

The problem is in \mathcal{NP} since we can exhibit a set X and check the size of its intersection with every set A_i .

We now show that $3\text{-Dimensional Matching} \leq_P \text{Intersection Inference}$. Suppose we are given an instance of $3\text{-Dimensional Matching}$, consisting of sets X , Y , and Z , each of size n , and a set T of m triples from $X \times Y \times Z$. We define the following instance of $\text{Intersection Inference}$. We define $U = T$. For each element $j \in X \cup Y \cup Z$, we create a set A_j of these triples that contain j . We then ask whether there is a set $M \subseteq U$ that has an intersection of size 1 with each set A_j .

Such sets are precisely those collections of triples for which each element of $X \cup Y \cup Z$ appears in exactly one: in other words, they are precisely the perfect three-dimensional matchings. Thus, our instance of $\text{Intersection Inference}$ has a positive answer if and only if the original instance of $3\text{-Dimensional Matching}$ does.

¹ex803.795.220