Morphological Image processing

morphology

- A branch of biology that deals with the form and structure of animals and plants.
- Same word here in the context of *mathematical morphology as a tool for extracting image components* that are useful in the representation and description of region shape, such as boundaries, skeletons, and the convex hull
- Thinning of objects, Thickening of objects
- Morphology Used in Image Pre proccessing and Post processing

Pre requisites to learn this

- Mathematics Set Theory
- Set Operations on Images
- Basic Image Terminology:
  - Pixel
  - Intensity
  - Operation
  - Image Arithmatic/Logical operations
  - Image processing algorithms using Masks

Basic concepts of SET Theory

- Let assume Image is represented in 2-D tuple then represented as z^2.
- z^2 X Co-oridnate and Y Co-ordinate as tuple (x,y)
- Lets assume if you want to process the image as 3-D tuple then represented as z^3.
- z^3 (x,y, Intensity)
- For study purpose we consider the Images in 2-D tuple for easy understanding.

Set Theory

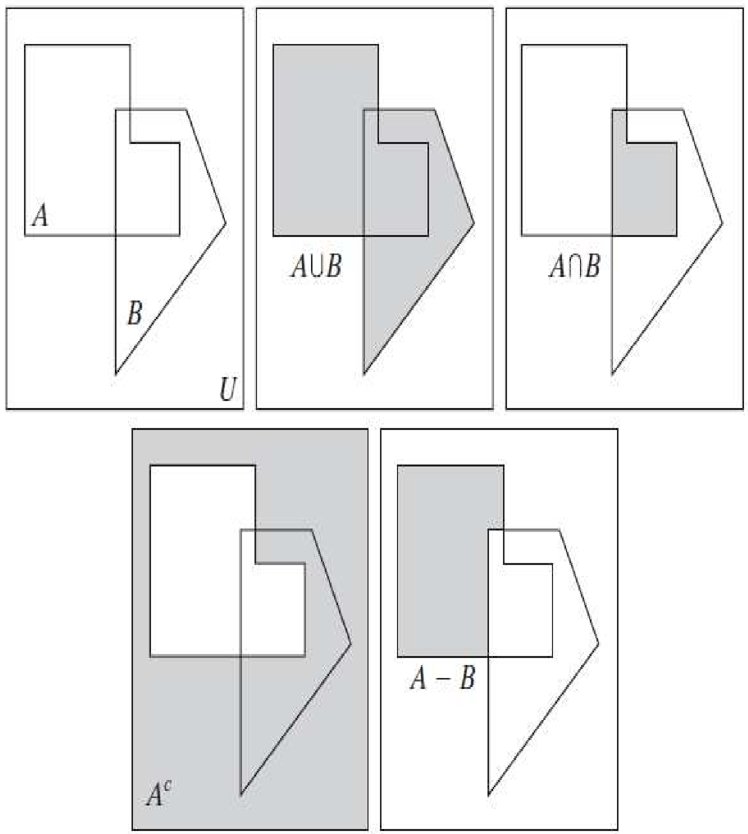
- z^2 is having all set of Pixes represented by an input Image (x,y)
- If A → Set in z^2 image
- Let a = (a1, a2) – element of A then a Ɛ A
- The Set with No Elements is NULL Set or {}
- **What are the elements of Set?**
- If every element of Set A is also an element of Set B then $A \subset B$.
- The Union of Two Set is C = A U B.
- The Intersection of Two Sets is $C = A \cap B$
  - If Two Sets are dis joint if no common elements $A \cap B = \square = \{ \}$

Co-ordinates of Pixels representing an Object

Set Operations on Images

a b c
d e

FIGURE 2.31 (a) Two sets of coordinates, *A* and *B*, in 2-D space. (b) The union of *A* and *B*. (c) The intersection of *A* and *B*. (d) The complement of *A*. (e) The difference between *A* and *B*. In (b)–(e) the shaded areas represent the members of the set operation indicated.



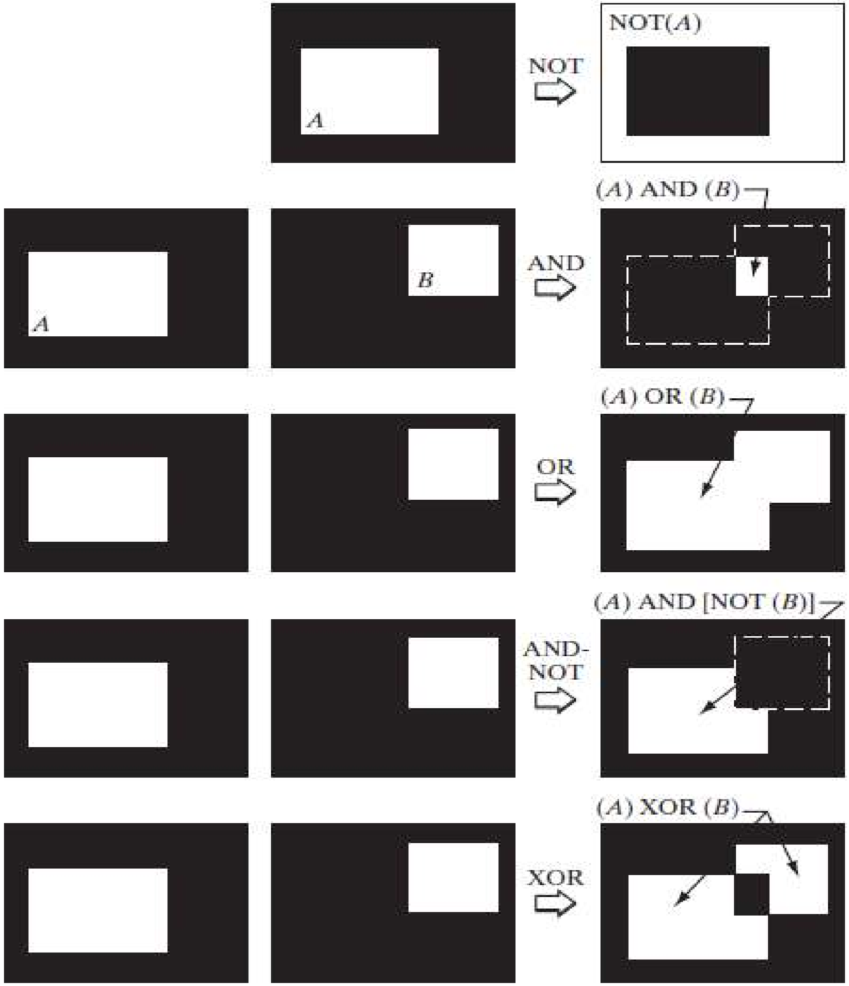
A's Compliment is given by Equation:
$A^c = \{w \mid w \notin A\}$ (doesnot belong to)

A-B : All pixels belog A but not in B
$A-B = \{w \mid w \in A, w \notin B\}$

Reflection of Set A: Â (A cap)
$\hat{A} = \{w \mid w = -a \text{ when } a \in A\}$
For all a in A return –a for Reflection of A

Logical Operations

FIGURE 2.33
Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.



Dilasion:

- Process of exapanding the Image
- Dilation increases brightness of the input Image
- Dilation is based on Set Operations
- **Expansion of Image:**
  - **By applying the mathematical operation on Original Image by a Structural Image.**
  - Image Sets in the *dilation of* ***A*** *by* ***C*** *is denoted*

$\mathbf{A} \oplus \mathbf{C}$

Dilation is defined as below where **C** is structuring Element and **A** is Image Set (Original Image)

- $\mathbf{A} \oplus \mathbf{C} = \{\mathbf{w} \mid [(\hat{\mathbf{C}})_{\mathbf{w}} \cap \mathbf{A}] \subset \mathbf{A}\}$ (**A** Dilation by **C**)

