Learning priors, likelihoods, or posteriors

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"Within the field of approximate Bayesian inference, **variational** *and* **Monte Carlo** *methods are currently the mainstay techniques."*

— `http://approximateinference.org/`

The Statistician (1987) 36, pp. 247–249

Monte Carlo is fundamentally unsound

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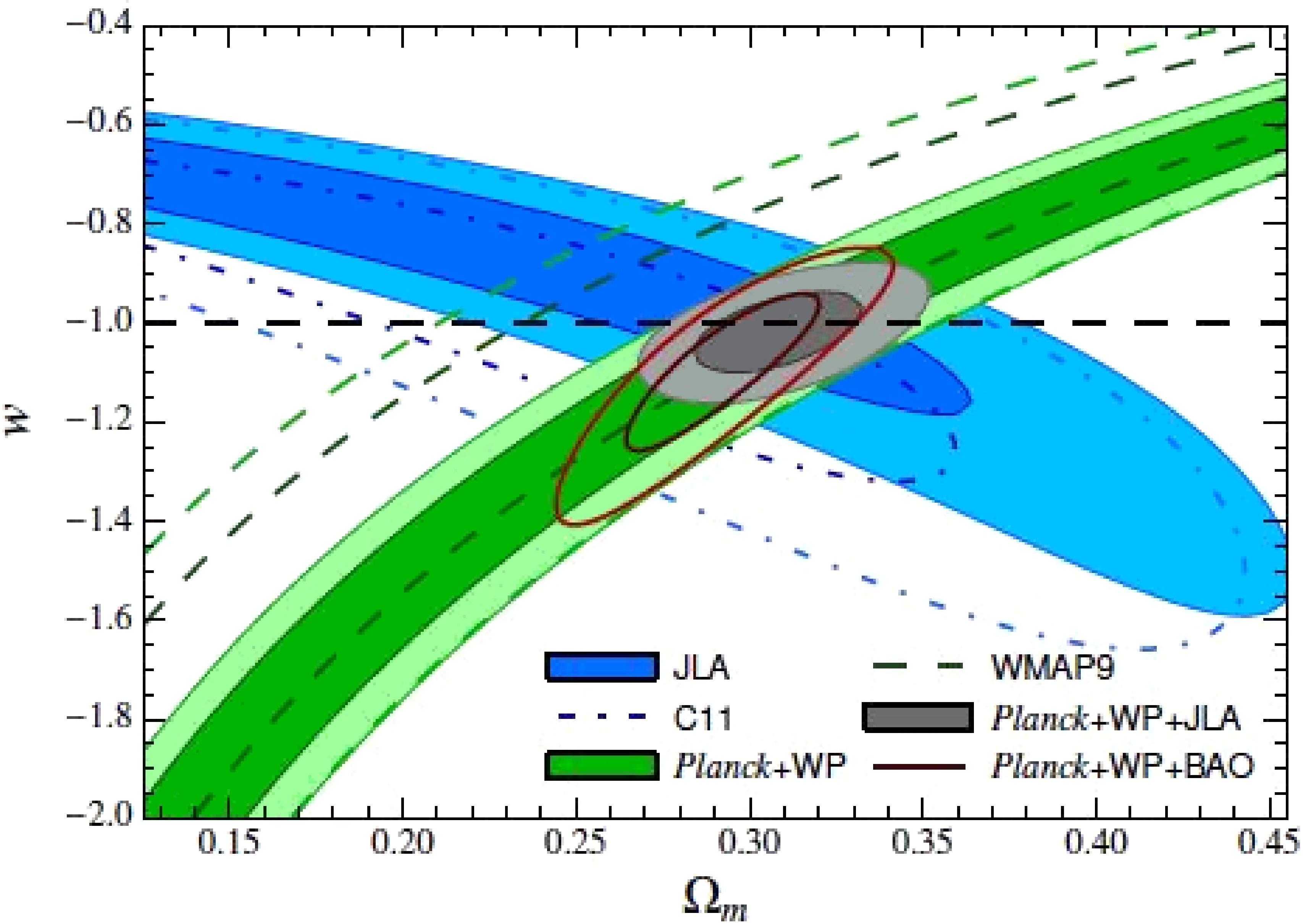
Abstract. We present some fundamental objections to the Monte Carlo method of numerical integration.

