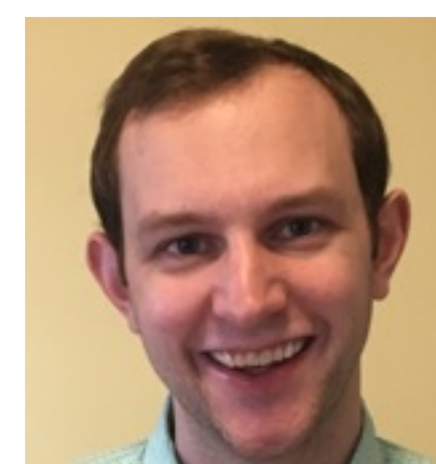
Automated Scalable Bayesian Inference via Data Summarization

Tamara Broderick
ITT Career Development
Assistant Professor,
MIT

With: Trevor Campbell, Jonathan H. Huggins



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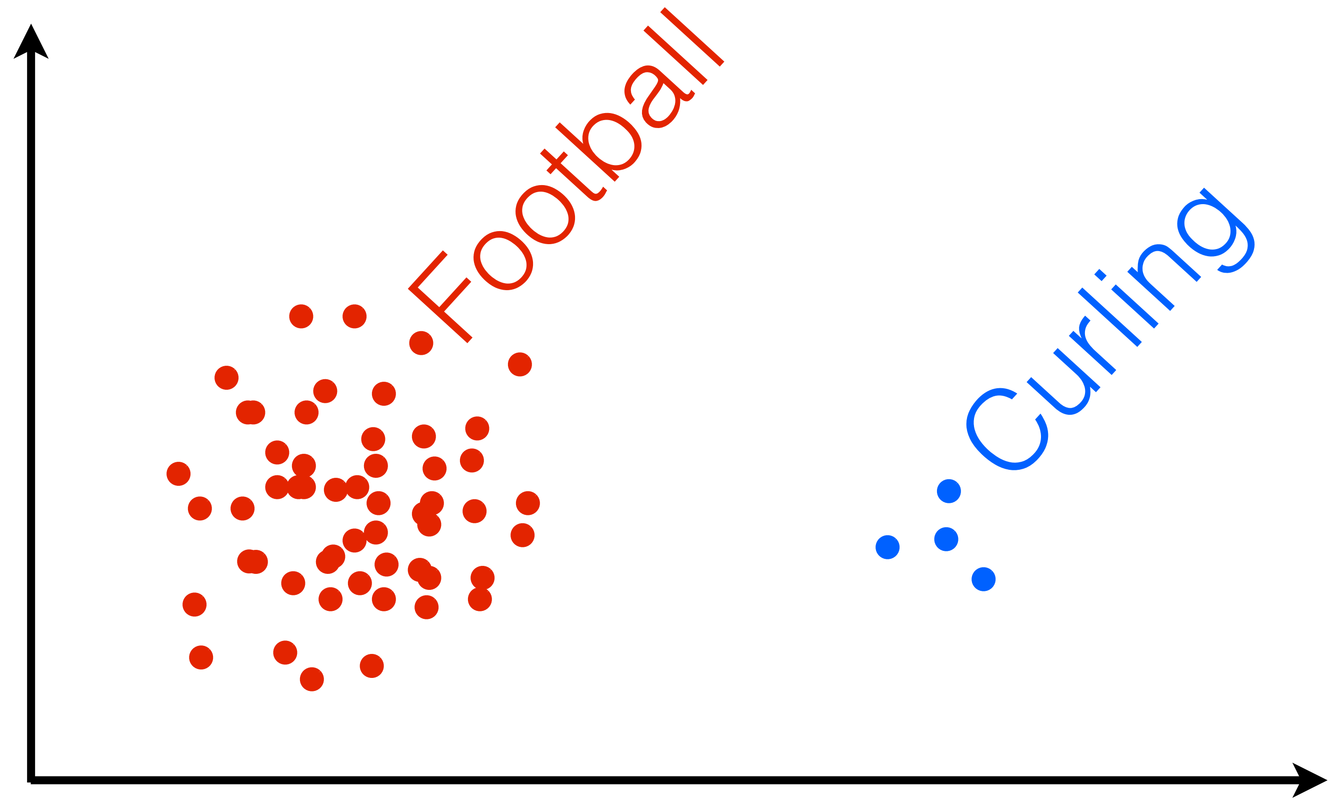

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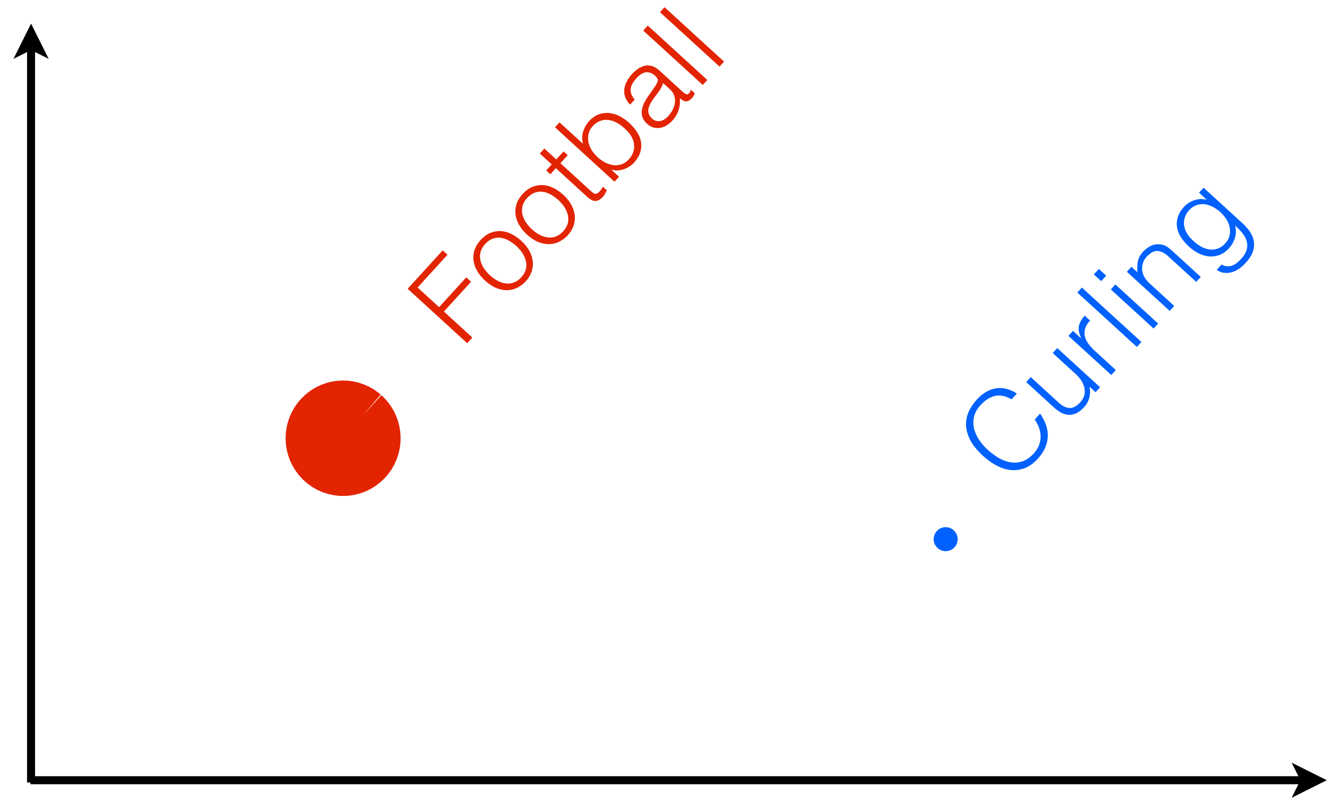

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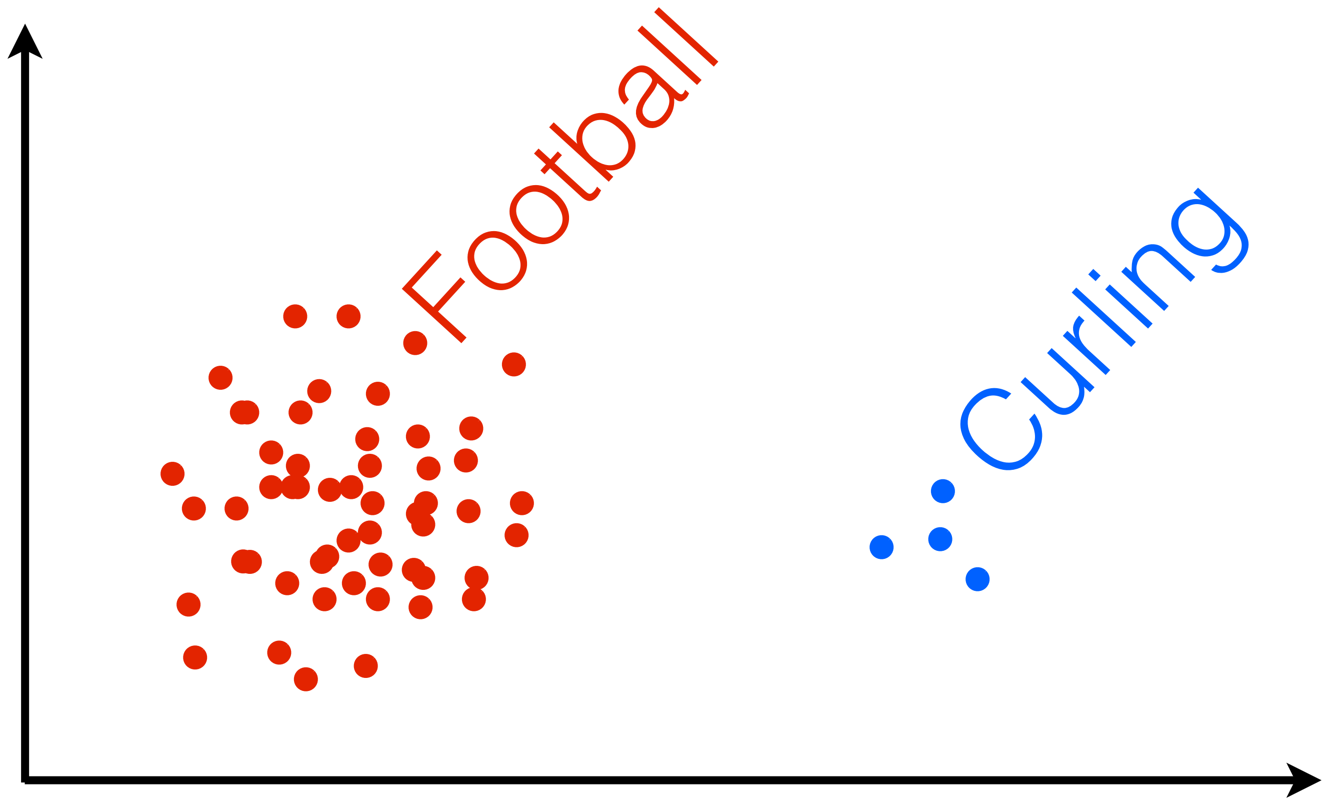

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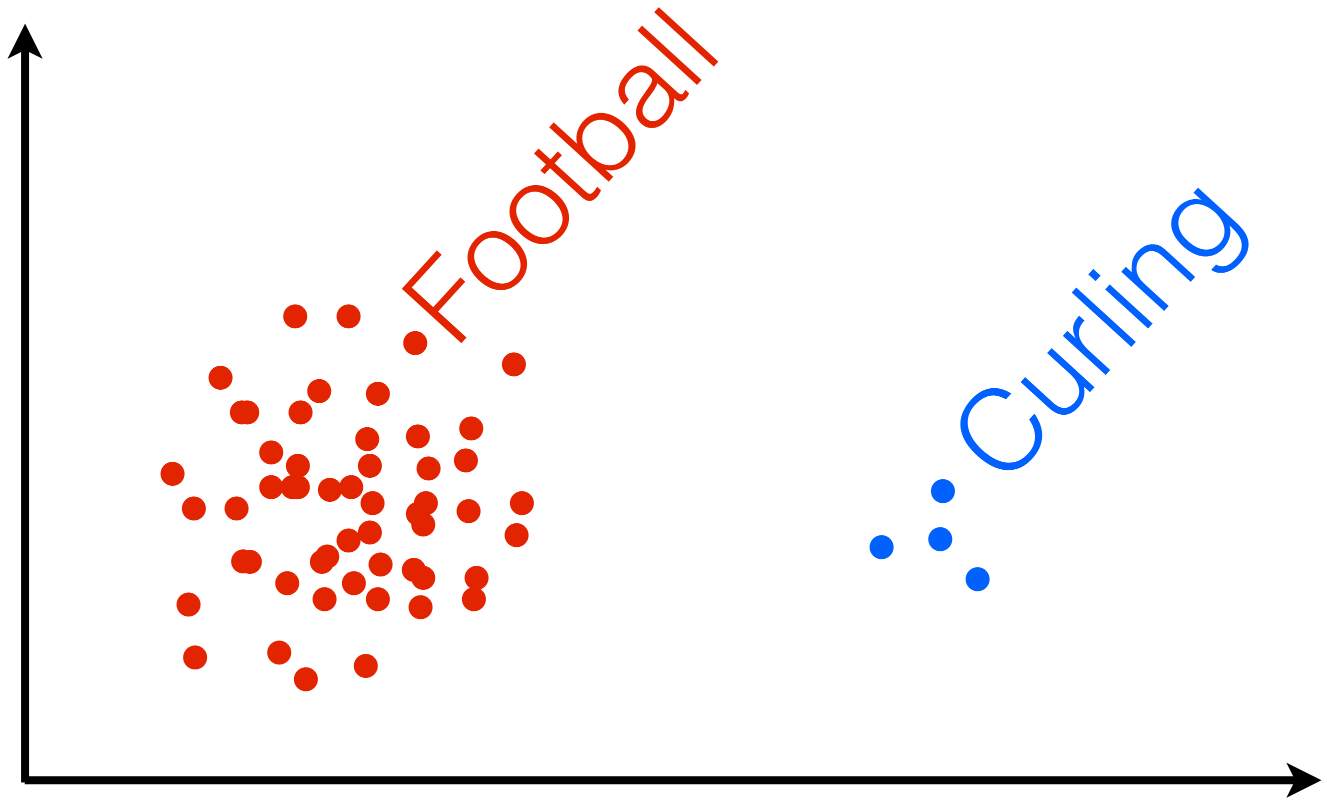
- Coresets: pre-process data to get a smaller, weighted data set



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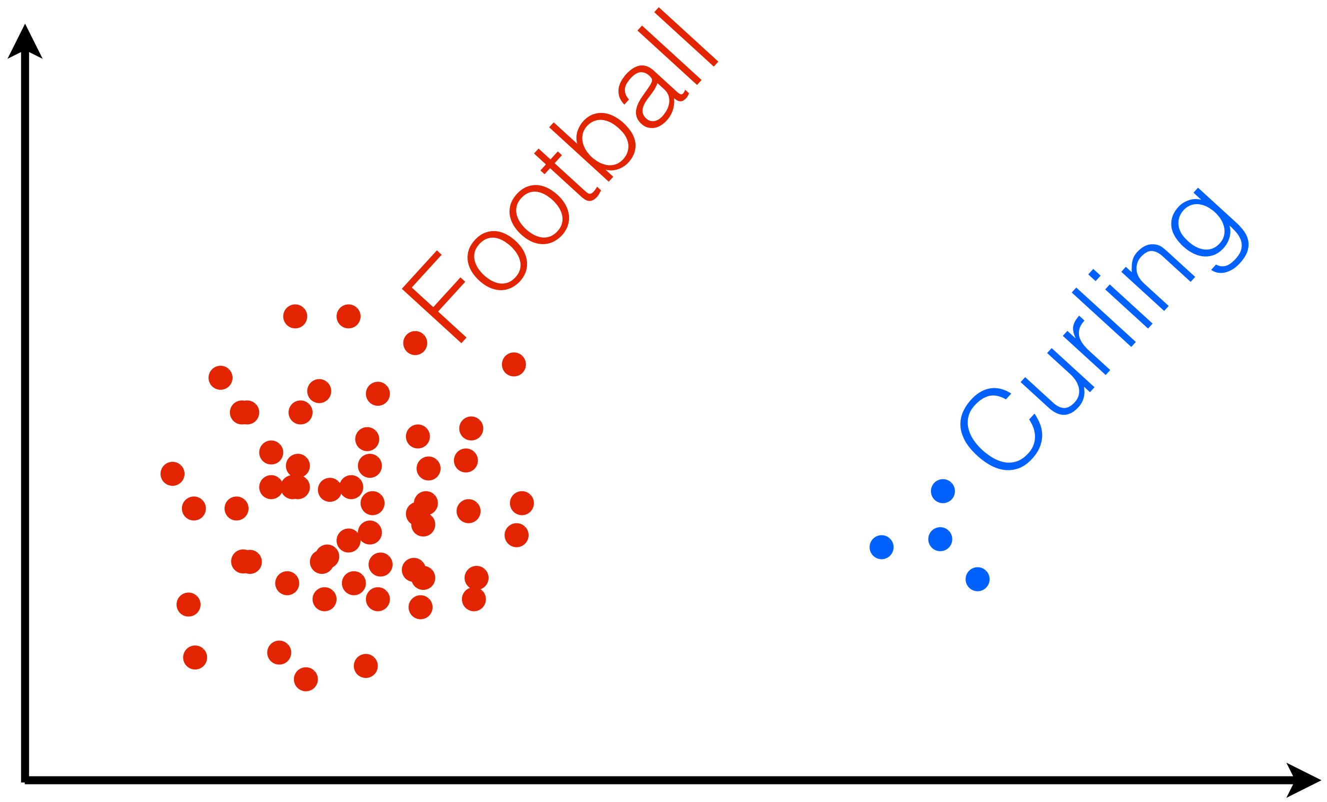

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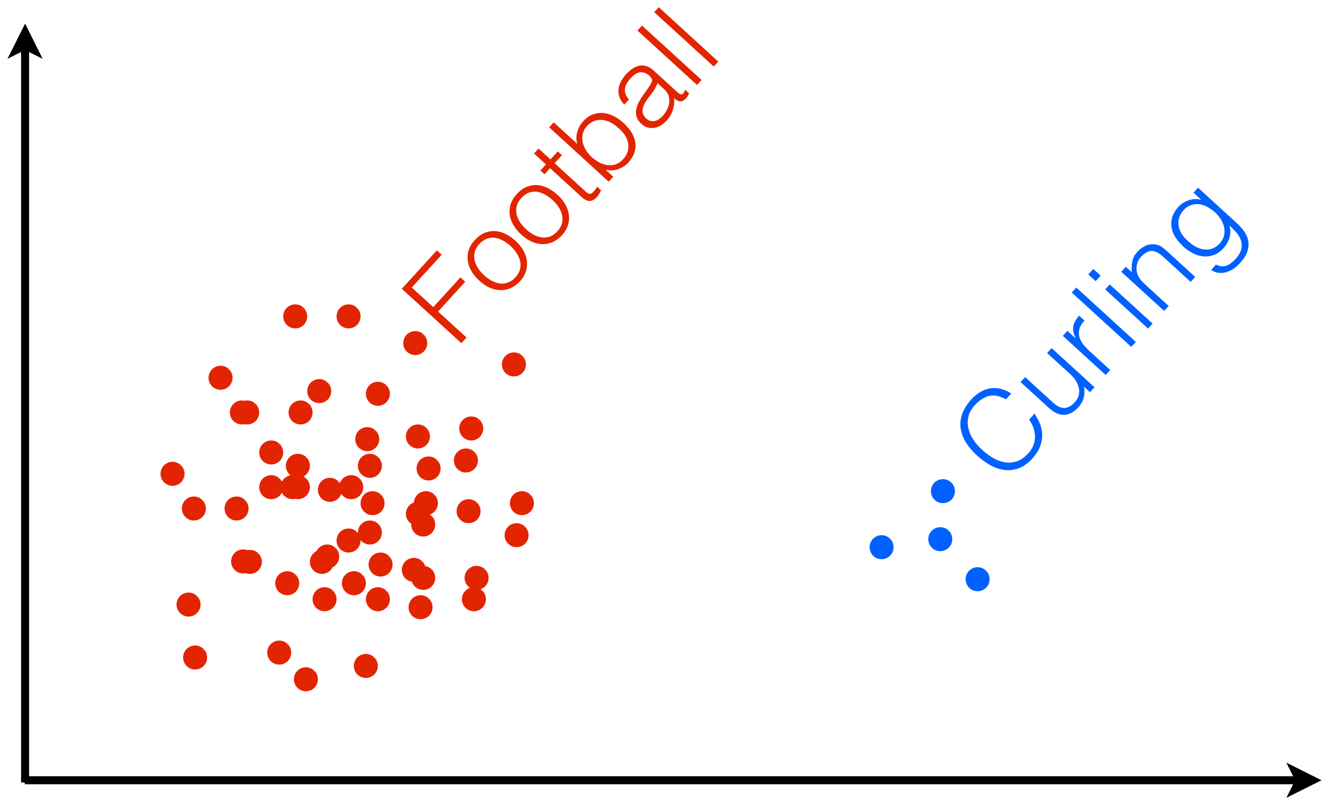

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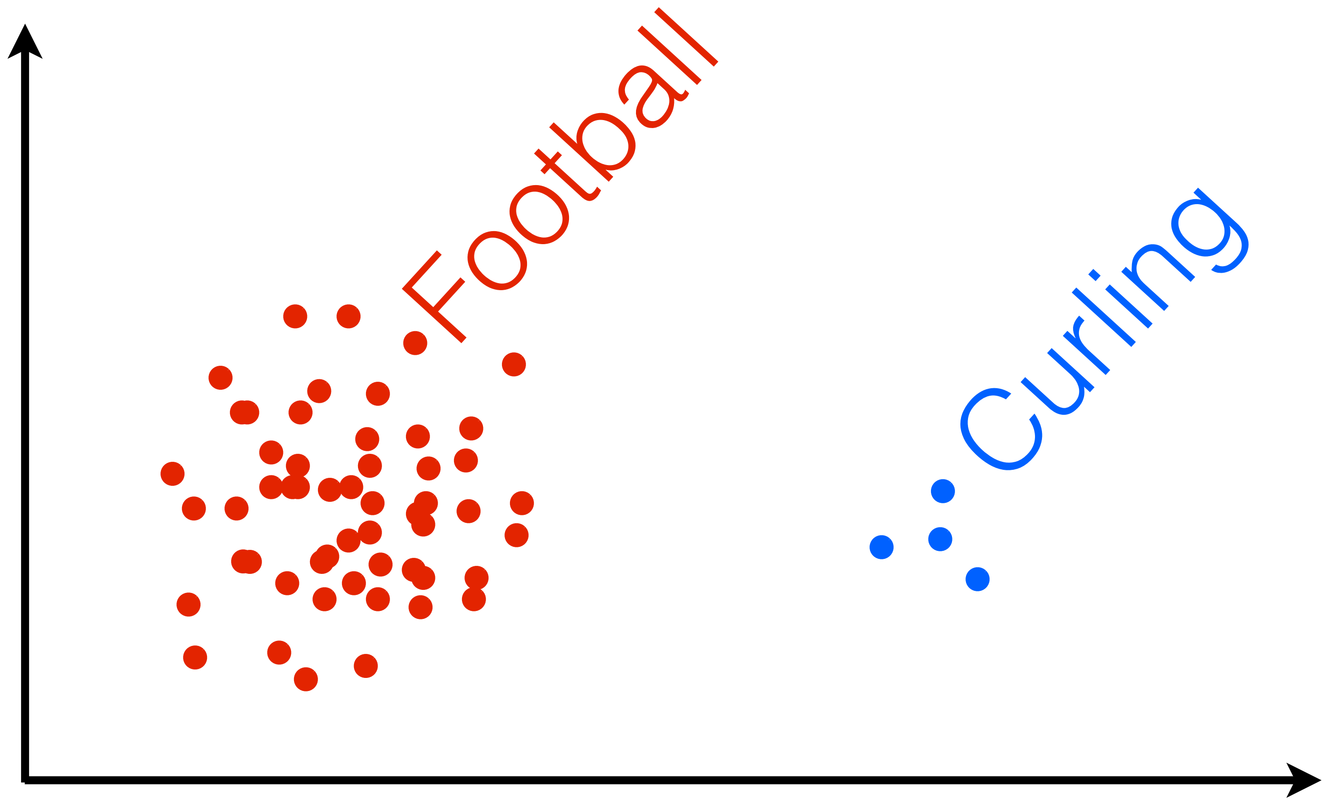

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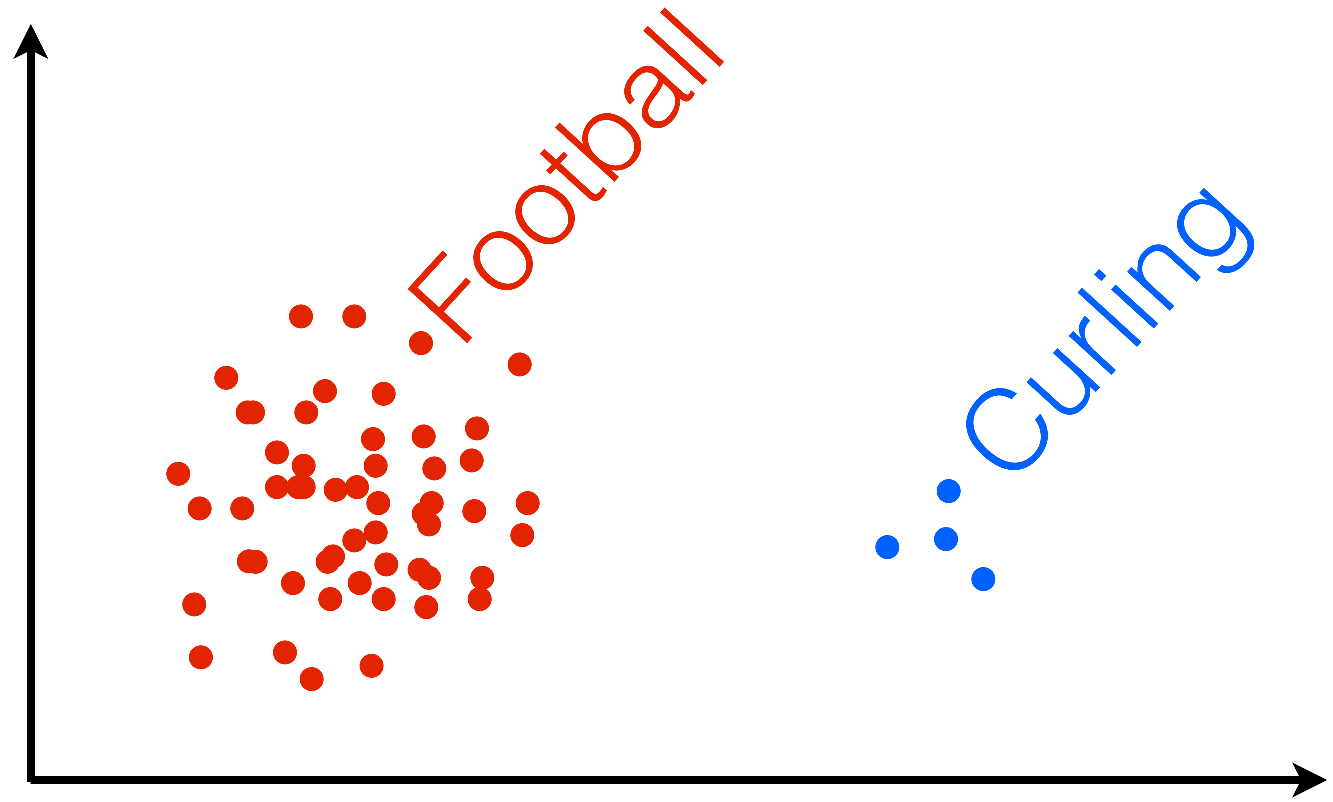

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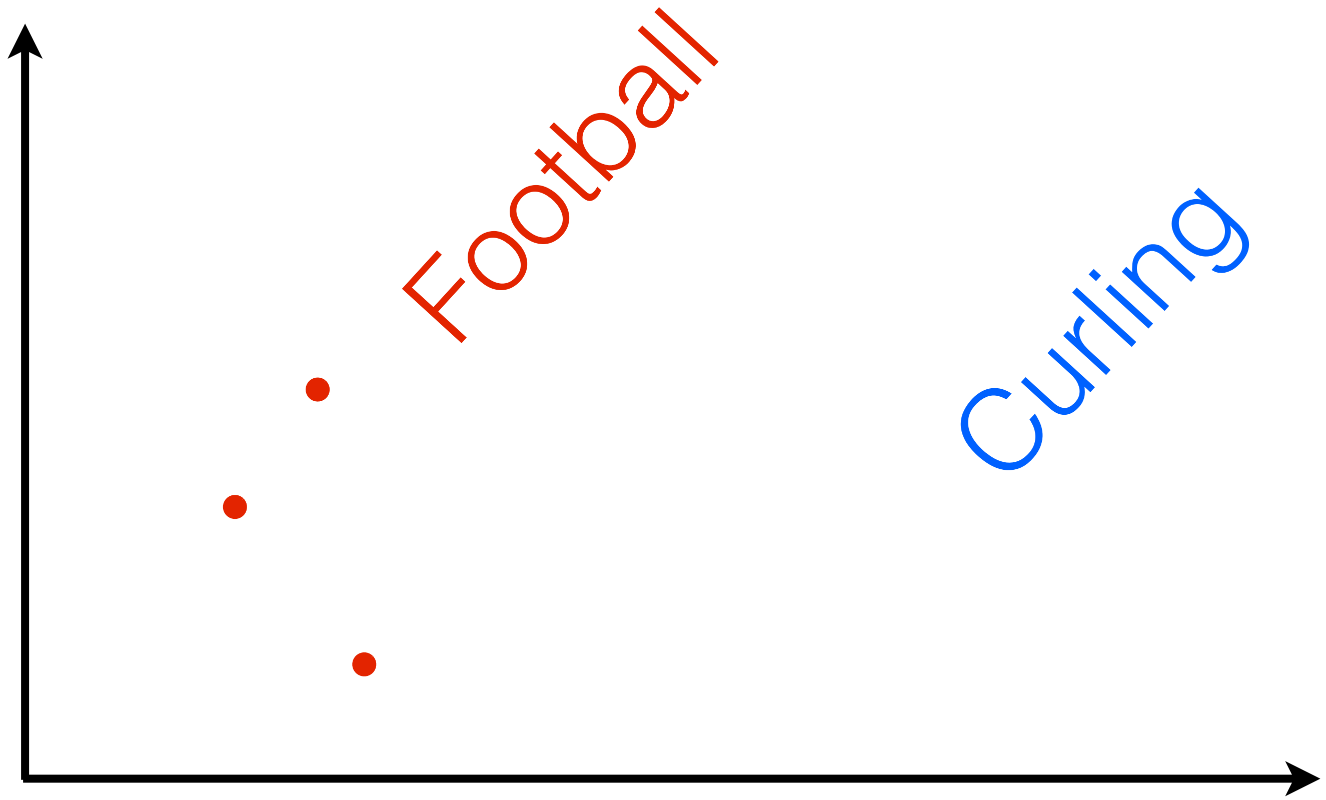

