The problem is in $\mathcal{NP}$, since we can exhibit a set of disjoint paths P_i, and it can be checked in polynomial time that they are paths in G, connect the corresponding nodes, and are disjoint.

Now we show *3-SAT* $\leq_P$ *Directed Disjoint Paths*. Consider a *3-SAT* problem given by a set of clauses $C_1, \ldots, C_k$, each of length 3, over a set of variables $X = \{x_1, \ldots, x_n\}$. To create the corresponding instance of the Directed Disjoint Paths problem, we will have $2n$ directed paths, each of length k, one paths P_i corresponding to variable x_i and one path P'_i corresponding to $\overline{x_i}$. We add n source-sink pairs corresponding to the n variables, and connect source s_i to the first node on paths P_i and P'_i and connect the last nodes in paths P_i and and P'_i to sink t_i. Note that there are two directed paths connecting s_i to t_i: the path s_i, P_i, t_i, and the path s_i, P'_i, t_i. We will think of selecting the first of these paths as setting the variable x_i to *false* (as the variable through the copies of $\overline{x_i}$ are left unused), and selecting the second path will correspond to setting the variable x_i to *true*.

Now will will add k additional source sink pairs, one corresponding to each clause C_j. Let S_j and T_j the source sink pair corresponding to clause C_j. We will claim that there is a path from S_j to T_j disjoint from the path selected to connect the s_i-t_i source-sink pairs if and only of clause C_j is satisfied by the corresponding assignment. Assume clause C_j contains the literal t_{j1}, t_{j2} and t_{j3}. Now we have a path P_i or P'_i corresponding to each of these variables or negated variables. The paths have n nodes each, let v_{j1}, v_{j2} and v_{j3} denote the jth node on the 3 corresponding paths. We add the edges $(S_j, v_{j\ell})$ and $(v_{j\ell}, T_j)$ for each of $\ell = 1, 2, 3$.

Now we claim that the resulting directed graph has node disjoint paths connecting the source-sink pairs s_i-t_i and S_j-T_j for $i = 1, 2, \ldots, n$ and $j = 1, \ldots, k$ if and only if the *3-SAT* instance is satisfiable. One direction is easy to see: if the *3-SAT* instance is satisfiable, then select the paths connecting s_i to t_i corresponding to the satisfying assignment, as suggested above. Then the source-sink pair S_j and T_j can be connected through the path using the true variable in the clause.

Finally, we need to show that if the disjoint paths exists, than the *3-SAT* formula has a satisfying assignment. Note that the paths P_i and P_j are disjoint, and the graph has no edges connecting different paths. The only edges outside these paths in the graph are edges entering one of the sinks, or leaving a source. As a result the only paths in the graph connecting an s_i-t_i pair are the two paths s_i, P_i, t_i and s_i, P'_i, t_i, and the only paths in G connecting S_j-T_j pairs are the three possible paths through each of the 3 variable nodes in C_j. Hence, sets of disjoint paths connecting the source-sink pairs, correspond to satisfying assignments.

[1] ex563.824.406