

DIGITAL IMAGE PROCESSING

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DIGITAL IMAGE Basic Relationships between Pixels PROCESSING

Neighbors of a Pixel

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) & f(0,4) & - & - & - & - & - \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) & f(1,4) & - & - & - & - & - \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) & f(2,4) & - & - & - & - & - \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) & f(3,4) & - & - & - & - & - \\ I & I & I & I & I & - & - & - & - & - \\ I & I & I & I & I & - & - & - & - & - \end{bmatrix}$$

Neighbors of a Pixel

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) & f(0,4) & - & - & - & - & - \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) & f(1,4) & - & - & - & - & - \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) & f(2,4) & - & - & - & - & - \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) & f(3,4) & - & - & - & - & - \\ I & I & I & I & I & - & - & - & - & - \\ I & I & I & I & I & - & - & - & - & - \end{bmatrix}$$

□ A Pixel p at coordinates (x, y) has 4 horizontal and vertical neighbors.

□ Their coordinates are given by:

$$\begin{matrix} (x+1, y) & (x-1, y) & (x, y+1) & \& \\ (x, y-1) & & & \\ f(2,1) & f(0,1) & f(1,2) & \end{matrix}$$

$$f(1,0)$$

□ This set of pixels is called the 4-neighbors of p denoted by $N_4(p)$.

□ Each pixel is unit distance from (x, y) .

Neighbors of a Pixel

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) & f(0,4) & - & - & - & - \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) & f(1,4) & - & - & - & - \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) & f(2,4) & - & - & - & - \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) & f(3,4) & - & - & - & - \\ I & I & I & I & I & - & - & - & - \\ I & I & I & I & I & - & - & - & - \end{bmatrix}$$

□ A Pixel p at coordinates (x, y) has 4 diagonal neighbors.

□ Their coordinates are given by:

$$\begin{matrix} (x+1, y+1) & (x+1, y-1) & (x-1, y+1) & & & & & & & & \\ (x-1, y-1) & & & & & & & & & & \\ f(2,2) & & f(2,0) & & f(0,2) & & & & & & \end{matrix}$$

$f(0,0)$

□ This set of pixels is called the diagonal-neighbors of p denoted by $N_D(p)$.

□ diagonal neighbors + 4-neighbors = 8-neighbors of p .

Adjacency, Connectivity

Adjacency: Two pixels are adjacent if they are neighbors and their intensity level 'V' satisfy some specific criteria of similarity.

e.g. $V = \{1\}$

$V = \{0, 2\}$

Binary image = $\{0, 1\}$

Gray scale image = $\{0, 1, 2, \dots, 255\}$

In binary images, 2 pixels are adjacent if they are neighbors & have some intensity values either 0 or 1.

In gray scale, image contains more gray level values in range 0 to 255.

Adjacency, Connectivity

4-adjacency: Two pixels p and q with the values from set ' V ' are 4-adjacent if q is in the set of $N_4(p)$.

e.g. $V = \{0, 1\}$

1	1	2
1	1	0
1	0	1

p in **RED** color

q can be any value in **GREEN** color.

Adjacency, Connectivity

8-adjacency: Two pixels p and q with the values from set ' V ' are 8-adjacent if q is in the set of $N_8(p)$.

e.g. $V = \{1, 2\}$

0	1	1
0	2	0
0	0	1

p in **RED** color

q can be any value in **GREEN** color

Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set ' V ' are m-adjacent if

(i) q is in $N_4(p)$ OR

(ii) q is in $N_D(p)$ & the set $\underline{N_4(p) \cap N_4(q)}$ have no pixels whose values are from ' V '.

e.g. $V = \{1\}$

0 _a	1 _b	1 _c
0 _d	1 _e	0 _f
0 _g	0 _h	1 _i

Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$

e.g. $V = \{1\}$

(i) b & c

0 _a	1 _b	1 _c
0 _d	1 _e	0 _f
0 _g	0 _h	1 _i

Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$

e.g. $V = \{1\}$

(i) b & c

0 _a	1 _b	1 _c
0 _d	1 _e	0 _f
0 _g	0 _h	1 _i

Soln: b & c are m-adjacent.

Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$

e.g. $V = \{1\}$

(ii) b & e

0 _a	1 _b	1 _c
0 _d	1 _e	0 _f
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Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

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Soln: b & e are m-adjacent.

Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$ OR

e.g. $V = \{1\}$

(iii) e & i

0 _a	1 _b	1 _c
0 _d	1 _e	0 _f
0 _g	0 _h	1 _i

Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. $V = \{1\}$

(iii) e & i

0 _a	1 _b	1 _c
0 _d	1 _e	0 _f
0 _g	0 _h	1 _i

Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. $V = \{1\}$

(iii) e & i

0 _a	1 _b	1 _c
0 _d	1 _e	0 _f
0 _g	0 _h	1 _i

Soln: e & i are m-adjacent.

Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$ OR

(ii) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. $V = \{1\}$

(iv) e & c

0 _a	1 _b	1 _c
0 _d	1 _e	0 _f
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Adjacency, Connectivity

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$ OR

(ii) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. $V = \{1\}$

(iv) e & c

0 _a	1 _b	1 _c
0 _d	1 _e	0 _f
0 _g	0 _h	1 _i

Soln: e & c are NOT m-adjacent.

Adjacency, Connectivity

Connectivity: 2 pixels are said to be connected if there exists a path between them.

Let 'S' represent subset of pixels in an image.

Two pixels p & q are said to be connected in 'S' if there exists a path between them consisting entirely of pixels in 'S'.

For any pixel p in S, the set of pixels that are connected to it in S is called a connected component of S.

Paths

Paths: A path from pixel p with coordinate (x, y) with pixel q with coordinate (s, t) is a sequence of distinct sequence with coordinates $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ where

$$(x, y) = (x_0, y_0)$$

$$\& (s, t) = (x_n, y_n)$$

Closed path: $(x_0, y_0) = (x_n, y_n)$

Paths

Example # 1: Consider the image segment shown in figure.
Compute length of the *shortest-4*, *shortest-8* & *shortest-m paths*
between pixels p & q where,
 $V = \{1, 2\}$.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

Paths

Example # 1:

Shortest-4 path:

$V = \{1, 2\}$.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

Paths

Example # 1:

Shortest-4 path:

$V = \{1, 2\}$.

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Paths

Example # 1:

Shortest-4 path:

$V = \{1, 2\}$.



Paths

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Shortest-4 path:

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Paths

Example # 1:

Shortest-4 path:

$V = \{1, 2\}$.



Paths

Example # 1:

Shortest-4 path:

$V = \{1, 2\}$.



So, Path does not exist.

Paths

Example # 1:

Shortest-8 path:

$V = \{1, 2\}$.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

Paths

Example # 1:

Shortest-8 path:

$V = \{1, 2\}$.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

Paths

Example # 1:

Shortest-8 path:

$V = \{1, 2\}$.



Paths

Example # 1:

Shortest-8 path:

$V = \{1, 2\}$.



Paths

Example # 1:

Shortest-8 path:

$V = \{1, 2\}$.



Paths

Example # 1:

Shortest-8 path:

$V = \{1, 2\}$.



So, shortest-8 path = 4

Paths

Example # 1:

Shortest-m path:

$V = \{1, 2\}$.

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Paths

Example # 1:

Shortest-m path:

$V = \{1, 2\}$.

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Paths

Example # 1:

Shortest-m path:

$V = \{1, 2\}$.

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Paths

Example # 1:

Shortest-m path:

$V = \{1, 2\}$.

4	2	3	2	q
3	3	1	3	
2	3	2	2	
p	2	1	2	3

Diagram illustrating a grid of numbers with a path highlighted in red. The path starts at 'p' (row 4, column 1), moves right to 2 (row 4, column 2), then up to 1 (row 3, column 2), and finally right to 2 (row 3, column 3). The path ends at 'q' (row 1, column 4). The red numbers are 2, 1, and 2.

Paths

Example # 1:

Shortest-m path:

$V = \{1, 2\}$.



Paths

Example # 1:

Shortest-m path:

$V = \{1, 2\}$.



Paths

Example # 1:

Shortest-m path:

$V = \{1, 2\}$.



So, shortest-m path = 5

Regions & Boundaries

Region: Let R be a subset of pixels in an image. Two regions R_i and R_j are said to be adjacent if their union form a connected set.

Regions that are not adjacent are said to be disjoint.

We consider 4- and 8- adjacency when referring to regions.

Below regions are adjacent only if 8-adjacency is used.

