

PROBLEM SET 10

ABIGAIL RODRIQUES

i. $x = \dot{y} + y$

$$x(t) \rightarrow h(t) \rightarrow (x * h)(t)$$

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}$$

$$\begin{array}{c} \downarrow \mathcal{L} \\ X(s) \end{array} \rightarrow \begin{array}{c} \uparrow \mathcal{L}^{-1} \\ H(s) \end{array} \rightarrow Y(s) = H(s)X(s)$$

$$x = \dot{y} + y$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1} \quad \text{Roc: } \text{Re}\{s\} > -1$$

$$\downarrow \mathcal{L}$$

$$X = sY + Y$$

$$Y(s+1) = X$$

$$Y(s) = H(s) \mathcal{L}\{u(t)\}$$

$$= \frac{1}{s+1} \cdot \frac{1}{s}$$

$$\leftarrow = \frac{1}{s(s+1)} = \frac{A}{s+0} + \frac{B}{s+1}$$

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$A(s+1) + B(s) = 1$$

$$\text{when } s = -1, B = -1$$

$$\text{when } s = 0, A = 1$$

$$y(t) = Ae^{-st}u(t) + Be^{-st}u(t)$$

$$= (1)e^{-0t}u(t) + (-1)e^{-(-1)t}u(t)$$

$$= u(t) - e^t u(t)$$

$$= (1 - e^t)u(t)$$

2. A. $H(s) = \frac{1/\tau}{s + 1/\tau}$ $K(s) = \frac{K_I}{s}$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{K_I}{s} \left(\frac{1/\tau}{s + 1/\tau} \right)}{1 + \frac{K_I}{s} \left(\frac{1/\tau}{s + 1/\tau} \right)}$$

$$= \frac{K_I/s \div \tau}{s + (K_I + 1)/\tau} = \frac{K_I/\tau}{s^2 + (K_I + 1)/\tau}$$

$$\lim_{s \rightarrow 0} \left(\frac{K_I/\tau}{s^2 + (K_I + 1)/\tau} \right) = 1$$

as $s \rightarrow 0$, DC gain $\rightarrow 1$

B. $s^2 + \frac{(K_I + 1)}{\tau} = 0$

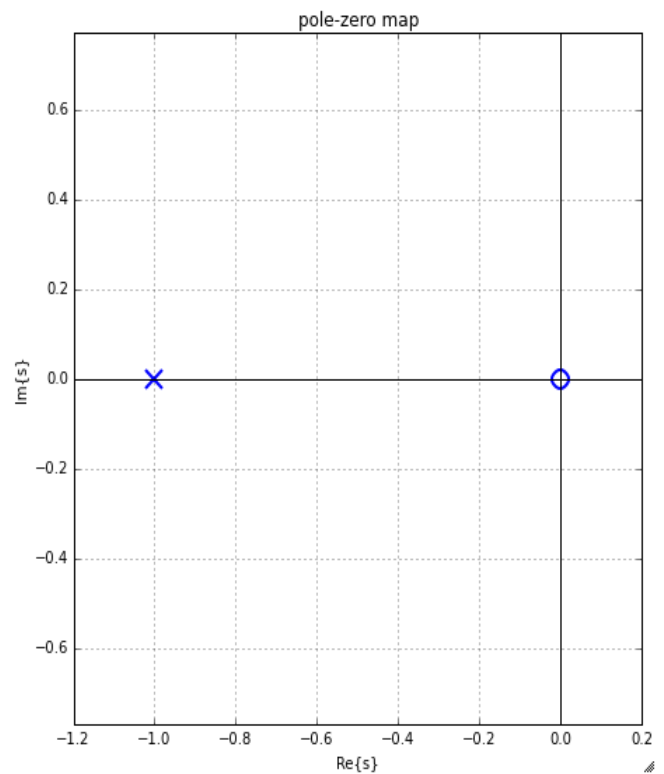
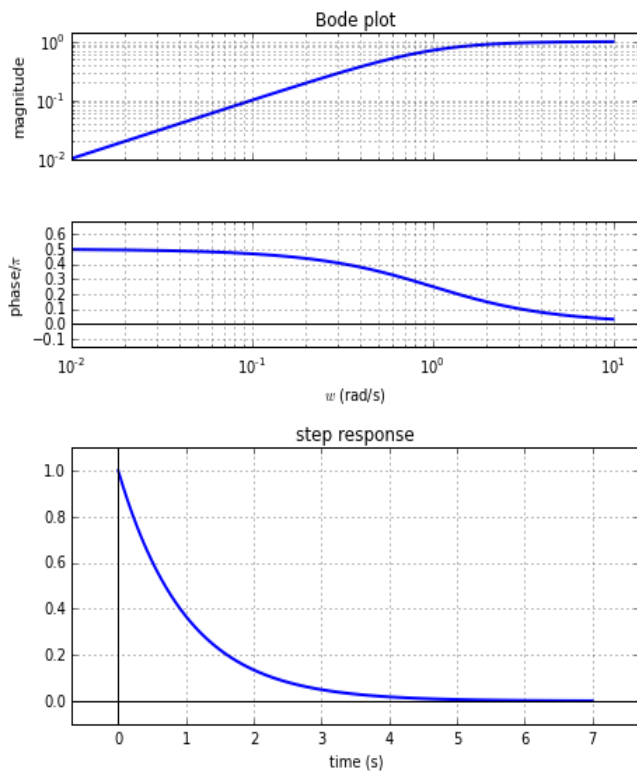
$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

if $K \gg 1/\tau$, poles are $\pm \sqrt{\frac{K_I}{\tau}}$

$$s = \frac{-\frac{1}{\tau} \pm \sqrt{\left(\frac{1}{\tau}\right)^2 - 4(1)\left(\frac{K_I}{\tau}\right)}}{2(1)}$$

