

We choose the problem Y to be \mathcal{B} -SAT. Take an instance of \mathcal{B} -SAT and define a real variable y_i in the polynomial for each Boolean variable x_i in the \mathcal{B} -SAT instance. Now, each clause becomes a product three terms in the variables $\{y_i\}$, where we represent x_i by y_i and $\overline{x_i}$ by $(1 - y_i)$. So for example, the clause $x_i \vee \overline{x_j} \vee x_k$ becomes the corresponding product $(1 - y_j)y_j(1 - y_k)$. Now, by the distributive law, each of these products can be written as a sum of at most eight monomials.

We define our polynomial to be the sum of all these monomials, and our bound B to be 0. For thinking about the reduction, it is useful to think about the polynomial in its form before we applied the distributive law, when it had one product for each clause. In order for its value to be ≤ 0 , each of these products must be 0; and the only way for this to happen is for one of the terms in the corresponding clause to be set so that it satisfies the clause. The polynomial can achieve a value ≤ 0 if and only if the original \mathcal{B} -SAT instance was satisfiable.

¹ex705.869.283