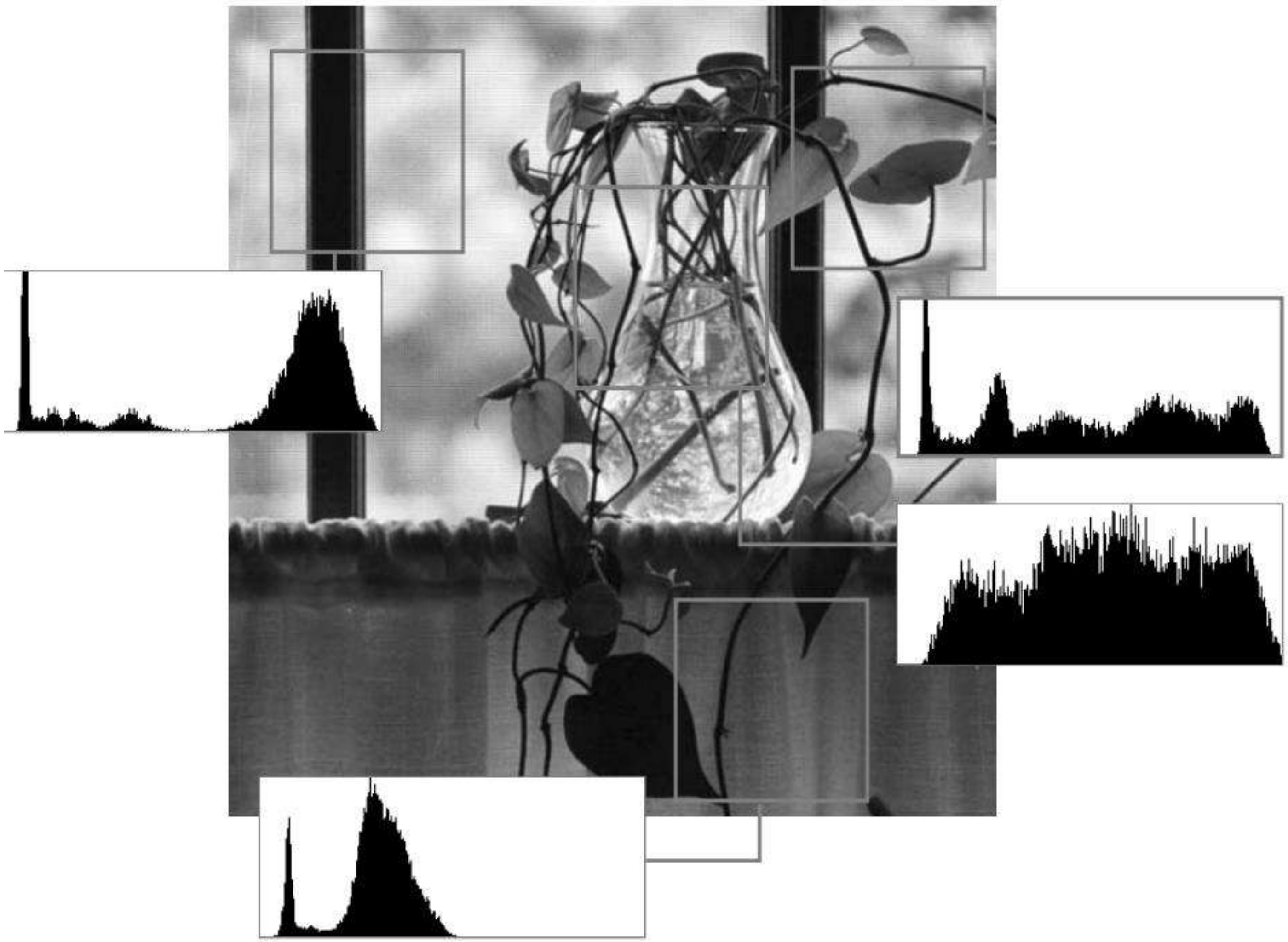
Wavelet and Multi Resolution Processing

NEED OF WAVELETS, MULTI RESOLUTION TRANSFORMATIONS

- When ever we look at the Image we see connected regions of Similar Texture and Grey levels are combined to form an Object.
  - In an image we have similar textures and grey levels are combined seen to perceive an Object.
- If Objects in image are very small in Size (or) low in contrast, we normally examine them using High Resolution.
- If Objects in image are Large in size or High in Contrast then coarse view is preferred.
- If image has small size objects and large size objects (or) Low in contrast objects & High in contrast objects it will be advantageous to study them at different Resolutions of an Image.
- This is the Fundamental motivation for Mutiple Resolution processing.

Complex image example



An image and A Local histogram variations are shown at different parts of image here.

So a statistical modelling is very difficult to do for image processing.

