

Given a set  $X$  of vertices, we can use depth-first search to determine if  $G - X$  has no cycles. Thus *undirected feedback set* is in NP.

We now show that *vertex cover* can be reduced to *undirected feedback set*. Given a graph  $G = (V, E)$  and integer  $k$ , construct a graph  $G' = (V', E')$  in which each edge  $(u, v) \in E$  is replaced by the four edges  $(u, x_{uv}^1)$ ,  $(u, x_{uv}^2)$ ,  $(v, x_{uv}^1)$ , and  $(v, x_{uv}^2)$  for new vertices  $x_{uv}^i$  that appear only incident to these edges. Now, suppose that  $X$  is a vertex cover of  $G$ . Then viewing  $X$  as a subset of  $V'$ , it is easy to verify that  $G' - X$  has no cycles. Conversely, suppose that  $Y$  is a feedback set of  $G'$  of minimum cardinality. We may choose  $Y$  so that it contains no vertex of the form  $x_{uv}^i$  — for it does, then  $Y \cup \{u\} - \{x_{uv}^i\}$  is a feedback set of no greater cardinality. Thus, we may view  $Y$  as a subset of  $V$ . For every edge  $(u, v) \in E$ ,  $Y$  must intersect the four-node cycle formed by  $u, v, x_{uv}^1$ , and  $x_{uv}^2$ ; since we have chosen  $Y$  so that it contains no node of the form  $x_{uv}^i$ , it follows that  $Y$  contains one of  $u$  or  $v$ . Thus,  $Y$  is a vertex cover of  $G$ .

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<sup>1</sup>ex867.590.603