Problem Set 10
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1. $x = \dot{y} + y$

$$\begin{array}{ccccc} x(t) & \longrightarrow & h(t) & \longrightarrow & (x*h)(t) \\ \downarrow \mathcal{L} & & & & \uparrow \mathcal{L}^{-1} \\ X(s) & \longrightarrow & H(s) & \longrightarrow & Y(s) = H(s)X(s) \end{array}$$

$u(t) \overset{\mathcal{L}}{\Longleftrightarrow} \frac{1}{s}$

$x = \dot{y} + y$

$\Downarrow \mathcal{L}$

$X = sY + Y$

$Y(s+1) = X$

$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$ ROC: $\text{Re}\{s\} > -1$

$Y(s) = H(s)\,\mathcal{L}\{u(t)\}$

$= \frac{1}{s+1} \cdot \frac{1}{s}$

$= \frac{1}{s(s+1)} = \frac{A}{s+0} + \frac{B}{s+1}$

$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$

$A(s+1) + B(s) = 1$

when $s = -1$, $B = -1$

when $s = 0$, $A = 1$

$y(t) = Ae^{-st}u(t) + Be^{-st}u(t)$

$= (1)e^{-(0)t}u(t) + (-1)e^{-(-1)t}u(t)$

$= u(t) - e^{t}u(t)$

$= (1 - e^{t})u(t)$

2. A. $H(s) = \frac{1/\tau}{s + 1/\tau}$ $K(s) = \frac{K_I}{s}$

$\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{K_I}{s}\left(\frac{1/\tau}{s+1/\tau}\right)}{1 + \frac{K_I}{s}\left(\frac{1/\tau}{s+1/\tau}\right)} = \frac{K_I/s \div \tau}{s + (\frac{K_I}{s} + 1)/\tau} = \frac{K_I/\tau}{s^2 + (K_I + s)/\tau}$

$\lim_{s \to 0} \left(\frac{K_I/\tau}{s^2 + (K_I + s)/\tau}\right) = 1$

as $s \to 0$, DC gain $\to 1$

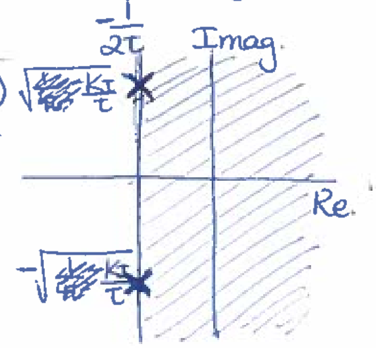
B. $s^2 + \frac{(K_I + s)}{\tau} = 0$

$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

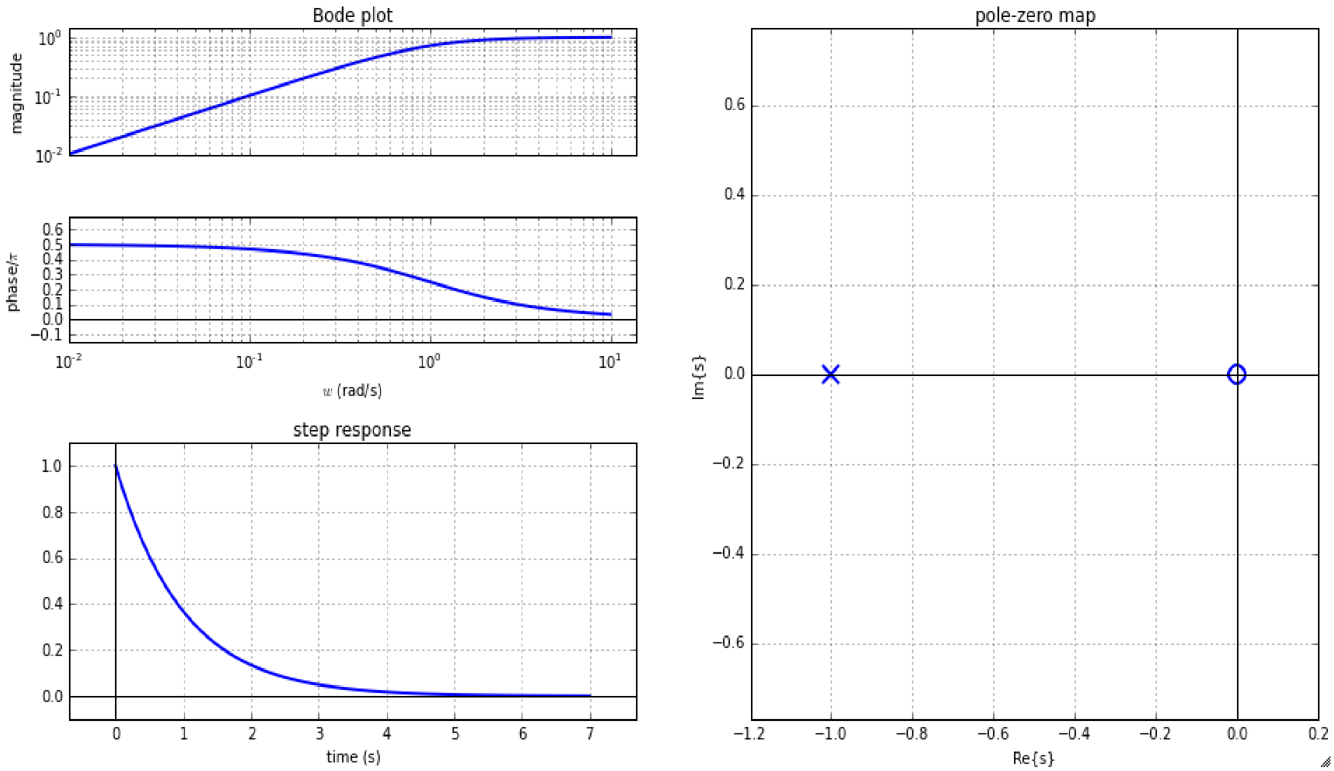
$\boxed{\text{if } K >> 1/\tau \text{, poles are } \pm\sqrt{\frac{K_I}{\tau}}}$