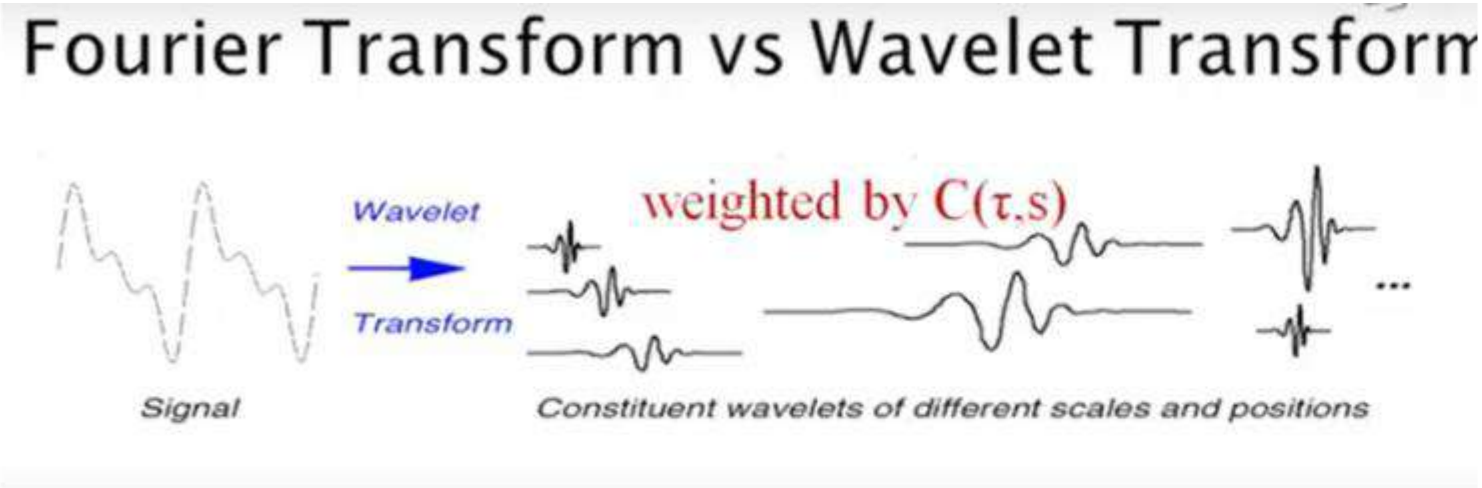
Wavelets history

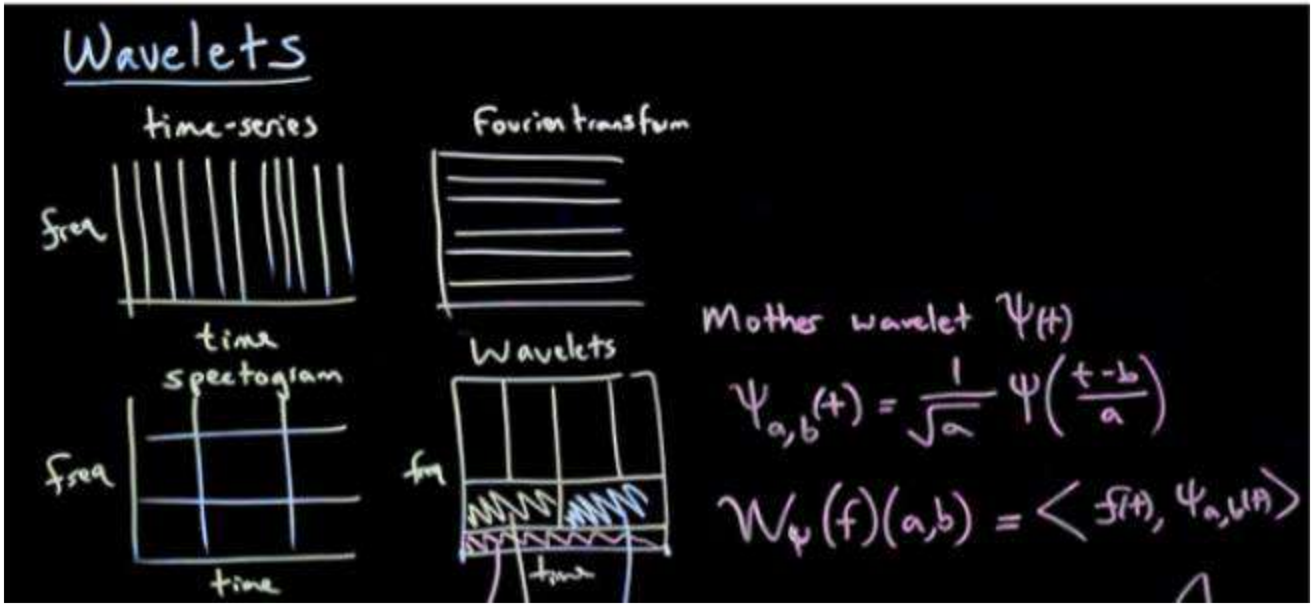
- Wavelet means a Waveform of Limited duration (some variable Bandwidth) that has an average value of ZERO
- In this lecture understand Wavelets
- F.T is used in Image processing since 1950's (for Frequecy domain)
- Wavelet transforms (WT) are used in Image processing because it capture both Temporal and Frequency aspects
- WT has made easier to compress, Analyse and Transmit images. This is the basic advantage of WT compared to F.T
- In F.T the basic functions are Sinusoids, and In W.T has small waves that are known as Wavelets of varying Frequencies and limited Duration.
- W.T uses Small Waves which are known as Wavelet of varying Frequencies and limited Duration.

Wavelets history

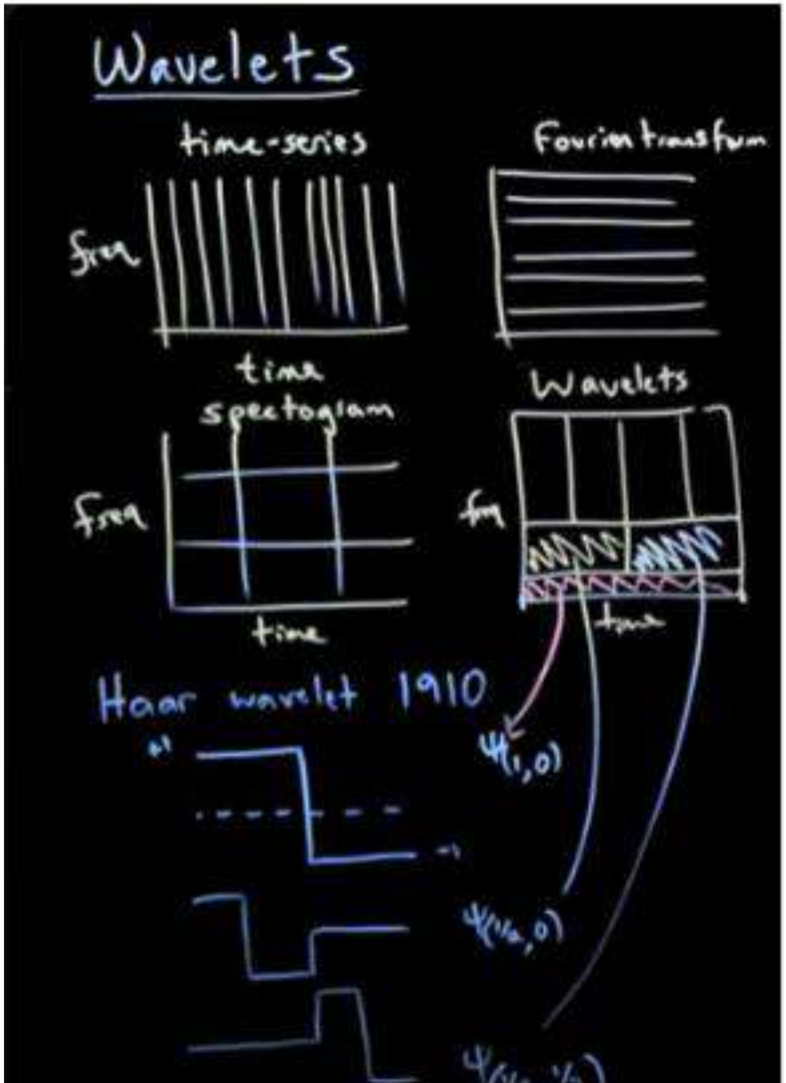
- In the year 1987, wavelets were shown as powerful approach to Signal processing and Analysis which is known as Multiresolution Theory.
- The multi resolution Theory incorporates → Sub band coding, Quadrature mirror Filtering, Pyramid Image processing, HAAR transformations
- The Mulit Resolution (MR) Theory i.e Multiple Resolution → As name implies the MR used in representation and analysis of signals at more than One Resolution

Background of Wavelets





Background of Haar Wavelets



- The HAAR Transform is based on a class of Orghogonal matrices whose elements are either 1, -1, 0 or multiplied by power of $x = \frac{\sqrt{2}}{1}$

The HAAR transform is computationally efficient transform as the Transform of an N-point vector requiresonly 2(N-1) additions and N multiplications.