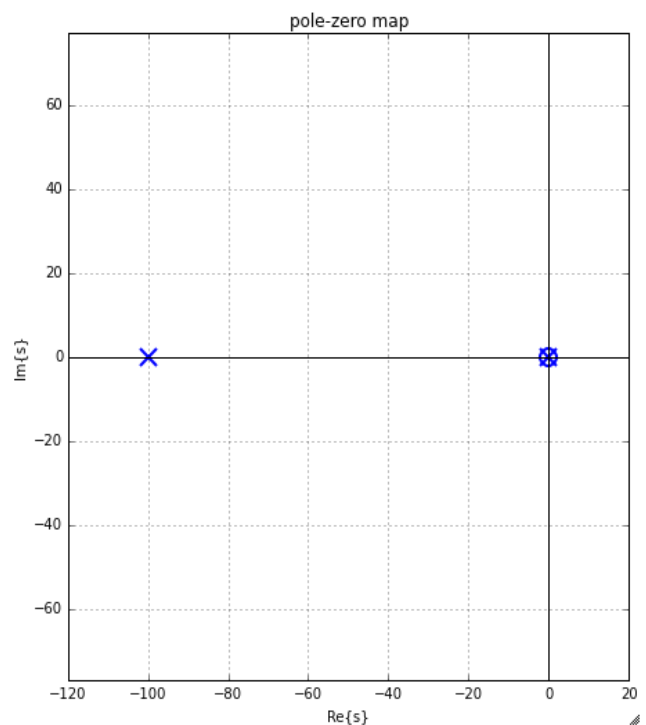
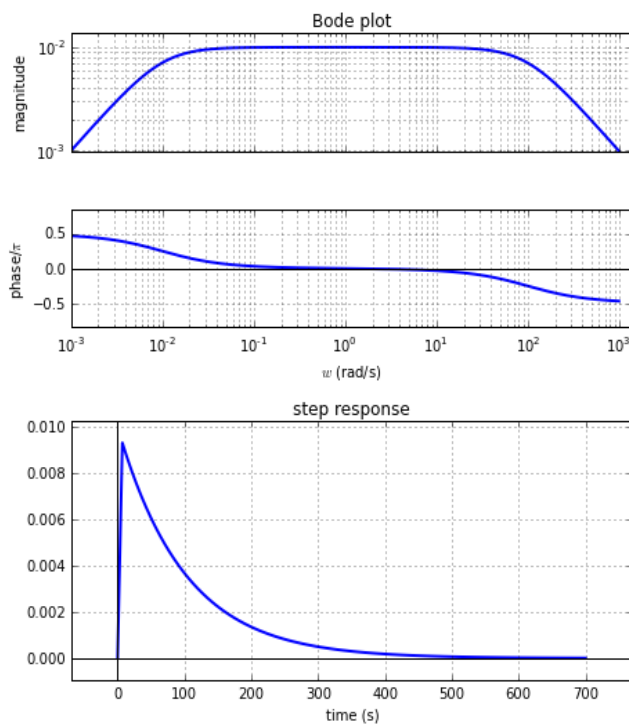
$$s = \underbrace{-\frac{1}{2\tau}}_{\text{Re}} \pm \underbrace{\sqrt{\frac{1}{4\tau^2} - \frac{K_I}{\tau}}}_{\text{Imag.}}$$

3.
A.



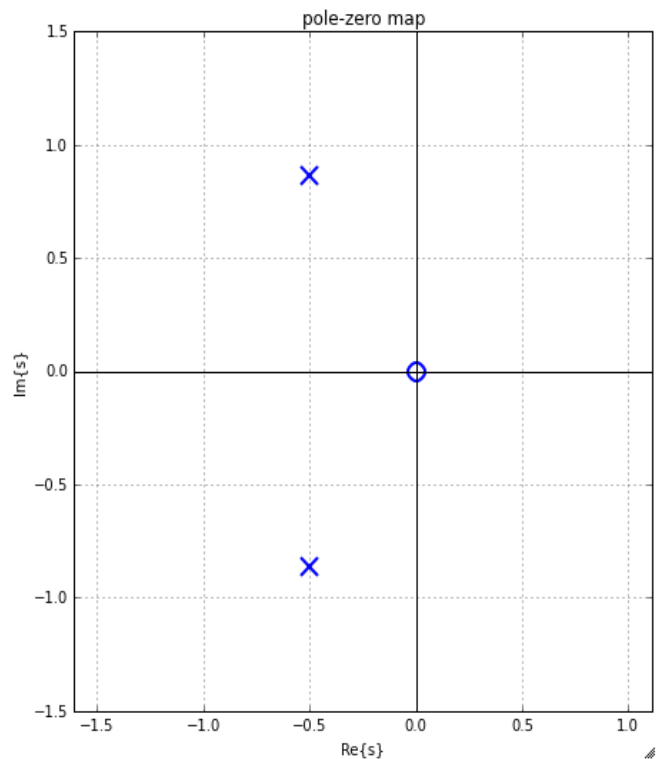
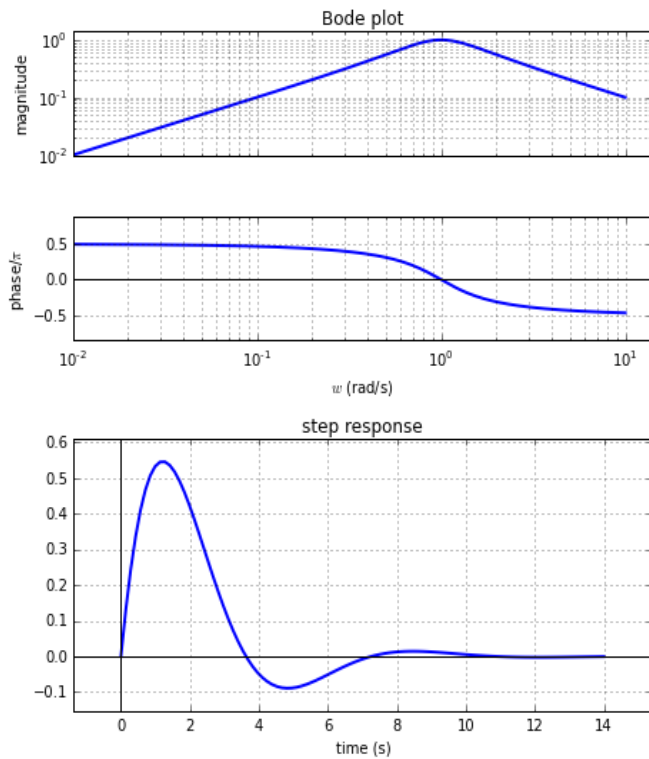
This system acts like a high-pass filter, since it cuts off lower frequencies (seen in the Bode plot). There is a pole at 0 and a zero at approximately 0. By looking at the step response, we can tell that the system stabilizes.

