

The problem is in \mathcal{NP} since we can exhibit a subset E' of the edges, and it can be checked in polynomial time that at most a nodes in X are incident to an edge in E' , and at least b nodes in Y are incident to an edge in E' .

We now show that *Set Cover* is reducible to this problem. Given a collection of sets S_1, \dots, S_m over a ground set U of size n , we define a bipartite graph in which the nodes in X correspond to the sets S_i , and the nodes in Y correspond to the elements in U . We join each set node to the nodes corresponding to elements that it contains. We also set $a = k$ and $b = n$. (In particular, this means that our (a, b) -skeleton must touch every node in Y .)

Now, if G has an (a, b) -skeleton E' , then the k nodes in X incident to edges in E' correspond to k sets that collectively contain all elements, so they form a set cover. Conversely, if there is a set cover of size k , then taking E' to be the set of all edges incident to the corresponding set nodes yields an (a, b) -skeleton.

¹ex748.182.100