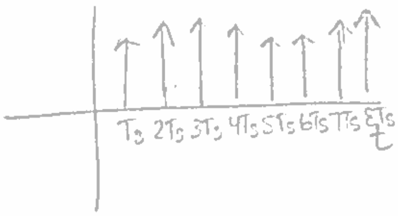
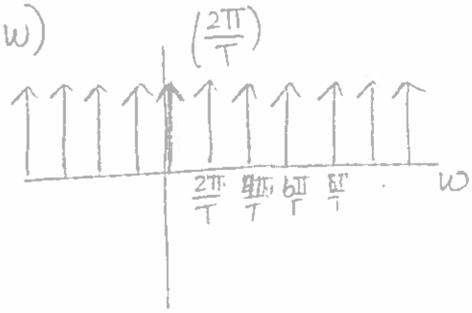
PROBLEM SET 8 ABIGAIL RODRIGUES

1) a) $x_p(t)$



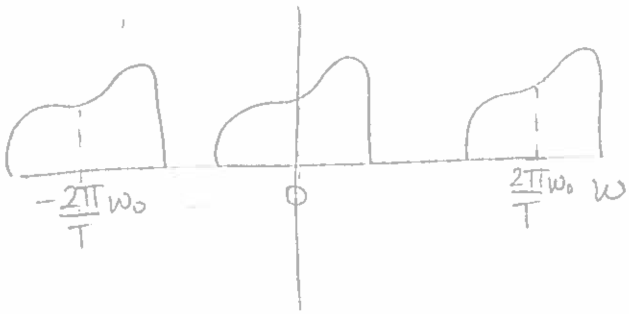
b) $P(w)$



c) $X_p(w) = \frac{1}{2\pi} X(w) * P(w)$

$= \frac{1}{\cancel{2\pi}} X(w) * \frac{\cancel{2\pi}}{T} \sum_{k=-\infty}^{\infty} \delta(w - \frac{2\pi}{T}k)$

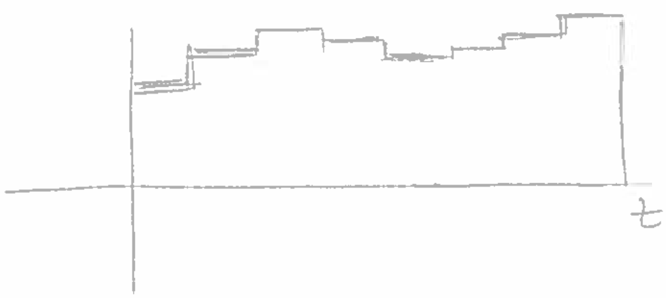
$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X(w - \frac{2\pi}{T}k)$



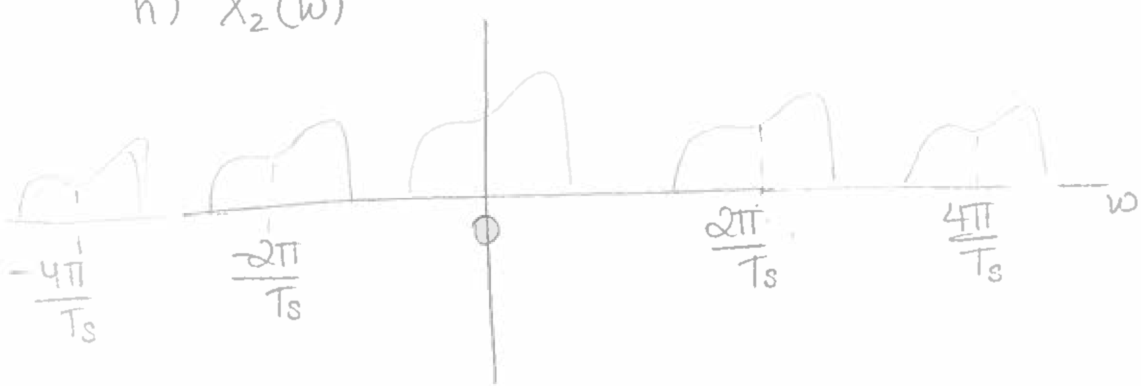
d) $\frac{2\pi}{T_s} > 2w_m$

e) Apply a filter - allowing values within the range $-w_m$ to w_m to pass through