B.



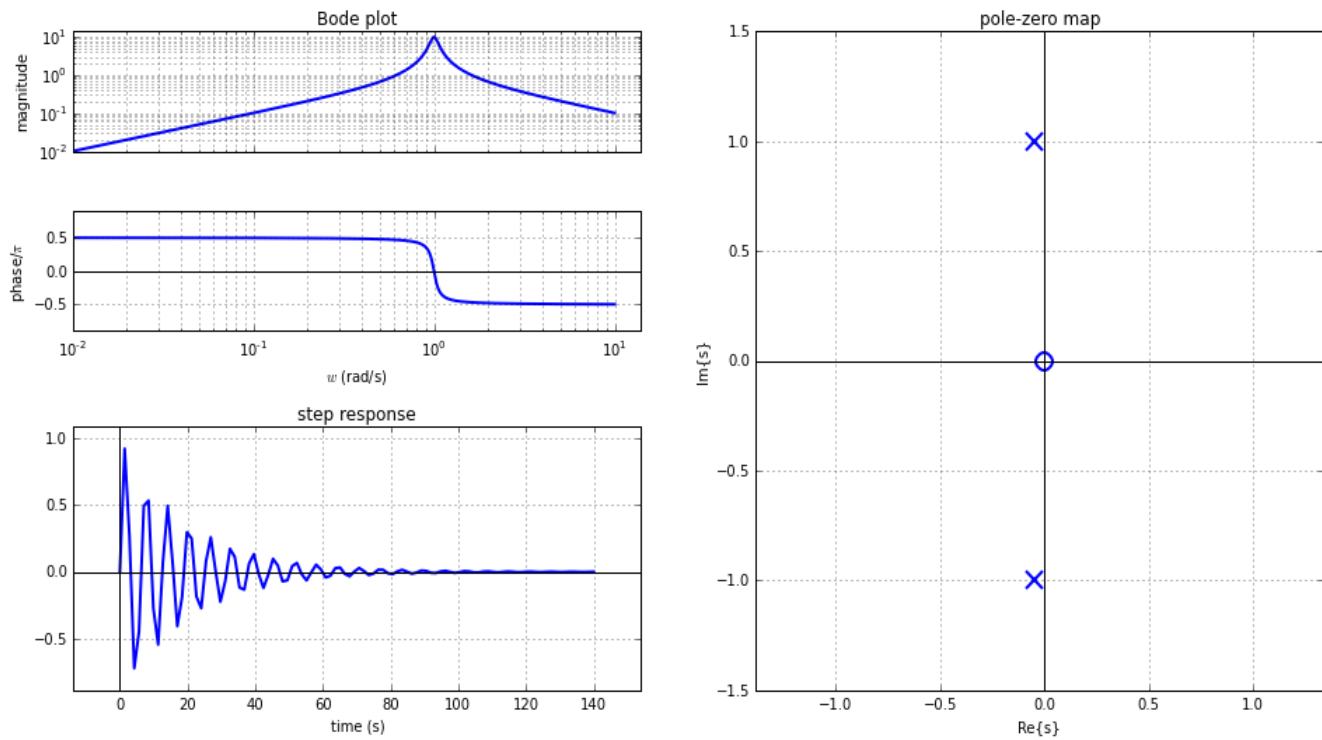
This system acts like a band-pass filter, since it allows only a certain range of frequencies (seen in the Bode plot). There is a pole at 0 and 0 and a zero at approximately 0. By looking at the step response, we can tell that the system stabilizes.

C.



