s-t cut and min cut

$A \subset V$ with $s \in A$ and $t \in \bar{A}$

$\text{cut}(A, \bar{A}) = \{ (x,y) \in E \mid x \in A \text{ and } y \in \bar{A} \text{ or } y \in A \text{ and } x \in \bar{A} \}$

capacity $c(A, \bar{A}) = \sum_{\substack{u \in A, v \in \bar{A} \\ (u,v) \in E}} c(u,v)$

Min cut $\Rightarrow$ cut of least capacity.

→ **Project Selection Problem**

Project $i \in P$ has profit $p_i \in R^+$
Project $j \in C$ has cost $c_j \in R^+$

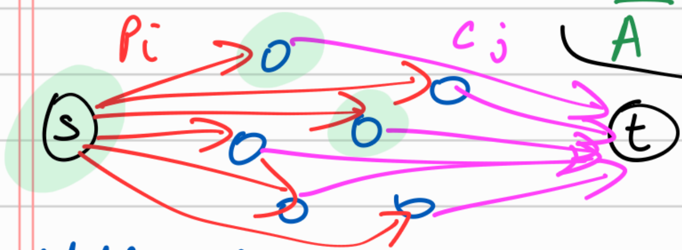
So $A \subseteq P \cup C$ st $\exists$ no project in $\bar{A}$ on which any project of A depends.

max^m $q(A) = \sum_{i \in P \cap A} p_i - \sum_{j \in C \cap A} c_j$

$= P_{tot} - \sum_{i \in P \cap \bar{A}} p_i - \sum_{j \in C \cap A} c_j$

$= P_{tot} - \left(\sum p_i + \sum c_j \right)$

$\bar{A}$ $\quad$ A $\quad$ min^m



middle edges → ∞

$q'(A) = \sum_{i \in P \cap \bar{A}} p_i + \sum_{j \in C \cap A} c_j$

→ Summary

subset of obj A st $q(A)$ is maximised

1) Formulate $q(A)$ clearly.
2) Manipulate $q(A)$ so as to get another func $q'(A)$ st maximising $q(A)$ is equivalent to minimising $q'(A)$.
3) Express $q'(A)$ as the capacity of a cut.
   - add s and t
   - connect s and t to vertices with suitable weights.
   - in $q'(A)$ if any term $x \in A \rightarrow y \in \bar{A}$ leave its weight as it is / or 1.
   - in $q'(A)$ if any term $x \in A \Rightarrow$ vertices to t that weight edge
   - in $q'(A)$ if any term $x \in \bar{A}$ then s to vertices that weight edge.
   - if you want no edge $A \rightarrow \bar{A}$ (except s and t ones) make them ∞.
4) State and prove the theorem that relates min-cut of the network to the subset A that minimises $q'(A)$.

→ Image Segmentation

each pixel either f_i or b_i such that sum of values is max, and if 2 diff pixel adj then $-p_{ij}$

$$q(A) = \sum_{i \in A} f_i + \sum_{j \in \bar{A}} b_j - \sum_{\substack{(i,j) \in E \\ i \in A,\, j \in \bar{A} \,/\, i \in \bar{A},\, j \in A}} p_{ij}$$

$$= F - \sum_{i \in \bar{A}} f_i + B - \sum_{j \in A} b_j - \sum p_{ij}$$

$$= (F + B) - \Big(\sum_{i \in \bar{A}} f_i + \sum_{i \in A} b_j + \sum_{i \in A,\, j \in A} p_{ij} \Big)$$

$\Downarrow$ $s \to v$ f_i $\qquad$ $\Downarrow$ $v \to t$ b_j