

The problem is in  $\mathcal{NP}$  since we can exhibit a set of  $k$  nodes and check that the distance between all pairs is at least 3.

We now show *Independent Set*  $\leq_P$  *Strongly Independent Set*. Given a graph  $G$  and a number  $k$ , we construct a new graph  $G'$  in which we replace each edge  $e = (u, v)$  by a path of length two: we add a new node  $w_e$ , and we add edges  $(u, w_e), (w_e, v)$ . We also include edges between every pair of new nodes.

Now suppose that  $G$  has an independent set of size  $k$ . Then in this new graph  $G'$ , all these  $k$  nodes are distance at least three from each other, so this is a strongly independent set of size  $k$ . Conversely, suppose  $G'$  has a strongly independent set of size  $k$ . Now, this set can't contain any of the new nodes, since all such nodes are within distance two of every node in the graph. Thus, it consists of nodes present in  $G$ . Moreover, no two of these nodes can be neighbors in  $G$ , since then they'd be at distance two in  $G'$ . Thus this set of nodes forms an independent set of size  $k$  in  $G$ .

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<sup>1</sup>ex900.39.43