

GitHub: <https://github.com/aprajitagaurav/HPC-Spring22>

Processor: 2.3 GHz Quad-Core Intel Core i5

Memory: 8 GB 2133 MHz LPDDR3

Matrix Matrix multiplication:

- For loop ordering n, k, m:

Dimension	Time	Gflop/s	GB/s	Error
16	0.350713	5.702672	3.385961	0.000000e+00
64	0.135720	14.737387	7.714101	0.000000e+00
112	0.135702	14.742714	7.568804	0.000000e+00
160	0.135451	14.817508	7.547668	0.000000e+00
208	0.165118	12.207977	6.192027	0.000000e+00
256	0.154081	13.066252	6.609686	0.000000e+00
304	0.169911	11.905084	6.011284	0.000000e+00
352	0.177155	11.324874	5.710696	0.000000e+00
400	0.171835	11.918435	6.003912	0.000000e+00
448	0.187824	11.489304	5.783121	0.000000e+00
496	0.193061	11.376858	5.722835	0.000000e+00
544	0.216735	10.399112	5.228230	0.000000e+00
592	0.236082	8.788236	4.416385	0.000000e+00
640	0.260282	8.057233	4.047500	0.000000e+00
688	0.347452	7.498269	3.765483	0.000000e+00
736	0.310936	7.693307	3.862333	0.000000e+00
784	0.395983	7.301679	3.664809	0.000000e+00
832	0.324535	7.098529	3.562062	0.000000e+00
880	0.386926	7.044994	3.534506	0.000000e+00
928	0.481542	6.638493	3.329977	0.000000e+00
976	0.586874	6.336716	3.178097	0.000000e+00
1024	0.346040	6.205883	3.112032	0.000000e+00
1072	0.478801	5.145874	2.580137	0.000000e+00
1120	0.452189	6.213892	3.115268	0.000000e+00
1168	0.565053	5.639872	2.827179	0.000000e+00
1216	0.713020	5.043462	2.527952	0.000000e+00
1264	0.786305	5.136651	2.574421	0.000000e+00
1312	0.787281	5.737220	2.875170	0.000000e+00
1360	0.916850	5.487168	2.749636	0.000000e+00
1408	1.011297	5.520257	2.766009	0.000000e+00
1456	1.117160	5.525844	2.768615	0.000000e+00
1504	1.481862	4.591617	2.300388	0.000000e+00
1552	1.407853	5.310652	2.660459	0.000000e+00
1600	1.528957	5.357902	2.683974	0.000000e+00
1648	1.640590	5.456341	2.733137	0.000000e+00
1696	1.906529	5.117573	2.563313	0.000000e+00
1744	1.961579	5.408336	2.708820	0.000000e+00
1792	2.125867	5.413872	2.711467	0.000000e+00
1840	2.317723	5.375538	2.692151	0.000000e+00
1888	3.379789	3.982414	1.994371	0.000000e+00
1936	2.913319	4.981475	2.494597	0.000000e+00
1984	3.698583	4.222986	2.114686	0.000000e+00

- For loop ordering k, n m:

Dimension	Time	Gflop/s	GB/s	Error
16	0.211304	9.465061	5.619880	0.000000e+00
64	0.138111	14.482286	7.580572	0.000000e+00
112	0.141750	14.113660	7.245852	0.000000e+00
160	0.168040	11.943851	6.083899	0.000000e+00
208	0.199795	10.089112	5.117314	0.000000e+00
256	0.209235	9.622041	4.867400	0.000000e+00
304	0.215297	9.395406	4.744062	0.000000e+00

