

We prove this by induction on the number of nodes in  $T$ . Let  $n_0(T)$  denote the number of leaves of a binary tree  $T$ , and let  $n_2(T)$  denote the number of nodes with two children.

The basis of the induction is a tree with a single node. This node is the only leaf, and there are no nodes with two children.

Now, let  $T$  be an arbitrary binary tree on more than one node, and let  $v$  be a leaf. Since  $T$  has more than one node,  $v$  is not the root, so it has a parent  $u$ . Let  $T'$  be the tree obtained by deleting  $v$ .

If  $u$  had no other child in  $T$ , then it becomes a leaf in  $T'$ , so we have  $n_0(T') = n_0(T)$  and  $n_2(T') = n_2(T)$ . Applying the induction hypothesis to  $T'$  completes the induction step in this case. On the other hand, if  $u$  had another child in  $T$ , then it does not become a leaf after the deletion; but it used to have two children and now it doesn't. Thus we have  $n_0(T') = n_0(T) - 1$  and  $n_2(T') = n_2(T) - 1$ . Again, applying the induction hypothesis to  $T'$  completes the induction step in this case.