The *Dominating Set* problem is in NP: Given a set of k proposed servers, we can verify in polynomial time that all non-server workstations have a direct link to a workstation that is a server.

To prove that the problem in NP-complete we use the *Vertex Cover* problem that is known to be NP-complete. We show that

Vertex Cover $\leq_P$ *Dominating Set*

It is worth noting that *Dominating Set* is not the *same* problem as *Vertex Cover*: for example, on a graph consisting of three mutually adjacent nodes, we need to choose only one node to have a copy of the database *adjacent* to all other nodes; but we need to choose two nodes to obtain a *vertex cover*.

Consider an instance of *Vertex Cover* with a graph G and a number k. Let r denote the number of nodes in G that are not adjacent to any edge; these nodes will be called *isolated*. To create the equivalent *Dominating Set* problem we create a new graph G' by adding a parallel copy to every edge, and adding a new node in the middle of each of the new edges. More precisely, if G has an edge (i, j) we add a new node v_{ij}, and add new edges (i, v_{ij}) and (v_{ij}, j). Now we claim that G contains a set S of vertices of size at most k so that every edge of G has at least one end in S if and only if G' has a solution to the *Dominating Set* problem with $k + r$ servers. Note again that this reduction takes polynomial time to compute.

Let I denote the set of isolated nodes in G. If G has a vertex cover S of size k, then placing servers on the set $S \cup I$ we get a solution to the server placement problem with at most $k + r$ servers. We need to show that this set is a solution to the server placement; that is, we must show that for each vertex that is not in S, there is an adjacent vertex is in S. First consider a new vertex v_{ij}. The set S is a vertex cover, so either i or j must be in the set S, and so S contains a node directly connected to v_{ij} in G'. Now consider a node i of the original graph G, and let (i, j) be any edge adjacent to i. Either i or j is in S, so again there is an node directly connected to i that has a server.

Now consider the other direction of the equivalence: Assume that there is a solution to the Dominating Set problem with at most $k + r$ servers. First notice that all isolated nodes must be servers. Next notice that we can modify the solution so that we only use the original nodes of the graph as servers. If a node v_{ij} is used as a server than we can replace this node by either i and j and get an alternate solution with the same number of servers: the node v_{ij} is can serve requests from the nodes v_{ij}, i, j, and either i or j can do the same.

Next we claim that if there is a solution to the *Dominating Set* problem where a set S of nodes of the original graph are used as servers, then S forms a vertex cover of size k. This is true, as for each edge (i, j) the new node v_{ij} must have a directly connected node with a server, and hence we must have that either i or j is in S.

[1] ex771.562.634