

The subproblems will represent the optimum way to satisfy orders $1, \dots, i$ with an inventory of s trucks left over after the month i . Let $OPT(i, s)$ denote the value of the optimal solution for this subproblem.

- The problem we want to solve is $OPT(n, 0)$ as we do not need any leftover inventory at the end.
- The number of subproblems is $n(S + 1)$ as there could be $0, 1, \dots, S$ trucks left over after a period.
- To get the solution for a subproblem $OPT(i, s)$ given the values of the previous subproblems, we have to try every possible number of trucks that could have been left over after the previous period. If the previous period had z trucks left over, then so far we paid $OPT(i - 1, z)$ and now we have to pay zC for storage. In order to satisfy the demand of d_i and have s trucks left over, we need $s + d_i$ trucks. If $z < s + d_i$ we have to order more, and pay the ordering fee of K .

In summary the cost $OPT(i, s)$ is obtained by taking the smaller of $OPT(i - 1, s + d_i) + C(s + d_i)$ (if $s + d_i \leq S$), and the minimum over smaller values of z , $\min_{z < \min(s + d_i, S)} (OPT(i - 1, z) + zC + K)$.

We can also observe that the minimum in this second term is obtained when $z = 0$ (if we have to reorder anyhow, why pay storage for any extra trucks?). With this extra observation we get that

- if $s + d_i > S$ then $OPT(i, s) = OPT(i - 1, 0) + K$,
- else $OPT(i, s) = \min(OPT(i - 1, s + d_i) + C(s + d_i), OPT(i - 1, 0) + K)$.

¹ex304.359.690