

We build the following flow network. There is a node  $v_i$  for each patient  $i$ , a node  $w_j$  for each hospital  $j$ , and an edge  $(v_i, w_j)$  of capacity 1 if patient  $i$  is within a half hour drive of hospital  $j$ . We then connect a super-source  $s$  to each of the patient nodes by an edge of capacity 1, and we connect each of the hospital nodes to a super-sink  $t$  by an edge of capacity  $\lceil n/k \rceil$ .

We claim that there is a feasible way to send all patients to hospitals if and only if there is an  $s$ - $t$  flow of value  $n$ . If there is a feasible way to send patients, then we send one unit of flow from  $s$  to  $t$  along each of the paths  $s, v_i, w_j, t$ , where patient  $i$  is sent to hospital  $j$ . This does not violate the capacity conditions, in particular on the edges  $(w_j, t)$ , due to the load constraints. Conversely, if there is a flow of value  $n$ , then there is one with integer values. We send patient  $i$  to hospital  $j$  if the edge  $(v_i, w_j)$  carries one unit of flow, and we observe that the capacity condition ensures that no hospital is overloaded.

The running time is the time required to solve a max-flow problem on a graph with  $O(n + k)$  nodes and  $O(nk)$  edges.

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<sup>1</sup>ex297.369.93