

# Kingmaker: A Simulation Analysis of Strategic Voting

Chance Addis

May 2025

*A thesis presented to  
The Division of Mathematical and Natural Sciences  
Reed College*

*Submitted in partial fulfillment of the requirements  
for the degree Bachelor of Arts*



*Approved for the Division*  
*(Mathematics - Statistics)*

---

Michael Pearce



# *Contents*

1. Introduction .....	7
2. Background .....	9
2.1. A History of Social Choice .....	9
2.2. A Formalization of Social Choice .....	9
2.2.1. Basic Framework .....	9
2.3. A Model of Voting .....	10
2.3.1. Conceptualization of Ballots .....	10
2.3.2. Strategic Voting .....	10
3. Methods .....	11
3.1. A Theory of Voting .....	11
3.2. Hyper-parameters .....	11
3.3. etc... .....	11
4. Results .....	13
4.1. Optimization .....	13
4.2. Method Analysis .....	13
5. Discussion .....	15
5.1. Review .....	15
5.2. Introspection & Conclusion .....	15
5.3. Future Research .....	15
A. Symbols and Definitions .....	17
References .....	19



# *Abstract*

In the field of social choice theory, The *Gibbard-Satterthwaite Theorem* tells us that any all *non-trivial, non-dictatorial* voting methods do *not* have a *dominant strategy* for any individual voter. Thus every voting system which follows these axioms (which they should) must be *manipulable*. Thus any voting method must consider how voters will *strategically vote* in order to benefit their *social welfare*. There have been efforts such as ... that aim to measure how resilient a voting method is to certain kinds of strategic voting.

My thesis expands upon this literature by simulating complex social conditions in order to:

1. Synthesize optimal (in some measurable sense) strategies (for some subset of voter base),
2. Use those novel (as well as known) voting strategies to compare the resilience of common voting methods, and
3. ...





# *Dedication*

To my parents, for their ceaseless support.



# *1. Introduction*

In this section, I'll introduce the premise of the thesis. I will begin by describing how is in the field of social choice theory. Then I will introduce the idea of strategic voting, and the premises of the paper. Lastly, I will briefly discuss the methods of the thesis: this is a simulation study, where fabricated data will be used to judge and verify the efficacy of strategic voting strategies and the resilience of voting methods against them.

This section must introduce the thesis questions and provide a hook (perhaps some surprising results) that hook the reader and informs them why this matters.



## 2. Background

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat voluptatem. Ut enim aequale doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari voluptas distinguere possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in quo a nobis philosophia defensa et collaudata est, cum id, quod maxime placeat, facere possimus, omnis voluptas assumenda est, omnis dolor repellendus. Temporibus autem quibusdam et.

### 2.1. A History of Social Choice

Social choice has been applied since humanity has existed. After all there has always been a need to make collective decisions. The theory however, is more modern. The two scholars typically credited with the development of social choice theory is Nicolas de Condorcet (1743–1794) and Kenneth Arrow (1921–2017).

### 2.2. A Formalization of Social Choice

At its core, social choice theory is concerned with the analysis of *preference aggregation*, understood to be the aggregation of individual preferences, each of which compares two or more social alternatives, into a single collective preference (or choice). The basic framework, which is still standard, was introduced by *Kenneth Arrow* in 1951.

#### 2.2.1. Basic Framework

Let  $N = \{1, 2, \dots\}$  be a set of  $n$  individuals ( $n \geq 2$ ), and  $A = \{a, b, \dots\}$  be a set of  $m$  social alternatives, such as candidates, policies, goods, etc. Each individual  $v_i$  has a *preference ordering*  $P_i$  over these alternatives. A *preference ordering* is defined by a complete, total order on  $X$  known as a *weak preference*. It is written with the symbol  $\preceq, \succeq$ , where  $a \preceq$

$\mathcal{C}$  is defined as  $a$  is preferred or indifferent to  $\mathcal{C}$ . There are also shorthands for *strict preference* ( $a \prec \mathcal{C} := a \preceq \mathcal{C} \wedge \mathcal{C} \not\preceq a$ ) and *strict indifference* ( $a \sim \mathcal{C} := a \preceq \mathcal{C} \wedge \mathcal{C} \preceq a$ )

<sup>1</sup> A collection of preference orderings across a set of individuals  $\{P_1, P_2, \dots, P_n\}$ , is called a *profile*. A *social welfare function* is a function  $W : P \rightarrow A$

continue here

## 2.3. A Model of Voting

### 2.3.1. Conceptualization of Ballots

So far, the only conception of voting has been with *preferences*. But do voters actually submit their ballots deterministically? No. It's unrealistic to assume that voters always vote rationally in a predefined way. There is an element of randomness in the voting process. Say that a voter has the following opinions:  $a = 65\%$  approval and  $\mathcal{C} = 35\%$  approval. The voter may submit a ballot with  $a \prec \mathcal{C}$  or  $\mathcal{C} \prec a$  depending on how they feel on the day of the election. Voting is a *stochastic process*.

This conceptualization facilitates the need to disambiguate a *preference* from a *ballot*.<sup>2</sup> A preference is redefined as a distribution over preference orderings, and a ballot is a realization of that distribution. Think of a preference like a superposition, and when the election is held, the preference collapses into a ballot.

Stochastic voting is a concept hardwired into the framework of this thesis, as all the methods for generating ballots are stochastic. This framing allows us to conceptualize these *synthesizers* as voters who non-deterministically submit their ballots via some set of rules. Granted those rules might be “randomly select an ordering of candidates”, but it's still of the same process as real voting.

