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subject : Operation Research.

Jun - 2018

① Write an algorithm to solve LPP using Vogels approximation method:

This method prefered over the North western Corner Method & Least cost method because the initial basic feasible solution obtained by method is either optional sol^n or near to ptimal solution.

Algorithm:

// step1: Find the cells having smallest cost in each column & write difference along the side of table in each column.

Step1: Find the cell having smallest and next smallest cost in each row and the write the difference along the side of the table row penalty.

Step 2: Find the cells having smallest cost in each column & write the difference along the side of table in each column.

Step 3: Select the row or column with the Maximum penalty & find the cell that has atleast cost in selected row & column allocate as much as possible in this cost.

If there is a tie in the values of penalties then select the cell where maximum allocation can be possible.

Step 4: Adjust the supply & demand & cross out the satisfied row or column //

(2)

June - 2018

(2) Determine an initial Basic feasible solⁿ to following transporation problem. Using North - West corner rule.

	A	B	C	D	Available
I	6	3	5	4	22
II	5	9	2	7	15
III	5	7	8	6	8
Demand	7	12	17	9	

Solution:-

Given

Given

	A	B	C	D	Available
I	6	3	5	4	22
II	5	9	2	7	15
III	5	7	8	6	8
Demand	7	12	17	9	45/45

Total Number of supply constraints = 3
Total Number of demand " = 4

Here Total supply = Total Demand
∴ The problem is balanced transportation problem.

Step 1: The minimum value I = 22 & A = 7 compared. The smaller of min (22, 7) = 7 is assigned AI. The meets complete Demand & leaves 22 − 7 = 15 in I.

Table 1.

	A	B	C	D	Available
I	6	3	5	4	15
II	5	9	2	7	15
III	5	7	8	6	8
Demand	0	12	17	9	

Step 2: The minimum value for I = 15 and B = 12 are compared. The min (15, 12) is assigned to BI & leaves 15 − 12 = 3 in I.

Table 2:

	A	B	C	D	Available
I	6(7)	3(2)	5	4	3
II	5	9	2	7	15
III	5	7	8	6	8
demand	0	0	17	9	

Step 3: The minimum value for I = 3 and C = 17 The min (3, 17) = 3 is assigned to CI & leaves 17-3 = 14 is in C

Table 3

	A	B	C	D	Available
~~I~~	~~6(7)~~	~~3(2)~~	~~5(3)~~	~~4~~	0
II	5	9	2	7	15
III	5	7	8	6	8
Demand	0	0	14	9	

Step 4 The minimum of II - 15 C = 14 are compared.
The min (15, 14) = 14 is assigned to IIC & leaves 15 - 14 = 1 in II

Table 4:

	A	B	C	D	Available
~~I~~	~~6(7)~~	~~3(2)~~	~~5(3)~~	~~4~~	0
II	5	9	2(14)	~~7~~	1
III	5	7	8	6	8
Demand	0	0	0	9	

Step 5: The minimum of II - 1 & D = 9 are compared.

The minimum (1, 9) = 1 assigned to IID & leaves 9 − 1 = 8 in D.

Table 5

	A	B	C	D	Availabale
~~I~~	~~6(7)~~	~~3(2)~~	~~5(3)~~	~~4~~	0
~~II~~	~~5~~	~~9~~	~~2~~	~~7(1)~~	0
III	5	7	8	6	8
demand	$\emptyset$	$\emptyset$	$\emptyset$	8	

step 6: The min of III = 8 & D = 8 are compared.

· The min (8, 8) = 8 assigned to IIID & leaves 8 − 8 = 0 in D & III

Table 6:

	A	B	C	D	Available
~~I~~	~~6(7)~~	~~3(2)~~	~~5(3)~~	~~4~~	0
~~II~~	~~5~~	~~9~~	~~2~~	~~7(1)~~	0
~~III~~	~~7~~	~~7~~	~~8~~	~~6(8)~~	0
Demand	0	0	0	0	

Final Table values:

	A	B	C	D	Available
I	6(7)	3(2)	5(3)	4	0
II	5	9	2	7(1)	0
III	5	7	8	6(8)	0
Demand	0	0	0	0	

Total minimum cost = $6\times7 + 3\times2 + 5\times3 + 7\times1$
6×8

$= 42 + 6 + 15 + 7 + 48$

$= 170$

(5) June - 2018

(3) A company has 4 machine on which 3 jobs, Each job can be assigned to one machine. The cost of each job one each machine is given in the following table.

		Machine			
		W	X	Y	Z
Jobs	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

What are the job assignedments which will minimize the cost.

Solution

The Given problem is unbalanced because the Namber of jobs are not equal to number machines.

To make it Balaned we have to add one job with cost zero so that Balanced problem is.