HAAR TRANSFORMATIONS

- This is used for Multi Resoution analysis of a Digital Image. This is another technique for analysis.
- HAAR uses simplest Orthonormal Wavelets
- HAAR Transform is seperable and Symmetric
- HAAR transform can be represented in the form of a Matrix
- We can represent Haar transform as T = HFH where F is NxN Matrix, H is NxN Transformation Matrix, T is NxN Resulting Matrix
- Here Transformation H contains the Haar basic Function $h_k(z)$,
- These functions are defined over $z \in [0,1]$ for $k = 0,1,2,\ldots N-1$, here $N = 2^n$

HAAR TRANSFORMATIONS: PROCEDURE

- First we need to find the Order N; n = $log_2 N$
- Find p & q where $0 \leq p \leq n-1$
- if p = 0 then q = 1 (or) 0
  $p \neq 0$ then $1 \leq q \leq 2^p$
- then
  - Find the valeu of k, where $k = 2^p + q - 1$ and x = 0/n, 1/n, 2/n,…n-1/n

Unique decomposition of Integer $k \Leftrightarrow (p,q)$

$k = 0,1,2,\ldots.N-1$ *with* $N = 2^n$, $0 <= p <= n-1$

$q = 0,1$ *(for* $p=0$*);*

$1 <= q <= 2^p$ *(for* $p>0$*)*

Ex: $k=0$ *then* $p=0$, $q=0$

$k=1$ *then* $p=0$, $q=1$; $k=2$ *then* $p=1$, $q=1$; $k=3$ *then* $p=1$, $q=2$ ……

$h_k(x) = h_{p,q}(x)$ *for* $x \in = [0,1]$

$h_0(x) = h_{0,0}(x) = 1/\sqrt{N}$ for $x \in [0,1]$

- if $k \neq 0$ $h_k(x) = h_{p,q}(x) = \begin{cases} 1/\sqrt{N}\ 2^{p/2} & \text{if } (q-1)/2^p \leq x \leq (q-1/2)/2^p \\ -1/\sqrt{N}\ 2^{p/2} & \text{if } (q-1/2)/2^p \leq x \leq q/2^p \\ 0 & \text{if } x \in (0,1) \end{cases}$

Haar transformation

- Lets us solve equation with example.
- n = Log N, For N=2, log 2 then n = 1
- for p = 0, ∴ q = 0 or 1
- k = 2^p + q - 1
- Construct a Logic table p, q, and k, Here we found 2 values of k because N=2
- when p = 0, q=0 then k=0
- when p = 0, q=1 then k=1
- Haar Matrix can be given as
- H2 =

$$\begin{array}{cc} & \begin{array}{cc} z=(0/2) & z=(1/2) \end{array} \\ \begin{array}{c} k=0 \\ k=1 \end{array} & \begin{bmatrix} h_0(0/2) & h_0(1/2) \\ h_1(0/2) & h_1(1/2) \end{bmatrix} \end{array}$$

These terms are written based on the vlaues of k and z

p	q	k
0	0	0
0	1	1

HAAR TRANSFORMATION (KERNEL/H-MATRIX)

- $$H2 = \begin{array}{c c} & \begin{array}{cc} z=(0/2) & z=(1/2) \end{array} \\ \begin{array}{c} k=0 \\ k=1 \end{array} & \begin{bmatrix} h_0(0/2) & h_0(1/2) \\ h_1(0/2) & h_1(1/2) \end{bmatrix} \end{array}$$

$\therefore$ $h_0(0/2) = 1/\sqrt{N} = 1/\sqrt{2}$,

$h_0(1/2) = 1/\sqrt{2}$

$h_1(0/2) = 2^{p/2}/\sqrt{N} = 2^{0/2}/\sqrt{2} = 1/\sqrt{2}$

$h_1(1/2) = -\ 2^{p/2}/\sqrt{N} = -2^{0/2}/\sqrt{2} = -1/\sqrt{2}$

$$\therefore H2 = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \Rightarrow H2 = 1/\sqrt{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

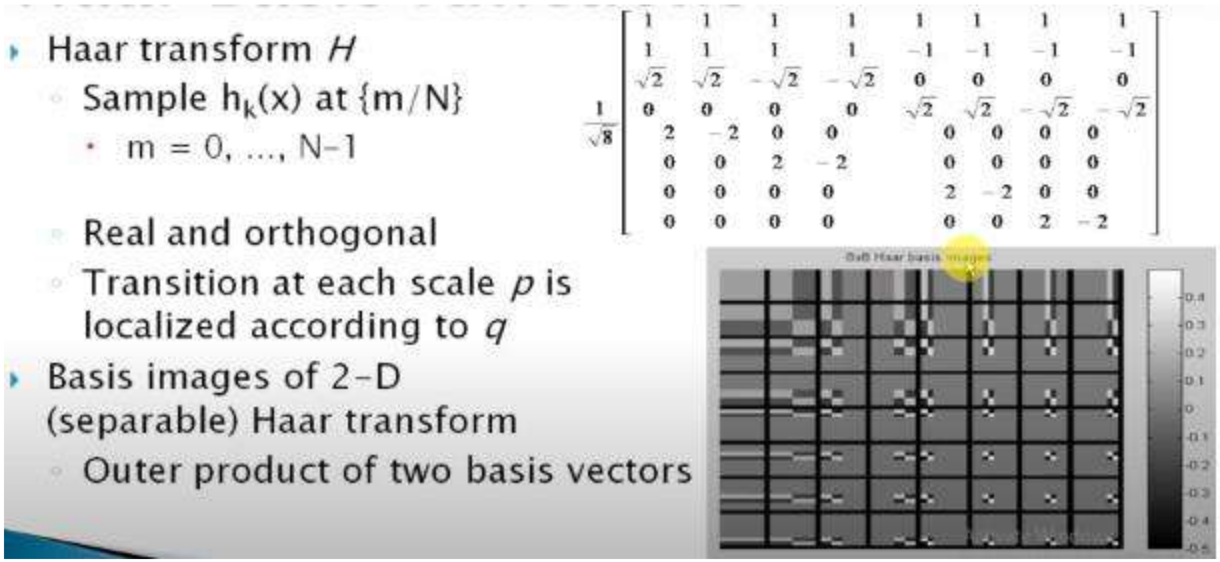
p	q	k
0	0	0
0	1	1

Similarly we can do for N=4…

Assignment 2 (option)

- Solve the Haar Transform matrix of order N=4?
- Solve the Haar Transform matrix of order N=8?
- Write a CV program to determine Haar Transform Matrix.
- Write a CV program to perform Haar Transformation T = HFH

HAAR TRANSFORMATION FOR ORDER N=8



The HAAR transformation matrices/Kernel can be used to extract features of the Image or objects of the image using varieties of the HAAR transformation matrices.

