

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2. **Gaussian Filter (Smoothest Cutoff)**
   - Formula (Low Pass):

$$H(u,v) = e^{-\frac{D(u,v)^2}{2D_0^2}}$$

   - Formula (High Pass):

$$H(u,v) = 1 - e^{-\frac{D(u,v)^2}{2D_0^2}}$$

Example: Step-by-Step Process

Step 1: Take an image

- Example → A grayscale image of size 256×256256×256 pixels

Step 2: Apply Fourier Transform

Convert the image into the frequency domain using:

Step 3: Apply Filter

Low Pass Filter – Apply filter mask that retains low frequencies near the center.
High Pass Filter – Apply filter mask that retains high frequencies near the edges.

Step 4: Apply Inverse Fourier Transform

Convert back to the spatial domain using:

Step 5: Output Result

Low Pass → Blurred image
High Pass → Edge-enhanced image