Kingmaker: A Simulation Analysis of Strategic Voting

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Abstract

In the field of social choice theory, The *Gibbard-Satterthwaite Theorem* tells us that any all *non-trivial, non-dictatorial* voting methods do *not* have a *dominant strategy* for any individual voter. Thus every voting system which follows these axioms (which they should) must be *manipulable.* Thus any voting method must consider how voters will *strategically vote* in order to benefit their *social welfare.* There have been efforts such as ... that aim to measure how resilient a voting method is to certain kinds of strategic voting.

My thesis expands upon this literature by simulating complex social conditions in order to:

1. Synthesize optimal (in some measurable sense) strategies (for some subset of voter base),
2. Use those novel (as well as known) voting strategies to compare the resilience of common voting methods, and
3. ...

Dedication

To my parents, for their ceaseless support.

1. Introduction

In this section, I'll introduce the premise of the thesis. I will begin by describing how is in the field of social choice theory. Then I will introduce the idea of strategic voting, and the premises of the paper. Lastly, I will briefly discuss the methods of the thesis: this is a simulation study, where fabricated data will be used to judge and verify the efficacy of strategic voting strategies and the resilience of voting methods against them.

    This section must introduce the thesis questions and provide a hook (perhaps some surprising results) that hook the reader and informs them why this matters.

2. Background

In this chapter, all the background theory will be introduced. This should cover: social choice, and likely game theory and optimization as well.

2.1. Election Systems

This section builds the foundational ideas of voting methods and the axioms and structure that we want them to have. It should also discuss some of the theorems that already bound what is possible to have in a voting system (*Arrow*, *Gibbard-Satterwaite*). It should be structured like so:

2.1.1. Axioms:

There are infinite ways to convert ballots into a "winner". But what characteristics do we want this "voting function" to have. This sub-section talks about some of the axioms that voting systems should ideally have, why we want them, and what they each mean. It should impress the significant of the *social welfare function* as a mathematical object that underpins the core of strategic voting.

2.1.2. Social Choice Theorems:

Having discussed some of the axioms that we'd ideally want to have, this sub-section breaks that idyllic view to discuss the boundaries of what is achievable in a voting system. Thus we introduce *Arrow's Impossibility Theorem*. We then turn to the social welfare function, and discuss the *Gibbard-Satterwaite Theorem* and why strategic voting is inherent to all forms of voting.

2.1.3. Voting Methods:

With some theory under our belts, this sub-section begins to introduce some of the actual methods that are used for real voting. We discuss their implementations, their characteristics (with respect to the social choice axioms), and their usage (how they are used and what inputs and outputs they give).

There is also room to add some less common, novel and unused methods which can then be compared to the more established methods.

2.2. How Voters Vote

In this section, we narrow in on strategic voting and the methods by which voters vote. This section asks how voters convert their complex preferences of each candidate and convert them into concrete ballots.

There should be discussion of the following types of voting: honest, strategic (deterministic), strategic (stochastic / noise), and mixed models.

We should of course reiterate why voters strategically vote and how then tend to do so.

2.2.1. Preferences → Ballots

This sub-section should talk about how preferences, which we describe as the jumble of opinions that a voter holds of each candidate, is converted into a (honest) ballot through a stochastic process. For example, we might conceptualize a voter who needs to rank the candidates as having some abstract distribution of "likability" for each candidate. Then on election day, they "draw" from that distribution and the order of those "likability" ratings is their ranked ballot.

2.2.2. Strategic Voting Interactions

This sub-section is where we discuss how the preferences to ballots process interacts with the process of strategic voting (conceptually). How should we consider these processes given that they happen in parallel and influence each other. A true preferences are likely to be influenced by knowledge of the environment, i.e. strategic voting.

2.2.3. History of Strategic Voting

This section breaks down some of the history behind strategic voting. We look back into past elections to determine (1) Historically, how have voters strategically altered their votes? What strategies do they tend to use? (2) When (if ever) has strategic voting influenced elections? How?

2.2.4. Optimal Strategy

This is the section where we dig into the idea of optimal strategy with respect to social welfare. Before the methods section, this section introduced the fundamental goals and characteristics that optimal strategies should have. hat does it mean for a strategic voting function to be "optimal"?

This section should also dovetail nicely into the methods section, which will discuss the implementation and framework that we will use the model strategic voting and analyze its efficacy.

3. Methods

This chapter should begin by restating the thesis questions, and explaining the overarching framework by which we aim to answer them. Mainly, via monte-carlo and simulation. It should explain that in the results section, we will:

1. Train computational models to "optimally vote"
2. Test those models (and more well-know ones) to find the best of the best
3. Test how manipulable various `methods` are
4. Determine which social conditions are most / least favorable for strategic voting
5. etc...

