

Thermostat-assisted Continuous-tempered Hamiltonian Monte Carlo for Multimodal Posterior Sampling

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Motivation & Solution

1. To sample multimodal posterior

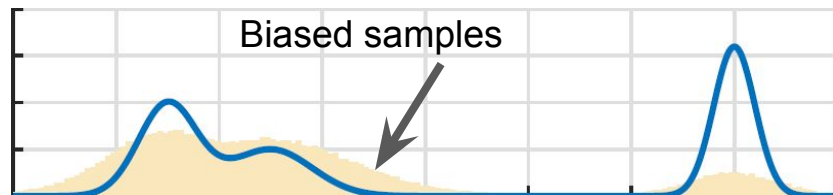
Continuous tempering

2. To leverage minibatches

Nose-Hoover thermostat

3. To establish a systematic integration

Fokker-Plank equation



Formulation & Simulation

$$\frac{d\boldsymbol{\theta}}{dt} = \mathbf{M}_{\boldsymbol{\theta}}^{-1} \mathbf{p}_{\boldsymbol{\theta}},$$

$$\frac{dp_{\xi}}{dt} = \frac{p_{\xi}}{m_{\xi}},$$

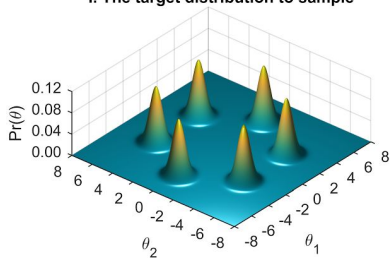
$$\frac{d\mathbf{p}_{\boldsymbol{\theta}}}{dt} = \frac{\tilde{\mathbf{f}}(\boldsymbol{\theta})}{\lambda(\xi)} - \frac{\mathbf{S}_{\boldsymbol{\theta}} \mathbf{p}_{\boldsymbol{\theta}}}{\lambda^2(\xi)},$$

$$\frac{dp_{\xi}}{dt} = \frac{\lambda'(\xi)}{\lambda^2(\xi)} \tilde{U}(\boldsymbol{\theta}) - W'(\xi) - \left[\frac{\lambda'(\xi)}{\lambda^2(\xi)} \right]^2 s_{\xi} p_{\xi},$$

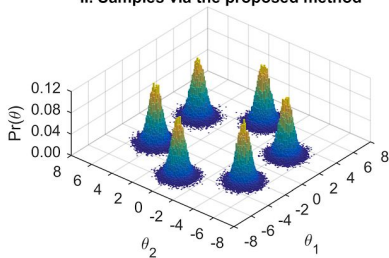
$$\frac{ds_{\theta}^{(i,j)}}{dt} = \frac{Q_{\theta}^{(i,j)}}{\lambda^2(\xi)} \left[\frac{p_{\theta_i} p_{\theta_j}}{m_{\theta_i}} - T \delta_{ij} \right],$$

$$\frac{ds_{\xi}}{dt} = \left[\frac{\lambda'(\xi)}{\lambda^2(\xi)} \right]^2 Q_{\xi} \left[\frac{p_{\xi}^2}{m_{\xi}} - T \right].$$

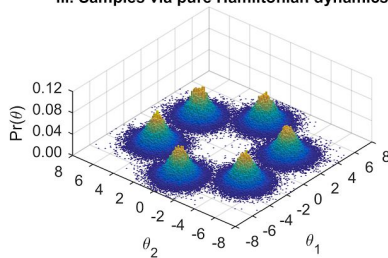
I. The target distribution to sample



II. Samples via the proposed method



III. Samples via pure Hamiltonian dynamics



IV. Sample via SGHMC w/ noise known

