(a) Let's look at a given machine p. In order for it to have no job, every job must be assigned to a different machine. As the jobs are assigned randomly and uniformly, the probability that a given job j is not assigned to p is $(1 - \frac{1}{k})$ and therefore the probability that p doesn't get any job is $(1 - \frac{1}{k})^k$. Therefore the expected number of machines with no jobs is $N(k) = k(1 - \frac{1}{k})^k$.

Finally $N(k)/k = (1 - \frac{1}{k})^k$, which goes to $1/e$ as k goes to infinity. Also notice that in the limit the number of machines with no jobs is k/e.

(b) There is a very simple solution to this problem. We notice that the number of rejected jobs (denote it by N_{rej}) is the number of total jobs k minus the number of accepted jobs N_{acc} ($N_{rej} = k - N_{acc}$). The number of jobs accepted is the k minus the number of machines with no jobs N_{nojob} (since the rest of the people do exactly 1 job). Therefore $N_{rej} = k - N_{acc} = k - (k - N_{nojob}) = N_{nojob}$. Therefore the answer to part **(b)** is the same as the answer to part **(a)**.

(c) This part will involve slight calculations. We know that the number of machines with no jobs is k/e (from the first part). We first calculate the number of machines with exactly one job. Again look at a machine p. The probability that only 1 job is assigned to that machine is $k\frac{1}{k}(1 - \frac{1}{k})^{k-1}$. (The chance of a given job j being assigned to p is $1/k$ and the probability that the remaining jobs will not be assigned to p is $(1 - \frac{1}{k})^{k-1}$. Finally there are k choices of the "given" job j which puts the coefficient k in the beginning). Notice that this also in the limit $1/e$ therefore the number of machines with exactly 1 jobs is also k/e.

Finally the remaining machines regardless of how many jobs they were assigned will perform exactly two jobs. There are $k - \frac{2k}{e}$ of these.

The final tally is k/e machines with one job and $k - \frac{2k}{e}$ people with two jobs. Subtracting this from k (the total number of jobs) we get that $\frac{k(3-e)}{e}$ jobs are rejected, which is approximately 11%.

[1] ex16.34.694