Posteriors in Cosmology



https://bitbucket.org/joezuntz/cosmosis/

Recognition networks

$$\theta^{(s)} \sim p(\theta)$$

$$\mathbf{x}^{(s)} \sim p\left(\mathbf{x} \,|\, \theta^{(s)}\right)$$

Training set: $\left\{\theta^{(s)}, \mathbf{x}^{(s)}\right\}_{s=1}^{S}$

Some of the relevant work

Hinton et al. (1995, Science) — Wake Sleep, Helmholtz machine

Morris (2001, UAI) — Recognition Networks

Blum & Francois (2010, S&C) — Conditional Gaussian, neural nets

Fan, Nott, Sisson (2012, Stat) — Mixture of experts

Mitrović, Dino Sejdinović, Teh (2016, ICML) — Kernel regression

...

Fast ϵ-free Inference of Simulation Models with Bayesian Conditional Density Estimation

Papamakarios and Murray (NIPS, 2016)

Lueckmann et al. (NIPS, 2017)

— Fit $\hat{p}(\theta \,|\, \mathbf{x})$ maximize $\sum_s \log \hat{p}(\theta^{(s)} \,|\, \mathbf{x}^{(s)})$

Mixture Density Networks (Bishop, 1994)

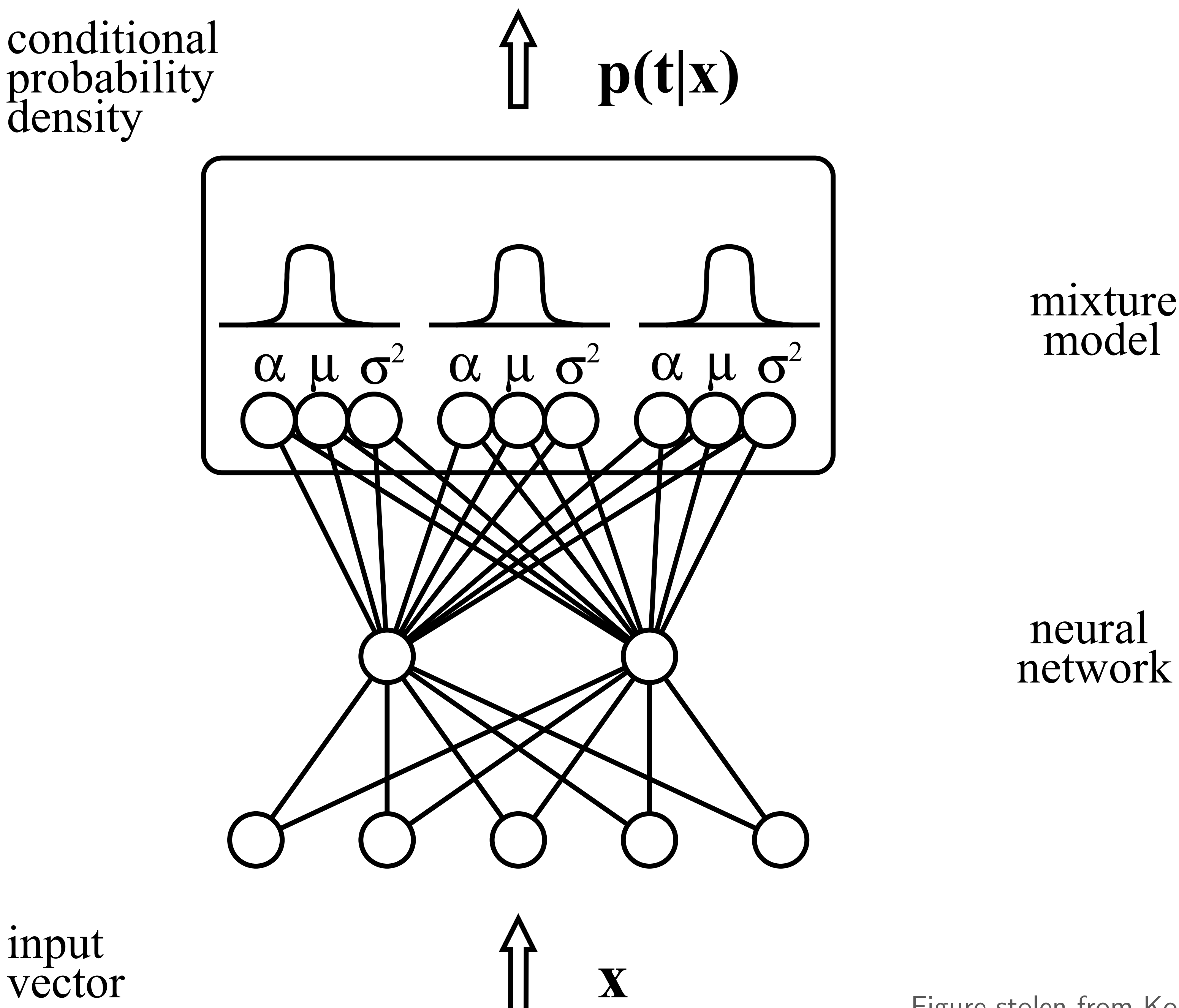


Figure stolen from Korin Richmond

Fast ϵ-free Inference of Simulation Models with Bayesian Conditional Density Estimation

Papamakarios and Murray (NIPS, 2016)
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— $\hat{p}(\theta \mid \mathbf{x}_{\text{observed}}) \rightarrow$ approx posterior

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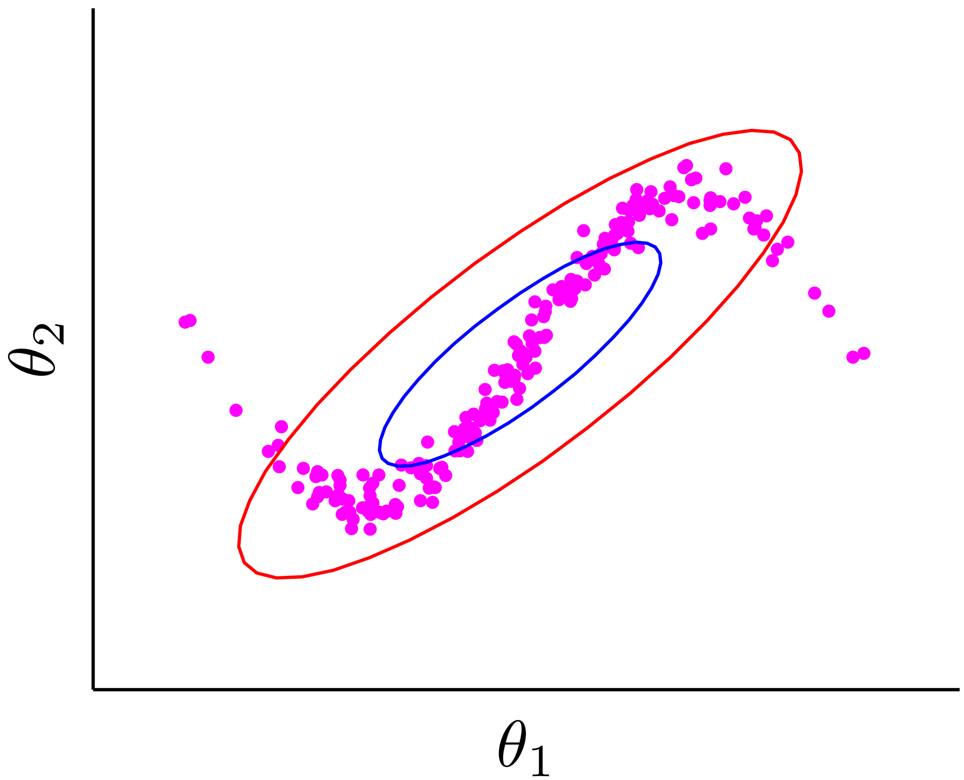
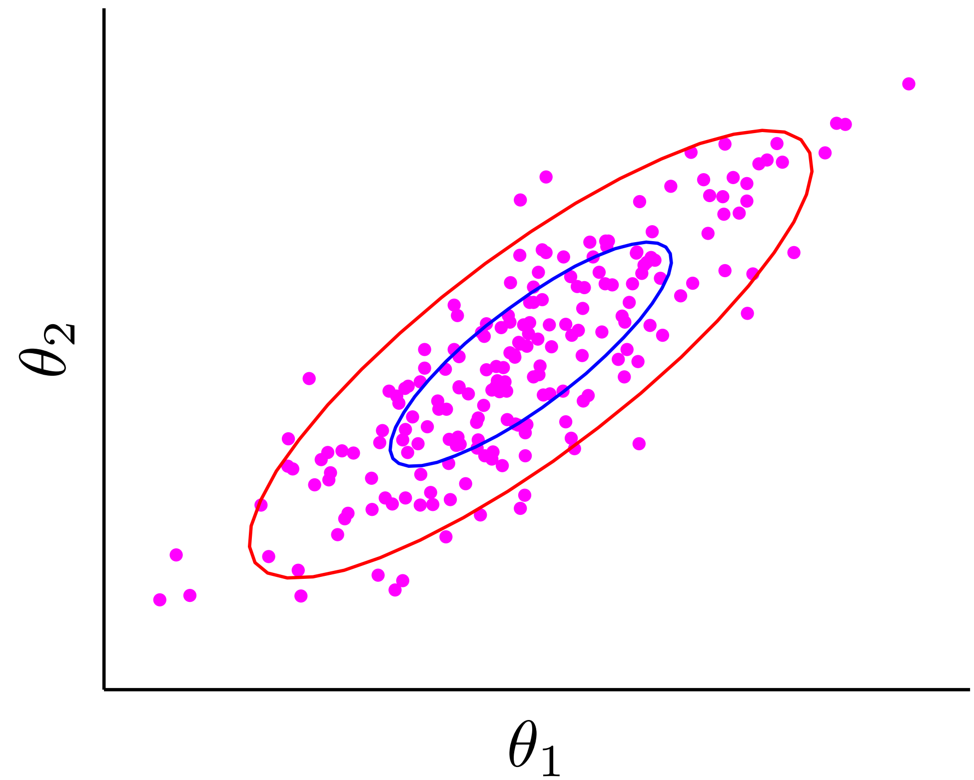
— Refine fit: more simulations