$s = \frac{-\frac{1}{\tau} \pm \sqrt{\left(\frac{1}{\tau^2}\right) - 4(1)\left(\frac{K_I}{\tau}\right)}}{2(1)}$

$s = \underbrace{-\frac{1}{2\tau}}_{\text{Re}} \pm \underbrace{\sqrt{\frac{1}{4\tau^2} - \frac{K_I}{\tau}}}_{\text{Imag.}}$

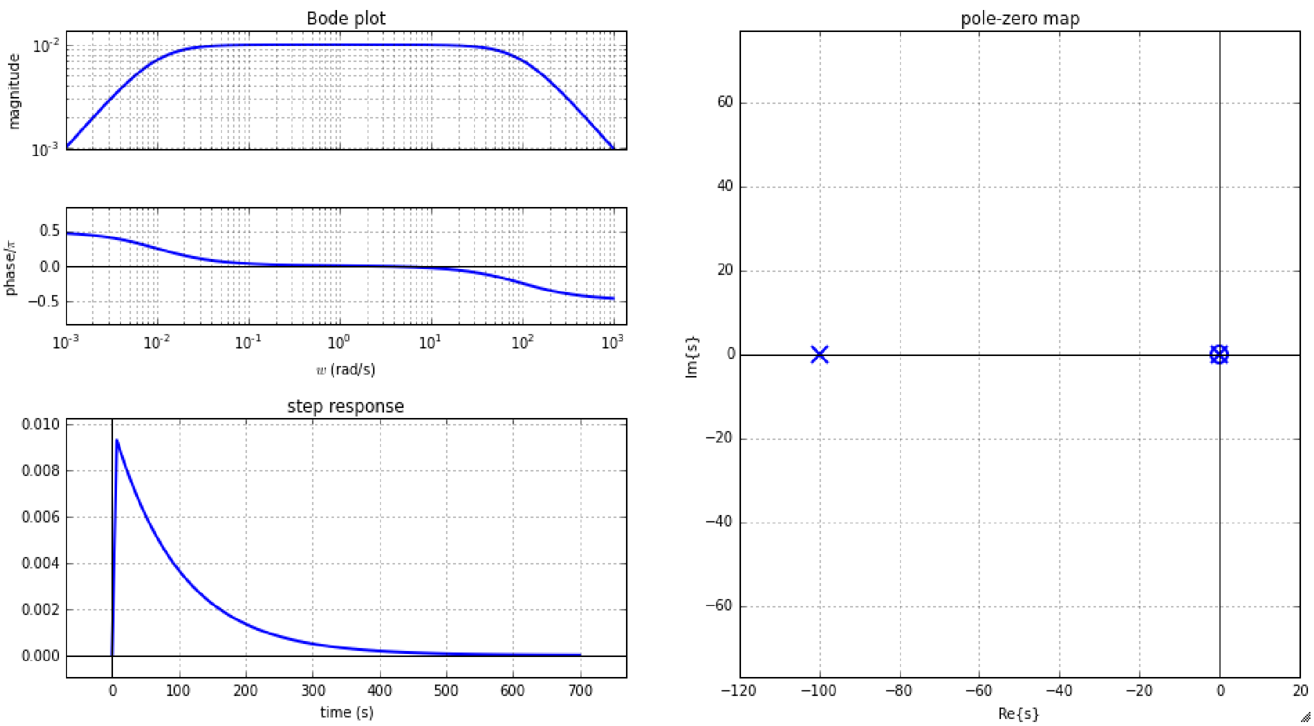


3.
A.



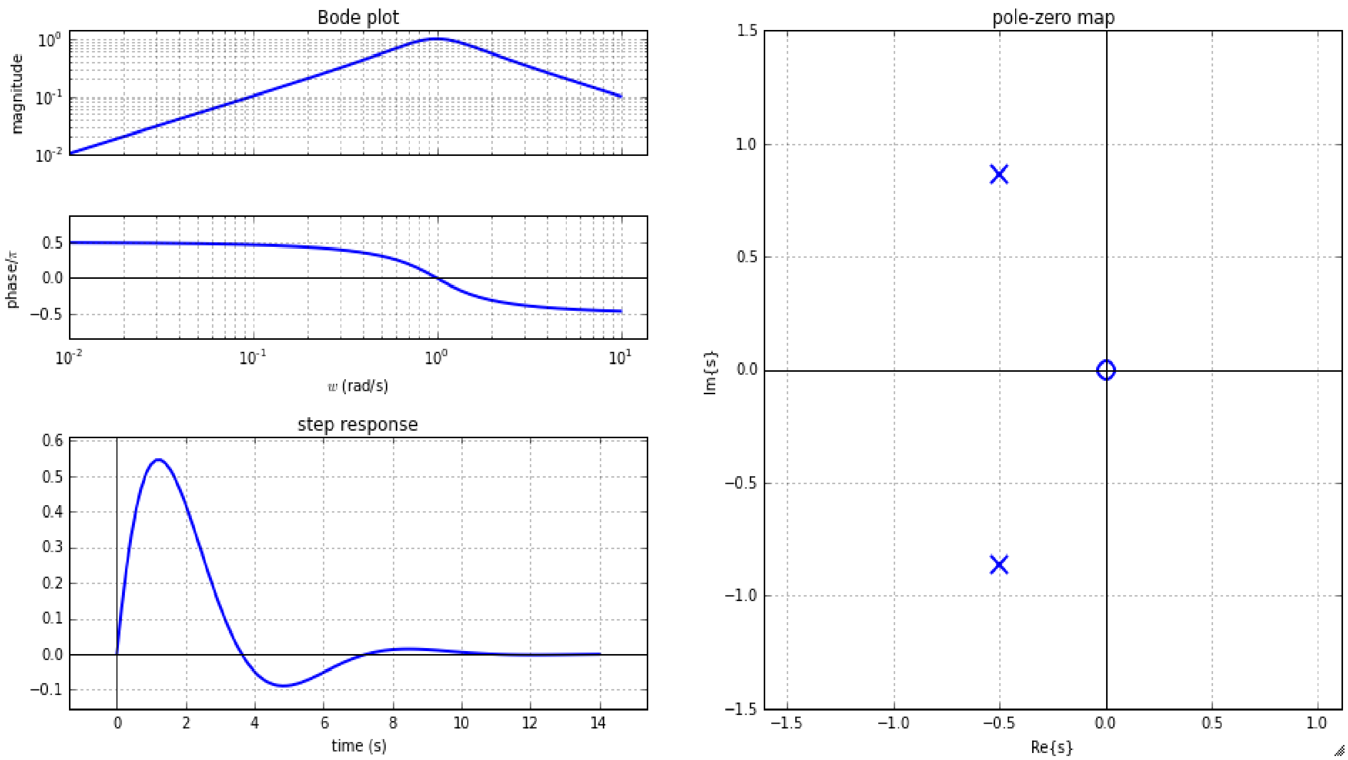
This system acts like a high-pass filter, since it cuts off lower frequencies (seen in the Bode plot). There