3.1. A Theory of Voting

In this section, we discuss how (and why) `Kingmaker` is set up the way it is, and the ways that this architecture will impact the way that simulations are run and results collected. The point of this section is to get the reader comfortable with the `Synthesizer` → `Strategy` → `Method` voting process.

3.2. Hyper-parameters

This section should describe the bounds of our analysis. It should specify which combinations of (`Synthesizer`, `Method`, `Strategy`, `SocialConditions`) we will be simulating.

3.3. etc...

There is more that needs to be added here

4. Results

This chapter should begin by introducing (1) the metrics that quantify manipulability of a voting method, and (2) the methods for judging the efficacy of a strategy (under some set of social conditions).

This should follow on explanation about which social conditions and combinations of `method`, `strategy`, and `synthesizer` will be used.

4.1. Optimization

This section should provide the results on "optimal" strategies for given `methods` or `social conditions`. This includes both simple known strategies and novel computational strategies (that we make). For each of these strategies, explain:

1. What general class of strategy it is
2. How it works conceptually
3. How is is implemented, and what are the hyper-parameters

For results, provide tables that give various metrics across the battery of `methods` and `social conditions` that its trained and ran on.

4.2. Method Analysis

This section should use the known + novel strategies to test the manipulability of various `methods`. The results should be from a range of (`synthesizer`, `social conditions`) pairs, and measured across each manipulability metric. Use ridge plots for this.

For analysis, ask (1) Which methods generally performed well or poorly—and what does that say about the metrics themselves, which ones should we put credence to—and (2) What were the strengths and weaknesses of each of the `methods`.

5. Discussion

5.1. Review

In this section, we tie everything together, reviewing the work that has been done. What are the most important results? What have we build conceptually?

Having completed analyzing the results, take the time to reflect on how well the results have followed the guidelines laid out in the methods section. What is the efficacy of these methods and our approach to simulation and analysis.

5.2. Introspection & Conclusion

This section is where the thesis winds down. It should step back and to consider the entire project as a whole. It should re-evaluate the thesis questions and the work that's been done. Have we succeeded in answering them?

Additionally, this section should swing back around to social choice to ask how this fits (or doesn't) with the existing literature. What have we gained by doing this analysis?

5.3. Future Research

This section discusses potential future research that follows from this work. In general, this falls into 3 buckets:

1. What is missing from the analysis that further analysis can fill in?
2. What are some conclusions that open new paths for further exploration?
3. What might be redone / expanded on to help clarify or revise any findings?

A. Symbols and Definitions

Symbol	Definition
$A, B, C, ...$	Candidates
$a, b, c, ...$	Voters
$\{a, b, c, ...\}$	The set of all voters
...	The preference of a voter x
...	The ballot of a voter x
$\prec, \succ, \preceq, \succeq$	$A \prec B \Rightarrow A$ is preferred to B, $A \preceq B \Rightarrow A$ is preferred or indifferent to B
$\mathcal{W}(x)$	Social welfare function for a voter x
$\mathcal{I}(h, ...)$	A ballot generator (synthesizer) with some hyper-parameters h
$\mathcal{S}(x)$	The strategy for a voter x
$\mathcal{M}(\{a, b, c, ...\})$	The outcome of a method, M, on a set of ballots.

References

[1] K. J. Arrow, "A Difficulty in the Concept of Social Welfare," *The Journal of political economy*, vol. 58, no. 4, pp. 328–346, 1950.

[2] A. Gibbard, "Manipulation of Voting Schemes: A General Result," *Econometrica*, vol. 41, no. 4, pp. 587–601, 1973.

[3] R. D. (. D. Luce, "Individual choice behavior; a theoretical analysis.." Wiley, New York, 1959.

[4] R. L. Plackett, "The Analysis of Permutations," *Journal of the Royal Statistical Society. Series C (AppliedStatistics)*, vol. 24, no. 2, pp. 193–202, 1975.

[5] J. G. Kemeny, "Mathematics without Numbers," *Daedalus (Cambridge, Mass.)*, vol. 88, no. 4, pp. 577–591, 1959.

[6] W. G. Ludwin, "Strategic Voting and the Borda Method," *Public Choice*, vol. 33, no. 1, pp. 85–90, 1978.

[7] C. L. Mallows, "Non-Null Ranking Models. I," *Biometrika*, vol. 44, no. 1/2, p. 114–, 1957.

[8] T. H. Hammond, "Rank Injustice?: How the Scoring Method for Cross-Country Running Competitions Violates Major Social Choice Principles," *Public choice*, vol. 133, no. 3/4, pp. 359–375, 2007.

[9] FairVote, "PR Library: Types of Voting Systems."