Localised Soliton Dynamics under central peak potentials

A thesis submitted in partial fulfillment of the requirement for the award of degree of

Bachelor of Science (Honours)

THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
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submitted by

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Abstract

We investigate the soliton formation and its dynamics inside a semiconductor laser cavity under central peak potentials. The nonlinear cavity comprises of a ventricle cavity surface emitting laser, coupled with saturable absorber and frequency selective feedback. We consider the cavity system is of (2+1) dimension. We achieved cavity soliton, which move randomly in absence of any externally applied potential, for only some selective ranges of system parameters. Further, we trap some of the cavity solitons using four different potentials. This trapping has important technological merits in the field of nonlinear optics, particularly in all-optical switching, all-optical logic gate and optical computing. This theoretical investigation involves numerical solution of a complex Ginzburg-Landau equation (CGLE); the governing equation of the cavity field, by Split-step Fourier Method (SSFM). Further, the stability of the cavity solitons has been investigated by using Lyapunov stability criteria.

Dedicated to,

my family and my mentors for their constant support and encouragement.

This thesis is also dedicated to the memory of my beloved grandmother,

Kiran Rastogi,

whose love, wisdom, and guidance have always been a source of inspiration.

Her unwavering confidence in me and my abilities have been instrumental in my life

She is dearly missed and forever cherished.

Declaration

I, Apratim Rastogi, hereby certify that work presented in this thesis entitled ”Localised Soliton Dynamics under central peak potentials” submitted in partial fulfillment of the requirement for the award of degree of Bachelor of Science (Honours) in the School of Physics and Materials Science, Thapar Institute of Engineering and Technology, Patiala is an authentic record of my own work carried out under supervision of Dr. Soumendu Jana. The matter embodied in this thesis has not been submitted in part or full to any other university or institute for the award of any degree.

Apratim Rastogi
(142000024)

This is to certify that the above statement made by the candidate is true to the best of my knowledge.

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Finally, my heartfelt gratitude go to my family for their unwavering support and understanding. To my parents, my sibling, and my grandfather, your love, patience, and constant encouragement have been my driving force.

To all those who have contributed to my academic and personal growth, I express my deepest gratitude.

Apratim Rastogi

Contents

Chapter 1

Introduction

Nonlinearity is what makes world possible. Nonlinearity can bring about the conditions into the existence that would otherwise be impossible. If we look around every fascinating object in the world around has an element of depth and chaos. It is this intriguing quality of the universe that captivates our attention and keep the world from being boring. From the interaction of fundamental forces of nature, complex organism's biology to morden day digital technology and artificial intelligence, nonlinearity is at play [1, 2, 3]. The central topic of this thesis is about one of the many captivating phenomenon of the nonlinearity in the world that has many potential application from fundamental physics to advancemenets in modern technology [4]. Our discussion here will be focused on cavity solitons which are special type of solitary waves of light [5, 4].

Soliton was discovered accidently, when John Scott Russell (1844) was taking a walk by the Union Canal near Edinburg, saw a solitary wave travelling in the canal [6]. Just take a moment and from a navie eye think about the phenomenon for a moment. It is extremely fascinating and weird. We usually get so captivated thinking about particles acting as waves from our quantum class, that we completely overlook to ask the question the other way around. Can a wave behave like a particle? There is a reason we never encounter any clue to the answer of this question in our usual physics classes. This is because the physics encountered in the classrooms are linear mostly while answer to these kind of questions lie in the more rich landscape of nonlinearity in nature.

One of the most interesting phenomena due to nonlinearity in nature, both for its applications and sheer mathematical beauty, is stable localized structures. Solitons are one such category of localized phenomena. Further, the cases of dissipative systems, which are usually far from equilibrium and close to the real world, also show the formation of Solitons. These kinds of solitons in the optical domain are broadly classified as Cavity Solitons (CS). They were first predicted in the early 1990s [7, 8] to exist in nonlinear optical resonators as localized structures of light that are maintained due to a very intricate balance between diffraction or dispersion, nonlinearity, and feedback. Since their existence was experimentally confirmed in semiconductor microresonators, they have found various applications in different domains, including optical information technologies, photonics, and optical sensing.