is a real pole at -1 and a zero at approximately 0. By looking at the step response, we can tell that the system stabilizes.

B.



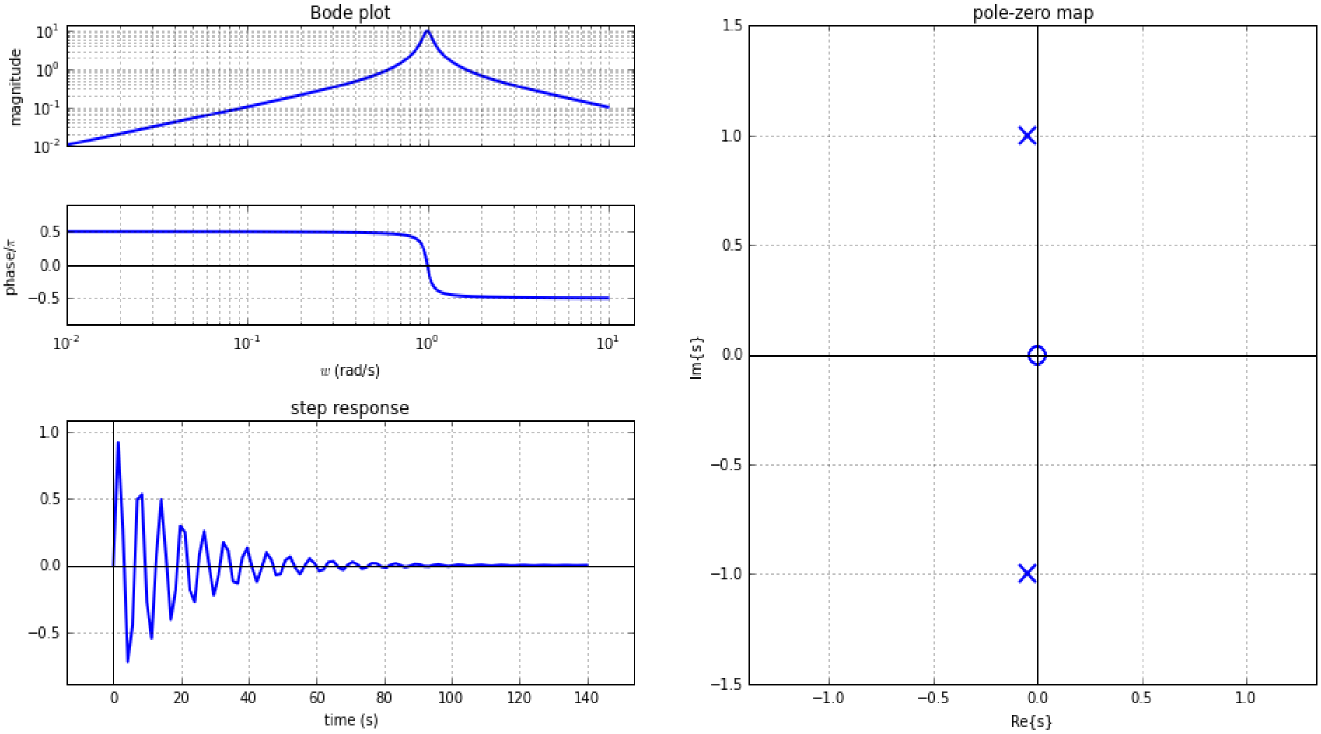
This system acts like a band-pass filter, since it allows only a certain range of frequencies (seen in the Bode plot). There is a real pole at -100 and a zero at approximately 0. By looking at the step response, we can tell that the system stabilizes.

C.



