Consider an ordered triple $(S, i, j)$, $1 \leq i, j \leq n$ and $S$ is a subset of the vertices that includes $v_i$ and $v_j$. Let $B[S, i, j]$ denote the answer to the question, "Is there a Hamiltonian path on $G[S]$ that starts at $v_i$ and ends at $v_j$?" Clearly, we are looking for the answer to $B[V, 1, n]$.

We now show how to construct the answers to all $B[S, i, j]$, starting from the smallest sets and working up to larger ones, spending $O(n)$ time on each. Thus the total running time will be $O(2^n \cdot n^3)$.

$B[S, i, j]$ is true if and only if there is some vertex $v_k \in S - \{v_i\}$ so that $(v_i, v_k)$ is an edge, and there is a Hamiltonian path from $v_k$ to $v_j$ in $G[S - \{v_i\}]$. Thus, we set $B[S, i, j]$ to be true if and only if there is some $v_k \in S - \{v_i\}$ for which $(v_i, v_k) \in E$ and $B[S - \{v_i\}, k, j]$ is true. This takes $O(n)$ time to determine.

[1] ex386.623.944