For ex: Extracting Right Eye, Left Eye etc

Image Pyramid (Multi resolution extraction)

- Image Pyramid is a representation of Image in more than One Resolution.
- We can say a powerful and simple structure to repsent the image at more than one Resolution and arranged in Pyramid shape as shown in the Image.
- An Image Pyramid is a collection of decreasing Image resolution arranged in a stack to take shape of Pyramid. With Apex as level 0 (image size (1x1). at the Base is at level J (image size NxN), all in between Levels are marked J-1, J-2…Level2, Level1 having correpsponding Image sizes : N/2XN/2, N/4XN/4, …4x4, 2x2 respectively.

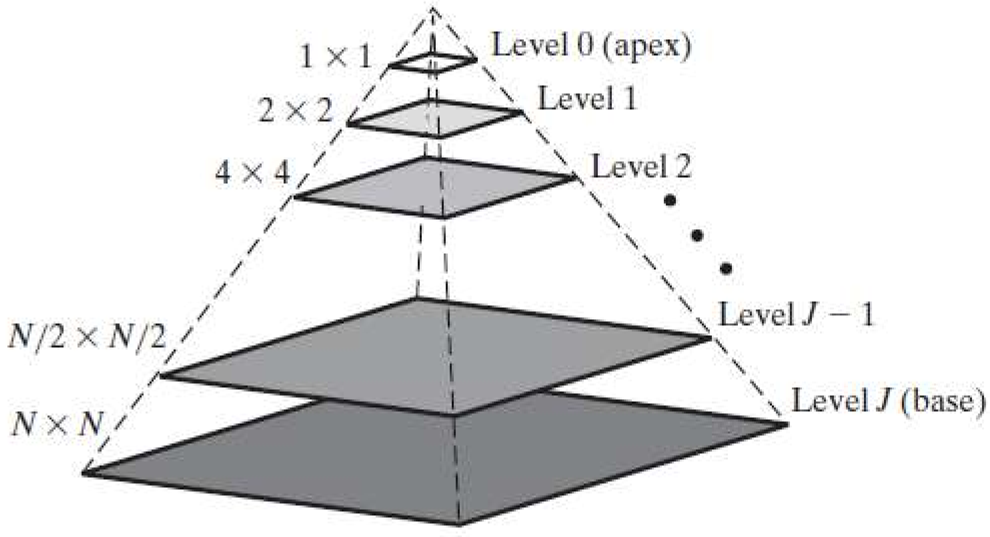


Image Pyramid

- This image shows the Base of Pyramid has High Resolution image and the Apex of the Pyramid is of Low approximation. If we move up in the Pyramid Both the Resolution and the size of the image is halved.
- Lets consider a Base level image is having NxN same as $2^J \times 2^J$ Where $J = \log_2 N$
- The intermediate levels is of size j ➔ $2^j \times 2^j$ where $0 <= j <= J$

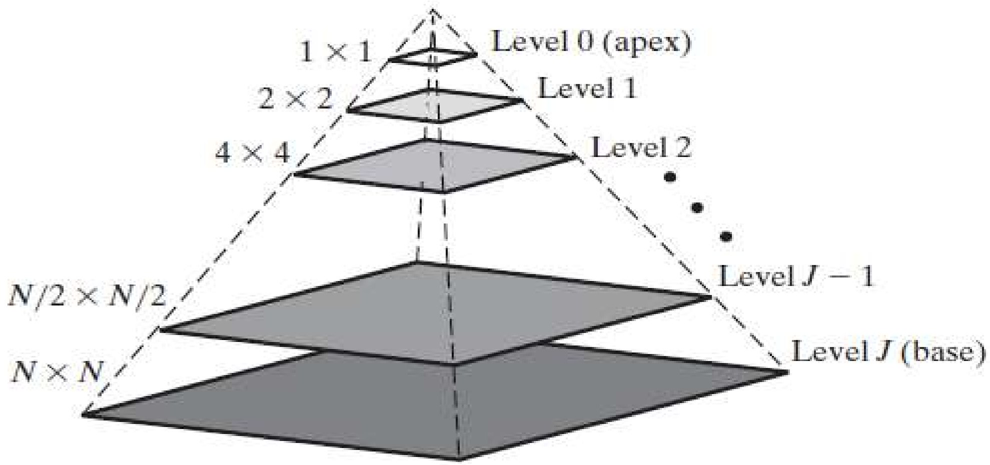


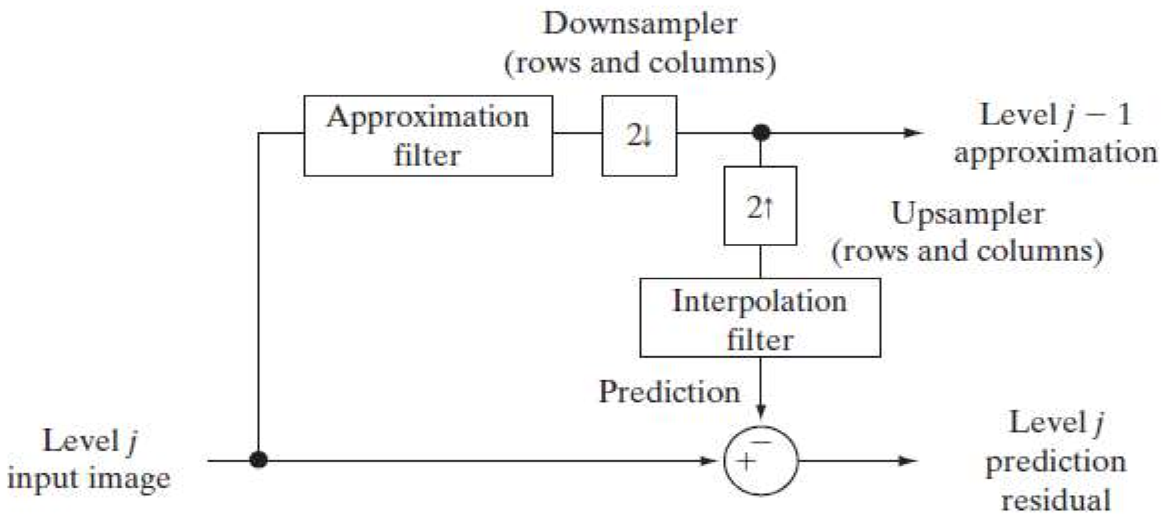
Image Pyramid

- Fully populated Pyramid are composed of J+1 resolution levels starting from 2^0 x 2^0 to 2^J x 2^J
- As you can see 2^0 x 2^0 ➔ 1x1 at Apex
- 2^1 x 2^1 ➔ 2x2 and so on..
- Most Pyramids truncated to P+1 levels. where j = J-P, …J-2, J-1,J where 1<= P <=J
- The total number of Piexels in P+1 level Pyramid, where P > 0 will be represented by using this Formula, which is given as