1	1	1	
1	0	1	R_i
0	1	0	
0	0	1	
1	1	1	R_j
1	1	1	

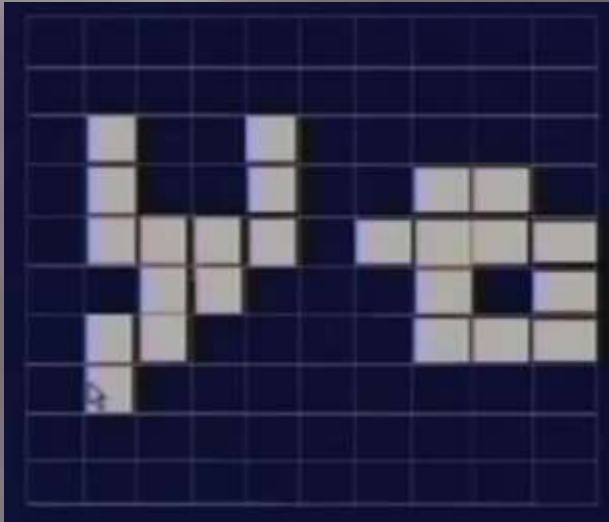
Regions & Boundaries

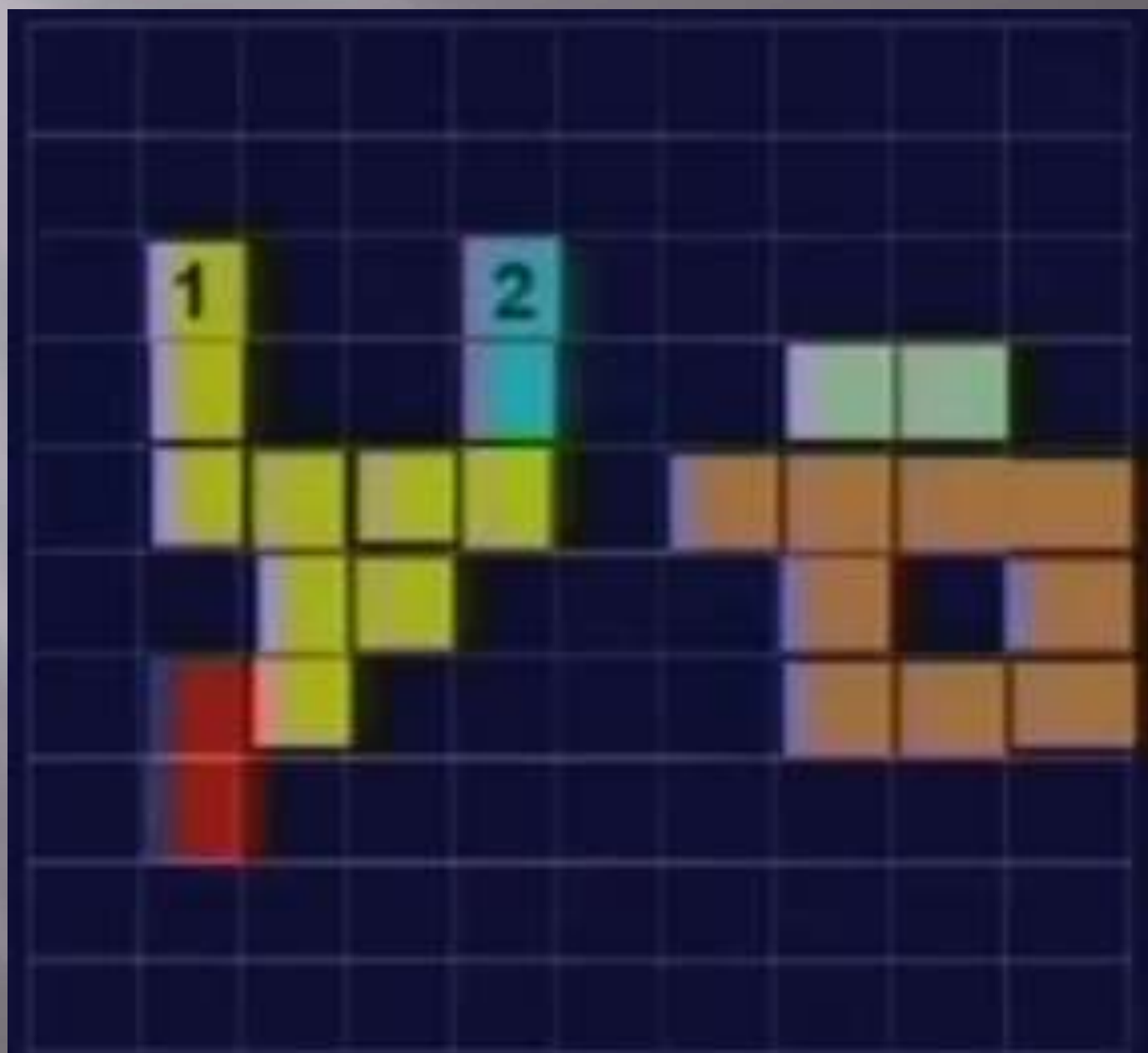
Boundaries (border or contour): The boundary of a region R is the set of points that are adjacent to points in the complement of R.

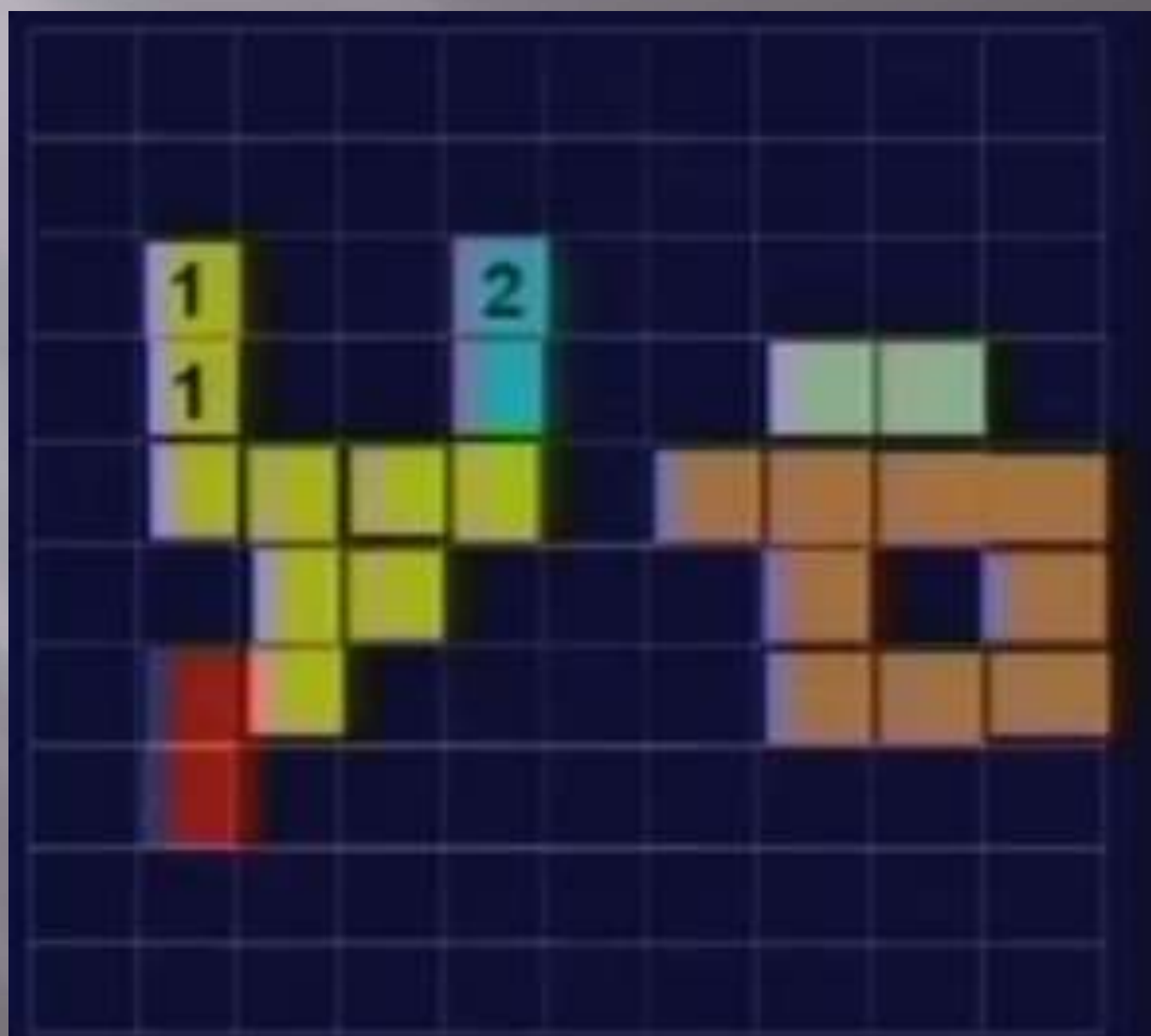
0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

