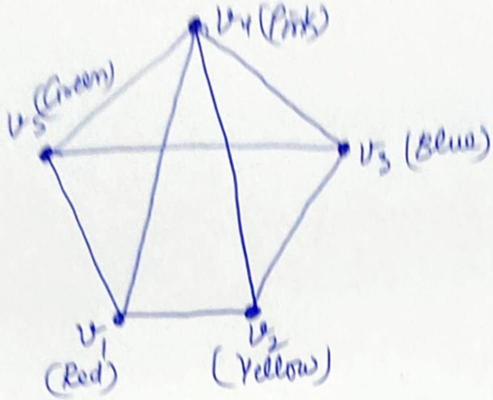
The chromatic number of some of the familiar graphs are easily determined.

(i) $\chi(K_n)=n$, where K_n is the complete graph of n vertices

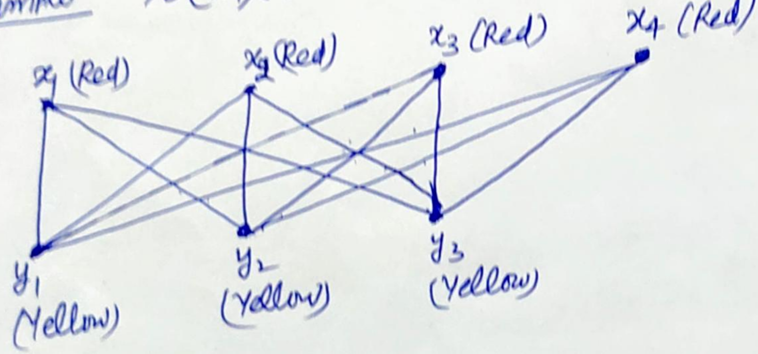
Since any two vertices are adjacent in a complete graph, no two vertices can receive the same color. It is clear that it can not be colored with fewer than n colors. For example: $\chi(K_5)=5$



(ii) $\chi(K_{m,n})=2$ i.e. the chromatic number of every bipartite graph is 2.

The vertex set can be partitioned into two subsets V_1 and V_2 such that no two vertices in V_1 (or V_2) are adjacent. One can give one color to all the vertices in one portion and another color to all vertices in the other partition.

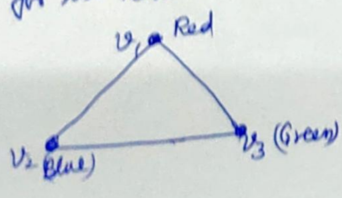
For example: $\chi(K_{4,3})=2$

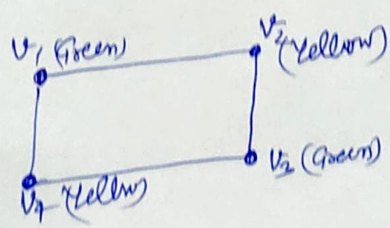


(iii) $\chi(C_n)=2$, if n is even

and $\chi(C_n)=3$, if n is odd

If n is even, we can alternate colors around the cycle; if n is odd we need a third color for the last vertex colored.





§. Homeomorphic Graphs

Definition: Two Graphs G_1 and G_2 are said to be homeomorphic if one graph can be obtained from the other by the creation of edges in series (in other words, by insertion of vertices of degree two) or by merging (mergers) of edges in series.

The following three graphs are homeomorphic.