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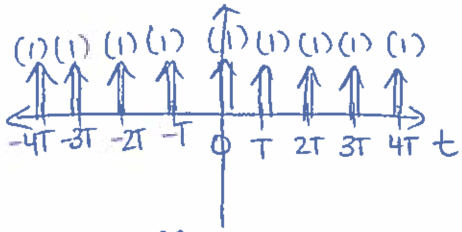
Problem Set 7

Due: 12/04/2015

(1) a)



b) Fourier Series Representation

$$\sum_{k=-\infty}^{\infty}\delta(t-kT)=p(t)$$

$$C_k=\frac{1}{T}\int_{-T/2}^{T/2}p(t)e^{-j\frac{2\pi}{T}kt}dt$$

$$=\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=-\infty}^{\infty}\delta(t-kT)e^{-j\frac{2\pi}{T}kt}dt$$

$$=\frac{1}{T}\int_{-T/2}^{T/2}\delta(t)e^{-j\frac{2\pi}{T}kt}dt$$

$$=\frac{1}{T}$$

c) $$x(t)=\sum_{k=-\infty}^{\infty}C_k e^{j\frac{2\pi}{T}kt}$$

$$X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$$

$$=\int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}C_k e^{j\frac{2\pi}{T}kt}e^{-j\omega t}dt$$

$$=\sum_{k=-\infty}^{\infty}C_k\int_{-\infty}^{\infty}e^{j(\frac{2\pi}{T}k-\omega)t}dt$$

$$=\sum_{k=-\infty}^{\infty}C_k 2\pi\delta(2\omega-\frac{2\pi}{T}k)$$

Sidenote: $e^{j\omega_0 t}$ ←

↓

$2\pi\delta(\omega-\omega_0)$

$\therefore\ \omega-(\frac{2\pi}{T}k-\omega)$

$=2\omega-\frac{2\pi}{T}k$

d) $$P(\omega)=\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(2\omega-\frac{2\pi}{T}k)$$

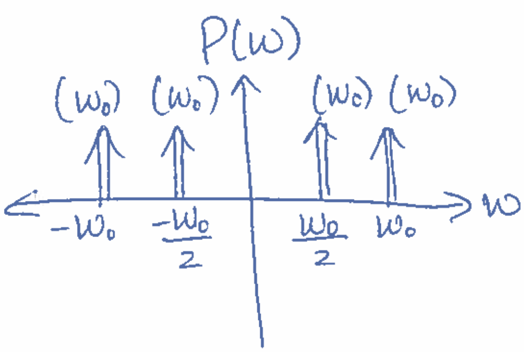
$$\omega_0=\frac{2\pi}{T}$$

$$P(\omega)=\omega_0\sum_{k=-\infty}^{\infty}\delta(2\omega-\omega_0 k)$$

e) Changing T:

→ as $T\to\infty$, $P(\omega)$ and $p(t)$ approach 0

→ as $T\to 0$, $P(\omega)$ and $p(t)$ approach ∞



(2) a) $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(w)e^{jwt}\,dw$

$h(t) = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwt}\,dw$

$-w_c < w < w_c \rightarrow H(w) = 1$

These are equal expressions because the signal is 0 after w_c and before $-w_c$

$h(t) = \frac{1}{2\pi}\left[\frac{1}{jt}e^{jwt}\right]_{-w_c}^{w_c}$

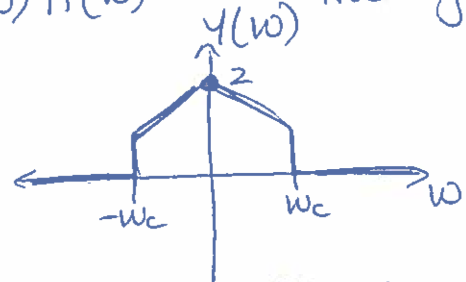
$h(t) = \frac{1}{\pi t}\left(\frac{1}{2j}e^{jw_ct} - \frac{1}{2j}e^{-jw_ct}\right)$