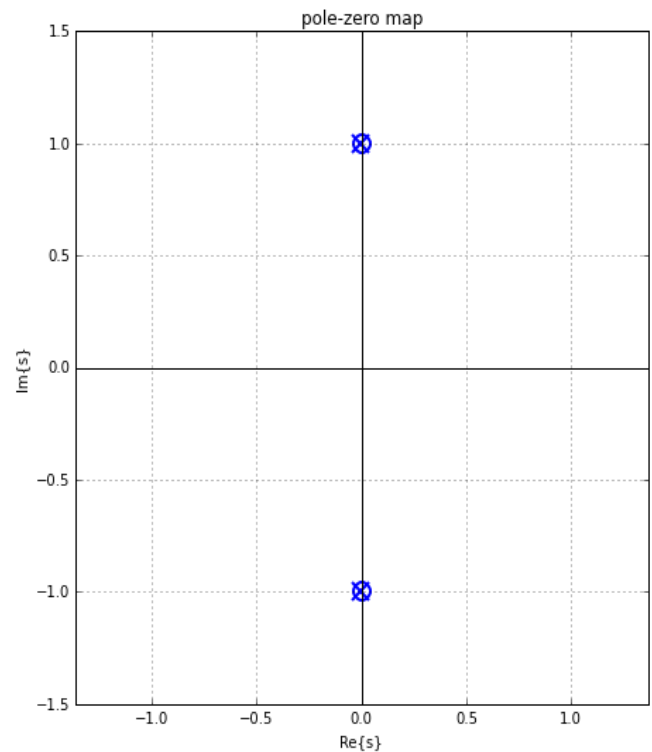
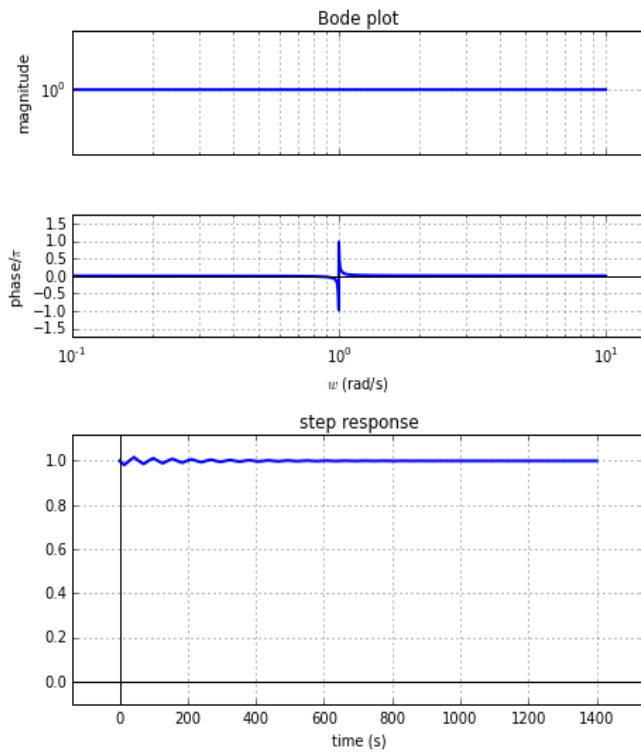
This system acts like a band-pass filter, since it allows only a certain range of frequencies (seen in the Bode plot). There are two poles at approximately 0.8 and -0.8 and a zero at approximately 0. By looking at the step response, we can tell that the system stabilizes.

D.



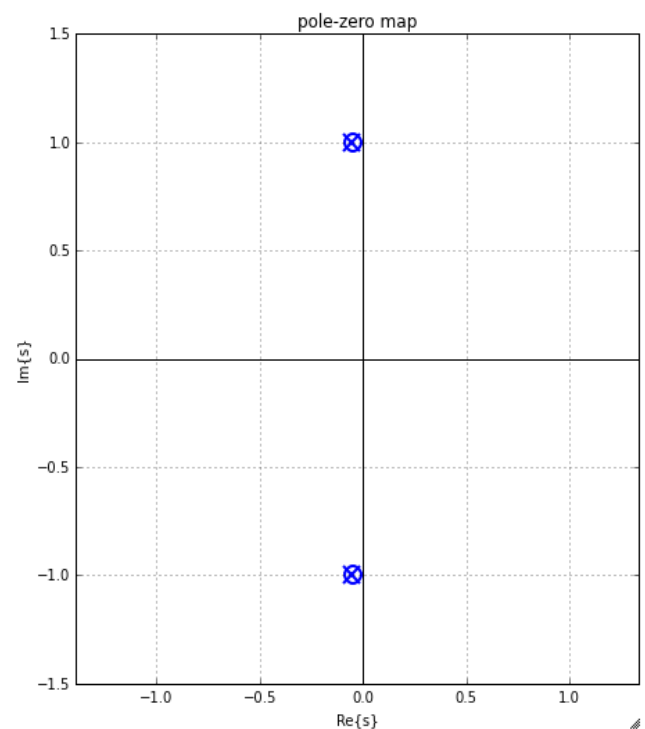
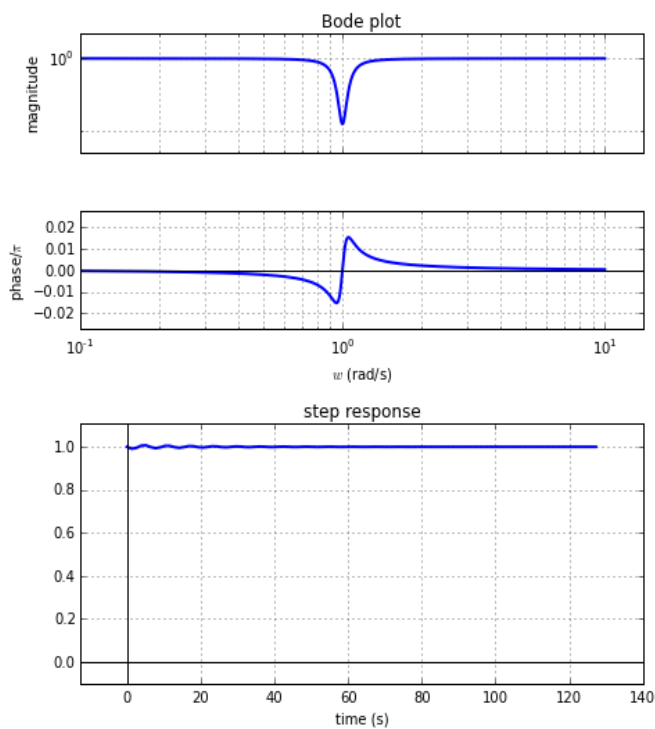
This system acts like a band-pass filter, since it allows only a certain range of frequencies (seen in the Bode plot). There are two poles at approximately 0.8 and -0.8 and a zero at approximately 0 . By looking at the step response, we can tell that the system stabilizes.

