We assume the graph G is connected; otherwise we work with the connected components separately (after computing them in $O(m+n)$ time).

We run BFS starting from an arbitrary node s, obtaining a BFS tree T. Now, if every edge of G appears in the BFS tree, then $G = T$, so G is a tree and contains no cycles. Otherwise, there is some edge $e = (v, w)$ that belongs to G but not to T. Consider the least common ancestor u of v and w in T; we obtain a cycle from the edge e, together with the u-v and u-w paths in T.

[1] ex858.281.666