Behavioral and Experimental Economics - Report II

Alessio Appruzzese

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Introduction

This report studies how people value lotteries under risk and ambiguity. In theory, Expected Utility Theory (EUT) says choices should depend only on outcomes and probabilities. If two lotteries have the same expected value, they should be valued the same. But in practice this often fails.

Ellsberg (1961) showed a clear pattern: people prefer known odds to unknown odds, even with the same expected value. This is ambiguity aversion. It breaks key parts of Savage's framework, like the Sure-Thing Principle.

Experimental work supports this idea. Maffioletti (1994) finds that when probabilities are unreliable or not transparent, subjects give lower valuations. Even with equal expected value, people react to how information is presented and who selects the bag. Framing and ambiguity both matter.

Our experiment uses different lotteries. Some have known, symmetric probabilities (pure risk). Others have uncertainty over the bag composition (ambiguity). Non-EU models, like MaxMin Expected Utility or Segal's Anticipated Expected Utility, predict a preference for the risky options when the set of possible probabilities is wide or includes extremes.

In the next sections I use these insights to read our data. I check how many subjects look EUT-consistent, whether Bayes' rule is applied across equivalent frames, and how ambiguity changes the stated prices.

1 Question 1

Consider three lotteries, F, L, and R. Each lottery involves drawing a ball from one of several opaque bags. The reward is £25 if the ball matches the color the participant has chosen.

- **Lottery F**: One bag contains 12 black balls, another contains 6 black and 6 white balls, and the third contains 12 white balls. The participant selects a color, then picks a bag, then draws a ball.
- **Lottery L**: The bags contain 4 black/8 white, 6 black/6 white, and 8 black/4 white balls, respectively. The participant selects a color, picks a bag, and draws a ball.
- **Lottery R**: Each of the three bags contains exactly 6 black and 6 white balls. The participant selects a color, picks a bag, and draws a ball.

A Non-EU maximizer does not base decisions solely on mathematical expectations but also considers the nature of uncertainty.

- **Ambiguity** (unknown probabilities) leads to discomfort or aversion.
- **Risk** (known probabilities) is typically preferred over ambiguous situations.

1. **Lottery R** is the most preferred because it involves pure risk: the probability of success is always known and equals $\frac{1}{2}$ regardless of the chosen color.
2. **Lottery L** is in the middle. It involves partial ambiguity since the bag compositions differ, but all bags have both black and white balls. The worst-case scenario for choosing a color results in a success probability of $\frac{4}{12} = \frac{1}{3}$, which is still better than zero.

3. **Lottery F** is the least preferred because it involves maximum ambiguity. For example, if one bets on black, it is possible to pick the all-white bag, resulting in a 0% chance of success. This worst-case possibility is highly aversive for an ambiguity-averse individual.

$$R > L > F$$

This ordering reflects a preference for situations where the probabilities are known (risk) over situations where the probabilities are uncertain (ambiguity). Non-EU maximizers may use decision rules such as MaxMin utility or Choquet Expected Utility, which take ambiguity aversion into account.

2 Question 2

We consider three lotteries: U, V, and B. Each involves drawing a ball from one of two opaque bags. One bag contains 12 black balls, the other 12 white balls. The participant chooses a color and earns £25 if the ball drawn matches the chosen color.

- **Lottery U**: The participant chooses the bag and draws a ball.
- **Lottery V**: The experimenter chooses the bag and the participant draws a ball.
- **Lottery B**: A fair coin decides which bag is selected, then the participant draws a ball.

A Non-EU maximizer does not only consider expected value but also the nature of the uncertainty.

- **Lottery B** involves pure risk. The bag is chosen at random with a fair coin, so the probability of success is exactly $\frac{1}{2}$. There is no ambiguity in the process.
- **Lottery U** involves ambiguity because the participant chooses the bag without knowing its contents. Some may consider this equivalent to random choice, but it introduces subjective uncertainty.
- **Lottery V** introduces the perception of **strategic uncertainty**. The experimenter chooses the bag, potentially raising concerns about adversarial selection, even if the process is unbiased. A Non-EU maximizer might evaluate this using a **MaxMin criterion**, considering the worst-case scenario.

$$B > U > V$$

This order reflects a preference for **known probabilities (risk)** over **unknown or potentially adversarial processes (ambiguity)**. Non-EU models such as MaxMin utility or ambiguity aversion predict this behavior.

3 Question 3

In this question we are asked to analyze and compare four lotteries: D, C, H, and A. They all have in common that we choose a color (black or white), a bag is selected at random among a set, and then we draw a ball. We win £25 if the ball matches the chosen color.