352	0.213032	9.417637	4.748950	0.000000e+00
400	0.225810	9.069554	4.568788	0.000000e+00
448	0.260898	8.271302	4.163345	0.000000e+00
496	0.292207	7.516700	3.781082	0.000000e+00
544	0.302409	7.452979	3.747040	0.000000e+00
592	0.294190	7.052407	3.544073	0.000000e+00
640	0.370030	5.667525	2.847046	0.000000e+00
688	0.421078	6.187177	3.107078	0.000000e+00
736	0.383192	6.242645	3.134045	0.000000e+00
784	0.495546	5.834663	2.928495	0.000000e+00
832	0.405581	5.680049	2.850265	0.000000e+00
880	0.509826	5.346700	2.682463	0.000000e+00
928	0.657559	4.861489	2.438603	0.000000e+00
976	0.634979	5.856665	2.937333	0.000000e+00
1024	0.365771	5.871114	2.944157	0.000000e+00
1072	0.437637	5.629893	2.822824	0.000000e+00
1120	0.507017	5.541935	2.778390	0.000000e+00
1168	0.588304	5.416978	2.715446	0.000000e+00
1216	0.715616	5.025172	2.518785	0.000000e+00
1264	0.771387	5.235992	2.624210	0.000000e+00
1312	0.891962	5.063898	2.537739	0.000000e+00
1360	1.023908	4.913441	2.462140	0.000000e+00
1408	1.144572	4.877472	2.443932	0.000000e+00
1456	1.398774	4.413333	2.211213	0.000000e+00
1504	1.401858	4.853661	2.431671	0.000000e+00
1552	1.647847	4.537205	2.272988	0.000000e+00
1600	2.015971	4.063551	2.035585	0.000000e+00
1648	1.983335	4.513417	2.260817	0.000000e+00
1696	2.070041	4.713338	2.360838	0.000000e+00
1744	2.371428	4.473625	2.240660	0.000000e+00
1792	2.716172	4.237277	2.122185	0.000000e+00
1840	2.753640	4.524559	2.265968	0.000000e+00
1888	2.988625	4.503649	2.255403	0.000000e+00
1936	3.206304	4.526279	2.266647	0.000000e+00
1984	3.454776	4.521006	2.263921	0.000000e+00

For loop ordering m, k n:

Dimension	Time	Gflop/s	GB/s	Error
16	0.507789	3.938646	2.338571	0.000000e+00
64	0.819860	2.439635	1.276996	0.000000e+00
112	0.587411	3.405825	1.748526	0.000000e+00
160	1.694681	1.184317	0.603262	0.000000e+00
208	1.270955	1.586017	0.804446	0.000000e+00
256	4.811506	0.418427	0.211665	0.000000e+00
304	1.557447	1.298793	0.655805	0.000000e+00
352	1.826754	1.098261	0.553811	0.000000e+00
400	1.671499	1.225248	0.617219	0.000000e+00
448	3.007135	0.717616	0.361211	0.000000e+00
496	1.804958	1.216887	0.612124	0.000000e+00
544	2.297397	0.981044	0.493227	0.000000e+00
592	2.193784	0.945739	0.475266	0.000000e+00
640	4.569211	0.458975	0.230563	0.000000e+00
688	6.117776	0.425855	0.213856	0.000000e+00
736	3.813407	0.627295	0.314926	0.000000e+00
784	8.029510	0.360089	0.180734	0.000000e+00
832	7.484018	0.307819	0.154464	0.000000e+00
880	9.585147	0.284387	0.142678	0.000000e+00
928	12.776593	0.250201	0.125505	0.000000e+00
976	17.030981	0.218358	0.109515	0.000000e+00
1024	11.613698	0.184910	0.092726	0.000000e+00
1072	11.375497	0.216593	0.108599	0.000000e+00
1120	12.680755	0.221584	0.111089	0.000000e+00
1168	16.097405	0.197971	0.099240	0.000000e+00
1216	17.780655	0.202247	0.101373	0.000000e+00

