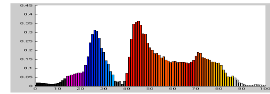
Dr. Qadri Hamarsheh



Digital Image Processing (750474)

Lecture 8

Basic Relationships between Pixels

Outline of the Lecture

- Neighbourhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries
- Distance Measures
- Matlab Example

Neighbors of a Pixel

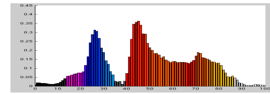
1. **N_4 (p) : 4-neighbors of p.**
   - Any pixel **p(x, y)** has two vertical and two horizontal neighbors, given by
     (x+1,y), (x-1, y), (x, y+1), (x, y-1)
   - This set of pixels are called the **<u>4-neighbors</u>** of **P, and is denoted by N_4(P)**
   - Each of them is at a **<u>unit distance</u>** from **P**.

2. **N_D(p)**
   - This set of pixels, called 4-neighbors and denoted by **N_D (p)**.
   - **N_D(p)**: four diagonal neighbors of **p** have coordinates:
     (x+1,y+1), (x+1,y-1), (x-1,y+1), (x-1,y-1)
   - Each of them are at **<u>Euclidean distance</u>** of **1.414** from **P**.

3. **N_8 (p):** 8-neighbors of **p**.
   - **N_4(P)**and **N_D(p)** together are called 8-neighbors of **p**, denoted by **N_8(p).**
   - **$N_8 = N_4 \cup N_D$**
   - Some of the points in the **N_4 , N_D** and **N_8** may fall **<u>outside</u>** image when P lies on the **<u>border</u>** of image.

F(x-1, y-1)	F(x-1, y)	F(x-1, y+1)
F(x, y-1)	F(x,y)	F(x, y+1)
F(x+1, y-1)	F(x+1, y)	F(x+1, y+1)

N_8 (p)



Adjacency

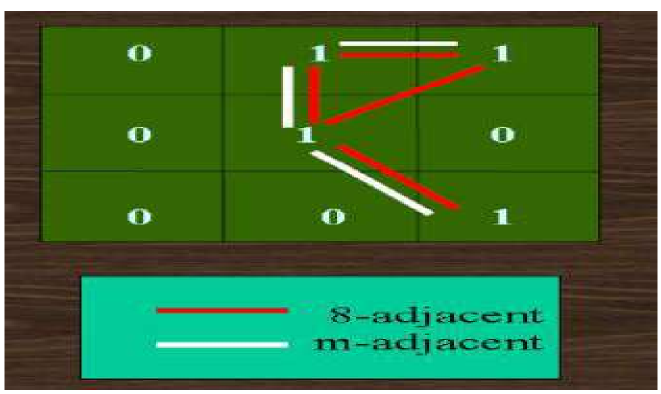
- Two pixels are **connected** if they are neighbors and their gray levels satisfy some specified criterion of similarity.
- For example, in a binary image two pixels are connected if they are 4-neighbors and have same value (0/1)
- **Let v:** a set of intensity values used to ***define adjacency*** and ***connectivity***.
- In a **binary Image** **v={1},** if we are referring to adjacency of pixels with value 1.
- In a **Gray scale image**, the idea is the same, but **v** typically contains more elements, for example **v= {180, 181, 182,....,200}.**
- If the possible intensity values 0 to 255, **v** set could be any subset of these 256 values.

Types of adjacency

1. **4-adjacency:** Two pixels **p** and **q** with values from v are **4-adjacent** if **q** is in the set N_4 **(p)**.
2. **8-adjacency:** Two pixels **p** and **q** with values from v are **8-adjacent** if **q** is in the set N_8 **(p).**
3. **m-adjacency (mixed):** two pixels **p** and **q** with values from v are **m-adjacent** if:
   - **q** is in N_4 **(p) or**
   - **q** is in N_D **(P) and**
   - **The set** $N_4(p) \cap N_4(q)$ has no pixel whose values are from v (**No intersection**).

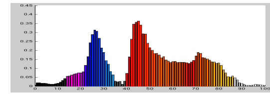
- **Mixed adjacency** is a modification of 8-adjacency "introduced to eliminate the ambiguities that often arise when 8- adjacency is used. (eliminate multiple path connection)
- Pixel arrangement as shown in figure for **v= {1}**

Example:



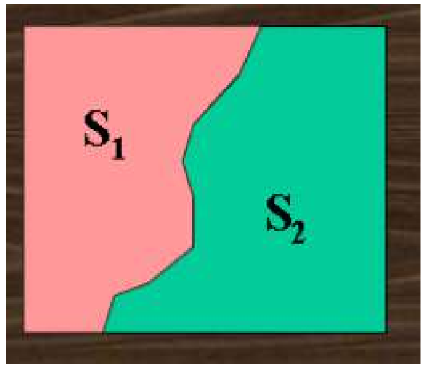
Path

- A ***digital path*** (or curve) from pixel **p** with coordinate **(x,y)** to pixel **q** with coordinate **(s,t)** is a sequence of ***distinct*** pixels with coordinates $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$, where $(x_0, y_0) = (x,y)$, $(x_n, y_n) = (s,t)$
- (x_i, y_i) is adjacent pixel (x_{i-1}, y_{i-1}) for $1 \le j \le n$,
- **n**- The ***length*** of the path.
- If $(x_0, y_0) = (x_n, y_n)$:the path is ***closed path***.
- We can define ***4- ,8- , or m-paths*** depending on the type of adjacency specified.



Connectivity

- Let **S** represent a subset of pixels in an image, Two pixels **p** and **q** are said to be connected in **S** if there exists a path between them.
- Two image subsets **S1** and **S2** are adjacent if some pixel in **S1** is adjacent to some pixel in **S2**

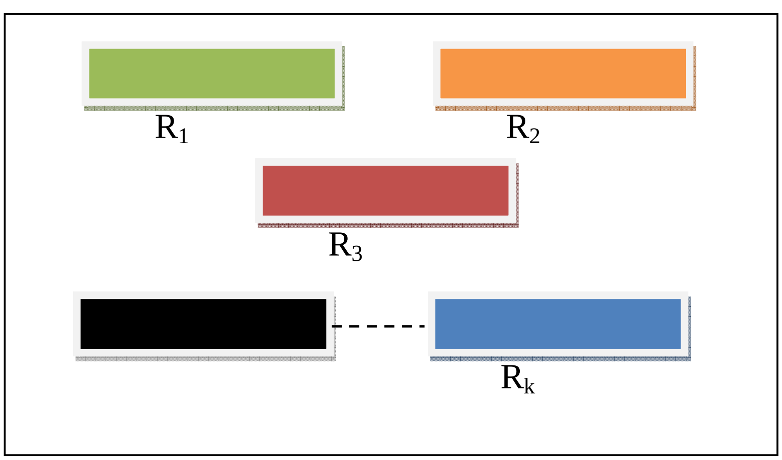


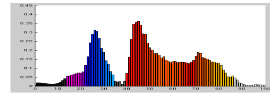
Region

- Let **R** to be a subset of pixels in an image, we call a **R** a region of the image. If **R** is a ***connected*** set.
- Region that are not adjacent are said to be **disjoint**.
- ***Example***: the two regions (of Is) in figure, are adjacent only if 8-adjacany is used.

1	1	1	
1	0	1	Ri
0	1	0	
0	0	1	
1	1	1	Rj
1	1	1	

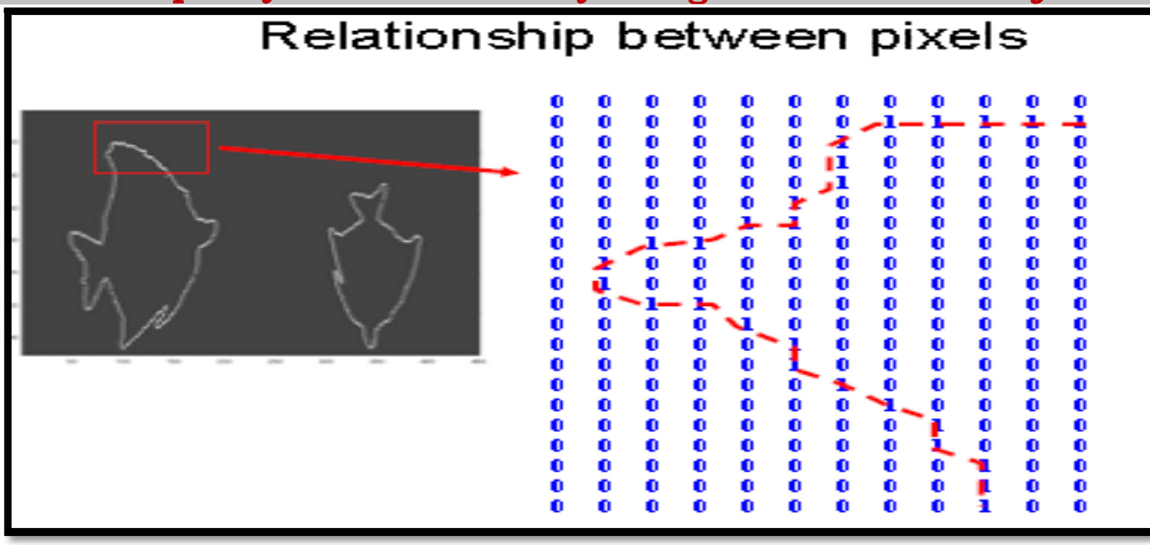
- ***4-path*** between the two regions does not exist, (so their union in not a connected set).
- **Boundary (border)** image contains **K** disjoint regions, R_k, **k=1, 2, …., k**, none of which touches the image border.