$h(t) = \frac{1}{\pi t}\sin(w_ct)$

b) $y(t) = x * h(t)$

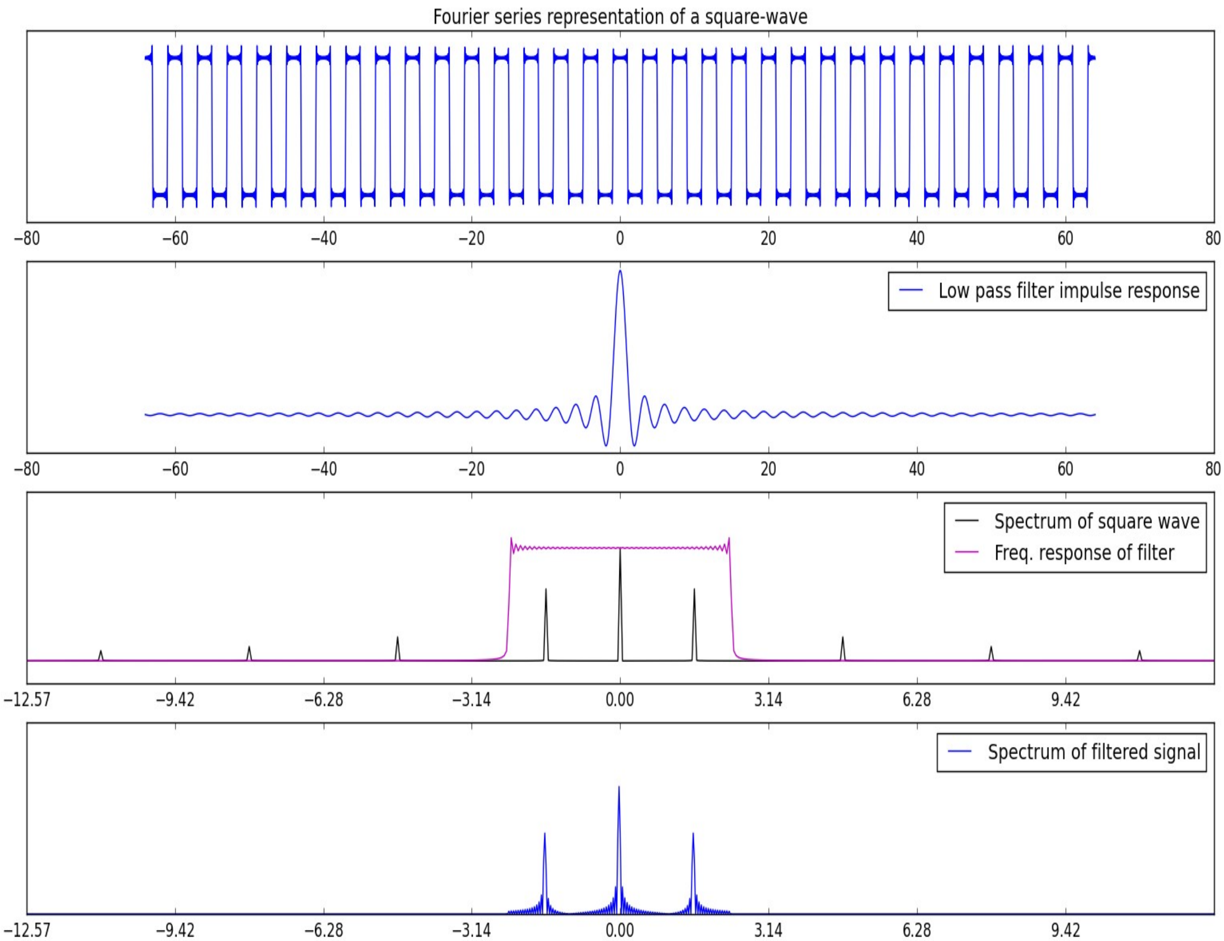
$Y(w) = X(w)H(w)$

$\therefore$ Between $-w_c$ and w_c, $Y(w)$ is the same as $X(w)$. Otherwise, the signal is 0.

