Let the sequence S consist of $s_1, \ldots, s_n$ and the sequence S' consist of $s'_1, \ldots, s'_m$. We give a greedy algorithm that finds the first event in S that is the same as s'_1, matches these two events, then finds the first event after this that is the same as s'_2, and so on. We will use $k_1, k_2, \ldots$ to denote the match have we found so far, i to denote the current position in S, and j the current position in S'.

```
Initially i = j = 1
While i ≤ n and j ≤ m
    If s_i is the same as s'_j, then
        let k_j = i
        let i = i + 1 and j = j + 1
    otherwise let i = i + 1
EndWhile
If j = m + 1 return the subsequence found:  k_1, ..., k_m
Else return that "S' is not a subsequence of S"
```

The running time is $O(n)$: one iteration through the while look takes $O(1)$ time, and each iteration increments i, so there can be at most n iterations.

It is also clear that the algorithm finds a correct match if it finds anything. It is harder to show that if the algorithm fails to find a match, then no match exists. Assume that S' is the same as the subsequence $s_{l_1}, \ldots, s_{l_m}$ of S. We prove by induction that the algorithm will succeed in finding a match and will have $k_j \leq l_j$ for all $j = 1, \ldots, m$. This is analogous to the proof in class that the greedy algorithm finds the optimal solution for the interval scheduling problem: we prove that the greedy algorithm is always ahead.

- *For each $j = 1, \ldots, m$ the algorithm finds a match k_j and has $k_j \leq l_j$.*

Proof. The proof is by induction on j. First consider $j = 1$. The algorithm lets k_1 be the first event that is the same as s'_1, so we must have that $k_1 \leq l_1$.

Now consider a case when $j > 1$. Assume that $j - 1 < m$ and assume by the induction hypothesis that the algorithm found the match k_{j-1} and has $k_{j-1} \leq l_{j-1}$. The algorithm lets k_j be the first event after k_{j-1} that is the same as s'_j if such an event exists. We know that l_j is such an event and $l_j > l_{j-1} \geq k_{j-1}$. So $s_{l_j} = s'_j$, and $l_j > k_{j-1}$. The algorithm finds the first such index, so we get that $k_j \leq l_j$. ∎

[1] ex876.936.4