		Machine			
		W	X	Y	Z
Jobs	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22
	D	0	0	0	0

Step I:

row reduction

Job/mach	W	X	Y	Z
A	0	6	10	14
B	0	5	9	11
C	0	5	9	12
D	0	0	0	0

step2 : column reduction

Job\machine	W	X	Y	Z
A	$\boxed{0}$	6	10	14
B	$\cancel{0}$	5	9	11
C	$\cancel{0}$	5	9	12
D	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

From above all the zeros are either assign or crossed out but the total number of assignment i.e. $2 < 4$
∴ we have to follow step 3.

Step3. J/m

J/m	W	X	Y	Z
A	$\boxed{0}$	6	10	14
B	0	5	9	11
C	0	5	9	12
D	0	$\boxed{0}$	0	0

smallest element $k = 5$

① add - smallest element which is uncovered to interselected cell.

② substract smallest element from uncovered cell.

③ Keep the elements unchanged to the values which covered by one line.

M\J	W	X	Y	Z
A	$\boxed{0}$	1	5	9
B	$\cancel{0}$	$\boxed{0}$	4	6
C	$\cancel{0}$	0	4	7
D	0	0	$\boxed{0}$	$\cancel{0}$

assigned from above table all the zeros are either assign or crossed out but the total number of assignement i.e 3<4

∴ we have to follow again step 3.

M|J	W	X	Y	Z
A	$\boxed{0}$	1	5	9
B	0	$\boxed{0}$	4	6
C	0	0	4	7
~~D~~	5	0	$\boxed{0}$	0

Smallest element K = 4.

① add K to interselected cells

② Substract K from uncovered cell

M|J	W	X	Y	Z
A	$\boxed{0}$	1	1	5
B	0	$\boxed{0}$	0	1
C	0	0	$\boxed{0}$	2
D	9	4	0	$\boxed{0}$

∴ A → W
B → X
C → Y
D → Z

and total minimum cost = 18 + 13 + 17 + 0
= 50 //

June-2016 & 2017

(iv) what is assignment problem? Discuss the mathematical model of assignment problem.

An assignment problem is a particular case of transportation problem where the objective is to assign a number of resources to an equal number of activities so as to minimize total cost or maximize total profit of allocation

Mathematical model:

Let there are n jobs & n persons are availalele with different skills If the cost of doing jth work by ith person is c_{ij} Then the cost matrix is given in the table below:-

Job/person	1	2	--- j ---	n
1	c_{11}	c_{12}	--- c_{1j} ---	c_{1n}
2	c_{21}	c_{22}	--- c_{2j} ---	c_{2n}
⋮	⋮	⋮	⋮	⋮
i	c_{i1}	c_{i2}	c_{ij}	c_{in}
⋮	⋮	⋮	⋮	⋮
n	c_{n1}	c_{n2}	c_{nj}	c_{nn}

Now the problem is which work to be assign to when so that the cost of completion of work will be minimum mathematically.

$$\text{To minimize } z(\text{cost}) = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij} \quad \text{— (1)}$$

$[i=1, 2, ---n; \ j=1,2 ---- n]$

where

$$x_{ij} = \begin{cases} 1, & \text{if } i\text{th person is assigned to } j\text{th work} \\ 0; & \text{if } i\text{th person is not assigned to the } j\text{th work.} \end{cases}$$

with the restrictions.

i) $\sum_{i=1}^{n} x_{ij} = 1;\ j = 1, 2 \ldots n$ i.e ith person will do only one work

ii) $\sum_{j=1}^{n} x_{ij} = 1;\ j = 1, 2 \ldots n$ i.e. jth work will be done only by one person //

June - 2016

(5) Five men available to do five different Jobs, From past records, The time (in hours) that each man takes to do job is known & is given in the following table:

Jobs

men	I	II	III	IV	V
A	2	9	2	7	1
B	4	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

Solution

Jobs

men	I	II	III	IV	V
A	2	9	2	7	1
B	4	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

Scanned with CamScanner

Step 1 : Row Reduction

Job/men	I	II	III	IV	V
A	1	8	1	6	0
B	3	7	6	5	0
C	3	5	4	2	0
D	3	1	6	2	6
E	4	2	8	4	0

Step 3: Job/men

Job/men	I	II	III	IV	V
A	$\boxed{0}$	7	0	4	0
B	2	6	5	3	$\boxed{0}$
C	2	4	3	$\boxed{0}$	0
D	2	$\boxed{0}$	5	0	0
E	3	1	7	2	$\boxed{0}$

From above table all the zeros are either assign or crossed out but the total number of assignment i.e. 4<5 ∴ we have to follow step 3:

Step 3:

Job/men	I	II	III	IV	V
A	$\boxed{0}$	7	0	4	0
B	2	6	5	3	0
C	2	4	3	0	0
D	2	$\boxed{0}$	5	0	0
E	3	1	7	2	0

k = 1 which is smallest element in table with not covered

1. add k=1 to intersect point & substract in uncovered values.