1264	20.138791	0.200557	0.100517	0.000000e+00
1312	21.897339	0.206272	0.103372	0.000000e+00
1360	24.764440	0.203151	0.101799	0.000000e+00
1408	25.989252	0.214805	0.107631	0.000000e+00
1456	29.819691	0.207019	0.103723	0.000000e+00
1504	35.075088	0.193988	0.097187	0.000000e+00
1552	38.540976	0.193991	0.097183	0.000000e+00
1600	43.320626	0.189102	0.094728	0.000000e+00
1648	48.727426	0.183708	0.092021	0.000000e+00
1696	52.106874	0.187246	0.093789	0.000000e+00
1744	61.574942	0.172292	0.086294	0.000000e+00
1792	69.461006	0.165693	0.082985	0.000000e+00
1840	69.341004	0.179677	0.089985	0.000000e+00
1888	78.092641	0.172356	0.086315	0.000000e+00
1936	79.624510	0.182263	0.091273	0.000000e+00
1984	86.850637	0.179838	0.090055	0.000000e+00

Due to following computations:

$C_{ij} = C[i+j*m]$,
 $A_{ip} = A[i+p*m]$, and
 $B_{pj} = B[p+j*k]$.

In order to read sequentially along the arrays, C and A clearly would do better to read sequentially along index i, and B along index p. Therefore the best order for the loops is $j \mid p \mid (n, k, m)$.

This is also done by MMult0.

- BLOCK_SIZE=16

Dimension	Time	Gflop/s	GB/s	Error
16	0.543631	3.678973	2.184390	0.000000e+00
64	0.551797	3.624812	1.897362	0.000000e+00
112	0.554523	3.607817	1.852227	0.000000e+00
160	0.557968	3.597054	1.832249	0.000000e+00
208	0.561783	3.588138	1.819945	0.000000e+00
256	0.607974	3.311433	1.675120	0.000000e+00
304	0.571045	3.542282	1.788619	0.000000e+00
352	0.568428	3.529474	1.779777	0.000000e+00
400	0.582130	3.518113	1.772249	0.000000e+00
448	0.603647	3.574888	1.799414	0.000000e+00
496	0.709172	3.097175	1.557954	0.000000e+00
544	0.639415	3.524858	1.772148	0.000000e+00
592	0.993318	2.088703	1.049644	0.000000e+00
640	0.645298	3.249897	1.632566	0.000000e+00
688	0.807777	3.225254	1.619659	0.000000e+00
736	0.720157	3.321676	1.667608	0.000000e+00
784	0.904489	3.196659	1.604445	0.000000e+00
832	0.667357	3.452007	1.732227	0.000000e+00
880	0.777426	3.506299	1.759126	0.000000e+00
928	0.923084	3.463083	1.737139	0.000000e+00
976	1.067521	3.483637	1.747172	0.000000e+00
1024	0.758374	2.831694	1.419995	0.000000e+00
1072	0.716984	3.436411	1.723014	0.000000e+00
1120	0.803281	3.497972	1.753671	0.000000e+00
1168	0.908583	3.507469	1.758239	0.000000e+00
1216	1.033792	3.478546	1.743564	0.000000e+00
1264	1.147473	3.519889	1.764121	0.000000e+00
1312	1.286273	3.511546	1.759788	0.000000e+00
1360	1.436292	3.502709	1.755218	0.000000e+00
1408	1.599786	3.489603	1.748519	0.000000e+00
1456	1.761263	3.505015	1.756118	0.000000e+00
1504	1.950373	3.488637	1.747798	0.000000e+00
1552	2.122670	3.522271	1.764540	0.000000e+00
1600	2.337970	3.503894	1.755232	0.000000e+00
1648	2.651463	3.376106	1.691126	0.000000e+00

1696	2.825128	3.453580	1.729844	0.000000e+00
1744	3.034746	3.495804	1.750909	0.000000e+00
1792	3.769686	3.053085	1.529098	0.000000e+00
1840	3.590571	3.469924	1.737791	0.000000e+00
1888	3.837813	3.507132	1.756352	0.000000e+00
1936	4.276745	3.393382	1.699320	0.000000e+00
1984	4.836607	3.229343	1.617113	0.000000e+00

