First we need to make sure that the problem is in NP. This is as in the previous cases easy to see. Given an initial set of size k, we can just run the influence algorithm mentioned in the assignment and obtain a final influence set, and then we can check whether the size of the influence set is at least b. The algorithm is polynomial, since at each iteration we are either terminating or at least adding another vertex to the already influenced list.

Now we need to prove that this problem is also NP-complete. We reduce from vertex cover. Let $G = (V, E)$ be a graph and k be a parameter for the instance of the vertex cover problem. We will create a node for each of the vertices v in G as well as for each of the edges e in G (we will call them vertex-nodes and edge-nodes). There is an edge (v, e) if v is one of the ends of the edge e. All the thresholds on nodes are $1/2$. We want to claim that a vertex cover of size k corresponds to a initial set of size k with the resulting influence set of size $(k + m)$ where m is the number of edges in G, and vice versa.

One direction is easy - if there is a vertex cover of size k then this vertex cover will infect all the edges, therefore we will have a influenced set of size $(k + m)$.

Now if there is an influenced set of size $(k + m)$ from an original set of k nodes, we claim that there is a vertex cover of size k. Notice that if there is an edge-node in the k size original set, then we can replace it by one of the vertex nodes adjacent to it, and the edge will get infected immediately. So therefore doing this operation repeatedly, we will get a set of size k consisting entirely of vertices that infects $(k + n)$ nodes. But this set would be a vertex cover, since each edge would have to have a neighbor among the original vertex nodes, otherwise it would not be infected.

[1] ex364.229.826