Let $I_1, \ldots, I_n$ denote the n intervals. We say that an *I_j-restricted solution* is one that contains the interval I_j.

Here is an algorithm, for fixed j, to compute an I_j-restricted solution of maximum size. Let x be a point contained in I_j. First delete I_j and all intervals that overlap it. The remaining intervals do not contain the point x, so we can "cut" the time-line at x and produce an instance of the Interval Scheduling Problem from class. We solve this in $O(n)$ time, assuming that the intervals are ordered by ending time.

Now, the algorithm for the full problem is to compute an I_j-restricted solution of maximum size for each $j = 1, \ldots, n$. This takes a total time of $O(n^2)$. We then pick the largest of these solutions, and claim that it is an optimal solution. To see this, consider the optimal solution to the full problem, consisting of a set of intervals S. Since $n > 0$, there is some interval $I_j \in S$; but then S is an optimal I_j-restricted solution, and so our algorithm will produce a solution at least as large as S.

[1] ex434.357.684