E.



This system doesn't act like a filter, since it allows all frequencies to pass through (seen in the Bode plot). There are two poles at approximately 1 and -1 and two zeros at approximately 0. By looking at the step response, we can see that the system stabilizes at 1.

F.

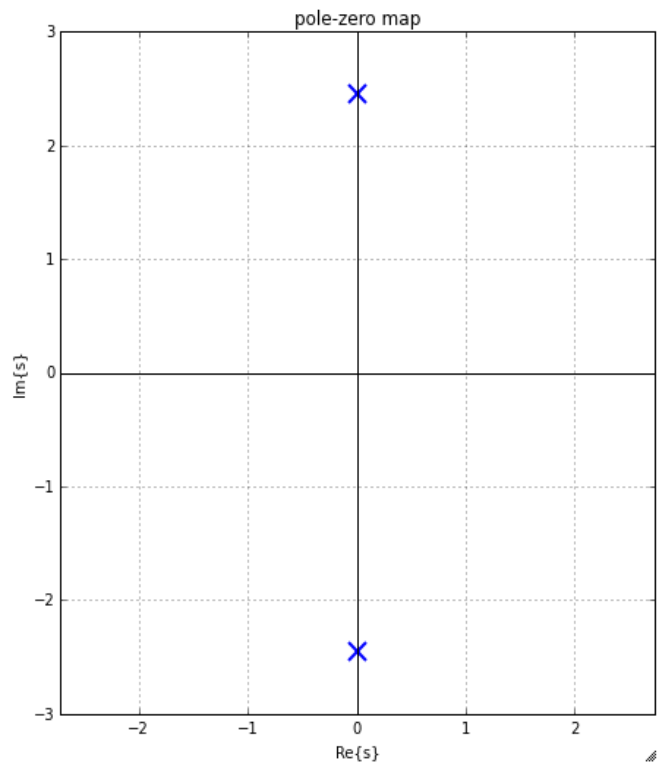
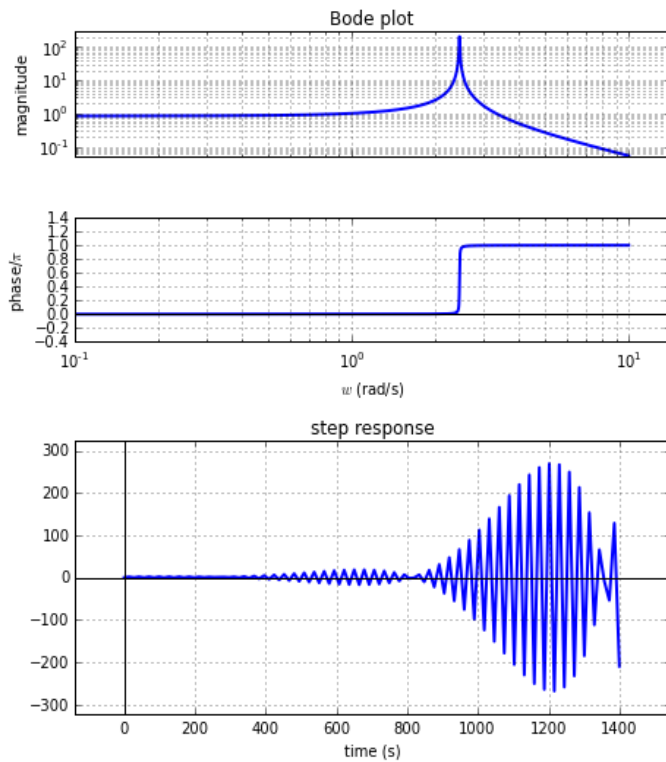


This system is a notch filter, since it only cuts off a specific frequency (seen in the Bode plot). There are two poles at approximately 1 and -1 and two zeros at approximately -0.1. By looking at the step response, we can see that the system stabilizes at 1.