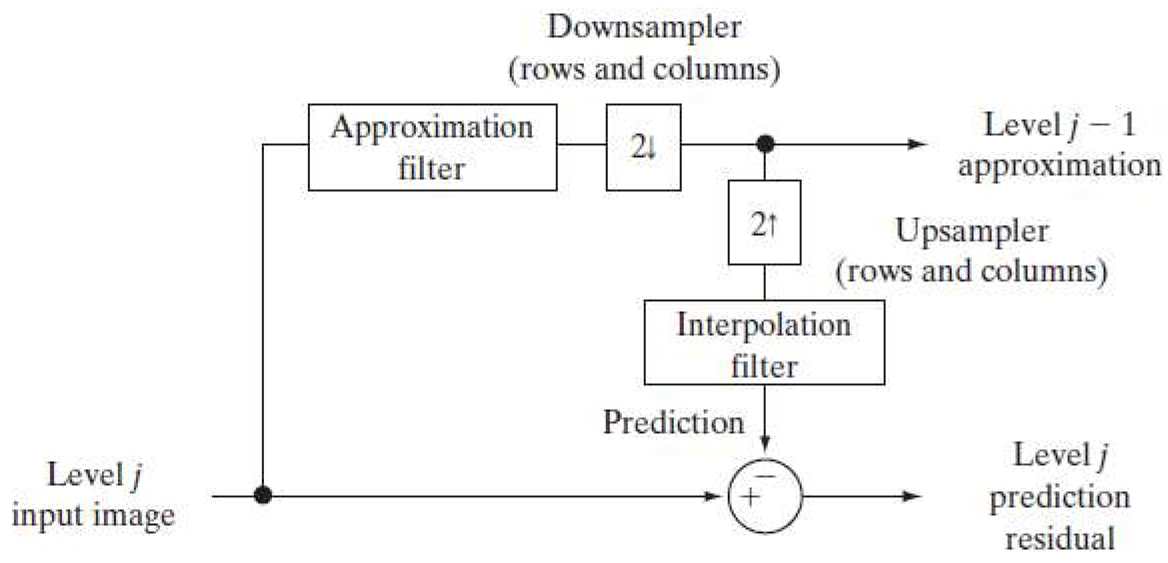
$$N^2\left(1+\frac{1}{(4)^1}+\frac{1}{(4)^2}+\cdots+\frac{1}{(4)^P}\right)\leq\frac{4}{3}N^2$$

Image pyramid construction : Steps

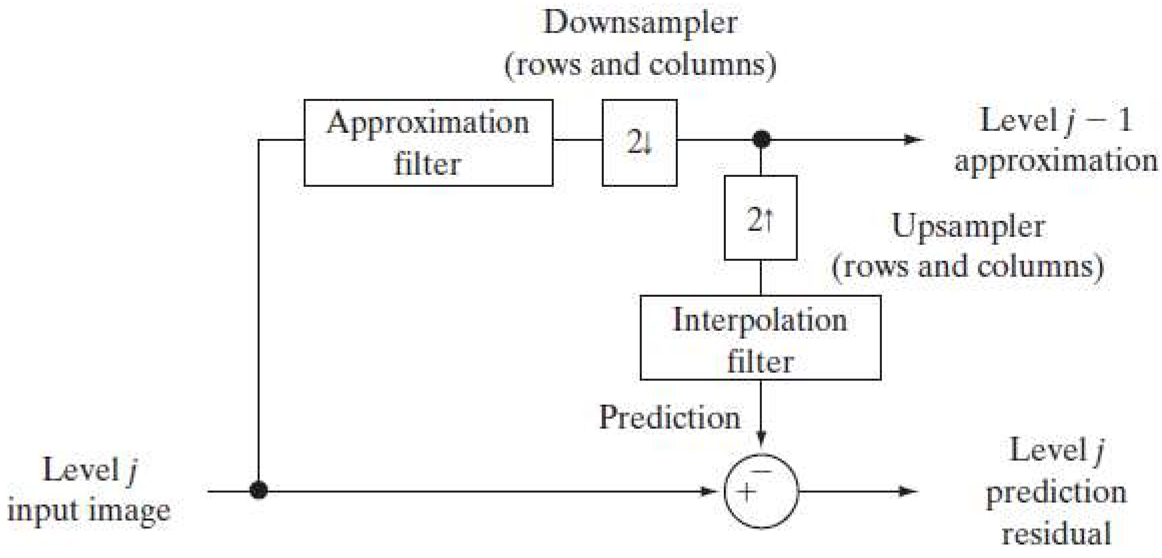
- Blocka Diagram is a simple system for constructing two intimately related image pyramids.
- The Level j – 1 approximation output provides the images needed to build an approximation pyramid (as described in the preceding paragraph)
- The Level J prediction residual output is used to build a complementary prediction residual pyramid
- At the top of the pyramid,(level J-P). All other levels contain prediction residuals, where the level J prediction residual (for J - P + 1 < j< J) is defined as the difference between the level J approximation and the estimate of level J approximation based on the level j-1 approximation
- The information at level j is the difference between Level j input image as well as the prediction that obtained from Level j-1 approximation.
- This difference can be coded and therefore Stored, also transmitted more efficiently.



- The Approximation and Prediction residual Pyramids are computed by Iterative functions as shown in this Figure.
- A P+1 level Pyramid is build by executing the operation in the block diagram P times.
- Before the first iteration, the image to be represented in pyramidal form is placed in level J of the approximation pyramid.
- The following three-step procedure is then executed P times—for j = J, J - 1,..., J - P + 1 (In that Order).
- Here Each pass is composed of 3 Steps.
- Step1: Compute a reduced-resolution approximation of the Level j input image [the input on the left side of the block diagram in Figure)].This is done by filtering and downsampling the filtered result by a factor of 2. Place the resulting approximation at level j - 1 of the approximation pyramid. (Which is nothing but Down sampler in Image)
- Step 2. Create an estimate of the Level j input image from the reduced resolution approximation generated in step 1. This is done by upsampling and the generated approximation. The resulting prediction image will have the same dimensions as the Level j input image.
- Step 3. Compute the difference between the prediction image of step 2 and the input to step 1. Place this result in level j of the prediction residual pyramid



- A variety of approximation and interpolation filters can be incorporated into the system. Filtering is performed in Spatial Domain.
- Varieties of Approximation Filters can be used,
- Using of Neighborhood Average Yeidls Mean Pyramid
- Gaussian Filters Yields, Gaussian Pyramid, No Filters used yields Subsampling Pyramids.
- Any type of Interpolation Filter can be used for interpolation filters.
- Upsampling adding o after every filtering. Downsampling means removing next line after filter.
- The Level j prediction residual that can be later used to reconstruct the original.
- The Prediction residual image can be used to generate the corresponding approximation pyramid including the Original Image without any error.
- So Executing these 3 steps for P time produces 2 intermedaite results related to P+1 level and Prediction residual level Pyramid.
- The Level j-1 approximation output are used to produce the Approximation Pyramid.
- This is Wavelet Transformations.



