Fractional Knapsack Problem. (17) (3)
(Greedy algo.)

June 14 (III)

Q2

item	1	2	3	4
Profit	10	10	12	18
Weight	2	4	6	9
Profit/w	5	2.5	2	2

1. Pick the items in the decreasing order of the profit/weight.
2. Break the tie among the items the same profit/weight by picking the item with lowest item index.
3. optimal sol. will be

item 1 (2 lb), item 2 (4 lb), item 3 (6 lb), item 4 (3 lb)

As $M = 15$

and item 1 (w) + item (2) w + item (3) w = 12

$\therefore$ item 4 (w) = 3

Max profit would be.

$= 2\times 5 + 4\times 2.5 + 6\times 2 + 3\times 2$

$= 10 + 10 + 12 + 6 = 38$ Ans (B)

Note:- If the items cannot be divided and we have to pick only either the full item or just leave it, it is called

a) integer Knapsack problem → Dynamic Programming

(0-1 Knapsack Problem)

Huffman Coding technique (optimal Coding technique)

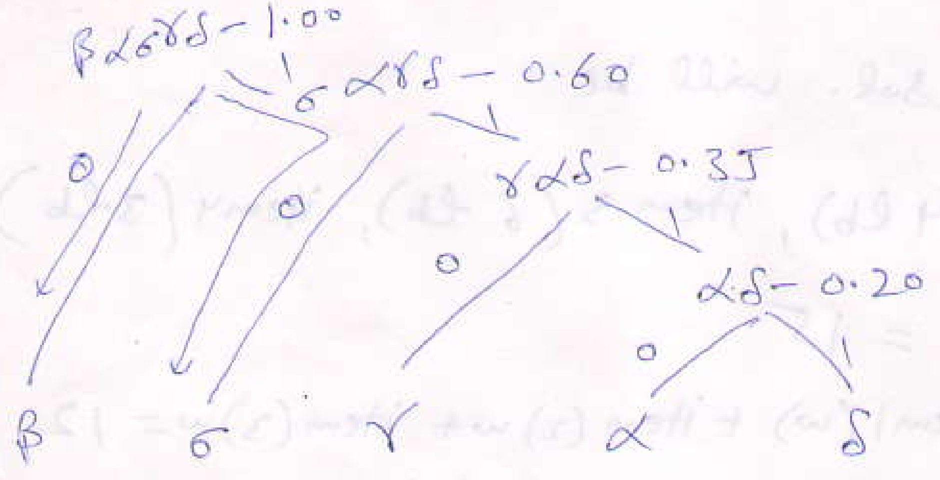
June 14 (11)

37 A text is made up of the characters $\alpha, \beta, \gamma, \delta$ and σ with the probability 0.12, 0.40, 0.15, 0.08 and 0.25 respectively. The optimal Coding technique will have the avg. length of.

a) 1.7 b) 2.15 c) 3.4 d). 3.8

Sol → Arrange the codes in descending order.

[In Huffman, we try to use a coding scheme s.t. a characters of Max Prob. (Max freq.) should use less no. of bits and characters of Min prob. (Min freq.) may use more no. of bits



0.40	0.25	0.15	0.12	0.08
0	10	110	1110	1111
(1 bit)	(2 bits)	(3 bits)	(4 bits)	(4 bits)

Now, combine least two probabilities.