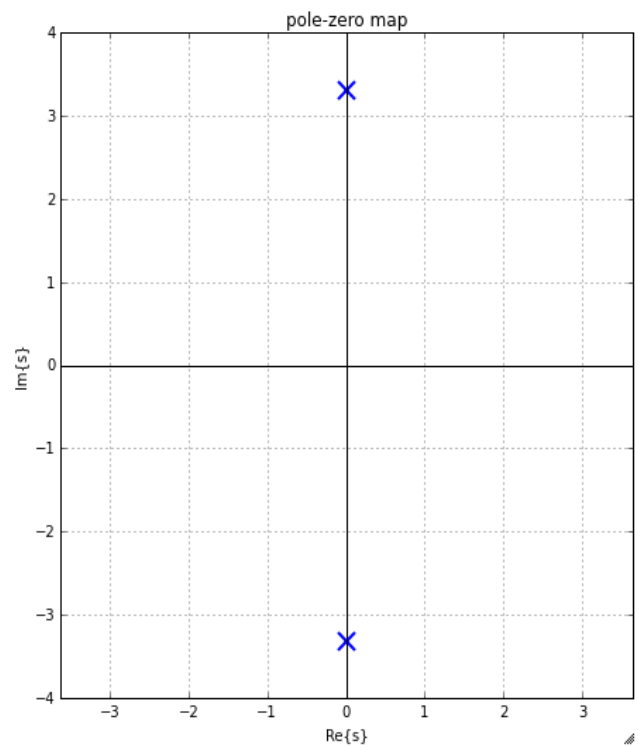
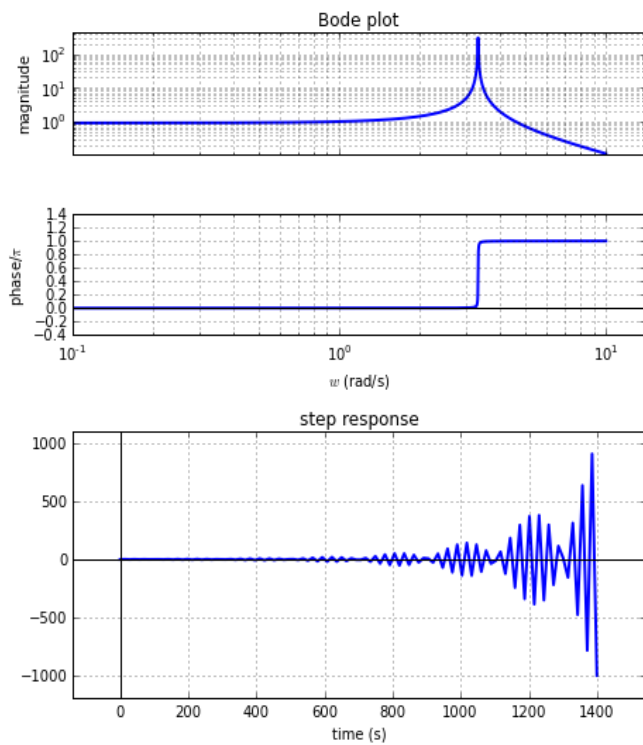
4.

B.

$K = 5$



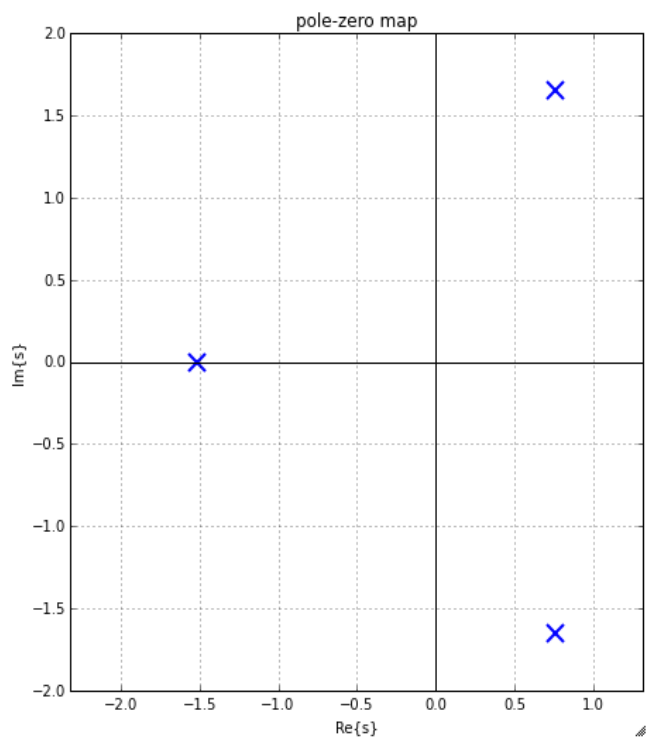
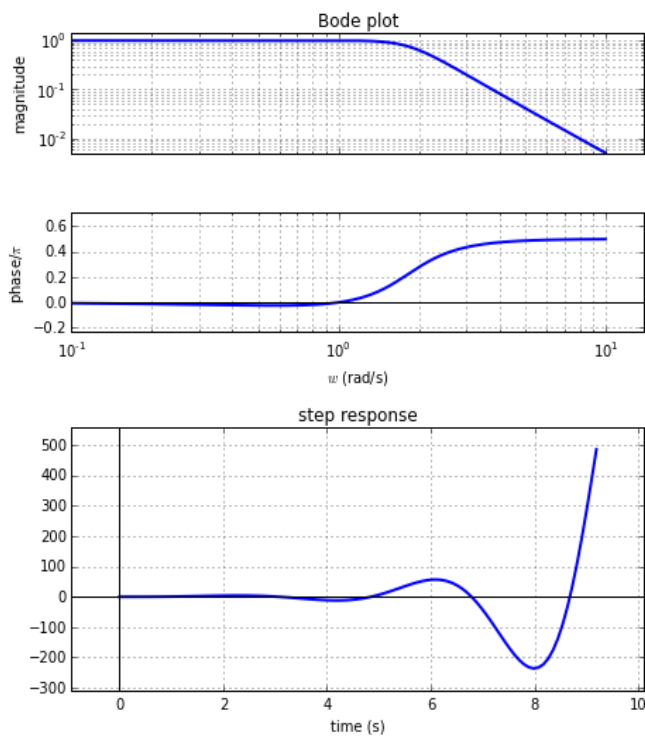
$K = 10$



