Definition of Signal Density

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Data tensor

For a given session, let the data tensor X_{nbt} have dimensions of size $[N, B, T]$ representing the number of neurons, number of bins, and number of trials. The value X_{nbt} is the mean DFF in neuron n when the mouse was on bin b during trial t.

Mean response

The mean response is then

$$\mu_{nb} = \frac{1}{T}\sum_t X_{nbt}$$

Signal vector

The signal vector is then $\Delta\mu_{nb} = \mu_{n(b+1)} - \mu_{nb}$, keeping in mind that bins b and $b+1$ must actually be spatially adjacent.

Signal direction

The signal direction unit vector is then

$$\hat{s}_{nb} = \Delta\mu_{nb} / \sqrt{\sum_i (\Delta\mu_{ib})^2}$$

Participation rate of the signal direction

The participation rate of the signal direction unit vector is then

$$PR_b = \sum_n (\hat{s}_{nb})^4$$

Inverse participation rate of the signal direction

The inverse participation rate of the signal direction unit vector is then

$$IPR_b = 1/PR_b$$

Signal density

The signal density is then

$$\rho_b = IPR_b/N$$

The expanded formula for signal density

If we expand all the terms, we can see that in terms of $\Delta\mu_{nb}$,

$$\rho_b = \frac{1}{N}\left(\sum_n (\Delta\mu_{nb})^2\right)^2 / \left(\sum_n (\Delta\mu_{nb})^4\right)$$

The mean signal density

We can also define the mean signal density over bins,

$$\bar{\rho} = \frac{1}{B-1}\sum_{b=1}^{B-1} \rho_b$$

This formula is easily extended to 2 directions of motion.

Noise density

Since I used noise density, I will mention that it is defined in the same way, just using $\sigma_{nb}^2 = Var_t\left[X_{nbt}\right]$.

Noise density

$$\nu_b = \frac{1}{N}\left(\sum_n \sigma_{nb}^2\right)^2 / \left(\sum_n \sigma_{nb}^4\right)$$

and mean noise density

$$\bar{\nu} = \frac{1}{B}\sum_{b=1}^{B} \nu_b$$