

(a) We can assume by symmetry that the first machine M_1 has the higher load. We need to prove that $T_1 \leq 2T_2$. Notice that our assumption that no single job takes more than half of the processing time implies the machine with higher load must have at least two jobs. Let job j be the smallest job on machine M_1 . Clearly, $T_1 \geq 2t_j$. The local search algorithm terminated, so moving job j from machine M_1 to machine M_2 does not decrease the difference between the processing times. This implies that $t_j \geq T_1 - T_2$. We get that $T_1 \leq t_j + T_2 \leq \frac{1}{2}T_1 + T_2$, and multiplying by two we get that $T_1 \leq 2T_2$.

(b) We observed above that if a job j satisfies $t_j \geq |T_1 - T_2|$ it cannot move. Further, the difference $|T_1 - T_2|$ decreases throughout the algorithm, so once this condition holds, job j will never move.

Now consider a job j , and we aim to prove that j will move at most once. Assume that job j starts on machine M_1 . So the first time it moves it will move from machine M_1 to machine M_2 . We always move the largest job, so at this point all remaining jobs on machine M_1 will have processing time at most t_j . Consider the sequence of consecutive moves all from machine M_1 to M_2 , and let $t_{j'}$ be the last job that moves in this direction (possibly $j = j'$). At this time $T_2 \geq T_1$ and $T_2 - T_1 \leq t_{j'}$ as before moving job j' machine M_1 had more work. Now we have that $t_j \geq t_{j'} \geq |T_1 - T_2|$, so by the observation above job j will not move again.

(c) As an example of a bad local optimum let machine M_1 have two jobs with processing time 3 each. While machine M_2 has two jobs with processing time 2 each. Now the difference in loads is 2, no single job can move, but swapping a pair of jobs yields a solution with identical loads.