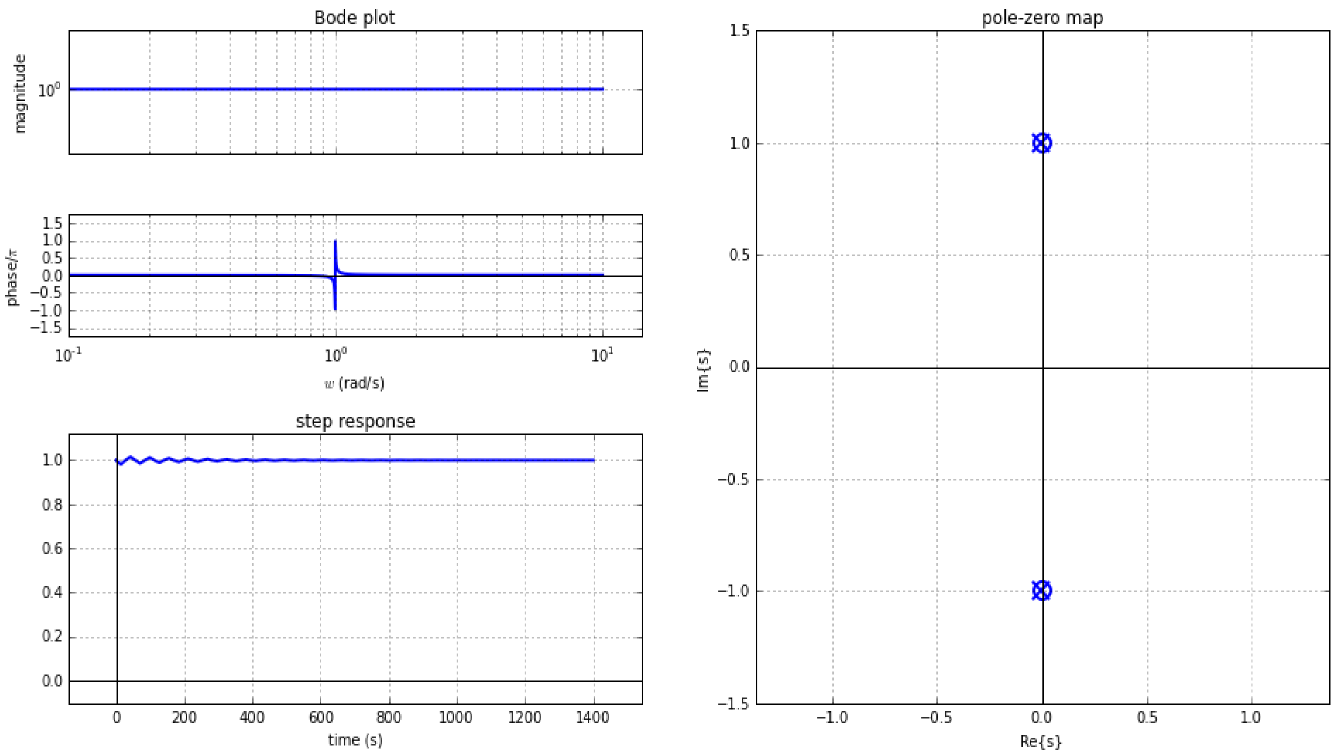
This system acts like a band-pass filter, since it allows only a certain range of frequencies (seen in the Bode plot). There are two imaginary poles at approximately 0.8j and -0.8 j, a real pole at -0.5 and a zero at approximately 0. By looking at the step response, we can tell that the system stabilizes.

D.



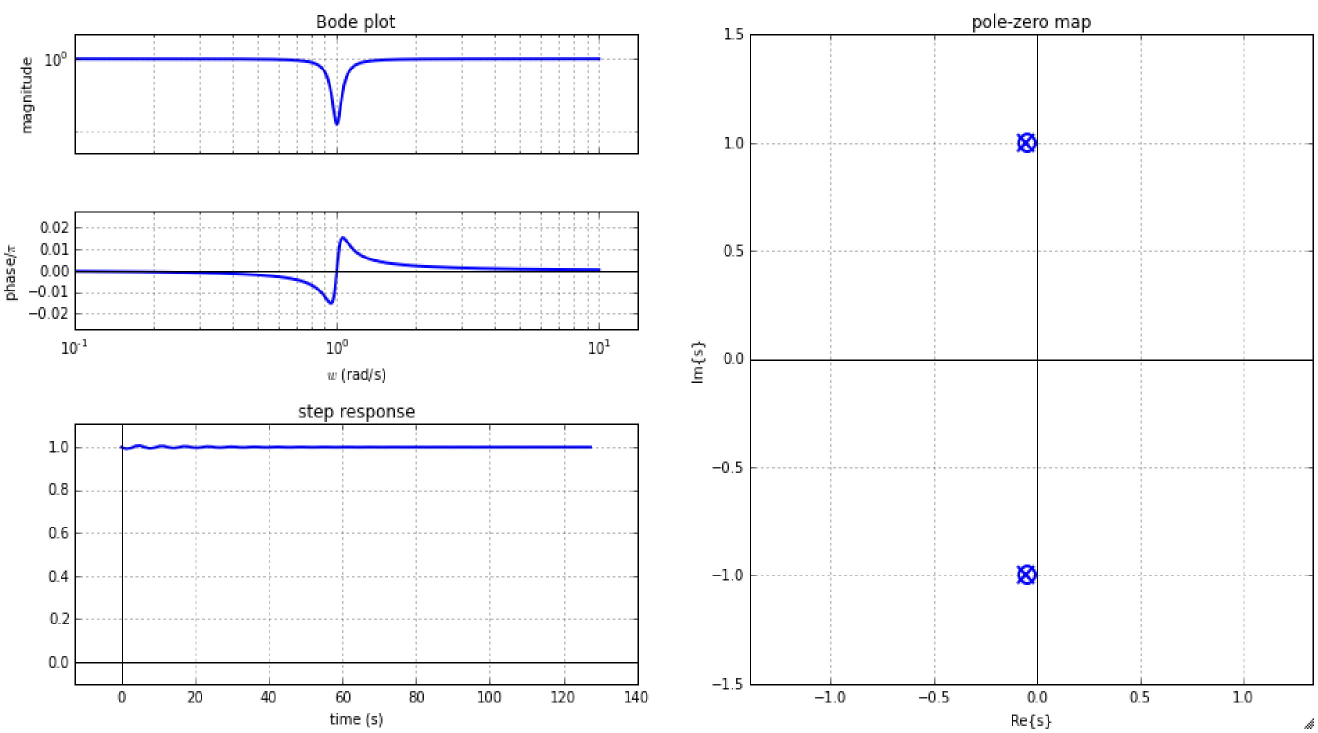
This system acts like a band-pass filter, since it allows only a certain range of frequencies (seen in the Bode plot). There are two imaginary poles at approximately 1.1j and -1.1j, a real pole at -0.1 and a real zero at approximately -0.01. By looking at the step response, we can tell that the system stabilizes.