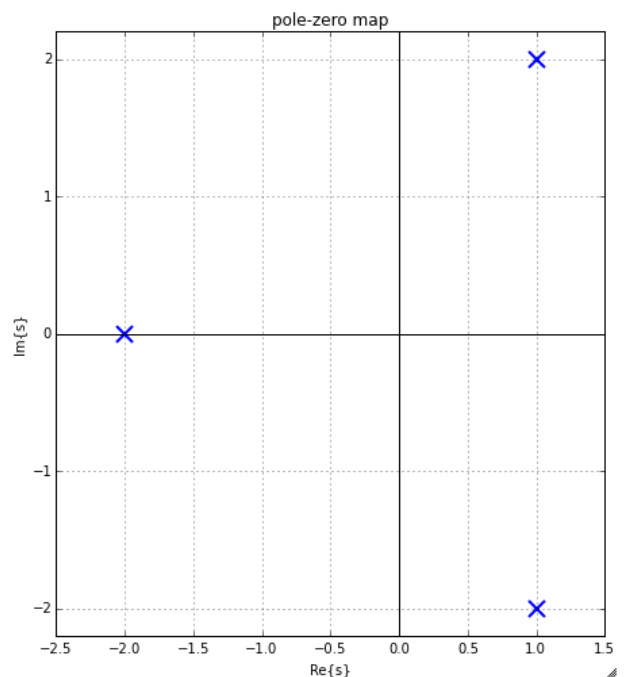
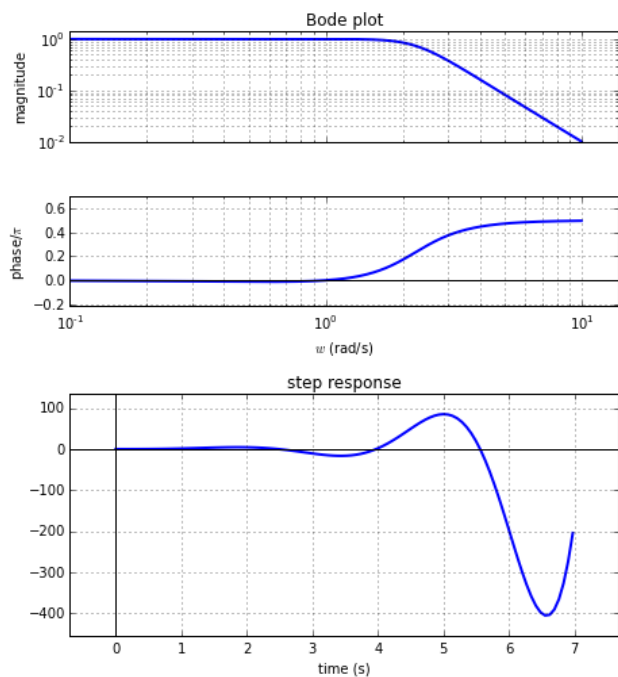
Looking at the step responses for both values of K , the system with proportional control does not stabilize.

C.

$K = 5$

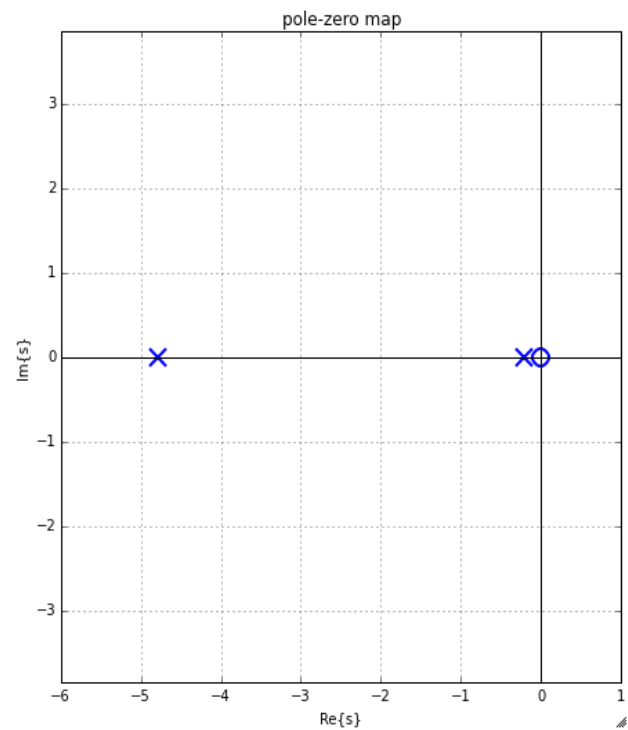
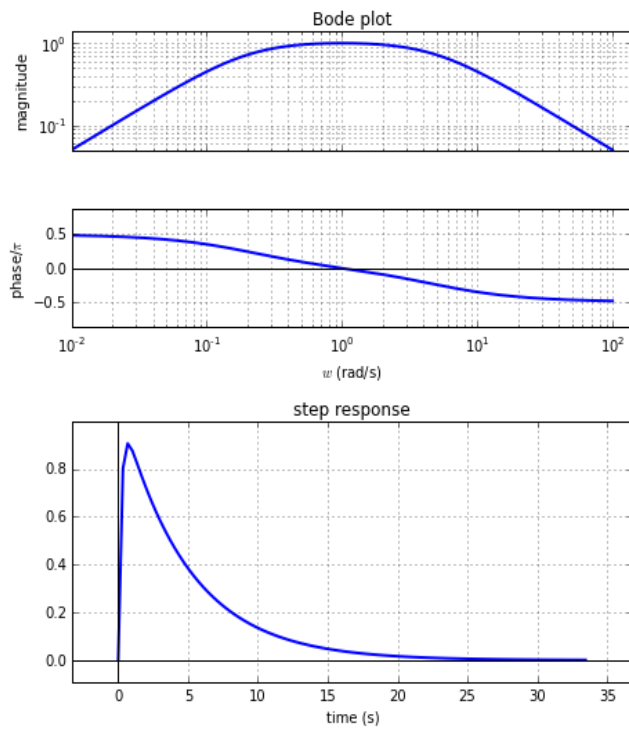


