(a) Consider a path with three edges, and an execution of the greedy algorithm in which the middle edge is added first.

(b) Consider the k connected components $C_1, \ldots, C_k$ of $M \cup M'$ — each is path or a cycle. Label a component C_i by an ordered pair $(|M \cap C_i|, |M' \cap C_i|)$. Now, if some C has a label of the form $(0, j)$, then it follows that $j = 1$, and this is an edge of M' that can be added to M. Otherwise, the labels are $\{x_i, y_i\}$, where $x_i \geq 1$ and $y_i \leq x_i + 1$ for each i. But then $|M'| - |M| = \sum_i (y_i - x_i) \leq k$ while $|M| = \sum_i x_i \geq k$, so we have $|M| \geq |M'| - |M|$. Rearranging this last inequality, we get $|M'| \leq 2|M|$.

Another way to prove this is the following. Since no edge of M' can be added to M, each $e \in M$ shares an endpoint with some $e' \in M'$. (It may share an endpoint with two edges in M'; then pick one arbitrarily.) Make the edge $e' \in M$ "pay for" the edge $e \in M$. Now, each edge $e \in M$ has been paid for by some edge $e' \in M'$, but each $e' \in M'$ has only two endpoints and hence pays for at most two edges in M. It follows that M' contains at most twice as many edges as M.

(c) Let M' be a matching of maximum size, and let M be the matching obtained by the greedy algorithm when it finally terminates. Then since there is no edge from M' that can be added to M, it follows from (a) that $|M| \geq \frac{1}{2}|M'|$.

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