

The strategy is as follows. The seller watches the first $n/2$ bids without accepting any of them. Let b^* be the highest bid among these. Then, in the final $n/2$ bids, the seller accepts any bid that is larger than b^* . (If there is no such bid, the seller simply accepts the final bid.)

Let b_i denote the highest bid, and b_j denote the second highest bid. Let S denote the underlying sample space, consisting of all permutations of the bids (since they can arrive in any order.) So $|S| = n!$. Let E denote the event that b_j occurs among the first $n/2$ bids, and b_i occurs among the final $n/2$ bids.

What is $|E|$? We can place b_j anywhere among the first $n/2$ bids ($n/2$ choices); then we can place b_i anywhere among the final $n/2$ bids ($n/2$ choices); and then we can order the remaining bids arbitrarily ($(n-2)!$ choices). Thus $|E| = \frac{1}{4}n^2(n-2)!$, and so

$$P[E] = \frac{n^2(n-2)!}{4n!} = \frac{n}{4(n-1)} \geq \frac{1}{4}.$$

Finally, if event E happens, then the strategy will accept the highest bid; so the highest bid is accepted with probability at least $1/4$.

¹ex437.89.251