Dilation Example

Origin element

Original Image

0	**1**	**0**
1	0	0
0	0	0

Structural Image

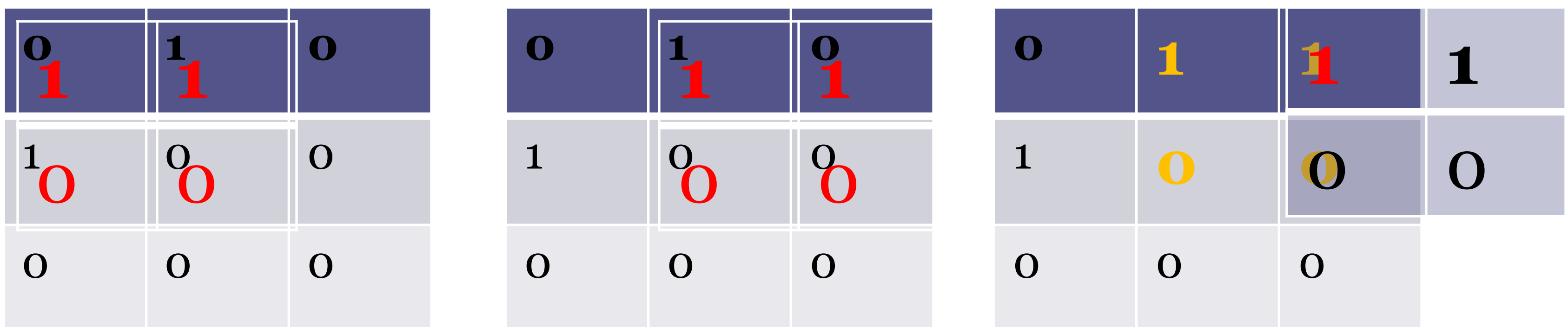
1	**1**
0	**0**

Let us consider an example to understand this, Original Image 3x3 and a structural image 2x2 which is also known as Mask. Will use this Structural image to slide over the Original Image from (0,0)

Dilation Example Working

Original Image

Structural Image



Step1: Place the Structural image a.k.a (SI) over Original Image at position (0,0). (SI elements are marked RED)
Step2: check Origin Element with corresponding Element in Image, if they don't match slide the SI to next postion (0,1) and Match Origin element.
Step3: If Origin Element Matches, then Pixel values of SI are updated correpsondingly in the Original image i.e. these position are udpated (0,1), (0,2), (1,1), (1,2) See Pic 3 (original image updated pixels are Colored in Orange).
Step4: Slide the SI to next position (0,3) and Match origin element of SI

Dilation Example Working

Original Image

Structural Image

0	1	1	1
1	0	0	0
0	0	0	0

0	1	1	1
1 1	0 1	0	0
0 0	0 0	0	0

0	1	1	1
1	1 1	0 1	0
0	0 0	0 0	0

Step5: Observe Mask is going beyond Image, origin element of SI Matches with the (0,2) then add a new column and update the correpsonding SI elements in Image, hence new image size is 3x4, New pixel values of (0,2), (0,3), (1,2), (1,3) are updated correspondingly marked orange, Next slide SI to (1,0).

Step6: Check Origin Element with corresponding Element in Image, if they match then copy values of set of Pixels (1,0), (1,1), (2,1), (2,2) with the corresponding SI pixels

Dilation Example Working

Dilated Image

0	1	1	1
1	1	1	1
0	0	0	0

Structural Image

1	1
0	0

Step6: Continue to do until all elements are slide by SI (mask)
Step7: Observe that 4th Row is not added because no element match in the Origin Element. Final Size: 3X4

Observations?

1. Image size incresed
2. Number of 1s increased hence brightness of picture is increased.

What happens when SI is changed by Elements or shape?

Solve below Dilation Example

Original Image

0	1	0
1	0	0
1	0	0

Structural Image

1	1
0	0

Origin element

Let us consider an example to understand this, Original Image 3x3 and a structural image 2x2 which is also known as Mask. Will use this Structural image to slide over the Original Image from (0,0)

Dilation Example Answer

Dilated Image

0	1	1	1
1	1	1	1
1	1	1	1
0	0	0	0

Structural Image

1	1
0	0

Solve below Dilation Example

Original Image

0	1	0
1	0	0
1	0	0

Structural Image

Origin element

0	1
1	1

Let us consider an example to understand this, Original Image 3x3 and a structural image SI which is also known as Mask. Will use this Structural image to slide over the Original Image from (0,0)

Solve below Dilation Example

Original Image

0	1	0
1	0	0
1	0	0

Structural Image

0	1
0	0

Origin element

Let us consider an example to understand this, Original Image 3x3 and a structural image SI which is also known as Mask. Will use this Structural image to slide over the Original Image from (0,0)

Solve below Dilation Example

Original Image

0	**1**	**0**
1	0	0
1	0	0

Structural Image

1	**1**
0	

Origin element

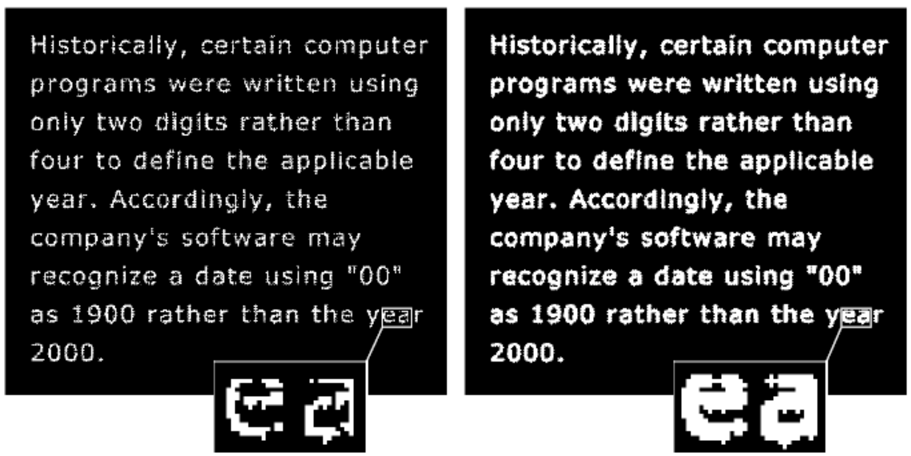
Let us consider an example to understand this, Original Image 3x3 and a structural image SI which is also known as Mask. Will use this Structural image to slide over the Original Image from (0,0)

Application of Dilation: Bridging the Gaps in input Image

Left image is having broken characters that we studied earlier in connection with lowpass filtering.
In the maximum length of the breaks is known to be two pixels.
Figure 9.7(b) shows a structuring image (SI) that can be used for repairing the gaps (note that instead of shading, we used 1^s to denote the elements of the SE and 0^s for the background; this is because the SI is now being treated as a subimage and not as a graphic).

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

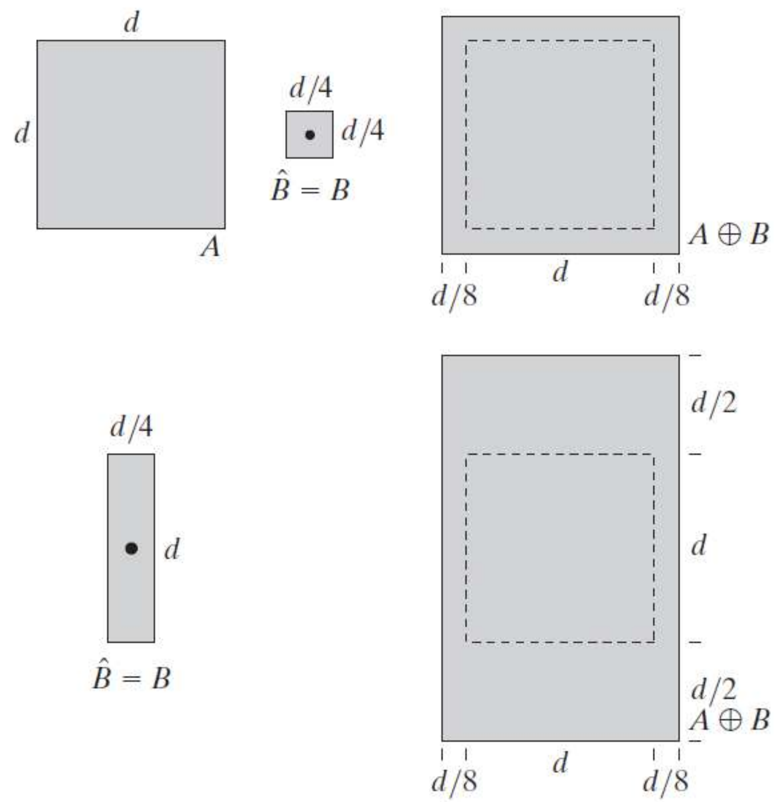


0	1	0
1	1	1
0	1	0

Dilation Other Examples:

a b c
d e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference



What is your Observations?

