Lecture 14

- Subsequence – $C[1\ldots m]$ is a subseq of $A[1\ldots n]$ if $\exists$ m integers $1 \le i_1 < i_2 \ldots < i_m \le n$ st $\forall\ 1 \le j \le m$, $C[j] = A[i_j]$

Base Case : $n = 0$ or $m = 0$

General Case :
$A[i] = B[j]$: $dp[i][j] = dp[i-1][j-1] + 1$
else $dp[i][j] = \max(dp[i][j-1], dp[i-1][j])$

use Bottom-Up Approach to reduce to polynomial TC.

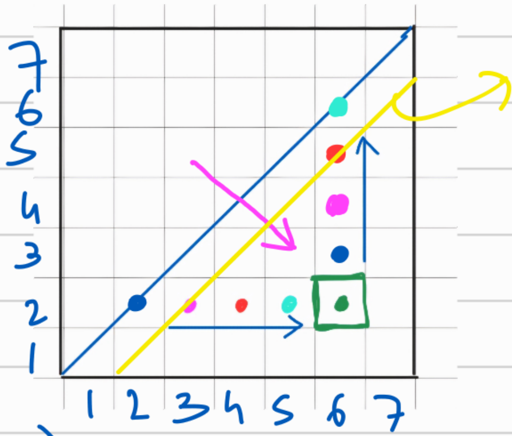
identify the number of sub-problems.

- Triangulation of convex polygon:

$w(i, j, K) \longrightarrow$ some $O(1)$ func

General: $T(i, j) = \min\limits_{i<K<j} \left(T(i, K) + T(K, j) + w(i, j, K) \right)$ $(j > i+1)$

Base: $T(i, i+1) = 0$



base case

Here we move in diagonal manner

$T[i, i+1] \longrightarrow 0$
for $(\Delta = 2$ to $n-1)$
 for $(i = 1$ to $n - \Delta)$
  $j = \Delta + i$;
   for($K = i+1$ to $j-1$)

return $T[1][n]$

Greedy & DP:

Optimal solution of a problem contains within it optimal solution for its smaller instances as well.

- ## Bitonic Tour

$\delta(p_i, p_j) \rightarrow$ distance between p_i and p_j.
A tour is bitonic if every vertical line cuts the tour exactly twice, except for the two extreme points.
cost of tour = Total dist travelled.

i) Travel backwards. If reached to some p_i from p_n and to some p_j from some p_{n+1}
Now, point p_{i-1} can either go to p_i or p_j

$T[i, j]$: least dist travelled to reach p_1 by 2 paths from p_i and p_j covering each point $p_2, \ldots p_{i-1}$ exactly once.

$$T[i,j] = \min\left(T[i-1, j] + \delta(p_{i-1}, p_i), \; T[i-1, i] + \delta(p_{i-1}, p_j) \right)$$

Base Case: $i = 1$ set $\delta(p_1, p_j)$;

Add one more point $p_{n+1} = p_n$;
for ($j=1$ to $n+1$) $T[1, j] = \delta(p_1, p_j)$;
for ($i=2$ to n): for ($j = i+1$ to $n+1$):
$T[i, j] = \min\left(T(i-1, j) + \delta(i, i-1)\right) + $ ———
return $T[n, n+1]$;