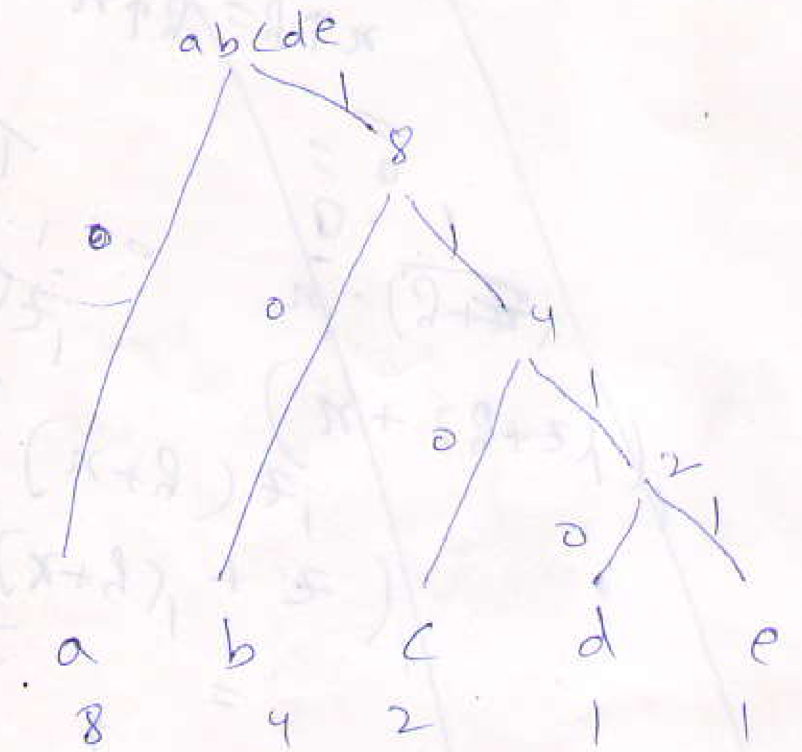
So. avg. length = $0.40 \times 1 + 0.25 \times 2 + 0.15 \times 3 + 0.12 \times 4 + 0.08 \times 4$

$= 2.15$

If instead of probability freq. is given.

e.g.

a	b	c	d	e	
8	4	2	1	1	= [Total = 16]



(1 bit) 2 bits 3 bits 4 bits 4 bits.

$$\text{Avg. length} = \frac{8\times1+4\times2+2\times3+4\times1+4\times1}{16} = \frac{\text{Total leng}}{\text{Total freq.}}$$

$$= \frac{8+8+6+4*4}{16} = \frac{30}{16} = \frac{15}{8} = 0.188 \text{ Ans}$$

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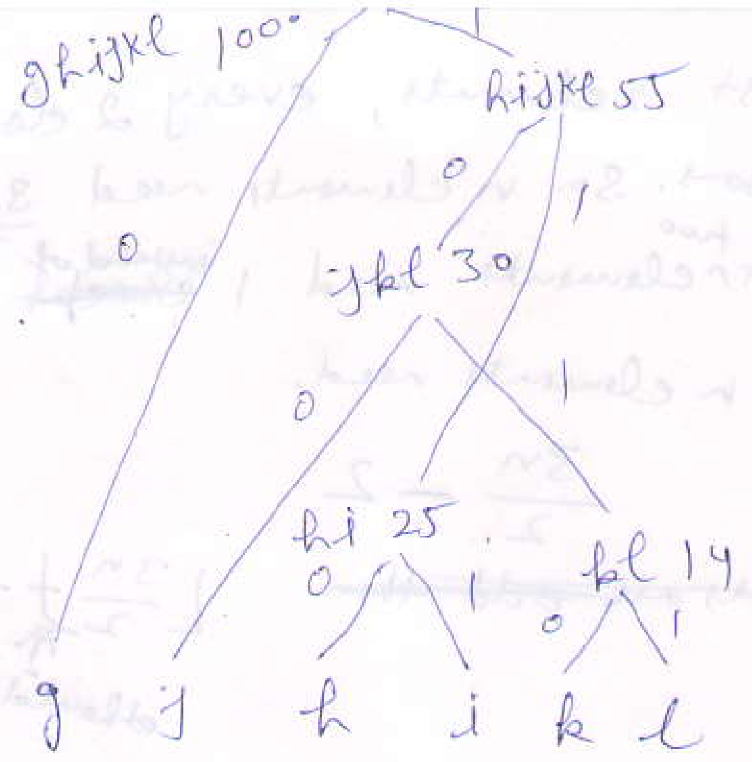
(14)



g	j	h	i	k	l
45	16	13	12	9	5
0	100	110	111	1010	1011
1 bit	3	3	3	4	4

Total bits $= 45\times1 + 16\times3 + 13\times3 + 12\times3 + 9\times4 + 5\times4$

$= 224$ K bits $= 224000$ bits Ans

Dec13(III)

(34) $A = \{a_1, a_2, a_3, a_4, a_5\}$

If $(a_1 > a_2)$
  max $= a_1$
  min $= a_2$
else
  ≡
} 1 comparison.