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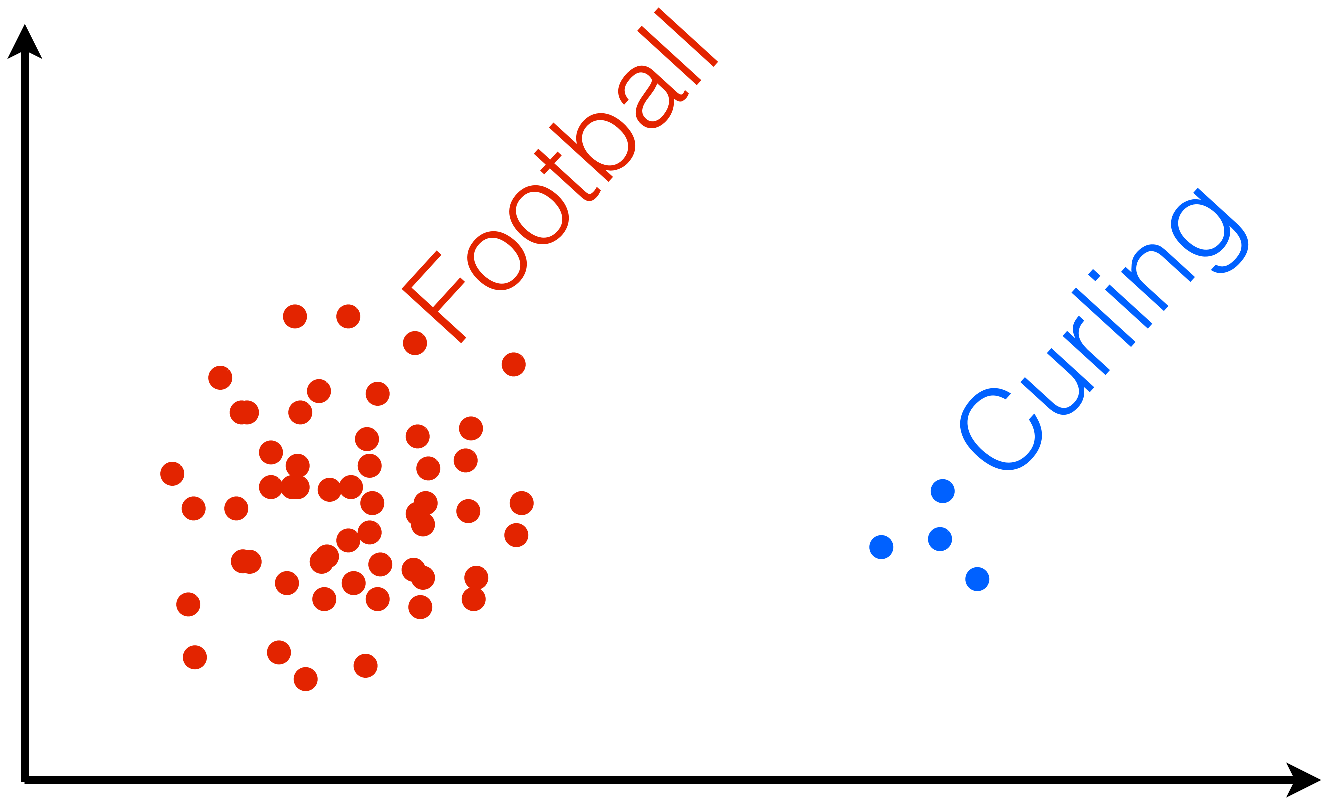

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Roadmap

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- Uniform data subsampling isn’t enough
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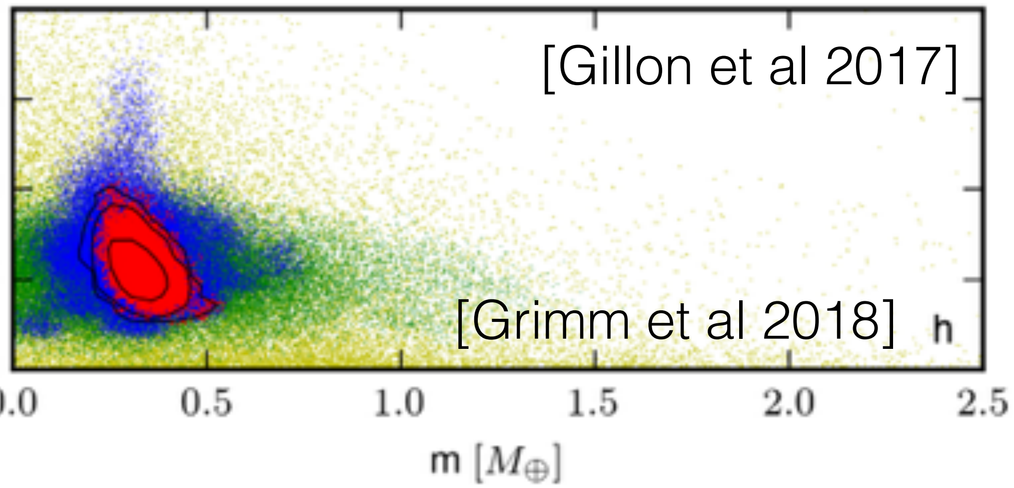

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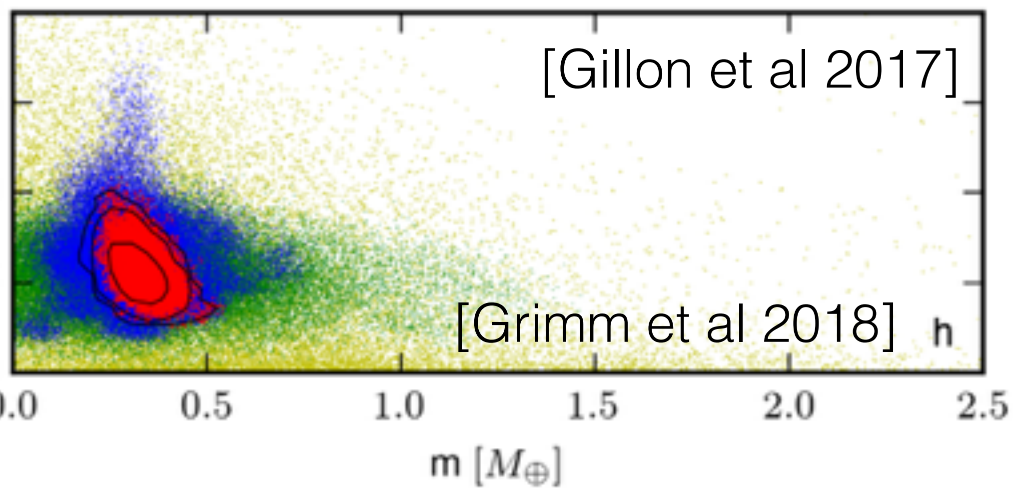

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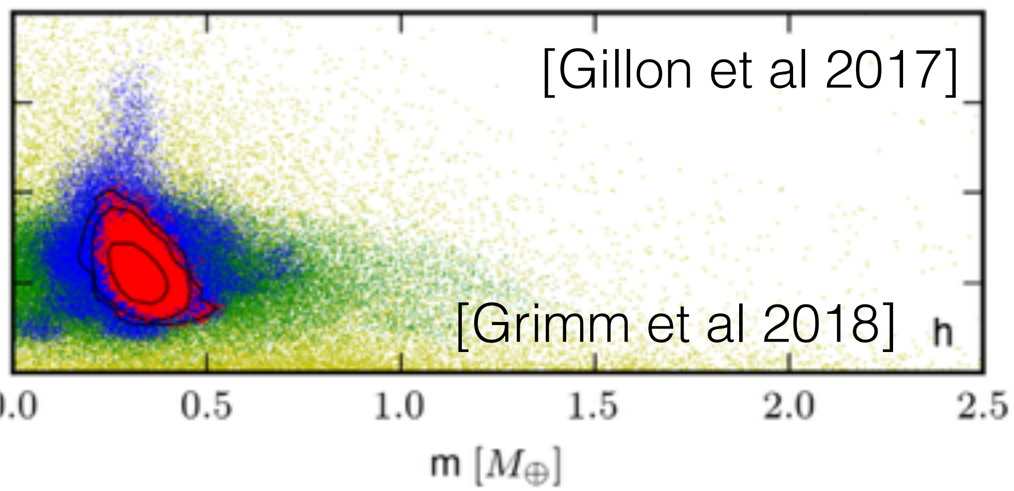

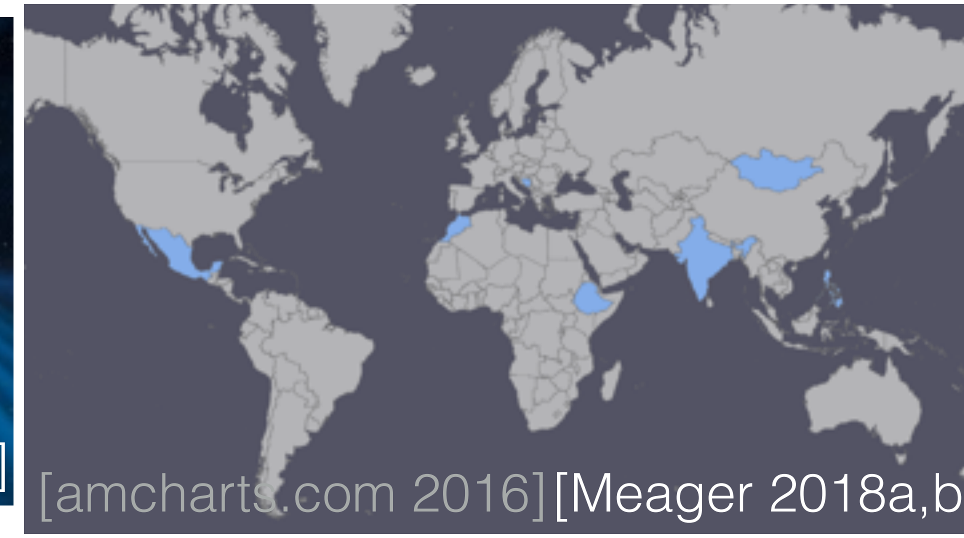

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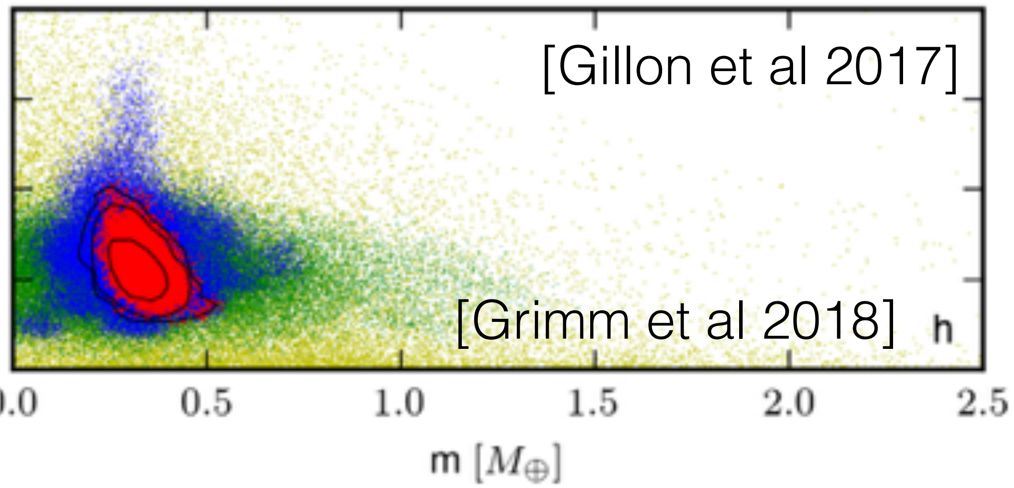

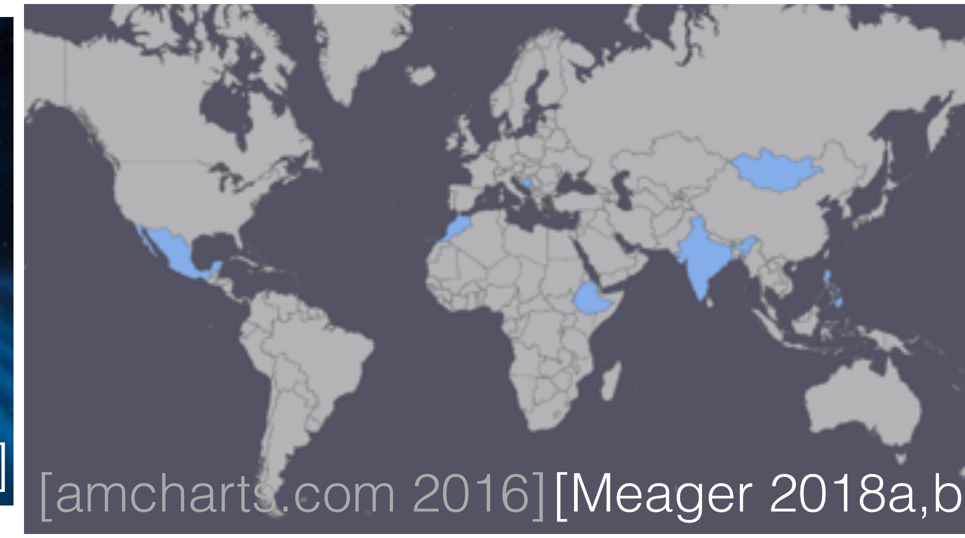

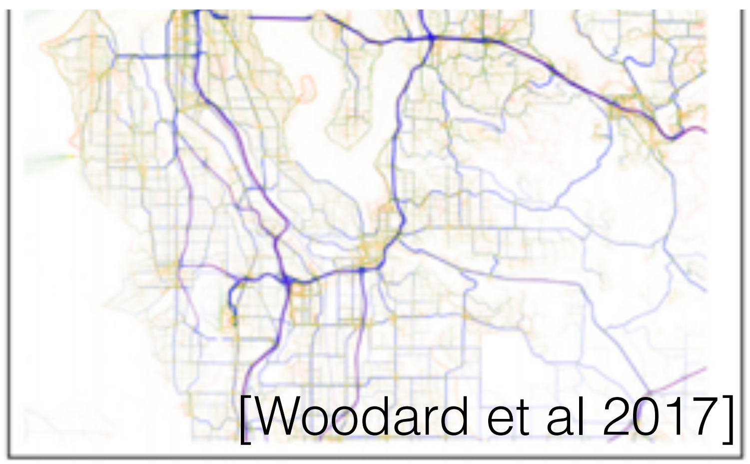

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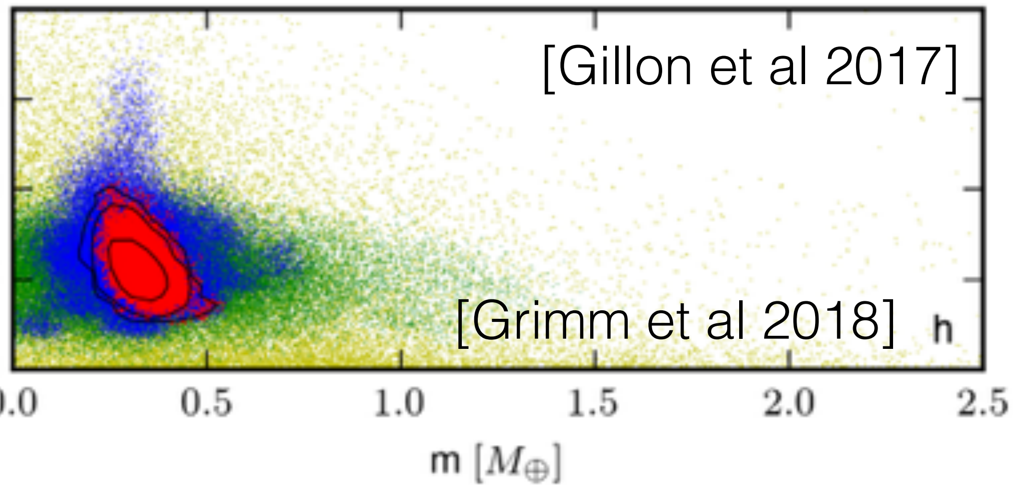

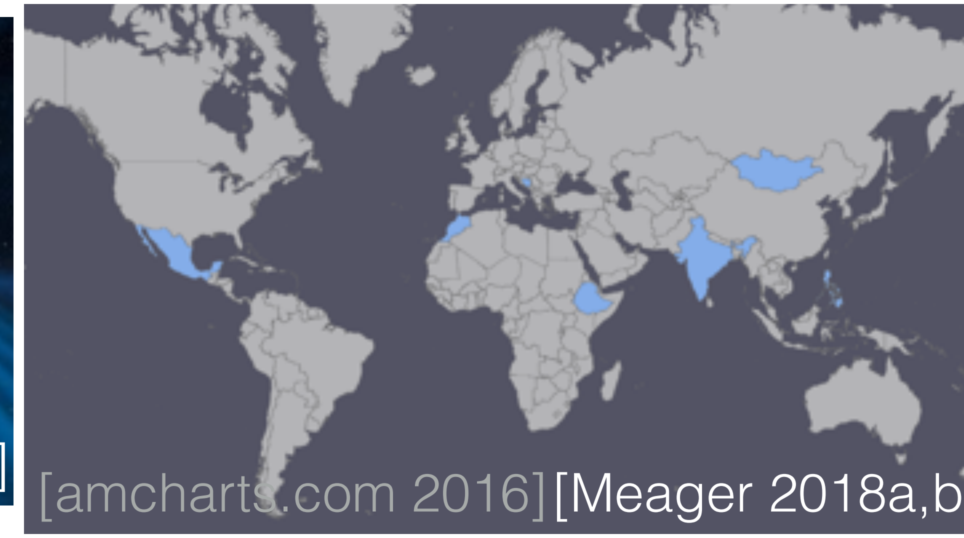

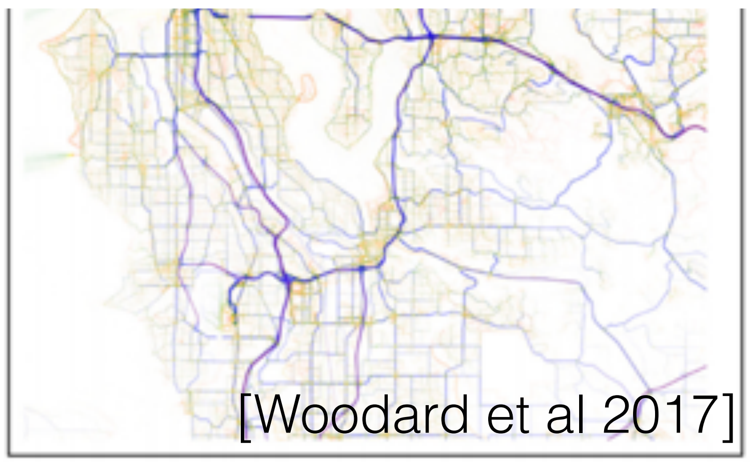

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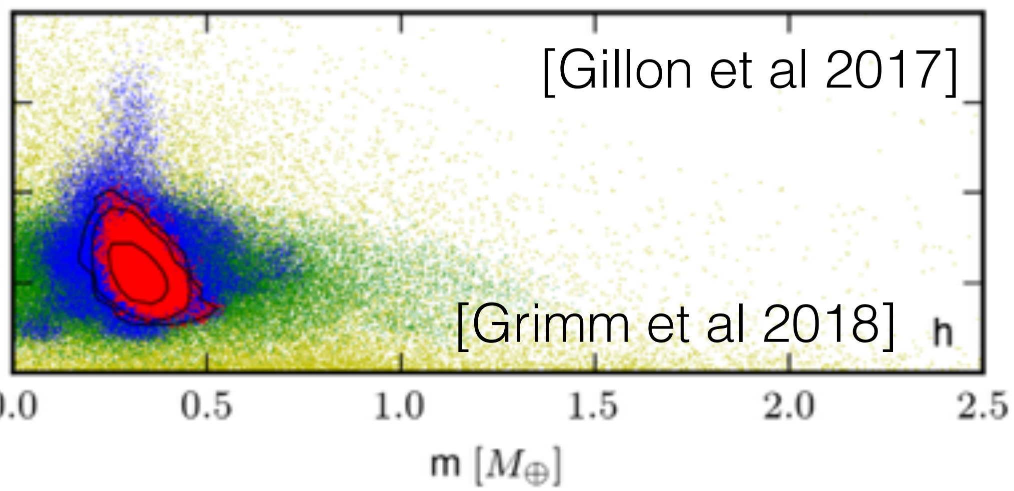

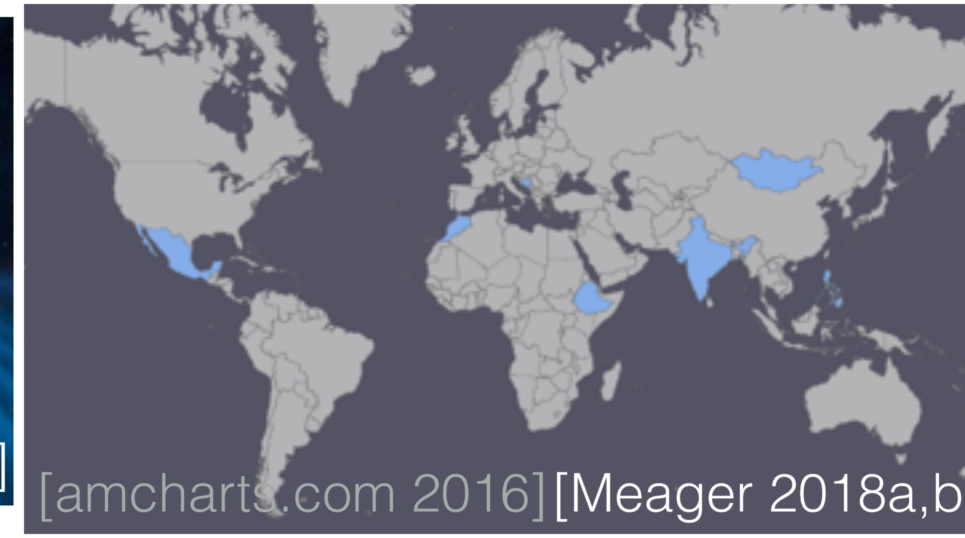

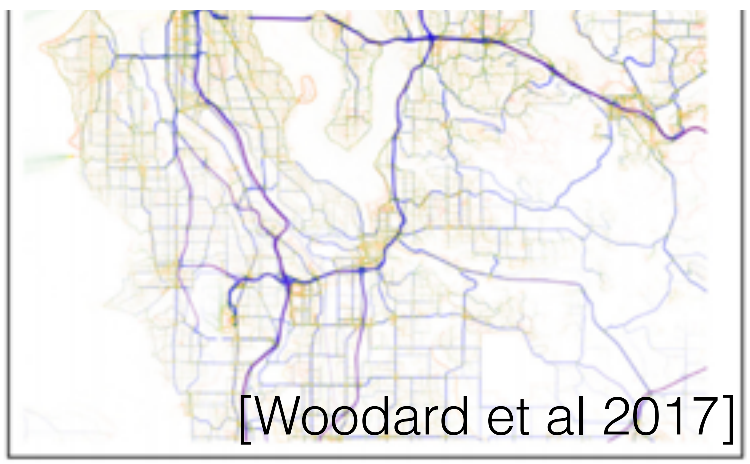

- Goal: Report point estimates, coherent uncertainties

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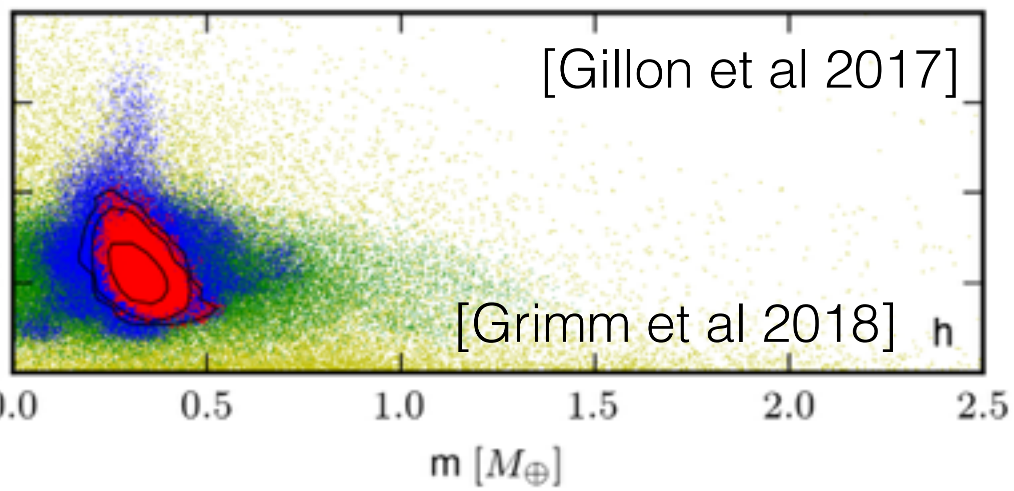

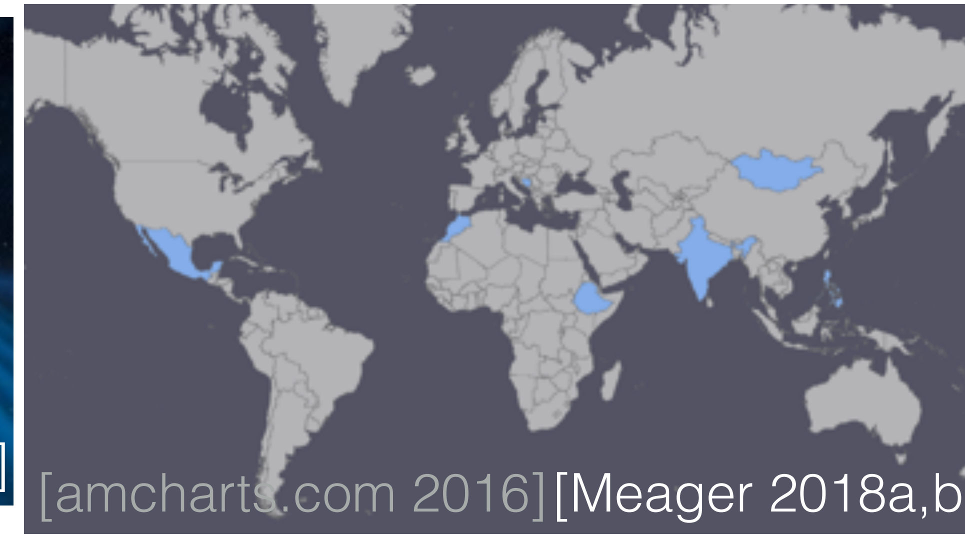

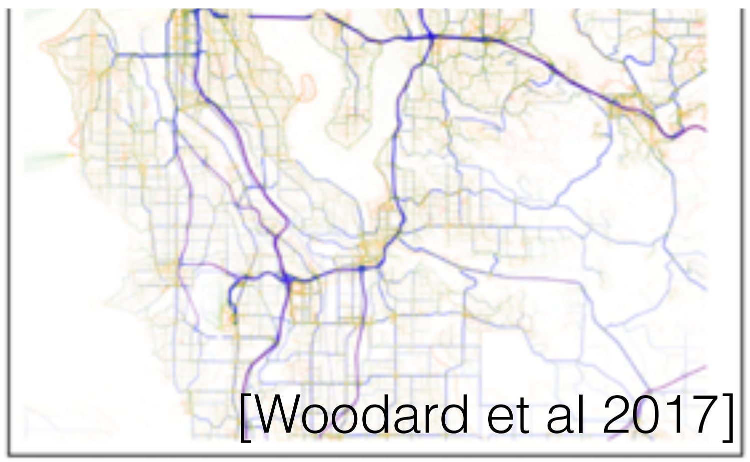

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- Challenge: existing methods can be slow, tedious, unreliable

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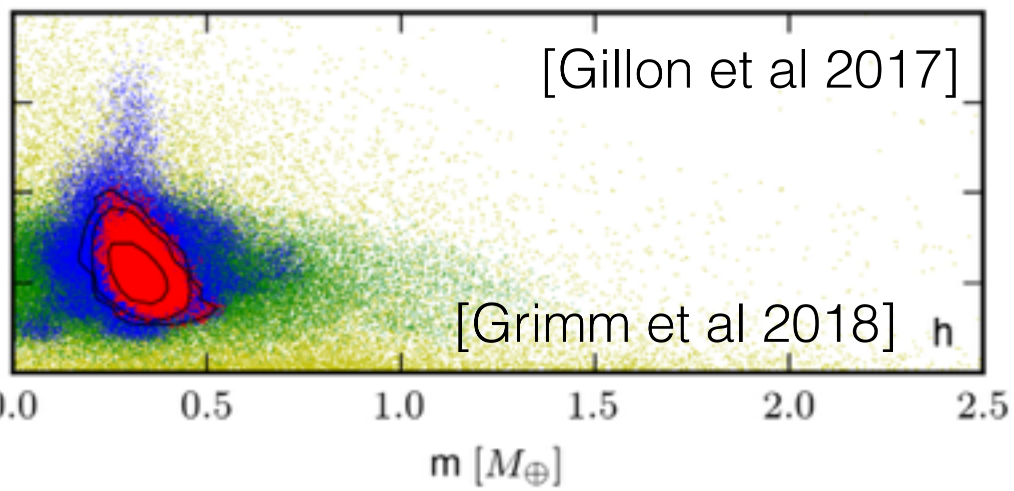

