(a) Let $a_1 = 1$ and $a_2 = 100$, and consider the bound $B = 100$. Only a_1 will be chosen, while an optimal solution would choose a_2.

(b) In fact, this can be done in $O(n)$ time. We first go through all the numbers a_i and delete any whose value exceeds B — such numbers cannot be used in any solution, including the optimal one, so we have not changed the value of the optimum by doing this.

We then go through the numbers $a_1, a_2, \ldots, a_n$ in order until the sum of numbers we've seen so far first exceeds B. Let a_j be the number on which this happens. Thus we have $\sum_{i-1}^{j} a_i \geq B$, but $\sum_{i-1}^{j-1} a_i \leq B$ and also $a_j \leq B$. Thus, one of the sets $\{a_1, a_2, \ldots, a_{j-1}\}$ or $\{a_j\}$ is at least $B/2$ and at most B; we select this set as our solution. Since the optimum has a sum of at most B, our solution is at least half the optimal value.

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[1] ex650.691.264