Trapping these cavity solitons is an important area of research. This is because trapping a CS gives us the ability to control and manipulate these localized light pulses within an optical cavity for application in advancements in technology. The real potential of the CS's entrapment is realised in photonics, where technologies such as optical switches can be used to make logic gates and effectively achieve optical computing. These optical switches are many orders of magnitude faster than regular semiconductor switches.

1.1 Background

In optics, cavity solitons (CS) are beams of light in which the nonlinearity gets counter-balanced by diffraction and gain gets counter-balanced by loss in the system to get a stable structure that maintains it shape and form while propagating in the nonlinear medium. They were first theorized in 1980s by Moloney and colleagues [7] while studying transverse effects in optical bistability (OB). These are generated by using laser pulses in an optical cavity that contains a nonlinear medium driven by a coherent beam (holding beam) [9]. The ability to switch cavity solitons on and off and to control their location and motion by applying laser pulses makes them interesting as potential 'pixels' for reconfigurable arrays or all-optical processing units [9]. Theoretical work on cavity solitons has stimulated a variety of experiments in macroscopic cavities and in systems with optical feedback [9]. The clear experimental realization has been hindered by boundary-dependence of the resulting optical patterns—cavity solitons should be self-confined. However, recent studies have demonstrated the generation of cavity solitons in vertical cavity semiconductor microresonators that are electrically pumped above transparency but slightly below lasing threshold [9].

The numerical simulations have allowed for clear interpretations of the experimental results and have served as an effective tool for theoretical study. In a study by Firth et al. (2002), the dynamics of two-dimensional Kerr cavity solitons were analyzed, and found to be absolutely stable over a substantial parameter domain, with regions of stable oscillation and of fivefold or sixfold azimuthal instability beyond the instability boundary [10]. McSloy et al. (2002) McSloy et al. (2002) applied quasi-exact numerical techniques to the calculation of stationary one- and two-dimensional, bound multipeaked cavity soliton solutions of a model describing a saturable absorber in a driven optical cavity [11]. Bache et al. (2005) studied a broad-area vertical cavity surface emitting laser (VCSEL) with a saturable absorber and numerically showed the presence of cavity solitons in the system [12]. These solitons existed as solitary structures formed through a modulationally unstable homogeneous lasing state that coexisted with a background with zero intensity [12].

Cavity solitons are intriguing localized intensity peaks that have been studied extensively in various systems, including semiconductor microcavities and fiber lasers [13, 14, 15]. The ability to control and manipulate these solitons makes them promising candidates for applications in all-optical processing units and other optical information systems [16, 4, 17, 18, 19, 20].

The complex Ginzburg-Landau equation (CGLE), is a close perturbative cousin of non-linear Schödinger's equation. The CGLE, initially devised for studying superconductivity [21], has found its way in nonlinear optics as a governing equation for soliton formation in nonlinear medium due to its pattern forming capabilities. The introduction of frequency-dependent losses has been shown to stabilize dissipative solitons in a complex Ginzburg-Landau model with simple cubic or saturable nonlinearity [22]. Analytic solutions for these solitons have been found in one dimension, and robust solitons have been identified numerically in two dimensions [22]. The research on cavity solitons in VCSELs and other systems using CGLE models has provided valuable and exciting insights which when confirmed using experimental data provides us with a robust theoretical tool [23, 24, 25, 22]

1.2 Motivation

Little work has been done in analysing the formation of cavity solitons in periodic potentials. During recent works from our group it was noticed that periodic potentials with central peak can entrap a CS in a very interesting manner. These CS gets, trapped on/near the central peak of the potential. This is particularly interesting because successful implementation would give us an impressive ability to use light pulses to make a memory storage system. Just by controlling the position of the peak of the potential, one would be able to control the CS.