Underfitting





True posterior samples

samples from Gaussian fit

— Modeling posteriors

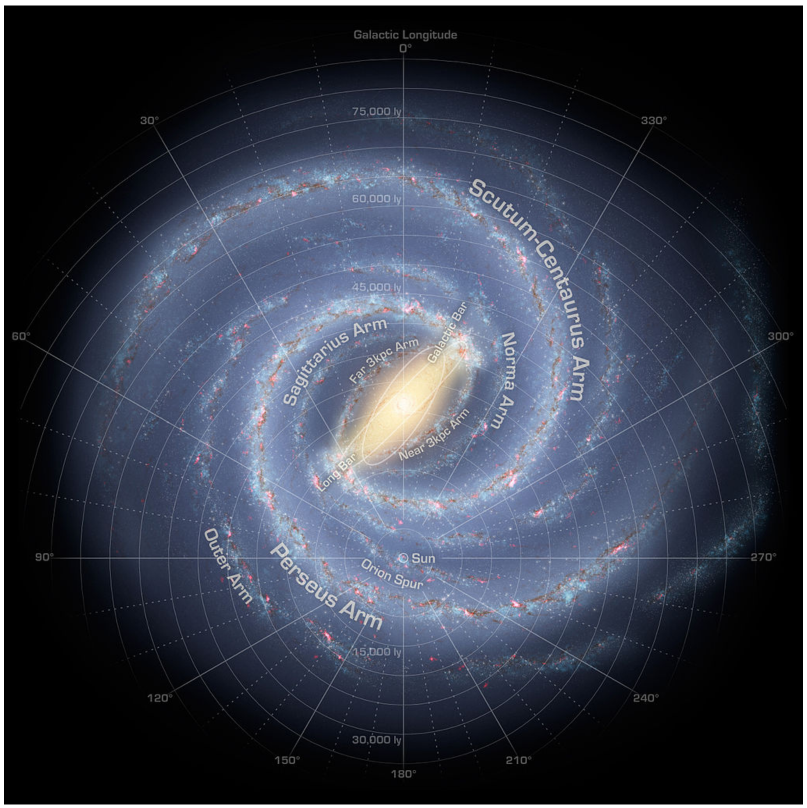
— **Modeling priors**

— Modeling likelihoods

Weighing the Milky Way

Busha, Marshall, Wechsler, Klypin and Primack (2011)