E.



This system doesn't act like a filter, since it allows all frequencies to pass through (seen in the Bode plot). There are two imaginary poles at approximately 1j and -1j, a real pole at -0.01 and two imaginary zeros at approximately 1j and -1j. By looking at the step response, we can see that the system stabilizes at 1.

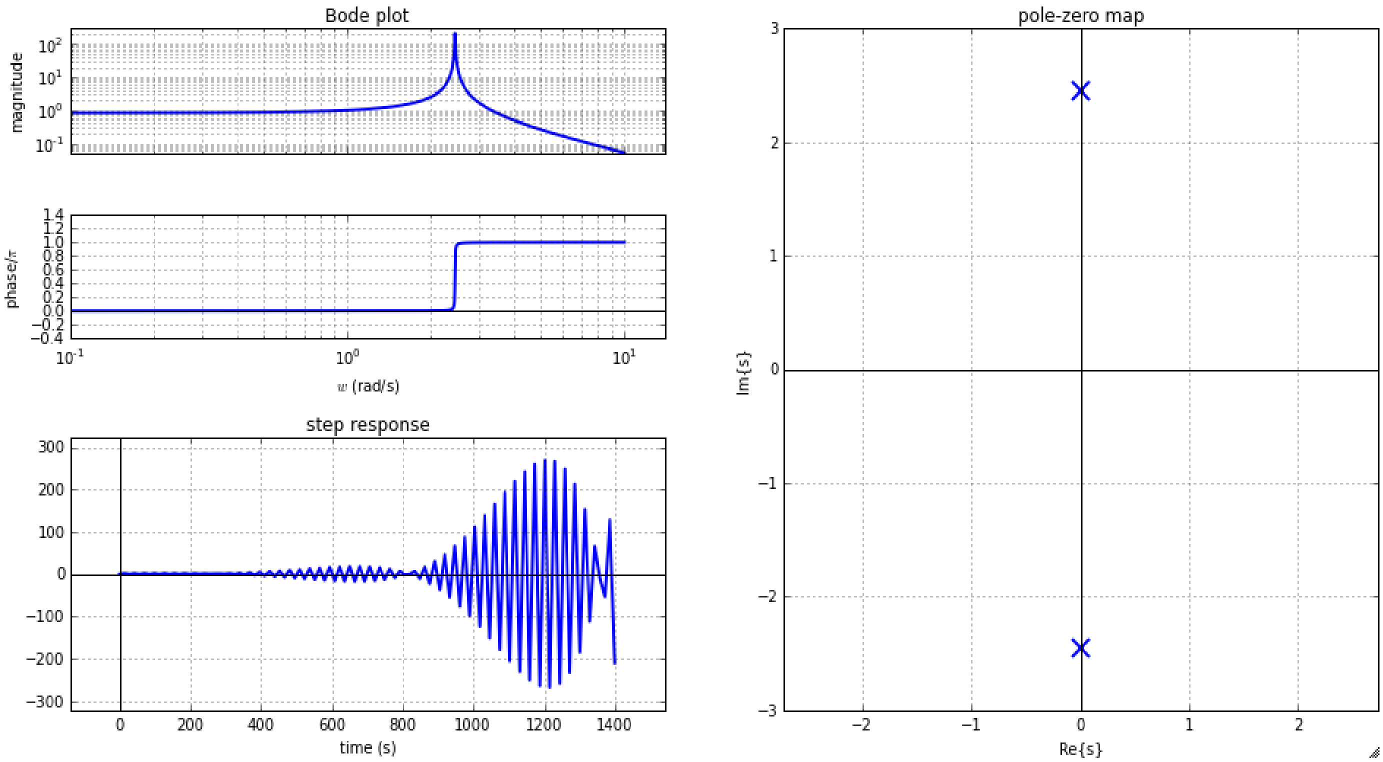
F.



