

We begin by noticing two facts related to the graph  $\mathcal{H}$  defined in the previous problem. First, if  $T$  and  $T'$  are neighbors in  $\mathcal{H}$ , then the number of  $X$ -edges in  $T$  can differ from the number of  $X$ -edges in  $T'$  by at most one. Second, the solution given above in fact provides a polynomial-time algorithm to construct a  $T$ - $T'$  path in  $\mathcal{H}$ .

We call a tree *feasible* if it has exactly  $k$   $X$ -edges. Our algorithm to search for a feasible tree is as follows. Using a minimum-spanning tree algorithm, we compute a spanning tree  $T$  with the minimum possible number  $a$  of  $X$ -edges. We then compute a spanning tree  $T'$  with the maximum possible number  $b$  of  $X$ -edges. If  $k < a$  or  $k > b$ , then there clearly there is no feasible tree. If  $k = a$  or  $k = b$ , then one of  $T$  or  $T'$  is a feasible tree. Now, if  $a < k < b$ , we construct a sequence of trees corresponding to a  $T$ - $T'$  path in  $\mathcal{H}$ . Since the number of  $X$ -edges changes by at most one on each step of this path, and overall it increases from  $a$  to  $b$ , one of the trees on this path is a feasible tree, and we return it as our solution.

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<sup>1</sup>ex708.930.216