: Unit : II :

Difference b/w classical & Quantum Mechar

Classical	Quantum
① Classical mechanics is a macroscopic physics	① Quantum mechanics is a microscopic physics
② Classical mechanics is based on newton's law	② Quantum mechanics is based on Plank's theory of radi
③ Classical mechanics is a definite physics.	③ Quantum mechanics is a probable physics.
④ Classical mechanics shows the wave nature of light.	④ Quantum mechanics shows wave nature as well as particle nature.
⑤ Classical mechanics is able to explain interference, diffraction, polarization etc.	⑤ Quantum mechanics is able to explain photo electric effect, crompton effect, Black body radiation etc.

Inadequency or failure of classical Mechanics :

⇒ Classical mechanics could not explain the stability of atom.

⇒ It could not explain black body radiation spectrum

⇒ It could not explain compton effect, photoelectric effect

⇒ It could not explain the discrete energy levels of atom.

Black body ~~radiation~~: A black body is one which completely absorbs the all incident radiation.

Eg :

Black body Radiation : The radiation emitted by the black body when it is placed in a constant high temperature known as black body radiation.

Plank's Theory of Quantum Mechanics

In 1900, Plank introduce the revolutionary theory of radiation known as quantum theory of radiation.

He was made the following assumptions :

(1) A black body consist of large number of small oscillating particles.

(2) The radiation emitted by the black body is in the form of discrete energy packets called Quanta / Photons

(3) The energy of one photon $(E) = h\nu = \frac{hc}{\lambda}$

(4) The black body either absorb or emit radiation which is integral multiple of '$h\nu$'.

i.e. $E = nh\nu$ $(n = 1, 2, 3, \ldots)$

Wave Particle Duality :

In some phenomena, like an interference, diffraction, polarization light behaves like a wave and in some other phenomena like in photoelectric effect, crompton effect, Black body radiation light behave like a particle. It means light has a dual nature which is called wave particle duality.

De-Broglie Hypothesis :-

"A moving particle is always associated with a wave" known as Matterwave. This is known as De-Broglie Hypothesis.

And, $\lambda = \frac{h}{mv} = \frac{h}{p}$ $\rightsquigarrow$ momentum of particle

~~According to de-Broglie~~

Other forms of De-Broglie wavelengths

(i) assume K.E. (kinetic energy of particle) = K

$$\therefore \lambda = \frac{h}{mv} = \frac{h}{\sqrt{m^2v^2}} = \frac{h}{\sqrt{\frac{1}{2} \times 2m^2v^2}}$$

$$\lambda = \frac{h}{\sqrt{2mK}}$$

(ii) If a particle is a charged particle of charge q . and accelerated by a potential difference V (volt).

$$\therefore K = qV$$

as $\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$

Ques :- find the De-Broglie wavelength of an electrons accelerated by a potential difference V volt.

for e^-, $\lambda_e = \sqrt{\frac{150}{V}}\ \text{Å} = \frac{12.26}{\sqrt{V}}\ \text{Å}$

(iii) If a particle is a gaseous molecule in thermal equilibrium,

K = Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K

$$K = \frac{3}{2} KT$$

Energy

↳ Boltzmann's constant.

$$\text{as } \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{3mKT}}$$

Boltzmann's constant

(iv) If a particle having rest mass m_0 moving with velocity 'v' which is comparable to 'c' then its relativistic mass is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\therefore \lambda = \frac{h}{mv} = \frac{h}{m_0 v}\sqrt{1 - \frac{v^2}{c^2}}$$

Properties of Matter wave :-

The properties of waves are :-

① The De-Broglie wavelength associated with lighter particle is greater than the heavier particle.

② The De-Broglie wavelength associated with slow moving particle is greater than fast moving particle.

③ If $v=0$, $\lambda=\infty$
& if $v=\infty$, $\lambda=0$ } i.e. particle should moving with finite velocity

④ Matter wave is associated with charged particle as well as neutral particles.