Example of Wavelet Transformations

a
b

FIGURE 7.3 Two image pyramids and their histograms: (a) an approximation pyramid; (b) a prediction residual pyramid.

The approximation pyramid in (a) is called a Gaussian pyramid because a Gaussian filter was used to construct it. The prediction residual pyramid in (b) is often called a Laplacian pyramid; note the similarity in appearance with the Laplacian filtered images in Chapter 3.

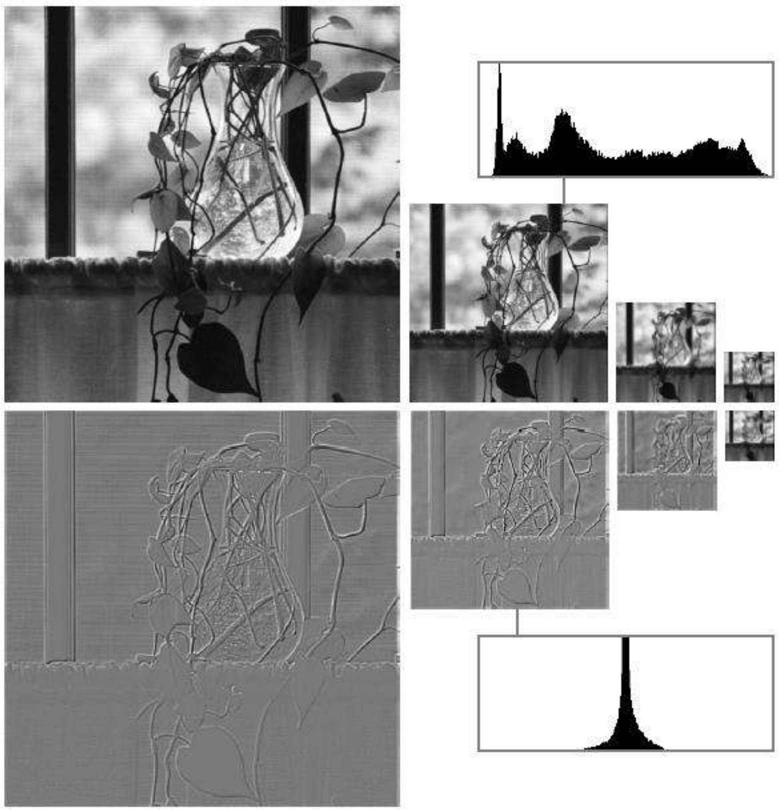


Figure 7.3 shows both an approximation pyramid and a prediction residual pyramid for the vase.
A lowpass Gaussian smoothing filter was used to produce the four-level approximation pyramid in Fig. 7.3(a). the resulting pyramid contains the original 512 * 512 resolution image (at its base) and three low-resolution approximations
(of resolution 256 * 256, 128 * 128, 64 * 64) Thus, is 3 and levels 9, 8, 7, and 6 out of a possible log2 (512) + 1 or 10 levels are present. The level 6 (i.e. 64 * 64,) approximation image is suitable for locating the window stiles (i.e., the window pane framing), for example, but not for finding the stems of the plant. In general, the lower-resolution levels of a pyramid can be used for the analysis of large structures or overall image context; the high-resolution images are appropriate for analyzing individual object characteristics. Such a coarse-to-fine analysis strategy is particularly useful in pattern recognition.

Example of Wavelet Transformations

a
b

FIGURE 7.3 Two image pyramids and their histograms: (a) an approximation pyramid; (b) a prediction residual pyramid.

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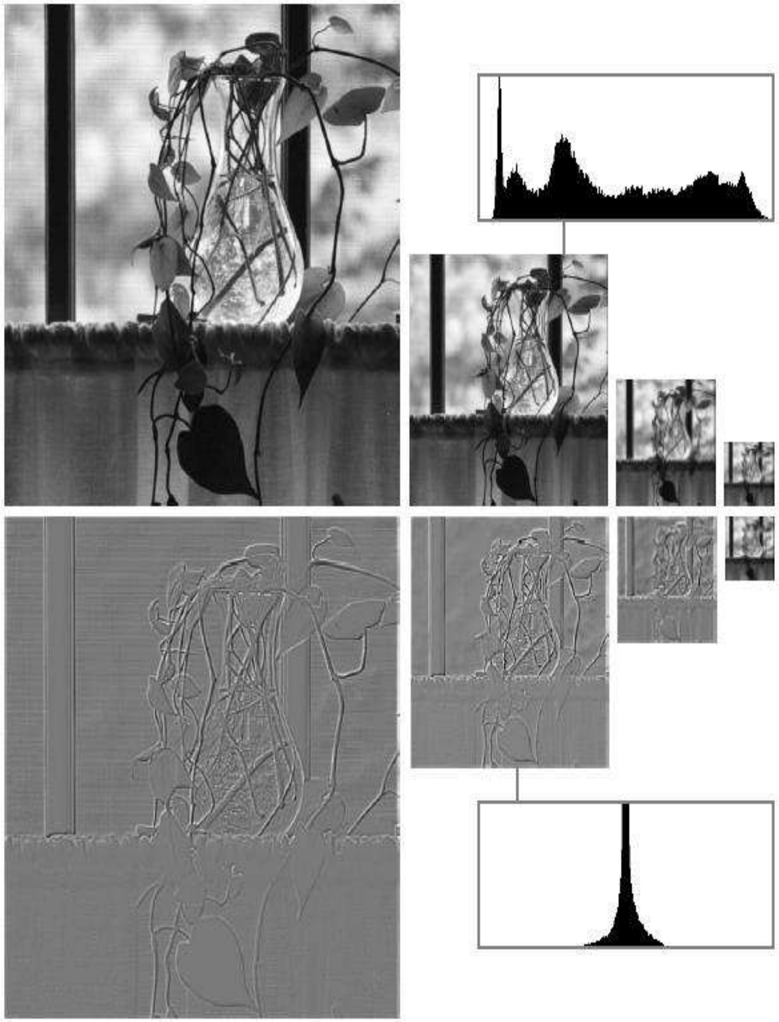


A bilinear interpolation filter was used to produce the prediction residual pyramid in Fig. 7.3(b).

In the absence of quantization error, the resulting prediction residual pyramid can be used to generate the complementary approximation pyramid in Fig. 7.3(a), including the original image, without error.