- **Let: R_u** - denote the **union** of all the **K** regions, **$(R_u)^c$**- denote its **complement**. (**Complement** of a set S is the set of points that are not in s).
  R_u - called **foreground**; **$(R_u)^c$** - called **background** of the image.
- **Boundary (border or contour)** of a region **R** is the set of points that are adjacent to points in the **complement** of **R** (another way: the border of a region is the set of pixels in the region that have at least are background neighbor).

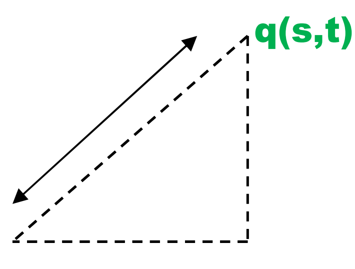
We must specify the connectivity being used to define adjacency



Distance Measures

- For pixels **p, q and z**, with coordinates **(x,y), (s,t)** and **(u,v)**, respenctively, **D** is a **distance function** or metric if:

D(p,q)≥0, D(p,q) = 0 if p=q
D(p,q) = D(q,p), and
D(p,z) ≤D(p,q) + D(q,z)



- The following are the different ***Distance measures***:

1. Euclidean Distance (D_e)

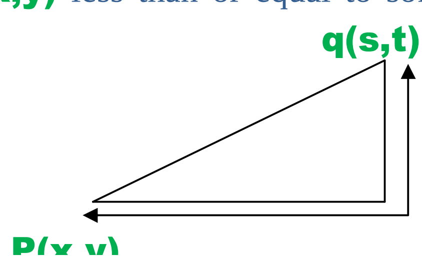
$$D_e\,(p,q) = \sqrt[2]{[(x-s)^2 + (y-t)^2]}$$

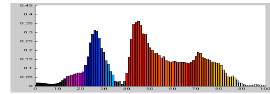
- The points contained in a **disk** of radius **r** centred at **(x,y).**

2. D_4 distance (city-block distance)

$$D_4\,(p,q) = |x-s| + |y-t|$$

- Pixels having a **D_4** distance from **(x,y)** less than or equal to some value **r** form a **Diamond** centred **(x,y)**,.





Example 1: the pixels with **D_4=1** are the **4-nighbors** of **(x, y).**

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

3. D_8 distance (chess board distance)

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

- **square** – centred at **(x, y)**
- **D_8 = 1** are 8-neighbors of **(x,y)**

Example: **D_8 distance** ≤ 2

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

4. D_m distance:

- Is defined as the **shortest m-path** between the points.
- The distance between pixels depends only on the values of pixels.

Example: consider the following arrangement of pixels

$$\begin{array}{ccc} & P_3 & P_4 \\ P_1 & P_2 & \\ P & & \end{array}$$

and assume that P, P_2 have value 1 and that P_1 and P_3 can have a value of 0 or 1

Suppose, that we consider adjacency of pixels value 1 (**v={1})**

a) ***if P_1 and P_3 are 0:***

Then **D_m** distance = 2

b) ***if P_1 = 1 and P_3 = 0***

m-distance = 3;

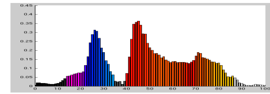
c) ***if P_1=0 ; and P_3 = 1***

d) ***if P1=P3 =1 ;***

m-distance=4 path = p p_1 p_2 p_3 p_4

$$\begin{array}{ccc} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{array} \qquad\qquad \begin{array}{ccc} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{array}$$

D_m=3 $\qquad$ D_m=2



Matlab Code

```
bw = zeros(200,200); bw(50,50) = 1; bw(50,150) = 1;
bw(150,100) = 1;
D1 = bwdist(bw,'euclidean');
D2 = bwdist(bw,'cityblock');
D3 = bwdist(bw,'chessboard');
D4 = bwdist(bw,'quasi-euclidean');
figure
subplot(2,2,1), subimage(mat2gray(D1)), title('Euclidean')
hold on, imcontour(D1)
subplot(2,2,2), subimage(mat2gray(D2)), title('City block')
hold on, imcontour(D2)
subplot(2,2,3), subimage(mat2gray(D3)), title('Chessboard')
hold on, imcontour(D3)
subplot(2,2,4), subimage(mat2gray(D4)), title('Quasi-Euclidean')
hold on, imcontour(D4)
```