If $(a_3 > a_4)$
  If $(\text{Max} > a_3)$
    max $=$
  If $(a_4 < \text{Min})$
    min $=$
} 3 Comparision.

i)

except first 2 elements, every 2 elements need 3 comparisons. So n elements need $\frac{3n}{2}$ comparisons but first two elements need 1 instead of ~~except~~ 3

$\therefore$ n elements need.

$$\frac{3n}{2} - 2$$

~~if elements are odd then~~ $\lfloor \frac{3n}{2} \rfloor - 2$

should be ceiling

$$\left.\begin{matrix}4\\11\end{matrix}\right] 1$$

$$\left.\begin{matrix}3\\7\end{matrix}\right] 3 \quad \text{Total comp} = 10$$

$$\left.\begin{matrix}5\\12\end{matrix}\right] 3. \quad \text{By formula} = \left\lceil \frac{3 \times 8}{2} \right\rceil - 2$$

$$\left.\begin{matrix}14\\10\end{matrix}\right] 3 \quad = 10$$

If elements are odd then.

$$\left\lfloor \frac{3N}{2} \right\rfloor - \left\lfloor \frac{3}{2} \right\rfloor$$

$$\left.\begin{matrix}4\\11\end{matrix}\right] 1$$

$$\left.\begin{matrix}3\\7\end{matrix}\right] 3 \quad \text{Total comp} = 9.$$

$$\left.\begin{matrix}5\\12\end{matrix}\right] 3 \quad \text{By formula} = \left\lceil \frac{21}{2} \right\rceil - \lfloor 1.5 \rfloor$$

$$14 \,] 2 \quad = 10 - 1 = 9$$

If 14 > min. If 14 ≯ Max MAX=14 } for 2 elements we need = 3 comp

" 1 " " " $= \frac{3}{2} = 1.5$ com

But in case of odd numbers last element take 2 comp which 0.5 more

$\frac{3n}{2} - 2 + 0.5$

1st element last element.

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(61) $E_1 : n^{K+\epsilon} + n^K \log n = \Theta(n^{K+\epsilon})$ $K \geq 0$ & $\epsilon > 0$

$E_2 : n^3 2^n + 6n^2 3^n = O(n^3 2^n)$

Which is true

Ans:→ E_1 is correct and E_2 is not correct.

(63) The sol. of the recurrence relation of

$T(n) = 3T(\lfloor (\frac{n}{4}) \rfloor) + n$ is

(a) $O(n^2)$ (b) $O(n \lg n)$ (c) $O(n)$ ✓ (d) $O(\lg \lg n)$

(66) Suppose that the splits at every level of quicksort are in the proportion $(1-\alpha)$ to α where $0 < \alpha \leq 1/2$ is a constant. The minimum depth of a leaf in the recursion tree is approx. given by. ~~f(n)~~

Sol:→ The minimum depth occurs for the path that always takes the smaller portion of the split. ie. the nodes that takes α portion of work from the parent node. The first node in the path gets α ~~porti~~ proportion of the work (the size of data processed by this node is αn), the second one get α^2 so on. The recursion bottoms out when the size of data becomes 1. Assume the recursion ends at level h

$$\alpha^h n = 1$$

$$h = \log_\alpha \frac{1}{n} = \log(1/n)/\log \alpha = \frac{-\log n}{\log \alpha}$$

Max. depth m.

$$(1-\alpha)^m n = 1$$

$$m = \log_{1-\alpha}\left(\frac{1}{n}\right) = \frac{\log\left(\frac{1}{n}\right)}{\log(1-\alpha)} = \frac{-\log n}{\log(1-\alpha)}$$

<u>Hashing</u> :→ In the division method for creating hash functions, we map a key k into one of the m slots by taking a remainder of k divided by m. ie. hash function is

$h(K) = K \bmod m$.

m = hash table size eg. 12.

K is the key e.g 100

then $h(K) = 4$

<u>Collision</u> :→ When two keys hash to same slot.

eg. $K = 52$

then $h(K) = 4$ which is collison with prev. one.

<u>Universal hashing</u> :→ In worst case, a fixed hash function as described above can be hashed to the same slot for all n keys. The only effective way to improve the situation is to choose the hash function randomly in a way that is independent of the keys that are actually going to be sorted. This approach is called universal hashing. The main idea behind universal hashing is to select the hash function at random at runtime from a carefully designed class of functions.

