

Given a set of n specimens and m judgments, we need to determine if the set of judgments are consistent. To be able to label each specimen either A or B , we construct an undirected graph $G = (V, E)$ as follows: Each specimen is a vertex. There is an edge between v_i and v_j if there is a judgment involving the corresponding specimens.

Once the graph is constructed, arbitrarily designate some vertex as the starting node s . Note that the graph G need not be connected in which case we will need starting nodes for each component. For each component G_i (with starting node s_i) of G label the specimen associated with s_i A . Now perform Breadth-First Search on G_i starting at s_i . For each node v_k that is visited from v_j , consider the judgment made on the specimens corresponding to (v_j, v_k) . If the judgment was “same,” label v_k the same as v_j and if the judgment was “different,” label v_k the opposite of v_j . Note that there may be some specimens that are not associated with any judgments. These specimens maybe labeled arbitrarily, but we shall label them A . Once the labeling is complete we may go through the list of judgments to check for consistency. More precisely (Refer to pg. 39 of the text)

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For each component C of G
  designate a starting node s and label it A
  Mark s as ‘‘Visited.’’
  Initialize R=s.
  Define layer L(0)=s.
  For i=0,1,2,...
    For each node u in L(i)
      Consider each edge (u,v) incident to v
      If v is not marked ‘‘Visited’’ (then v is also not labeled)
        Mark v ‘‘Visited’’
        If the judgment (u,v) was ‘‘same’’ then
          label v the same as u
        else (the judgment was ‘‘different’’)
          label v the opposite of u
        Endif
      Add v to the set R and to layer L(i+1)
    Endif
  Endfor
Endfor
Endfor
For each edge (u,v) (for each judgment (u,v))
  If the judgment was ‘‘same’’
    If u and v have different labels
      there is an inconsistency
    Endif
  Else (the judgment was ‘‘different’’)

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        If u and v have the same labels
            there is an inconsistency
        Endif
    Endif
Endfor

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First note that the running time of this algorithm is $O(m + n)$: Constructing G takes $O(m + n)$ since it has n vertices and m edges. Performing *BFS* on G takes $O(m + n)$ and going through the list of judgments to check consistency takes $O(m)$. Thus the running time is $O(m + n)$.

It is easily shown that if the labeling produced by the *BFS* is inconsistent, then the set of judgments is inconsistent. Note that this *BFS* labeling uses a subset of the judgments (the edges of the resulting *BFS* tree). Further the *BFS* labeling is the only possible labeling with the exception of inverting the labeling in each component of G , i.e. switching A and B . Thus if an inconsistency is found in this labeling then surely the entire set of m judgments cannot be consistent. On the other hand if the labeling is consistent with respect to the m judgments, we are done.