Divide and Conquer

→ Overview of the paradigm :

i) divide it into 2 or more instances of the same problem.

ii) solve each smaller instance recursively.
(imp to define base case)

iii) Combine the solutions of the smaller instances to get solution of the original problem.

- Merge Sort
- Multiplication of two n-bit integers.
- Count inversions in an array.
- Median in linear time.

Closest Pair of Points $O(n \log n)$

```
CP-Dist (P) {
  if (|P| = 1) return (∞, P);          → base case
    Pmed = x-median of P;
    (PL, PR) = split by x-med;
  (δL, PL) = CP-Dist (PL);
   (δR, PR) = CP-Dist (PR);
    δ = min (δL, δR);
    P' = merge (PL, PR);              ← merge step
    SL = strip of PL;
    SR = strip of PR;
  while (SL ≠ φ and SR ≠ φ) {
     a = first (SL) ; b = first(SR);
```

if $(y(a) \leq y(b))$ {
{ Compute dist from a to the first 4 points in S_R
update δ;
Remove a from S_L; }
else {
}

return sorted
{ return (δ, P'); }

→ Convex Hull $O(n \log n)$

→ Non Dominated $O(nh)$ $O(n \log n)$
$O(n^2)$:) $O(n \log h)$

→ Multiplying two polynomials

$$A(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1}$$
$$B(x) = b_0 + b_1 x + \dots + b_{n-1} x^{n-1}$$
(Coefficient representation)

$$C(x) = \sum_{0 \leq i \leq 2n-2} c_i x^i \qquad C(x) = A(x) \times B(x)$$

$$c_i = a_0 b_i + a_1 b_{i-1} + \dots + a_i b_0 \quad (x^i \text{ terms})$$

→ Point Value representation

$$\{(r_0, t_0), (r_1, t_1), \dots (r_{m-1}, t_{m-1})\}$$
m pairs with all r_i's distinct
$A(x) \longrightarrow$ deg $\leq m-1$ st
$A(r_i) = t_i \quad \forall i$

$\{A(r_0), \ldots A(r_{m-1})\} \times \{B(r_0), \ldots B(r_{m-1})\}$

$\downarrow$

$\{C(r_0) \ldots C(r_{m-1})\}$

$\Downarrow \qquad\qquad \Downarrow$

$A(r_0) \times B(r_0) \qquad A(r_{m-1}) \times B(r_{m-1})$

$A, B \Rightarrow (n-1)$ each $\quad C \Rightarrow 2n-2$ max

$\hookrightarrow 0$ to $2n-2$

$=$ total $(2n-1)$ terms

$$\boxed{A(x) = A_{even}(x^2) + x A_{odd}(x^2)}$$

$\rightarrow$ <u>n^{th} roots of unity</u>

$\omega_n^{j} = e^{i\left(\frac{2\pi j}{n}\right)} \qquad arg(z) \Rightarrow \left(0, \frac{2\pi}{n}, \frac{4\pi}{n} \ldots\right)$

$R_n = \{1, \omega_n, \omega_n^2, \omega_n^3, \ldots \omega_n^{n-1}\}$

If $n =$ odd $\quad \boxed{R_n^2 = R_n}$

If $n =$ even $\quad \boxed{R_n^2 = R_{\frac{n}{2}}}$