

(a) Suppose that  $n = 6$  and the coordinates are  $3, 2, -3, -2, -1, 0$ . Then the greedy algorithm would observe events  $2, 5, 6$ , while an optimal solution would observe events  $3, 4, 5, 6$ .

(b) Let  $OPT(j)$  denote the maximum number of events that can be observed, subject to the constraint that event  $j$  is observed. Note that  $OPT(n)$  is the value that we want.

To define a recurrence for  $OPT(j)$ , we consider the previous event before  $j$  that is observed in an optimal solution. If it is  $i$ , then we need to have  $|d_j - d_i| \leq j - i$ , and we behave optimally up through observing event  $i$ . Thus we have

$$OPT(j) = 1 + \min_{i: |d_j - d_i| \leq j - i} OPT(i).$$

The values of  $OPT$  can be built up in order of increasing  $j$ , in time  $O(j)$  for iteration  $j$ , leading to a total running time of  $O(n^2)$ . The value we want is  $OPT(n)$ , and the configuration can be found by tracing back through the array of  $OPT$  values.