- **Lottery D**: 2 bags, both with 6 black and 6 white balls. So no ambiguity at all: in every bag, the probability of drawing the chosen color is exactly 0.5.
- **Lottery C**: 13 bags, going from 12 black balls to 12 white balls in steps of 1 (e.g., one bag has 11 black + 1 white, another 10B + 2W, and so on). This gives us all possible mixes from $p = 0$ to $p = 1$ in steps of 1/12.
- **Lottery H**: 11 bags, similar to C but with fewer intermediate levels (some combinations are missing, like 8B–4W or 4B–8W), so the set of possible probabilities is a bit less smooth.
- **Lottery A**: 5 bags only — one with 12 black, one with 9B–3W, one with 6B–6W, one with 3B–9W, and one with 12 white. So just 5 possible success probabilities: 0, 0.25, 0.5, 0.75, and 1.

A standard EU maximizer would treat all these lotteries as equivalent because the expected probability of winning is 0.5 in every case. But someone who is sensitive to ambiguity — a non-EU decision-maker — might not be indifferent.

For a non-EU agent, it's not just about the expected value but also about how uncertain the probability itself is. In this context, we are talking about *second-order uncertainty*, meaning the uncertainty over what the actual chance of winning is.

The more the possible values of p are spread out (especially if they include extremes like 0 or 1), the more ambiguity we have. So lotteries with more intermediate values and fewer extreme jumps will be perceived as *less ambiguous*. In our case:

- **D** has no ambiguity at all: every bag gives $p = 0.5$.
- **C** is the most balanced among the ambiguous ones: it includes all intermediate compositions between black and white, so even if we don't know which bag we are drawing from, the chances are well-distributed.
- **H** is similar to C but skips a few middle points, so it is slightly more ambiguous than C.
- **A** is the most "extreme": it only includes five bags and has values at both ends (0 and 1), making it more ambiguous.

We can imagine that a non-EU agent uses something like the **MaxMin Expected Utility** or the **Smooth Ambiguity** model (Klibanoff–Marinacci–Mukerji). In both cases, the agent prefers lotteries where the distribution of success probabilities p is tighter and avoids those with wide or extreme spreads. Therefore the lotteries are ranked as follows:

$$\boxed{D > C > H > A}$$

So even though the expected payoff is the same for all, D is clearly the safest, and C is the "best" among the ambiguous ones. H comes next, while A is the most ambiguous and would be ranked lowest.

This result fits well with Segal's two-stage model: first, the decision-maker forms expectations over the possible p values (given by the bag compositions), and then evaluates the expected utility. If they are ambiguity-averse, they prefer lotteries where p is more stable.

3.1 Decision criteria and decision tree

Beyond ambiguity aversion, we can think of **at least two decision criteria** a non-EU agent might follow to rank lotteries D, C, H, and A:

1. **Segal's Anticipated Expected Utility (AEU)**: this is a two-stage decision model. First, the agent considers the distribution of success probabilities p (which depends on which bag is drawn). Then, they compute the expected utility given p. Finally, they apply a second-level utility function ϕ, which captures ambiguity attitude (e.g. ambiguity aversion if ϕ is concave).

   In this framework, lotteries with more dispersed p (i.e. more ambiguity) are less preferred when ϕ is concave. This leads to the ranking:

   $$\boxed{D > C > H > A}$$

   since D has zero dispersion (no ambiguity), C is smooth, H is slightly less smooth, and A is coarsest and includes extreme values $p = 0$ and $p = 1$.

2. **Worst-case criterion (MaxMin EU)**: here, the agent only considers the worst possible outcome of p, regardless of how likely it is. In our setting:

   - Lottery D: worst-case $p = 0.5$
   - Lotteries A, H, C: all have $p = 0$ among possible outcomes

   So the agent ranks:

   $$\boxed{D > A \sim C \sim H}$$

   since only D guarantees a minimum of 50% chance of winning. This is a more pessimistic criterion.

All four lotteries follow the same general tree structure. The only difference is the set of possible p's.