24 3

Let H be a finite collection of hash functions that map a given universe U of keys into the range $\{0, 1 \ldots m-1\}$. Such a collection is said to be universal if for each pair of distinct keys $x, y \in U$, the no. of hash functions H for which $h(x) = h(y)$ is precisely $|H| = m$. In other words, with a hash function randomly chosen from H the chance of a collision b/w x and y when $x \neq y$ is exactly $1/m$, which is exactly the chance of a collision if $h(x)$ and $h(y)$ are randomly chosen from the set $\{0, 1 \ldots m-1\}$

Theorem:→ If h is chosen from a universal collection of hash functions and is used to has n keys into a table of size m where $n \leq m$, the expected number of collisions involving a particular key x is less than 1.

June 14 (11)

(24) Big O estimate for the factorial function and the logarithm of the factorial fn.

Sol. $\lfloor n = n \cdot (n-1) \cdot (n-2) \ldots n-(n-1)$
$\underset{\simeq 1}{}$

$\simeq n^n = O(n^n)$

$\log(\lfloor n) = \simeq O(\log(n^n)) = O(n \log n)$] Ans (B)

Hashing → If collisions occur alternate cells are tried until $h_i(x) = (Hash(x) + f(i)) \bmod Hsize$

$$f(0) = 0$$

For linear probing → [suffers from primary clustering]

$$f(i) = i$$

For quadratic probing → [suffers from secondary clustering]

$$f(i) = i^2$$

[Has lesser collision than linear probing]

e.g. $H(x) = x \bmod 10$ → (1)

3, 5, 13, 24, 33, 45, 54

0	
1	
2	
3	3
4	13
5	5
6	24
7	33
8	45
9	54

for 54 need to search 4 cells.

Linear →

$h_0(x) = (3+0) \bmod 10$
$= 3$

$h_0(x) = (5+0) \bmod 10 = 5$

$h_0(x) = (hash(13) + 0) \bmod 10$
$= (3+0) \bmod 10$ [using (1) $hash(13) = 3$]
$= 3$

Collision occurs.

$h_1(x) = (3+1) \bmod 10 = 4$

for 24 $h_0(x) = (4+0) \bmod 10 = 4$
$h_1(x) = (4+1) \bmod 10 = 5$
$h_2(x) = (4+2) \bmod 10 = 6$

quadratic probing :→

$h_1(x) = (H(x) + i^2) \bmod 10$

$= (3 + 0) \% 10$

$= 3$

0	54
1	
2	
3	3
4	13
5	5
6	45
7	33
8	24
9	

for 13

$h_0(x) = (3 + 0) \bmod 10$

$= 3$

Collision occurs.

$h_1(x) = (3 + 1^2) \bmod 10$

$= 4$

for 24 $h_0(x) = (4 + 0) \bmod 10$

$= 4$

$h_1(x) = (4 + 1^2) \bmod 10$

$= 5$

$h_2(x) = (4 + 2^2) \bmod 10$

$= 8$

for 33 $h_0(x) = (3 + 0) \bmod 10$

$= 3$

$h_1(x) = 4$

$h_2(x) = (3 + 2^2) \bmod 10$

$= 7$

$h_4(x) = (4 + 4^2) \bmod 10$

$= 20 \bmod 10$

$= \underline{\underline{0}}$

for 45 $h_0(x) = 5$

$h_1(x) = (5 + 1^2) \bmod 10 = 6$

for 54 $h_0(x) = 4$ $h_1(x) = (4 + 1^2) \bmod 10 = 5$

$h_2(x) = 8$ $h_3(x) = 13 \bmod 10 = 3$

For double hashing:— [Best open addressing] 27 3

$$f(i) = i * hash_2(x)$$

$hash_2(x) = R - x \bmod R$.

(*) where R is a prime no < size of Hash table

(*) The function $hash_2(x)$ must never evaluate to zero.

(*) e.g.

$h(x) = x \bmod 10$

89 18 49 58 69

89

$h_i(x) = (Hash(x) + f(i)) \bmod hsize$

$h_0(x) = (9 + 0) \bmod 10$
$= 9$

18

$h_0(x) = (8+0) \bmod 10 = 8$

49

$h_0(x) = (9+0) \bmod 10 = 9$

collision occurs.