This and many other essential technological applications are motivating factors for our current work. The prospects of the work lies in the fact that it can we can have a very effective way to mediate the data transfer and storage using Cavity Soliton. The theoretical modeling is the first step in physical prototyping. And this work aims to achieve the initial.

1.3 Objective

What follows is the theoretical modelling of cavity soliton formation in a well used laser cavity system VCSEL (more on this later) which can be modelled with complex Ginzburg-Landau equation. We aim to achive the following objectives by the end:

1. Study the existence and stability of cavity solitons in various potential with central peak.

2. Effective modelling of such phenomena using approtiate numerical techniques.

The thesis is structured as follows. Chapter 2 discusses some basics, methodology behind the approach and the system we have defined. Here we also take a brief look at the analytical formulation of CGLE and why we need to resort to numerical method in the light of the kind of systems we are dealing with. Further, in chapter 3, we start by talking about our simulation, the assumptions we have taken, and finally mentions the results we have had. Finally, in chapter 3.2 we summarize the report and conclude.

Chapter 2

System and Methodology

In this chapter we describe the system, which has been used for the generation of CS, the governing equations, and the methodology to solve and analyze the governing equation.

2.1 System

We have used broad-area semiconductor laser cavity comprised of a Vertical-Cavity Surface Emitting Laser (VCSEL) with a saturable absorber (SA) and coupled to a frequency-selective feedback element(FSF) to study the formation of CS [23]. VCSELs are a class of semiconductor lasers characterized by their vertical emission relative to the surface of the semiconductor substrate. They offer several advantages over traditional edge-emitting lasers, including lower threshold currents, circular beam profiles, and the ability to be fabricated in dense two-dimensional arrays. When integrated with a saturable absorber, the VCSEL can exhibit a range of nonlinear dynamical behaviors, crucial for the formation of cavity solitons.

A saturable absorber is a material whose absorption decreases with increasing light intensity. This nonlinearity is essential for the self-organization processes leading to soliton formation. In the context of VCSELs, the saturable absorber enables the laser to support super-stable localized cavity solitons that can persist indefinitely under constant pumping conditions.

The addition of a frequency-selective feedback element further refines the system. The FSF element selectively reflects specific frequencies back into the laser cavity, effectively narrowing the emission spectrum and enhancing the stability of the cavity solitons. This selective feedback can be implemented using various optical components, such as Bragg gratings or Fabry-Pérot interferometers.

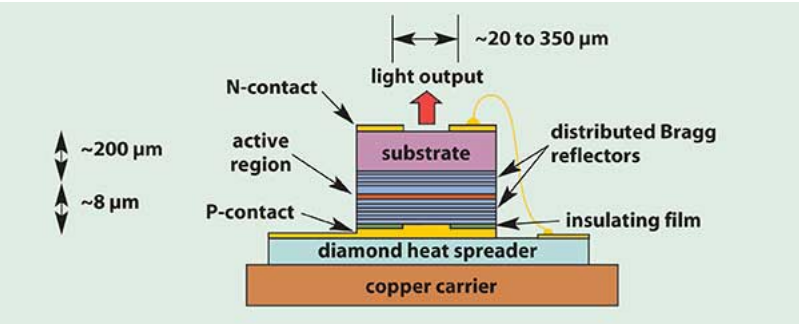
The dynamics of this complex system are often modeled by the Complex Ginzburg-Landau Equation (CGLE) [26, 27, 28]. The CGLE is a partial differential equation that describes the evolution of a complex field envelope and encompasses key physical processes such as dispersion, nonlinearity, gain, and loss. In the context of VCSELs with a saturable absorber and FSF [12, 22], the CGLE captures the interplay between diffraction, nonlinearity, and feedback, which governs the formation and stability of cavity solitons [29, 30].