⑤ The velocity of matter wave is not constant like electro-magnetic wave.

To prove: $v_p = \dfrac{c^2}{v}$

v_p = matter wave velocity

c = speed of light

v = particle velocity

The energy of a photon $E = h\nu = \frac{hc}{\lambda}$

$$\therefore \nu = \frac{E}{h}$$

$$\therefore \nu = \frac{mc^2}{h}$$ [by Einstein energy mass relation $E = mc^2$]

if v_p is the wave velocity,

then $v_p = \nu\lambda = \frac{mc^2}{h} \times \frac{h}{mv}$

$$\therefore v_p = \frac{c^2}{v}$$

Since, $v < c$

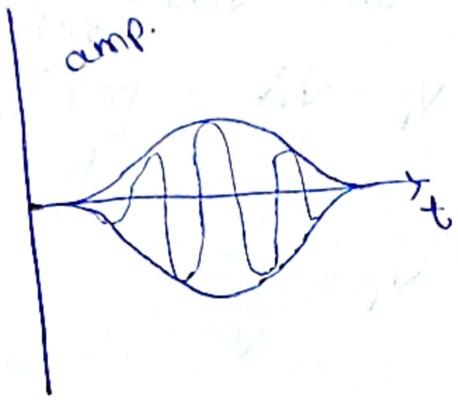
$\therefore v_p = \frac{c^2}{v} > c$ (which is not possible)

Since v is less than c, therefore $v_p = \frac{c^2}{v}$ which is always greater than c which is not possible,

Hence, it shows ~~matter~~ moving particle/wave is not associated with single wave, i.e. moving particle is associated with wave group or wave packet.

Wave Group or Wave packet :

The superposition of number of waves ~~associate~~ of slightly different frequencies and wavelength associated with moving particle known as wave group or wave packet.



⊛ Particle certainly line the wave packet

Wave Velocity or Phase velocity (V_p)

when a monochromatic wave travels in a medium then the velocity of that wave is known as wave velocity or Phase velocity (V_p)

where $\boxed{V_p = \frac{\omega}{K}}$

→ $\omega \rightarrow$ angular frequency $= 2\pi\nu$

→ $K \rightarrow$ propagation constant $= \frac{2\pi}{\lambda}$

as $\omega = 2\pi\nu$, $K = \frac{2\pi}{\lambda}$

$$V_p = \frac{\omega}{K} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda$$

Group Velocity (V_g):-

The velocity of the wave group associated with moving particle is known as group velocity.

$$\boxed{V_g = \frac{\partial \omega}{\partial k} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\text{difference in } \omega}{\text{difference in } k}}$$

Relation b/w Group velocity (V_g) & wave velocity (V_p)

$$V_p = \frac{\omega}{k}$$

$$\omega = V_p k$$

$$d\omega = V_p dk + k dV_p$$

$$\frac{d\omega}{dk} = V_p + k\frac{dV_p}{dk}$$

$$V_g = V_p + k\frac{dV_p}{d\lambda}\cdot\frac{d\lambda}{dk} \quad \text{--- (1)}$$

as $\lambda = \frac{2\pi}{k}$

$$\therefore \frac{d\lambda}{dk} = \frac{-2\pi}{k^2}$$

put in (1)

$$V_g = V_p + k\frac{dV_p}{d\lambda}\left(\frac{-2\pi}{k^2}\right) = V_p - \frac{2\pi}{k}\frac{dV_p}{d\lambda}$$

$$\boxed{V_g = V_p - \lambda \frac{dV_p}{d\lambda}}$$

$\Downarrow$

$\therefore (V_g < V_p \text{ always})$

$\Rightarrow$ for Non-dispersive Medium,

$$\left. \frac{dV_p}{d\lambda} = 0 \qquad \therefore V_g = V_p \right\}$$

To prove $V_g = V$

Consider a particle having rest mass 'm_0' moving with velocity v which is comparable to 'c'