APJ 743:40



Milky Way diagram, NASA

Magellanic Clouds, ESO/S. Brunier

http://en.wikipedia.org/wiki/File:236084main_MilkyWay-full-annotated.jpg
http://www.eso.org/public/images/b01/

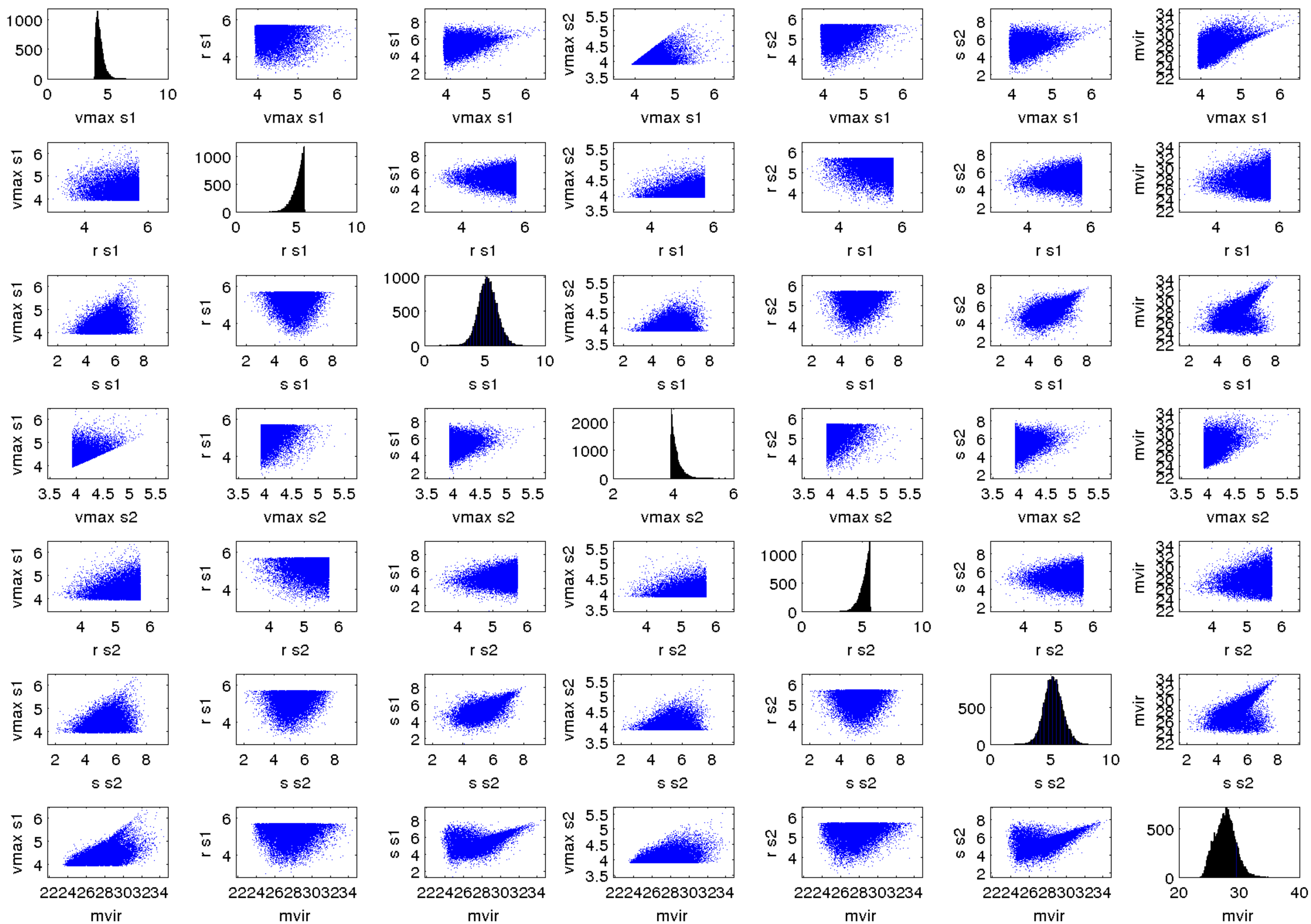
Bayesian Inference

$$p(\mathbf{x} \mid \mathbf{y}) \propto p(\mathbf{y} \mid \mathbf{x})\, p(\mathbf{x})$$