The general form of the CGLE used in this context is:

$$\frac{\partial E}{\partial t} = E + (1 + i\alpha)\nabla^2 E - (1 + i\beta)|E|^2 E + \gamma E \tag{2.1}$$

where E is the complex field envelope, α and β are real coefficients representing the linear and nonlinear contributions to the refractive index, respectively, and γ represents the gain/loss term modulated by the saturable absorber and the feedback.

The theoretical framework provided by the CGLE has been instrumental in predicting and explaining the rich variety of dynamical behaviors observed in experiments, including the formation, interaction, and stability of cavity solitons. The solitonic solutions of the CGLE, known as dissipative solitons, are particularly relevant for optical systems where gain and loss are inherently present. This configuration has garnered considerable attention due to its potential applications in optical communication and information processing.



2.2 Mathematical modeling

The cavity lasers, as described above can be modeled by, 2D CGLE which is given as below (2.3).

$$\iota\frac{\partial E}{\partial t} + \left[\theta - \frac{\mu\alpha}{1+g_1|E|^2} + \frac{\gamma\beta}{1+sg_2|E|^2} + \Delta\right]E - bE + V(x,y)E$$
$$= \iota\left[-1 + \frac{\mu}{1+g_1|E|^2} - \frac{\gamma}{1+sg_2|E|^2} + \alpha_{ns} + a\right]E \tag{2.2}$$

$$\frac{\partial E}{\partial t} = \left[-(1-i\theta) + \frac{\mu(1-\iota\alpha)}{1+g_1|E|^2} - \frac{\gamma(1-\iota\beta)}{1+sg_2|E|^2} + \alpha_{ns} + a + -ib + i\Delta\right]E + iV(x,y)E \tag{2.3}$$

Where a and b are given as follows:

$$a = \left(\frac{\sigma\lambda^2}{(\lambda^2+\omega^2)}\right)$$
$$b = \left(\frac{\sigma\lambda\omega}{(\lambda^2+\omega^2)}\right)$$

and the the different parameters used to model VCSEL are: θ detuning parameter ; μ : pumping parameter for active medium ; α : Linewidth Factor for active medium ;

g_1, g_2: coefficients of saturation of active and passive media respectively. ; s: saturation parameter ; β: linewidth factor for passive medium ; γ: pumping parameter for passive medium ; α_{ans}: non-saturable loss ; λ: bandwidth ; σ: Feedback-strength ; ω: Resonance Frequency ; $(a + ib)$: represents the feedback field ; $V(x, y)$: is the spatially varying potential function .

The values for the above parameters for which the soliton solutions can exist, can be found analytically using Lagrangian Variational Method [30, 31] for simple system without the potential. The analytical approach is limited to very simple systems. As the system becomes complex, it becomes extremely difficult to solve analytically.

2.3 Approach

Analytically solutions to PDEs is extremely limited to special and simple cases. It is more parsimonious to use different approximate methods to get understanding of the nature of PDE in question. Amongst many different numerical methods, Split Step Fourier method is one popular choice for its reasonable demands of computational resources. This method is used in various senarios such as in light pulse propagation in optical fibers, dynamics of optical microresonator, and finding soliton solutions.

Split-step Fourier method (SSFM) is based on seperating the linear and nonlinear part of the equations and dealing with them seperately in small steps. Linear part is dealt in frequency domain by Fourier tranforming it, while nonlinearity is dealt in time domain.

2.3.1 Split-step Fourier Method (SSFM)

Split-step Fourier Method (SSFM) is an effective and powerful numerical technique for solving partial differential equations (PDEs) which have linear and non-linear terms and can be separated into them [32, 33]. This method combines the advantages of Fourier transforms for handling spatial derivatives and explicit time-stepping schemes for non-linear terms. The method involves splitting the equation into linear and nonlinear parts and solving each part sequentially within a small time step Δt. In the present case, with CGLE, we can split it into dispersion terms and non-linear terms as in (2.4) and (2.5) and then proceed with the steps as described below.

$$\mathbf{D} = \left(-1 + i\theta + a - ib - iK_x^2 - iK_y^2\right)\frac{h}{2} \tag{2.4}$$

$$\mathbf{N} = \left(iV(x,y) + \frac{\mu(1-i\alpha)}{1+g_1|E|^2} - \frac{\gamma(1-i\beta)}{1+sg_2|E|^2} + \alpha_{ns}\right)h \tag{2.5}$$

where h is the step size.

