On Exploration, Exploitation and Learning in Adaptive Importance Sampling

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$\rightarrow$ Trade off between **Exploitation** and **Exploration**.

AdaIS as a Bandit Problem

Partition the sample space $\mathcal{X}$ into K disjoint subspaces $\mathcal{X}_1, \cdots, \mathcal{X}_K$.
Proposal:

$$q_t(x) = \sum_{a=1}^{K} q_{at} g_a(x) \tag{1}$$

where q_{at} are the mixing proportions at time t, $g_a(x)$ is the importance proposal on $\mathcal{X}_a$.

- Arm: Subspace
- Reward: Importance weights
- Regret: $R(q_t) = KL(\pi \| q_t) - KL(\pi \| q^*) = \sum_a \pi_a \log \frac{\pi_a}{q_{at}}$

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Proposal:

$$q_t(x) = \sum_{a=1}^{K} \underbrace{q_{at}}_{\substack{\text{boost when} \\ \text{under exploration}}} g_a(x) \tag{2}$$

where q_{at} are the mixing proportions at time t, $g_a(x)$ is the importance proposal on $\mathcal{X}_a$.

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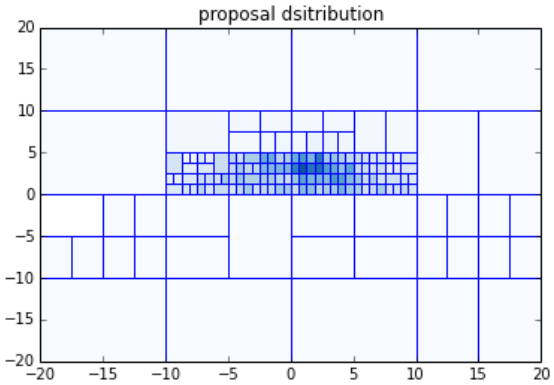
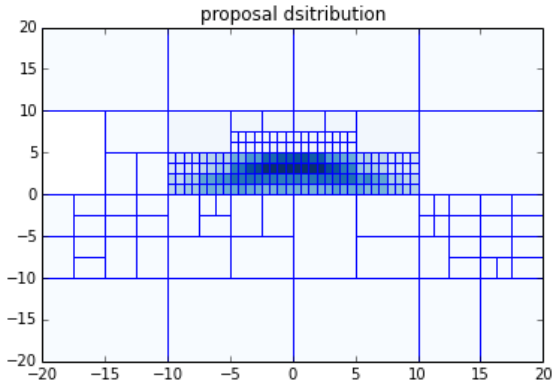
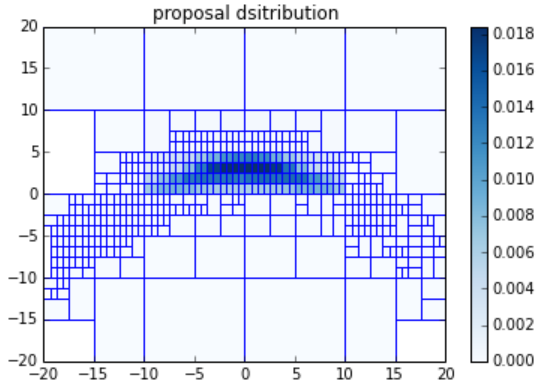
Banana-shaped Example



(a) 1000th iteration



(b) 10000th iteration



(c) 100000th iteration