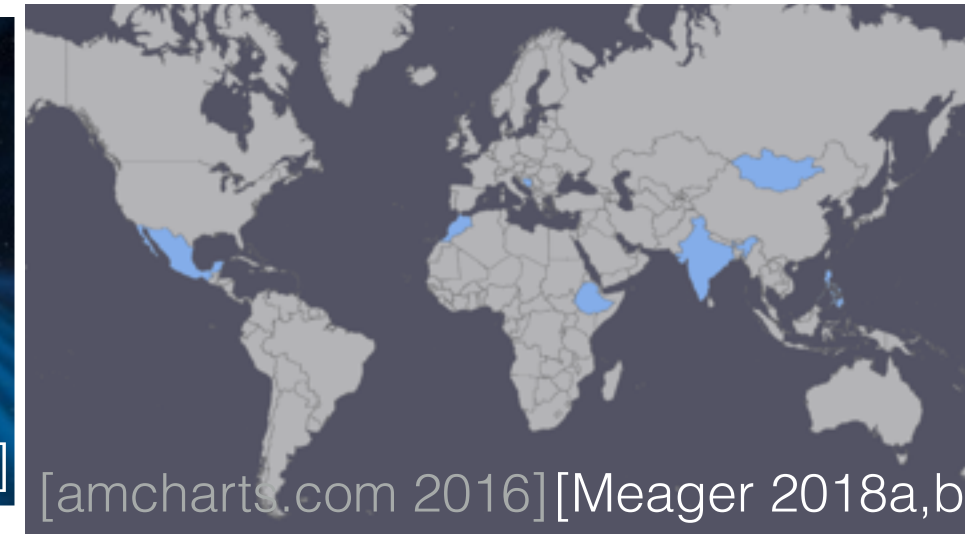

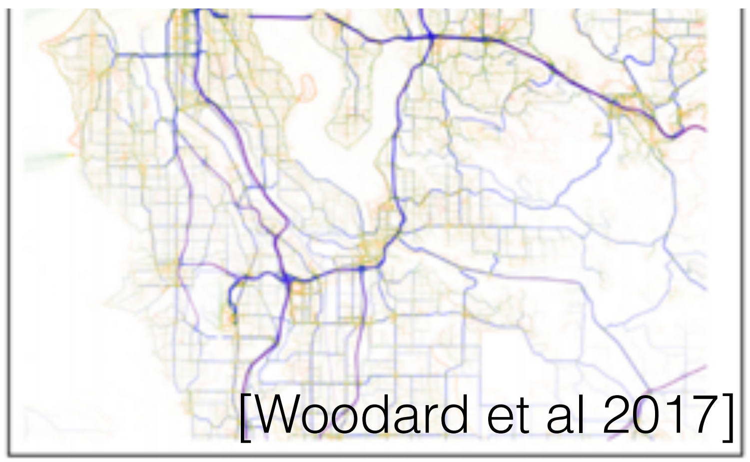

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- Challenge: existing methods can be slow, tedious, unreliable
- Our proposal: use *efficient data summaries* for **scalable**, **automated** algorithms with **error bounds for finite data**

Bayesian inference

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$p(\theta)$

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$p(y|\theta)p(\theta)$

Bayesian inference $p(\theta|y) \propto_\theta p(y|\theta)p(\theta)$

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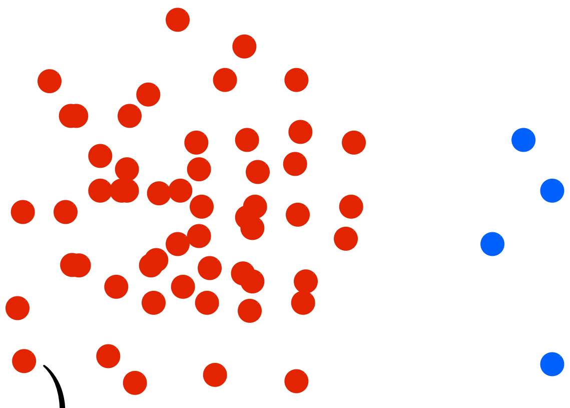

(x_n, y_n)

Bayesian inference

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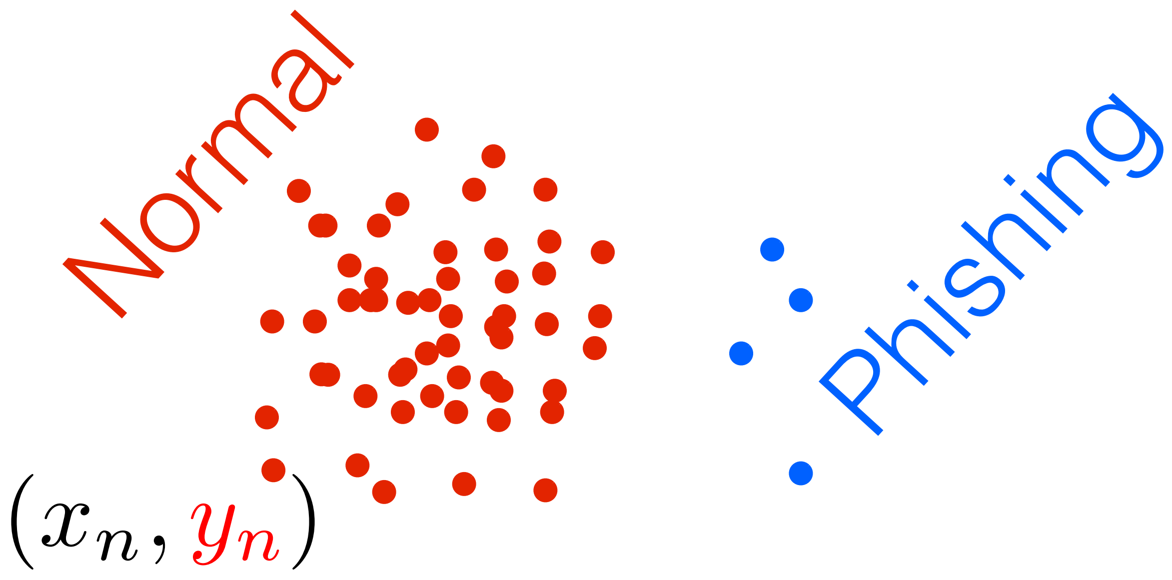

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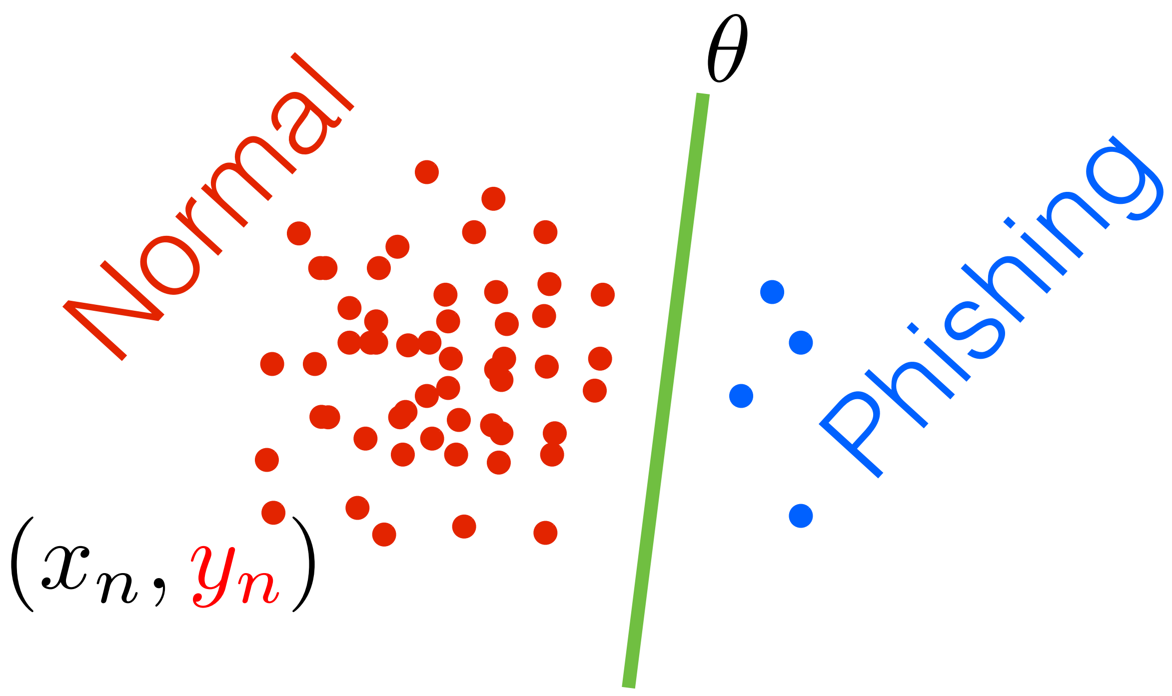

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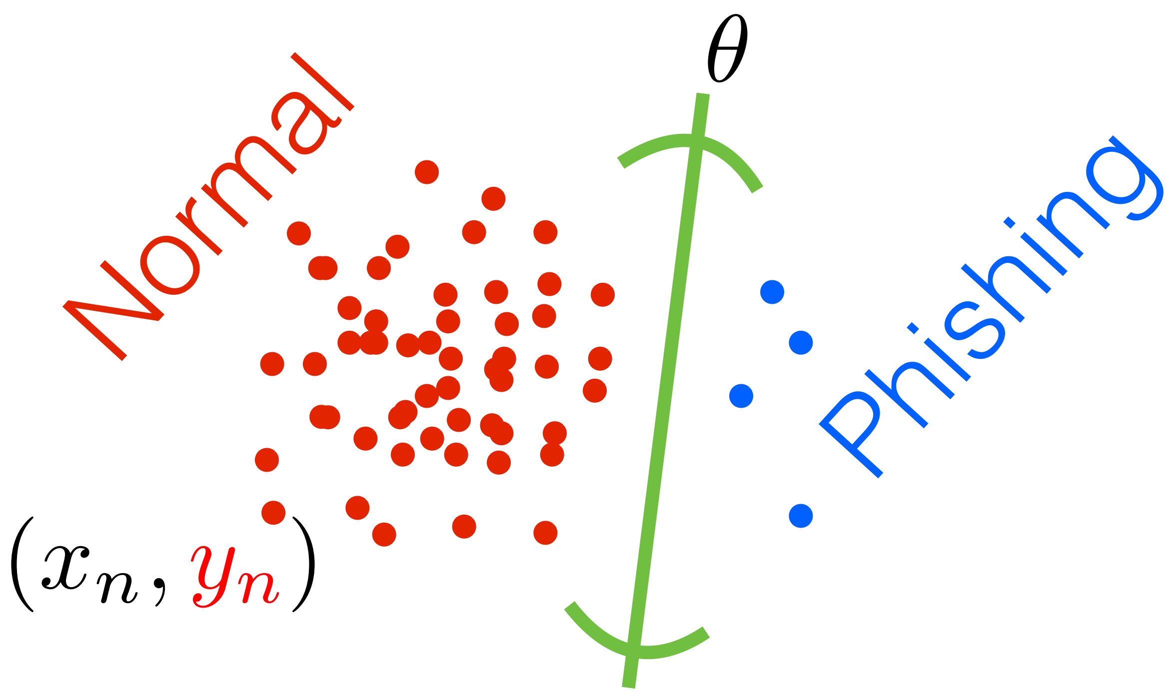

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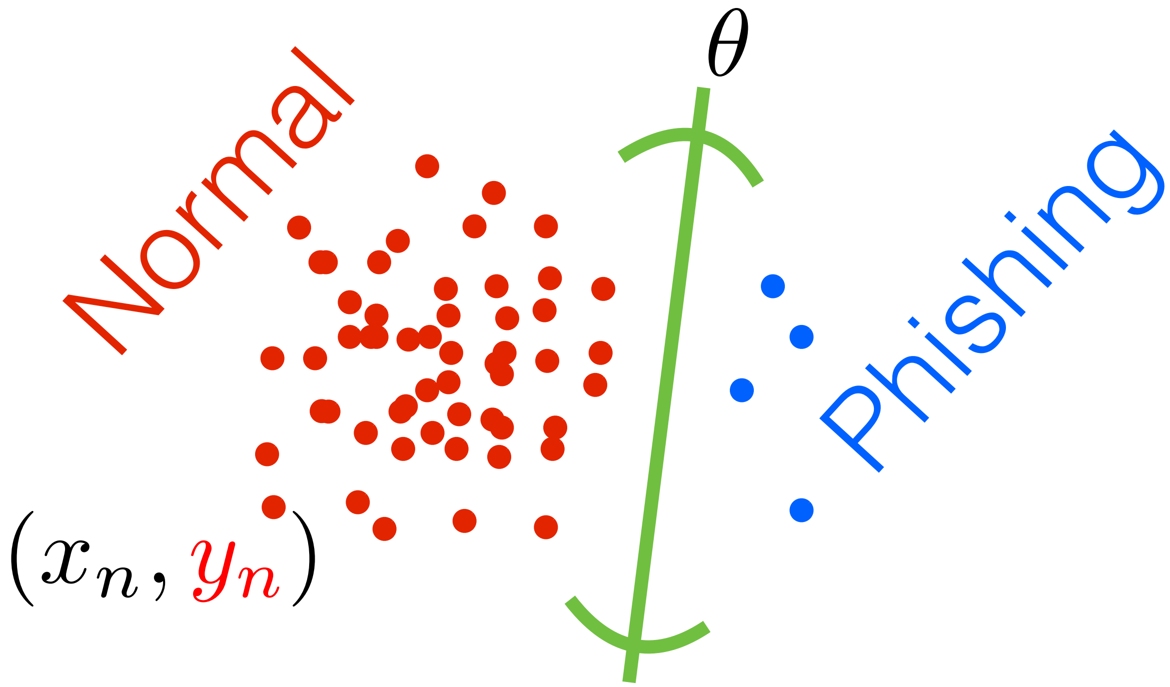

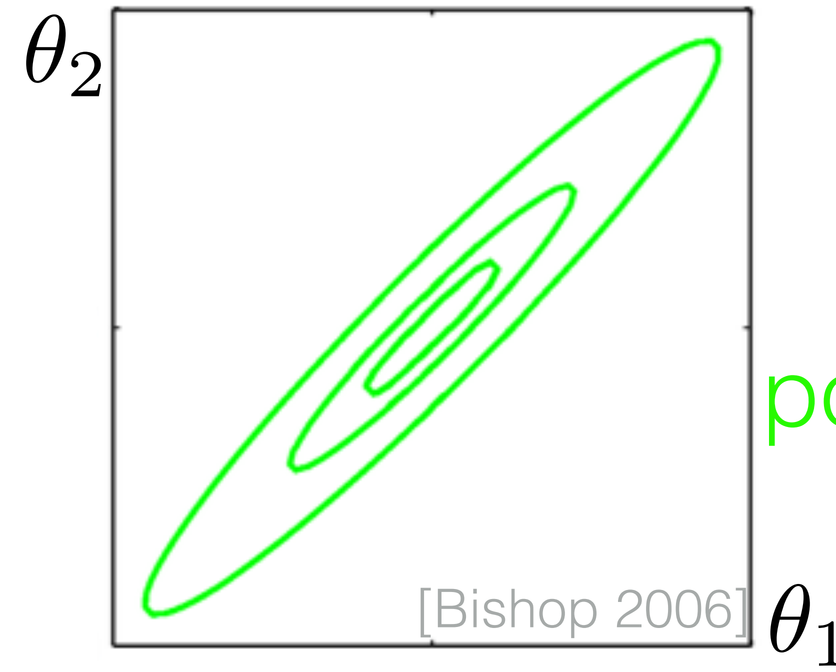

Exact posterior

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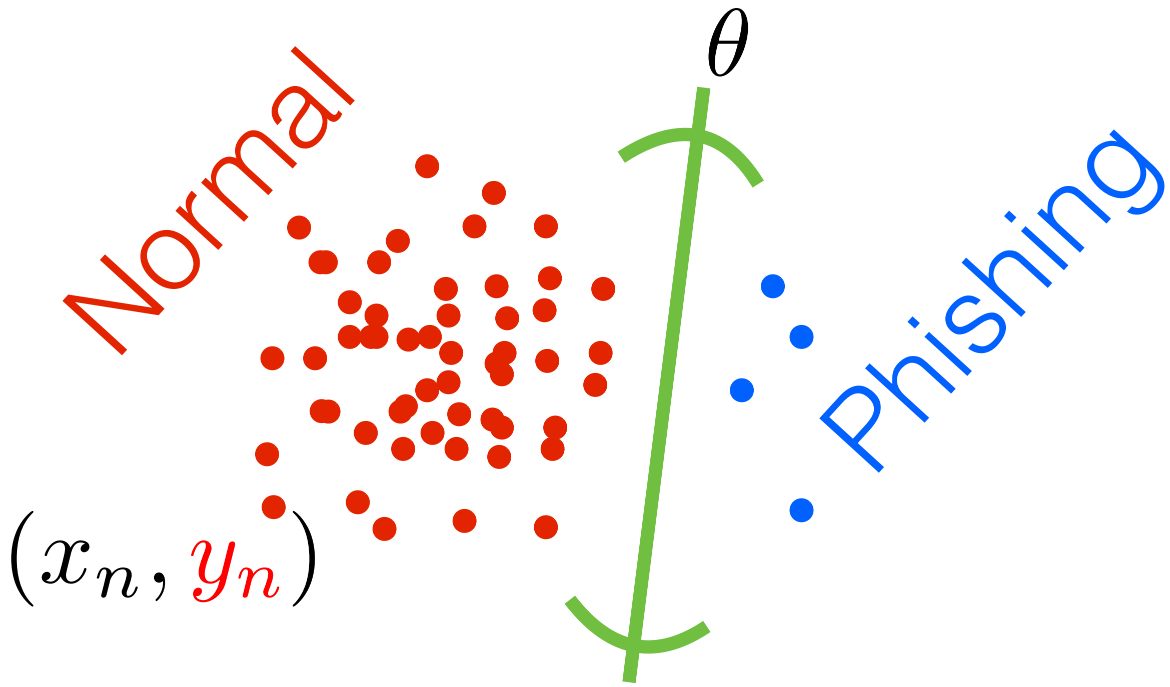

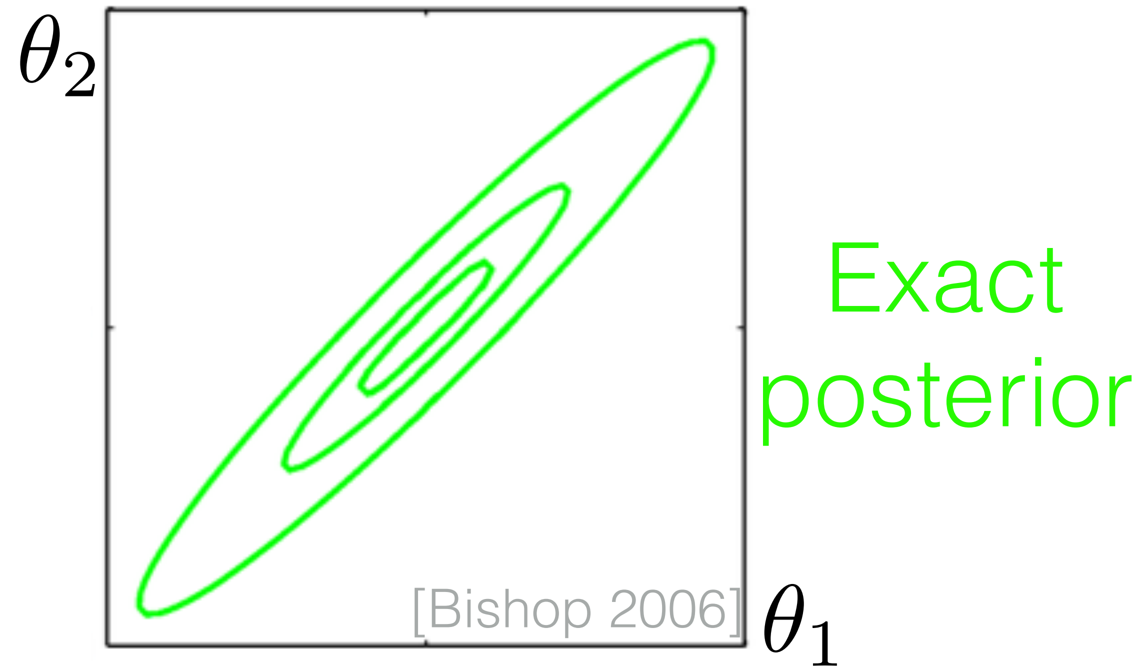

- MCMC: Eventually accurate but can be slow [Bardenet, Doucet, Holmes 2017]

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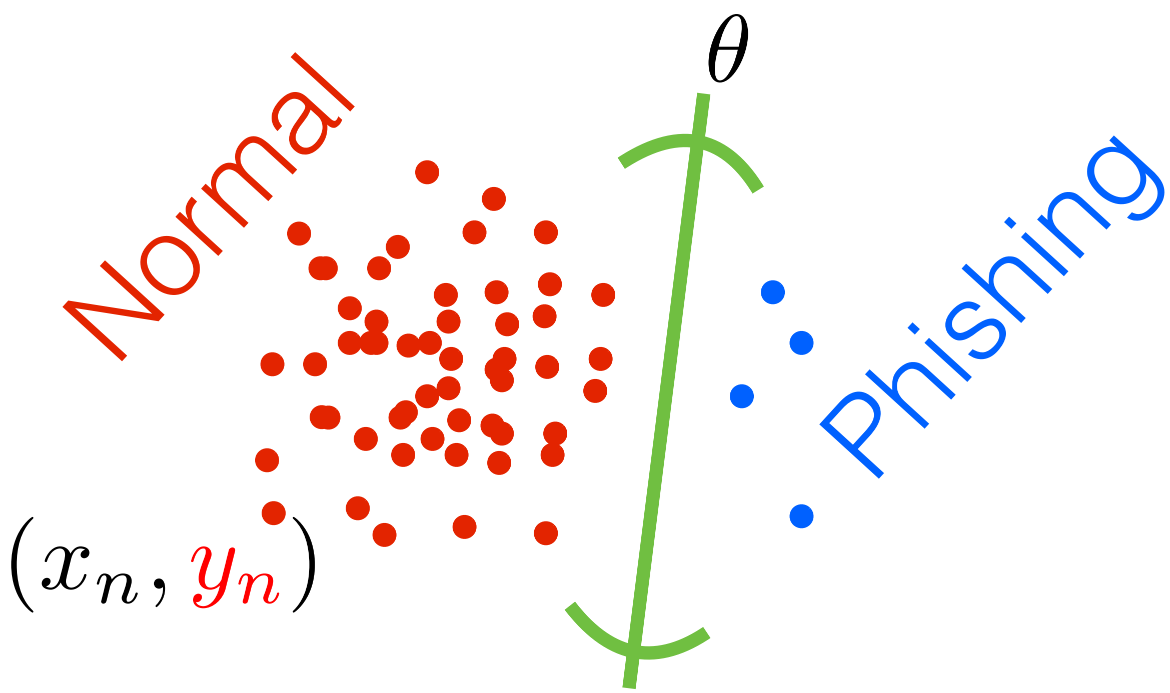

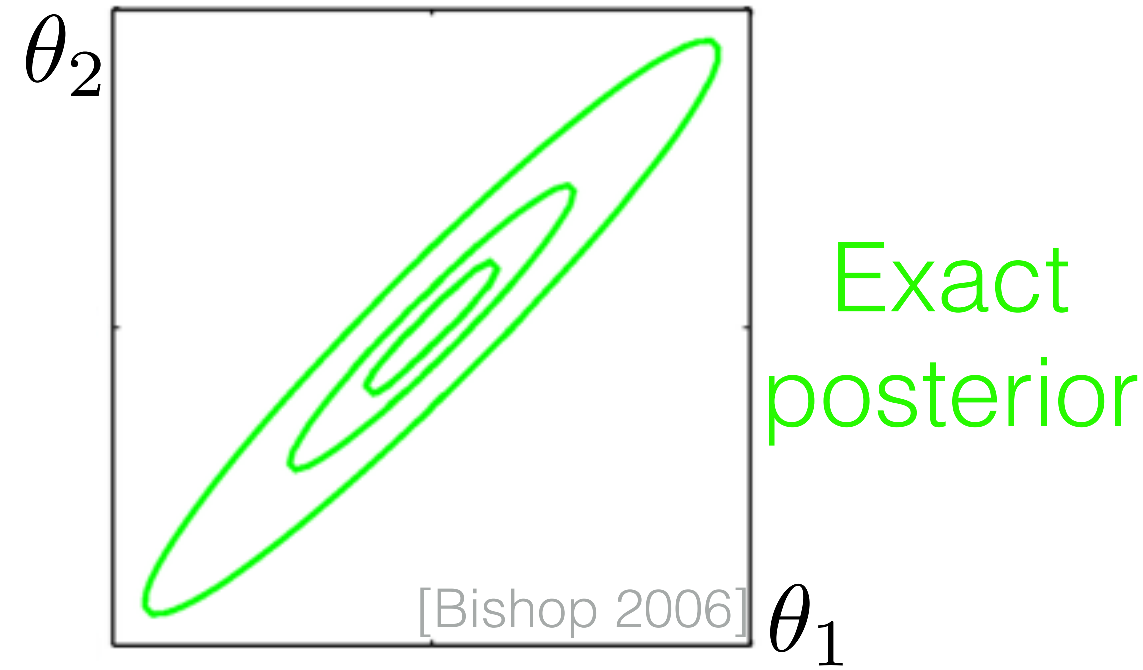

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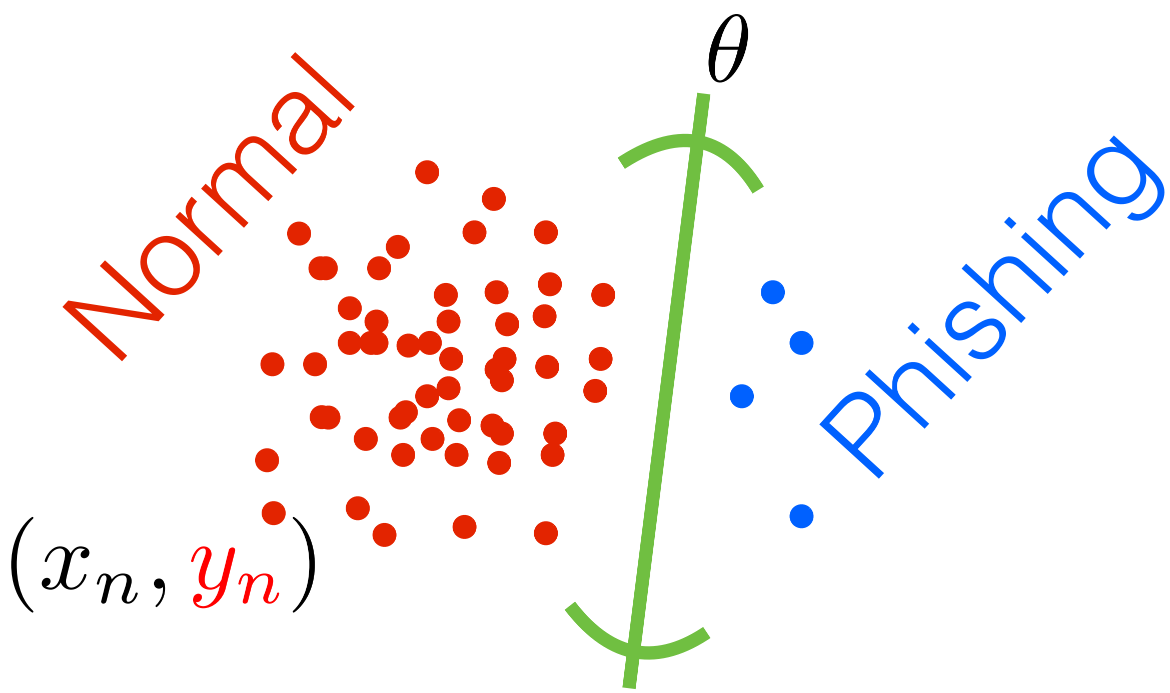

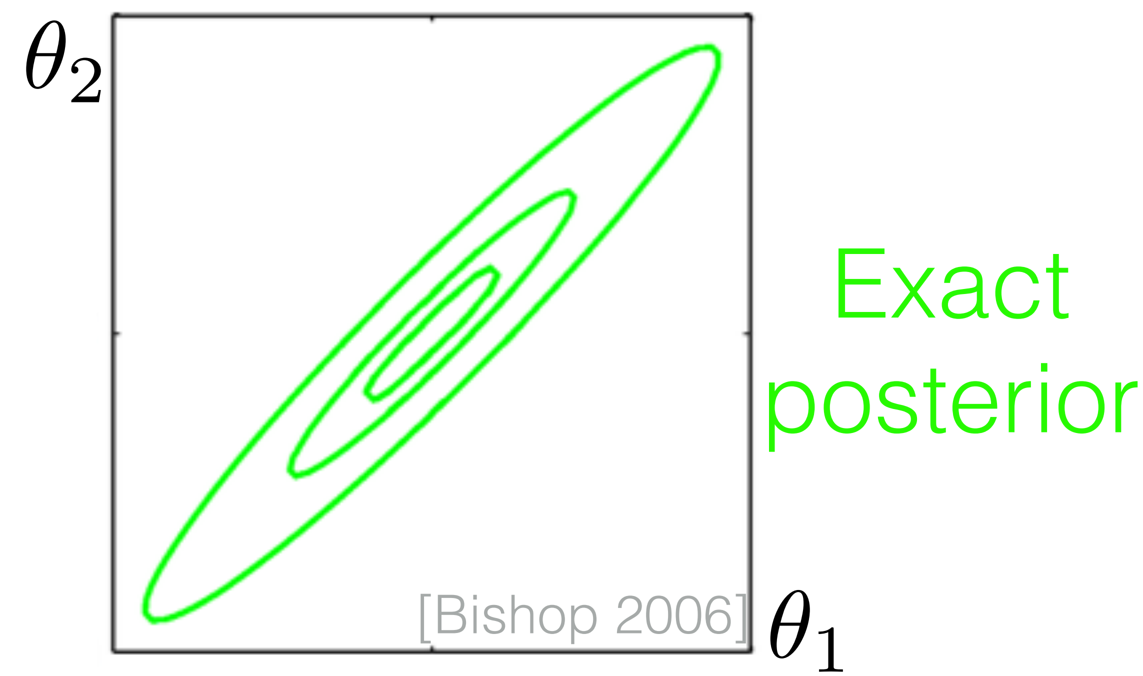

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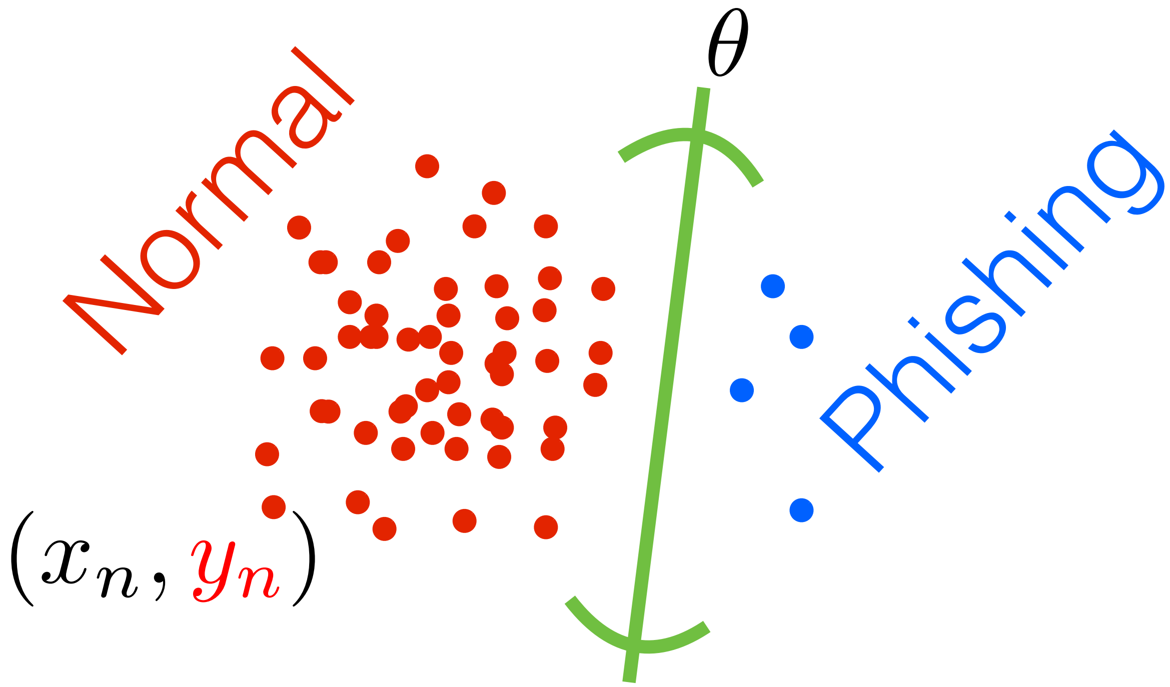