than $\Rightarrow$ $m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ —①

if ω is the angular velocity,

$$\omega = 2\pi\nu = \frac{2\pi E}{h}$$

$$\omega = \frac{2\pi mc^2}{h} \rightarrow \omega = \frac{2\pi m_0 c^2}{h\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h} \frac{d}{dv}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h}\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2} \quad \text{——— (2)}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h}$$

$$k = \frac{2\pi m_0 v}{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}$$

$$\frac{dk}{dv} = \frac{d}{dv}\left[\frac{2\pi m_0 v}{h(1-v^2/c^2)^{1/2}}\right]$$

$$\frac{dk}{dv} = \frac{2m_0\pi}{h}\frac{d}{dv}\left[\frac{v}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right]$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} + v\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)\right]$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}} \quad \text{——— (3)}$$

then $v_g = \dfrac{d\omega}{dk} = \dfrac{d\omega/dv}{dk/dv} = \dfrac{\frac{2\pi m_0 v}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}}{\frac{2\pi m_0}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}}$

$\therefore \boxed{v_g = v} \rightarrow$ which shows that group velocity is equal to particle velocity.

100%

Wave function (ψ) and its physical significance

$\Downarrow$

or Probability Amplitude

$\Downarrow$

it is complex Quantity

$$\psi = A + iB$$

$\downarrow$ complex conjugate of ψ

$$\psi^* = A - iB$$

$$|\psi|^2 = \psi\psi^*$$

$$|\psi|^2 = (A+iB)(A-iB)$$

$$|\psi|^2 = A^2 + B^2 \Rightarrow \text{real}$$

$\downarrow$

name $\Rightarrow$ Probability density $\Rightarrow$ Probability of finding a particle in wave packet at a particular time, at a particular point.

i.e. $$\iiint_{-\infty}^{\infty} |\psi|^2 \, dx\,dy\,dz = 1$$

$\Rightarrow$ The quantity whose variation with matter wave known as wave function (ψ), it is also probability amplitude of matter wave.

it is a complex quantity and related to $|\psi|^2$ known as probability density.

$|\psi|^2 = \psi\psi^*$ which is a real quantity.

according to max. Born, $|\psi|^2$ gives the probability of finding the particle in a wave packet at a particular time and at a particle point.

Since, particle certainly line the wave packet.

$$\boxed{\iiint_{-\infty}^{\infty} |\psi|^2 \, dx\,dy\,dz = 1}$$, this condition is known as normalization condition.

Conditions for Acceptable wave function :-

(a) ψ must be finite.

(b) ψ must be single valued.

(c) ψ must be continuous.

Schrodinger's wave Equation :-

1. Schrodinger's time independent wave equation.
2. Schrodinger's time dependent wave equation.

→ To study the wave nature of moving particle, Schrodinger derived an equation known as Schrodinger's wave equation.

It is of two type written previously.

① Schrodinger's Time independent wave equation:

$\vec{\nabla}$: Del operator (3-dimensional differential operator)

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla}\cdot\vec{\nabla} = (\nabla)^2 = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

general wave equation,

$$\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

The differential wave equation for matter is:

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \quad \text{— ①}$$

solution of equation ① is

$$\psi = \psi_0 e^{-i\omega t} \quad \text{— ②}$$

differentiating eq (2) with respect to 't' ;

$$\frac{\partial \psi}{\partial t} = \psi_0 \frac{\partial}{\partial t} e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = -\psi_0 \, i\omega \, e^{-i\omega t}$$

differentiating again wrt 't' ,

$$\frac{\partial^2 \psi}{\partial t^2} = \psi_0 (-i\omega)^2 e^{-i\omega t} = (-i\omega)^2 \underline{\psi_0 e^{-i\omega t}}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \quad \text{—— (3)}$$

put here

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\boxed{\nabla^2 \psi = \frac{1}{v^2} (-\omega^2 \psi)}$$

$$(\nabla^2) \psi = \frac{1}{v^2} (-4\pi^2 \nu^2) \psi$$

$$= \frac{1}{v^2} \left(-4\pi^2 \frac{v^2}{\lambda^2} \psi \right)$$

$$\nabla^2 \psi = \frac{-4\pi^2}{\lambda^2} \psi$$

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0$$

$$\nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \quad \text{—— (4)}$$

Ques: (i) (ii) $\psi = e^{-ikx}$

Normalize both equations.

