At all times, some intervals will be marked (they're already intersected) and some won't. Iteratively, we look at the unmarked interval that ends earliest, and among the intervals that intersect it, we choose the interval I that ends the latest. We add I to our set and mark all intervals intersected by I.

Suppose we select $i_1, i_2, ..., i_k$, and an optimal solution selects $j_1, j_2, ..., j_m$. First note that no interval in either solution is "nested" inside another, so we can assume our two lists of indices are sorted both by start as well as finish time. Let x_t be the earliest-finishing unmarked interval in iteration t: this is the one that caused us to select i_t.

We claim that intervals $j_1, ..., j_{t-1}$ do not intersect x_t. It will then follow that we cannot have $m \leq k-1$, for then x_k wouldn't be intersected by the optimal solution. The base case is trivial; in general, suppose we know the claim to be true up to x_t. Then the earliest optimal interval j_u that does intersect x_t has $u \geq t$. But i_t does not intersect x_{t+1}, and it is the latest-ending interval that intersects x_t; hence j_u does not intersect x_{t+1} either. So none of $j_1, ..., j_u$ intersect x_{t+1}, and $u \geq t$, so this completes the induction step.

[1] ex624.87.982