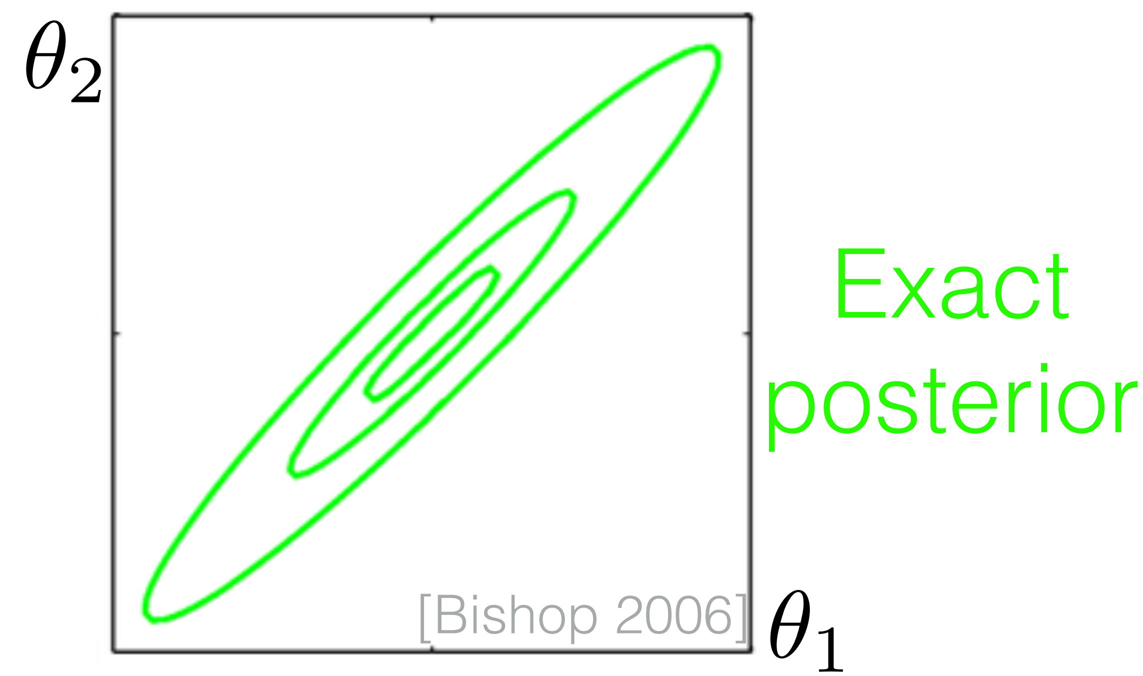

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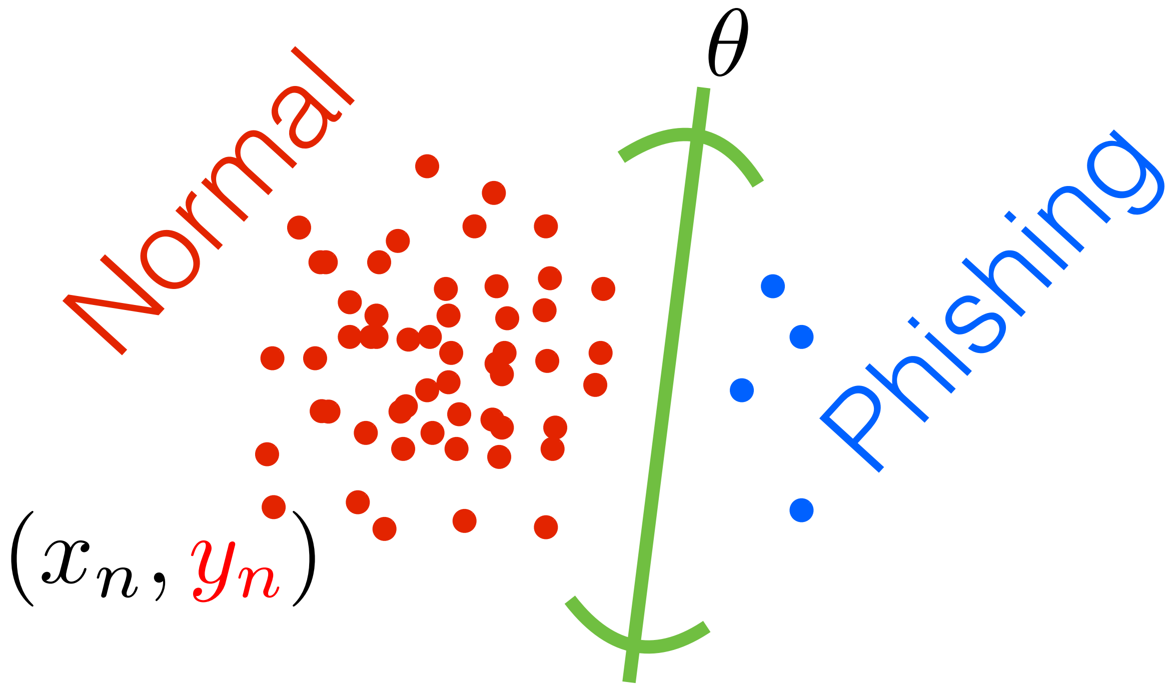

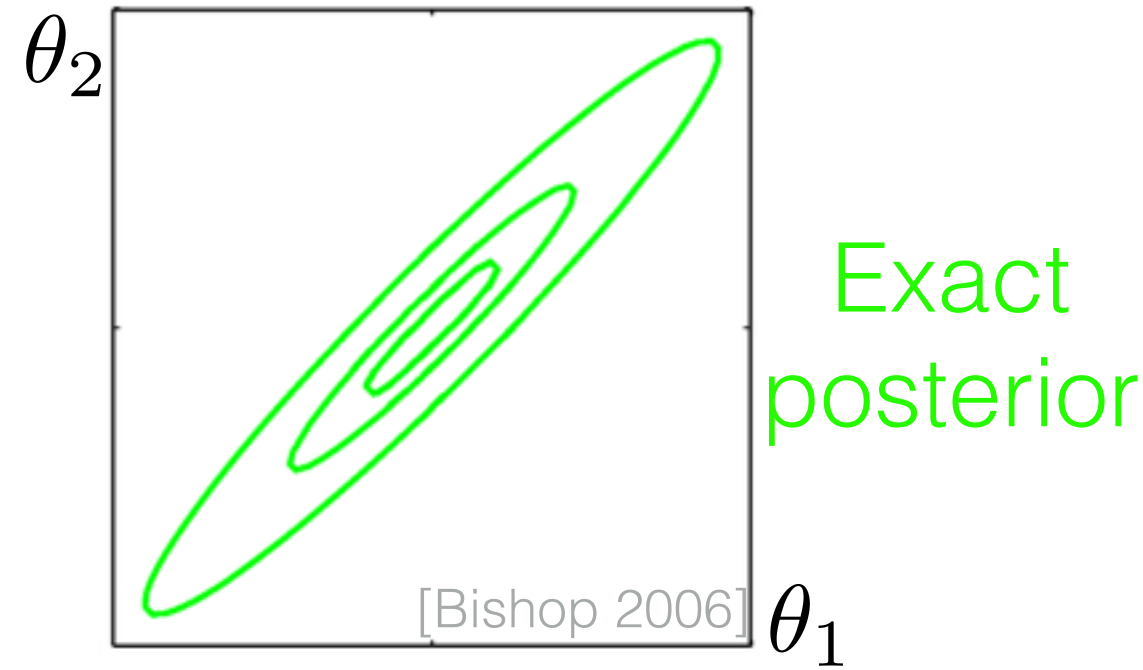

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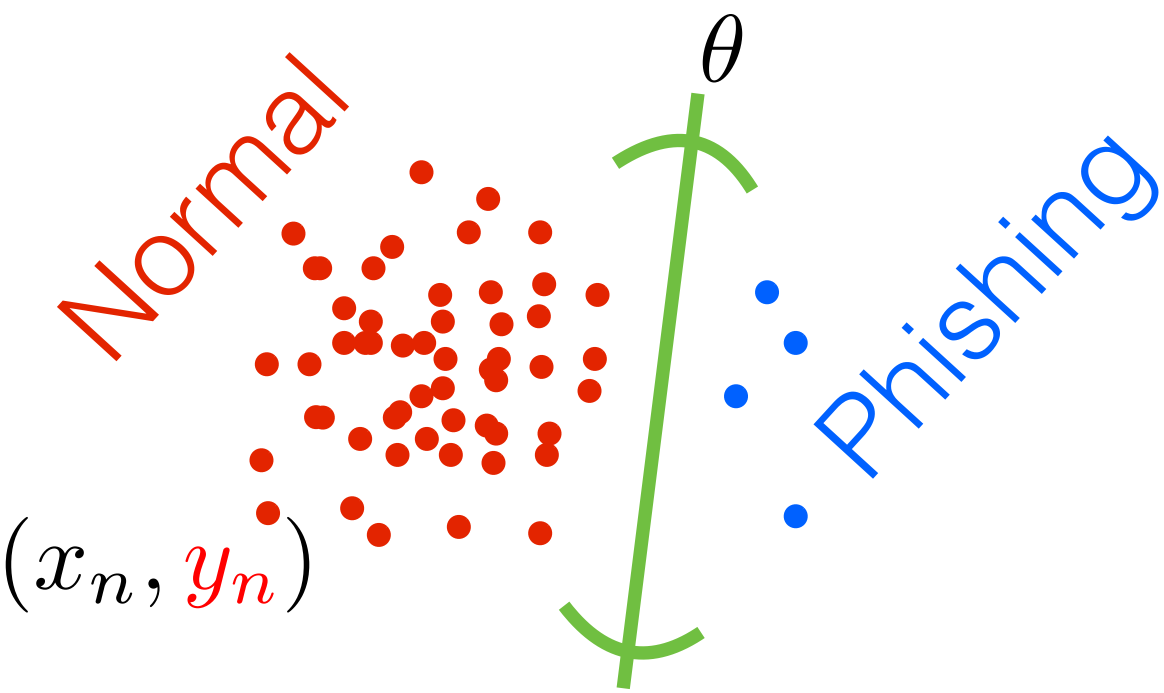

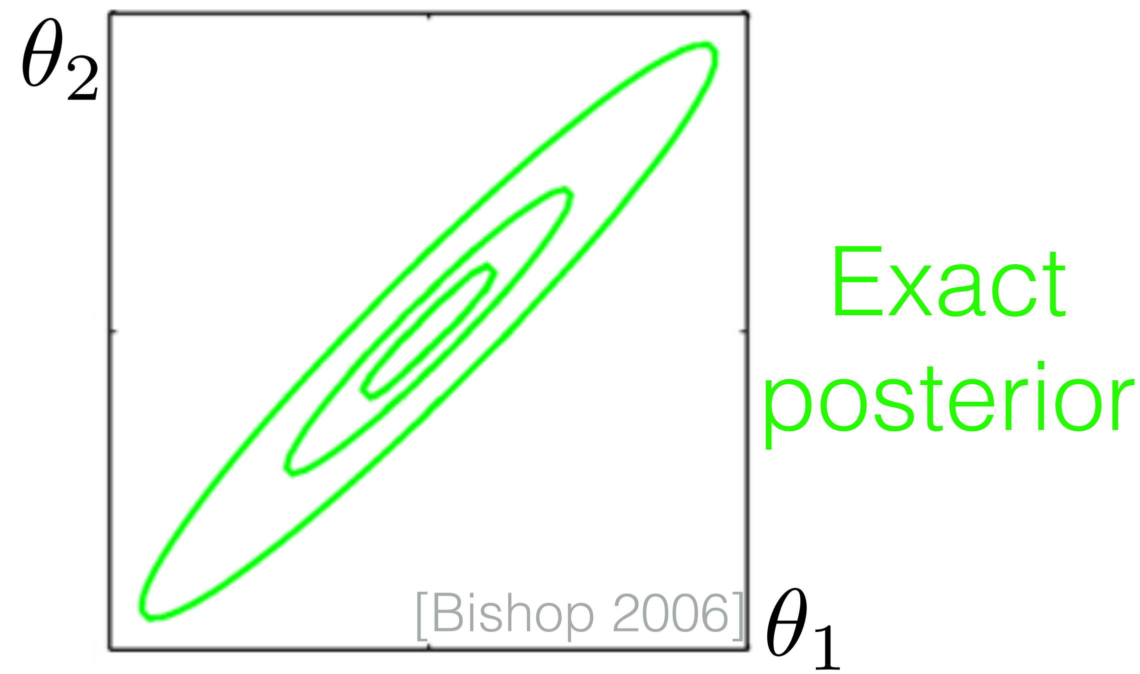

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[MacKay 2003; Bishop 2006; Wang, Titterington 2004; Turner, Sahani 2011; Fosdick 2013; Dunson 2014; Bardenet, Doucet, Holmes 2017; Opper, Winther 2003; Giordano, Broderick, Jordan 2015, 2017; Huggins, Campbell, Kasprzak, Broderick 2018]

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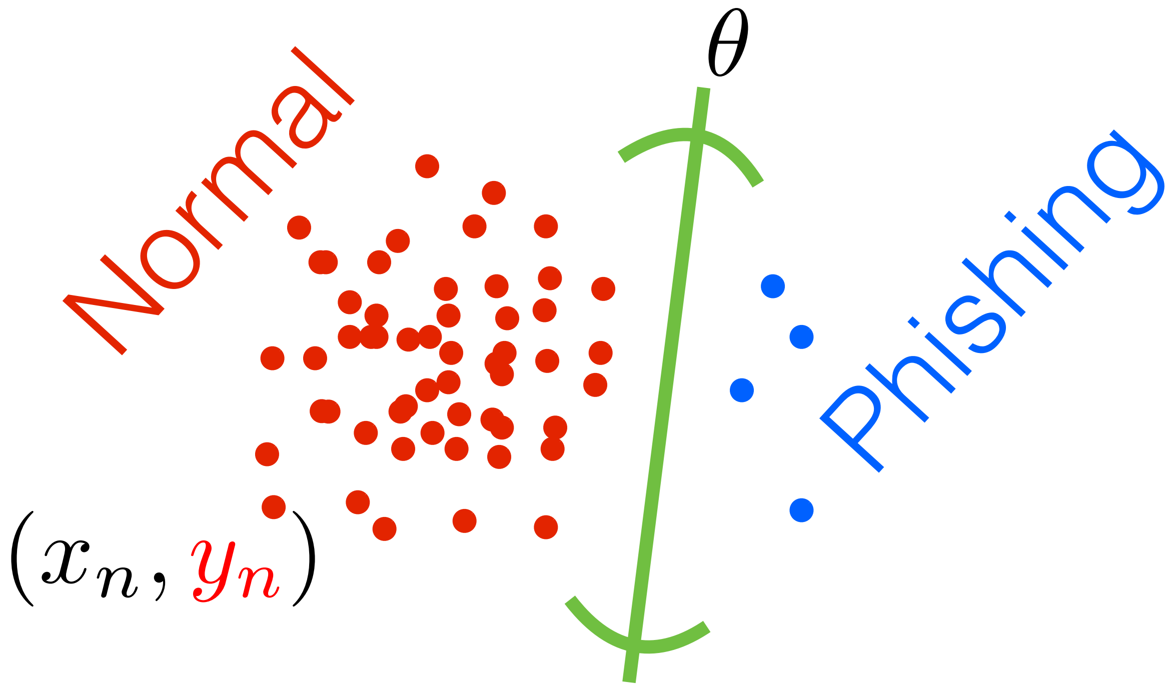

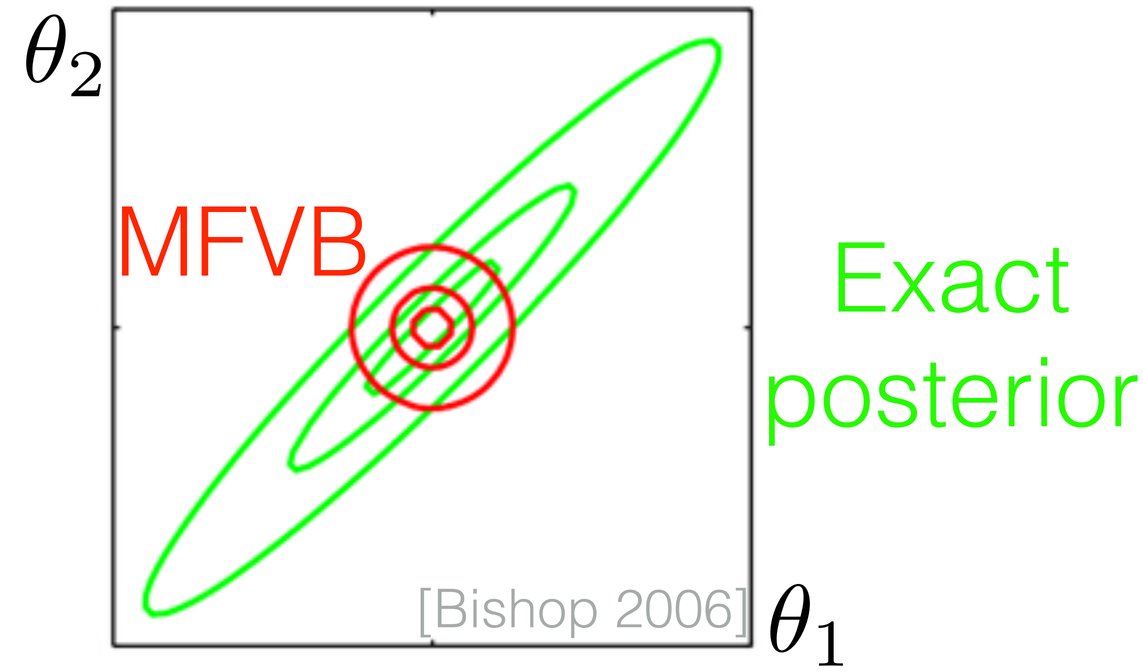

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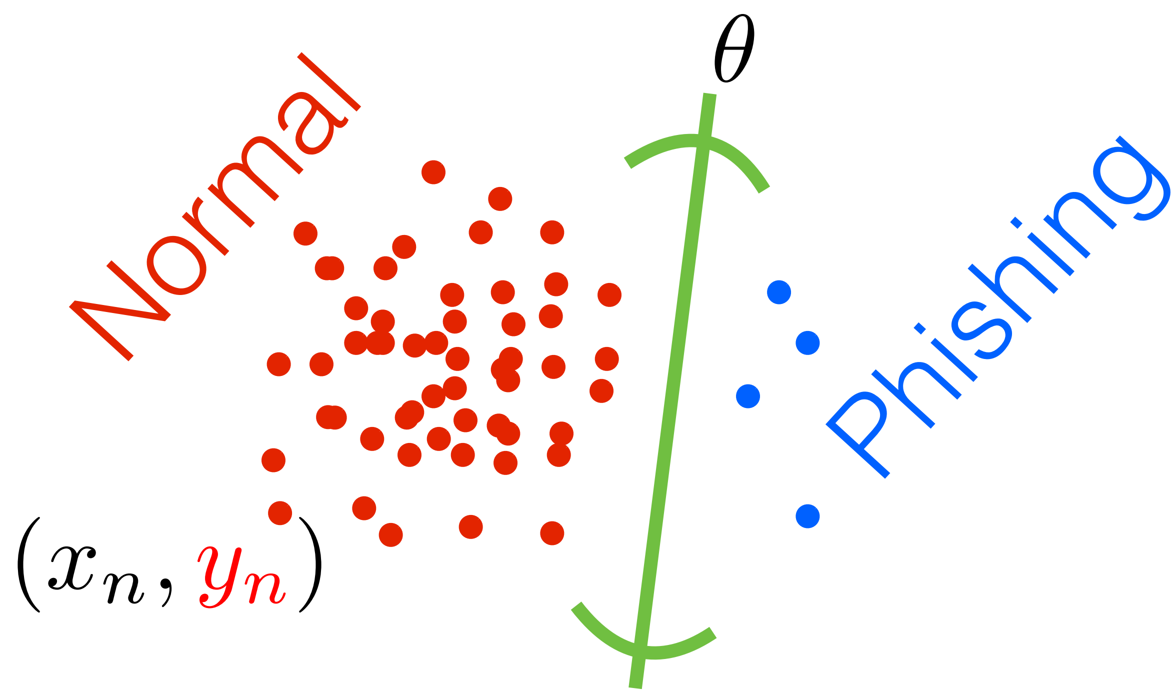

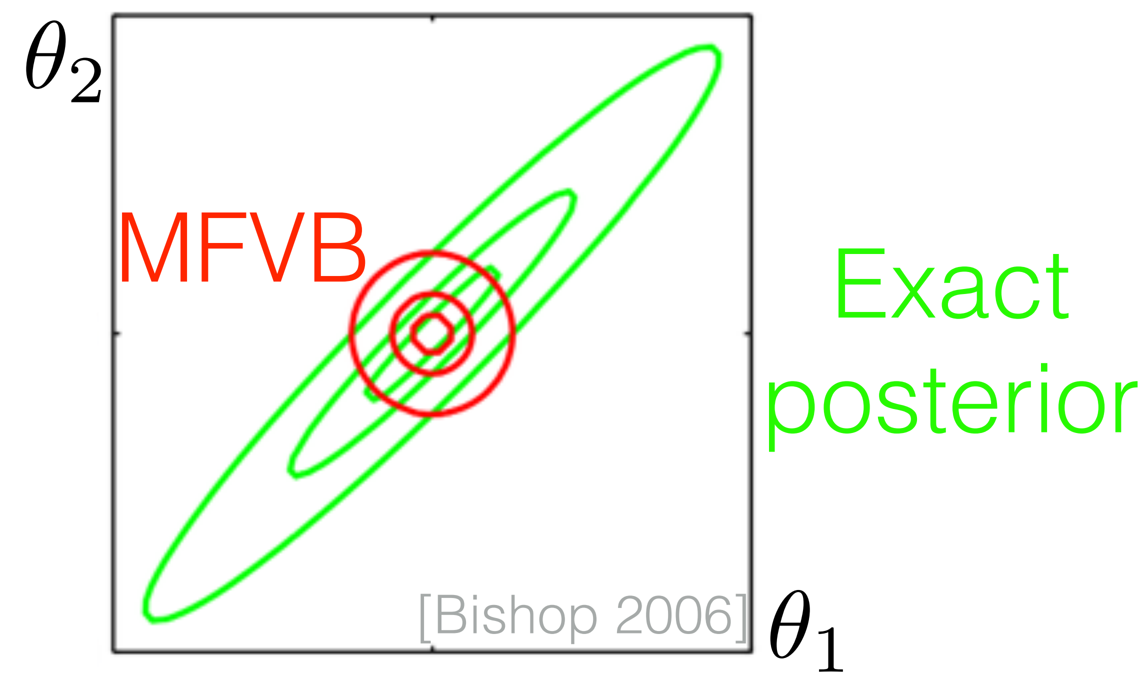

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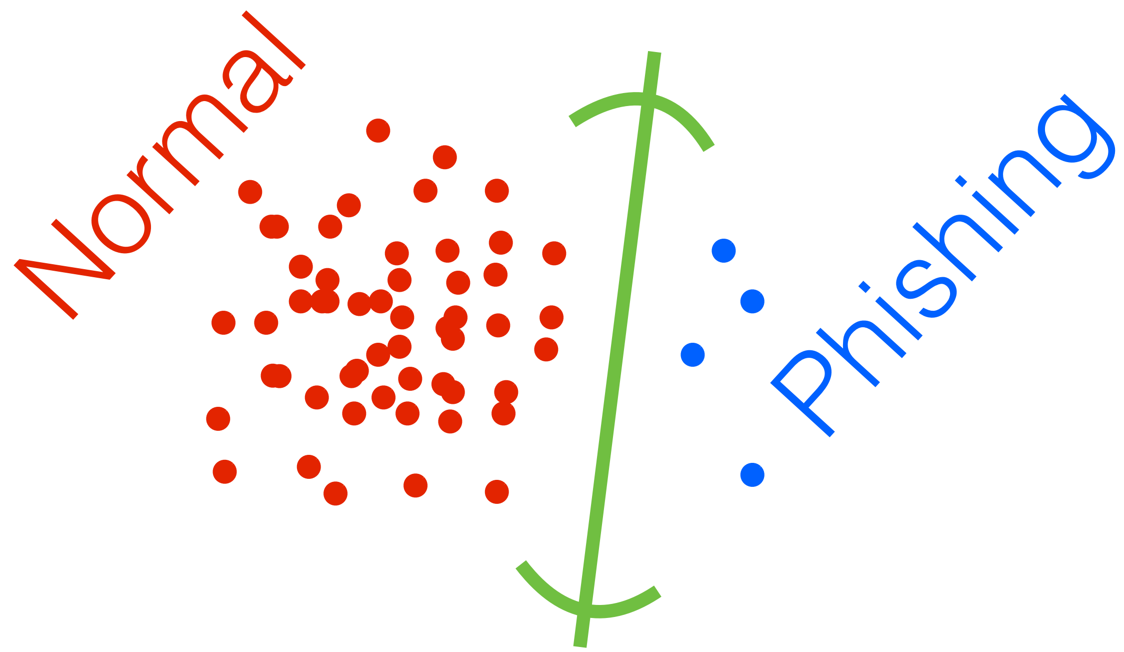
- Automation: e.g. Stan, NUTS, ADVI

[http://mc-stan.org/ ; Hoffman, Gelman 2014; Kucukelbir, Tran, Ranganath, Gelman, Blei 2017]

Bayesian coresets

- Posterior $p(\theta|y) \propto_\theta p(y|\theta)p(\theta)$



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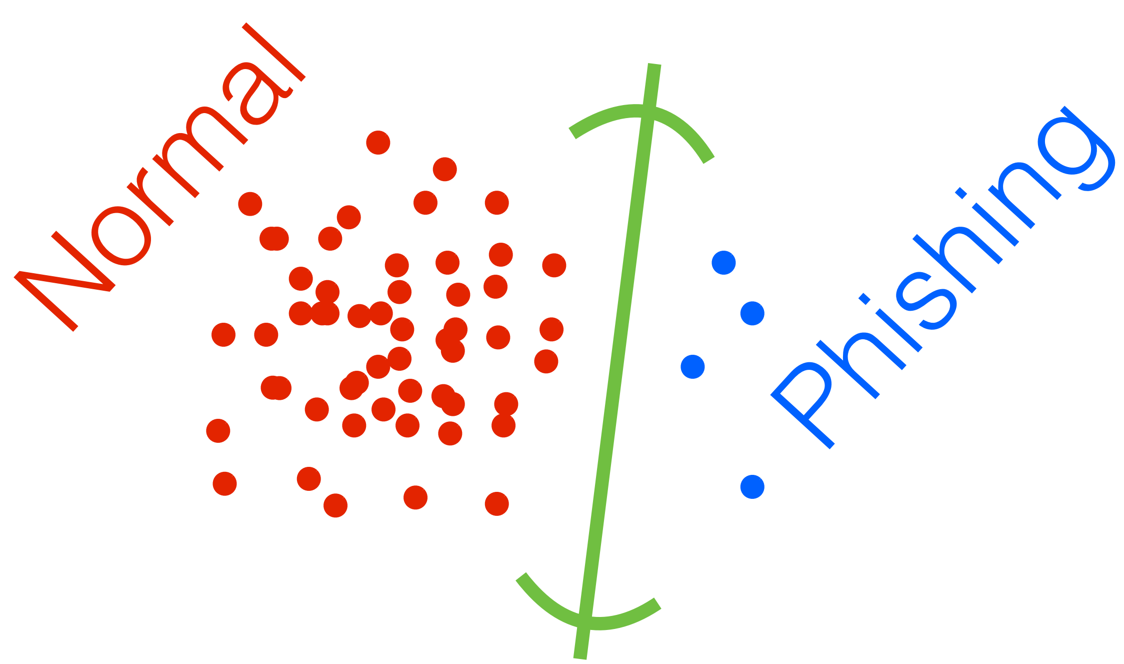