if E = Total energy

V = Potential energy

$\therefore$ kinetic energy $= E - V$

$\frac{1}{2}mv^2 = (E-V)$

$m^2v^2 = 2m(E-V)$

Putting on equation (4), we get

$$\nabla^2\psi + \frac{4\pi^2}{h} \times \frac{2m(E-V)}{(\hbar)^2}\psi = 0$$

$$\therefore \quad \boxed{\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0}$$ Standard form of Shrodinger's wave equation.

for free particle, $V = 0$

$$\therefore \quad \nabla^2\psi + \frac{8\pi^2 m}{h^2}E\psi = 0$$

define $\hbar = \frac{h}{2\pi}$

$h = \hbar(2\pi)$

$h^2 = \hbar^2(4\pi^2)$

Now, $$\boxed{\nabla^2\psi + \frac{8\pi^2 m(E-V)}{\hbar^2(4\pi^2)} = 0}$$

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

Schrodinger's time dependent wave Eqⁿ :

The differential wave equation for matter is :

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{—— (1)}$$

Solution of equation (1) is :

$$\psi = \psi_0 e^{-i\omega t} \quad \text{—— (2)}$$

partial differentiating (2) w.r.t. 't',

$$\frac{\partial\psi}{\partial t} = \psi_0(-i\omega)e^{-i\omega t}$$

$$\frac{\partial\psi}{\partial t} = -i\omega\psi$$

$$\frac{\partial\psi}{\partial t} = -i(2\pi\nu)\psi$$

$$\frac{\partial\psi}{\partial t} = -i\left(\frac{2\pi E}{h}\right)\psi$$

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi$$

$$E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}$$

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \quad \text{—— (3)}$$

we know that Schrodinger's wave equation is

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\nabla^2 \psi = -\frac{2m}{\hbar^2}(E\psi - V\psi) = 0$$

$$-\frac{\hbar^2}{2m}\nabla^2 \psi = E\psi - V\psi$$

$$V\psi - \frac{\hbar^2}{2m}\nabla^2 \psi = E\psi$$

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2 \psi + V\psi = i\hbar\frac{\partial \psi}{\partial t}} \rightarrow \text{time dependent of Schrodinger's wave equation.}$$

Heisenberg's Uncertainity Principle :

According to Heisenberg Uncertainity Principle,

It is impossible to measure, the exact position and momentum of a microscopic particle simultaneously.

acc. to Heisenberg,

$$\Delta x \cdot \Delta P \geq h/4\pi$$

$$\Delta E \cdot \Delta t \geq h/4\pi$$

$$\Delta J \cdot \Delta \theta \geq h/4\pi$$

$\Delta x \rightarrow$ uncertainity in position

$\Delta P \rightarrow$ uncertainity in momentum

$\Delta E \rightarrow$ uncertainity in energy

$\Delta t \rightarrow$ uncertainity in time

$\Delta J \rightarrow$ uncertainity in angular momentum

$\Delta \theta \rightarrow$ uncertainity in angle

Ques: If the uncertainity in position of e^- is 4×10^{-10} m, calculate uncertainity in momentum.

Sol

$$\Delta x \cdot \Delta p = h/4\pi$$

$$\Delta p = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 4 \times 10^{-10}} = 1.31 \times 10^{-25} \text{ kg m/sec}$$

Ques: The lifetime of an excited state of nucleus is 10^{-12} sec. what is the uncertainity in energy.

