Latex Assignment6

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31 August,2023

Example:-1-28 (12.3)

1. Consider the following information regarding the number of men ad women workers in three factories I, II and III

	Men Workers	Women Workers
I	30	27
II	25	31
III	27	26

Table 1:

Represent the above information in the form of a 3×2 matrix. What does the entry in the third row and second column represent ?

2. If a matrix has 8 elements, what are the possible orders it can have ?

3. Construct a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|1 - 3j|$

4. If $\begin{pmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{pmatrix}$. Find the values of a, b, c, x, y and z.

5. Find the values of a, b, c and d from the following equation :

$$\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix} \tag{1}$$

6. Given $A = \begin{pmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & \frac{1}{2} \end{pmatrix}$, find $A + B$.

7. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{pmatrix}$, then find $2A - B$.

8. If $A = \begin{pmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{pmatrix}$ then find that X, such that $2A + 3X = 5B$

9. Find X and Y, if $X + Y = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix}$ and $X - Y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$

10. Find the values of x and y from the following equations:

$$2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix} \tag{2}$$

11. Two farmers Ramkishan and Gurucharan Singh cultivate only three varities of rice namely Basmati, Permal and Naura. The sale (in Rupees) of these three varities of rice by both the farmers in the month of September and October are given by the following matrices A and B.
September Sales (in Rupees)

$$A = \begin{pmatrix} \text{Basmati} & \text{Permal} & \text{Naura} \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ \text{Ramkishan} \\ \text{Gurucharan Singh} \end{matrix} \tag{3}$$

October Sales (in Rupees)

$$B = \begin{pmatrix} \text{Basmati} & \text{Permal} & \text{Naura} \\ 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{pmatrix} \begin{matrix} \\ \text{Ramakishan} \\ \text{Gurucharan Singh} \end{matrix} \tag{4}$$

(i) Find the combined sales in Sepember and October for each farmer in each variety.

(ii) Find the decrease in sales from September to October.

(iii) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.

12. Find AB, if $A = \begin{pmatrix} 6 & 9 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 6 & 0 \\ 7 & 9 & 8 \end{pmatrix}$.

13. If $A = \begin{pmatrix} 1 & -2 & -3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$, then find AB, BA. Show that $AB \neq BA$.

14. If $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then $AB = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $BA = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ clearly $AB \neq BA$. Thus matrix multiplication is not commutative.

15. Find AB, if $A = \begin{pmatrix} 0 & -1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 5 \\ 0 & 0 \end{pmatrix}$.

16. If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 0 & 2 \\ -1 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{pmatrix}$ find $A(BC), (AB)C$ and show that $(AB)C = A(BC)$.

17. If $A = \begin{pmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}, C = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$ Calculate AC, BC and $(A+B)C$. Also, verify that $(A + B)C = AC + BC$.

18. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$, then show that $A^3 - 23A - 40I = 0$.

19. In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three ways: telephone, housecalls and letters. The cost per contact (in paise) is given in matrix A as
cost per contact

$$A = \begin{pmatrix} 40 \\ 100 \\ 50 \end{pmatrix} \begin{matrix} \text{Telephone} \\ \text{Housecall} \\ \text{Letter} \end{matrix} \tag{5}$$

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The number of contacts of each type made in two cities X and Y is given by

$$B = \begin{pmatrix} \text{Telephone} & \text{Housecall} & \text{Letter} \\ 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{matrix} X \\ Y \end{matrix} \tag{6}$$

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Find the total amount spent by the group in the two cities X and Y.

20. If $A = \begin{pmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$, verify that

    (a) $(A^1)^1 = A$
    (b) $(A + B)^1 = A^1 + B^1$
    (c) $(kB)^1 = kB^1$, where k is any constant.

21. If $A = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 & -6 \end{pmatrix}$, verify that $(AB)^1 = B^1A^1$.

22. Express the matrix $B = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ as the sum of symmetric and a skew symmetric matrix.

23. By using elementary operations, find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.

24. Obtain the inverse of the following matrix using elementary operations.

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} \tag{7}$$

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25. Find P^{-1}, if it exists, given $P = \begin{pmatrix} 10 & -2 \\ -5 & 1 \end{pmatrix}$.

26. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, then prove that $A^n = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}, n\varepsilon N$.

27. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute, that is $AB = BA$.

28. Let $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}, C = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$. Find a matrix D such that $CD - AB = 0$.