Prediction residual images can be highly compressed by assigning fewer bits to the more probable values. Finally,we note that the prediction residuals in Fig. 7.3(b) are scaled to make small prediction errors more visible;

Assignment 2 (option)

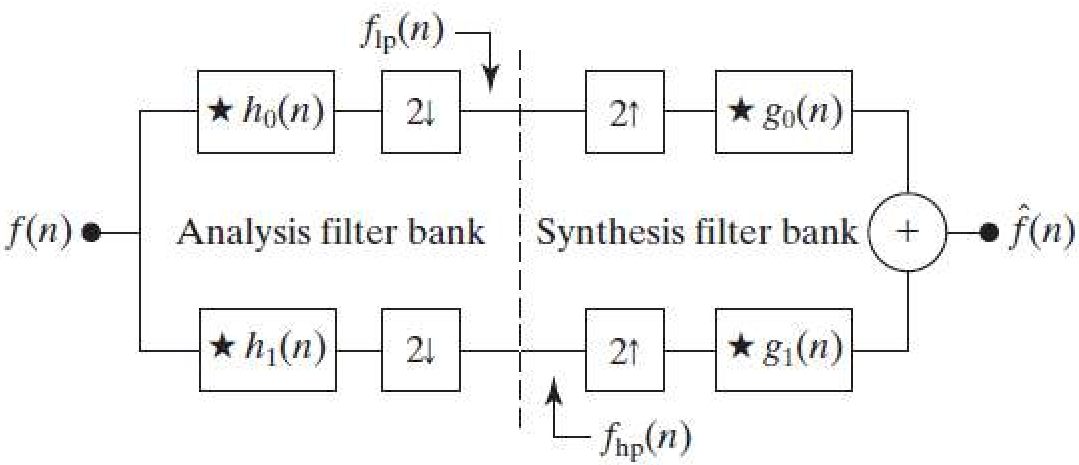
- Write a program to display a Image pyramids for a Image resolution of 512x512

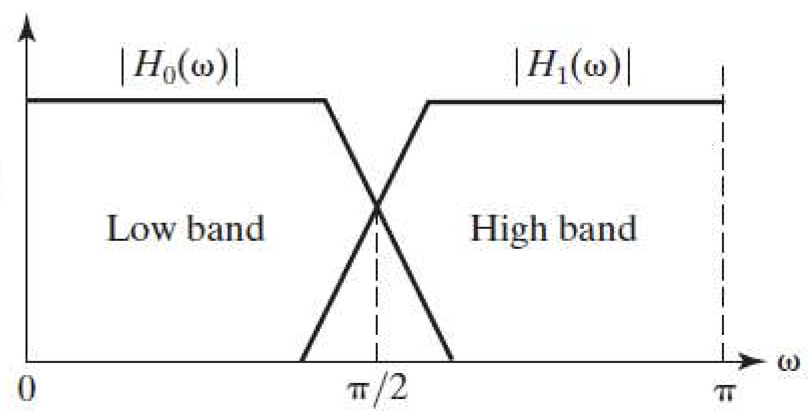
Subband coding

- This is used for Multiresolution Analysis of an Image.
- In Subband coding an Image is decomposed into set of band limited components which are known an Subbands.
- These Subbands can be reassembeld and to reconstruct the original Image without any Error.
- This technique is used for Speech and Image compression where each subband is generated by bandpass filtering input image. ➔ So the An input image is pass through a bandpass filter and generate subbands.
- Since the bandwidth of Subband is smaller than the original image the subbands can be downsample without loss of information.

SUBBAND CODING

- The reconstruction of Original image can be done by Downsampling, Filtering, and summing the Subbands.
- All these process is shown in this Figure. This figure the Principal component of 2 band subband coding and decoding system. Here the Input x(n) is 1-D band limited discrete signal wher n can have value from 0.1,2,3… and so on.
- The output sequeuence $\dot{f}$ (n) is formed through decomposition of f(n) into $f_{lp}(n)$ and $f_{hp}(n)$ using Analysis Filter $h_0(n)$ and $h_1(n)$ and Synthesis Filter $g_0(n)$ and $g_1(n)$.
- Here Filter $h_0(n)$ and $h_1(n)$ are Half band digital Filters whose idealised transformed characteristics is shown in the Second Figure. This Figure shows the spectrum splitting properties of the Filter where $H_0(\omega)$ is a Low Pass Filter whose output is an approximation of X(n) and $H_1(\omega)$ is High Pass filter whose output is High Frequency.
- The goal in subband coding is to select $h_0(n)$ and $h_1(n)$, $g_0(n)$ and $g_1(n)$ so that $\dot{f}$ (n) = f(n). That is, so that the input and output of the subband coding and decoding system are identical. When this is accomplished, the resulting system is said to employ *perfect reconstruction filters.*