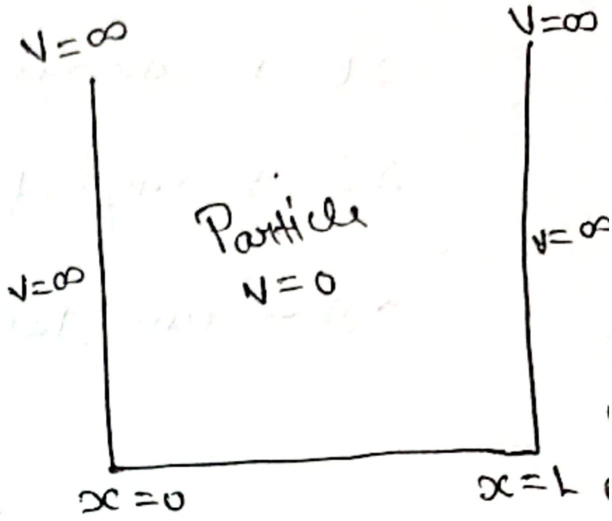
Sol

$$\Delta E = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 10^{-12}} = 5.25 \times 10^{-23} \text{ J}$$

Particle in 1 Dimensional Infinite Potential box :

$V = 0$ (Inside the box $x > 0$ & $x < L$)

$V = \infty$ at walls & outside the box (i.e. $x \leq 0$ & $x \geq L$)



Consider a 1 dimensional infinite potential box of width 'L'. Suppose particle is inside the one dimensional box and it is free to move in one dimensional box, then the expression for potential energy for a particle is given by :

$V = 0$ (Inside the box) (i.e $x > 0$ & $x < L$)

and $V = \infty$ at walls and outside the box (i.e. $x \leq 0$ & $x \geq L$)

The Schrodinger's wave for the given system can be written as,

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\,\psi = 0$$

Inside the box, $V=0$

$$\therefore \nabla^2\psi + \frac{8\pi^2 m}{h^2}E\psi = 0$$

$$\nabla^2\psi + \frac{8\pi^2 mE}{h^2}\psi = 0$$

$$\nabla^2\psi + K^2\psi = 0 \quad \text{——} (2)$$

$$K^2 = \frac{8\pi^2 m}{h^2}E \quad \text{——} (3)$$

⇒ The solution of eqn (2) can be written as

$$\boxed{\psi = A\sin Kx + B\cos Kx} \quad \text{——} (4)$$

where A and B are the constants and can be determined by the boundary conditions

* Applying first boundary condition:

when $x=0$, $\psi=0$

as $\psi = A\sin Kx + B\cos Kx$

$$0 = A\sin 0 + B\cos 0$$

$$B = 0$$

$$\therefore \boxed{\psi = A\sin Kx} \quad \text{——} (5)$$

* Applying Second boundary condition,

when $x = L$, $\psi = 0$

as $\psi = A \sin Kx$

$0 = A \sin(Kx)$

$\therefore \sin(Kx) = 0$ (as B already 0, hence, A can't be zero.)