First Row: Dilated Image is increased size by size of SE

Second Row: Dilated Image elongated verticaly by the Vertical size of SE.

Assignment 2: (option)

- Solve above Dilation examples
- Write programs to Dilate by SE with different shapes, sizes and Pixel values.

Erosion

- Opposite process of Dilation
- Image shrinking is obtained.
- Erosion can be expressed as

$$A \ominus B = \{z \mid (B)_z \cap A^c = \varnothing\} \qquad (9.2\text{-}2)$$

where A^c is the complement of A and Φ is the empty set.

1	**1**	**1**	**1**
1	1	0	1
0	1	1	1
0	1	0	0

1	**1**
1	**0**

Origin element

Erosion: procedure

SI is slide through the Original Image and Match the all pixels in SI image with original Image, if any such pattern is found then Origin is kept same but rest of all elements are marked as 0's corresponding elements in Image. Final image is shown below Figure 3.

1	1 1	1 1	1
1	1 1	0 0	1
0	1	1	1
0	1	0	0

1	1	0	1
1	0	0	1
0	1 1	1 1	1
0	1 1	0 0	0

1	1	0	1
1	0	0	1
0	1	0	1
0	0	0	0

Number of 1's reduced
Image is also shrinked,
Brighness also reduced.
Thins object

Erosion: Examples

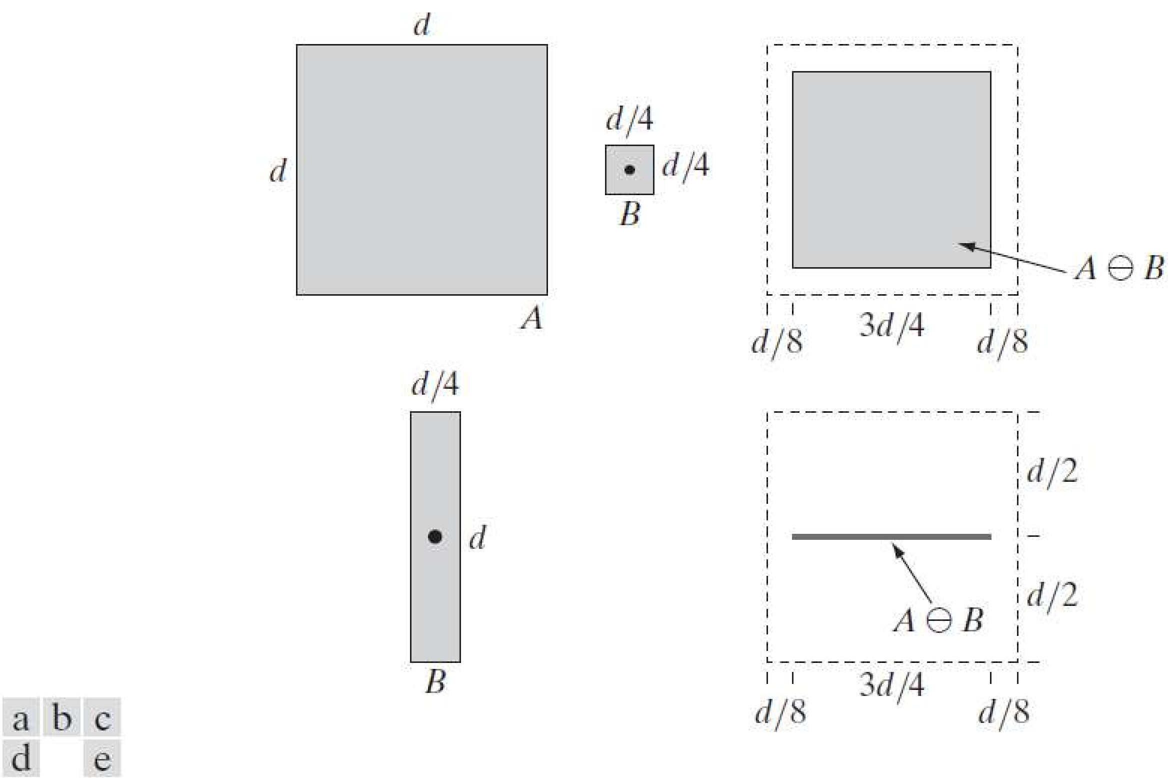
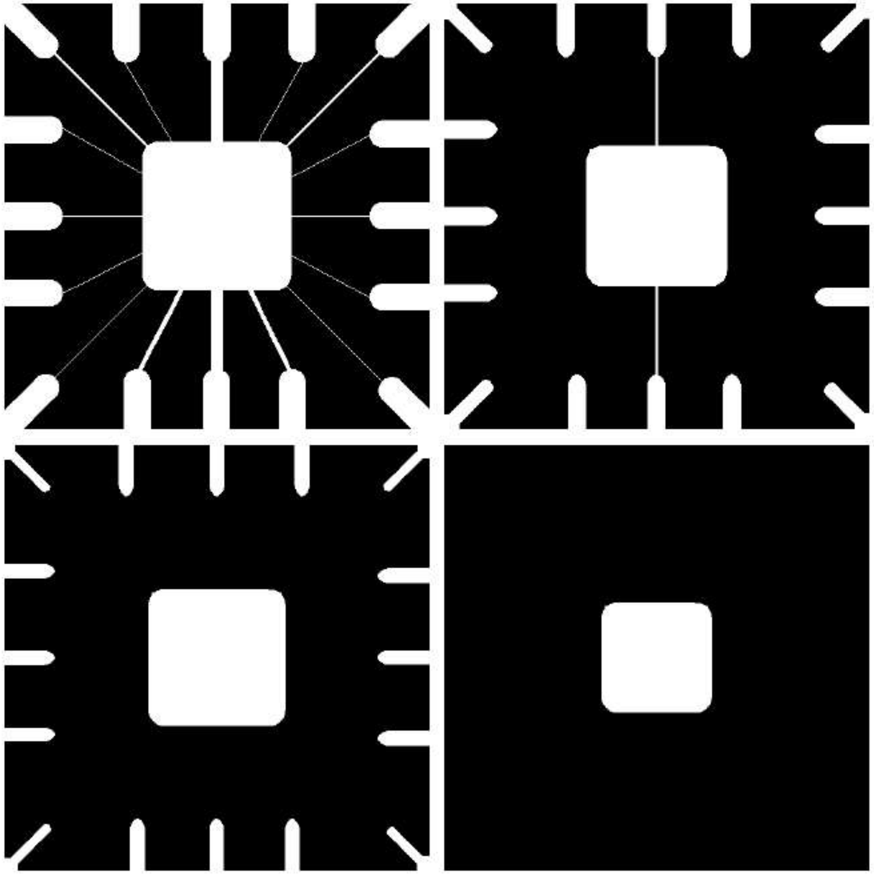


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Erosion : more examples

a	b
c	d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.



Opening and Closing of an Image

- Morphological Operations
- Openings:
  - Smooths the contour of an object
  - Breaks narrow isthmuses (bridge or joint)
  - Eliminates thin protrusions
- *Closing:*
  - Smooth sections of contours
  - Fuses narrow breaks and long thin gulfs,
  - Eliminates small holes, and
  - Fills gaps in the contour.

