

evasive path is in NP since we may check, for an s - t path P , whether $|P \cap Z_i| \leq 1$ for each i .

Now let us suppose that we have an instance of \mathcal{B} -sat with n variables x_1, \dots, x_n and t clauses. We create the following directed graph $G = (V, E)$. The vertices will be partitioned into *layers*, with each node in one layer having edges to each node in the next. Layer 0 will consist only of the node s . Layer i will have two nodes for $i = 1, \dots, n$, three nodes for $i = n+1, \dots, n+t$, and only the node t in layer $n+t+1$. Each node in layers $1, 2, \dots, n+t$ will also be assigned a *label*, equal to one of the $2n$ possible literals. In layer i , for $1 \leq i \leq n$, we label one node with the literal x_i and one with the literal $\overline{x_i}$. In layer $n+i$, for $1 \leq i \leq t$, we label the three nodes with $\ell_{i1}, \ell_{i2}, \ell_{i3}$, where $\{\ell_{ij}\}$ are the three literals appearing in clause i . Finally, for every pair of nodes whose labels correspond to a variable and its negation, we define a distinct zone Z .

Now, if there is a satisfying assignment for the \mathcal{B} -sat instance, we can define an s - t path P that passes only through nodes with the labels of literals set to *true*; P is thus an evasive path. Conversely, consider any evasive path P ; we define variable x_i to be *true* if P passes through the vertex in layer i with label x_i , and *false* if P passes through the vertex in layer i with label $\overline{x_i}$. Since P does not visit any zone a second time, it must therefore visit only nodes whose labels correspond to literals set to *true*, and hence the \mathcal{B} -sat instance is satisfiable.

¹ex636.680.130