The problem is in  $\mathcal{NP}$  because we can exhibit a set of k customers, and in polynomial time is can be checked that no two bought any product in common.

We now show that  $Independent\ Set \leq_P Diverse\ Subset$ . Given a graph G and a number k, we construct a customer for each node of G, and a product for each edge of G. We then build an array that says customer v bought product e if edge e is incident to node v. Finally, we ask whether this array has a diverse subset of size k.

We claim that this holds if and only if G has an independent set of size k. If there is a diverse subset of size k, then the corresponding set of nodes has the property that no two are incident to the same edge — so it is an independent set of size k. Conversely, if there is an independent set of size k, then the corresponding set of customers has the property that no two bought the same product, so it is diverse.

 $<sup>^{1}</sup>$ ex640.690.659