Procedure for Opening

- Step1: Erosion on Image
- Step2: Dilation on step-1 output image
- The Opening procedure is given below for input image A and Structural image (B) / The Opening of Set **A** by Structuring element **B** is denoted by:

$$A \circ B = (A \ominus B) \oplus B \qquad (9.3\text{-}1)$$

Appications:

1. Identify gaps in an Image
2. Edges become sharp (or) smooth
3. Isolates objects which are touching one another

Processing of Opening

Origin element

1	1	1	0	1	1	1
1	1	1	1	1	1	1
1	1	1	0	1	1	1

1	1	1
1	1	1
1	1	1

0	0	0	0	0	0	0
0	1	0	1	0	1	0
0	0	0	0	0	0	0

A θ B

1	1	1	0	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

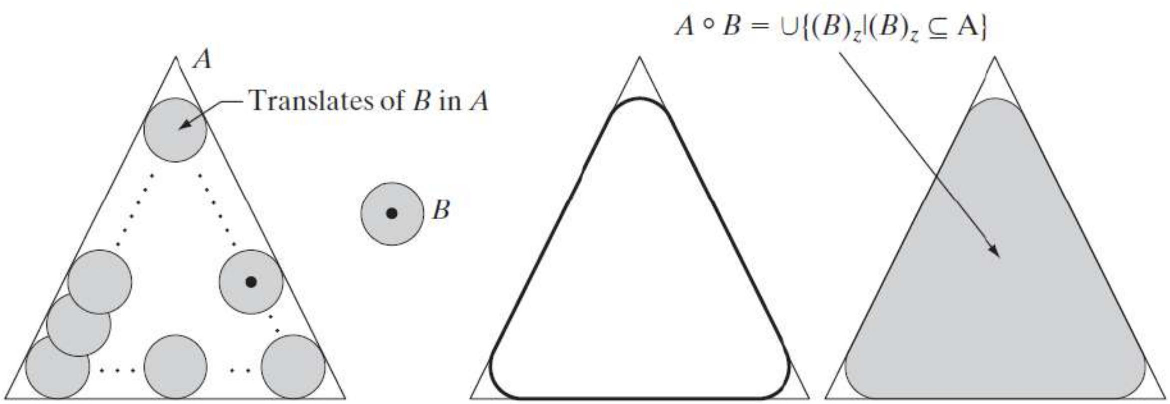
A o B = ((A θ B) + B)

How to interpret the Opening operation?

The opening operation has a simple geometric interpretation (Fig. 9.8). Suppose that we view the structuring element as a (flat) “rolling ball.” The *boundary of is then established by the points in that reach the farthest into the boundary of as is rolled around the inside of this boundary*. This geometric *fitting property of the opening operation leads to a settheoretic* formulation, which states that the opening of by is obtained by taking the union of all translates of that fit into That is, opening can be expressed as a fitting process such that

$$A \circ B = \bigcup\{(B)_z \mid (B)_z \subseteq A\} \qquad (9.3\text{-}3)$$

where $\bigcup\{\cdot\}$ denotes the union of all the sets inside the braces.



a b c d

FIGURE 9.8 (a) Structuring element B “rolling” along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Closing

Step1: Dilation
Step2: Erosion applied on output of Step1

Closing of an Image A by Structural element B is given by

$$A \bullet B = (A \oplus B) \ominus B \qquad (9.3\text{-}2)$$

Used to Fuse narrow breaks and elinate small holes.

1	1	1	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	1	1	1

1	1	1
1	1	1
1	1	1

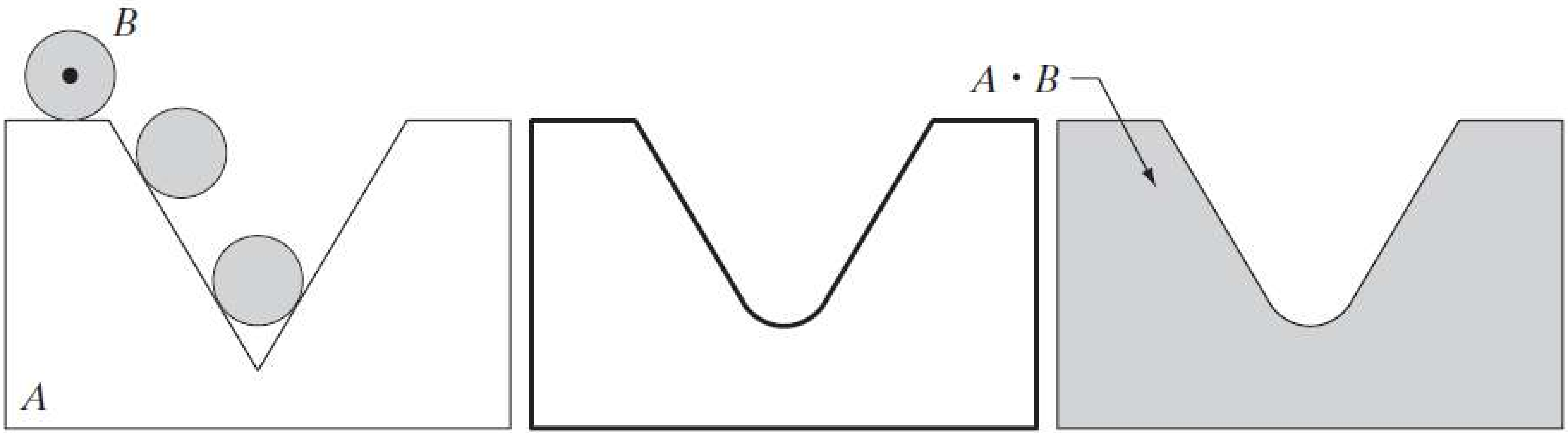
A + B

1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1

A ●B = ((A + B) θ B)

1	1	1	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	1	1	1

Interpretation of Closing operation

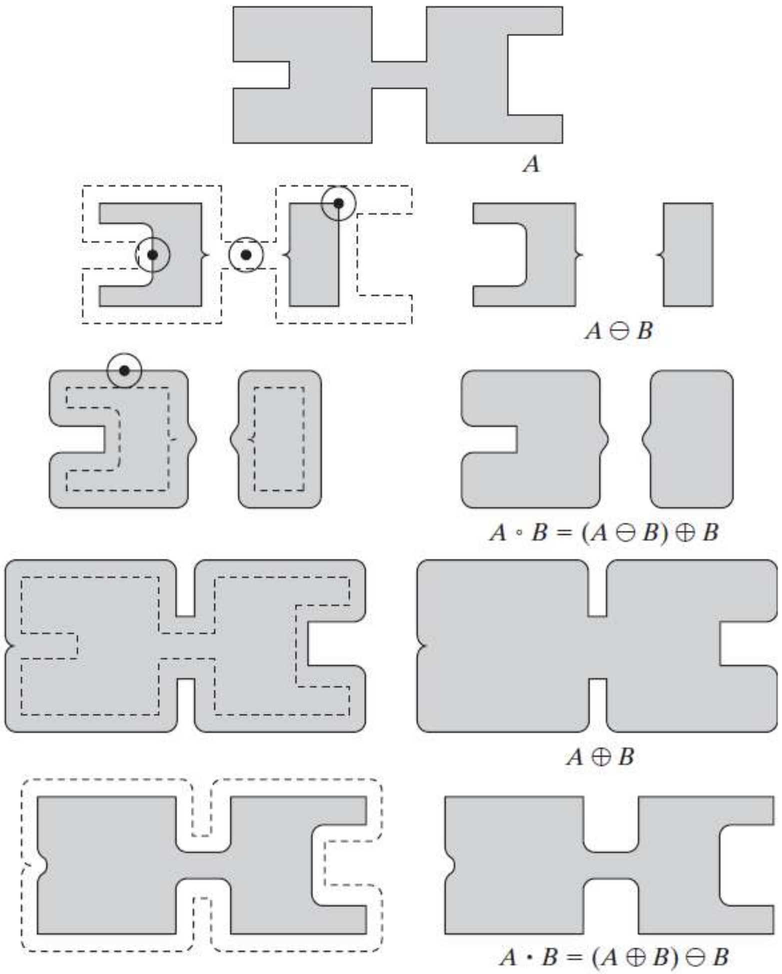


a b c

FIGURE 9.9 (a) Structuring element *B* "rolling" on the outer boundary of set *A*. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade *A* in (a) for clarity.