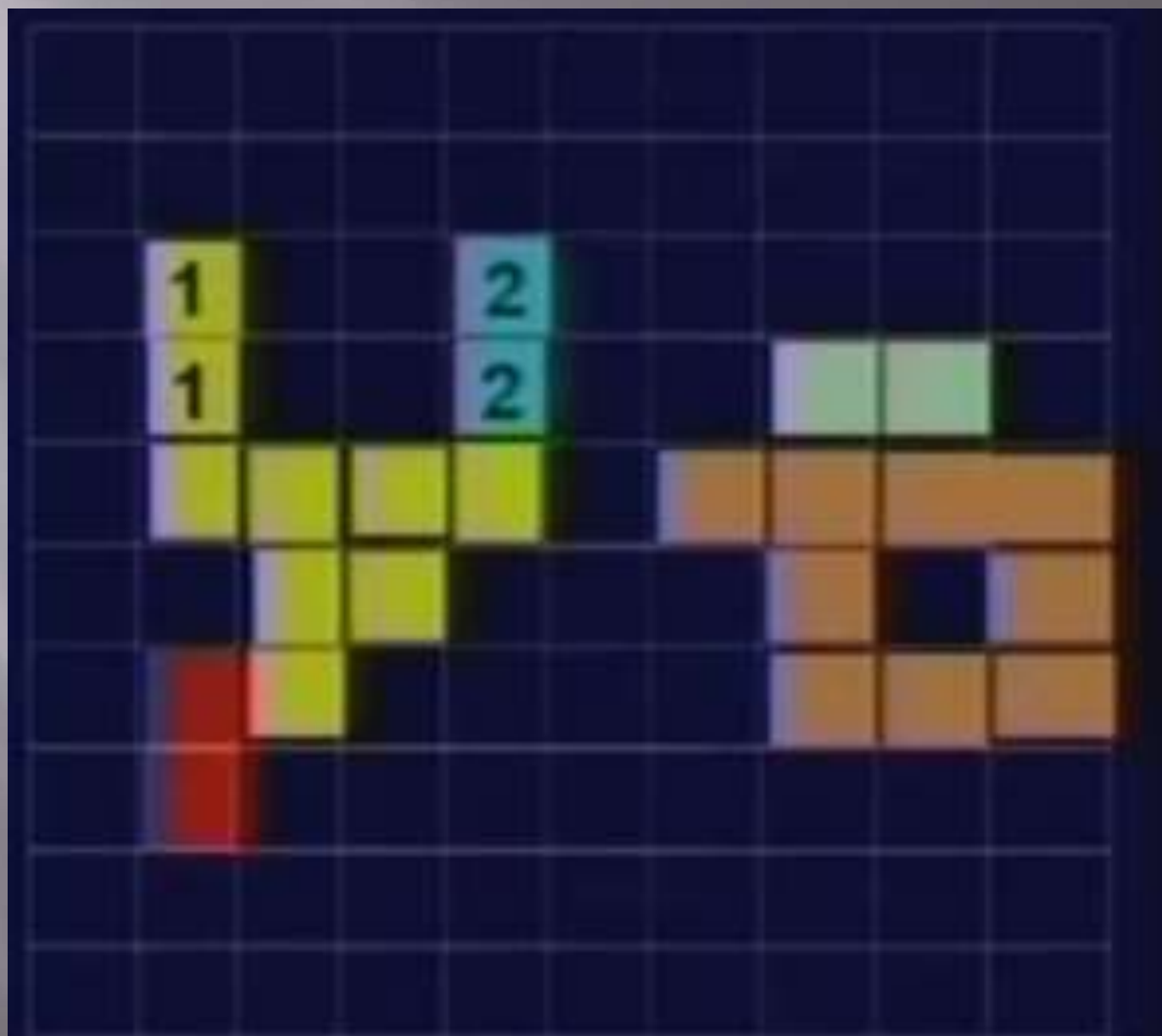
RED colored 1 is NOT a member of border if 4-connectivity is used between region and background. It is if 8-connectivity is used.

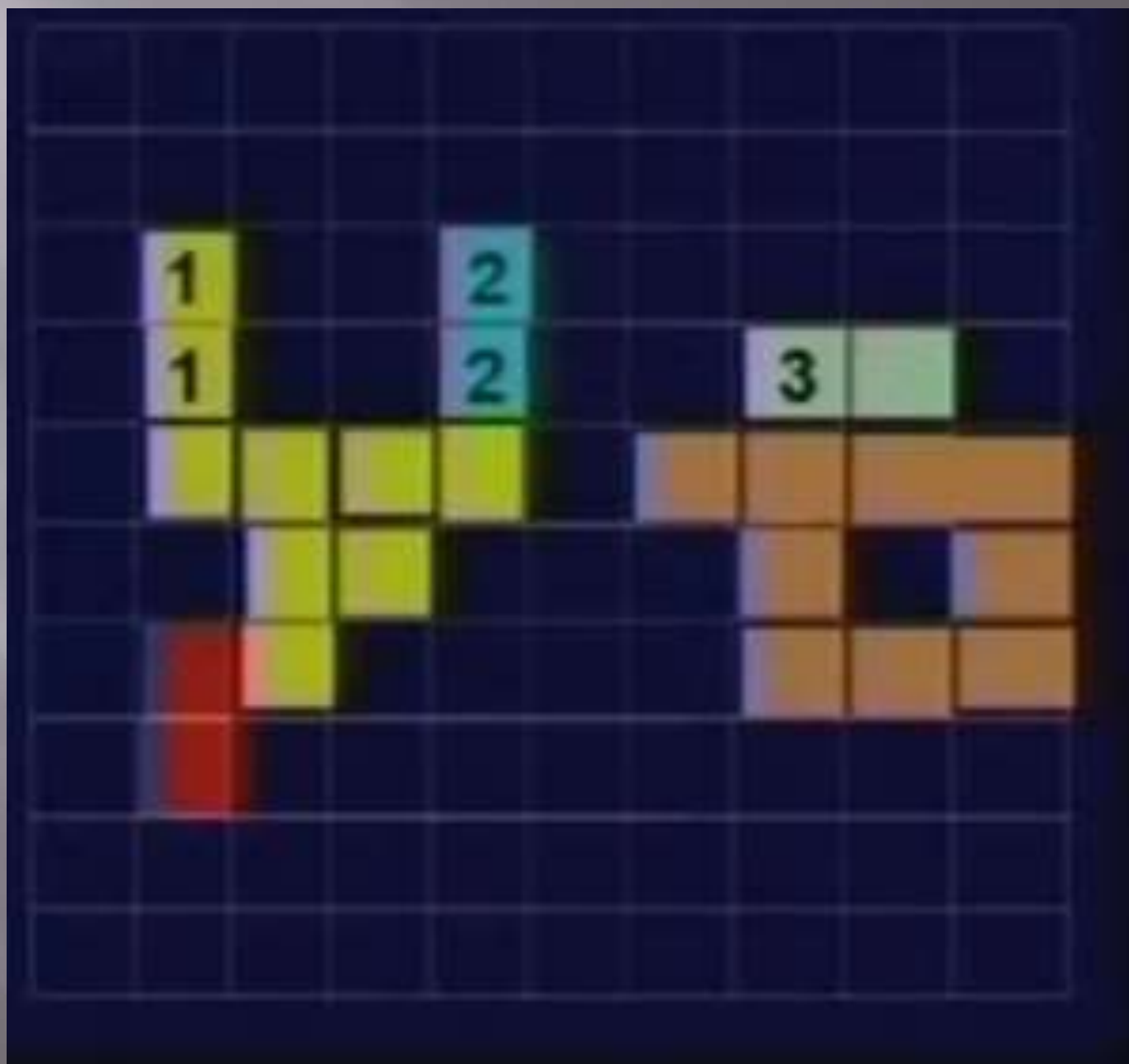
Example:

