We will use the following simple algorithm. Consider triples of T in any order, and add them if they do not conflict with previously added triples. Let M denote the set returning by this algorithm and M^* be the optimal three-dimensional matching.

(1) *The size of M is at least $1/3$ of the size of M^*.*

Proof. Each triple (a, b, c) in M^* must intersect at least one triple in our matching M (or else we could extend M greedily with (a, b, c)). One triple in M can only be in conflict with at most 3 triples in M^* as edges in M^* are disjoint. So M^* can have at most 3 times as many edges as M has. ∎

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[1] ex271.721.76

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