Closing has a similar geometric interpretation, except that now we roll on the outside of the boundary (Fig. 9.9). As discussed below, opening and closing are duals of each other, so having to roll the ball on the outside is not unexpected. Geometrically, a point **w** is an element of **A • B** if and only if **(B)z ∩ A ‡ Φ** for any translate of **(B)z** that contains w

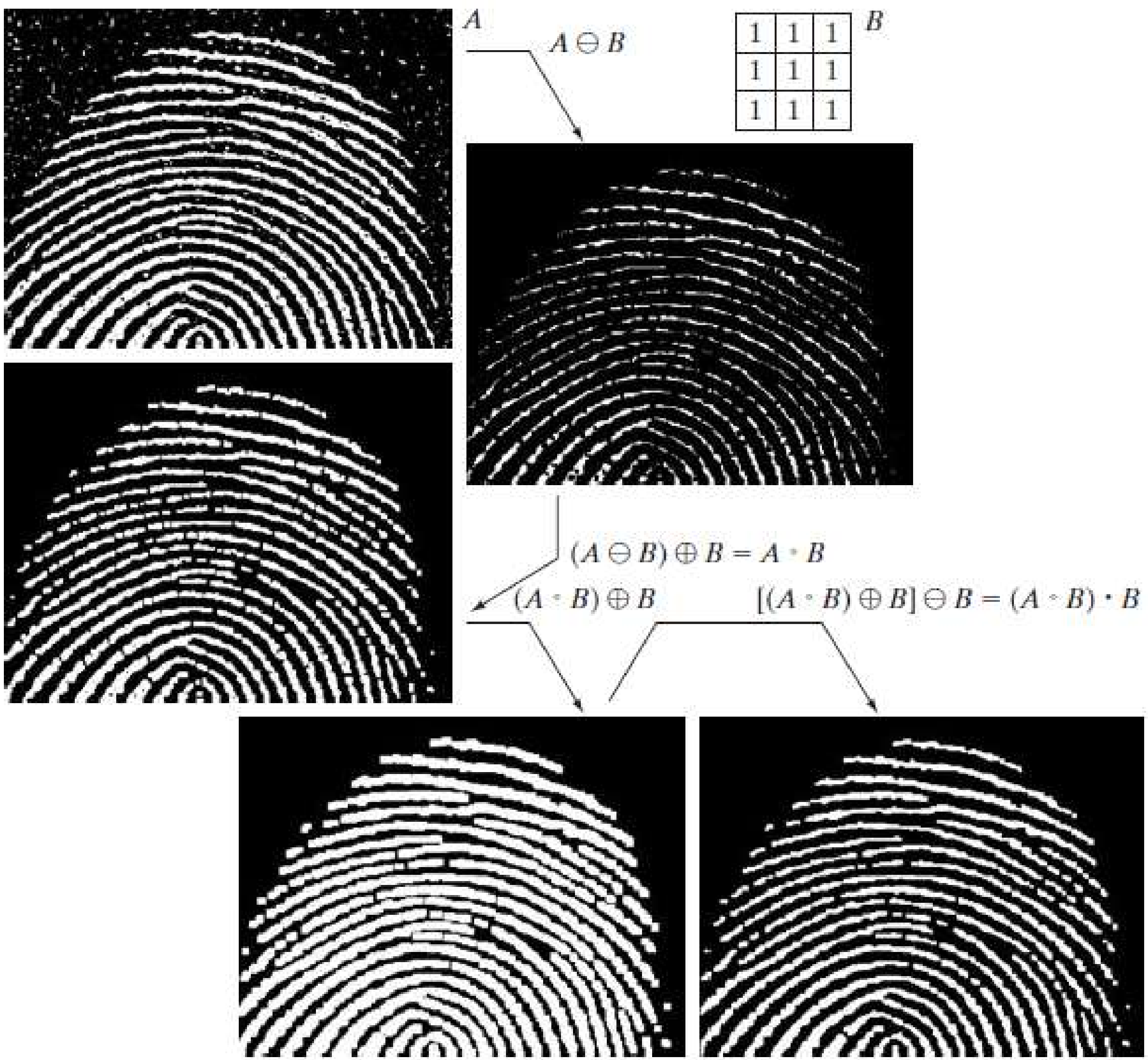
Practical exampels of Opening, Closing, Erosion and Dilation



	a	
	b	c
	d	e
	f	g
	h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Practical examples:



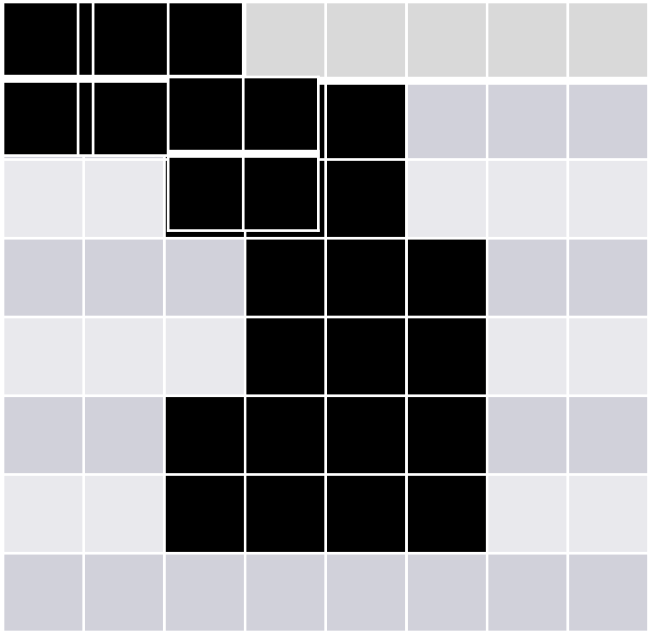
a b c d e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Hit, Miss or Fit

- In Morpohology Hit, Miss or Fit is an operation that detects the patterns in a binary image using a Structuring Image (SI)/Structural Element (SE).
- Miss: Not a single element of SE match
- Hit: Atleast one element of SE match with image
- Fit: All SI elements match with Image.

Miss, Hit and Fit:



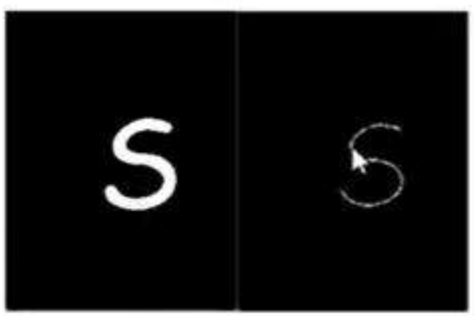
Miss

Hit: No change in the image

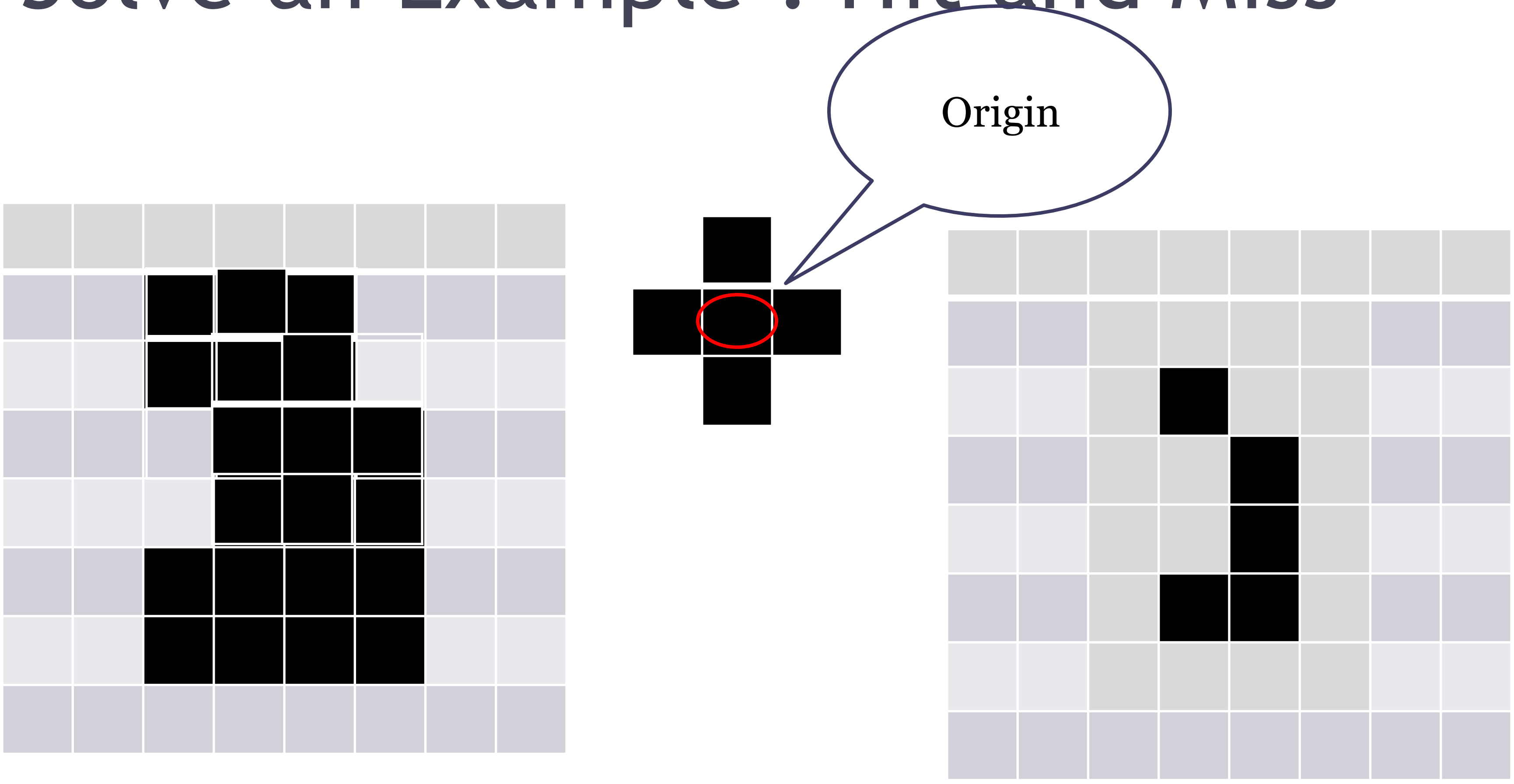
Fit: Retain the Origin Element

Use of Hit and Miss

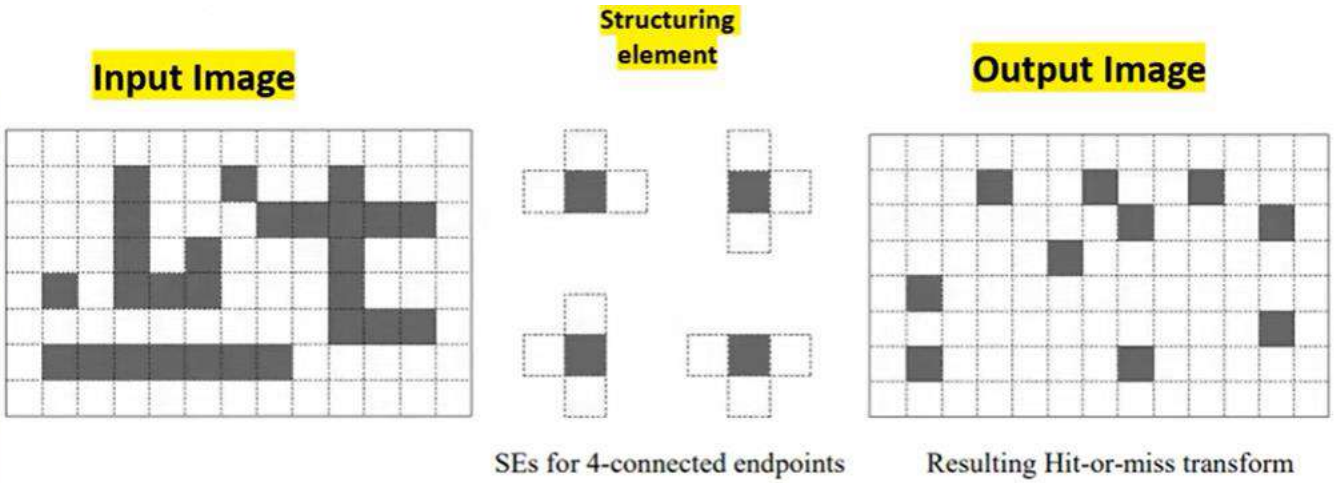
- Make the Image thin
- Skeletonize the shape in Binary Image



Solve an Example : Hit and Miss

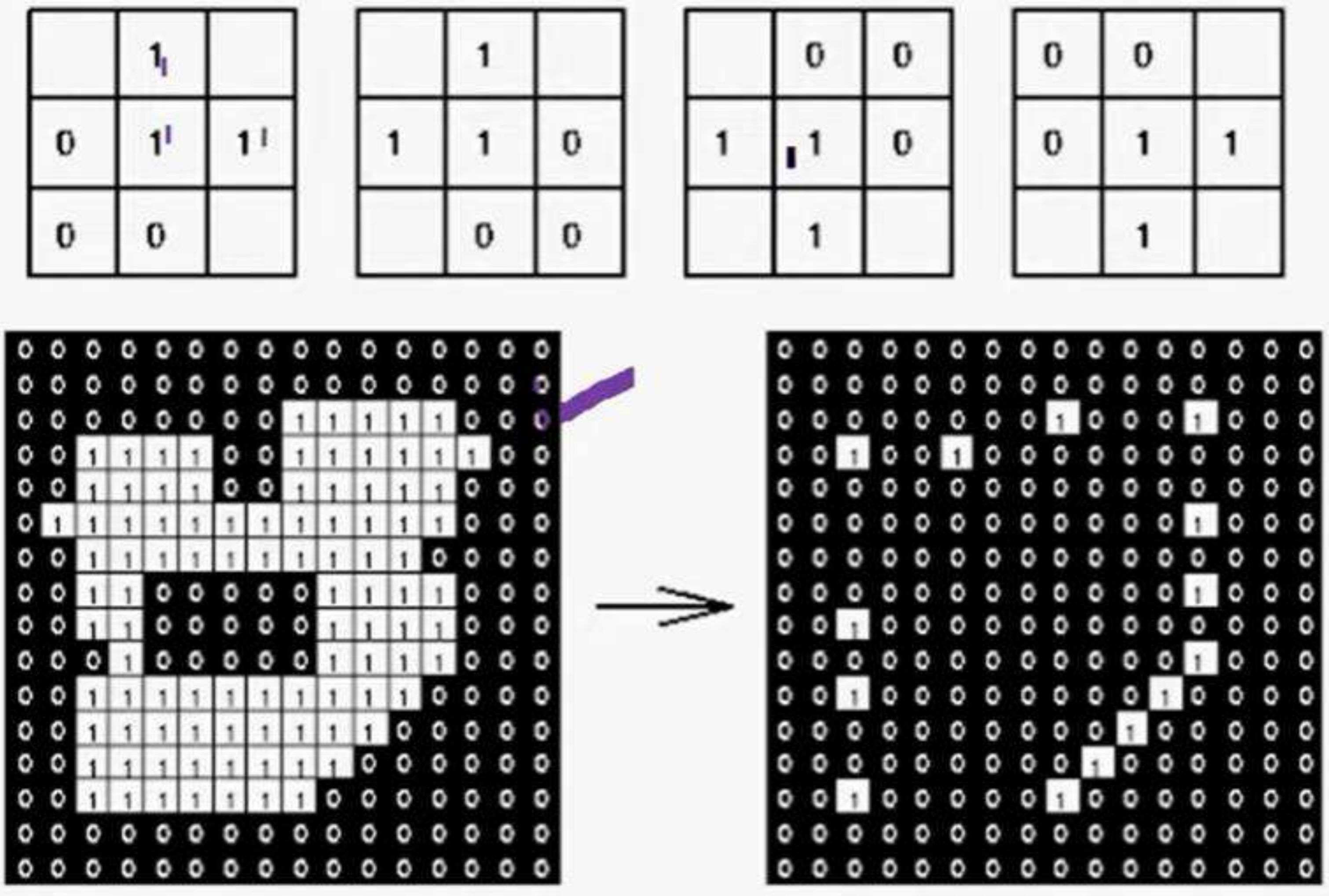


Hit and Miss is an Iterative process



In this example we consider a input image having multiple shapes.
We consider multiple Structuring Elements and applying on Input image iteratively (one after another)
We get Output Image

Example : We use below 4 Structural elements SE to Thinning operation and Pattern detection.



Practical applications:

- Boundary Extraction
- Hole Filling
- Extraction of connected components
- Convex Hull
- Thinning
- Thickening
- Skeletons
- Pruning
- Morphological Reconstruction

Practical application of Morphology

Boundary extraction: Consider Boundary of A is β(A) given by:

β(A) = A – (A θ B) – Eq 9.5-1 where A is input image, B is Suitable Structural Images.