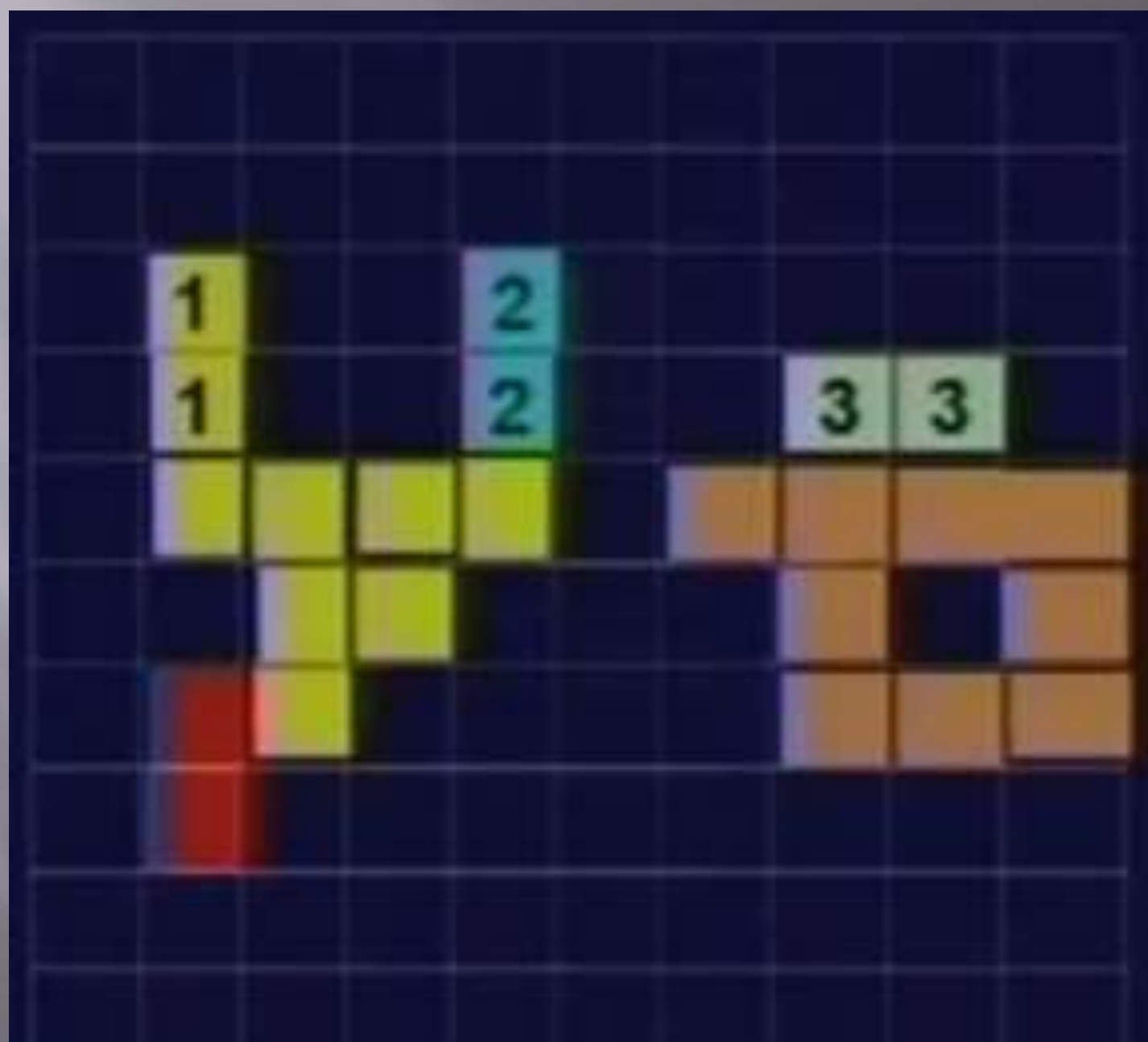




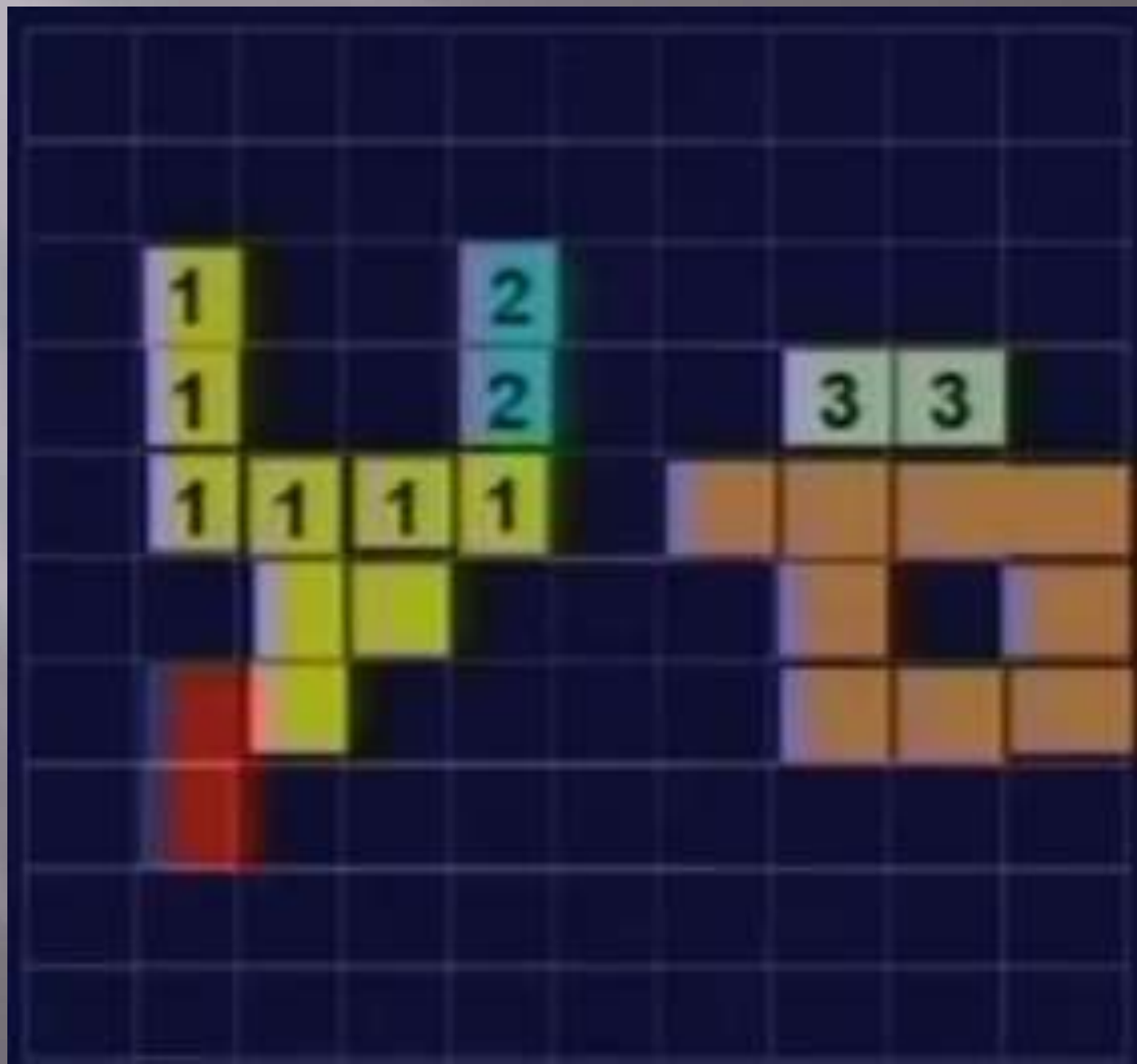


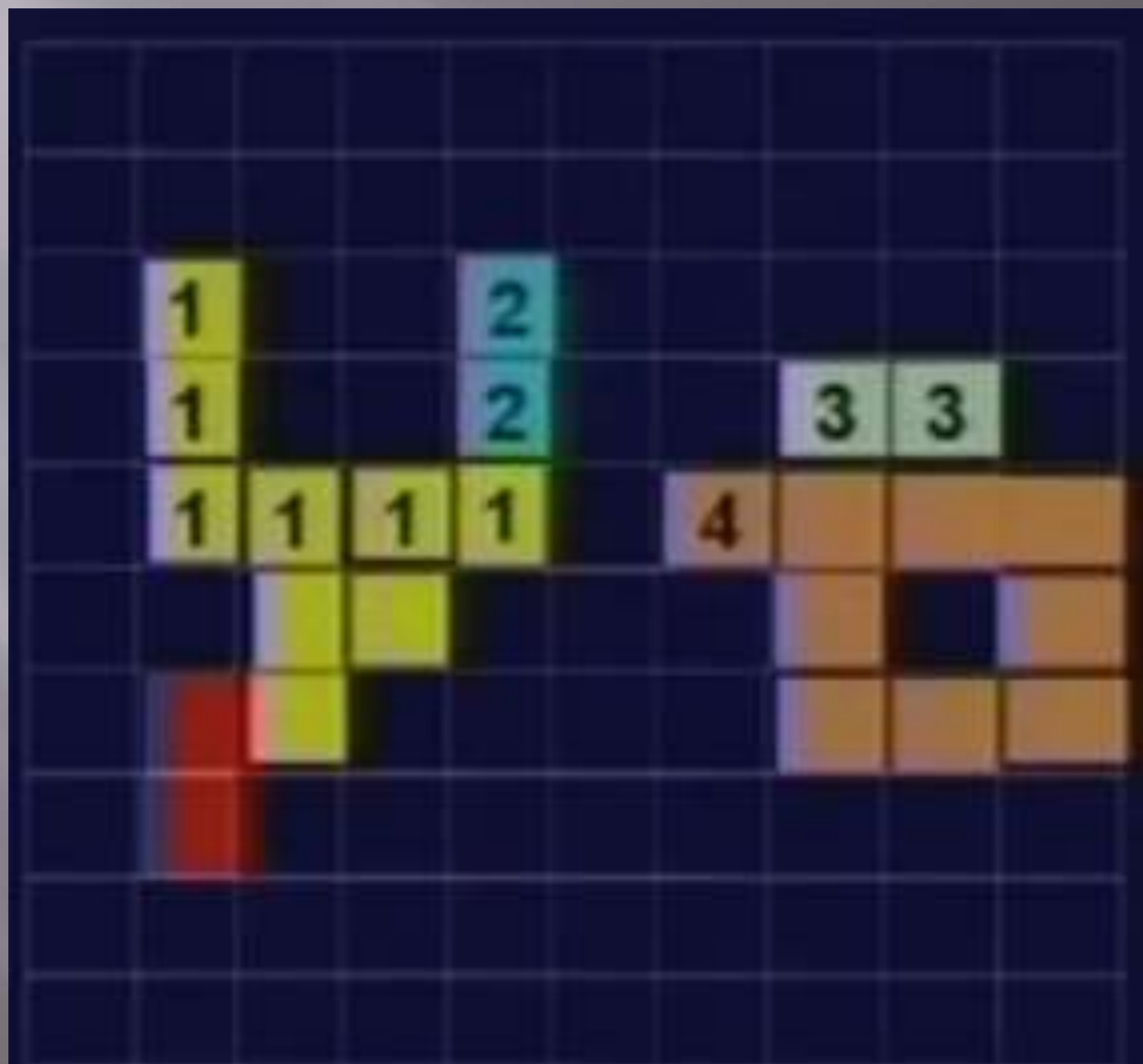




