

The basic idea is to ask: How should we gerrymander precincts 1 through  $j$ , for each  $j$ ? To make this work, though, we have to keep track of a few extra things, by adding some variables. For brevity, we say that the  $A$ -votes in a precinct are the votes for party  $A$ , and  $B$ -votes are the votes for party  $B$ . We keep track of the following information about a partial solution.

- How many precincts have been assigned to district 1 so far?
- How many  $A$ -votes are in district 1 so far?
- how many  $A$ -votes are in district 2 so far?

So let  $M[j, p, x, y] = \text{true}$  if it is possible to achieve at least  $x$   $A$ -votes in district 1 and  $y$   $A$ -votes in district 2, while allocating  $p$  of the first  $j$  precincts to district 1. ( $M[j, p, x, y] = \text{false}$  otherwise.) Now suppose precinct  $j + 1$  has  $z$   $A$ -votes. To compute  $M[j + 1, p, x, y]$ , you either put precinct  $j + 1$  in district 1 (in which case you check the results of sub-problem  $M[j, p - 1, x - z, y]$ ) or in precinct 2 (in which case you check the results of sub-problem  $M[j, p, x, y - z]$ ). Now to decide if there's a solution to the whole problem, you scan the entire table at the end, looking for a value of "true" in any entry of the form  $M[n, n/2, x, y]$ , where each of  $x$  and  $y$  is greater than  $mn/4$ . (Since each district gets  $mn/2$  votes total.)

We can build this up in order of increasing  $j$ , and each sub-problem takes constant time to compute, using the values of smaller sub-problems. Since there are  $n^2m^2$  sub-problems, the running time is  $O(n^2m^2)$ .

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<sup>1</sup>ex706.269.18