BLOCK_SIZE=20

Dimension	Time	Gflop/s	GB/s	Error
20	0.579249	3.452777	1.985347	0.000000e+00
60	0.559784	3.573094	1.875874	0.000000e+00
100	0.557862	3.588699	1.848180	0.000000e+00
140	0.561706	3.566133	1.821275	0.000000e+00
180	0.560603	3.578661	1.819153	0.000000e+00
220	0.627423	3.190549	1.617028	0.000000e+00
260	0.616724	3.248884	1.643185	0.000000e+00
300	0.582974	3.519883	1.777541	0.000000e+00
340	0.649594	3.146285	1.587023	0.000000e+00
380	0.638175	3.267340	1.646568	0.000000e+00
420	0.591798	3.505358	1.765198	0.000000e+00
460	0.658345	3.252691	1.636952	0.000000e+00
500	0.676935	3.323807	1.671875	0.000000e+00
540	0.638538	3.452411	1.735796	0.000000e+00
580	0.724328	3.232435	1.624577	0.000000e+00
620	0.731009	3.260259	1.638017	0.000000e+00
660	0.698246	3.293924	1.654448	0.000000e+00
700	0.619045	3.324475	1.669361	0.000000e+00
740	0.722851	3.363547	1.688591	0.000000e+00
780	0.851054	3.345631	1.679250	0.000000e+00
820	0.664852	3.317239	1.664688	0.000000e+00
860	0.729979	3.485338	1.748748	0.000000e+00
900	0.836288	3.486836	1.749229	0.000000e+00
940	0.975220	3.406754	1.708814	0.000000e+00
980	1.084350	3.471911	1.741270	0.000000e+00
1020	0.624725	3.397362	1.703677	0.000000e+00
1060	0.682529	3.490010	1.749944	0.000000e+00
1100	0.790462	3.367652	1.688418	0.000000e+00
1140	0.852641	3.475187	1.742166	0.000000e+00
1180	2.448707	1.341959	0.672685	0.000000e+00
1220	1.108592	3.275954	1.642005	0.000000e+00
1260	1.258974	3.177788	1.592677	0.000000e+00
1300	1.553600	2.828270	1.417398	0.000000e+00
1340	1.470048	3.273504	1.640416	0.000000e+00
1380	1.541445	3.409881	1.708647	0.000000e+00
1420	1.709812	3.349244	1.678160	0.000000e+00
1460	1.867029	3.333785	1.670317	0.000000e+00
1500	2.092400	3.225961	1.616206	0.000000e+00
1540	2.185747	3.341891	1.674201	0.000000e+00
1580	2.323411	3.395277	1.700862	0.000000e+00
1620	2.767272	3.072722	1.539206	0.000000e+00
1660	2.733746	3.346541	1.676295	0.000000e+00
1700	3.198452	3.072111	1.538766	0.000000e+00
1740	3.012492	3.497453	1.751742	0.000000e+00
1780	3.232809	3.489072	1.747476	0.000000e+00
1820	3.445915	3.498965	1.752366	0.000000e+00
1860	3.947086	3.260561	1.632910	0.000000e+00
1900	4.082218	3.360428	1.682867	0.000000e+00
1940	4.260699	3.427317	1.716309	0.000000e+00
1980	4.568867	3.397951	1.701550	0.000000e+00

BLOCK_SIZE=200

Dimension	Time	Gflop/s	GB/s	Error
200	0.213760	9.431140	4.786304	0.000000e+00
400	0.262342	7.806606	3.932578	0.000000e+00
600	0.286193	7.547345	3.792541	0.000000e+00
800	0.255261	8.023151	4.026619	0.000000e+00

1000	0.510312	7.838335	3.930925	0.000000e+00
1200	0.456022	7.578581	3.798763	0.000000e+00
1400	0.828954	6.620388	3.317287	0.000000e+00
1600	1.079274	7.590288	3.802260	0.000000e+00
1800	1.586762	7.350820	3.681536	0.000000e+00