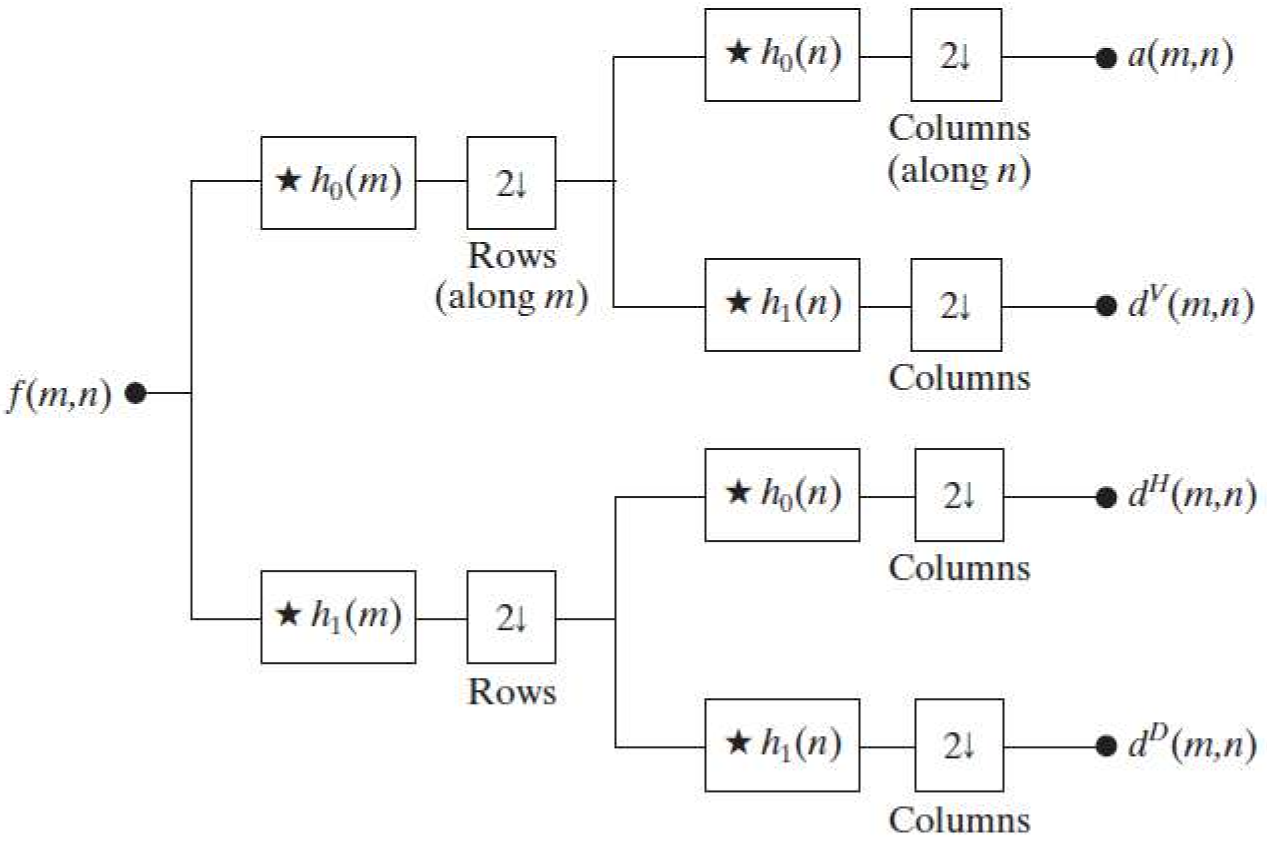


Subband coding

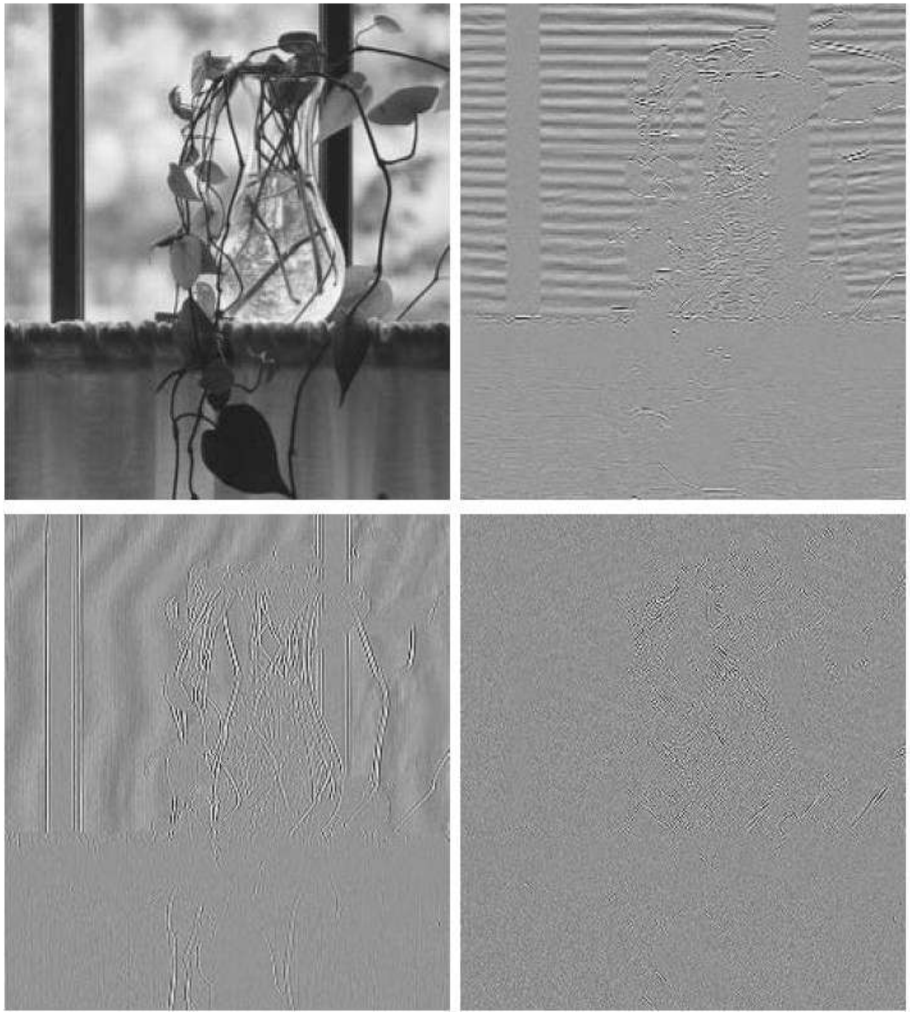
- All the Filtering is performed in Time Domain by coverting each Filer input to an Impulse response $\delta(n)$. Inorder to reconstruct the input we have to select $h_0(n)$, $h_1(n)$, along with $g_0(n)$ and $g_1(n)$, Here to analyze subband coding Z-transform is sued. Z-transform is ideal tool to analyse Discrete tool transform system.

2-D Subband coding

- The separable filters are first applied in one dimension (e.g., vertically) and then in the other (e.g., horizontally). Moreover, downsampling is performed in two stages—once before the second filtering operation to reduce the overall number of computations. The resulting filtered outputs,denoted a(m, n), d^V(m, n), d^H(m, n), and d^D(m, n) n Fig. 7.7, are called the ***approximation, vertical detail, horizontal detail, and diagonal detail*** subbands of the input image, respectively.
- These subbands can be split into four smaller subbands, which can be split again, and so on.



2D Subband coding



A four-band split of the 512 * 512 image of a vase in Fig. 7.1, based on the filters in Fig. 7.8, is shown in Fig. 7.9. Each quadrant of this image is a subband of size 256 * 256. Beginning with the upper-left corner and proceeding in a clockwise manner, the four quadrants contain approximation subband $\boldsymbol{a}$, horizontal detail subband $\boldsymbol{d}^H$ diagonal detail subband $\boldsymbol{d}^D$ and vertical detail subband $\boldsymbol{d}^V$ respectively.

All subbands, except the approximation subband in Fig. 7.9(a), have been scaled to make their underlying structure more visible. Note the visual effects of aliasing that are present in Figs. 7.9(b) and (c)—the $\boldsymbol{d}^H$ and $\boldsymbol{d}^V$ subbands.

The wavy lines in the window area are due to the downsampling of a barely discernable window screen in Fig. 7.1. Despite the aliasing, the original image can be reconstructed from the subbands in Fig. 7.9 without error.

The required synthesis filters, $\boldsymbol{g_0(n)}$ ***and*** $\boldsymbol{g_1(n)}$ are determined. and incorporated into a filter bank that roughly mirrors the system in Fig. 7.7. In the new filter bank, filters $\boldsymbol{h_i(n)}$ ***for*** $\boldsymbol{i=\{0,1\}}$ are replaced by their counterparts, and upsamplers and summers are added.