Latex Assignment 8

APARNA ANAND

6,september 2023

Example:-1-30 (12.10)

1. Represent graphically a displacement of 40 km, 30° west of south.
2. Classify the following measures as scalars and vectors.
   (a) 5 Seconds.
   (b) $1000cm^3$.
   (c) 10 Newton
   (d) $30km/hr$.
   (e) $10g/cm^3$
   (f) $20m/s$ towards north.
3. In Fig. 1, which of the vectors are :
   (a) Collinear
   (b) Equal
   (c) Coinitial.
4. Find the values of x, y and z so that the vectors $\vec{a} = x\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b} = 2\hat{i}+y\hat{j}+\hat{k}$ are equal.
5. Let $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j}$. Is $\left|\vec{a}\right| = \left|\vec{b}\right|$? Are the vectors $\vec{a}$ and $\vec{b}$ equal?
6. Find unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{K}$.
7. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units.
8. Find the unit vector in the direction of the sum of the vectors, $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$.
9. Write the direction ratios of the vector $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and hence calculate its direction cosines.

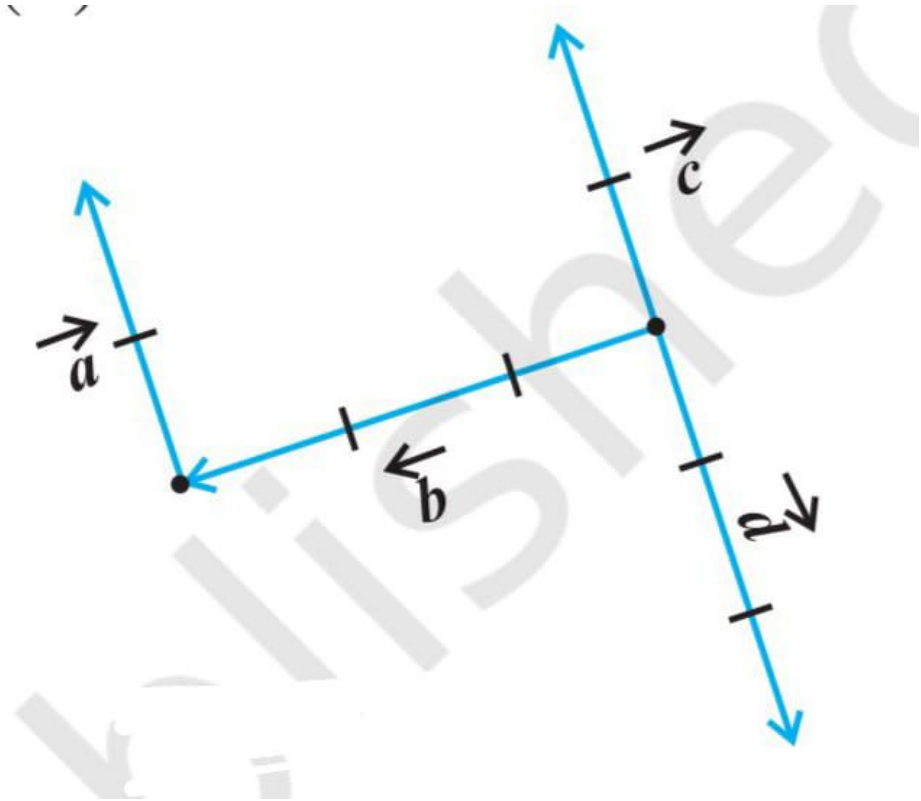

Figure 1: 10.5

10. Find the vector joining the points $P(2, 3, 0)$ and $Q(-1, -2, -4)$ directed from P to Q.

11. Consider two points P and Q with position vectors $\overrightarrow{OP} = 3\overrightarrow{a} - 2\overrightarrow{b}$ and $\overrightarrow{OQ} = \overrightarrow{a} + \overrightarrow{b}$. Find the position vector of a point R which divides the line joining P and Q in the ratio 2 : 1,

    (i) internally, and

    (ii) externally.

12. Show that the points $A(2\hat{i} - \hat{j} + \hat{k}), B(\hat{i} - 3\hat{j} - 5\hat{k}), C(3\hat{i} - 4\hat{j} - 4\hat{k})$ are the vertices of a right angled triangle.

13. Find the angle between two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ with magnitudes 1 and 2 respectively and when $\overrightarrow{a} \cdot \overrightarrow{b} = 1$.

14. Find angle θ between the vectors $\overrightarrow{a} = \hat{i} + \hat{j} - \hat{k}$ and $\overrightarrow{b} = \hat{i} - \hat{j} + \hat{k}$.

15. If $\vec{a} = 5\hat{i} - \hat{j} - 3k$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

16. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

17. Find $\left|\vec{a} - \vec{b}\right|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3$ and $\vec{a} \cdot \vec{b} = 4$.

18. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, then find $\left|\vec{x}\right|$.

19. For any two vectors $\vec{a}$ and $\vec{b}$, we always have $\left|\vec{a} \cdot \vec{b}\right| \leq \left|\vec{a}\right|\left|\vec{b}\right|$ (Cauchy - Schwartz inequality).

20. For any two vectors $\vec{a}$ and $\vec{b}$, we always have $\left|\vec{a} + \vec{b}\right| \leq \left|\vec{a}\right| + \left|\vec{b}\right|$ (triangle inequality).

21. Show that the points $A(-2\hat{i} + 3\hat{j} + 5\hat{k})$, $B(\hat{i} + 2\hat{j} + 3\hat{k}$ and $C(7i - \hat{k})$ are collinear.

22. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$.

23. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

24. Find the area of a triangle having the points $A(1, 1, 1)$, $B(1, 2, 3)$ and $C(2, 3, 1)$ as its vertices.

25. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

26. Write all the unit vectors in XY-plane.

27. If $\hat{i} + \hat{j} + \hat{k}, 2\hat{i} + 5\hat{j}, 3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ are the position vectors of points A, B, C and D respectively, then find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

28. Let $\vec{a}, \vec{b}$, and $\vec{c}$ are three vectors such that $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4\left|\vec{c}\right| = 5$ and each one of them being perpndicular to the sum of the other to, find $\left|\vec{a} + \vec{b} + \vec{c}\right|$.

29. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4$ and $\left|\vec{c}\right| = 2$.

30. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}, \vec{\alpha} = 3\hat{i} - \hat{j}, \vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$ where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.