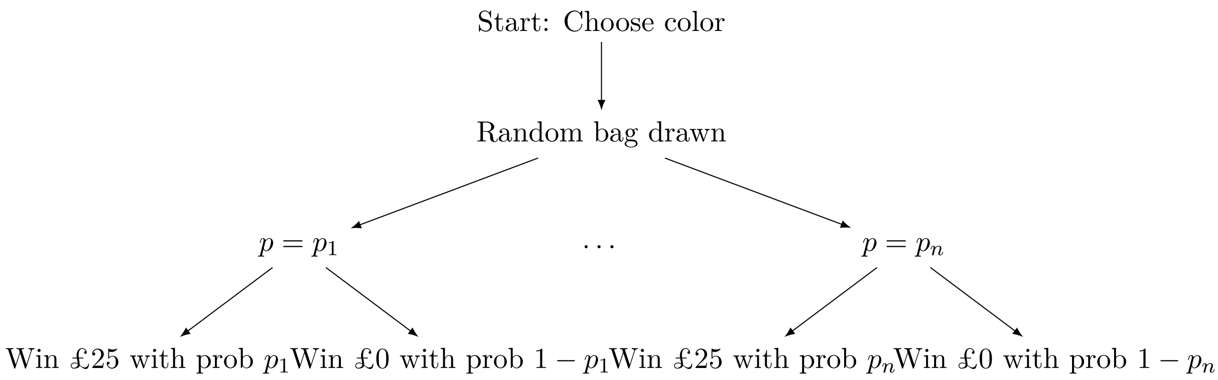

Figure 1: Generic decision tree for lotteries D, A, H, C.

For lottery D: $n = 2$ and all $p = 0.5$. For A: $n = 5$, with $p \in \{0, 0.25, 0.5, 0.75, 1\}$. For C: $n = 13$, with $p \in \{0, \frac{1}{12}, ..., 1\}$. For H: $n = 11$, skipping a few intermediate p values.

Prefer lotteries that avoid extreme outcomes like $p = 0$ or $p = 1$. These extreme cases create a feeling of unfairness or exposure. According to this simple intuition:

- Lottery D is safest (no extremes).
- Lottery C includes all cases but smooths out the extremes.
- H is less smooth than C, but still has many intermediate values.
- A includes the extremes and few middle points.

So the ranking again becomes:

$$\boxed{D > C > H > A}$$

This ordering holds across most models where ambiguity is penalized. But if someone were ambiguity-seeking (they like "pure chances"), they might reverse the ranking among the ambiguous lotteries.

4 Question 4

In order to evaluate how many participants in our experimental dataset behave consistently with *Expected Utility Theory* (EUT), we analyzed the stated minimum selling prices across a series of ambiguous lotteries. Under EUT, each lottery should be evaluated by computing its expected value (EV), and this should guide the certainty equivalent (CE) that a rational decision-maker is willing to accept. Given the binary structure of the payoffs — typically £25 if the correct color is drawn, £0 otherwise — the expected value for a fair 50/50 lottery is £12.5.

To operationalize this, we classify participants as approximately EUT-consistent if their **average stated CE** falls within the interval £11.0–£14.0. This buffer accounts for minor inconsistencies due to noise or imprecision. Based on this criterion:

- 6 out of 17 participants (approximately 35%) appear to follow Expected Utility Theory;
- 10 participants (59%) reported prices systematically *below* the EV, suggesting **risk aversion**;
- 1 participant (6%) reported prices *above* the EV, indicating **risk seeking** preferences.

In addition, we examined whether participants applied **Bayes' rule** — that is, whether they reduced compound lotteries (lotteries involving multiple stages of uncertainty) to a single equivalent probability. According to the EUT + Bayes framework, different framings of the same probabilistic structure (e.g., random draw by subject vs. random draw by experimenter) should yield the same valuation.

However, several participants gave significantly different valuations to formally equivalent lotteries, violating Bayesian reduction. This suggests the presence of *non-EU behavior*, possibly due to ambiguity aversion, second-order uncertainty, or framing effects.

While a non-negligible fraction of the sample exhibits behavior consistent with Expected Utility maximization, the majority deviates from this framework. Most participants appear risk averse, and a significant share fails to apply Bayes' rule. These findings support the use of behavioral models to better capture real-world decision-making under uncertainty.

4.1 Risk Attitudes and Application of Bayes' Rule

To assess the risk attitudes of participants, we compared their average minimum selling prices for the lotteries to the expected value of a fair binary gamble. Since most lotteries involved a £25 gain with a probability around 0.5, a risk-neutral agent should assign a certainty equivalent (CE) of approximately £12.5.

Based on their average prices:

- **10 out of 17 participants** (59%) reported average CEs significantly *below* £12.5. This indicates **risk aversion**, as they undervalue the lotteries compared to their expected value.
- **6 participants** (35%) stated average prices close to £12.5, which is compatible with **risk-neutral** preferences.
- Only **1 participant** (6%) showed signs of **risk-seeking** behavior, reporting average prices *above* the expected value.

In terms of Bayesian reasoning, we evaluated whether participants treated lotteries with formally equivalent probabilities in a consistent way. According to *Bayes' rule*, different descriptions of the same probabilistic event (e.g., choosing a bag yourself vs. the experimenter choosing it) should be valued equally by a rational agent. This is because the objective probability of winning remains unchanged under EUT with Bayesian updating.

