Yes, $\mathcal{H}$ will always be connected. To show this, we prove the following fact.

(1) *Let $T = (V, F)$ and $T' = (V, F')$ be two spanning trees of G so that $|F - F'| = |F' - F| = k$. Then there is a path in $\mathcal{H}$ from T to T' of length k.*

Proof. We prove this by induction on k, the case $k = 1$ constituting the definition of edges in $\mathcal{H}$. Now, if $|F - F'| = k > 1$, we choose an edge $f' \in F' - F$. The tree $T \cup \{f'\}$ contains a cycle C, and this cycle must contain an edge $f \notin F'$. The tree $T \cup \{f'\} - \{f\} - T'' - (V, F'')$ has the property that $|F'' - F'| = |F' - F''| = k - 1$. Thus, by induction, there is a path of length $k - 1$ from T'' to T'; since T and T'' are neighbors, it follows that there is a path of length k from T to T'. ∎

[1] ex135.857.224