Let c_e denote the cost of the edge e and we will overload the notation and write c_{st} to denote the cost of the edge between the nodes s and t.

This problem is by its nature quite similar to the shortest path problem. Let us consider a two-parameter function $Opt(i, s)$ denoting the optimal cost of shortest path to s using *exactly* i edges, and let $N(i, s)$ denote the number of such paths.

We start by setting $Opt(i, v) = 0$ and $Opt(i, v') = \infty$ for all $v' \neq v$. Also set $N(i, v) = 1$ and $N(i, v') = 0$ for all $v' \neq v$. Intuitively this means that the source v is reachable with cost 0 and there is currently one path to achieve this.

Then we compute the following recurrence:

$$Opt(i, s) = min_{t,(t,s)\in E}\{Opt(i-1, t) + c_{ts}\}. \tag{1}$$

The above recurrence means that in order to travel to node s using exactly i edges, we must travel a predecessor node t using exactly $i-1$ edges and then take the edge connecting t to s. Once of course the optimal cost value has been computed, the number of paths that achieve this optimum would be computed by the following recurrence:

$$N(i, s) = \sum_{t,(t,s)\in E \ \ and \ \ Opt(i,s)=Opt(i-1,t)+c_{ts}} N(i-1, t). \tag{2}$$

In other words, we look at all the predecessors from which the optimal cost path may be achieved and add all the counters.

The above recurrences can be calculated by a double loop, where the outside loops over i and the inside loops over all the possible nodes s. Once the recurrences have been solved, our target optimal path to w is obtained by taking the minimum of all the paths of different lengths to w - that is:

$$Opt(w) = min_i\{Opt(i, w)\}. \tag{3}$$

And the number of such paths can be computed by adding up the counters of all the paths which achieve the minimal cost.

$$N(w) = \sum_{i,Opt(i,w)=Opt(w)} N(i, w). \tag{4}$$

[1] ex720.859.203