We imagine dividing the set S into 20 *quantiles* $Q_1, \ldots, Q_{20}$, where Q_i consists of all elements that have at least $.05(i-1)n$ elements less than them, and at least $.05(20-i)n$ elements greater than them. Choosing the sample S' is like throwing a set of numbers at random into bins labeled with $Q_1, \ldots, Q_{20}$.

Suppose we choose $|S'| = 40,000$ and sample with replacement. Consider the event $\mathcal{E}$ that $|S' \cap Q_i|$ is between 1800 and 2200 for each i. If $\mathcal{E}$ occurs, then the first nine quantiles contain at most $19,800$ elements of S', and the last nine quantiles do as well. Hence the median of S' will belong to $Q_{10} \cup Q_{11}$, and thus will be a $(.05)$-approximate median of S.

The probability that a given Q_i contains more than 2200 elements can be computed using the Chernoff bound (4.1), with $\mu = 2000$ and $\delta = .1$; it is less than

$$\left[\frac{e^{.05}}{(1.05)^{(1.05)}}\right]^{10000} < .0001.$$

The probability that a given Q_i contains fewer than 1800 elements can be computed using the Chernoff bound (4.2), with $\mu = 2000$ and $\delta = .1$; it is less than

$$e^{-(.5)(.1)(.1)2000} < .0001.$$

Applying the Union Bound over the 20 choices of i, the probability that $\mathcal{E}$ does not occur is at most $(40)(.0001) = .004 < .01$.

[1] ex835.763.619