$\therefore Kx = n\pi$

$$\boxed{K = \frac{n\pi}{L}} \quad \text{——— (6)}$$

squaring on both side

$K = \frac{n\pi}{L}$

$$K^2 = \frac{n^2\pi^2}{L^2}$$

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{L^2}$$

$$\boxed{E = \frac{n^2h^2}{8mL^2}}$$

⇒ for ground state, $n = 1$

$$E_1 = \frac{n^2h^2}{8mL^2} = \frac{h^2}{8mL^2}$$

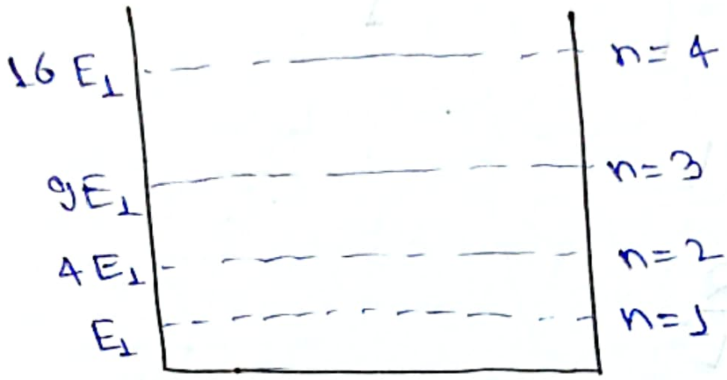
$$E_1 = \frac{h^2}{8mL^2}$$

→ for first excited state (i.e. $n=2$)

$$E_2 = \frac{4h^2}{8mL^2} = 4E_1$$

→ for second excited state (i.e. $n=3$)

$$E_3 = \frac{9h^2}{8mL^2} = 9E_1$$



Constant A can be determined with the help of normalization condition :

$$\psi = A\sin(kx)$$

$$\psi = A\sin\left(\frac{n\pi x}{L}\right)$$

$$\therefore \psi^* = A\sin\left(\frac{n\pi x}{L}\right)$$

$$|\psi|^2 = \psi\psi^* = A^2\sin^2\left(\frac{n\pi x}{L}\right)$$

* Applying Second boundary condition,

when $x = L$, $\psi = 0$

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$\therefore kx = n\pi$

$$\boxed{k = \frac{n\pi}{L}} \quad \text{—— (6)}$$

squaring on both side

$k = \frac{n\pi}{L}$

$k^2 = \frac{n^2\pi^2}{L^2}$

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{L^2}$$

$$\boxed{E = \frac{n^2h^2}{8mL^2}}$$

⇒ for ground state, $n = 1$

$E_1 = \frac{n^2h^2}{8mL^2} = \frac{h^2}{8mL^2}$

$E_1 = \frac{h^3}{8mL^2}$

→ for first excited state. (i.e. $n=2$)

$$E_2 = \frac{4h^2}{8mL^2} = 4E_1$$

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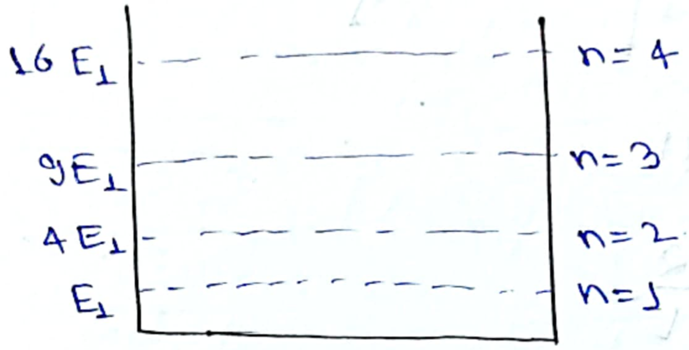

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$$\psi = A\sin\left(\frac{n\pi x}{L}\right)$$

$$\therefore \psi^* = A\sin\left(\frac{n\pi x}{L}\right)$$

$$|\psi|^2 = \psi\psi^* = A^2\sin^2\left(\frac{n\pi x}{L}\right)$$

$$|\psi|^2 = \frac{A^2}{2}\left[1 - \cos\left(\frac{2n\pi x}{L}\right)\right]$$

$$\int_0^L |\psi|^2 dx = \frac{A^2}{2}\left[x - \frac{\sin\left(\frac{2n\pi x}{L}\right)}{\frac{2n\pi}{L}}\right]_0^L$$