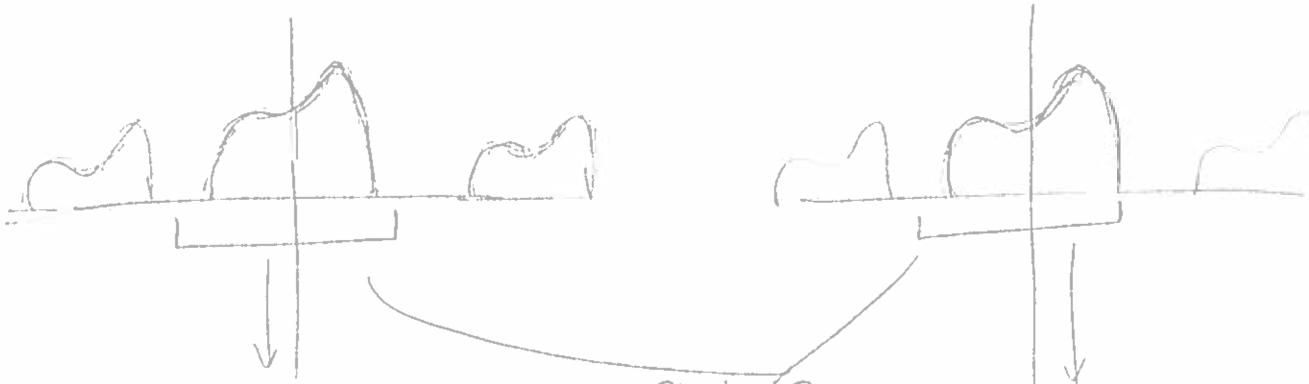
g) $x_2(t) = x_p * z(t)$ $Z(w) \rightarrow$ sinc function



h) $X_2(w)$



i) $\bar{X}(\omega) = X_2(\omega) H(\omega)$ $\qquad$ $\hat{X}(\omega) = X_p(\omega) H(\omega)$



j) more rounded

this is the only part that differs between the two.

less rounded

k) $\omega_m = \frac{\pi}{T_s} = \omega_c$

$x_2(t) = x_p * z(t) \rightarrow X_2(\omega) = X_p(\omega) Z(\omega)$

$x_p(t) = x(t) p(t) \longrightarrow X_p(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\omega - \frac{2\pi k}{T})$

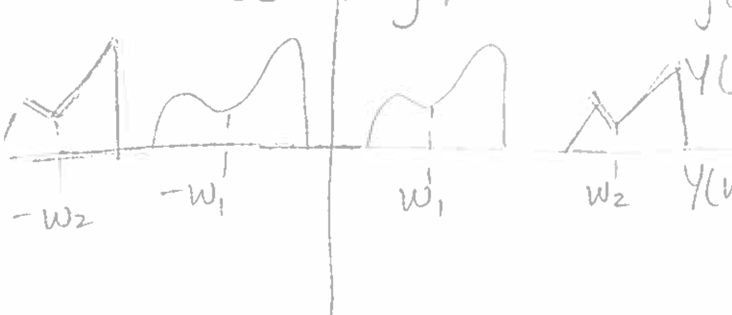
$Z(\omega) = \frac{1}{\sqrt{2\pi}} \operatorname{sinc}(\omega/2)$

$$\frac{\bar{X}(\omega)}{\hat{X}(\omega)} = \frac{X_2(\omega)\cancel{H(\omega)}}{X_p(\omega)\cancel{H(\omega)}} = \frac{\cancel{X_p(\omega)}Z(\omega)}{\cancel{X_p(\omega)}} = Z(\omega)$$

Ratio is $Z(\omega)$

2 a) $Y(\omega)$

scaled by 1/2



keeps repeating

$\cos(\omega_0 t) \longrightarrow \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$

$y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$

$Y(\omega) = \frac{1}{2\cancel{\pi}} X_1(\omega) * (\cancel{\pi}\delta(\omega - \omega_1) + \cancel{\pi}\delta(\omega + \omega_1)) + \ldots$

$Y(\omega) = \frac{1}{2} X_1(\omega) (\delta(\omega - \omega_1) + \delta(\omega + \omega_1)) + \ldots$