$K = 10$

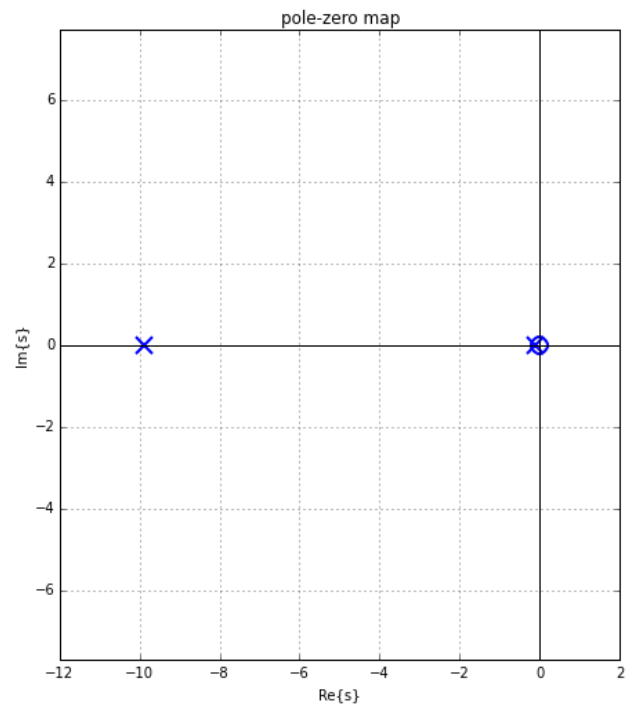
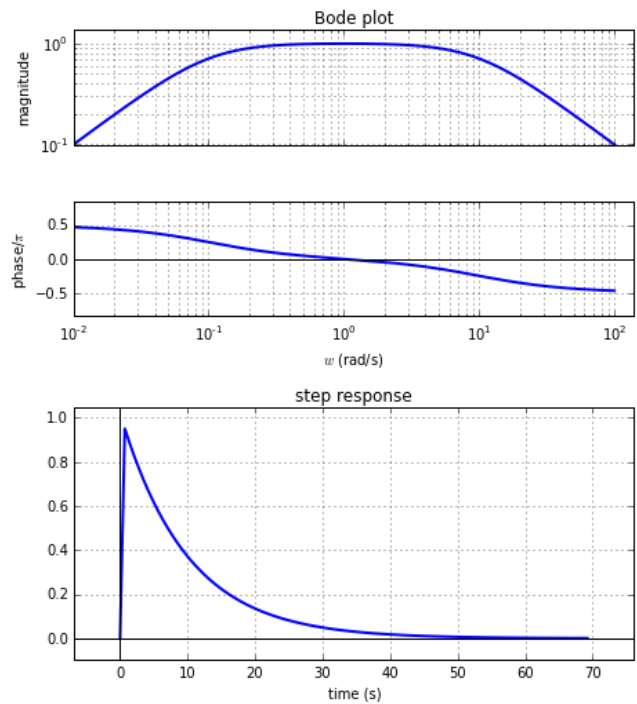


Looking at the step responses for both values of K , the system with integral control does not stabilize.
D.

$K = 5$



$K = 10$



Looking at the step responses for both values of K , the system with differential/derivative control is the only system that stabilizes.

$$4. b) \quad H(s) = \frac{1}{s^2 - 0.01s + 1}$$

$$\begin{aligned} \frac{Y}{Y_{sp}} &= \frac{KH}{1+KH} = \frac{K}{s^2 - 0.01s + 1} \div \left(1 + K \left(\frac{1}{s^2 - 0.01s + 1} \right) \right) \\ &= \frac{K}{s^2 - 0.01s + 1 + K} \end{aligned}$$

$$s^2 - 0.01s + 1 + K = 0$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-0.01) \pm \sqrt{(-0.01)^2 - 4(1)(K+1)}}{2(1)}$$

$$= \frac{0.01 \pm \sqrt{0.0001 - 4(K+1)}}{2}$$

$$= 0.005 \pm \sqrt{0.000025 - (K+1)}$$

$$= 0.005 \pm \sqrt{-K - 0.000075}$$

$$= 0.005 \pm \sqrt{K + 0.000075} j$$

c) Integral Control

$$K(s) = \frac{K}{s}$$

$$\frac{Y}{Y_{sp}} = \frac{\frac{K}{s}}{s^2 - 0.01s + 1 + \frac{K}{s}}$$

$$= \frac{K}{s^3 - 0.01s^2 + s + K}$$

d) Differential Control

$$K(s) = K \times s$$

$$\frac{Y}{Y_{sp}} = \frac{K \times s}{s^2 - 0.01s + 1 + Ks}$$

$$= \frac{K(s)}{s^2 + (0.01 + K)s + 1}$$