In the SSFM, we approximate the solution over a time step Δt by first solving the linear part in the Fourier domain, then solving the nonlinear part in the spatial domain. The Split-Step Fourier method for solving the CGLE involves the following steps:

1. **Initialization**: Start with the initial condition $u(x, y, 0)$.

2. **Fourier Transform**: Convert the spatial domain field $u(x, y, t)$ to the Fourier domain using the Fast Fourier Transform (FFT):

$$\hat{u}(k_x, k_y, t) = \mathcal{F}\{u(x, y, t)\}, \tag{2.6}$$

   where k_x and k_y are the wave numbers in the x and y directions, respectively.

3. **Linear Evolution**: Solve the linear part in the Fourier domain:

$$\hat{u}(k_x, k_y, t + \Delta t) = \hat{u}(k_x, k_y, t)\exp\left\{(1 + i\alpha)k^2\Delta t\right\}, \tag{2.7}$$

where $k^2 = k_x^2 + k_y^2$.

4. **Inverse Fourier Transform:** Convert the field back to the spatial domain:

$$u(x, y, t + \Delta t/2) = \mathscr{F}^{-1}\{\hat{u}(k_x, k_y, t + \Delta t)\}. \tag{2.8}$$

5. **Nonlinear Evolution**: Solve the nonlinear part in the spatial domain:

$$u(x, y, t + \Delta t) = u(x, y, t + \Delta t/2) \exp\left\{(1 - (1 + i\beta)|u(x, y, t + \Delta t/2)|^2)\Delta t\right\}. \tag{2.9}$$

6. **Repeat**: Iterate the process over multiple time steps until the desired time T is reached.

This method efficiently separates the linear and nonlinear components of the CGLE, allowing for stable and accurate numerical solutions. It is widely used in computational physics and nonlinear optics for studying pattern formation, wave propagation, and turbulence.

Advantages and Applications

The SSFM is particularly advantageous due to its efficiency and accuracy in handling both dispersion and nonlinearity in wave equations. It is widely used in the study of optical solitons, wave propagation in nonlinear media, and various other fields where the CGLE plays a pivotal role [34, 35, 32].

The method's ability to decouple the linear and nonlinear components allows for larger time steps than would be feasible with traditional finite-difference methods, making it a powerful tool in computational physics and applied mathematics.

Chapter 3

Results

In this chapter, we will look at the results we found and talk about stability analysis of the CGLE with four different potential functions.

3.1 Generation of Cavity solitons

First, we generate cavity solitons by numerically solving the governing CGLE following SSFM. A typical set of CS is presented in figure below Fig: 3.1.