BLOCK_SIZE=300

Dimension	Time	Gflop/s	GB/s	Error
300	0.196518	10.441808	5.273113	0.000000e+00
600	0.243925	8.855193	4.449735	0.000000e+00
900	0.317941	9.171505	4.601038	0.000000e+00
1200	0.377251	9.161018	4.591960	0.000000e+00
1500	0.878820	7.680752	3.848057	0.000000e+00
1800	1.255126	9.293093	4.654291	0.000000e+00

BLOCK_SIZE=380

Dimension	Time	Gflop/s	GB/s	Error
380	0.188817	11.043163	5.565173	0.000000e+00
760	0.389281	6.765957	3.396332	0.000000e+00
1140	0.325228	9.110804	4.567390	0.000000e+00
1520	0.927646	7.571440	3.793192	0.000000e+00
1900	2.866644	4.785387	2.396472	0.000000e+00

BLOCK_SIZE=400

Dimension	Time	Gflop/s	GB/s	Error
400	0.177352	11.547687	5.817147	0.000000e+00
800	0.203670	10.055507	5.046607	0.000000e+00
1200	0.371818	9.294877	4.659057	0.000000e+00
1600	0.849558	9.642657	4.830368	0.000000e+00

BLOCK_SIZE=420

Dimension	Time	Gflop/s	GB/s	Error
420	0.187238	11.079288	5.579213	0.000000e+00
840	0.240346	9.864178	4.949704	0.000000e+00
1260	0.415104	9.637955	4.830451	0.000000e+00
1680	0.945068	10.034482	5.026200	0.000000e+00

BLOCK_SIZE=500

Dimension	Time	Gflop/s	GB/s	Error
500	0.211634	10.631544	5.347667	0.000000e+00
1000	0.437331	9.146390	4.586914	0.000000e+00
1500	0.689970	9.783028	4.901297	0.000000e+00

BLOCK_SIZE=600

Dimension	Time	Gflop/s	GB/s	Error
600	0.225109	9.595340	4.821658	0.000000e+00
1200	0.410852	8.411779	4.216404	0.000000e+00
1800	1.505790	7.746100	3.879505	0.000000e+00

BLOCK_SIZE=800

Dimension	Time	Gflop/s	GB/s	Error
800	0.270774	7.563495	3.795929	0.000000e+00
1600	1.320067	6.205744	3.108690	0.000000e+00

Therefore, BLOCK_SIZE of around 400 is optimal.

- Peak performance with BLOCK_SIZE = 400 with -O2 flag

Dimension	Time	Gflop/s	GB/s	Error
400	0.187500	10.922678	5.502299	0.000000e+00

Previous peak performance:

With NREPEATS = 50, PFIRST = 20, PLAST = 600, PINC = 20 and -O3 optimisation:

Dimension	Time	Gflop/s	GB/s
20	0.000210	19.040728	31.988423

Percentage peak performance achieved = $10.922678 / 19.040728 * 100 = 57.364\%$

Gauss-Seidel:

N: 7, NUM_THREADS: 2
residue 115 : 9.94508e-07
time: 0.000388

N: 25, NUM_THREADS: 4
residue 1223 : 9.97125e-07
time: 0.0236545

N: 205, NUM_THREADS: 6
residue 5000 : 74.4251
time: 1.0181

N: 2005, NUM_THREADS: 8
residue 5000 : 2611.97
time: 40.9997

Jacobi:

N: 7, NUM_THREADS: 2, time: 0.00225886,
residue 199 : 9.82778e-07

N: 25, NUM_THREADS: 4,
residue 2306: 9.93562e-07
time: 0.0744037

N: 205, NUM_THREADS: 6,
residue 5000 : 93.4599
time: 0.905635

N: 2005, NUM_THREADS: 8,
residue 5000 : 1893.17
time: 30.927