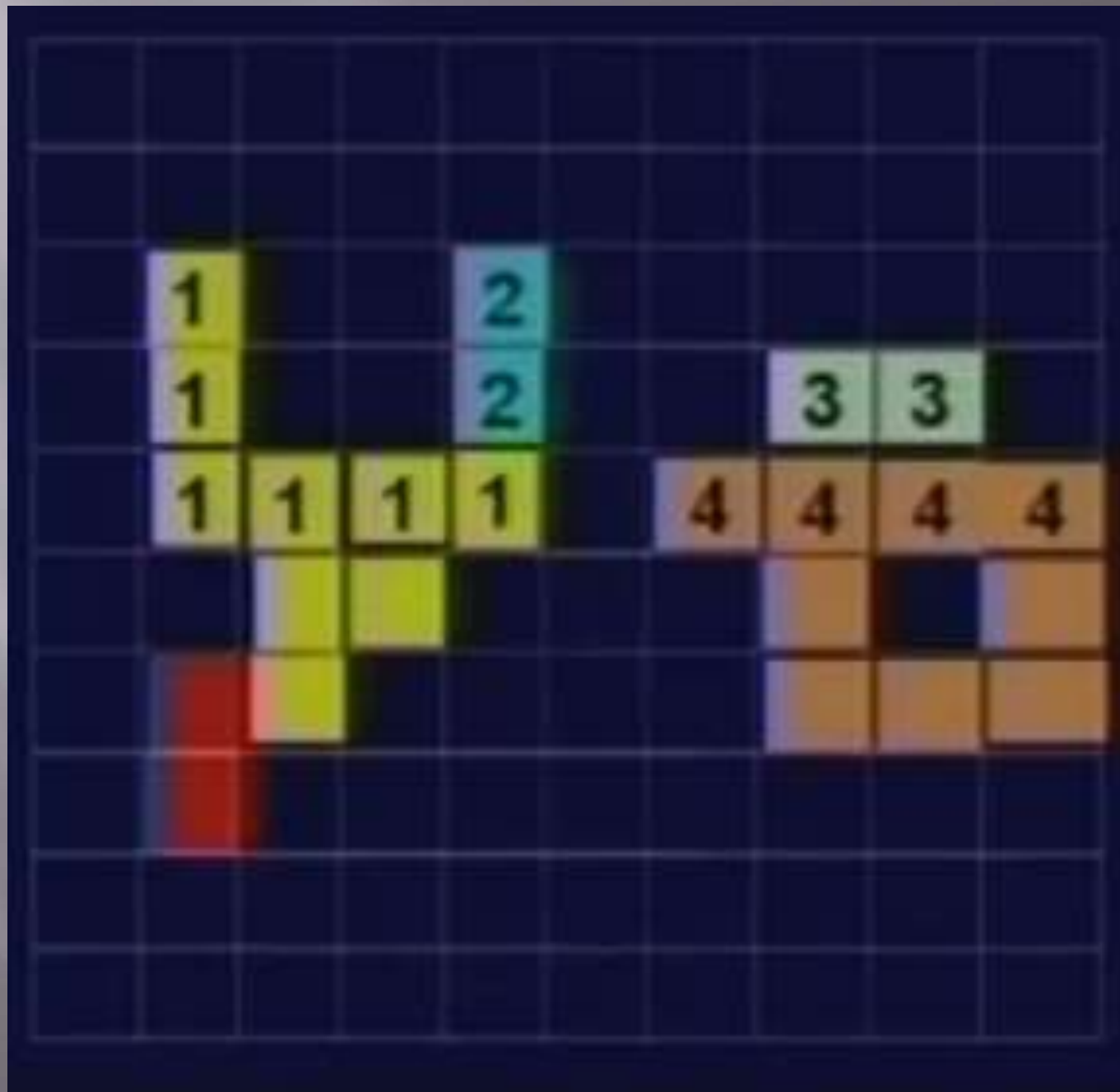


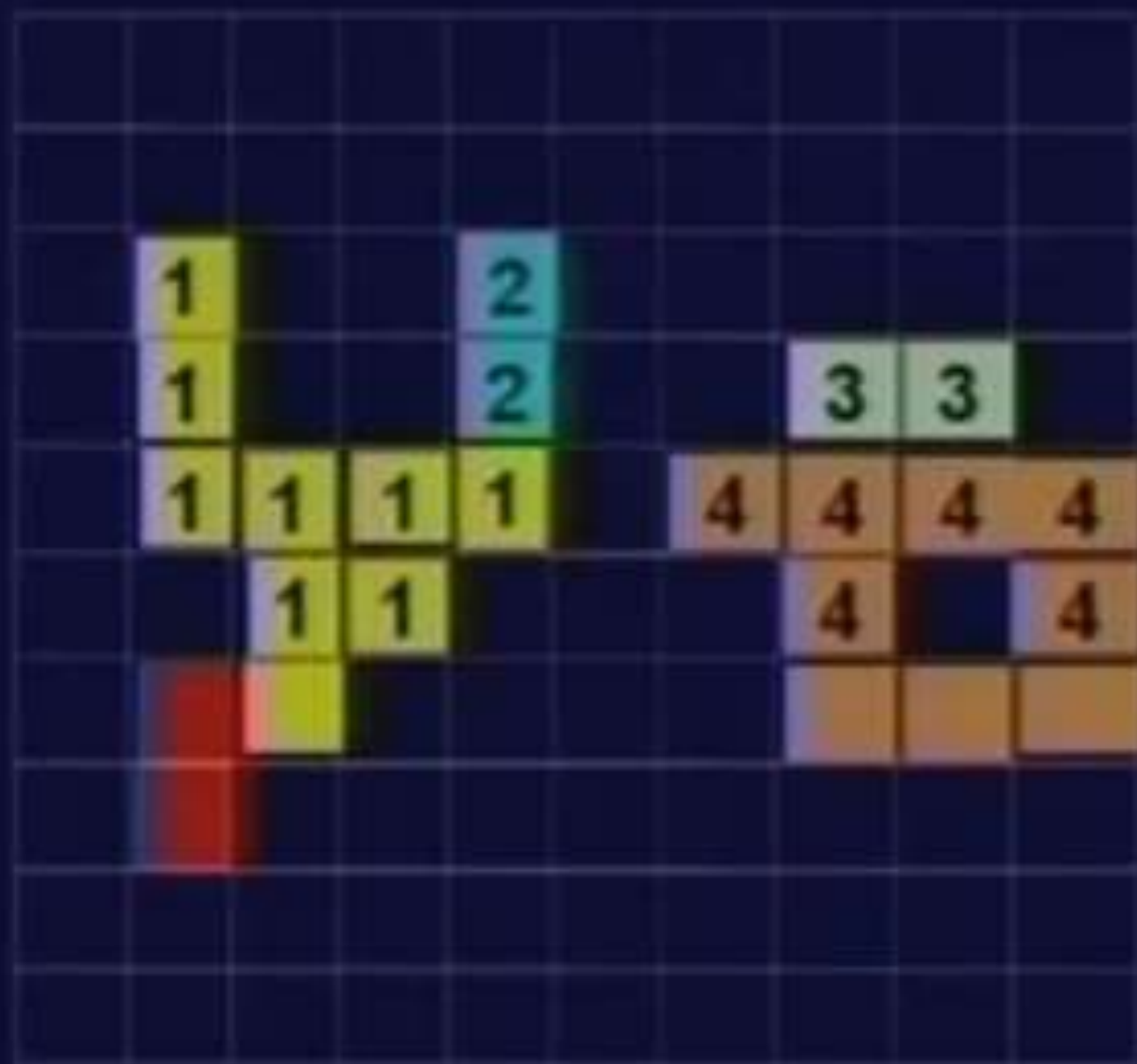
(1=2)