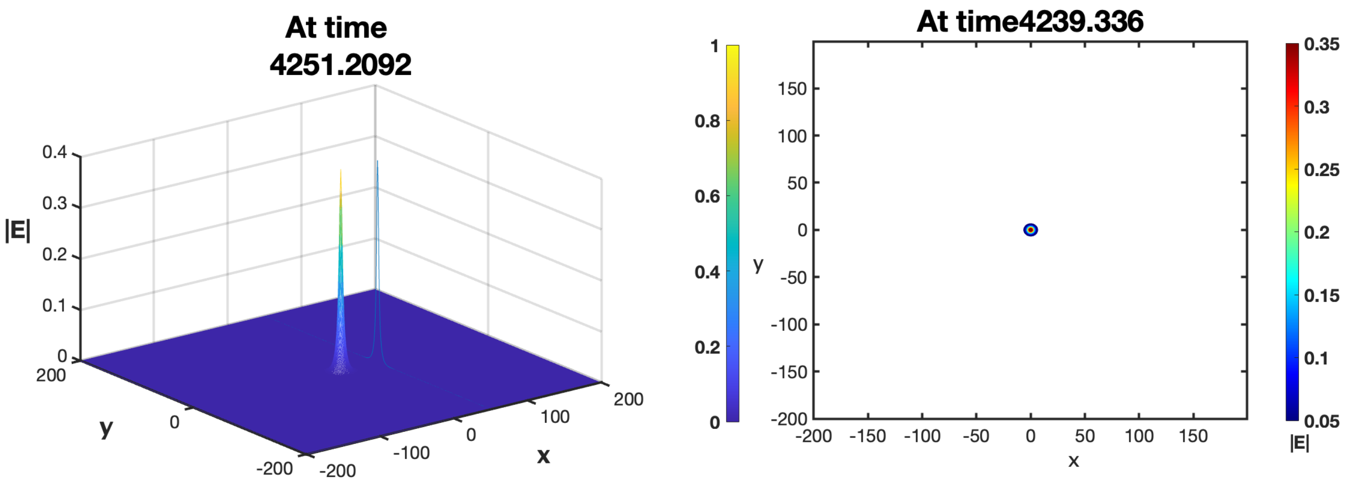


Figure 3.1: Cavity Soliton generation without any potential

Now, we use a potential to trap the generated CS. Four such different potentials have been used. The following are the numerical simulation of CGLE with different spatially varying transverse potential functions, all of which have central peaks. The aim here is to trap CS near or on the central peak of the applied potential. This helps in effective manipulation of CS which have many technological applications.

Assumptions: The following parameter values were initially found using analytical approximation like discussed above lagrangian variational method. $\theta = 1.1$, $\mu = 1.37$, $\alpha = 2.7$,$\gamma = 0.55$, $\beta = 1.4$,$s = 10$, $g_1 = g_2 = G = 4$, feedback bandwidth $\lambda = 0.6$, feedback strength $\sigma = 0.4$, resonance frequency $\omega_0 = 1.2$, Amplitude of CS $A = 0.25$, unsaturable absorption parameter $\alpha_{NS} = 0.020$.

These values are used to define our system and only on subsequent numerical simulations were altered based on the need. Otherwise they were mostly kept constant so that overall our system is unchanged except for the change in potential which is introduced by us.

We have also considered a symmetric system as far as the spatial geometry is concerned. This is a valid assumption because we generally have the ability to make the geometry of our instruments precise. That is exactly what has been assumed for our VCSEL system.

3.1.1 Sinc potential with Gaussian wave profile:

In this configuration, we have CGLE with Sinc potential function. Sinc function in 2D has a central peak at $(0, 0)$ with concentric rings of diminishing amplitude surrounding it as can be seen in (3.2).

The Gaussian driving beam is used

1. Here we have four stable peaks (CS) that maintain their position and then we have another which is trapped aroud the peak of the potential and keeps rotating around it.

