To solve this problem, construct a graph $G = (V, E)$ as follows: Let the set of vertices consist of a super-source node, four supply nodes (one for each blood type) adjacent to the source, four demand nodes and a super sink node that is adjacent to the demand nodes. For each supply node u and demand node v, construct an edge (u, v) if type v can receive blood from type u and set the capacity to ∞ or the demand for type v. Construct an edge (s, u) between the source s and each supply node u with the capacity set to the available supply of type u. Similarly, for each demand node v and the sink t, construct an edge (v, t) with the capacity set to the demand for type v.

Now compute an (integer-valued) maximum flow on this graph. Since the graph has constant size, the scaling max-flow algorithm takes time $O(\log C)$, where C is the total supply, and the Preflow-Push algorithm takes constant time.

We claim that there is sufficient supply for the projected need. if and only if the edges from the demand nodes to the sink are all saturated in the resulting max-flow. Indeed, if there is sufficient supply, in which s_{ST} of type S are used for type T, then we can send a flow of s_{ST} from the supply node of type S to the demand node of type T, and respect all capacity conditions. Conversely, if there is a flow saturating all edges from demand nodes to the sink, then there is an integer flow with this property; if it sends f_{ST} units of flow from the supply node for type S to the demand node for type T, then we can use f_{ST} nodes of type S for patients of type T.

(b) Consider a cut containing the source, and the supply and demand nodes for B and A. The capacity of this cut is $50 + 36 + 10 + 3 = 99$, and hence all 100 units of demand cannot be satisfied.

An explanation for the clinic administrators: There are 87 people with demand for blood types O and A; these can only be satisfied by donors with blood types O and A; and there are only 86 such donors.

Note. We observe that part (a) can also be solved by a greedy algorithm; basically, it works as follows. The O group can only receive blood from O donors; so if the O group is not satisfied, there is no solution. Otherwise, satisfy the A and B groups using the leftovers from the O group; if this is not possible, there is no solution. Finally, satisfy the AB group using any remaining leftovers. A short explanation of correctness (basically following the above reasoning) is necessary for this algorithm, as it was with the flow algorithm.

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