

ESO207A: Data Structures and Algorithms

Theoretical Assignment 1

Due Date: 4th September, 2023

Total Number of Pages: 3

Total Points 160

Instructions-

1. For submission typeset the solution to each problem and compile them in a single pdf file. Hand-written solutions will not be accepted. You can use L^AT_EX or Word for typesetting.
 2. Start each problem from a new page. Write down your Name, Roll number and problem number clearly for each problem.
 3. For each question, give the pseudo-code of the algorithm with a clear description of the algorithm. Unclear description will receive less marks. Less optimal solutions will receive only partial marks.
 4. Assume that sorting would have $O(n\log(n))$ complexity.
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Question 1. Ideal profits

(40 points)

In the X world, companies have a hierarchical structure to form a large binary tree network (can be assumed to be a perfect binary tree). Thus every company has two sub companies as their children with the root as company X. The total number of companies in the structure is N . The wealth of each company follow the same general trend and doubles after every month. Also after every year, half of the wealth is distributed to the two child companies (i.e. one fourth to each) if they exist (i.e. the leaf node companies do not distribute their wealth). You can assume that at the end of the year, the month (doubling) operation happens before the year (distribution) operation. Also the sharing operation at the end of the year happens simultaneously for all the companies.

Given the initial wealth of each of the N companies, you want to determine the final wealth of each company after m months.

(A perfect binary tree is a special tree such that all leaf nodes are at the maximum depth of the tree, and the tree is completely filled with no gaps. Detailed explanation *here*)

- (a) (20 points) Design an algorithm in $O(n^3\log(m))$ complexity to find the final wealth of each company after m months.
- (b) (10 points) Analyze the time complexity of your algorithm and briefly argue about the correctness of your solution.
- (c) (10 points) Consider the case of a single company (i.e. only root) in the tree. Give a constant time solution to find the final wealth after m months.

Question 2. Moody Friends

(40 points)

P friends arrive at a hotel after a long journey and want rooms for a night. This hotel has n rooms linearly arranged in form of an array from left to right where array values depict the capacities of the rooms. As these are very close friends they will only consider consecutive rooms for staying. As you are the manager of the hotel you are required to find cheapest room allocation possible for them (sum of the capacities of selected rooms should be greater than or equal to P). Cost of booking every room is same and is equal to C .

- (a) (15 points) Design an algorithm in $O(n)$ time complexity for determining the minimum cost room allocation. The allocated rooms should be consecutive in the array and their capacities should sum to at least P . You will get 5 points if you design an $O(n \log n)$ time algorithm.
- (b) (15 points) Now suppose they don't care about the cost and total capacity anymore. But they came up with a beauty criteria for an allocation. According to them, an allocation is beautiful if GCD (Greatest Common Divisor) of capacities of all rooms in the allocation is at least equal to or greater than a constant K . And they want to take maximum number of contiguous rooms possible. Your task is to design an algorithm in $O(n \log(n))$ time complexity for determining the maximum number of contiguous rooms they can get which satisfy the beauty constraints. You can assume access to a blackbox GCD algorithm which can give you GCD of two numbers in constant $O(1)$ time.
- (c) (10 points) Give proof of correctness and time complexity analysis of your approach for part (a).

Question 3. BST universe

(30 points)

You live in a BST world where people are crazy about collecting BSTs and trading them for high values. You also love Binary Search Trees and possess a BST. The number of nodes in your BST is n .

- (a) (10 points) The Rival group broke into your lab to steal your BST but you were able to stop them. But still they managed to swap exactly two of the vertices in your BST. Design an $O(n)$ algorithm to find which nodes are swapped and the list of their common ancestors.
- (b) (20 points) Seeing you were able to easily revert the damage to your tree, they attacked again and this time managed to rearrange exactly k of your nodes in such a way that none of the k nodes remain at the same position after the rearrangement. Also all the values inside this BST are positive integers and upper bounded by a constant G . Your task is to determine the value of k and which nodes were rearranged. Design an algorithm of complexity $O(\min(G + n, n \log(n)))$ for the same. (Hint : Consider two cases for $G < n \log(n)$ and $G > n \log(n)$)

Question 4. Helping Joker

(20 points)

Joker was challenged by his master to solve a puzzle. His master showed him a deck of n cards. Each card has value written on it. Master announced that all the cards are indexed from 1 to n from top to bottom such that $(a_1 < a_2 < \dots < a_{n-1} < a_n)$. Then his master performed an operation on this deck invisible to Joker (Joker was not able to see what he did), he picked a random number k between 0 and n and shifted the top k cards to the bottom of the deck. So after the operation arrangement of cards from top to bottom looks like $(a_{k+1}, a_{k+2} \dots a_n, a_1, a_2 \dots a_k)$ where $(k+1, k+2 \dots n, 1, 2 \dots k)$ are original indices in the sorted deck. Joker's task is to determine the value of k . Joker can make a query to his master. In a query, joker can ask to look at the value of any card in the deck. Joker asked you for help because he knew you were taking an algorithms course this semester.

- (a) (15 points) Design an algorithm of complexity $O(\log(n))$ for Joker to find the value of k .
- (b) (5 points) Provide time complexity analysis for your strategy.

Question 5. One Piece Treasure

(30 points)

Strawhat Luffy and his crew got lost while searching for One piece (worlds largest known treasure). It turns out he is trapped by his rival Blackbeard. In order to get out he just needs to solve a simple problem. You being the smartest on his crew are summoned to help. On the gate out, you get to know about a hidden string of lowercase english alphabets of length n . Also, an oracle is provided which accepts an input query of format (i, j) , and returns true if the *substring* (i, j) of the hidden string is a palindrome and false otherwise in $O(1)$ time. But there is a catch, the place will collapse killing all the crew members, if you ask any more than $n * \log^2(n)$ queries to the oracle. The string is hidden and you can't access it. (Assume the string is very big i.e. n is a large number)

- (a) (30 points) You need to design a strategy to find number of palindromic substrings in the hidden string so your crew can safely escape from this region. Please state your algorithm clearly with pseudocode.

(A contiguous portion of the string is called a substring)