$hash_2(x) = \{ R - x \bmod R$.
say $R = 7$
$= 7 - 49 \bmod 7 = 7$

$h_1(x) = (9 + 1 \times 7) \bmod 10 = 16 \bmod 10 = 6$

58

$h_0(x) = 8$ (collision)

$hash_2(x) = 7 - 58 \bmod 7 = 5$

$h_1(x) = (8+5) \bmod 10 = 3$

89 $h_0(x) = 9$ (Collision)

4

28

$hash_2(x) = 7 - 69 \bmod 7$

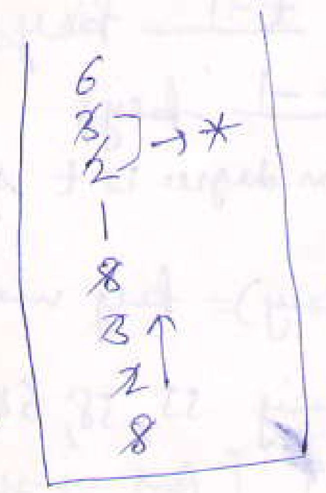
$= 7 - 6 = 1$

$h_1(x) = (9+1) \bmod 10 = 0$

(38) The following postfix exp. is evaluated using a stack (29) (6)

$$\overrightarrow{8\,2\,3\,\wedge\,/\,2\,3\,*\,+\,5\,1\,*\,-}$$

The top two elements of the stack after first * is evaluated.



Ans. (6, 1)

Dec 13 (III)

(37) The longest common subsequence of the sequences

$X = \langle A, B, C, B, D, A, B \rangle$

$Y = \langle B, D, C, A, B, A \rangle$ has length.

(A) 2 (B) 3 (C) 4 (D) 5

Sol:→ Substrings are consecutive parts of a string while subsequences need not be. Substring of a string is always a subsequence of the string, but a subsequence of a string is not always a substring of the string.

LCS. A subsequence is a sequence that can be

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derived from another sequence by deleting some elements without changing the order of the remaining elements.

L.C.S = {B, D, A B}, {B, C, A, B}

So. length = 4 Ans.

Dec 13 (II)

(24) For any B tree of minimum degree child $t \geq 2$, every node other than the root must have atleast t-1 keys and every node can have at most 2t-1 keys

No. of keys = No. of child − 1 ; As min. degree is t so max degree = 2t

[~~So~~ $\lceil \frac{m}{2} \rceil \leq n \leq m$]

Dec 12 (II)

(26) A hash function f defined as f(key) = key mod 13, with linear probing is used to insert key 55, 58, 68, 91, 27, 145. What will be the location of 79. [Ans 5th location]

(45) What is the result of the following exp.

$(1 \& 2) + (3 \& 4)$

(a) 1 (b) 3 (c) 2 (d) 0

Sol. Bitwise and of 1 & 2

01 (1) 10 (2) Bitwise And 00	011 (3) 100 (4) 000 (Bitwise And)

00 + 00 = 0 Ans

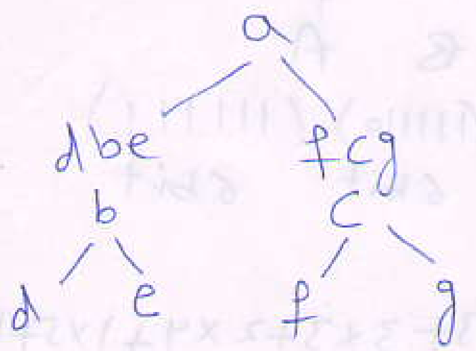
June 15 (II)

(22) The inorder and preorder Traversal of binary tree are dbeafcg and abdecfg respect. The post-order Traversal is