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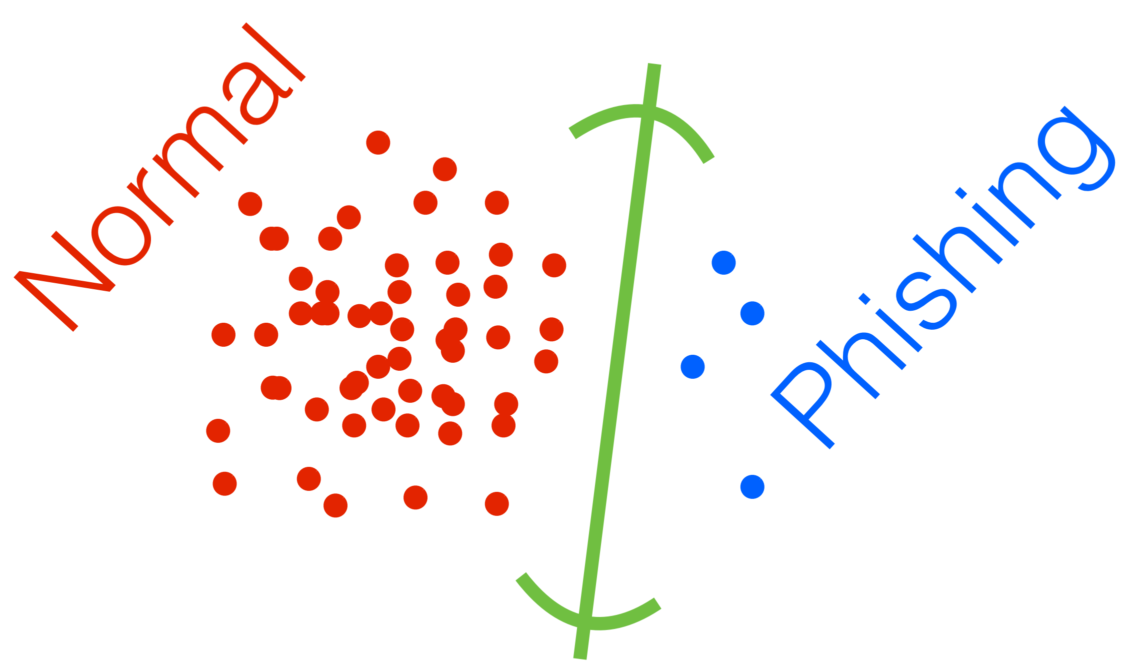

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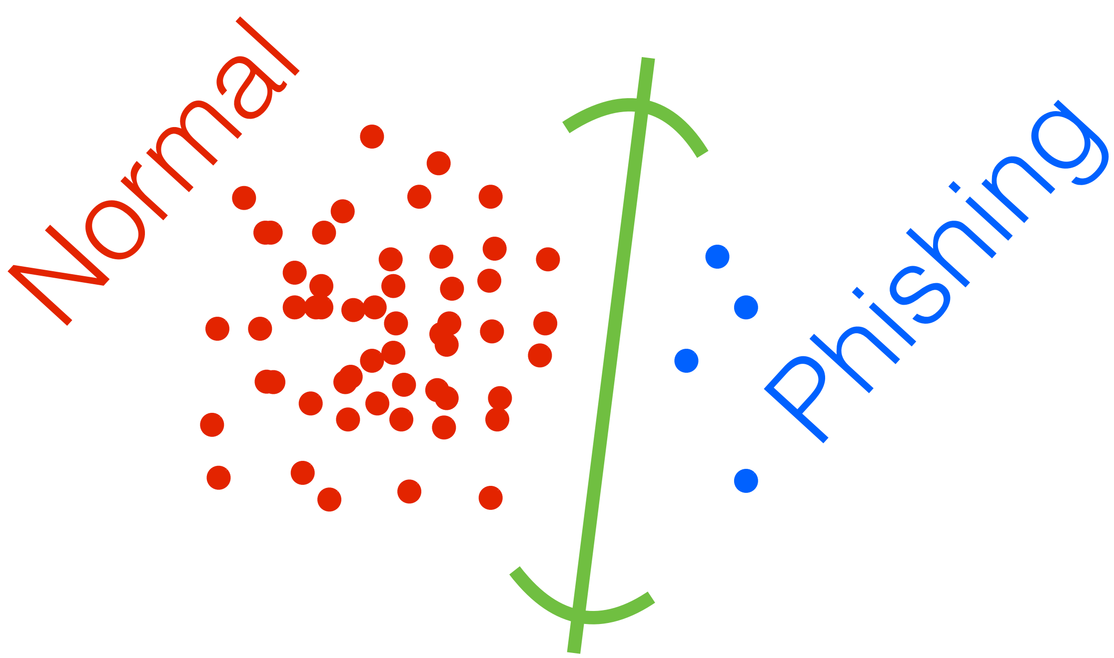
$$\|w\|_0 \ll N$$



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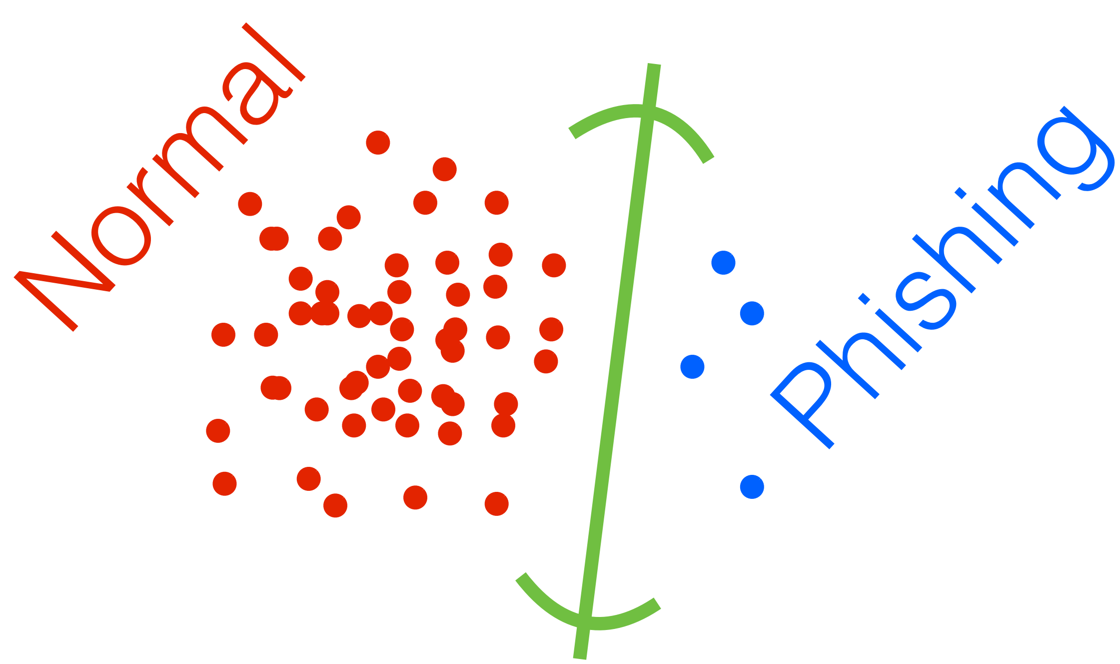

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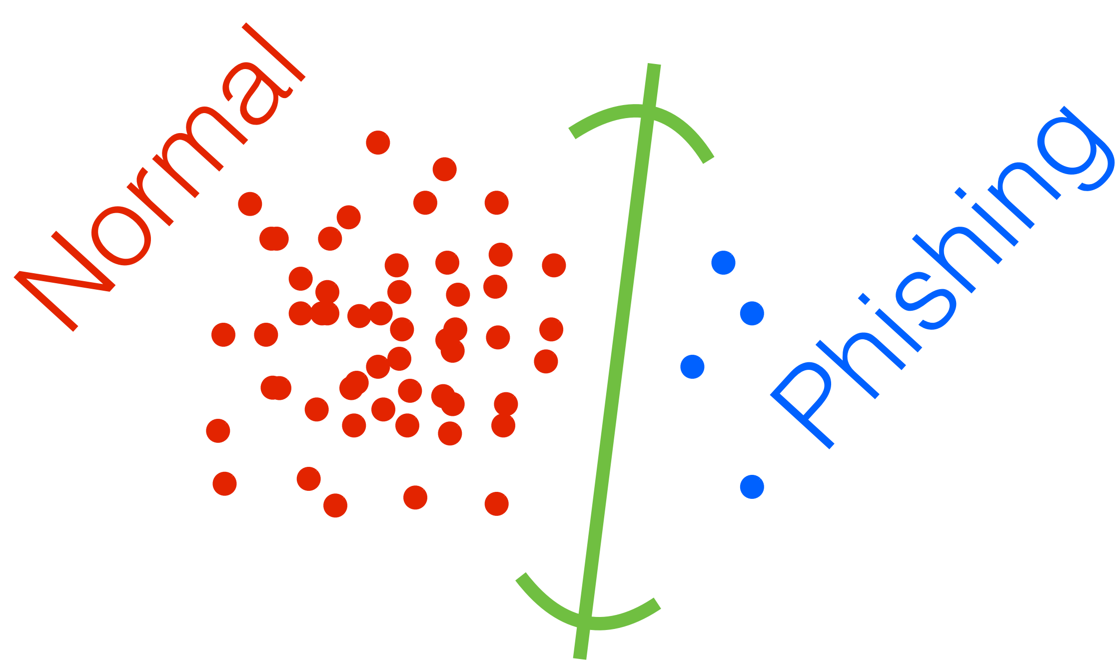

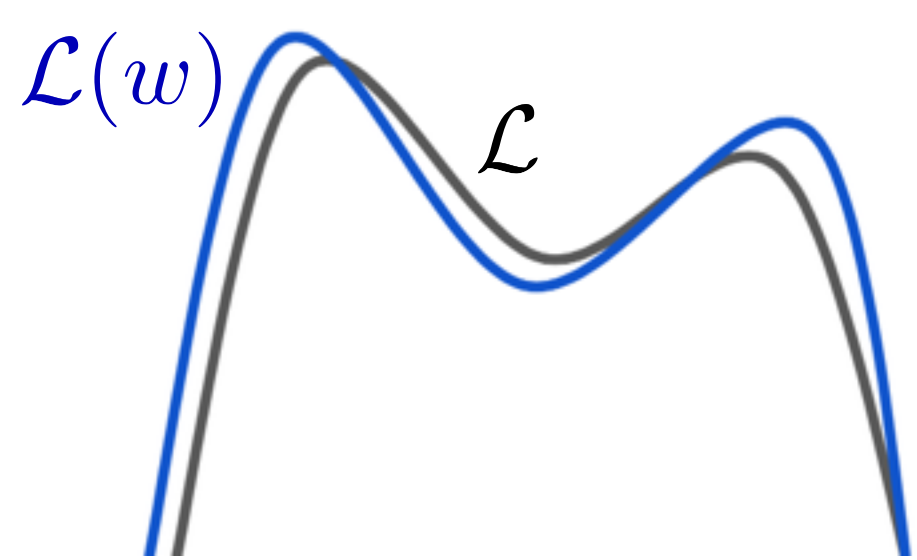

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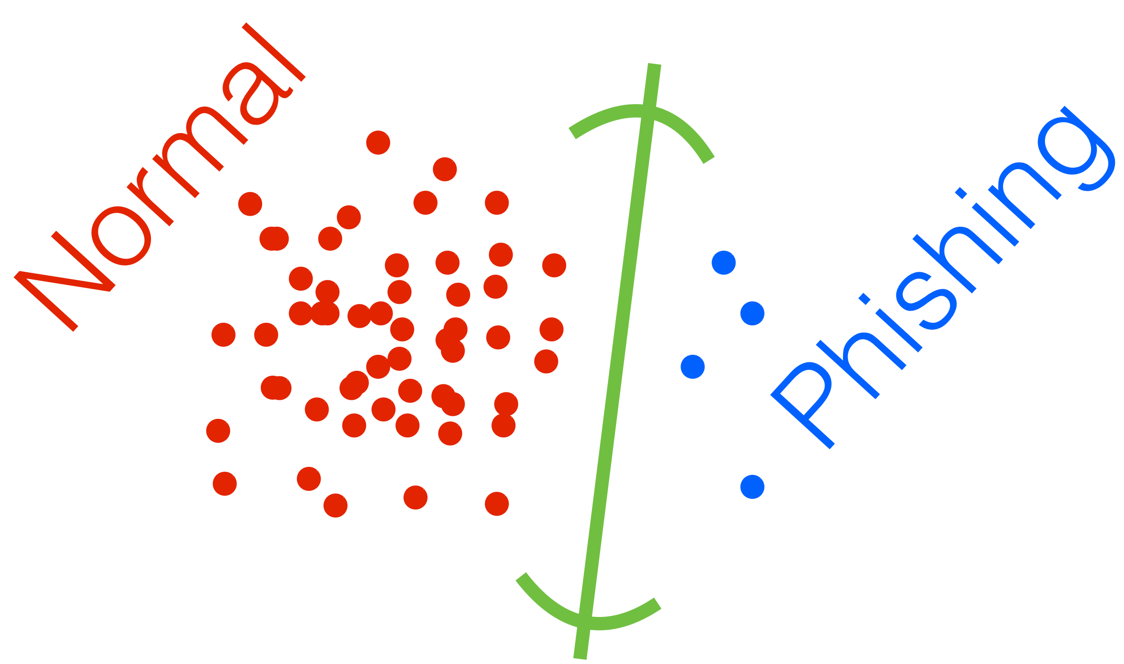

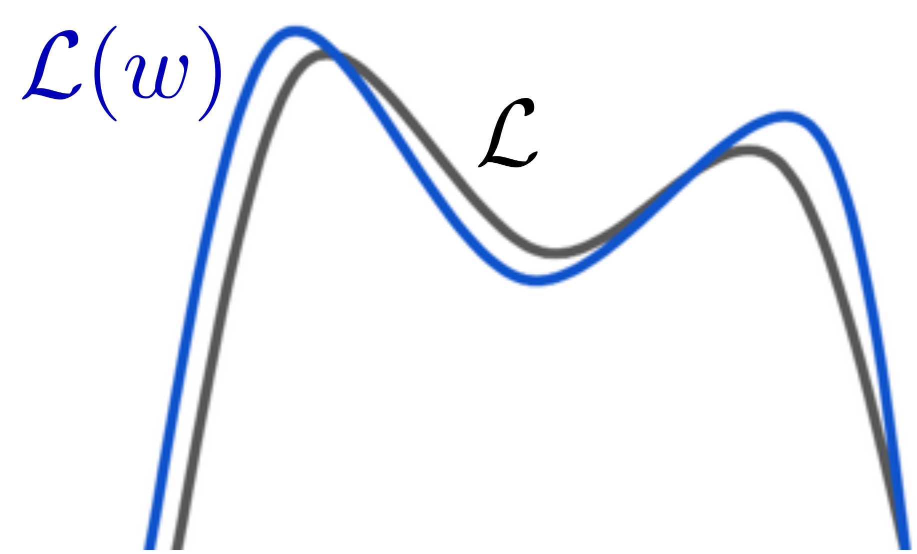

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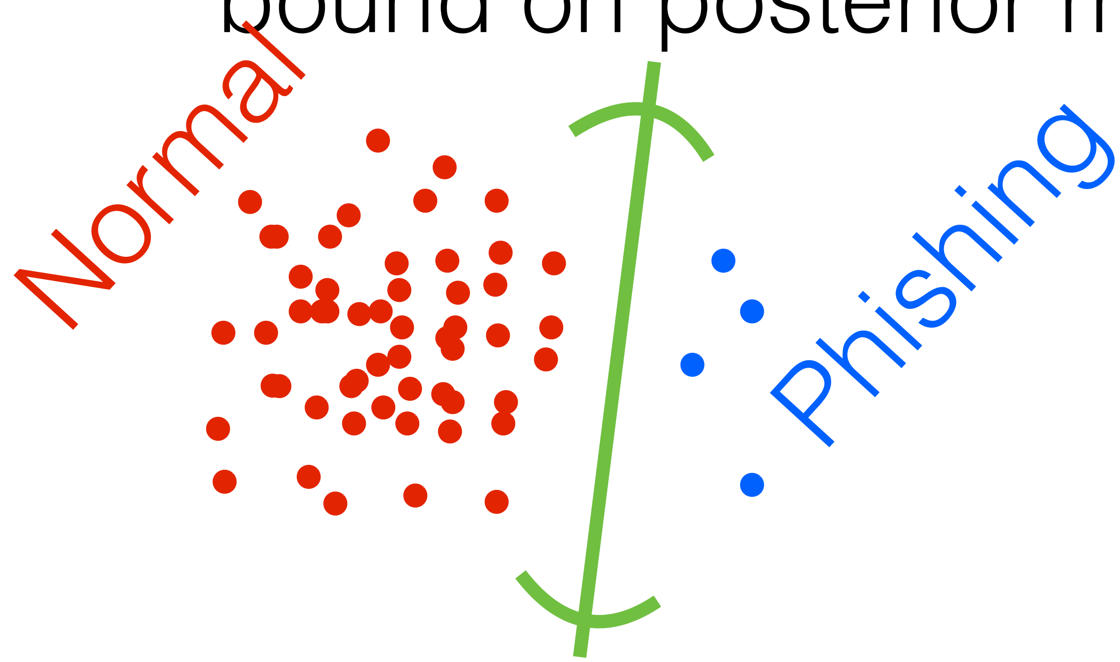
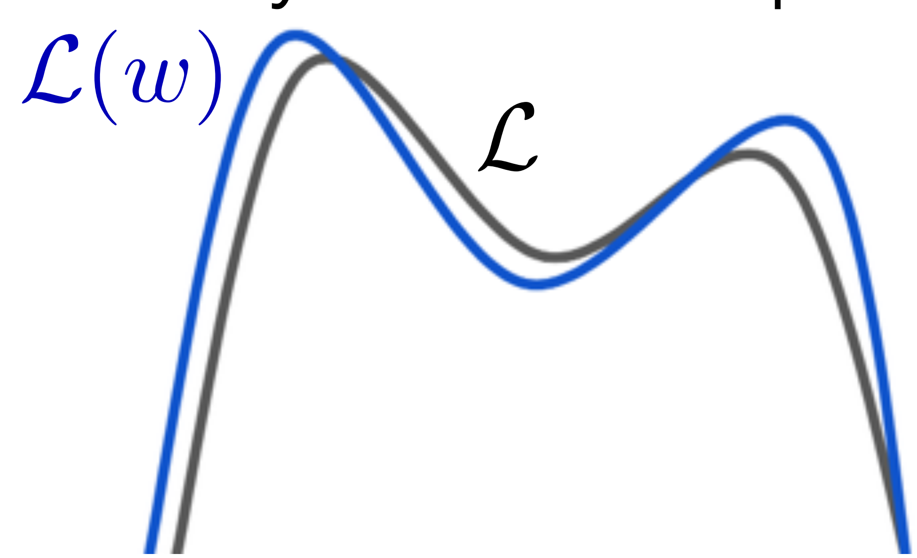
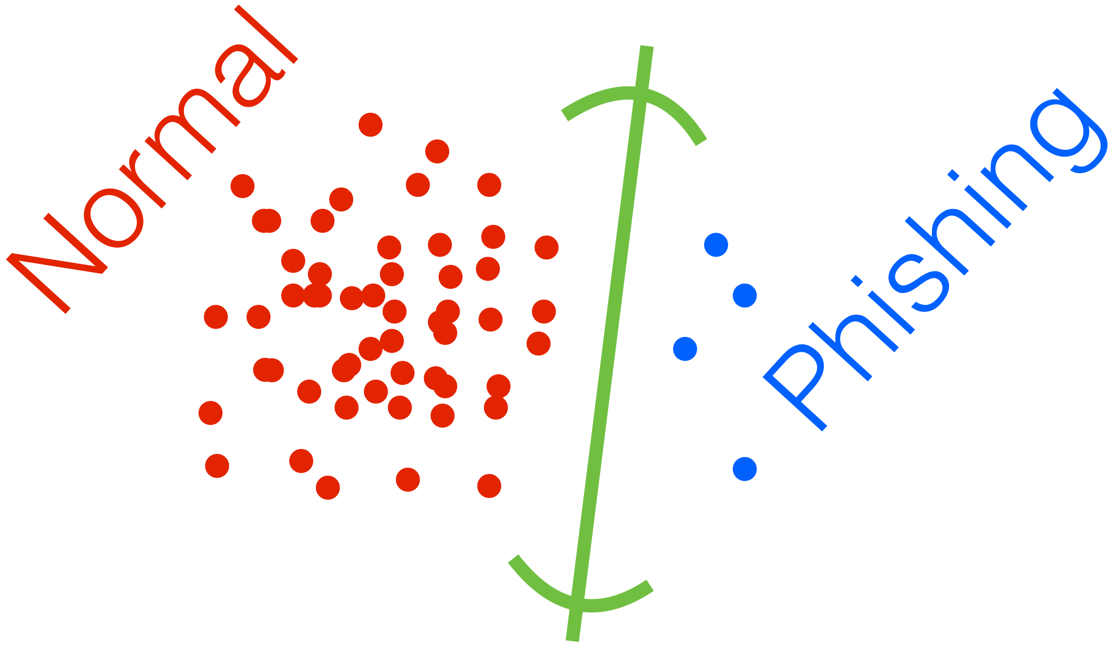
  - Bound on Wasserstein distance to exact posterior ➔ bound on posterior mean/uncertainty estimate quality





[Huggins, Campbell, Kasprzak, Broderick 2018; Broderick 2018]

Uniform subsampling



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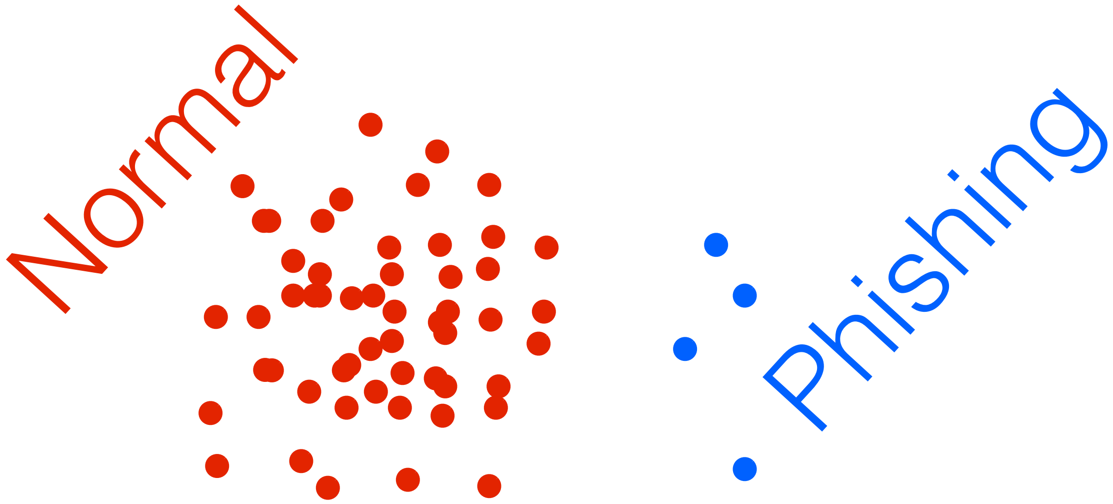

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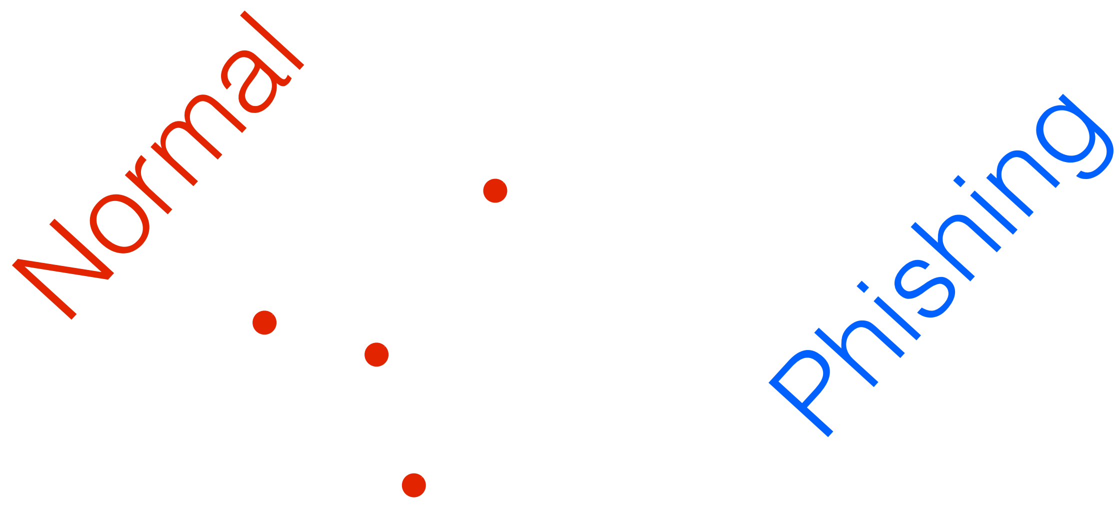

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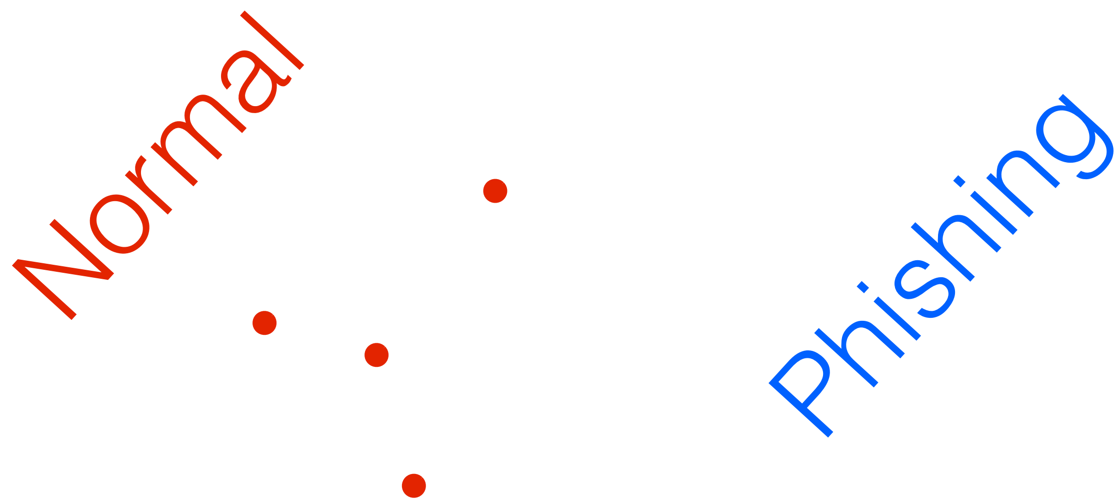
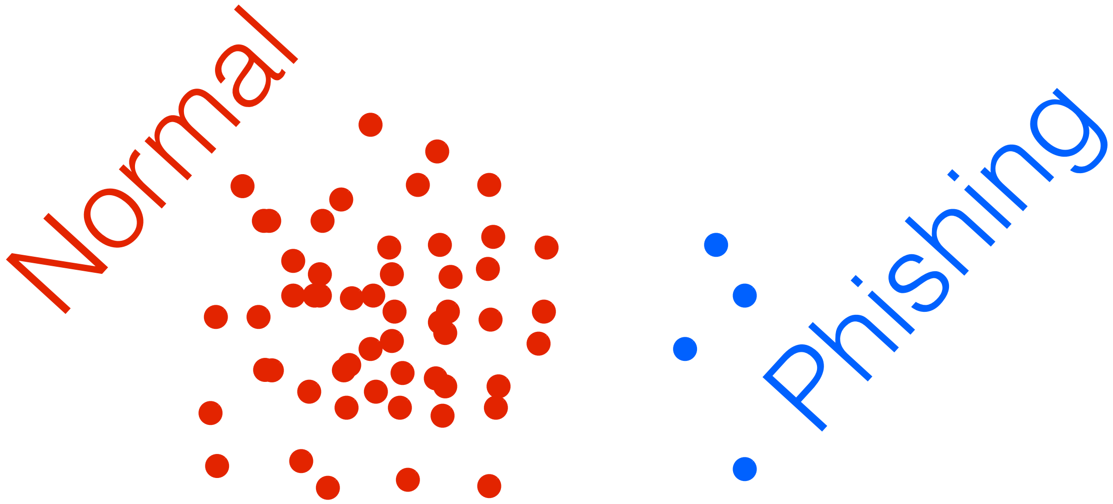


- Might miss important data

Uniform subsampling



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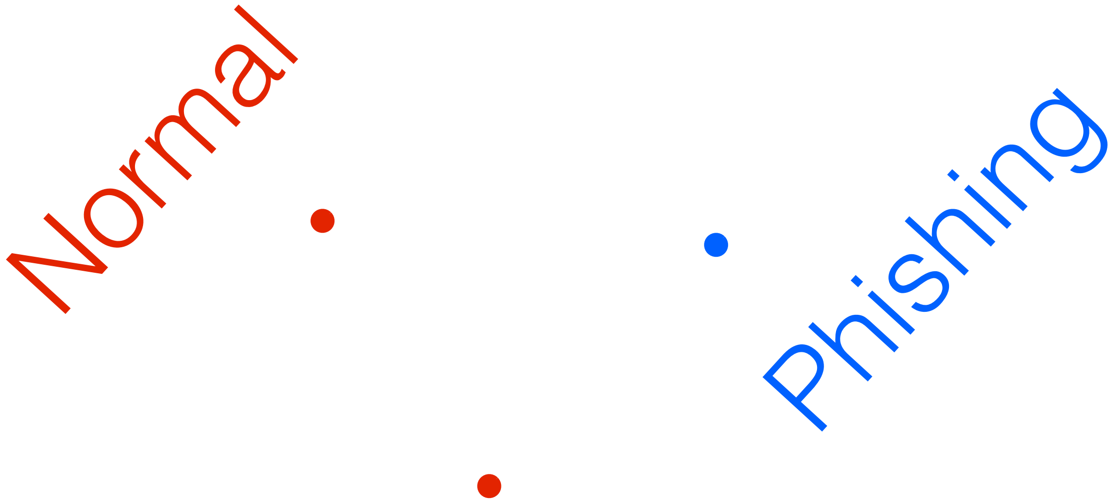

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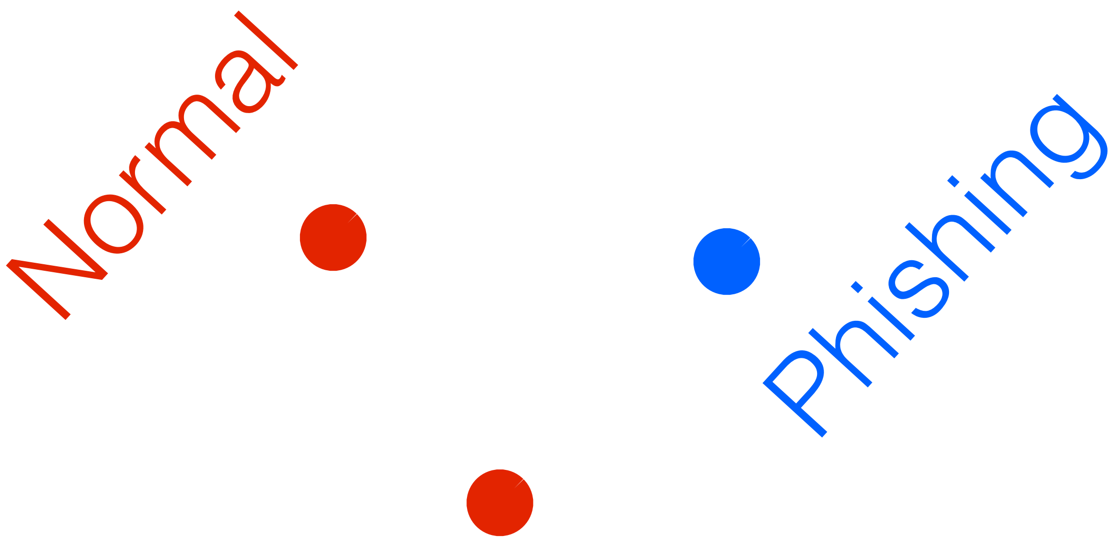

