(a) This problem can be solved using network flow. We construct a graph with a node v_i for each cannister, a node w_j for each truck, and an edge (v_i, w_j) of capacity 1 whenever cannister i can go in truck j. We then connect a super-source s to each of the cannister nodes by an edge of capacity 1, and we connect each of the truck nodes to a super-sink t by an edge of capacity k.

We claim that there is a feasible way to place all cannisters in trucks if and only if there is an s-t flow of value n. If there is a feasible placement, then we send one unit of flow from s to t along each of the paths s, v_i, w_j, t, where cannister i is placed in truck j. This does not violate the capacity conditions, in particular on the edges (w_j, t), due to the capacity constraints. Conversely, if there is a flow of value n, then there is one with integer values. We place cannister i in truck j if the edge (v_i, w_j) carries one unit of flow, and we observe that the capacity condition ensures that no truck is overloaded.

The running is the time required to solve a max-flow problem on a graph with $O(m+n)$ nodes and $O(mn)$ edges.

(b) When there are conflicts between pairs of cannisters, rather than between cannisters and trucks, the problem becomes NP-complete.

We show how to reduce *3-Coloring* to this problem. Given a graph G on n nodes, we define a cannister i for each node v_i. We have three trucks, each of capacity $k = n$, and we say that two cannisters cannot go in the same truck whenever there is an edge between the corresponding nodes in G.

Now, if there is a 3-coloring of G, then we can place all the cannisters corresponding to nodes assigned the same coloring in a single truck. Conversely, if there is a way to place all the cannisters in the three trucks, then we can use the truck assignments as colors; this gives a 3-coloring of the graph.

[1] ex460.602.46