### 2.3.2. Strategic Voting

continue here

---

<sup>1</sup>Here I forgo the more general formalism of *preference aggregation rule* in favor of the more specific case of *social welfare functions*. The term *social welfare function* is a specific type of preference aggregation rule that *always* produces a complete social ranking of alternatives. For the scope of this thesis, social welfare functions are more suitable.

<sup>2</sup>Here I redefine preference to a new definition and define ballot in its place.

## 3. *Methods*

This chapter should begin by restating the thesis questions, and explaining the overarching framework by which we aim to answer them. Mainly, via monte-carlo and simulation. It should explain that in the results section, we will:

1. Train computational models to “optimally vote”
2. Test those models (and more well-know ones) to find the best of the best
3. Test how manipulable various methods are
4. Determine which social conditions are most / least favorable for strategic voting
5. etc...

### 3.1. *A Theory of Voting*

In this section, we discuss how (and why) Kingmaker is set up the way it is, and the ways that this architecture will impact the way that simulations are run and results collected. The point of this section is to get the reader comfortable with the Synthesizer → Strategy → Method voting process.

### 3.2. *Hyper-parameters*

This section should describe the bounds of our analysis. It should specify which combinations of (Synthesizer, Method, Strategy, SocialConditions) we will be simulating.

### 3.3. *etc...*

There is more that needs to be added here





## *4. Results*

This chapter should begin by introducing (1) the metrics that quantify manipulability of a voting method, and (2) the methods for judging the efficacy of a strategy (under some set of social conditions).

This should follow on explanation about which social conditions and combinations of method, strategy, and synthesizer will be used.

### *4.1. Optimization*

This section should provide the results on “optimal” strategies for given methods or social conditions. This includes both simple known strategies and novel computational strategies (that we make). For each of these strategies, explain:

1. What general class of strategy it is
2. How it works conceptually
3. How it is implemented, and what are the hyper-parameters

For results, provide tables that give various metrics across the battery of methods and social conditions that it trained and ran on.

### *4.2. Method Analysis*

This section should use the known + novel strategies to test the manipulability of various methods. The results should be from a range of (synthesizer, social conditions) pairs, and measured across each manipulability metric. Use ridge plots for this.

For analysis, ask (1) Which methods generally performed well or poorly—and what does that say about the metrics themselves, which ones should we put credence to—and (2) What were the strengths and weaknesses of each of the methods.



## *5. Discussion*

### *5.1. Review*

In this section, we tie everything together, reviewing the work that has been done. What are the most important results? What have we build conceptually?

Having completed analyzing the results, take the time to reflect on how well the results have followed the guidelines laid out in the methods section. What is the efficacy of these methods and our approach to simulation and analysis.

### *5.2. Introspection & Conclusion*

This section is where the thesis winds down. It should step back and to consider the entire project as a whole. It should re-evaluate the thesis questions and the work that's been done. Have we succeeded in answering them?

Additionally, this section should swing back around to social choice to ask how this fits (or doesn't) with the existing literature. What have we gained by doing this analysis?

### *5.3. Future Research*

This section discusses potential future research that follows from this work. In general, this falls into 3 buckets:

1. What is missing from the analysis that further analysis can fill in?
2. What are some conclusions that open new paths for further exploration?
3. What might be redone / expanded on to help clarify or revise any findings?



## A. Symbols and Definitions

Symbol	Definition
$A, B, C, \dots$	Candidates
$a, b, c, \dots$	Voters
$\{a, b, c, \dots\}$	The set of all voters
$\dots$	The preference of a voter $x$
$\dots$	The ballot of a voter $x$
$\prec, \succ, \preceq, \succeq$	$A \prec B \Rightarrow A$ is preferred to $B$ , $A \preceq B \Rightarrow A$ is preferred or indifferent to $B$
$\mathcal{W}(x)$	Social welfare function for a voter $x$
$\mathcal{J}(h, \dots)$	A ballot generator (synthesizer) with some hyper-parameters $h$
$\mathcal{S}(x)$	The strategy for a voter $x$
$\mathcal{M}(\{a, b, c, \dots\})$	The outcome of a method, $M$ , on a set of ballots.



# References

- [1] K. J. Arrow, “A Difficulty in the Concept of Social Welfare,” *The Journal of political economy*, vol. 58, no. 4, pp. 328–346, 1950.
- [2] A. Gibbard, “Manipulation of Voting Schemes: A General Result,” *Econometrica*, vol. 41, no. 4, pp. 587–601, 1973.
- [3] R. D. ( . D. Luce, “Individual choice behavior; a theoretical analysis..” Wiley, 1959.
- [4] R. L. Plackett, “The Analysis of Permutations,” *Journal of the Royal Statistical Society. Series C (AppliedStatistics)*, vol. 24, no. 2, pp. 193–202, 1975.
- [5] J. G. Kemeny, “Mathematics without Numbers,” *Daedalus (Cambridge, Mass.)*, vol. 88, no. 4, pp. 577–591, 1959.
- [6] W. G. Ludwin, “Strategic Voting and the Borda Method,” *Public Choice*, vol. 33, no. 1, pp. 85–90, 1978.
- [7] C. L. Mallows, “Non-Null Ranking Models. I,” *Biometrika*, vol. 44, no. 1/2, p. 114–, 1957.
- [8] Hammond and Thomas H., “Rank Injustice?: How the Scoring Method for Cross-Country Running Competitions Violates Major Social Choice Principles,” *Public choice*, vol. 133, no. 3/4, pp. 359–375, 2007.
- [9] Niclas Boehmer *et al.*, “Guide to Numerical Experiments on Elections in Computational Social Choice,” 2024, [Online]. Available: <https://arxiv.org/abs/2402.11765>
- [10] C. List, “Social Choice Theory,” *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, 2022.
- [11] FairVote, “PR Library: Types of Voting Systems.”