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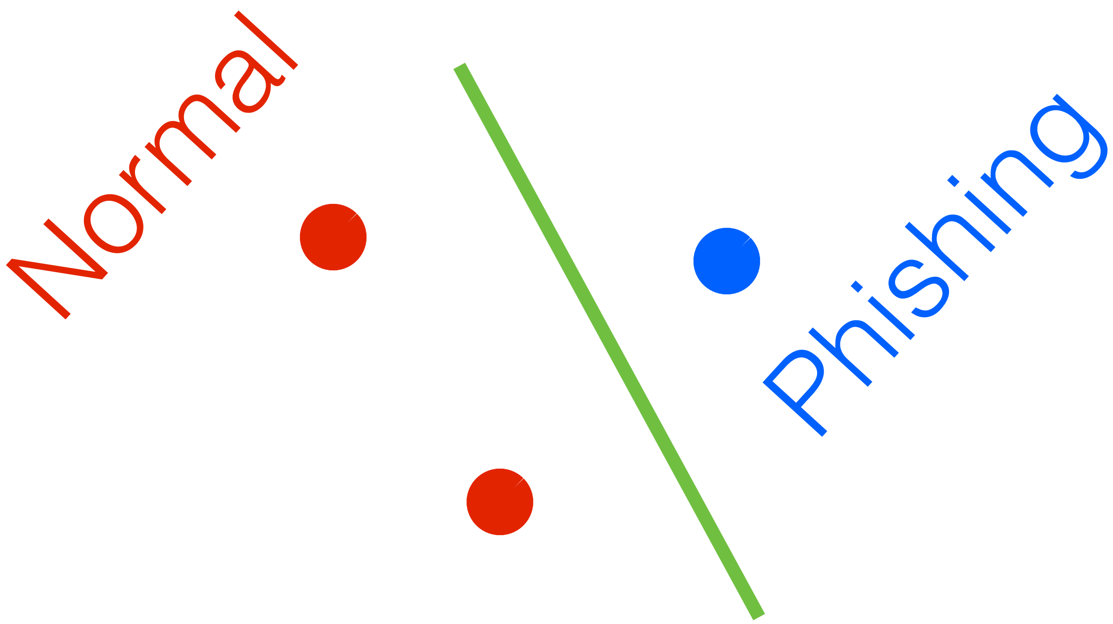

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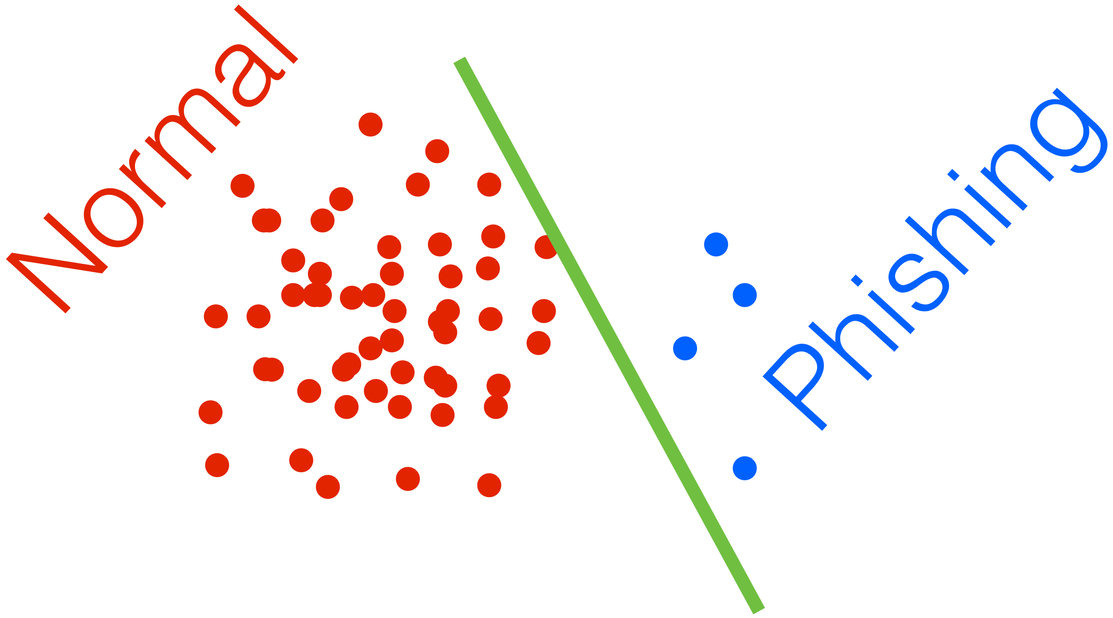

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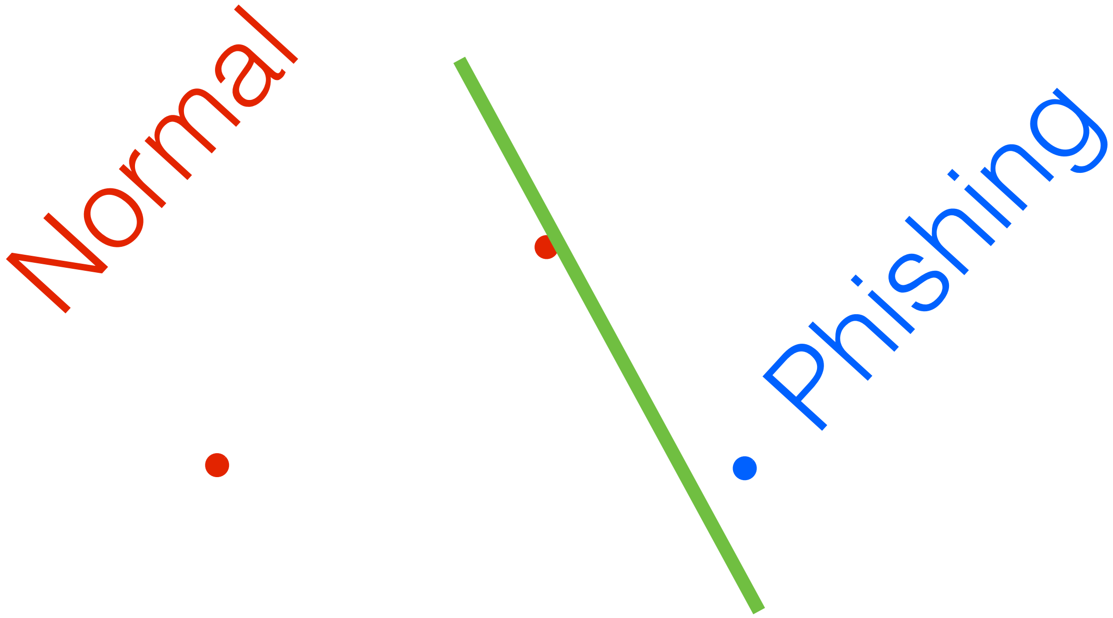

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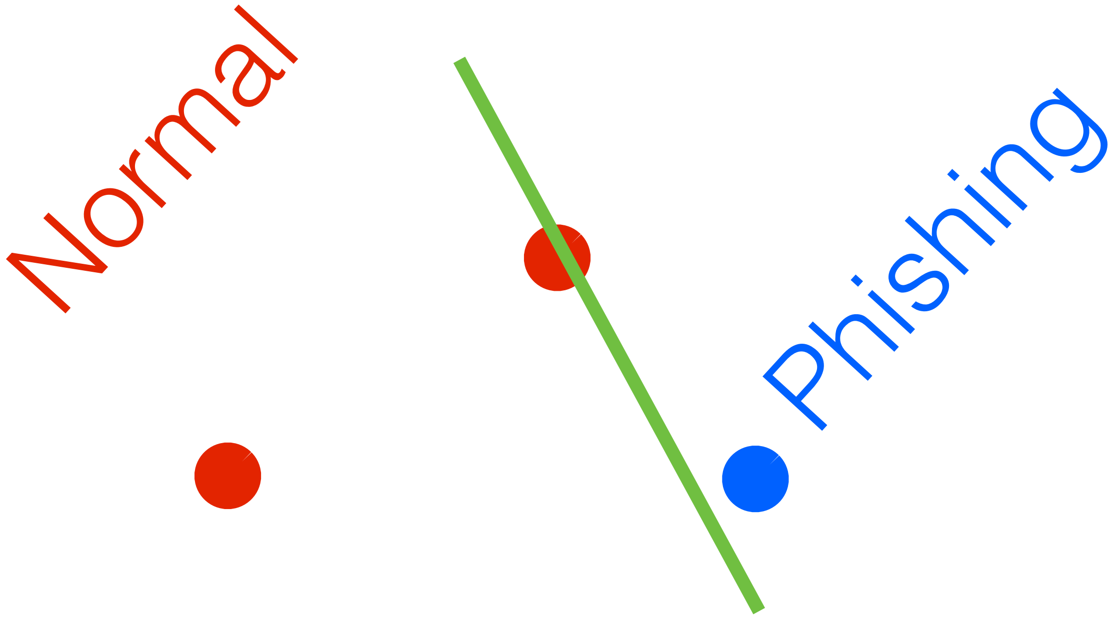

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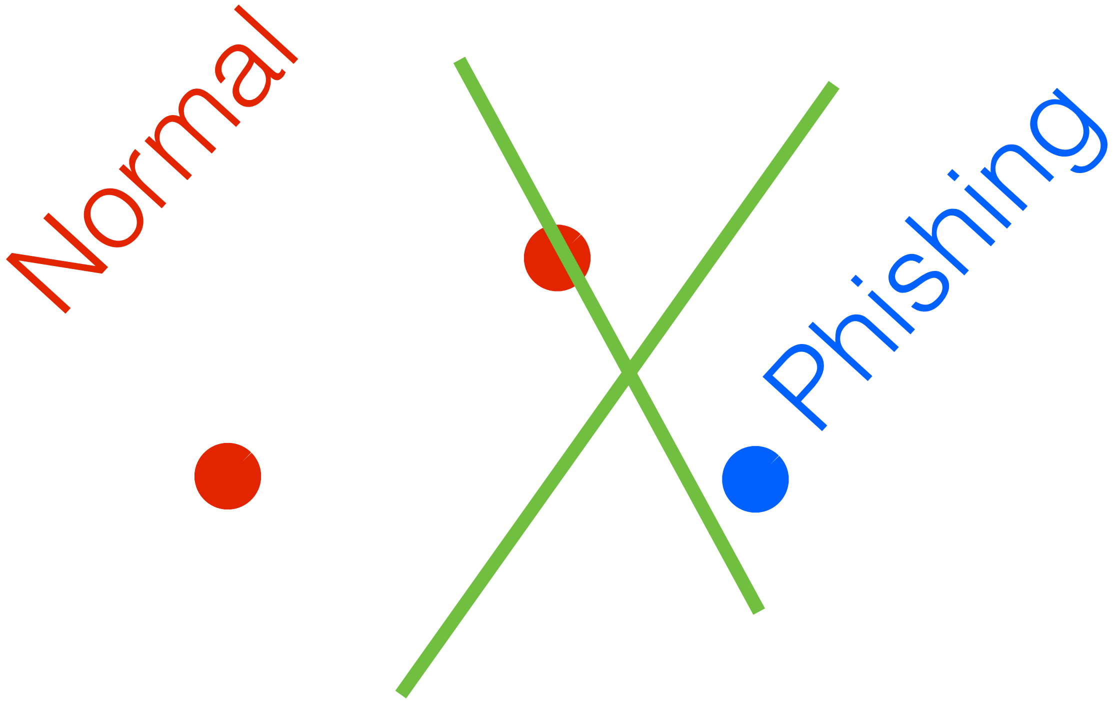

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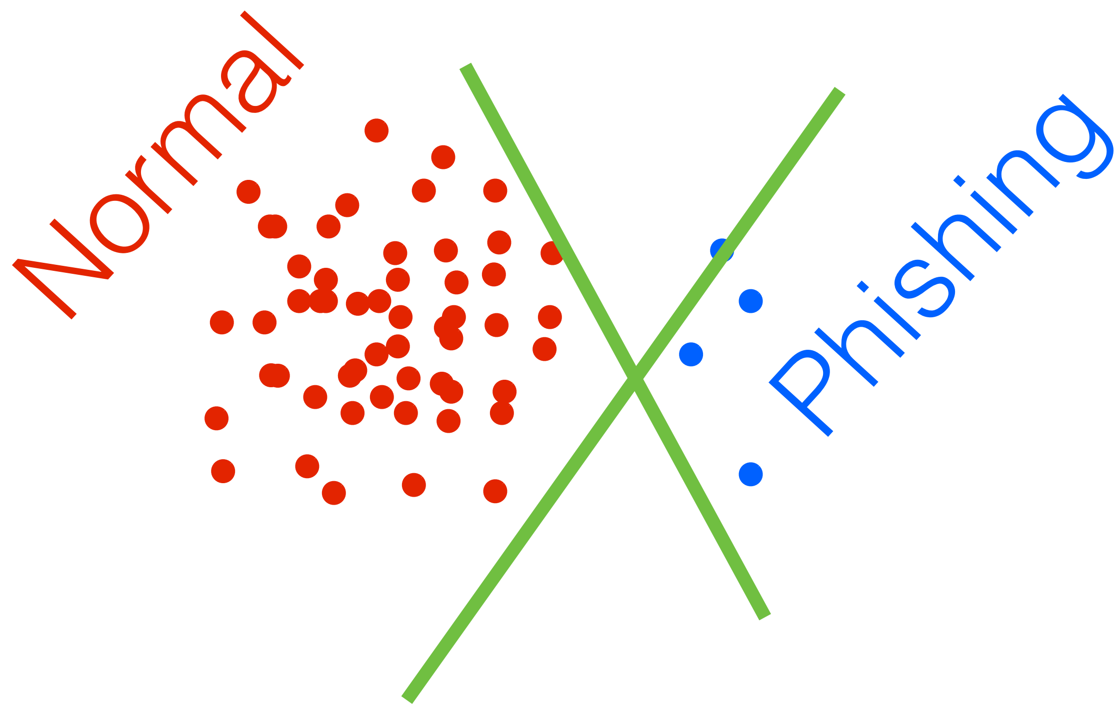

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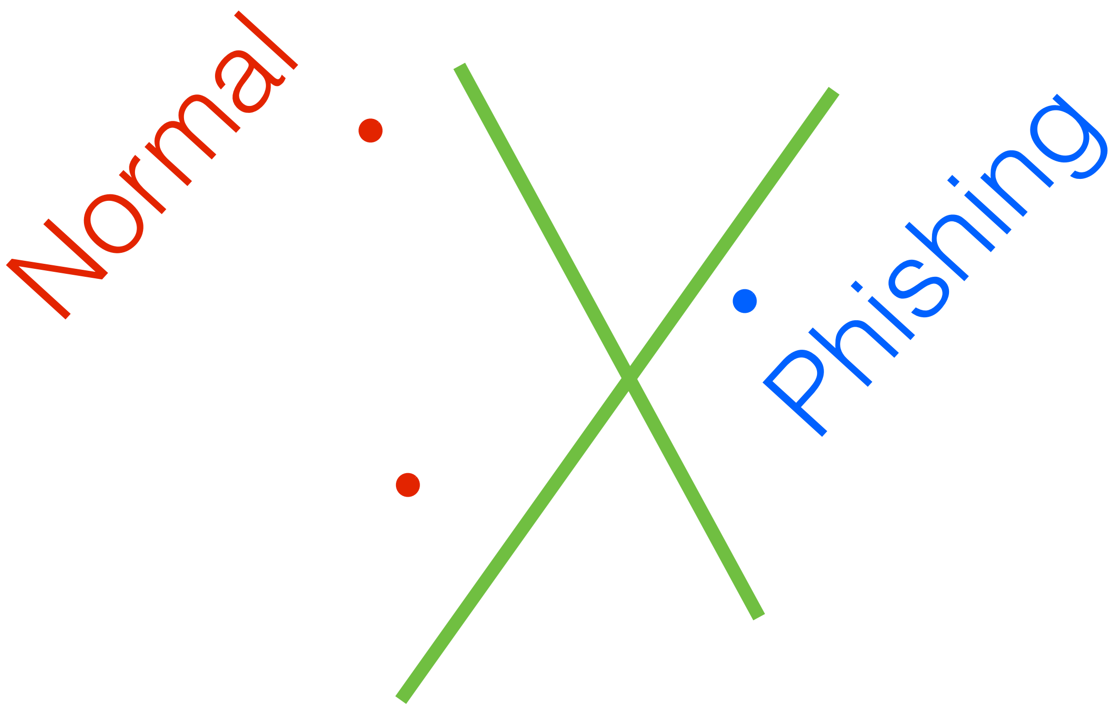

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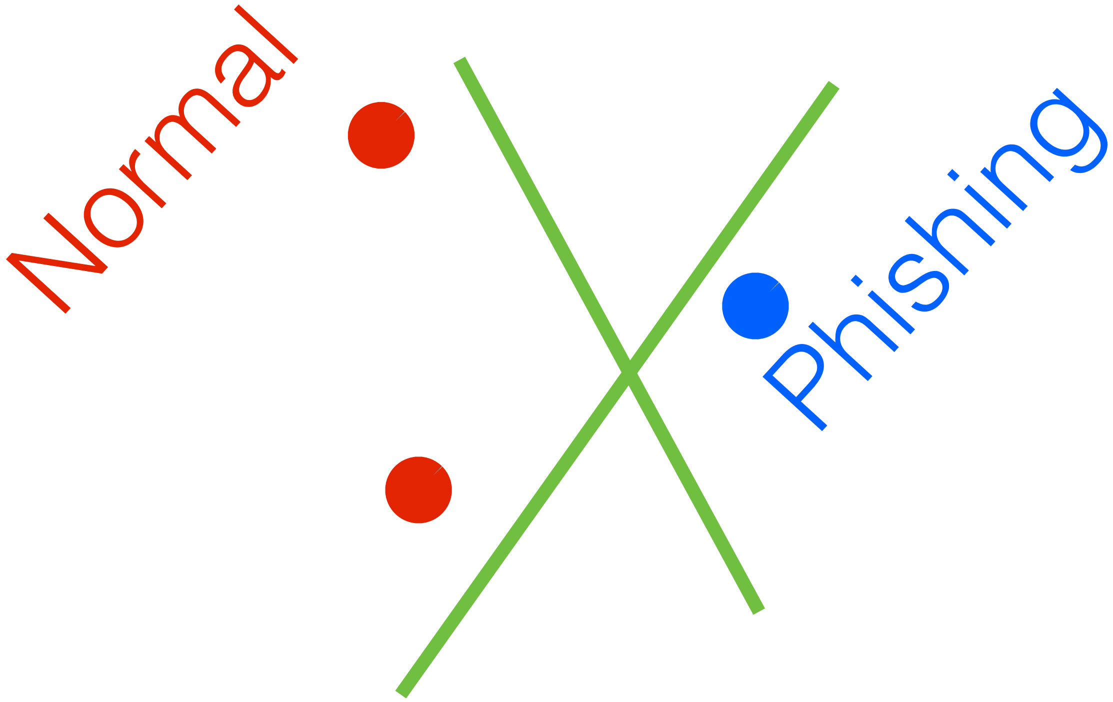

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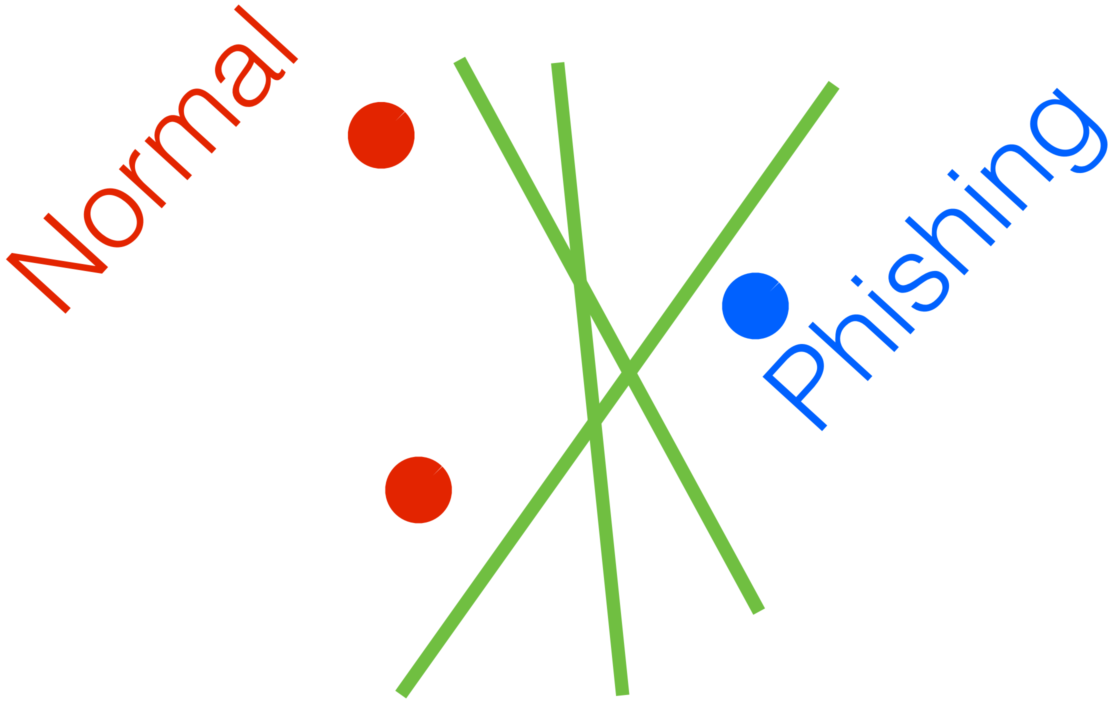

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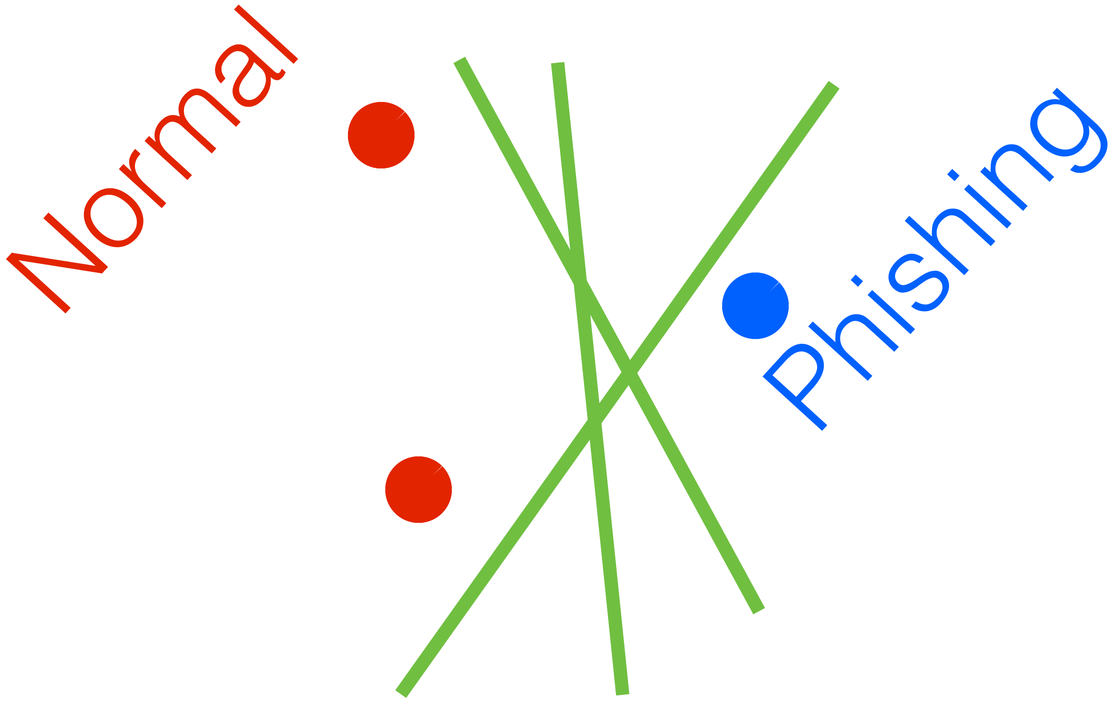

- Might miss important data
- Noisy estimates

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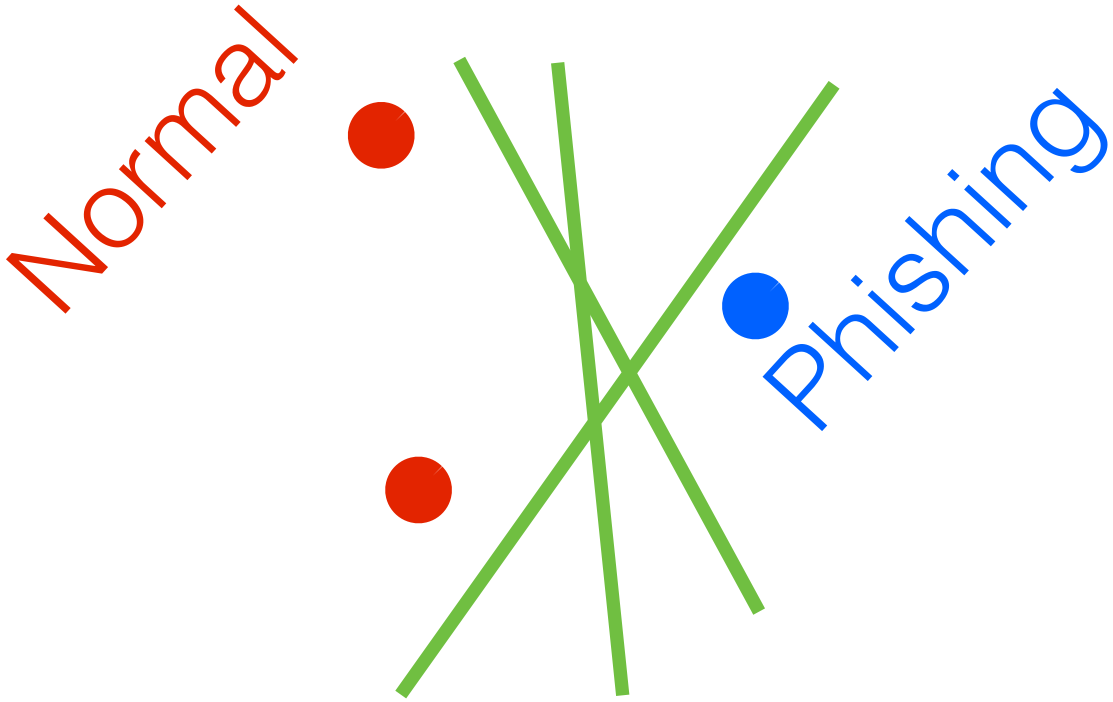

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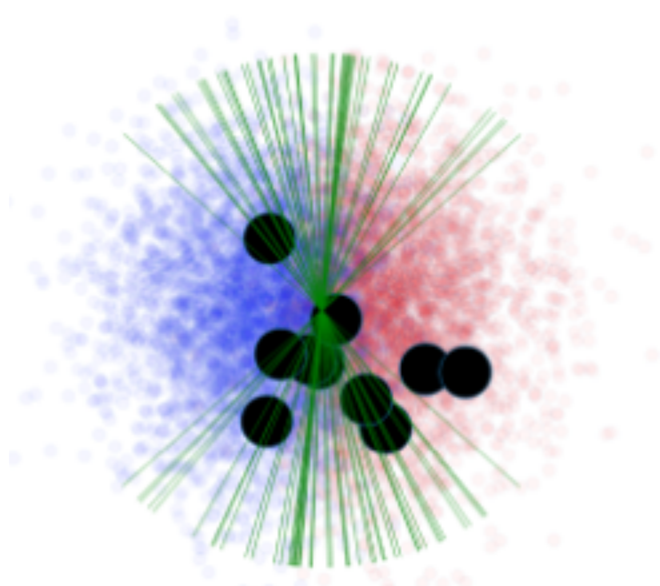
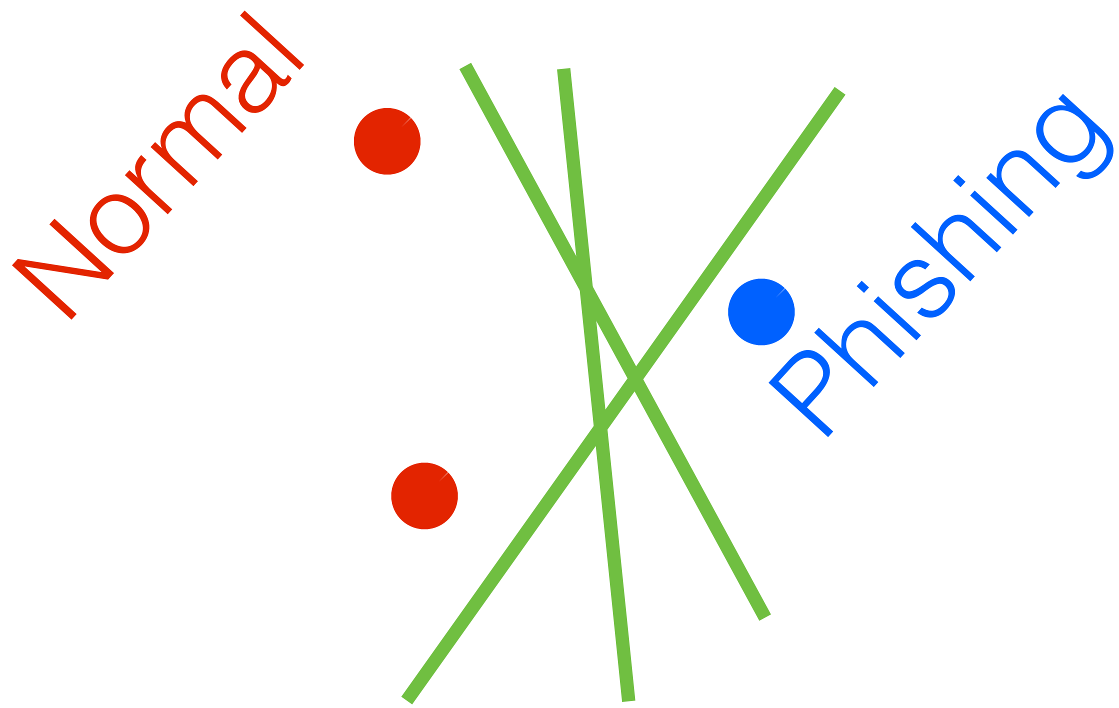


