First, we observe the following fact. If we consider two different sets of costs $\{c_e\}$ and $\{c'_e\}$ on the edge set of E, with the property that the sorted order of $\{c_e\}$ and $\{c'_e\}$ is the same, then Kruskal's algorithm will output the same minimum spanning tree with respect to these two sets of costs.

It follows that for our time-changing edge costs, the set of edges in the minimum spanning tree only changes when two edge costs change places in the overall sorted order. This only happens when two of the parabolas defining the edge costs intersect. Since two parabolas can cross at most twice, the structure of the minimum spanning tree can change at most $2\binom{m}{2} \leq m^2$ times, where m is the number of edges. Moreover, we can enumerate all these crossing points in polynomial time.

So our algorithm is as follows. We determine all crossing points of the parabolas, and divide the "time axis" $\mathbf{R}$ into $\leq m^2$ intervals over which the sorted order of the costs remains fixed. Now, over each interval I, we run Kruskal's algorithm to determine the minimum spanning tree T_I. The cost of T_I over the interval I is a sum of $n-1$ quadratic functions, and hence is itself a quadratic function; thus, having determined the sum of these $n-1$ quadratic functions, we can determine its minimum over I in constant time.

Finally, minimizing over the best solution found in each interval gives us the desired tree and value of t.

[1] ex696.856.903