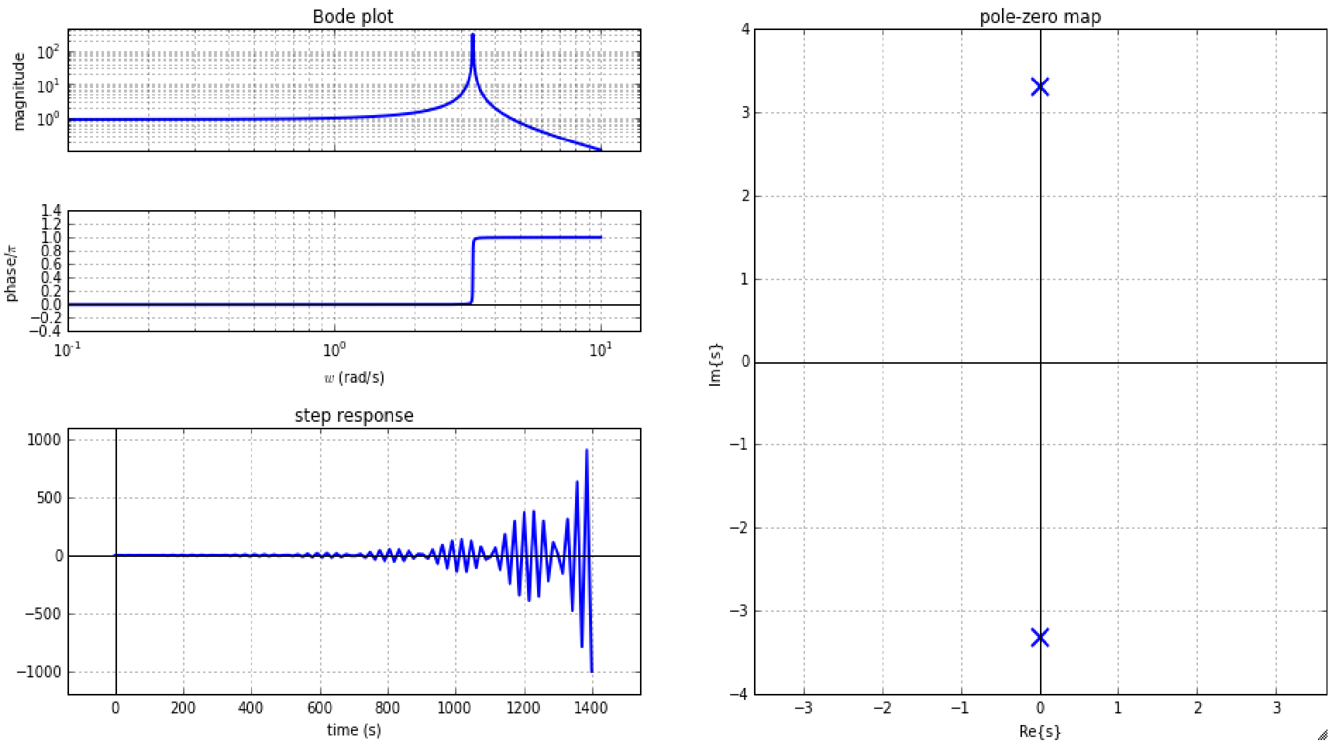
This system is a notch filter, since it only cuts off a specific frequency (seen in the Bode plot). There are two imaginary poles at approximately 1j and -1j, a real pole at -0.11, a real zero at -0.1 and two zeros at approximately -0.1. By looking at the step response, we can see that the system stabilizes at 1.