$\mathbf{x} = [r, v, m]$, vector of galaxy properties

$\mathbf{y} = [\hat{r}, \hat{v}]$, noisily observe part of $\mathbf{x}$

The prior: simulation samples

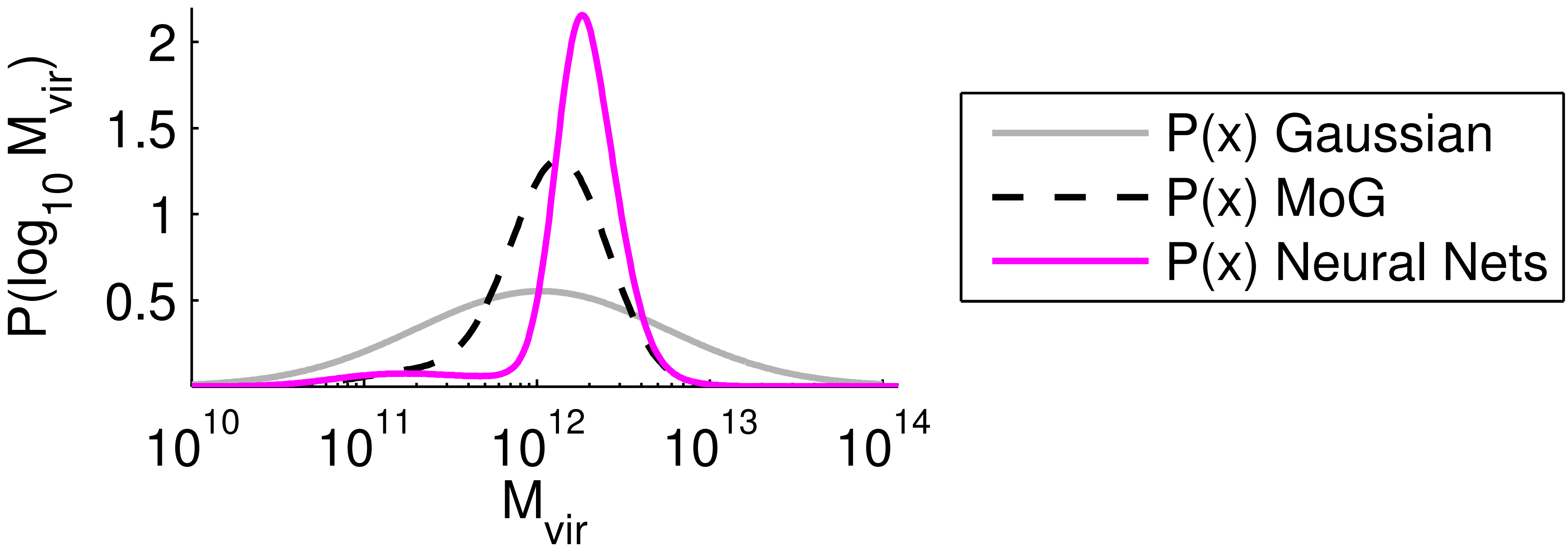


Milky Way mass

$p(\mathbf{x})$ theory: simulated galaxy properties

$p(\mathbf{y} \mid \mathbf{x})$ observations of Milky Way

$p(\mathbf{x} \mid \mathbf{y}) \rightarrow p(x_1 \mid \mathbf{y})$, posterior of mass

