Clearly if any of the putative degrees d_i is equal to 0, then this must be an isolated node in the graph; thus we can delete d_i from the list and continue by recursion on the smaller instance.

Otherwise, all d_i are positive. We sort the numbers, relabeling as necessary, so that $d_1 \geq d_2 \geq \cdots \geq d_n > 0$. We now look at the list of numbers

$$L = \{d_1 - 1, d_2 - 1, \ldots, d_{d_n} - 1, d_{d_n+1}, \ldots, d_{n-2}, d_{n-1}\}.$$

(In other words, we subtract 1 from the first d_n numbers, and drop the last number.) We claim that there exists a graph whose degrees are equal to the list $d_1, \ldots, d_n$ if and only if there exists a graph whose degrees form the list L. Assuming this claim, we can proceed recursively.

Why is the claim true? First, if there is a graph with degree sequence L, then we can add an n^{th} node with neighbors equal to nodes $v_1, v_2, \ldots, v_{d_n}$, thereby obtaining a graph with degree sequence $d_1, \ldots, d_n$. Conversely, suppose there is a graph with degree sequence $d_1, \ldots, d_n$, where again we have $d_1 \geq d_2 \geq \cdots \geq d_n$. We must show that in this case, there is in fact such a graph where node v_n is joined to precisely the nodes $v_1, v_2, \ldots, v_{d_n}$; this will allow us to delete node n and obtain the list L. So consider any graph G with degree sequence $d_1, \ldots, d_n$; we show how to transform G into a graph where v_n is joined to $v_1, v_2, \ldots, v_{d_n}$. If this property does not already hold, then there exist $i < j$ so that v_n is joined to v_j but not v_i. Since $d_i \geq d_j$, it follows that there must be some v_k not equal to any of v_i, v_j, v_n with the property that (v_i, v_k) is an edge but (v_j, v_k) is not. We now replace these two edges by (v_i, v_n) and (v_j, v_k). This keeps all degrees the same; and repeating this transformation will convert G into a graph with the desired property.

[1] ex168.851.857