

# Latex Assignment1

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## Example:-1-34 (12.4)

1. Evaluate  $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$

2. Evaluate  $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

3. Evaluate the determinant  $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$

4. Evaluate  $\Delta = \begin{vmatrix} 0 & \sin \alpha & -\cos \alpha \\ -\sin \alpha & 0 & \sin \beta \\ \cos \alpha & -\sin \beta & 0 \end{vmatrix}$

5. Find the values of  $x$  for which  $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ .

6. Verify Property 1 for  $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

7. Verify Property 2 for  $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

8. Evaluate  $\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$

9. Evaluate  $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$ .

10. Show that  $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0$
11. Prove that  $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$
12. Without expanding, prove that  $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$
13. Evaluate  $\Delta \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$
14. Prove that  $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$
15. If  $x, y, z$  are different and  $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ , then show that  $1 + xyz = 0$ .
16. Show that  $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left| 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right| = abc + bc + ca + ab.$
17. Find the area of the triangle whose vertices are  $(3, 8)$ ,  $(-4, 2)$  and  $(5, 1)$ .
18. Find the equation of the line joining  $A(1, 3)$  and  $B(0, 0)$  using determinants and find  $k$  if  $D(k, 0)$  is a point such that area of triangle  $ABD$  is 3 sq. units.
19. Find the minor of element 6 in the determinant  $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}.$
20. Find minors and cofactors of all the elements of the determinant  $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}.$
21. Find minors and cofactors of the elements  $a_{11}, a_{21}$  in the determinant  $\delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$
22. Find minors and cofactors of the elements of the determinant  $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$  and verify that  $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$ .

23. Find  $\text{adj } A$  for  $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ .

24. If  $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ , then verify that  $A \text{ adj } A = |A|$ . Also find  $A^{-1}$ .

25. If  $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$ , then verify that  $(AB)^{-1} = B^{-1}A^{-1}$ .

26. Show that the matrix  $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$  satisfies the equation  $A^2 - 4A + I = O$ , where  $I$  is  $2 \times 2$  identity matrix and  $O$  is  $2 \times 2$  zero matrix. Using this equation, find  $A^{-1}$ .

27. Solve the system of equations

$$2x + 5y = 1, \quad (1)$$

$$3x + 2y = 7 \quad (2)$$

.

28. Solve the following system of equations by matrix method.

$$3x - 2y + 3z = 8 \quad (3)$$

$$2x + y - z = 1 \quad (4)$$

$$4x - 3y + 2z = 4 \quad (5)$$

29. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

30. If  $a, b, c$  are positive and unequal, show that value of the determinant  $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$  is negative.

31. If  $a, b, c$  are in A.P find the value of  $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+ & 10y+c \end{vmatrix}$ .

32. Show that  $\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$ .

33. Use product  $\begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 3 & 2 & 4 \end{vmatrix} \begin{vmatrix} 2 & 0 & 1 \\ 9 & 2 & 3 \\ 6 & 1 & 2 \end{vmatrix}$  to solve the system of equations.

$$x - y + 2z = 1 \quad (6)$$

$$2z - 3z = 1 \quad (7)$$

$$3x - 2y + 4z = 2 \quad (8)$$

34. Prove that  $\Delta = \begin{vmatrix} a+bx & c+dx+p+qx & \\ ax+b & cx+d+px+q & \\ u & v & w \end{vmatrix} = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$ .