$M = 10$

Uniform subsampling



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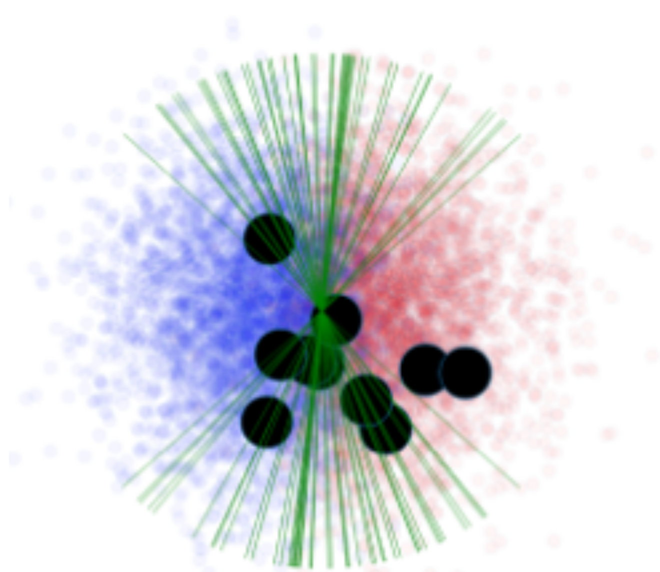

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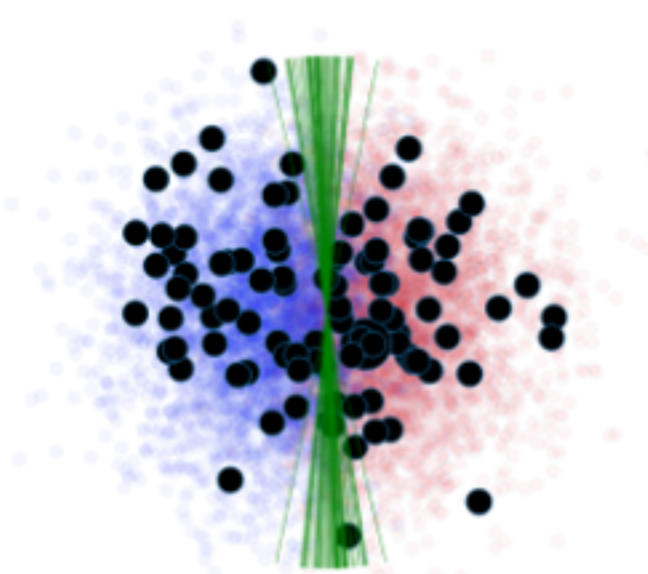

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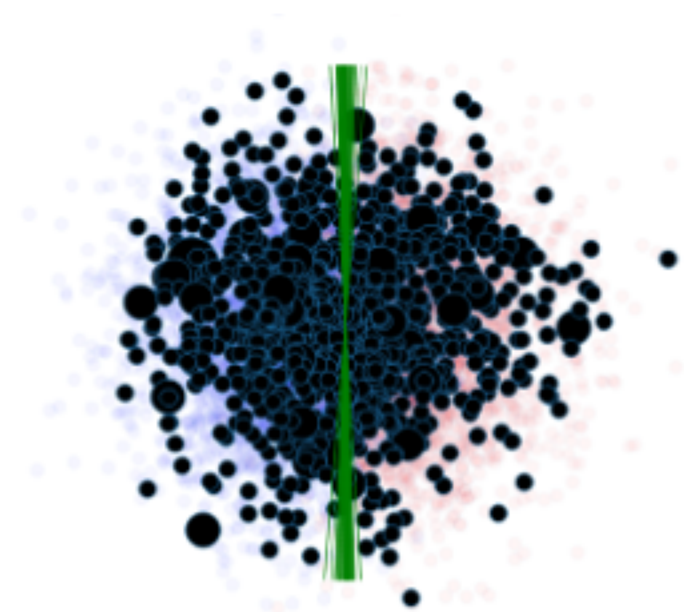

$M = 1000$

Importance sampling

- "Optimal" importance weights

Thm (Campbell, B). δ ∈ (0,1). W.p. ≥1 - δ, after *M* iterations,

$$\|\mathcal{L}(w) - \mathcal{L}\| \leq \frac{\sigma\bar{\eta}}{\sqrt{M}}\left(1 + \sqrt{2\log\frac{1}{\delta}}\right)$$

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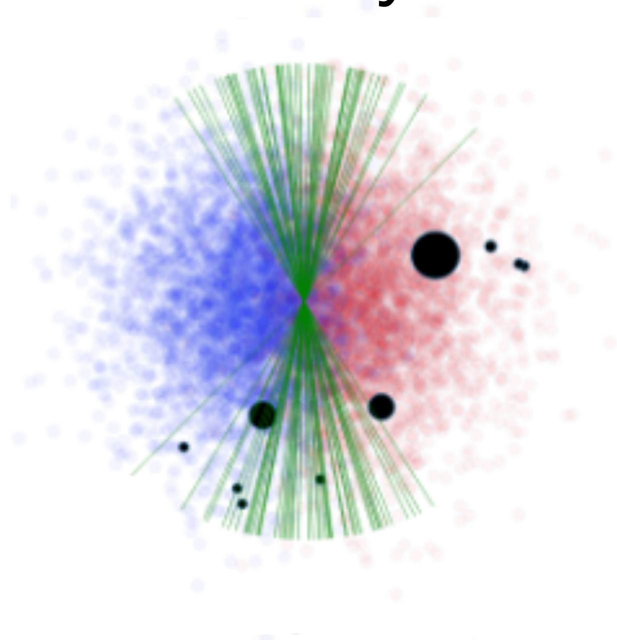

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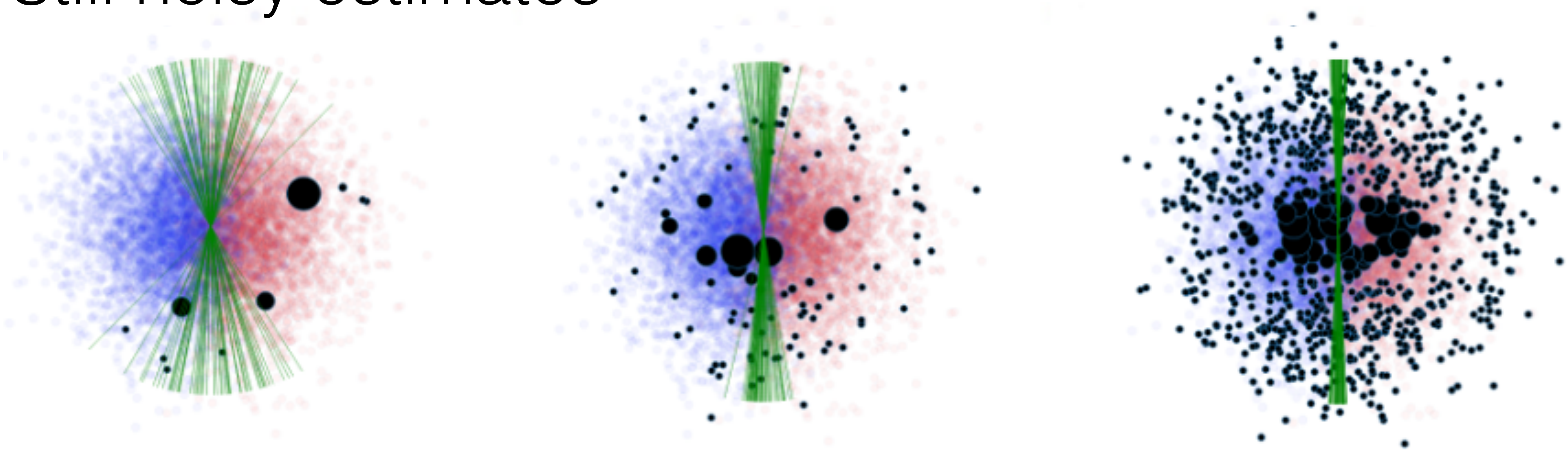
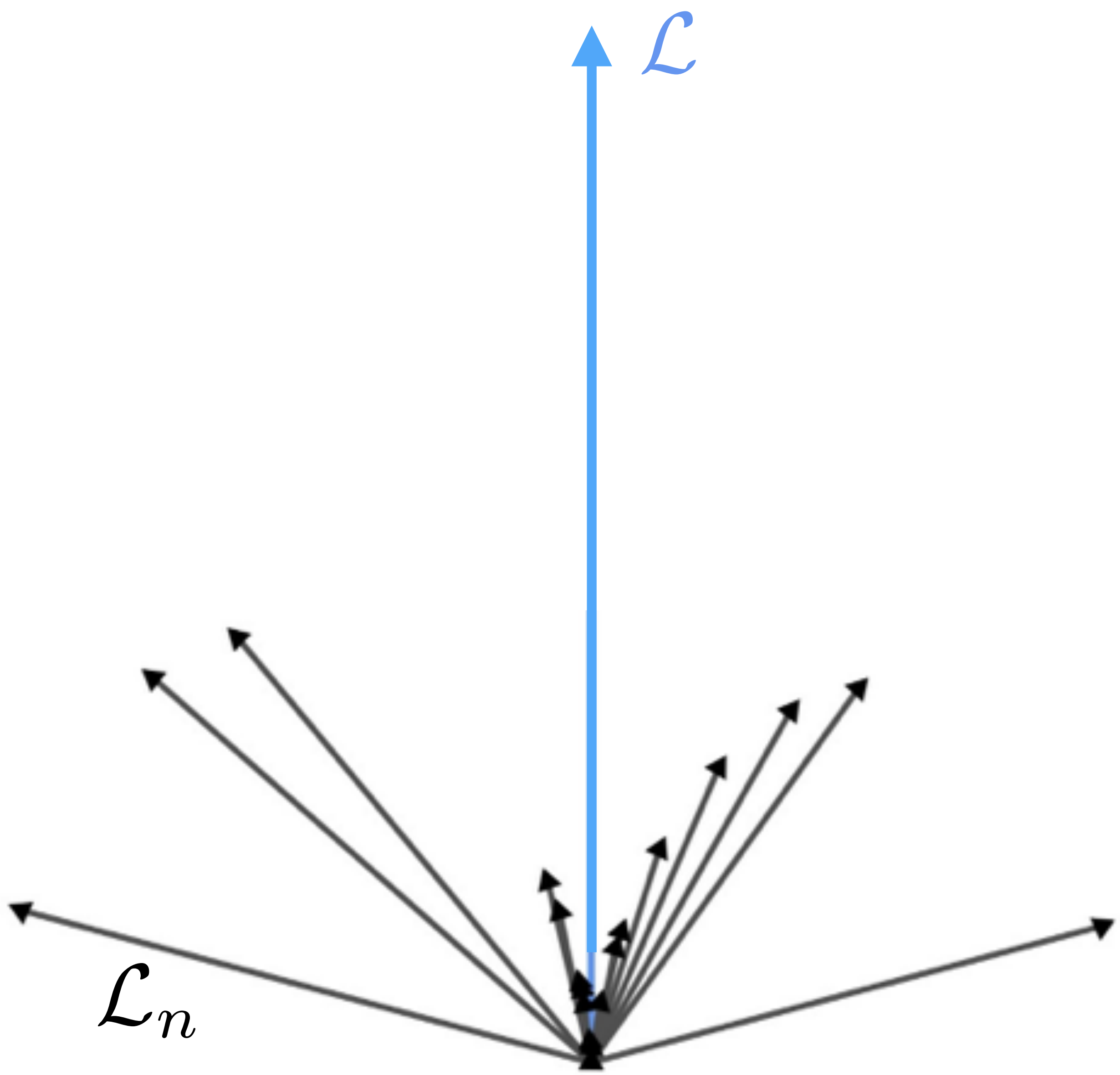


M = 10 *M* = 100 *M* = 1000

How to get a good Bayesian coreset?

- Want: Small error with few coreset points



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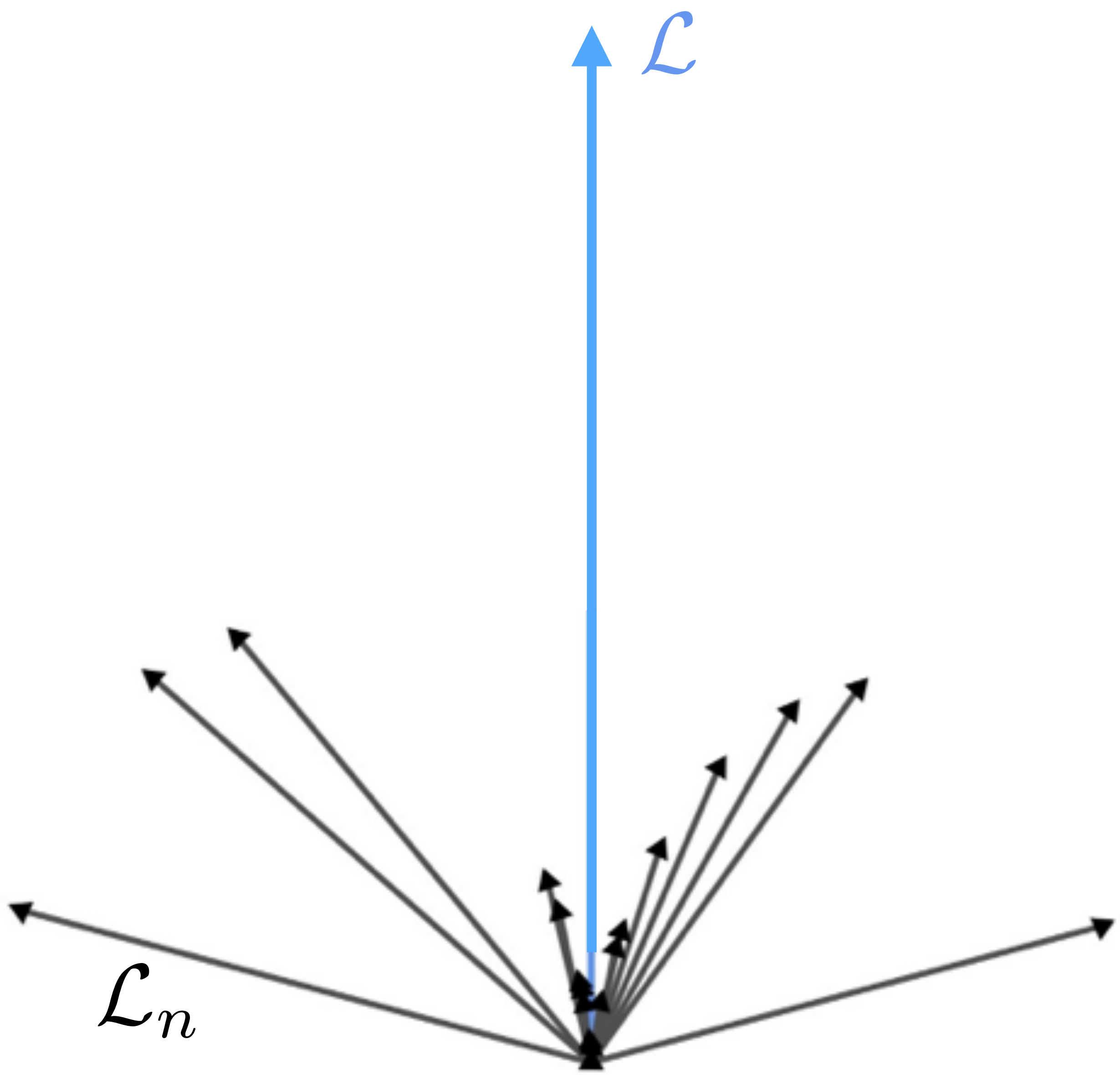
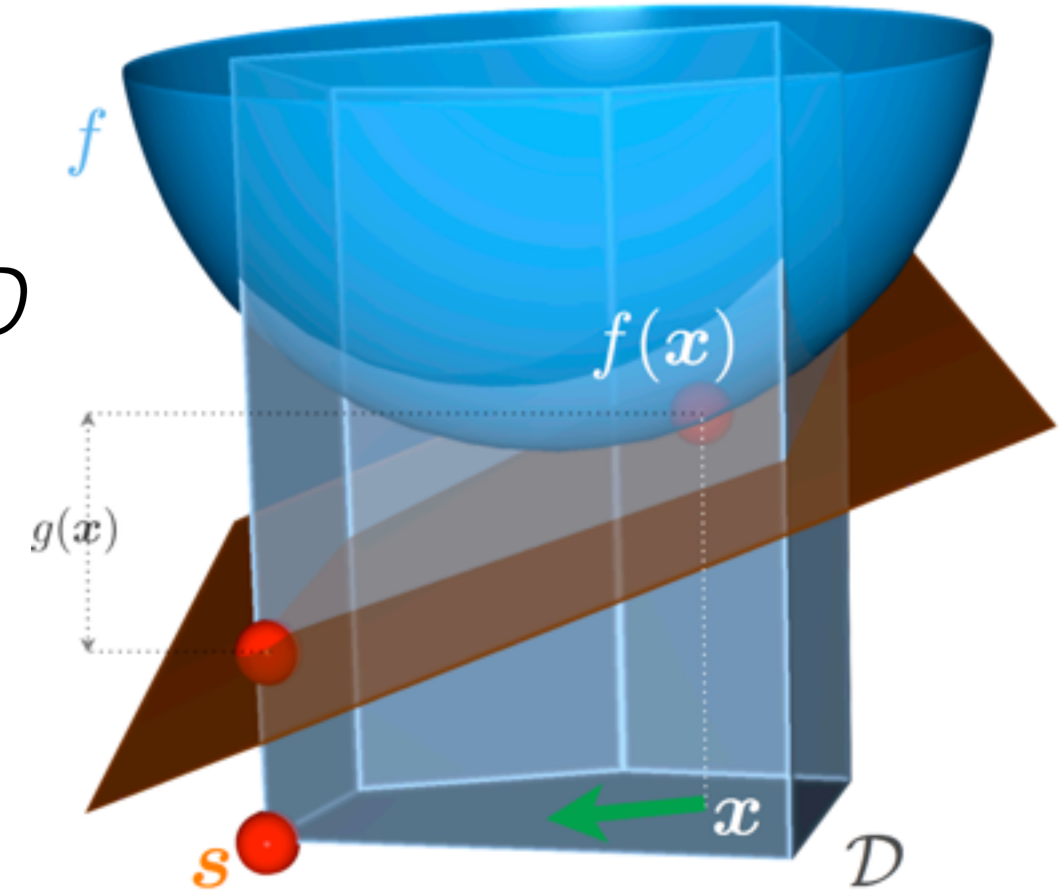


- need to consider (residual) error direction
- sparse optimization

Frank-Wolfe

Convex optimization on a polytope *D*



[Jaggi 2013]

Frank-Wolfe

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- Repeat:
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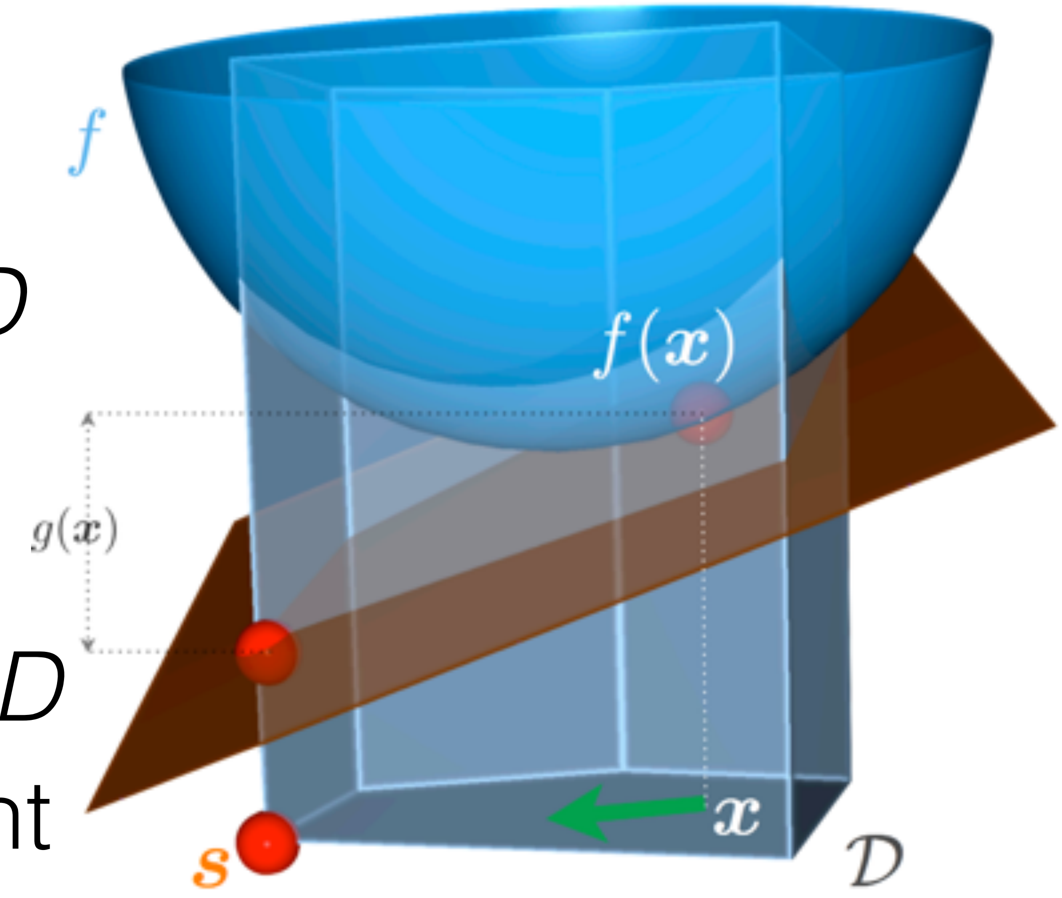

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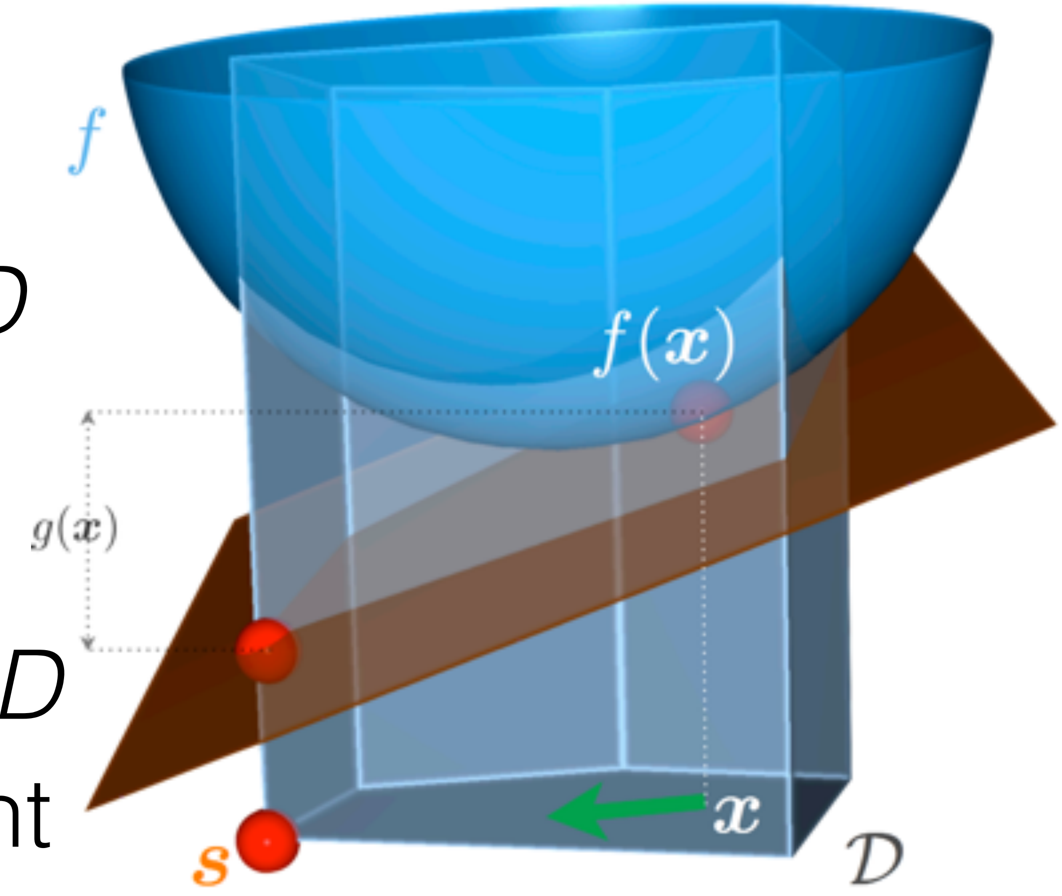

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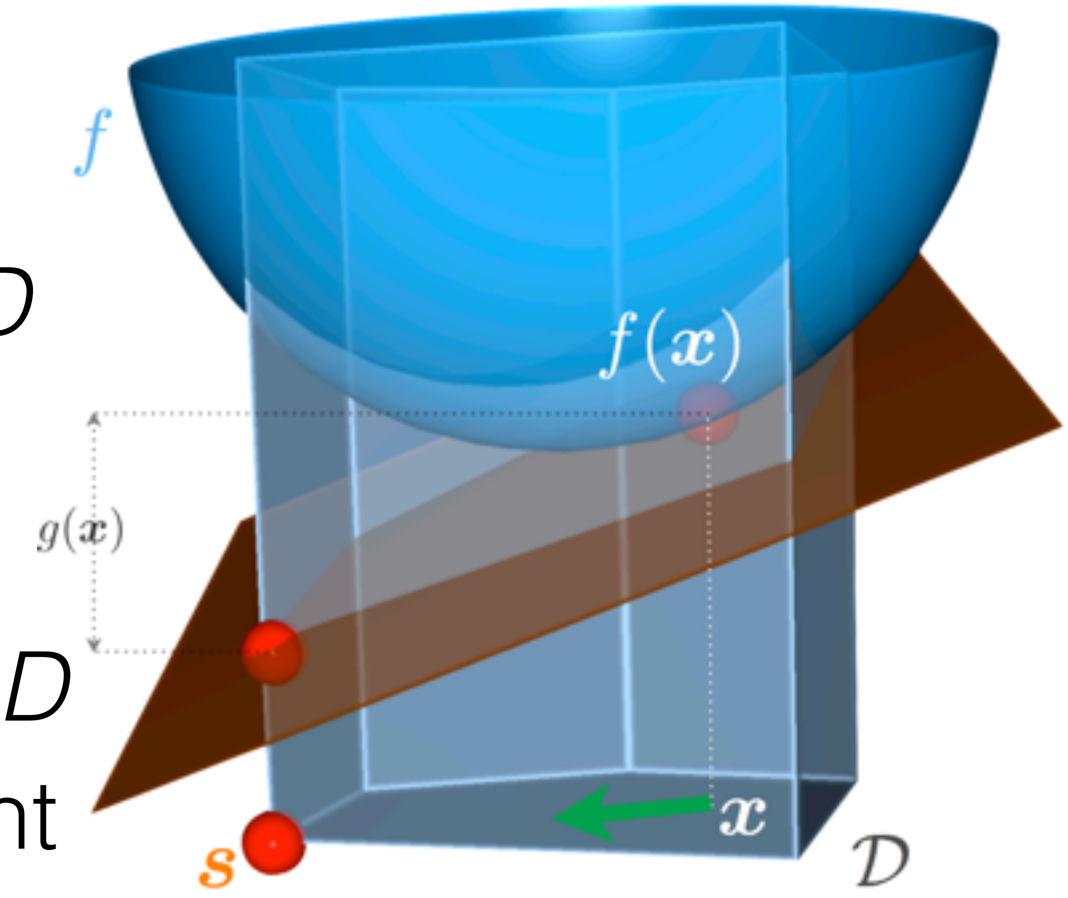

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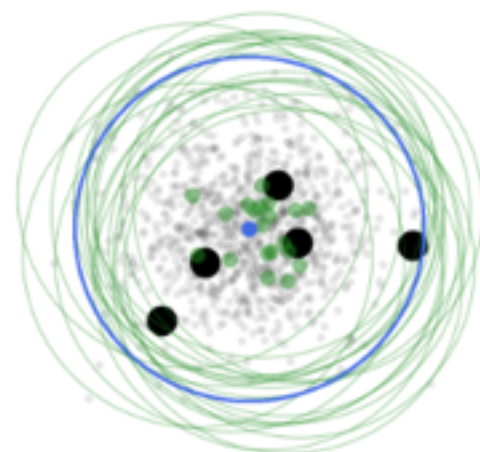
Gaussian model (simulated)

- 10K pts; norms, inference: closed-form

Uniform subsampling

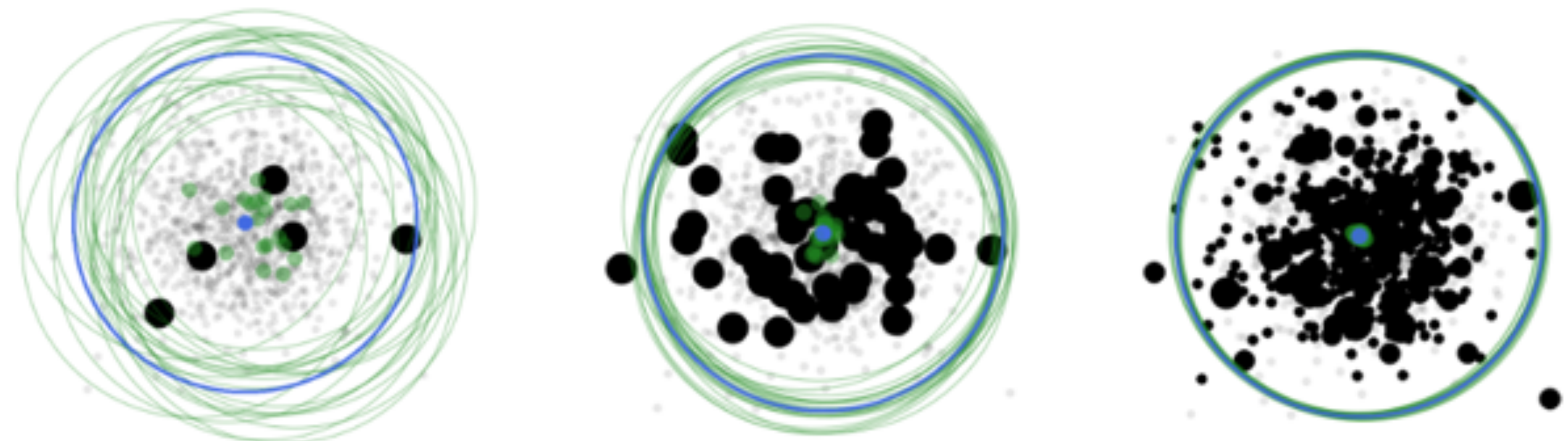


$M = 5$

Gaussian model (simulated)