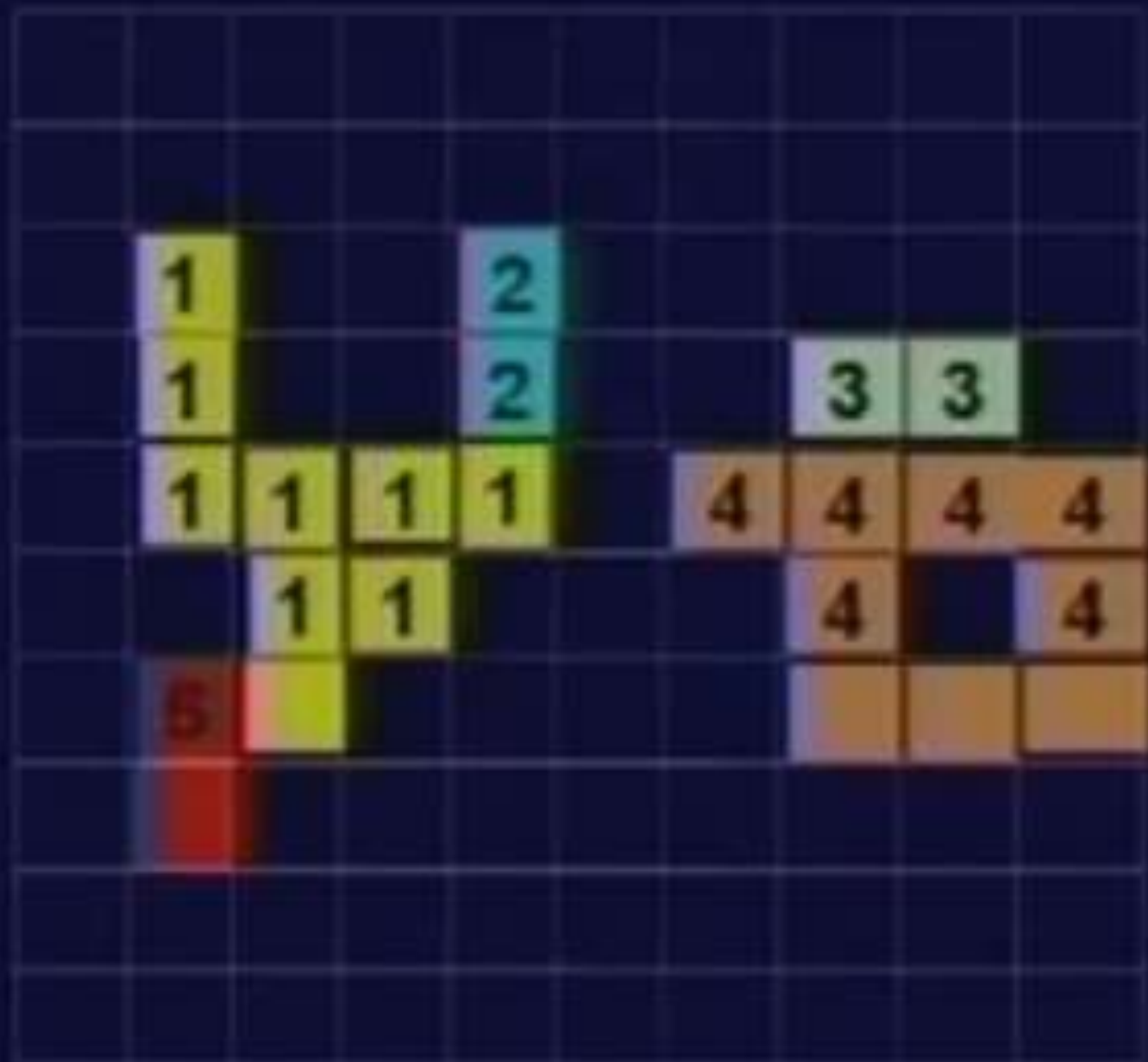




(3=4)