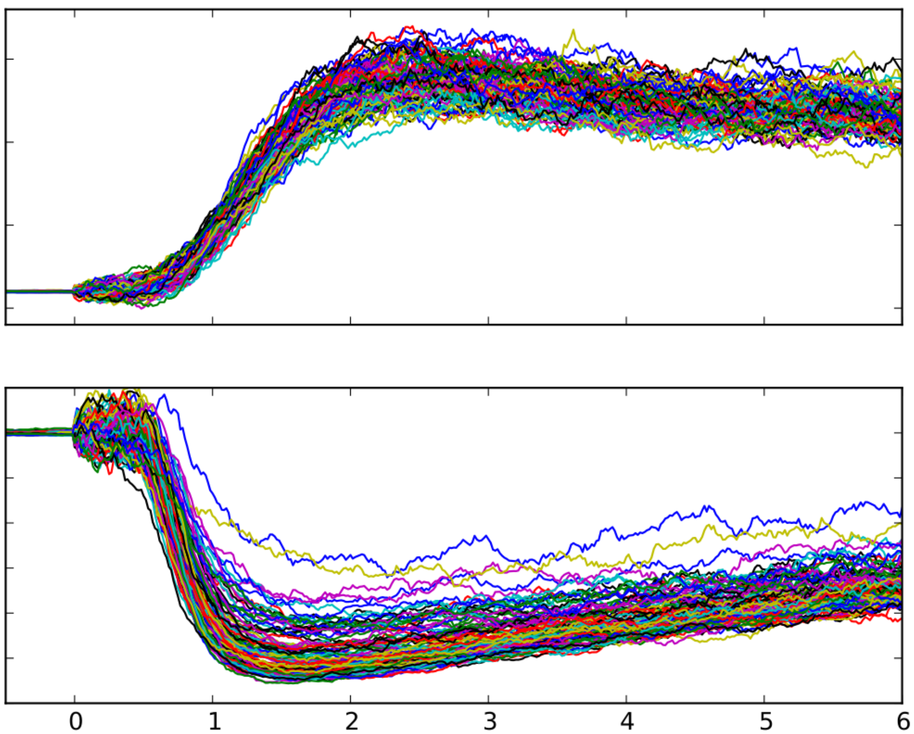


— Modeling posteriors

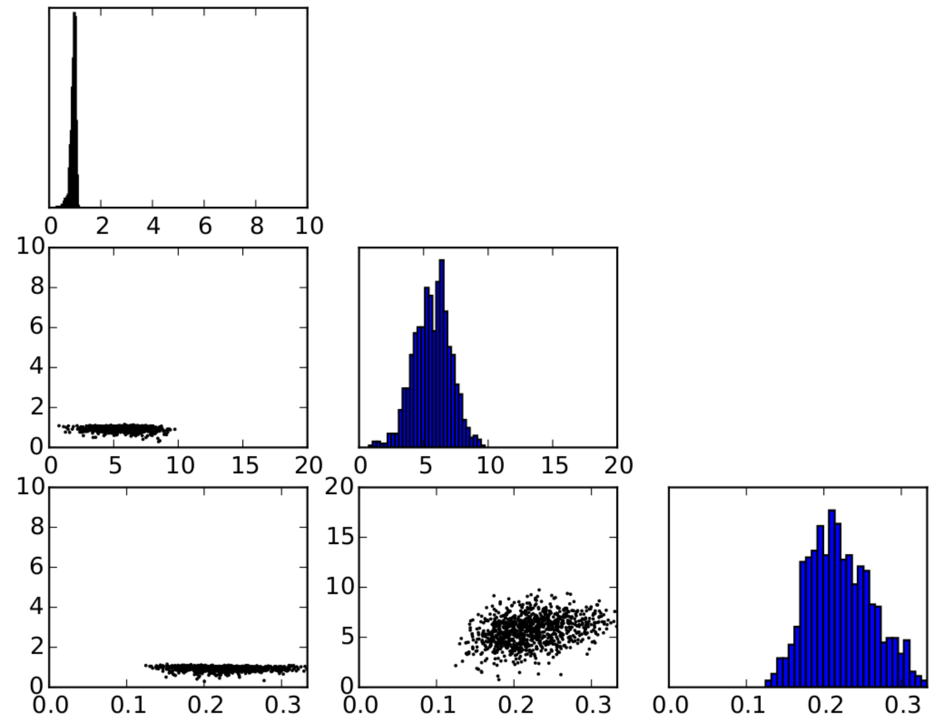
— Modeling priors

— **Modeling likelihoods**

Surrogate modeling / emulation



data $\mathbf{x}$



parameters $\boldsymbol{\theta}$

$$p(\theta \,|\, \mathcal{D}) \;\propto\; p(\theta) \prod_n p(\mathbf{x}^{(n)} \,|\, \theta)$$

Cf Cranmer, Pavez, Louppe (2016) arXiv:1506.02169

Thanks!

`http://iainmurray.net`

NADE variants, MADE, and MAF
ϵ-free ABC, pseudo-marginal slice sampling