To show that *Number Partitioning* is in NP we use the subset $S \subset \{1, \ldots, n\}$ such that $\sum_{i \in S} w_i = \frac{1}{2} \sum_{i-1}^{n} w_i$ as the certificate. Given a certificate we can add the corresponding numbers in polynomial time and test if the claimed equation holds.

To prove that *Number Partitioning* is NP-complete, we show that *Subset-Sum* $\leq_P$ *Number Partitioning*. Consider an arbitrary instance of *Subset Sum* with numbers $w_1, \ldots, w_n$ and target sum W. We will construct an equivalent instance of *Number Partitioning*. Let $T = \sum_{i=1}^{n} w_i$ be the total sum of all numbers. Add two numbers $w_{n+1} = W + 1$ and $w_{n+2} = T + 1 - W$. Note that the sum of all $n + 2$ numbers is $2T + 2$. We claim that the partition problem with these $n + 2$ numbers is equivalent to the original *Subset Sum* instance. To prove this, assume first that the answer in the *Subset Sum* problem is "yes", there is a subset S such that $\sum_{i \in S} w_i = W$. Now we can create a partition solution by adding w_{n+2} to the subset S, and using all other numbers as the other part. Now assume conversely that the answer in the *Number Partitioning* problem is "yes", there is a partition where the two parts have equal sums, that is, they both sum to $T + 1$. Note that w_{n+1} and w_{n+2} cannot be in the same part as $w_{n+1} + w_{n+2} > T + 1$. Consider the part that contains w_{n+2}. The sum of all numbers in this part is $T + 1$, so the numbers other than w_{n+2} must sum to W.

Note that it was important to add the $+1$ in both w_{n+1} and w_{n+2}. A natural first idea would have been to use $w_{n+1} = W$ and $w_{n+2} = T - W$. However, this instance of *Number Partitioning* is always "yes", independent of the answer in the original *Subset Sum* problem, as now the total sum is $2T$ and $w_{n+1} + w_{n+2} = T$.

[1] ex123.267.365