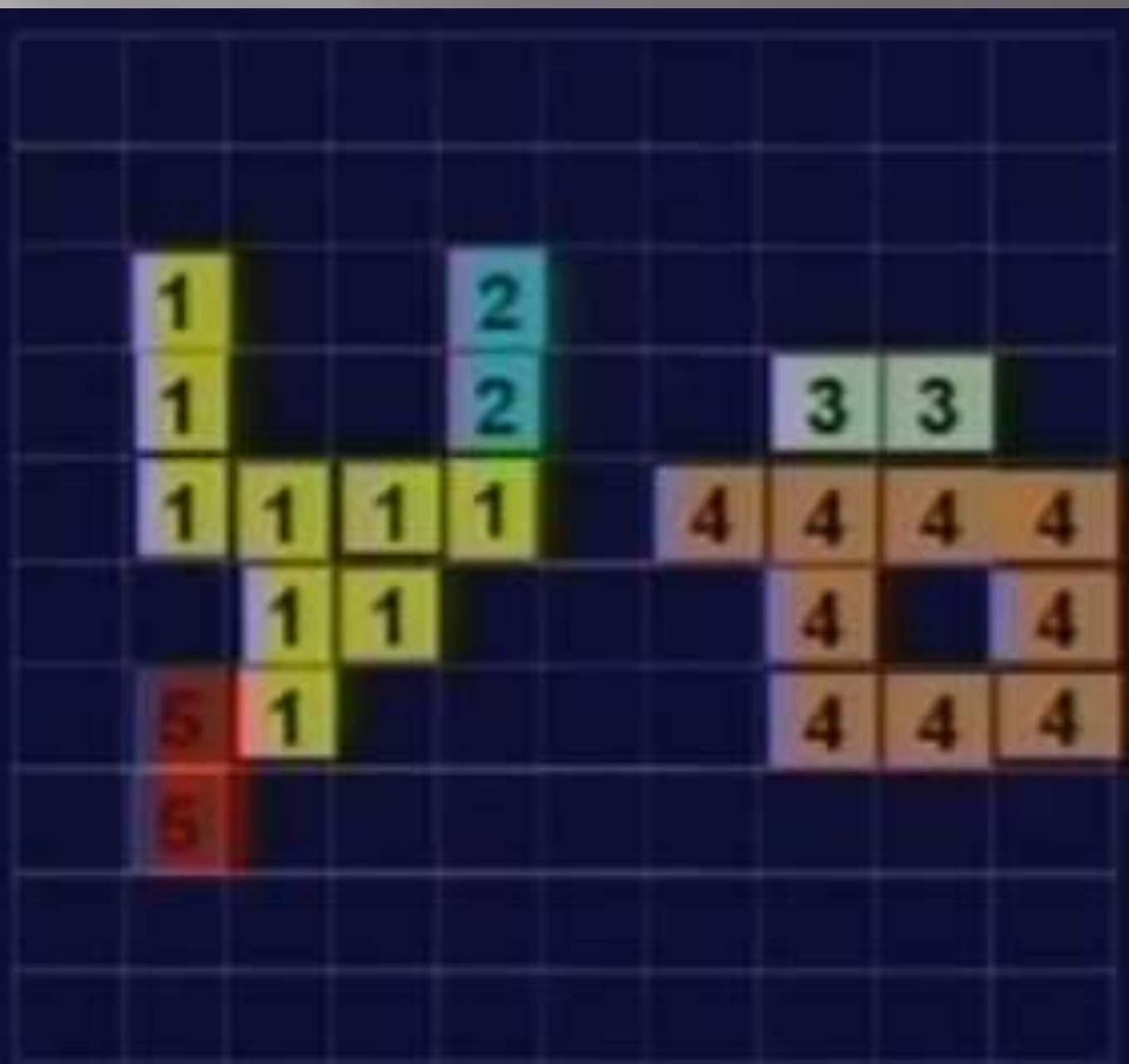


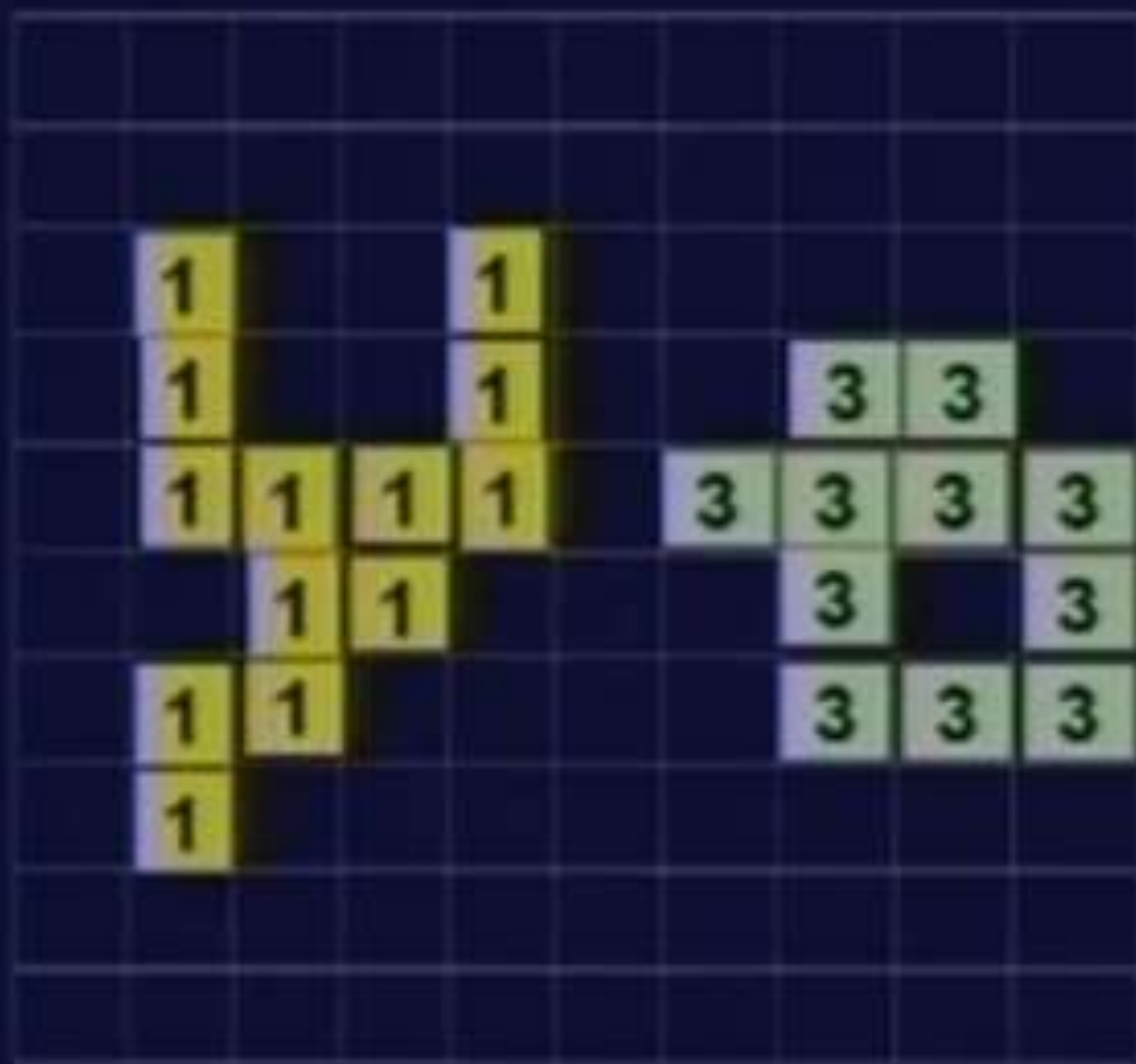




(1=5)







Distance Measures

Distance Measures: Distance between pixels p , q & z with co-ordinates (x, y) , (s, t) & (v, w) resp. is given by:

- a) $D(p, q) \geq 0$ [$D(p, q) = 0$ if $p = q$]called reflexivity
- b) $D(p, q) = D(q, p)$ called symmetry
- c) $D(p, z) \leq D(p, q) + D(q, z)$ called transmitivity

Euclidean distance between p & q is defined as-

$$D_e(p, q) = [(x - s)^2 + (y - t)^2]^{1/2}$$

Distance Measures

City Block Distance: The D_4 distance between p & q is defined as

$$D_4(p, q) = |x - s| + |y - t|$$

In this case, pixels having D_4 distance from (x, y) less than or equal to some value r form a diamond centered at (x, y) .



Pixels with D_4 distance ≤ 2 forms the following contour of constant distance.

Distance Measures

Chess-Board Distance: The D_8 distance between p & q is defined as

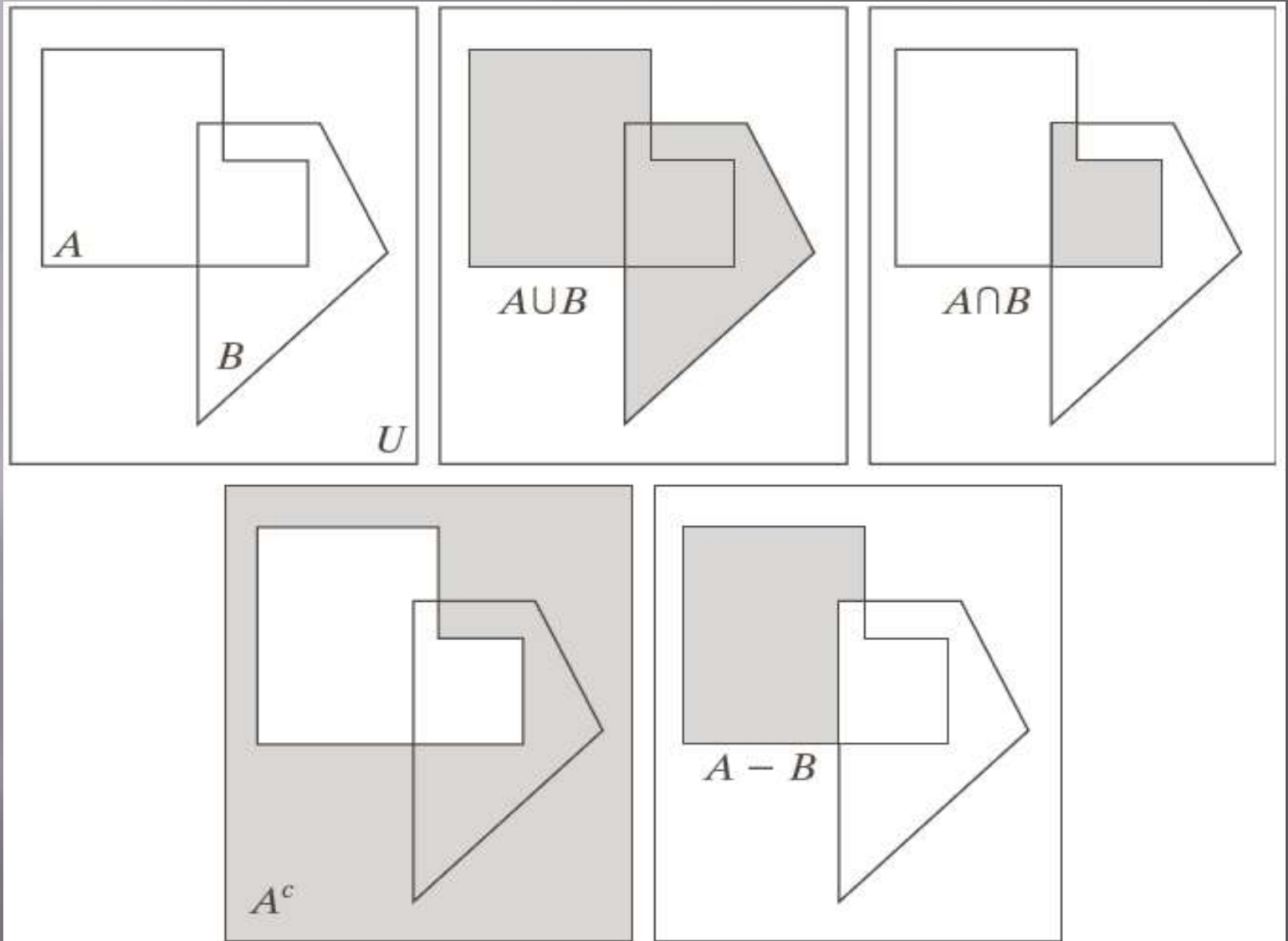
$$D_8(p, q) = \max(|x - s|, |y - t|)$$

In this case, pixels having D_8 distance from (x, y) less than or equal to some value r form a square centered at (x, y) .

