The claim is false; we show that for every natural number c, there exists a graph G so that $diam(G)/apd(G) > c$. First, we fix a number k (the relation to c will be determined later), and consider the following graph. We take a path on $k-1$ nodes $u_1, u_2, \ldots, u_{k-1}$ in this order. We then attach $n-k+1$ additional nodes $v_1, v_2, \ldots, v_{n-k+1}$, each by a single edge, to u_1; the number n will also be chosen below.

The diameter of G is equal to $dist(v_1, u_{k-1}) = k$. It is not difficult to work out the exact value of $apd(G)$; but we can get a simple upper bound as follows. There are at most kn 2-element sets with at least one element from $\{u_1, u_2, \ldots, u_{k-1}\}$. Each of these pairs is at most distance $\leq k$. The remaining pairs are all distance at most 2. Thus

$$apd(G) \leq \frac{2\binom{n}{2} + k^2 n}{\binom{n}{2}} \leq 2 + \frac{2k^2}{n-1}.$$

Now, if we choose $n-1 > 2k^2$, then we have $apd(G) < 3$. Finally, choosing $k > 3c$, we have $diam(G)/apd(G) > 3c/3 = c$.

[1] ex85.422.171