Latex Assignment9

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Example 1-30 (12.11)

1. If a line makes angle $90°, 60°$ and $30°$ with the positive direction of x, y and z-axes respectively, find its direction cosines.

2. If a line has direction ratios $2, -1, -2$, determine its direction cosines.

3. Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.

4. Find the direction cosines of x, y and z-axis.

5. Show that the points $A(2, 3, -4)$, $B(1, -2, 3)$ and $C(3, 8, -11)$ are collinear.

6. Find the vector and the cartesian equations of the line through the point $(5, 2, -4)$ and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

7. Find the vector equation for the line passing through the points $(-1, 0, 2)$ and $(3, 4, 6)$.

8. The Cartesian equation of a line is

$$\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2} \tag{1}$$

   Find the vector equation for the line.

9. Find the angle between the pair of lines given by

$$\overrightarrow{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \tag{2}$$
$$\text{and } \overrightarrow{r} = 5\hat{i} + 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k}) \tag{3}$$

10. Find the angle between the pair of lines:

$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4} \tag{4}$$
$$\text{and } \frac{x+1}{1} = \frac{y-4}{1} = \frac{z+5}{2} \tag{5}$$

11. Find the shorter distance between the lines l_1 and l_2 whose vector equations are

$$\overrightarrow{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k}) \tag{6}$$

$$\text{and} \overrightarrow{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}) \tag{7}$$

12. Find the distance between the lines l_1 and l_2 given by

$$\overrightarrow{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{8}$$

$$\text{and} \overrightarrow{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{9}$$

13. Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin $2\hat{i} - 3\hat{j} + 4\hat{k}$. Also find its cartesian form.

14. Find the direction cosines of the unit vector perpendicular to the plane $\overrightarrow{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) + 1 = 0$ passing through the origin.

15. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin.

16. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$.

17. Find the vector and cartesian equations of the plane which passes through the point $(5, 2, -4)$ and perpendicular to the line with direction ratios $2, 3, -1$.

18. Find the vector equations of the plane passing through the points $R(2, 5, -3)$, $S(-2, -3, 5)$ and $T(5, 3, -3)$.

19. Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z - axis respectively.

20. Find the vector equation of the plane passing through the intersection of the planes $\overrightarrow{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\overrightarrow{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$, and the point $(1, 1, 1)$.

21. Show that the lines

$$\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5} \text{ and } \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5} \text{ are coplanar .} \tag{10}$$

22. Find the angle between the two planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$ using vector method.

23. Find the angle between the two planes $3x - 6y + 2z = 7$ and $2x + 2y - 2z = 5$.

24. Find the distance of a point $(2, 5, -3)$ from the plane $\overrightarrow{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$.

25. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

26. A line makes angles α, β, γ and δ with the diagonals of a cube, prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}. \tag{11}$$

27. Find the equation of the plane that contains the point $(1, -1, 2)$ and is perpendicular to each of the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$.

28. Find the distance between the point $P(6, 5, 9)$ and the plane determined by the points $A(3, -12)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$

29. Show that the lines

$$\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta} \tag{12}$$

$$\text{and } \frac{x-a+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma} \text{ are coplanar .} \tag{13}$$

30. Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses XY-plane.