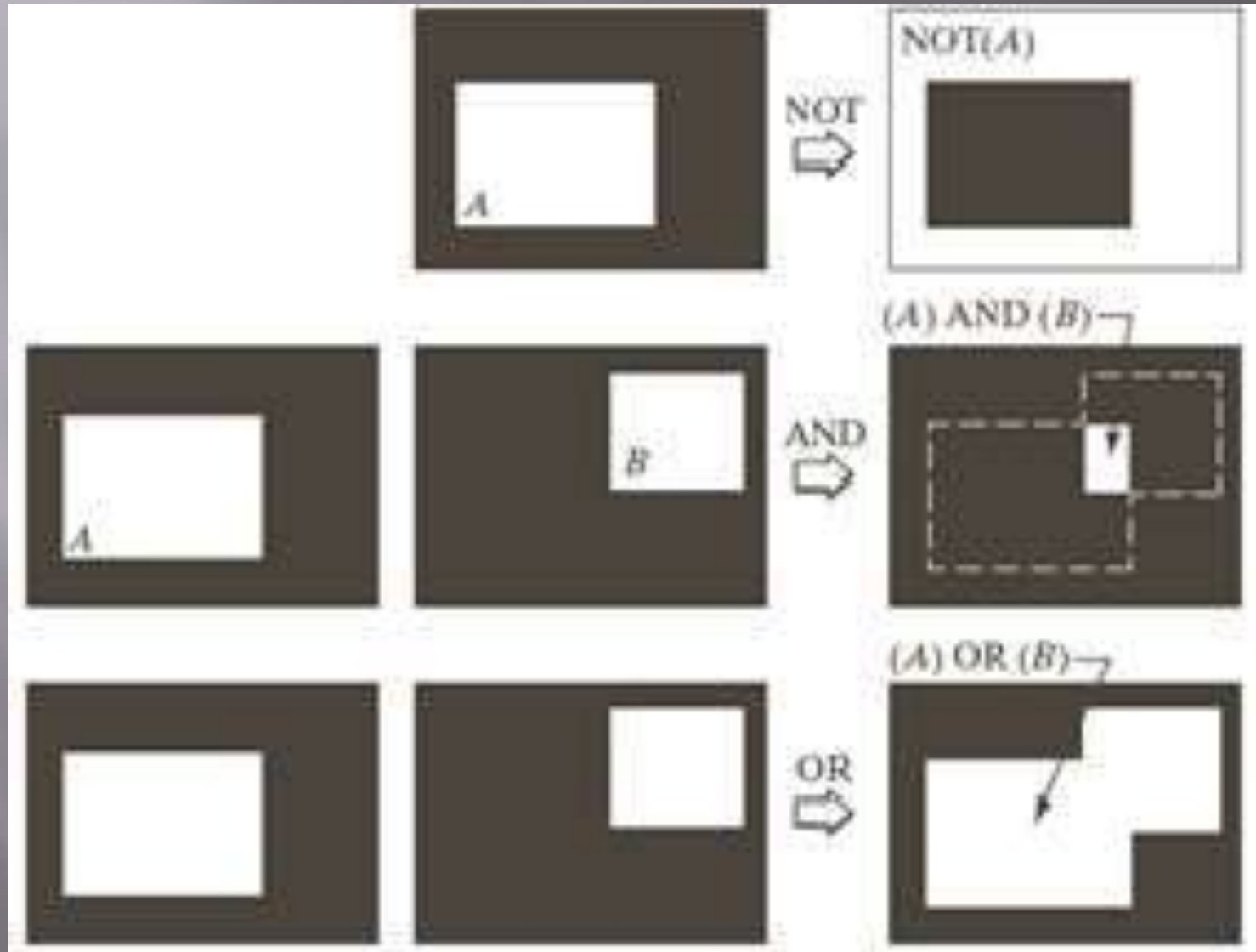
2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with D_8 distance ≤ 2 forms the following contour of constant distance.

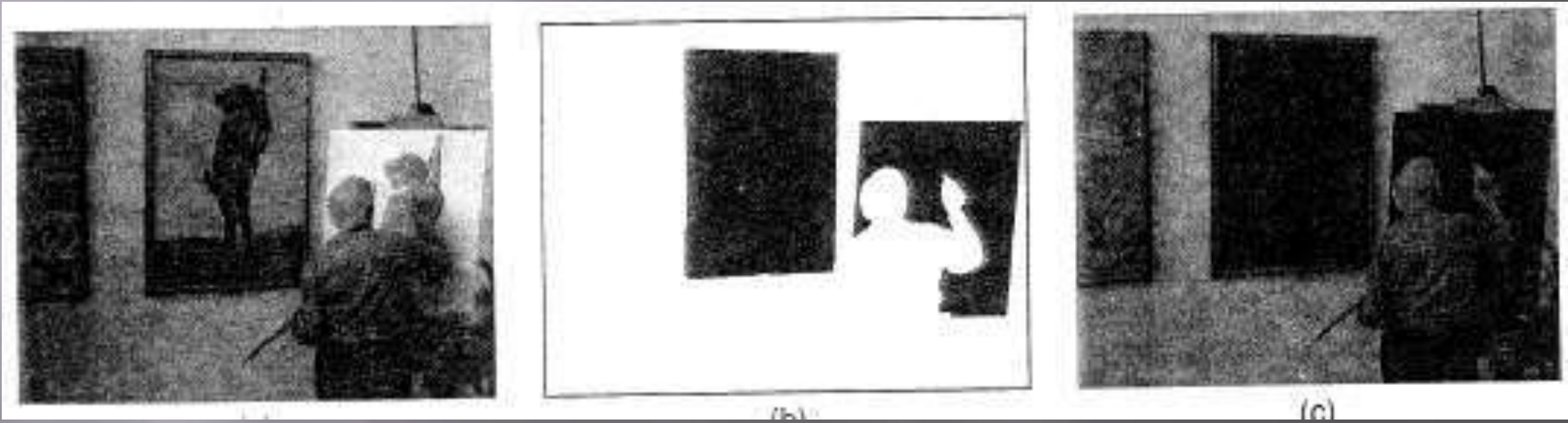
Set operations



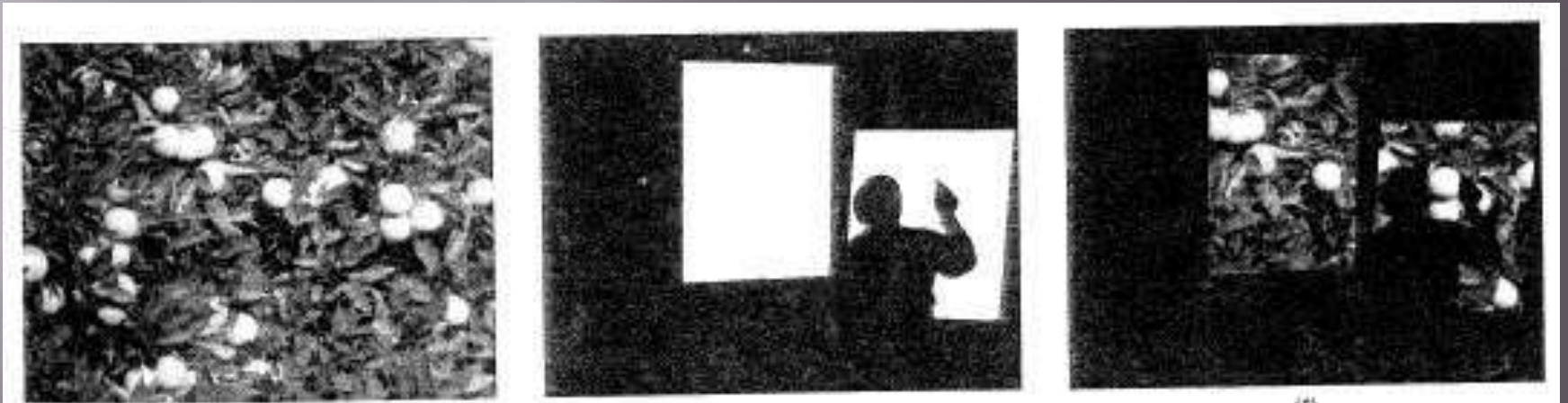
Logical operations



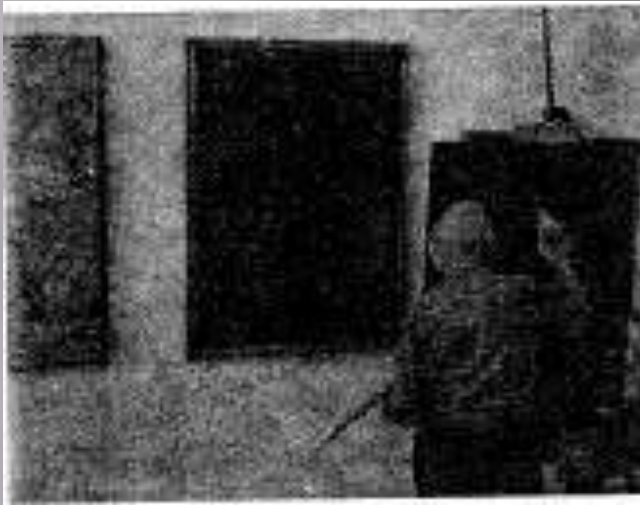
- ▣ The AND operator is usually used to mask out part of an image.



- ▣ Parts of another image can be added with a logical OR operator.



Result of AND



Result of OR



OR





(A) AND (B)



AND-
NOT
⇒

$(A) \text{ AND } [\text{NOT } (B)] \rightarrow$



(A) XOR (B)



XOR
⇒

$(A) \text{ XOR } (B) \rightarrow$