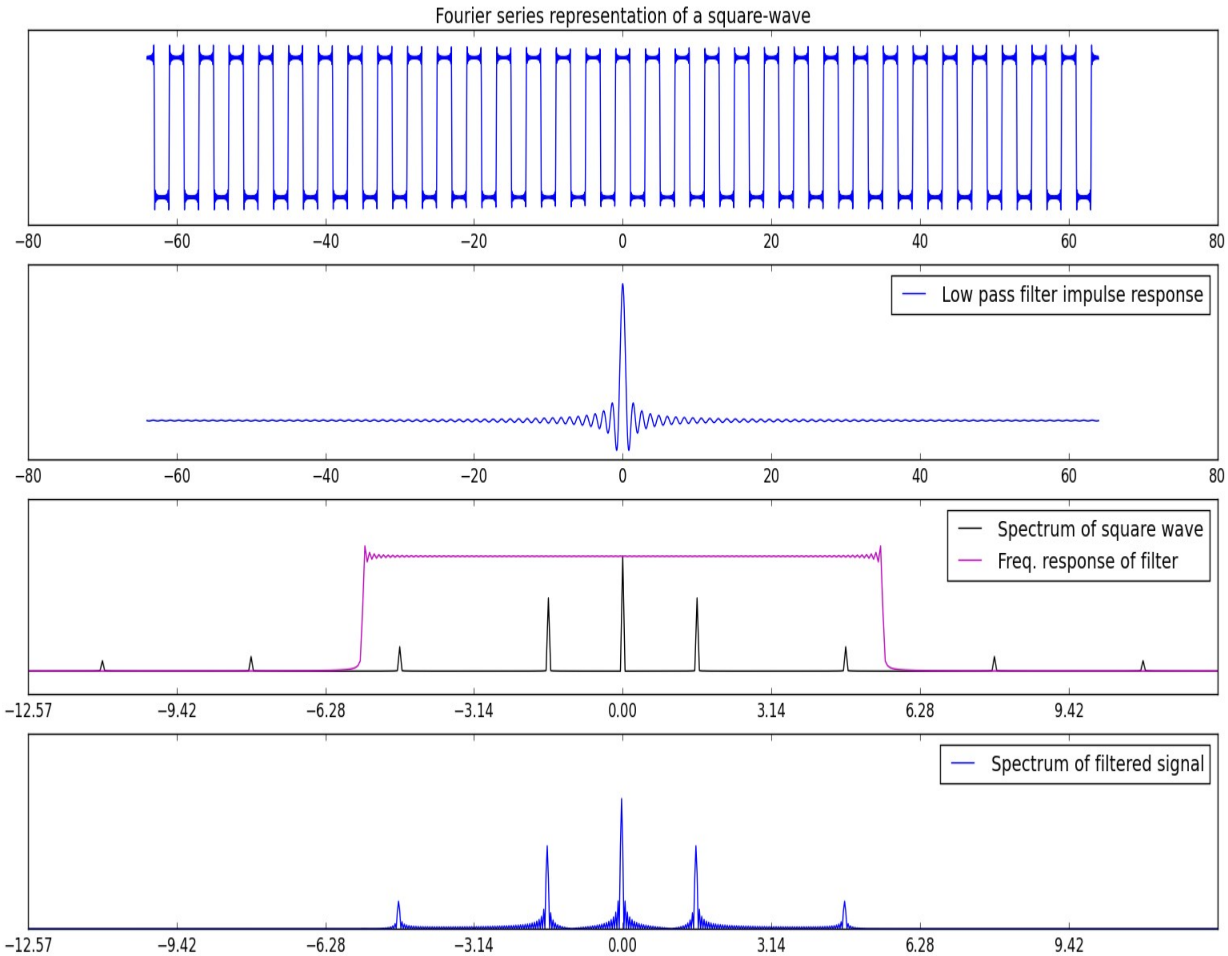


c) Because it is able to cutoff higher frequencies ($>|w_c|$) while keeping the original signal the same.

d) Plots are attached (next pages)

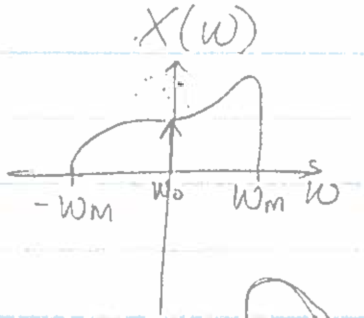


The graphs above show my results when I input my h(t) function calculated earlier on in the question. The value for the cutoff frequency is set to 0.75*pi, which is validated by the third graph where you can see that the frequency cuts off slightly under the value of pi.

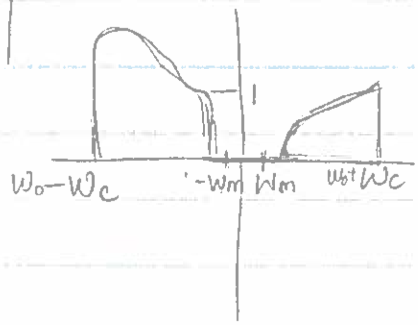


I inputted the h(t) function calculated earlier on in the question with the value of the cutoff frequency set to 1.75*pi, which is can be seen in the third graph. You can see that the frequency cuts off slightly between the values of pi and 2*pi (closer to the value of 2*pi).

(3) $y(t) = x(t)\cos(w_c t)$



Multiplication in the time domain is convolution in the frequency domain.



$$Y(w) = \frac{1}{2\pi} X * H(w)$$

$$\cos(w_0 t) \longrightarrow \pi\delta(w - w_0) + \pi\delta(w + w_0)$$

$$H(w) = \pi\delta(w - w_c) + \pi\delta(w + w_c)$$

Looking at the graphs drawn, since $w_c >> w_m$, and assuming that the points between $-w_m$ and w_m don't overlap within the graph, $X(w) = Y(w)$ but if the graph overlaps then the graph will be scaled by 0.5.

<u>Radio</u>

$X(w)$ - radio signal being sent

$Y(w)$ - signal received

$H(w)$ - reproduces many signals at different frequency levels