Plot Fulfillment is in NP: Given an instance of the problem, and a proposed s-t path P, we can check that P is a valid path in the graph, and that it meets each set T_i.

Plot Fulfillment also looks like a covering problem; in fact, it looks a lot like the *Hitting Set* problem from the previous question: we need to "hit" each set T_i. However, we have the extra feature that the set with which we "hit" things is a path in a graph; and at the same time, there is no explicit constraint on its size. So we use the path structure to impose such a constraint.

Thus, we will show that *Hitting Set* $\leq_P$ *Plot Fulfillment*. Specifically, let us consider an instance of *Hitting Set*, with a set $A = \{a_1, \ldots, a_n\}$, subsets $S_1, \ldots, S_m$, and a bound k. We construct the following instance of *Plot Fulfillment*. The graph G will have nodes s, t, and

$$\{v_{ij} : 1 \leq i \leq k,\ 1 \leq j \leq n\}.$$

There is an edge from s to each v_{1j} $(1 \leq j \leq n)$, from each v_{kj} to t $(1 \leq j \leq n)$, and from v_{ij} to $v_{i+1,\ell}$ for each $1 \leq i \leq k-1$ and $1 \leq j, \ell \leq n$. In other words, we have a *layered graph*, where all nodes v_{ij} $(1 \leq j \leq n)$ belong to "layer i", and edges go between consecutive layers. Intuitively the nodes v_{ij}, for fixed j and $1 \leq i \leq k$ all represent the element $a_j \in A$.

We now need to define the sets T_ℓ in the *Plot Fulfillment* instance. Guided by the intuition that v_{ij} corresponds to a_j, we define

$$T_\ell = \{v_{ij} : a_j \in S_\ell,\ 1 \leq i \leq k\}.$$

Now, we claim that there is a valid solution to this instance of *Plot Fulfillment* if and only if our original instance of *Hitting Set* had a solution. First, suppose there is a valid solution to the *Plot Fulfillment* instance, given by a path P, and let

$$H = \{a_j : v_{ij} \in P \text{ for some } i\}.$$

Notice that H has at most k elements. Also for each ℓ, there is some $v_{ij} \in P$ that belongs to T_ℓ, and the corresponding a_j belongs to S_ℓ; thus, H is a hitting set.

Conversely, suppose there is a hitting set $H = \{a_{j_1}, a_{j_2}, \ldots, a_{j_k}\}$. Define the path $P = \{s, v_{1,j_1}, v_{2,j_2}, \ldots, v_{k,j_k}, t\}$. Then for each ℓ, some a_{j_q} lies in S_ℓ, and the corresponding node v_{q,j_q} meets the set T_ℓ. Thus P is a valid solution to the *Plot Fulfillment* instance.

[1] ex425.710.356