$$1 = \frac{A^2}{2}\left[L - 0\right]$$

$$\Rightarrow \quad 1 = \frac{A^2}{2} \times L$$

$$\Rightarrow \quad A^2 = \frac{2}{L}$$

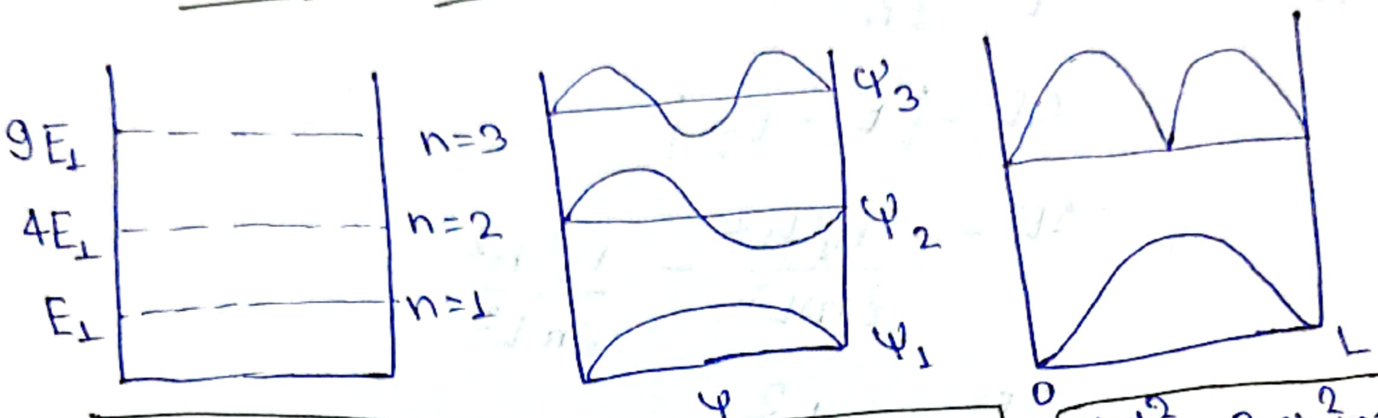
$$\Rightarrow \quad A = \sqrt{\frac{2}{L}}$$

$\therefore$ as $\psi = A \sin kx$

$$\therefore \quad \boxed{\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}$$

$$\boxed{|\psi|^2 = \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right)}$$

The Energy (E), wave function (ψ) and probability density $|\psi|^2$ of a particle in one - dimensional infinite potential boxes represented as :-



ψ

$$\boxed{E = \frac{n^2 h^2}{8mL^2}}$$

$$\boxed{\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}$$

$$\boxed{|\psi|^2 = \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right)}$$

⇒ when the particle is in least energy state (i.e. $n=1$), than the probability of finding the particle is maximum at $\frac{L}{2}$.

and minimum at 0 and L.

⇒ and when the particle is in first excited state (i.e. $n=2$) than the probability of finding the particle is maximum at $\frac{L}{4}$ and $\frac{3L}{4}$. and minimum at $\frac{L}{2}$.

Ques: Calculate the energy difference between the ground state and first excited state for an electron in one-dimensional rigid box of length 1Å.

Sol:

$$L = 1\text{Å}$$

$$\Delta E = |E_1 - E_2|$$

$$\Delta E = \frac{n_1^2 h^2}{8mL^2} - \frac{n_2^2 h^2}{8mL^2}$$

$$\Delta E = \frac{h^2}{8mL^2}[1-4]$$

$$\Delta E = \frac{3h^2}{8mL^2} = \frac{3 \times (6.6)^2 \times (10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 10^{-20}} J$$

$$\Delta E = \frac{3 \times (6.6)^2 \times 10^{-68+51}}{9.1 \times 8} J$$

$$\Delta E = \frac{3 \times (6.6)^2 \times 10^{-17}}{9.1 \times 8} J$$

$$\Delta E = \frac{3 \times (6.6)^2 \times 10^{-17}}{9.1 \times 8 \times (1.6 \times 10^{-19})} eV$$

$$\Delta E = 112.1 \ eV$$