$\omega_1 + 2\omega_m < \omega_2$

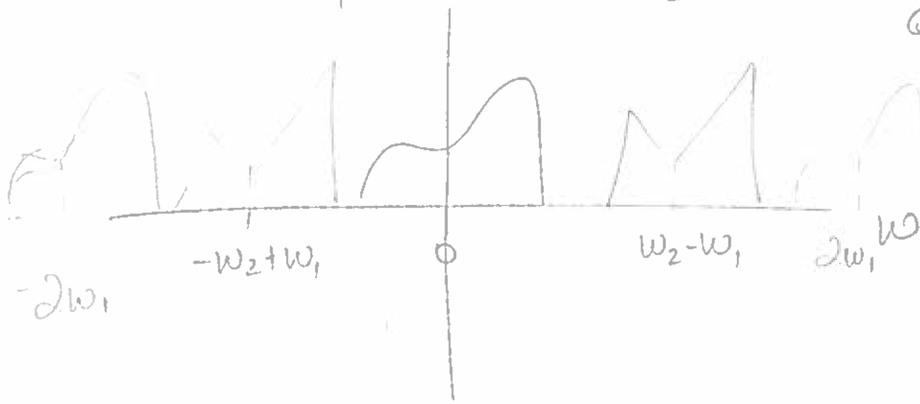
$\omega_2 - \omega_1 > 2\omega_m$

b) $y(t)\cos(\omega_1 t) \rightarrow \frac{1}{2\pi} Y(\omega) * (\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1))$

FT of $y(t)\cos(\omega_1 t)$

original scaled by 1/4

Since $\omega_2 > 2\omega_m + \omega_1$ there shouldn't be any interference



width $= 2\omega_m$

$\omega_1 + \omega_2 = \omega_1 + (2\omega_m + \omega_1)$

$= 2\omega_1 + 2\omega_m$

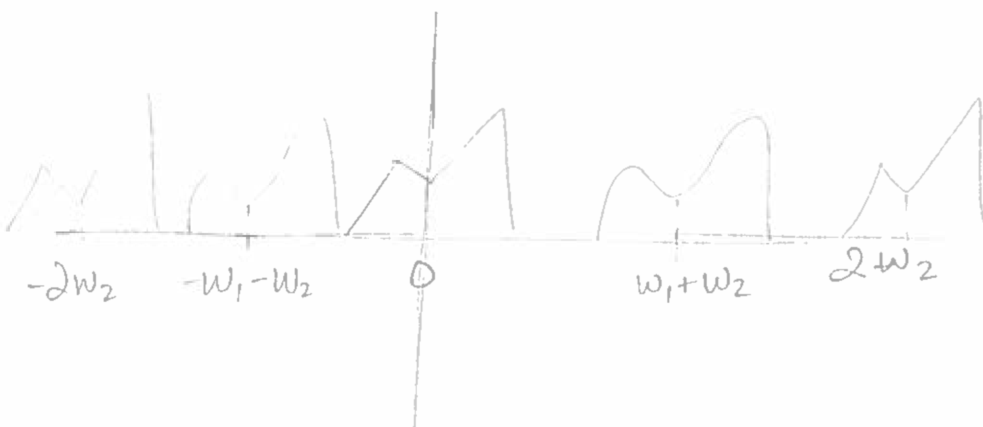
FT of $y(t)\cos(\omega_2 t)$

original scaled by 1/4

$\omega_1 < \omega_2 - 2\omega_m$

$\omega_1 + \omega_2 = \omega_2 - 2\omega_m + \omega_2$

$= 2\omega_2 - 2\omega_m$



c) Using filters with a bandwidth of $2\omega_m$ centered around 0 for each

(3) a)

$$i(t) = C\frac{d\,V_{out}(t)}{dt}$$

$$V_L(t) = L\frac{d\,i(t)}{dt}$$

$$V_{in}(t) = V_R(t) + V_L(t) + V_{out}(t)$$

$$V_{in}(t) = i(t)R + L\frac{d\,i(t)}{dt} + V_{out}(t)$$

$$V_{in}(t) = RC\frac{d\,V_{out}(t)}{dt} + L\frac{d\left(C\frac{d\,V_{out}(t)}{dt}\right)}{dt} + V_{out}(t)$$

$$V_{in}(t) = RC\frac{dV_{out}(t)}{dt} + LC\frac{d\left(\frac{d\,V_{out}(t)}{dt}\right)}{dt} + V_{out}(t)$$

b) $V_{in}(w) = jwRC\,V_{out}(w) + (jw)^2 LC\,V_{out}(w) + V_{out}(w)$

$$H(w) = \frac{V_{out}(w)}{V_{in}(w)} = \frac{\cancel{V_{out}(w)}}{jwRC\cancel{V_{out}(w)} + (jw)^2LC\cancel{V_{out}(w)} + \cancel{V_{out}(w)}}$$

