Suppose $m \leq n$, and let $L$ denote the maximum length of any string in $A \cup B$. Suppose there is a string that is a concatenation over both $A$ and $B$, and let $u$ be one of minimum length. We claim that the length of $u$ is at most $n^2L^2$.

For suppose not. First, we say that position $p$ in $u$ is of *type* $(a_i, k)$ if in the concatenation over $A$, it is represented by position $k$ of string $a_i$. We define *type* $(b_i, k)$ analogously. Now, if the length of $u$ is greater than $n^2L^2$, then by the pigeonhole principle, there exist positions $p$ and $p'$ in $u$, $p < p'$, so that both are of type $(a_i, k)$ and $(b_j, k)$ for some indices $i, j, k$. But in this case, the string $u'$ obtained by deleting positions $p, p+1, \ldots, p'-1$ would also be a concatenation over both $A$ and $B$. As $u'$ is shorter than $u$, this is a contradiction.

[1] ex690.144.299