Job/men	I	II	III	IV	V
A	$\boxed{0}$	7	0	4	1
B	1	5	4	2	0
C	2	4	3	0	1
D	2	0	5	0	1
E	2	0	6	1	0

From above table all zero are either assign or crossed out but the total number of assignment i.e. 4<5 ∴ we have to report Step 3.

Job/men	I	II	III	IV	V
A	$\boxed{0}$	7	$\boxed{0}$	4	1
B	1	5	4	2	$\boxed{0}$
C	2	4	3	$\boxed{0}$	
D	2	$\boxed{0}$	5	0	1
E	2	0	6 6	1	0

k=1

∴ add k=1 to intersected cell & substract in uncovered cell.

Job/men	I	II	III	IV	V
A	$\boxed{0}$	8	0	5	2
B	0	5	3	2	$\boxed{0}$
C	1	4	2	0	1
D	1	$\boxed{0}$	4	0	1
E	1	0	5	1	0

From above table all zero are either assign or crossed out but the total no of assignment i.c 4<5 ∴ we have to repeat Step 3.

Job/men	I	II	III	IV	V
A	0	8	0	5	2
B	0	6	3	2	0
C	1	4	2	0	1
D	1	0	4	0	1
E	1	0	5	1	0

Small element $k = 2$.
add $k=2$ into intersected cell & substract in uncovered cell.

Job/men	I	II	III	IV	V
A	2	10	$\boxed{0}$	7	4
B	$\boxed{0}$	5	2	2	$\cancel{0}$
C	1	4	$\cancel{0}$	$\boxed{0}$	1
D	1	$\boxed{0}$	2	$\cancel{0}$	1
E	1	$\cancel{0}$	3	1	$\boxed{0}$

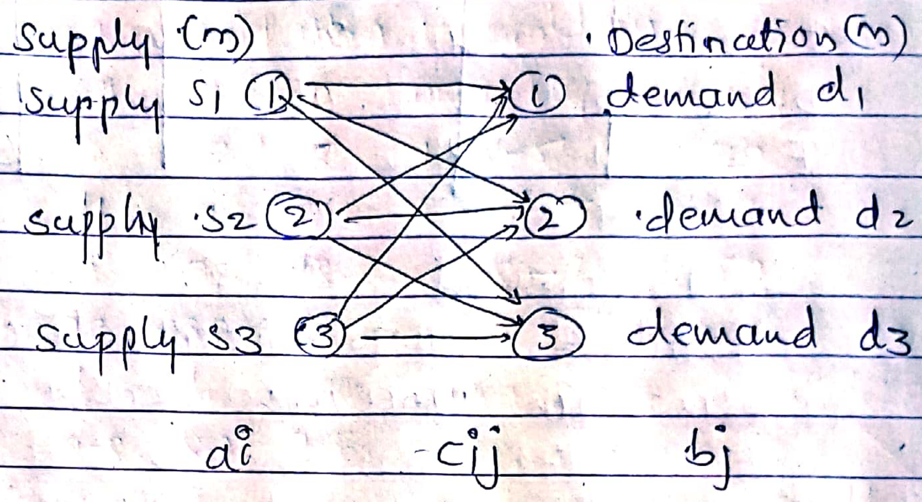
$\therefore$ A → III   Total time taken.
B → I $= 4+2+2+3+1$
C → IV $= 12$ hrs
D → II
E → V

June - 2016

(6) Mathemetical model of transporation model.

The transportation ~~model~~ problem is a special type of LPP where the objective is to minimize the cost of Distributing a product from a number of sources or ~~or~~ organise to a number of destination

The problem is represented by the n/w is



There are m sources and n destination each represented by a node the arc represented the row linking The source and destination.

Arc (ij) joining source i to destination j carries two piece of information.

(1) The transportation cost per unit c_{ij}

(2) The amount shifted x_{ij} The amount of supply of the source is a_i & the amount of demand b_j

The objective of the model is to minimize the total transportation cost while satisfying all the supply & demand interactions.

$$\text{minimize } f = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Two Types

① Balanced transporatation
where the total supply equal to total demand

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

② Un balanced transportation:
where the total supply not equal to total demand.

$$\sum_{i=1}^{m} a_i <> \sum_{j=1}^{n} b_j$$

June - 2016

⑦ find the optimal solution using VAM of the following transportation problem.