Below Figure illustrates the mechanics of boundary extraction. It shows a simple binary object, a structuring element and the result of using Eq. (9.5-1). Although the structuring element in Fig. 9.13(b) is among the most frequently used, it is by no means unique. For example, using a structuring element of 1s would result in a boundary between 2 and 3 pixels thick

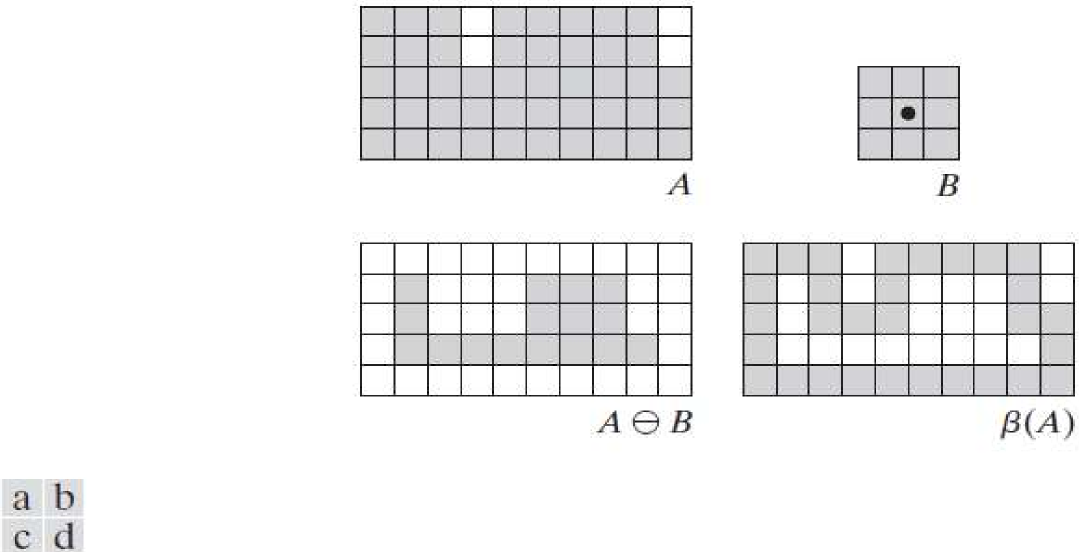
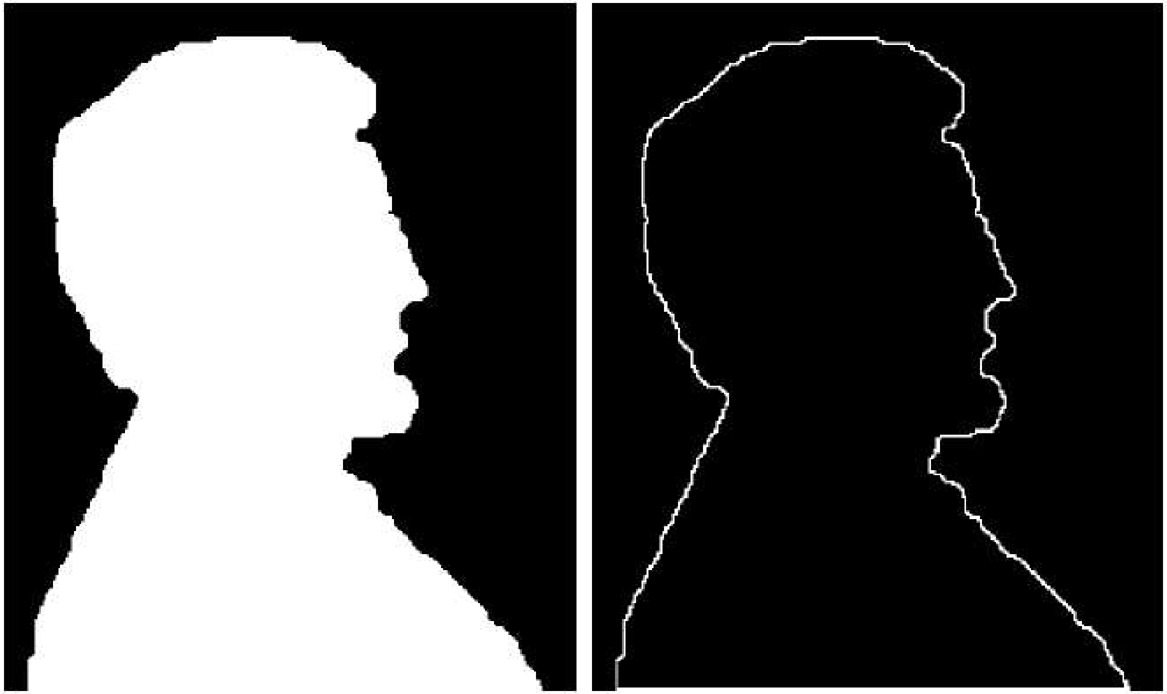


FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Boundary Extraction



a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

EXAMPLE 9.5: Boundary extraction by morphological processing.

Figure 9.14 further illustrates the use of Eq. (9.5-1) with a 3×3 structuring element of 1s. As for all binary images in this chapter, binary 1s are shown in white and 0s in black, so the elements of the structuring element, which are 1s, also are treated as white. Because of the size of the structuring element used, the boundary in Fig. 9.14(b) is one pixel thick.

sample applications

- Opening by Reconstruction
- Filling Holes
- Border Cleaning

Opening by Reconstruction

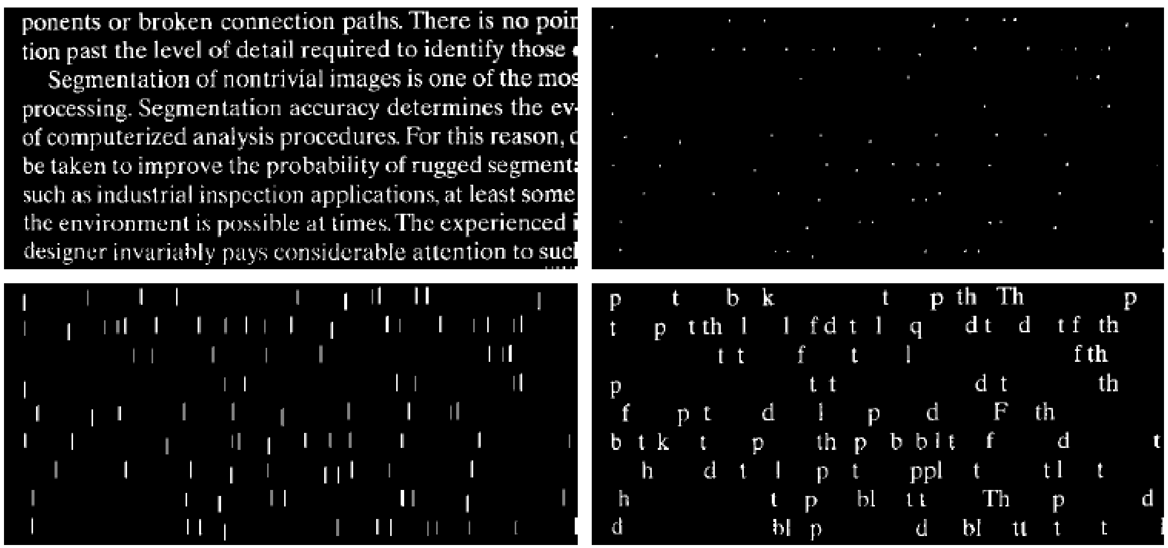
In a morphological opening, erosion removes small objects and the subsequent dilation attempts to restore the shape of objects that remain. However, the accuracy of this restoration is highly dependent on the similarity of the shapes of the objects and the structuring element used.

Opening by reconstruction restores exactly the shapes of the objects that remain after erosion.The opening by reconstruction of size of an image is defined as the reconstruction by dilation of F from the erosion of size n of F that is,

$$O_R^{(n)}(F) = R_F^D\left[(F \ominus nB)\right] \qquad (9.5\text{-}27)$$

Where FΘnB indicates n Erosions of F by B.