However, in our dataset, many participants assigned noticeably different values to such equivalent lotteries. This suggests a widespread violation of **Bayesian reduction**, likely due to behavioral factors such as *ambiguity aversion*, *framing effects*, or *source preference*. For instance, some participants devalued lotteries where they did not have control over the bag selection, despite the statistical equivalence.

In conclusion, most participants appear to be risk-averse, and a significant portion do not apply Bayes' rule consistently. These findings highlight the limits of classical expected utility models in explaining real-world behavior and reinforce the relevance of behavioral theories in decision-making under uncertainty.

5 Question 5:

A rational Bayesian evaluates lotteries by applying **Bayes' rule** to reduce compound uncertainty to a single-stage probability. Therefore, lotteries that allow this reduction clearly and without ambiguity are those where Bayesian updating is most straightforward and thus most informative.

In the context of the experiment, the following lotteries are the most relevant from a Bayesian perspective:

- **Lottery R**: All three bags have the same composition (6 black, 6 white). Regardless of which bag is chosen, the probability of drawing the chosen color is objectively 0.5. A Bayesian should evaluate this as a classic 50/50 gamble, and any deviation in valuation would suggest a violation of reduction of compound lotteries.
- **Lottery B, V, U**: These three lotteries involve identical physical compositions (two bags: one all black, one all white), but differ only in *who chooses the bag*

(self, experimenter, or random device). A rational Bayesian should evaluate all three identically, since in each case the probability of winning is exactly 0.5. Any discrepancy in valuation would indicate behavioral effects such as *source preference* or *ambiguity aversion*.

- **Lottery L**: The bags have slightly different probabilities (e.g., 4/12, 6/12, 8/12 black balls), but the structure is symmetric and the expected probability is still 0.5. A Bayesian would average over these compositions to compute the expected value, making this a good test of second-order belief integration.
- **Lottery C and H**: These involve a rich set of bags with varying probabilities, and require the agent to integrate over many possible probability distributions. This setting is especially informative because it allows us to observe whether participants apply probabilistic averaging (as a Bayesian would), or are affected by ambiguity and complexity.

These lotteries isolate the Bayesian component of decision-making. Since they share the same expected winning probability (or at least, a computable one), any difference in participants' evaluations cannot be attributed to differences in objective risk, but must instead reflect subjective beliefs, cognitive limitations, or attitudes toward ambiguity.

Thus, by comparing how participants evaluate these structurally equivalent lotteries, we can test whether they are truly acting as rational Bayesians — consistently applying the rule of probabilistic reduction — or whether their valuations are driven by non-EU factors such as ambiguity aversion, lack of trust in randomness, or framing effects.

Conclusion: Lotteries like R, B, U, V, and C provide the cleanest tests of Bayesian rationality because they allow clear expectations to be computed. They should therefore be prioritized in evaluating whether individuals follow Expected Utility + Bayesian updating or diverge toward behavioral alternatives.

6 Question 6:

While participating in the experiment, my choices were primarily motivated by the principle of **expected value maximization**. For each lottery, I attempted to compute the objective probability of winning and multiply it by the monetary reward (£25), in order to state a selling price that reflected the lottery's expected value. In cases where the structure involved multiple bags or stages of uncertainty, I applied **Bayes' rule** to reduce the compound uncertainty into a single probability. When the ambiguity became excessive — such as in lotteries with many unknown distributions — I occasionally adopted simplified heuristics (e.g., assuming uniform randomness across bags).

Moreover, I tried to remain internally consistent across lotteries that shared the same underlying probabilities, regardless of the way in which the uncertainty was presented (self-choice vs. experimenter-choice, for example). This reflects an effort to adhere to the Expected Utility framework with Bayesian updating.

Comparison with other participants:

By examining the open-ended responses in the dataset, it appears that many participants were influenced by *qualitative or emotional heuristics*, rather than strictly probabilistic reasoning. Several patterns emerged:

- Some participants explicitly mentioned being **overwhelmed** by complex lotteries with many bags, leading them to assign arbitrary or rounded values.
- Others reported treating all lotteries as roughly 50/50 gambles, regardless of their structure, which indicates reliance on **simplification heuristics** rather than probabilistic reasoning.
- A few participants assigned lower values to lotteries with more "random steps" or experimenter involvement, indicating **ambiguity aversion** or a discomfort with indirect control over the outcome.

Overall, while my approach was grounded in formal economic theory, the majority of participants appeared to rely on intuitive, heuristic-based strategies. This highlights the gap between normative (rational) models and actual decision-making behavior, consistent with findings in behavioral economics.

My decision process aimed to be fully consistent with Bayesian rationality and expected utility, but the diversity of motivations observed among participants reflects the complexity of human reasoning under uncertainty — ranging from deliberate computation to affective simplification.

References

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