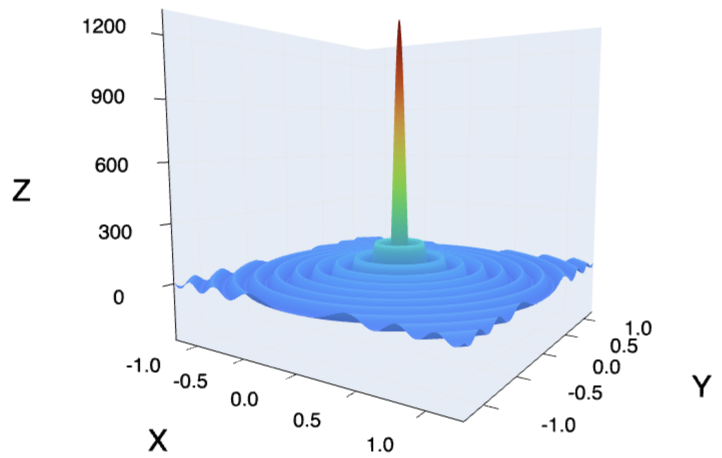


Figure 3.2: Sinc function in 2D with central peak

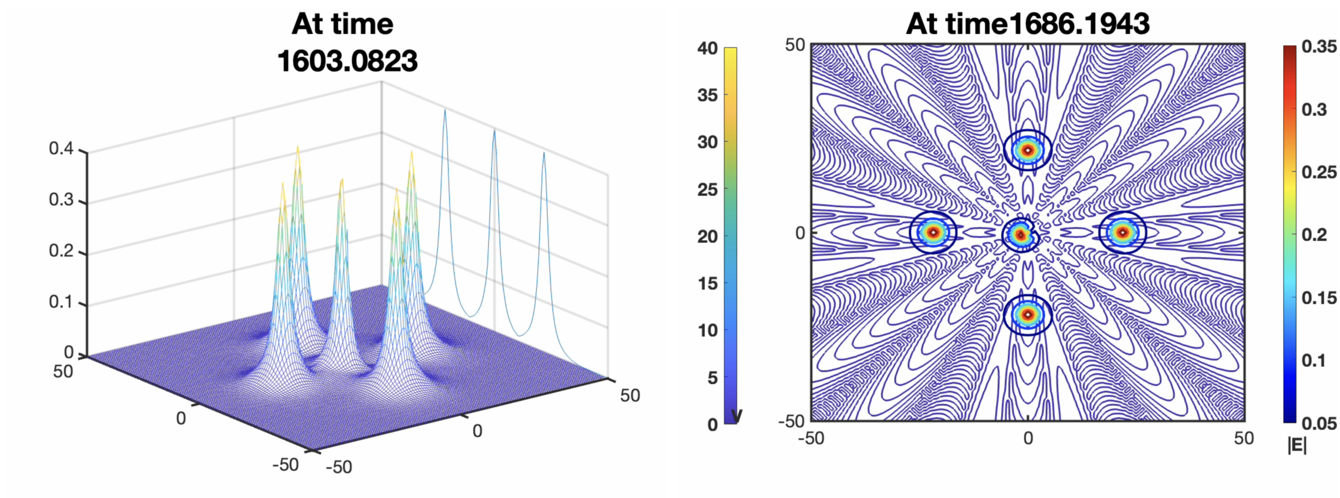


Figure 3.3: Solitons in Sinc Potential with gaussian wave profile showing five pillars

2. We have also found that this system is highly sensitive to the frequence of concentric peaks occuring in the applied potential and relatively less sensitive to the hight of the potential.

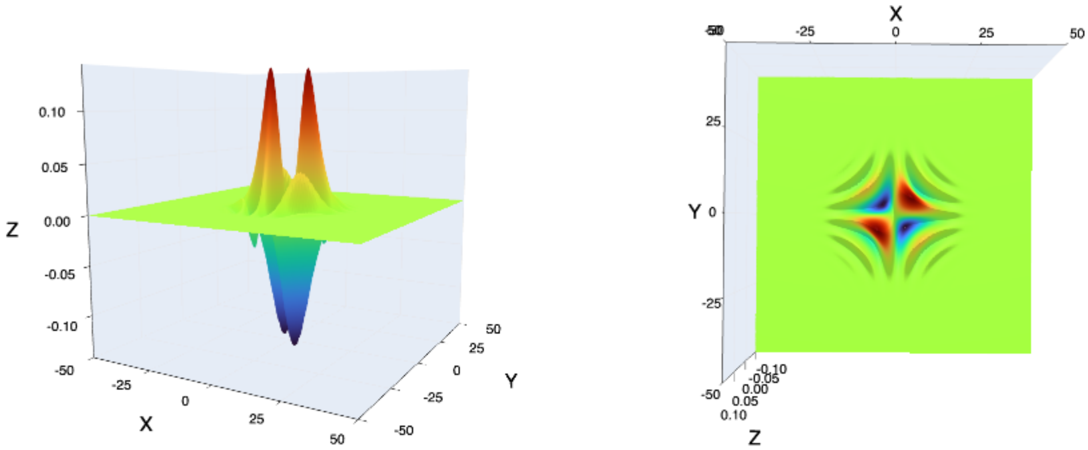
3. For the given parameters mentioned above, the range of potential height in which the phenomenon occurs is **43.300- 49.723**