Inorder : d b e a f c g

Preorder : a b d e c f g

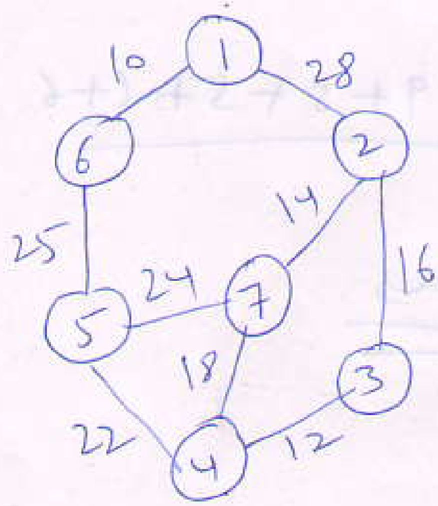
root



Postorder : L R Root

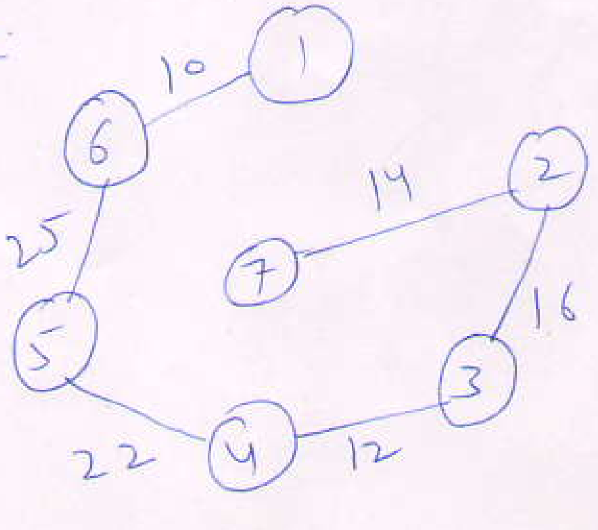
debfgca Ans

(24)



Minimum Spanning tree.

Use Kruskal's algo.



June 15 (III)

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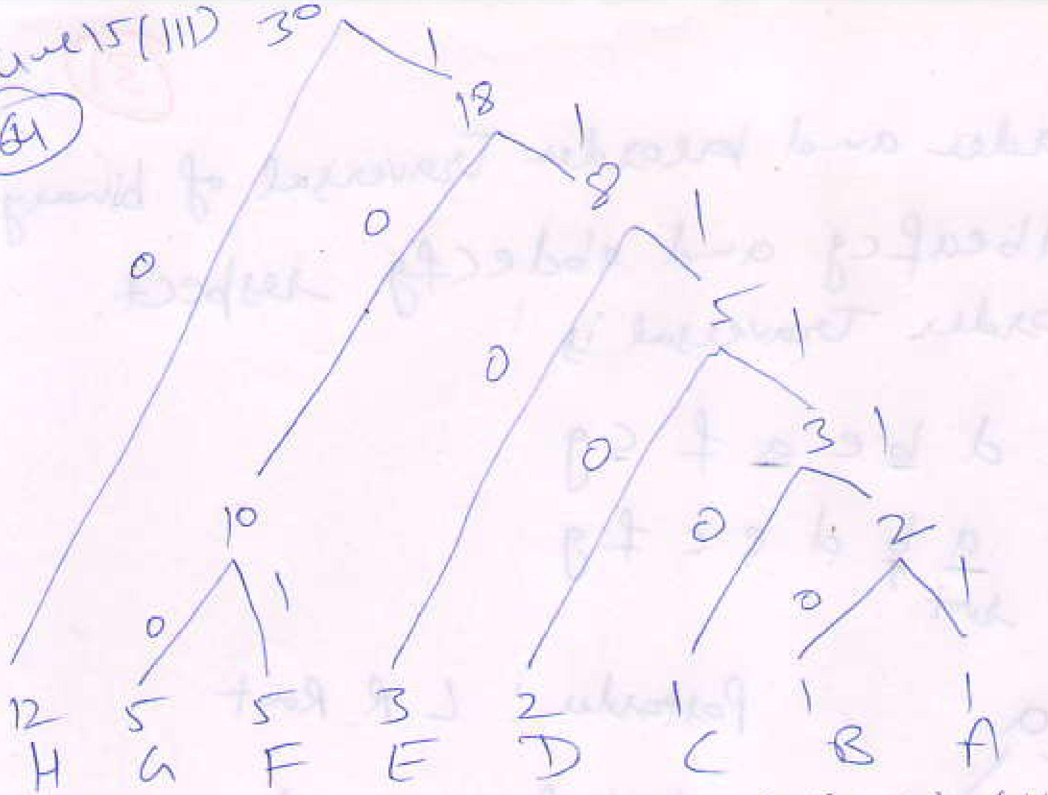

(0) (100) (101) (110) (1110) (11110) (111110) (111111)

1 bit 3bit 3bit 3bit 4bit 5bit 6bit 6bit

$$\text{Avg. size} = \frac{12\times1+5\times3+5\times3+3\times3+2\times4+1\times5+1\times6+1\times6}{30}$$

$$= \frac{12+15+15+9+8+5+6+6}{30}$$

$$= \frac{76}{30} \quad \underline{\text{Ans}}$$