Latex Assignment1

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Example:-1-34 (12.4)

1. Evaluate $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$

2. Evaluate $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

3. Evaluate the determinant $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$

4. Evaluate $\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$

5. Find the values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.

6. Verify Property 1 for $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

7. Verify Property 2 for $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

8. Evaluate $\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$

9. Evaluate $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$.

10. Show that $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0$

11. Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

12. Without expanding, prove that $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$

13. Evaluate $\Delta \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

14. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

15. If x, y, z are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$.

16. Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left|1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right| = abc + bc + ca + ab$.

17. Find the area of the triangle whose vertices are $(3, 8), (-4, 2)$ and $(5, 1)$.

18. Find the equation of the line joining $A(1, 3)$ and $B(0, 0)$ using determinants and find k if $D(k, 0)$ is a point such that area of triangle ABD is 3 sq. units.

19. Find the minor of element 6 in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$.

20. Find minors and cofactors of all the elements of the determinant $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$.

21. Find minors and cofactors of the elements a_{11}, a_{21} in the determinant $\delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$.

22. Find minors and cofactors of the elements of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ and verify that $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$.

23. Find adj A for $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$.

24. If $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then verify that A adj $A = |A|$. Also find A^{-1}.

25. If $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$.

26. Show that the matrix $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ satisfies the equation $A^2 - 4A + 1 =)$ where 1 is 2×2 zero matrix. Using this equation find A^{-1}.

27. Solve the system of equations

$$2x + 5Y = 1, \tag{1}$$
$$3x + 2y = 7 \tag{2}$$

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28. Solve the following system of equations by matrix method.

$$3x - 2y + 3z = 8 \tag{3}$$
$$2x + y - z = 1 \tag{4}$$
$$4x - 3y + 2z = 4 \tag{5}$$

29. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. B adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

30. If a, b, c are positive and unequal, show that value of the determinant $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

31. If a, b, c are in $A.P$ find the value of $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+ & 10y+c \end{vmatrix}$.

32. Show that $\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$.

33. Use product $\begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 3 & 2 & 4 \end{vmatrix} \begin{vmatrix} 2 & 0 & 1 \\ 9 & 2 & 3 \\ 6 & 1 & 2 \end{vmatrix}$ to solve the system of equations.

$$x - y + 2z = 1 \tag{6}$$
$$2z - 3z = 1 \tag{7}$$
$$3x - 2y + 4z = 2 \tag{8}$$

34. Prove that $\Delta = \begin{vmatrix} a + bx & c + dx + p + qx & \\ ax + b & cx + d + px + q & \\ u & v & w \end{vmatrix} = (1 - x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$.