This problem can be decided in polynomial time. It helps to view the $QSAT$ instance as a $CSAT$ instance instead: Player 1 controls the set A of odd-indexed variables while Player 2 controls the set B of even-indexed variables. Our question then becomes: can Player 1 force a win?

We claim that Player 1 can force a win if and only if each clause C_i contains a variable from A. If this is the case, Player 1 can win by setting all variables in A to 1. If this is not the case, then some clause C_i has no variable from A. Player 2 can then win by setting all variables in B to 0: in particular, this will cause the clause C_i to evaluate to 0.

[1] ex63.946.695