- 10K pts; norms, inference: closed-form

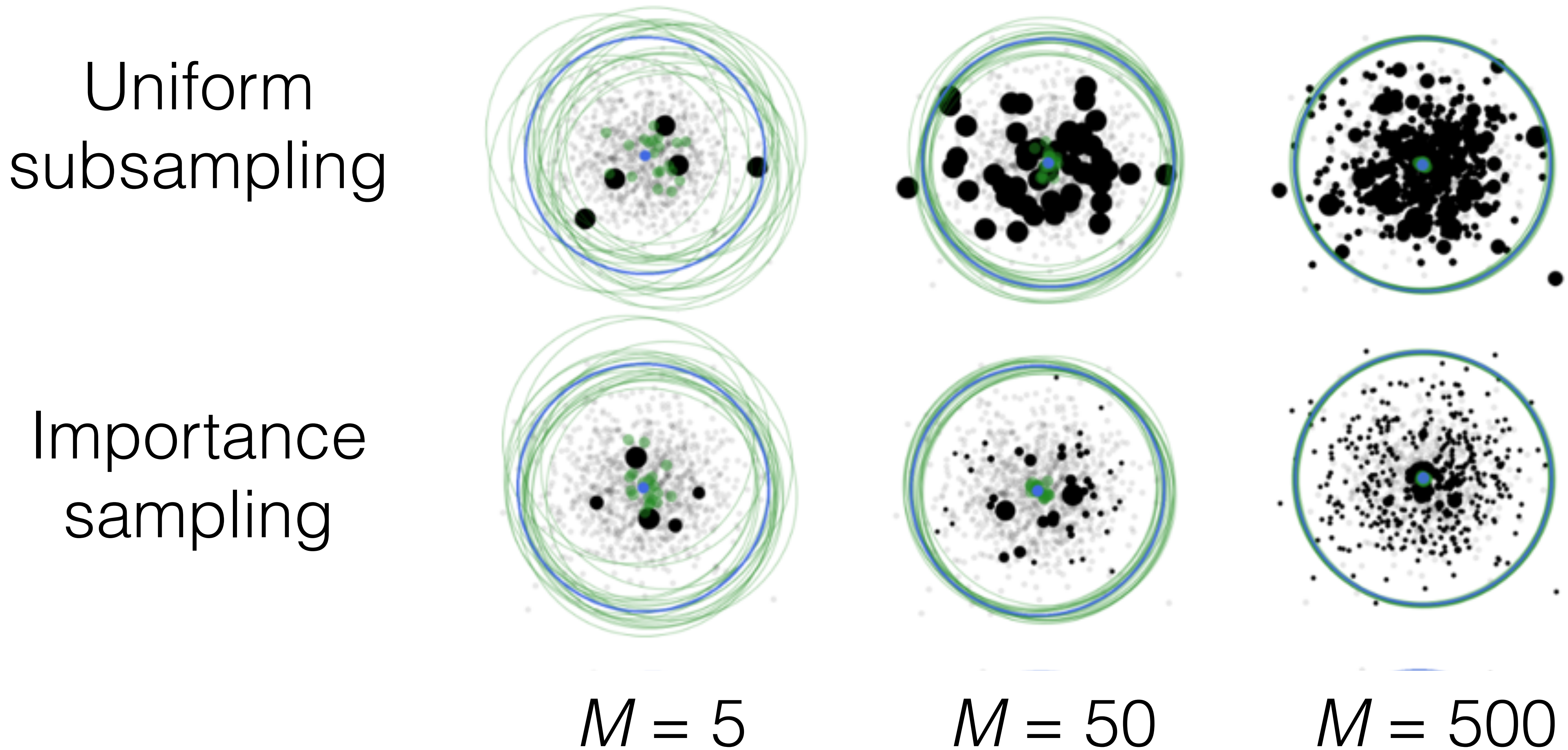
Uniform subsampling



$M = 5$ $M = 50$ $M = 500$

Gaussian model (simulated)

- 10K pts; norms, inference: closed-form



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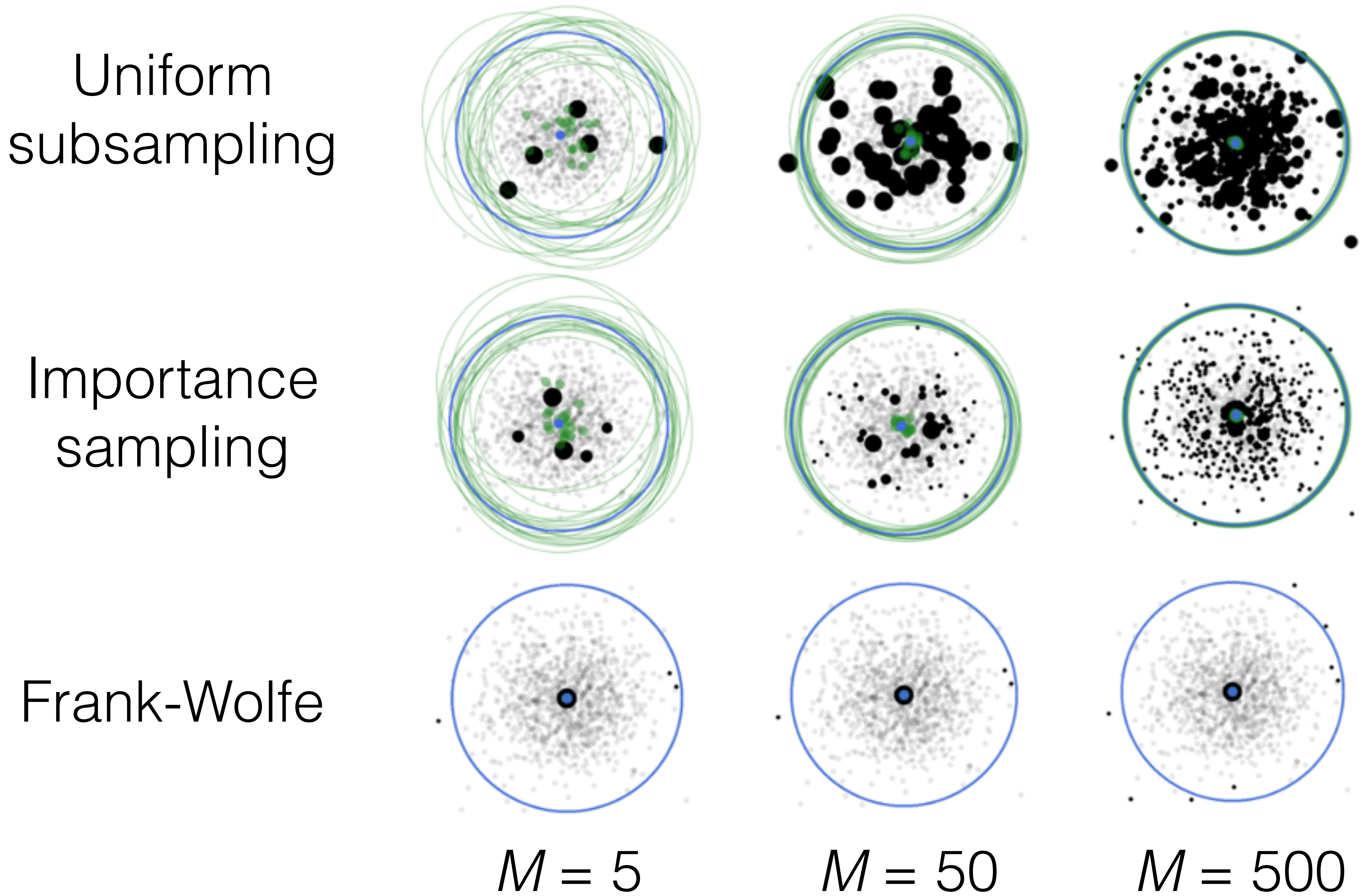


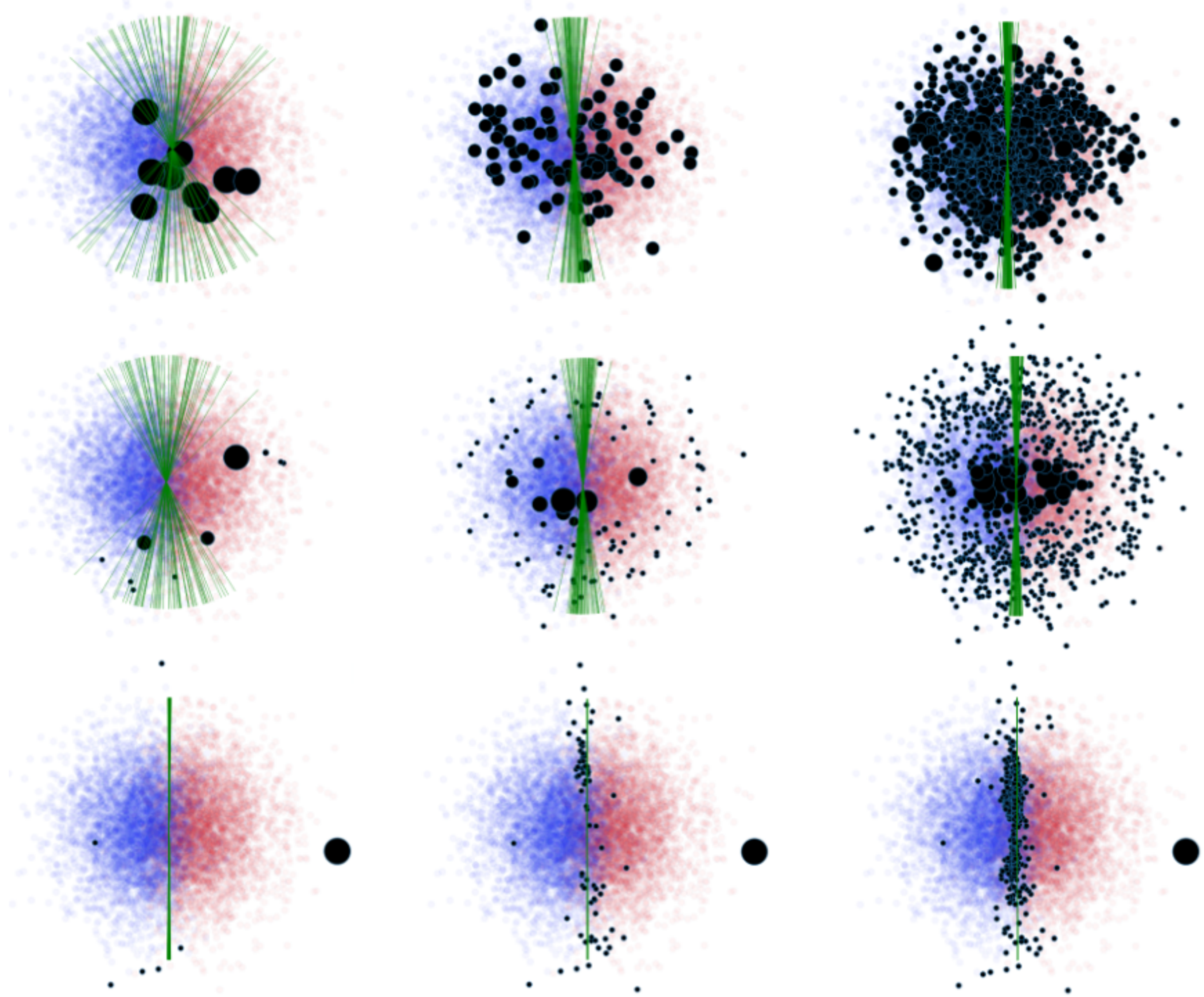
Logistic regression (simulated)

- 10K data points

Uniform subsampling

Importance sampling

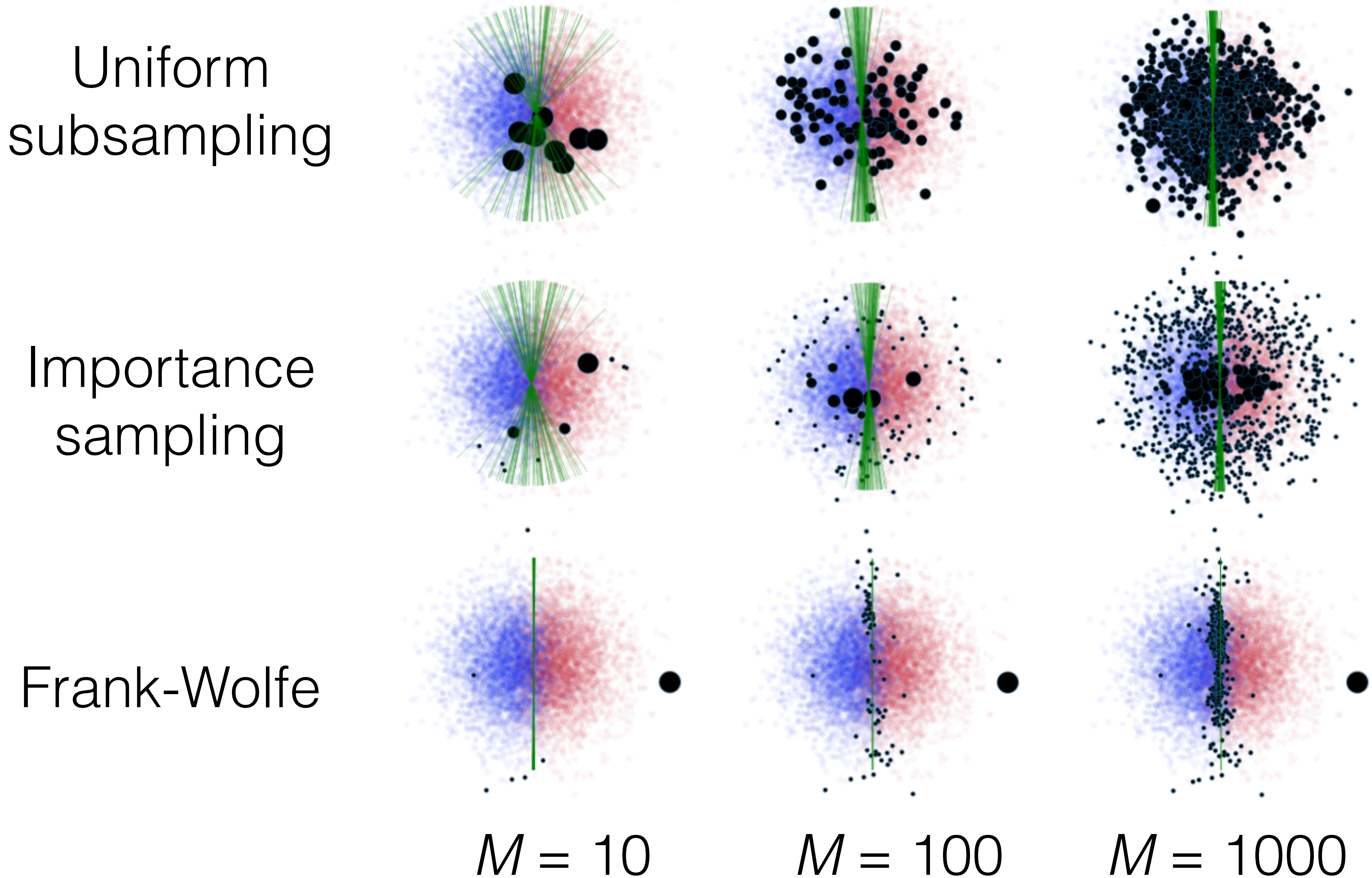
Frank-Wolfe



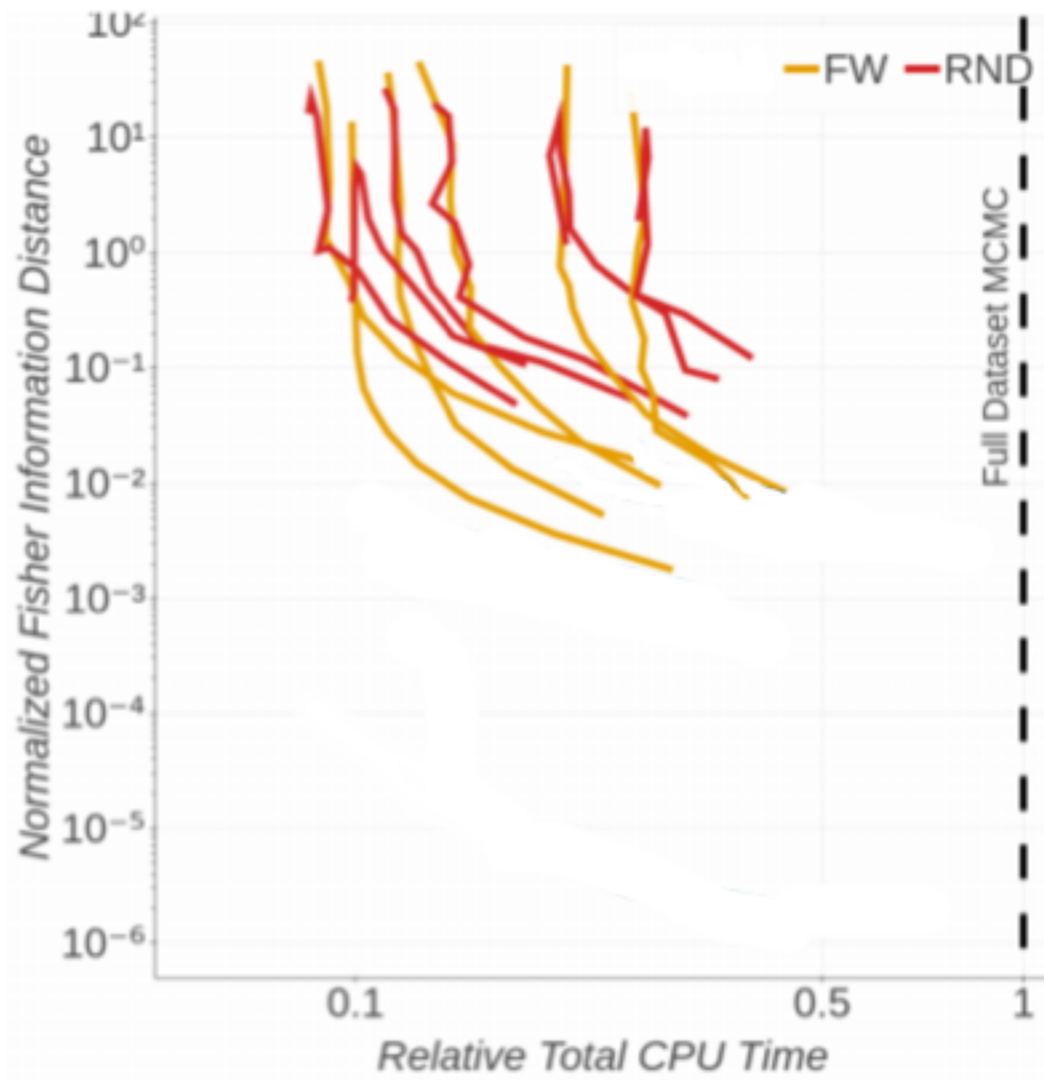
$M = 10$ $M = 100$ $M = 1000$

Logistic regression (simulated)

- 10K data points
- similar for Poisson regression, spherical clustering



Real data experiments



lower error

less total time

Uniform subsampling

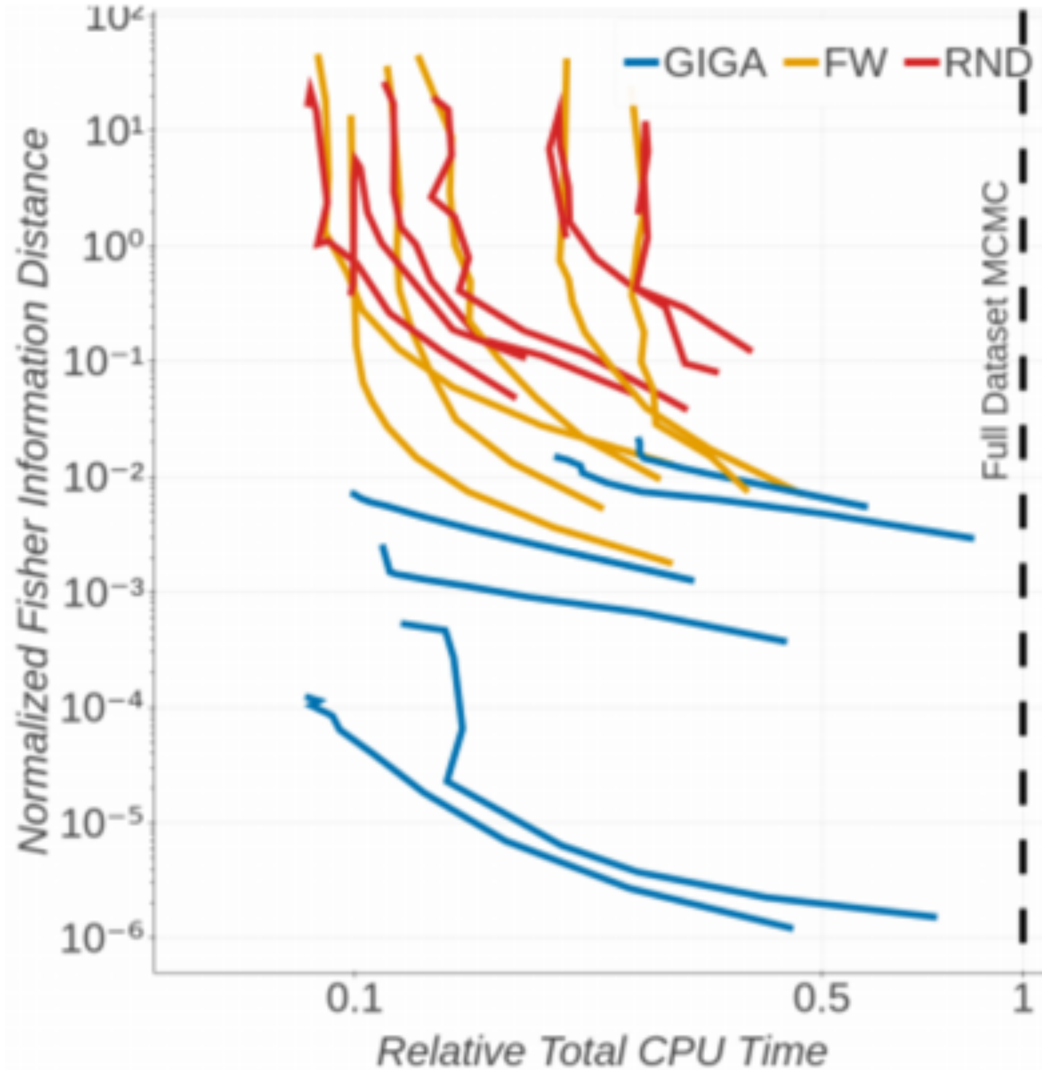
Frank Wolfe coresets

Data sets include:

- Phishing
- Chemical reactivity
- Bicycle trips
- Airport delays

Real data experiments

lower error



less total time

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Frank Wolfe coresets

GIGA coresets

Data sets include:

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Conclusions

- *Data summarization* for **scalable**, **automated** approx. Bayes algorithms with **error bounds on output quality (for finite data)**
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R Agrawal, T Campbell, JH Huggins, and T Broderick. Data-dependent compression of random features for large-scale kernel approximation. ArXiv:1810.04249

T Campbell and T Broderick. Automated scalable Bayesian inference via Hilbert coresets. ArXiv:1710.05053.

T Campbell and T Broderick. Bayesian coreset construction via Greedy Iterative Geodesic Ascent. *ICML* 2018.

JH Huggins, T Campbell, and T Broderick. Coresets for scalable Bayesian logistic regression. *NeurIPS* 2016.

JH Huggins, RP Adams, and T Broderick. PASS-GLM: Polynomial approximate sufficient statistics for scalable Bayesian GLM inference. *NeurIPS* 2017.

JH Huggins, T Campbell, M Kasprzak, and T Broderick. Practical bounds on the error of Bayesian posterior approximations: A nonasymptotic approach. ArXiv: 1809.09505.

References (2/6)

PK Agarwal, S Har-Peled, and KR Varadarajan. Geometric approximation via coresets. *Combinatorial and Computational Geometry* 52 (2005): 1-30.

M Bădoiu, S Har-Peled, and P Indyk. Approximate clustering via core-sets. *Proceedings of the 34th Annual ACM Symposium on Theory of Computing*, 2002.

R Bardenet, A Doucet, and C Holmes. On Markov chain Monte Carlo methods for tall data. *The Journal of Machine Learning Research* 18.1 (2017): 1515-1557.

M Bauer, M van der Wilk, and CE Rasmussen. Understanding probabilistic sparse Gaussian process approximations. *NeurIPS* 2016.

T Broderick. “Variational Bayes and beyond: Bayesian inference for big data” ICML Tutorial, 2018. http://www.tamarabroderick.com/tutorial_2018_icml.html

T Broderick, N Boyd, A Wibisono, AC Wilson, and MI Jordan. Streaming variational Bayes. *NeurIPS* 2013.

CM Bishop. *Pattern Recognition and Machine Learning*. Springer-Verlag New York, 2006.

Y Chen, M Welling, and A Smola. Super-samples from kernel herding. *UAI* 2010.

W DuMouchel, C Volinsky, T Johnson, C Cortes, and D Pregibon. Squashing flat files flatter. In *Proceedings of the fifth ACM SIGKDD international conference on Knowledge discovery and data mining*, pp. 6-15. ACM, 1999.

D Dunson. Robust and scalable approach to Bayesian inference. Talk at *ISBA* 2014.

D Feldman, and M Langberg. A unified framework for approximating and clustering data. In *Proceedings of the forty-third annual ACM symposium on Theory of computing*, pp. 569-578. ACM, 2011.

References (3/6)

B Fosdick. *Modeling Heterogeneity within and between Matrices and Arrays*, Chapter 4.7. PhD Thesis, University of Washington, 2013.

RJ Giordano, T Broderick, and MI Jordan. Linear response methods for accurate covariance estimates from mean field variational Bayes. *NeurIPS* 2015.

R Giordano, T Broderick, R Meager, J Huggins, and MI Jordan. Fast robustness quantification with variational Bayes. *ICML 2016 Workshop on #Data4Good: Machine Learning in Social Good Applications*, 2016.

MD Hoffman, and A Gelman. The No-U-turn sampler: adaptively setting path lengths in Hamiltonian Monte Carlo. *Journal of Machine Learning Research* 15, no. 1 (2014): 1593-1623.

JH Huggins, T Campbell, M Kasprzak, and T Broderick. Scalable Gaussian Process inference with finite-data mean and variance guarantees. Under review. ArXiv:1806.10234.

M Jaggi. Revisiting Frank-Wolfe: Projection-Free Sparse Convex Optimization. *ICML* 2013.

A Kucukelbir, R Ranganath, A Gelman, and D Blei. Automatic variational inference in Stan. *NeurIPS* 2015.

A Kucukelbir, D Tran, R Ranganath, A Gelman, and DM Blei. Automatic differentiation variational inference. *The Journal of Machine Learning Research* 18.1 (2017): 430-474.

DJC MacKay. *Information Theory, Inference, and Learning Algorithms*. Cambridge University Press, 2003.

D Madigan, N Raghavan, W Dumouchel, M Nason, C Posse, and G Ridgeway. Likelihood-based data squashing: A modeling approach to instance construction. *Data Mining and Knowledge Discovery* 6, no. 2 (2002): 173-190.

References (4/6)

M Opper and O Winther. Variational linear response. *NeurIPS* 2003.

G Rosman, M Volkov, D Feldman, JW Fisher III, D Rus. Coresets for k-segmentation of streaming data. *NeurIPS* 2014.

RE Turner and M Sahani. Two problems with variational expectation maximisation for time-series models. In D Barber, AT Cemgil, and S Chiappa, editors, *Bayesian Time Series Models*, 2011.

B Wang and M Titterington. Inadequacy of interval estimates corresponding to variational Bayesian approximations. In *AISTATS*, 2004.

Application References (5/6)

Abbott, Benjamin P., et al. "Observation of gravitational waves from a binary black hole merger." *Physical Review Letters* 116.6 (2016): 061102.

Abbott, Benjamin P., et al. "The rate of binary black hole mergers inferred from advanced LIGO observations surrounding GW150914." *The Astrophysical Journal Letters* 833.1 (2016): L1.

Chati, Yashovardhan Sushil, and Hamsa Balakrishnan. "A Gaussian process regression approach to model aircraft engine fuel flow rate." *Cyber-Physical Systems (ICCPS), 2017 ACM/IEEE 8th International Conference on*. IEEE, 2017.

Gillon, Michaël, et al. "Seven temperate terrestrial planets around the nearby ultracool dwarf star TRAPPIST-1." *Nature* 542.7642 (2017): 456.

Grimm, Simon L., et al. "The nature of the TRAPPIST-1 exoplanets." *Astronomy & Astrophysics* 613 (2018): A68.

Meager, Rachael. "Understanding the impact of microcredit expansions: A Bayesian hierarchical analysis of 7 randomized experiments." *AEJ: Applied*, to appear, 2018a.

Meager, Rachael. "Aggregating Distributional Treatment Effects: A Bayesian Hierarchical Analysis of the Microcredit Literature." Working paper, 2018b.

Woodard, Dawn, Galina Nogin, Paul Koch, David Racz, Moises Goldszmidt, and Eric Horvitz. "Predicting travel time reliability using mobile phone GPS data." *Transportation Research Part C: Emerging Technologies* 75 (2017): 30-44.

Additional image references (6/6)

amCharts. Visited Countries Map. https://www.amcharts.com/visited_countries/ Accessed: 2016.

J. Herzog. 3 June 2016, 17:17:30. Obtained from: https://commons.wikimedia.org/wiki/File:Airbus_A350-941_F-WWCF_MSN002_ILA_Berlin_2016_17.jpg (Creative Commons Attribution 4.0 International License)