$$= \frac{1}{jwRC + (jw)^2LC + 1} = \frac{1}{jwRC - w^2LC + 1}$$

$$\theta = \frac{imag}{real} = \frac{wRC}{-w^2LC+1}$$

c) $$|H(w)| = \frac{1}{\sqrt{(w)^2(RC)^2 + (1 - w^2(LC)^2)^2}}$$

d) $|H(w)|$ is maximized, when $\frac{d|H(w)|}{dw} = 0$

$$\frac{d|H(w)|}{dw} = -\frac{1}{2}\left((RCw)^2 + (1-w^2(LC)^2)^2\right)^{-3/2}\left(2(RC)^2w + 2(1-w^2(LC)^2)(-2w(LC)\right.$$

$$2(RC)^2w - 4(w(LC)^2)(1-w^2(LC)^2) = 0$$

Con't

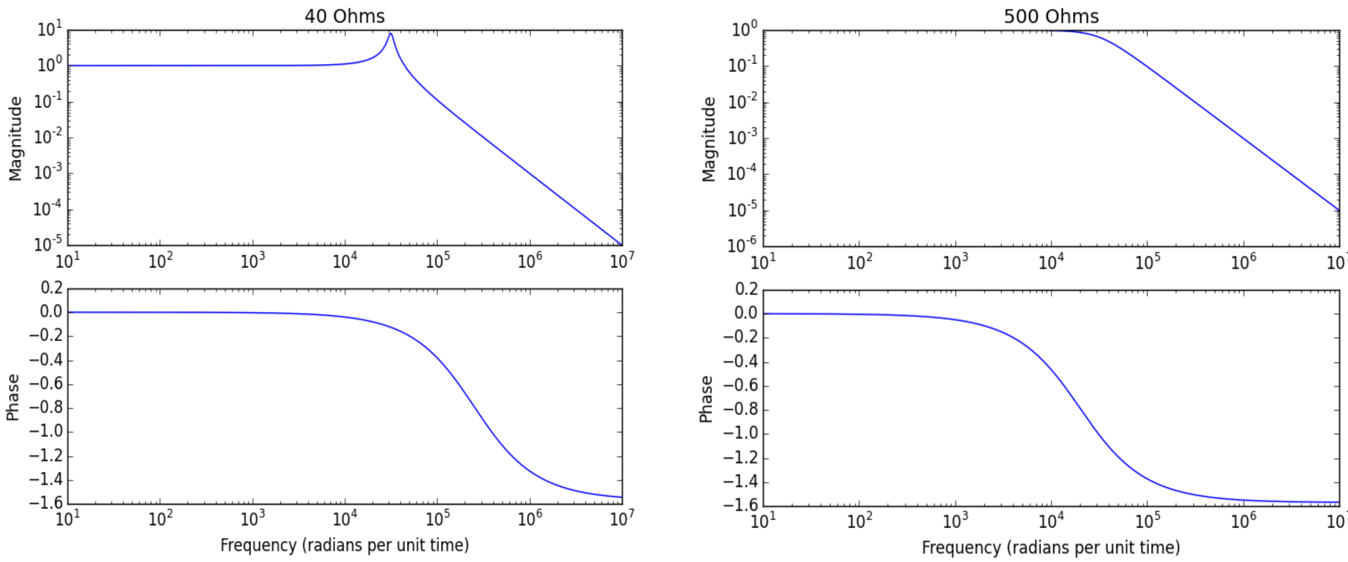
$$2(R\cancel{C})^2\cancel{w} = 4\cancel{w}(L\cancel{C})^2 + 4w^{\cancel{3}}(L\cancel{C})^4$$

$$2R = 4L^2 + 4w^2L^{\cancel{4}}C^2$$

$$\sqrt{\frac{{}^{1}\cancel{2}R - \cancel{4}^{2}L^2}{{}_{2}\cancel{4}L^4C^2}} = w$$

$$\boxed{w = \sqrt{\frac{R - 2L}{2L^4C^2}}}$$

Both graphs display low pass filters, where the higher frequencies are cut off. However, changing the resistance changes where the frequency is cutoff.



By comparing the two graphs, we can see that the graph with the higher resistance (500 Ohms) is cutoff at a lower frequency than the lower resistance (40 Ohms). As a result, we can see that the cutoff frequency is inversely proportional to the resistance, when the capacitance and inductance are kept constant. Another trend we can see is that the phase shift corresponds to the cutoff frequency.

For the lower resistance, we can also see that the magnitude suddenly spikes before it drops. This relates to the maximum value for omega, the equation of which was calculated in the previous question.