Zero-weight-cycle is in $\mathcal{NP}$ because we can exhibit a cycle in G, and it can be checked that the sum of of the edge weights on this cycle are equal to 0.

We now show that *Subset Sum* $\leq_P$ *Zero-weight-cycle*. We are given numbers $w_1, \ldots, w_n$, and we want to know if there is a subset that adds up to exactly W. We construct an instance of *Zero-weight-cycle* in which the graph has nodes $0, 1, 2, \ldots, n$, and an edge (i, j) for for all pairs $i < j$. The weight of edge (i, j) is equal to w_j. Finally, there is an edge $(n, 0)$ of weight $-W$.

We claim that there is a subset that adds up to exactly W if and only if G has a zero-weight cycle. If there is such a subset S, then we define a cycle that starts at 0, goes through the nodes whose indices are in S, and then returns to 0 on the edge $(n, 0)$. The weight of $-W$ on the edge $(n, 0)$ precisely cancels the sum of the other edge weights. Conversely, all cycles in G must use the edge $(n, 0)$, and so if there is a zero-weight cycle, then the other edges must exactly cancel $-W$ — in other words, their indices must form a set that adds up to exactly W.

[1] ex642.498.819