Suppose that to obtain n words, we need L lines (most of which will get repeated many times, as described above). We write the script as follows

```
line 1 = <text of line 1 here>
line 2 = <text of line 2 here>
...
line L = <text of line L here>
For i = 1, 2, ..., L
  For j = 1, 2, ..., i
    Sing lines j through 1
  Endfor
Endfor
```

Now, the nested `For` loops have length bounded by a constant c_1, so the real space in the script is consumed by the text of the lines. Each of these lines in the script has length at most c_2 (where c_2 is the maximum line length c plus the space to write the variable assignment). So in total, the space required by the script is $S = c_1 + c_2 L$.

Recall that n denotes the number of words this produces when sung. n is at least $1 + 2 + \cdots + L = \frac{1}{2}L(L-1)$; hence, $\frac{1}{2}(L-1)^2 \leq n$, and so $L \leq 1 + \sqrt{2n}$. Plugging this into our bound on the length of the script, we have $f(n) = S \leq c_1 + c_2\sqrt{2n} = O(\sqrt{n})$.

[1] ex434.486.949