Opening by Reconstruction



a b
c d

FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

Filling Holes

Filling holes: In Section 9.5.2, we developed an algorithm for filling holes based on knowing a starting point in each hole in the image. Here, we develop a fully automated procedure based on morphological reconstruction. Let $I(x, y)$ denote a binary image and suppose that we form a marker image F that is 0 everywhere, except at the image border, where it is set to $1 - I$; that is,

$$F(x, y) = \begin{cases} 1 - I(x, y) & \text{if } (x, y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases} \tag{9.5-28}$$

Then

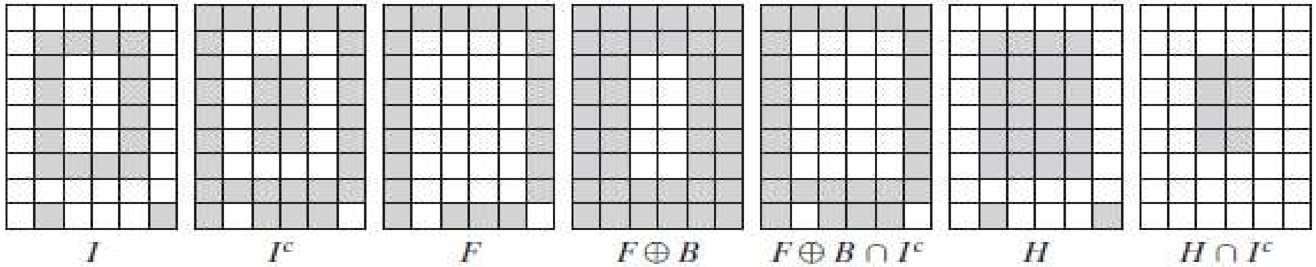
$$H = \left[R_{I^c}^{D}(F) \right]^c \tag{9.5-29}$$

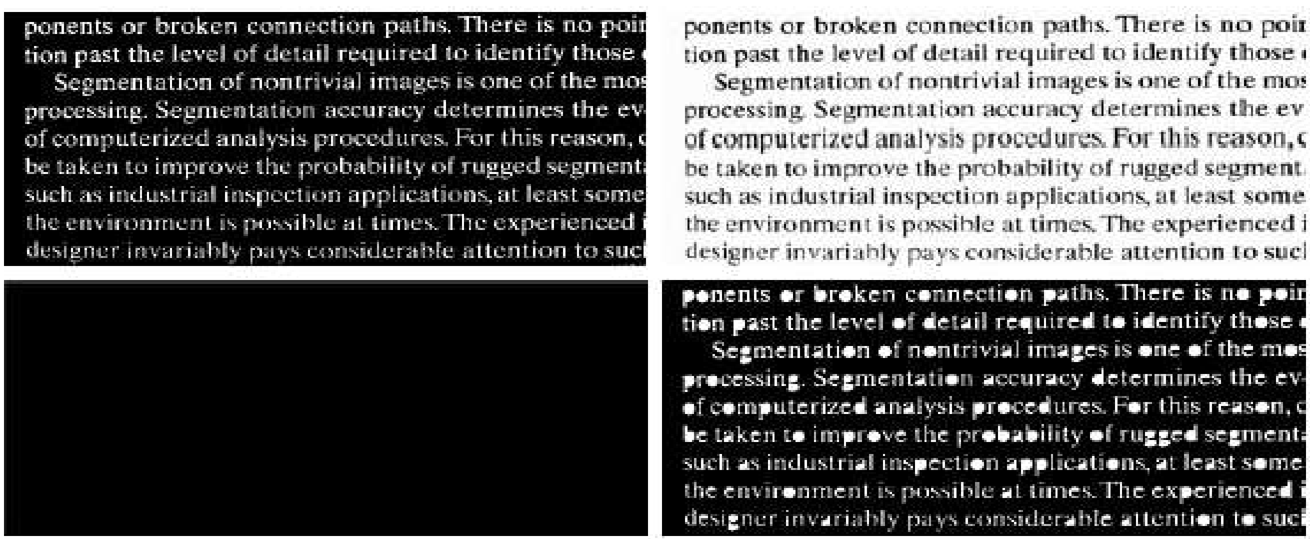
is a binary image equal to I with all holes filled.

Filling Holes

a b c d e f g

FIGURE 9.30 Illustration of hole filling on a simple image.





a b
c d

FIGURE 9.31 (a) Text image of size 918 × 2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).

Border cleaning

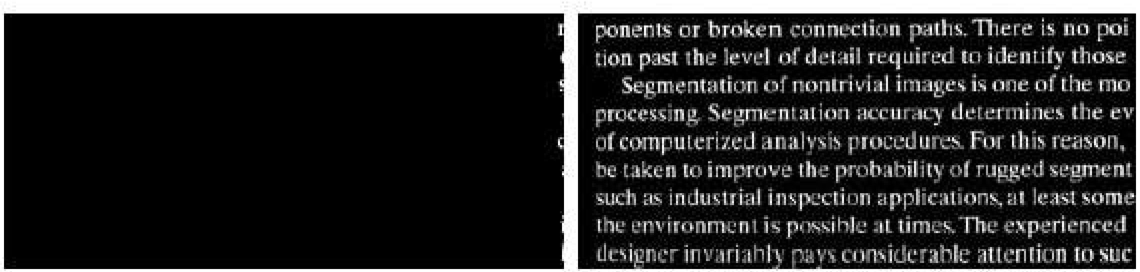
The extraction of objects from an image for subsequent shape analysis is a fundamental task in automated image processing. An algorithm for removing objects that touch (i.e., are connected to) the border is a useful tool because (1) it can be used to screen images so that only complete objects remain for further processing, or (2) it can be used as a signal that partial objects are present in the field of view. As a final illustration of the concepts introduced in this section, we develop a border-clearing procedure based on morphological reconstruction. In this application, we use the original image as the mask and the following marker image:

$$F(x, y) = \begin{cases} I(x, y) & \text{if } (x, y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases} \tag{9.5-30}$$

The border-Eclearing algorithm first computes the morphological reconstruction $R_I^D(F)$ (which simply extracts the objects touching the border) and then computes the difference

$$X = I - R_I^D(F) \tag{9.5-31}$$

to obtain an image, X, with no objects touching the border.



a b

FIGURE 9.32 Border clearing. (a) Marker image. (b) Image with no objects touching the border. The original image is Fig. 9.29(a).

Extraction of Connected Components

Extraction of connected components from a binary image is central to many automated image analysis applications.

Let A be a set containing one or more connected components

X_0 form an array (of the same size as the array containing A) whose elements are 0_s (background) .

The objective is to start with X_0 and find all the connected components.

$X_k = (X_{k-1} \oplus B) \cap A$, where k= 1, 2, 3... Eq 9.5.-3 and B is a suitable structuring element.

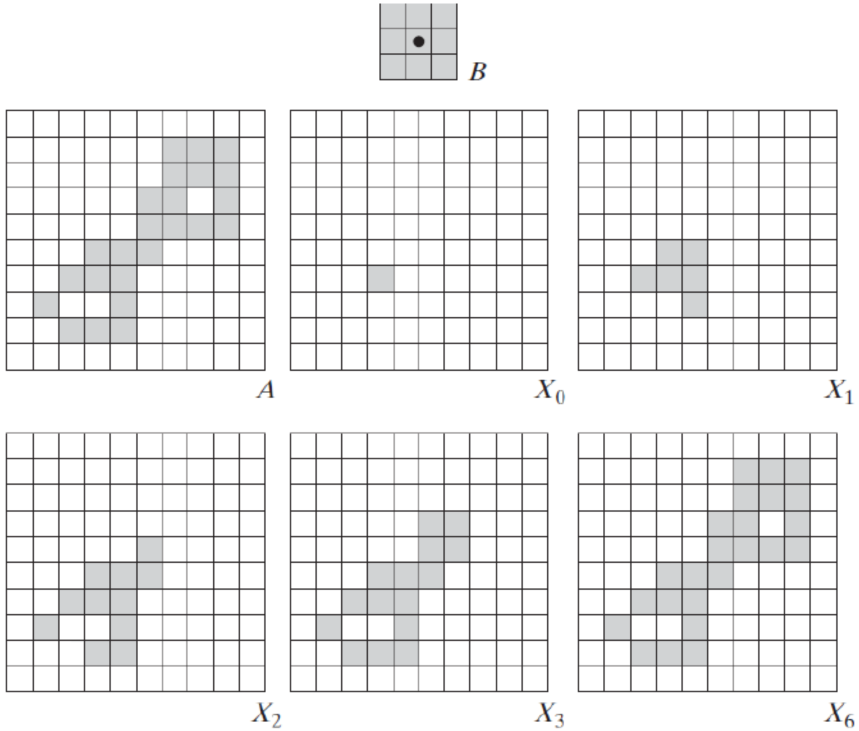


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).

Connected components: Application

- Connected components are used frequently for automated inspection. Figure 9.18(a) shows an X-ray image of a chicken breast that contains bone fragments.

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

For more concepts on further image processing read following chapters in Text book

- Text Book:

Digital image processing: Rafel C Gonzales, Chapter 9