4.
B.

K = 5



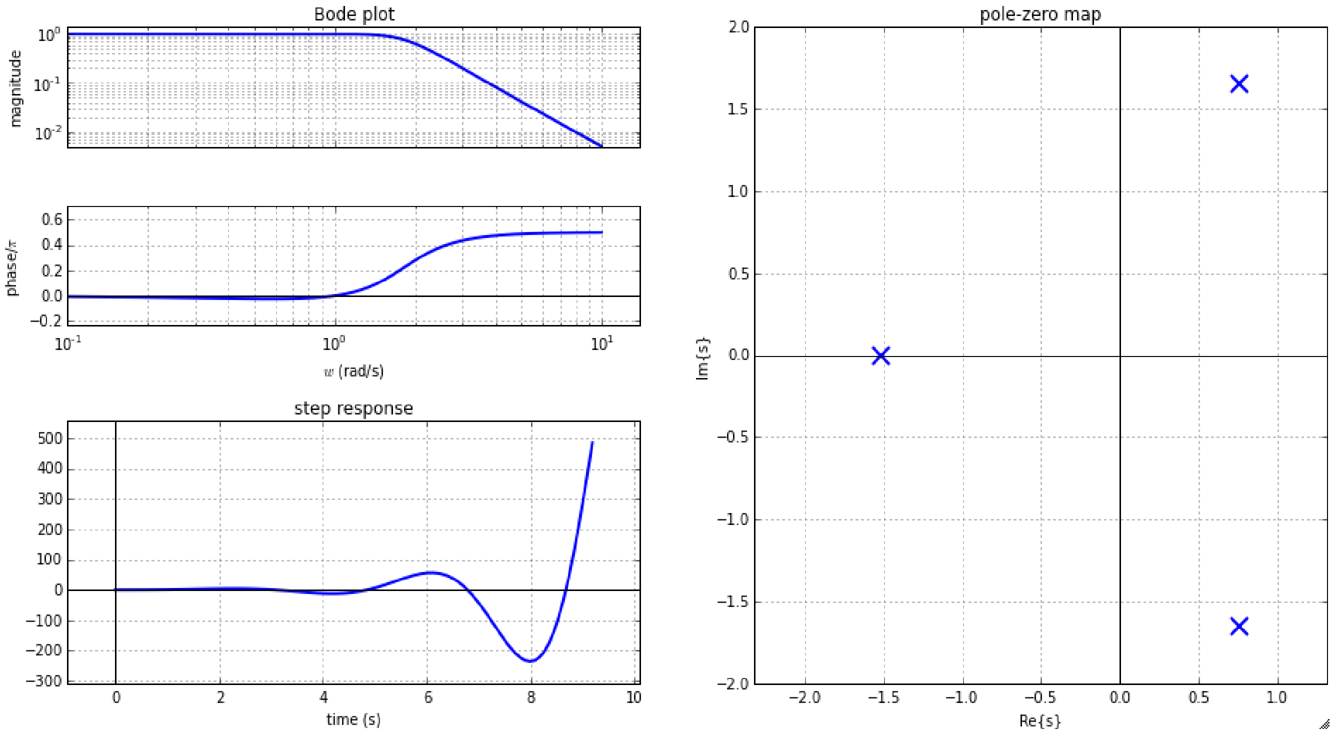
K = 10



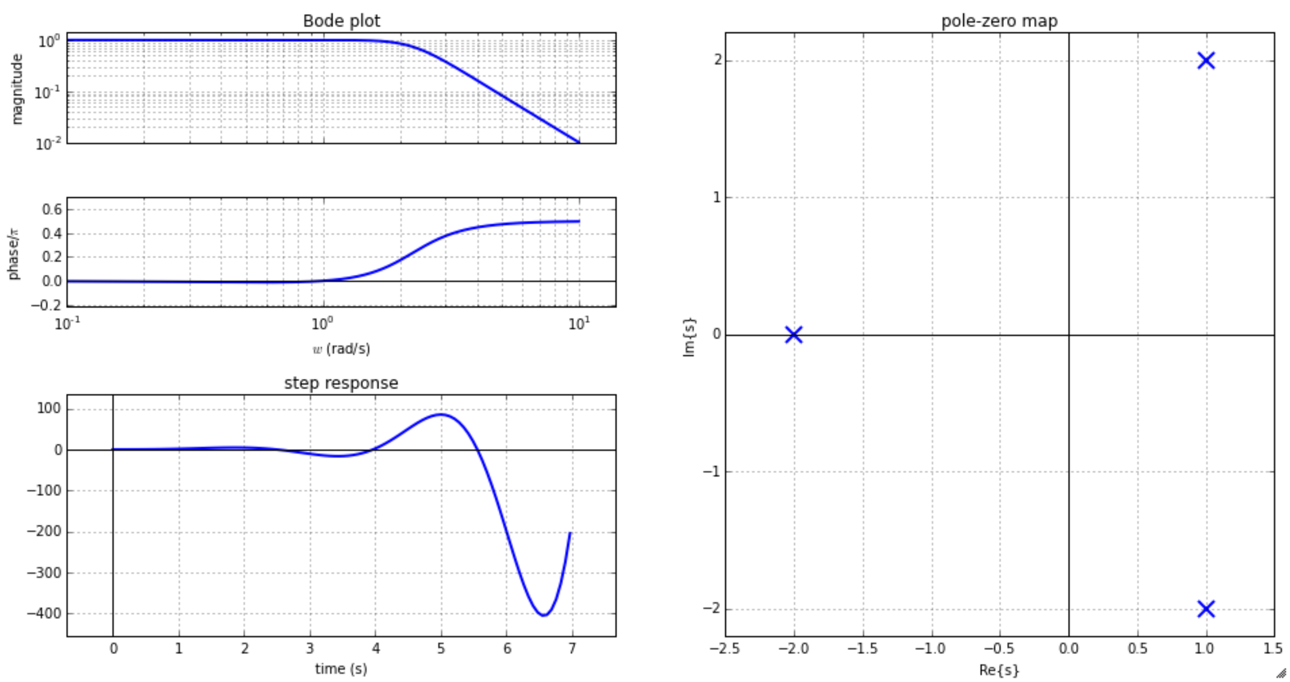
Looking at the step responses for both values of K, the system with proportional control does not stabilize.

C.