	A	B	C	D	Supply
X	6	8	8	5	30
Y	5	11	9	7	40
Z	8	9	7	13	50
Demand	35	28	32	25	

Solution: Given

	A	B	C	D	Supply
X	6	8	8	5	30
Y	5	11	9	7	40
Z	8	9	7	13	50
Dmnd	35	28	32	25	120/120

Here Total demand = Total supply
120 = 120

∴ Given problem is balanced transportation problem.

	A	B	C	D	row penalty	supply
X	6	8	8	5	1 = 6−5	30
Y	5	11	9	7	2 = 7−5	40
Z	8	9	7	13	1 = 8−7	50
Demand	35	28	32	25		
colum penalty	1 = 6−5	1 = 9−8	1 = 8−7	2 = 7−5		

Maximum penalty occurs in D column. The maximum c_{ij} in this column is $DX = 5$

The maximum allocation in this cell is min (30, 25)

It satisfy Demand D & leaves 30 − 25 = 5 in X

	A	B	C	D	Supply	row Penalty
X	6	8	8	5(25)	5	2 = 8−6
Y	5	11	9	7	40	4 = 5−9
Z	8	9	7	13	50	1 = 8−7
Demnd	35	28	32	0		
column Penalty	1 = 6−5	1 = 9−8	1 = 8−7			

The maximum penalty occurs in Y row minimum c_{ij} in this row is $AY = 5$. The maximum allocation in this cell is min (40, 35)

It satisfy Demand A & leaves
40-35 = 5 in supply Y

	A	B	C	D	supply	row. P.
X	6	8	8	5(25)	5	0=8=8
Y	5(35)	11	9	7	5	2=9-11
Z	8	9	7	13	50	2=9-7
Dem	0	28	32	0		
Column Penalty	-	1 = 9-8	1 = 8-7	-		

Maximum penalty 2 in Y row minimum cost = 9 in YC

allocation in this cell is min (32, 5) = 5

It satisfies the supply D &
levers 32-5 = 27 in C

	A	B	C	D	su	row p
X	6	8	8	5(25)	5	0=8-8
Y	5(35)	11	9	7	0	-
Z	8	9	7	13	50	2=9-7
Dem	0	28	27	0		
col penalty	-	1 = 9-8	1 = 8-7	-		

The maximum penalty = 2 in Z
minimum - cost in Z = 7 in ZC
allocation for ZC is min (50, 27) =
& it leaves 50-27 = 23 in
Z & demand C = 0

	A	B	C	D	Supply	row.P
X	6	8	8	5(25)	5	3=8-5
Y	5(35)	11	9(5)	7	0	-
Z	8	9	7(27)	13	23	9
Demand	0	28	0	0		
col Penalty	-	1 9-8	-			

The Maximum penalty = 9 Z
minimum cost in Z = 9 in ZB
allocation for ZB is min (28, 23) = 23
& it leaves 28 - 23 = 5 in B &
Satisfy Z.

	A	B	C	D	Su	row.P
X	6	8	8	5(25)	5	8
Y	5(35)	11	9(5)	7	0	
Z	8	9(23)	7(27)	13	23	
Demand	0	5	0	0		
col penalty	-	8	-			

The maximum penalty = 8 in B
Minimum cost in B = 8 in XB
allocation for XB is min
(5, 5) = 5 & it leaves
5 - 5 = 0 in both Demand
& Supply.

	A	B	C	D	Su	row.P
X ~~A~~	6	8(5)	8	5(25)	0	–
y ~~B~~	5(35)	11	9(5)	7	0	–
Z ~~C~~	8	9(23)	7(27)	13		–
Demand	0	0	0	0		
col penalty	–					

So the optimal solution:

$8\times5 + 5\times25 + 5\times35 + 9\times5 + 9\times23 + 7\times27$

$= 40 + 125 + 175 + 45 + 207 + 189$

$= 781$ //

July - 2017

A computer center has three expert programmers. The center wants three appl^n programmers to be developed. The head of the computer center after carefully studying the prgms to be developed estimates the computer time in minutes required by expert for the application program as follows ~~for the application~~ ~~program~~ as

		programmers		
		A	B	C
program	1	120	100	80
	2	80	90	110
	3	110	140	120

Assign the programmer to the program in such a way that the total computer application development is minimum.

Given

	A	B	C
1	120	100	80
2	80	90	110
3	110	140	120

Step 1 : row direction

	A	B	C
1	40	20	0
2	0	10	30
3	0	30	10

Step 2 : Column direction

	A	B	C
1	40	10	$\boxed{0}$
2	$\boxed{\cancel{0}}$	$\boxed{\cancel{0}}$	30
3	$\boxed{0}$	20	10

∴ Programers assigned to program

1 program → C programer

2 " → B programmer

3 " → A "

Total = 110 + 80 + 90

= 280 //