

Kingmaker: A Simulation Analysis of Strategic Voting

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Contents

1 INTRODUCTION	1
2 BACKGROUND	2
2.1 Election Methods	2
2.1.1 Social Welfare	2
3 METHODS	2
4 RESULTS	2
5 DISCUSSION	2
1 MATHEMATICAL SYMBOLS AND DEFINITIONS	2
REFERENCES	4

Abstract

In the field of social choice theory, The *Gibbard-Satterthwaite Theorem* tells us that any all *non-trivial, non-dictatorial* voting methods do *not* have a *dominant strategy* for any individual voter. Thus every voting system which follows these axioms (which they should) must be *manipulable*. Thus any voting method must consider how voters will *strategically vote* in order to benefit their *social welfare*. There have been efforts such as ... that aim to measure how resilient a voting method is to certain kinds of strategic voting.

My thesis expands upon this literature by simulating complex social conditions in order to:

1. Synthesize optimal (in some measurable sense) strategies (for some subset of voter base),
2. Use those novel (as well as known) voting strategies to compare the resilience of common voting methods, and
3. ...

Dedication

To my parents, for their ceaseless support.

1

Introduction

2

Background

...

1.1 Election Methods

2.1 Election Methods

Elections of some kind have existed forever. In ancient civilizations there has always been a form of elections, from dictatorial to oligarchical to consensus. The best-known ancient civilization that had voted by consensus had to be Athens, which voted by democracy, though it's worth noting that democracy included only free adult men. Additionally, it is not remotely the first such democracy, not only in Greece, but in general. There is evidence of many older civilizations which voted by consensus.

All of this to point out that the modern underpinnings of voting have existed for a long time almost unchanged. Pluracracy as a voting method has existed for thousands of years. If anything, the modern revolution of voting has been in increased representation.

What this brief history tells us is that while the purpose of voting is to make (collective) decisions via some sort of procedure, what characteristics of the procedure, as well as the social conditions that surround it vastly impact its outcomes.

1.1 Social Welfare

2.1.1 Social Welfare

This brings us to the *social welfare function*, denoted $\mathcal{W}(v)$. The social welfare function is a measure of a voter v 's "happiness" at a particular outcome or result. The goal of any voting method is to maximize social welfare for a given collection of preferences through some deterministic process which is *not* dependent of the preferences of the voters.

Generally we'll take the social welfare function to be the sum of the welfare over all voters, where all voters is representative of the entire population.

So we have some abstract metric that can tell us how well

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Methods

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Results

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Discussion

1

Mathematical Symbols and Definitions

Symbol	Definition
A, B, C, \dots	Candidates
a, b, c, \dots	Voters
$\{a, b, c, \dots\}$	The set of all voters
\succsim	The preference of a voter x
\mathcal{B}_x	The ballot of a voter x
$\prec, \succ, \preceq, \succeq$	$A \prec B \Rightarrow A$ is preferred to B , $A \preceq B \Rightarrow A$ is preferred or indifferent to B
$\mathcal{W}(x)$	Social welfare function for a voter x
$\mathcal{I}(h, \dots)$	A ballot generator (synthesizer) with some hyper-parameters h
$\mathcal{S}(x)$	The strategy for a voter x
$\mathcal{M}(\{a, b, c, \dots\})$	The outcome of a method, M , on a set of ballots.

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