K = 5

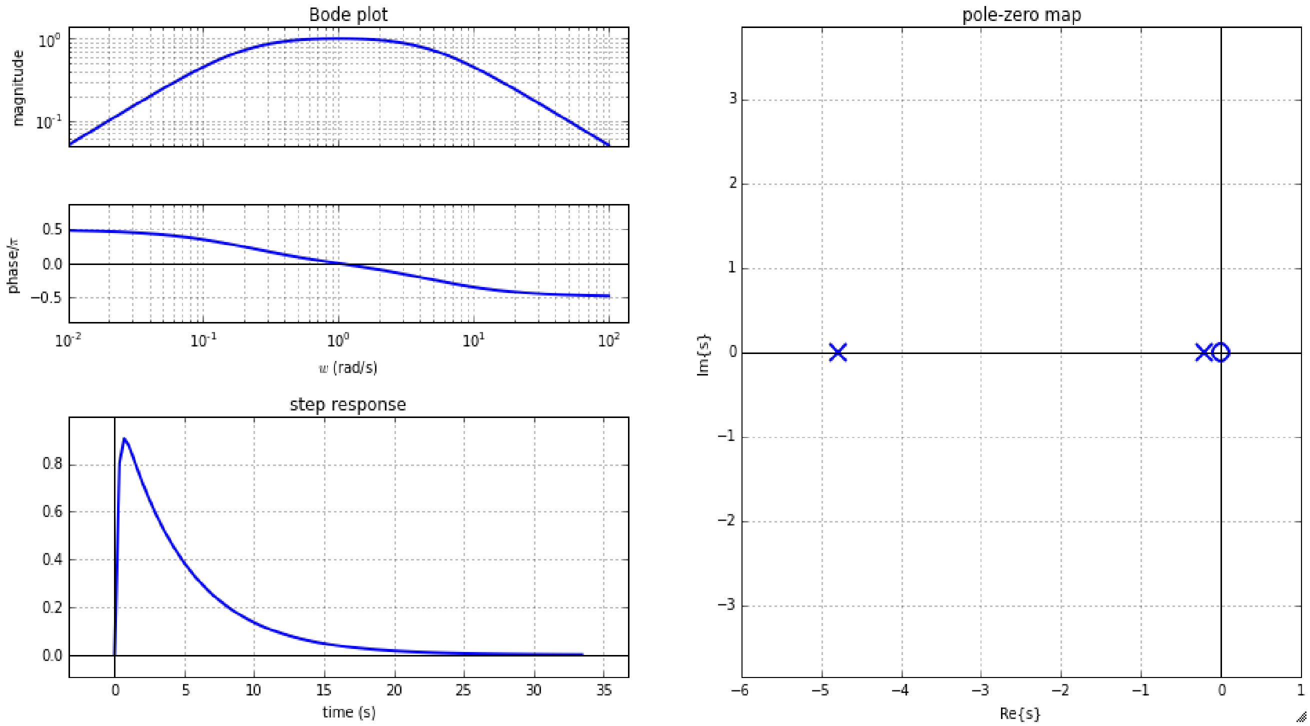


K = 10

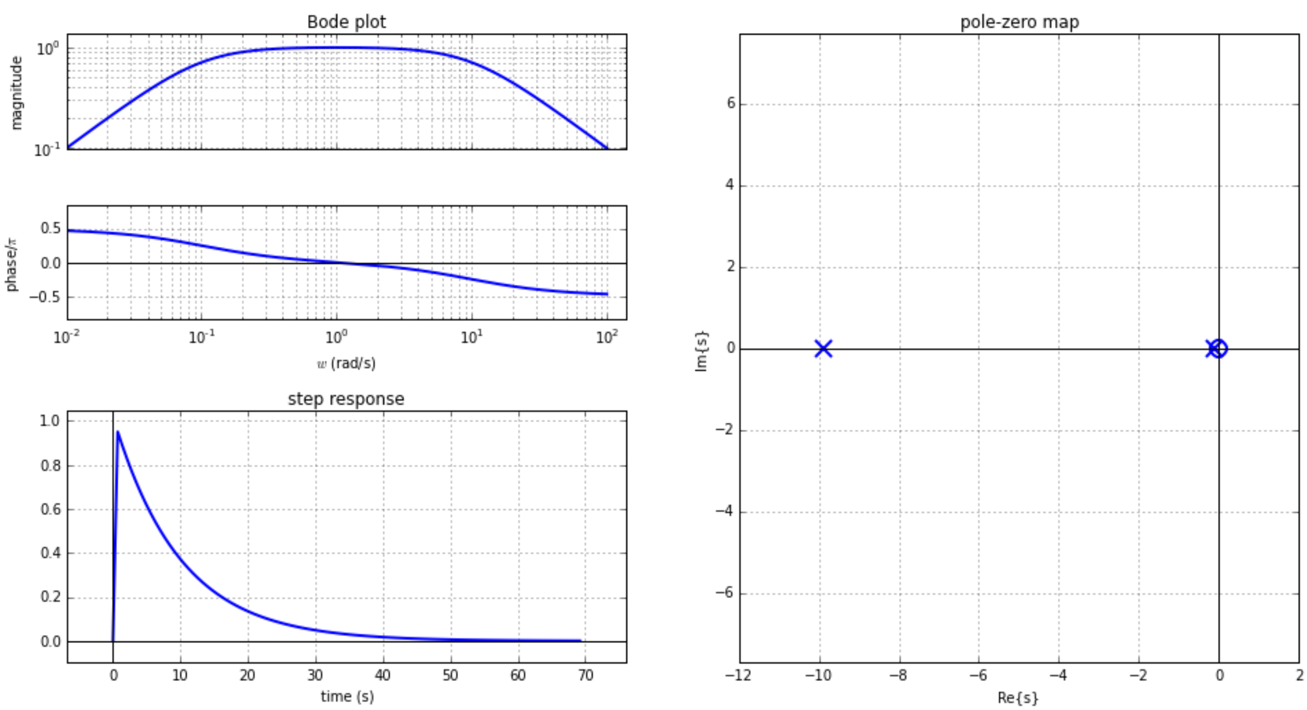


Looking at the step responses for both values of K, the system with integral control does not stabilize.
D.

K = 5



K = 10



Looking at the step responses for both values of K, the system with differential/derivative control is the only system that stabilizes.

4. b) $H(s) = \dfrac{1}{s^2 - 0.01s + 1}$

$$\frac{Y}{Y_{sp}} = \frac{KH}{1+KH} = \frac{K}{s^2-0.01s+1} \div \left(1 + K\left(\frac{1}{s^2-0.01s+1}\right)\right)$$

$$= \frac{K}{s^2-0.01s+1+K}$$

$$s^2 - 0.01s + 1 + K = 0$$

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-0.01) \pm \sqrt{(-0.01)^2 - 4(1)(K+1)}}{2(1)}$$

$$= \frac{0.01 \pm \sqrt{0.0001 - 4(K+1)}}{2}$$

$$= 0.005 \pm \sqrt{0.000025 - (K+1)}$$

$$= 0.005 \pm \sqrt{-K - 0.000075}$$

$$= 0.005 \pm \sqrt{K + 0.000075}\, j$$

c) Integral Control

$$K(s) = \frac{K}{s}$$

$$\frac{Y}{Y_{sp}} = \frac{\frac{K}{s}}{s^2 - 0.01s + 1 + \frac{K}{s}}$$

$$= \frac{K}{s^3 - 0.01s^2 + s + K}$$

d) Differential Control

$$K(s) = K \times s$$

$$\frac{Y}{Y_{sp}} = \frac{K \times s}{s^2 - 0.01s + 1 + Ks}$$

$$= \frac{K(s)}{s^2 + (0.01 + K)s + 1}$$