First, we claim the problem is in NP. For consider any set of k of the functions $f_{i_1}, \ldots, f_{i_k}$. If q is the maximum number of "break-points" in the piecewise linear representation of any one of them, then $F = \max(f_{i_1}, \ldots, f_{i_k})$ has at most k^2q^2 break-points. Between each pair of break-points, we can compute the area under F by computing the area of a single trapezoid; thus we can compute the integral of F in polynomial time to verify a purported solution.

We now show how the Vertex Cover problem could be solved using an algorithm for this problem. Given an instance of Vertex Cover with graph $G = (V, E)$ and bound k, we write $V = \{1, 2, \ldots, n\}$ and $E = \{e_0, \ldots, e_{m-1}\}$. We construct a function f_i for each vertex i as follows. First, let $t = 2m - 1$, so each f_i will be defined over $[0, 2m - 1]$. If e_j is incident on i, we define $f_i(x) = 1$ for $x \in [2j, 2j + 1]$; if e_j is not incident on i, we define $f_i(x) = 0$ for $x \in [2j, 2j + 1]$. We also define $f_i(x) = \frac{1}{2}$ for each x of the form $2j + \frac{3}{2}$. Finally, to define $f_i(x)$ for $x \in [2j + 1, 2j + 2]$ for an integer $j \in \{0, \ldots, m - 2\}$, we simply connect $f_i(2j + 1)$ to $f_i(2j + \frac{3}{2})$ to $f_i(2j + 2)$ by straight lines.

Now, if there is a vertex cover of size k, then the pointwise maximum of these k functions has covers an area of 1 on each interval of the form $[2j, 2j + 1]$ and an area of $\frac{3}{4}$ on each interval of the form $[2j + 1, 2j + 2]$, for a total area of $B = m + \frac{3}{4}(m - 1)$. Conversely, any k functions that cover this much area must cover an area of 1 on each interval of the form $[2j, 2j + 1]$, and so the corresponding nodes constitute a vertex cover of size k.

[1] ex561.283.906