

Consider a graph G with nodes s and t , and $n - 2$ other nodes v_1, \dots, v_{n-2} . There are two parallel edges from s to each v_i , and one edge from v_i to t . The minimum s - t cut is to separate t by itself.

If we run the version of the contraction algorithm described in the problem, it will independently contract each of the length-2 paths from s to t in some order. In order for it to find the minimum s - t cut, it must contract each v_i into s , not into t . There is a $2/3$ chance of this happening for each i , so the probability that the minimum s - t cut is found is $(2/3)^{n-2}$, an exponentially small quantity.

(Note that this example poses no problem for the global minimum cut, which consists of any of the nodes v_i on its own.)

¹ex242.186.32