4. the range for potential frequency is **12.998-13.125**

3.1.2 TCMSG potential with Gaussian wave profile:

The TCMSG profile [36] is a compound function has two peaks and two valleys around the center as shown in fig (3.4a) and (3.4b) is given as:

$$V(x,y) = A\sin\left(\delta\frac{xy}{\omega_0^2}\right)\exp\left(\frac{-x^2-y^2}{\omega_0^2}\right) \tag{3.1}$$



(a) (b)

Figure 3.4: TCMSG Plots (a) shows the side view (b) shows the ariel view

For this system, with input profile being Gaussian, we see that solitons are nicely trapped around the center on the peaks an valleys as shown in fig (3.5). The ones which are in valleys, exhibit more stability than those that are on the peak. This is evident in the simulation where the solitons on the peaks can be seen to be shaking throughout the cycle and are first to decay upon any change in either parameters or external interference.

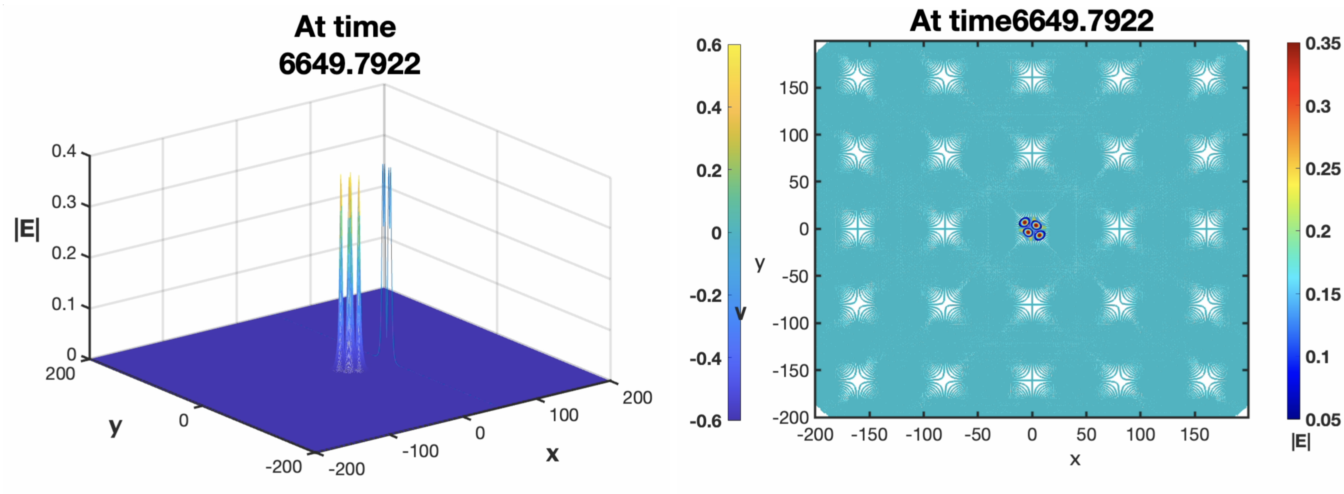


Figure 3.5: Soliton Trapping in TCMSG Potential

3.1.3 Sinc potential with TCMSG wave profile:

Again we used the ring shaped Sinc potential 3.2, but this time with TCMSG wave profile 3.4a instead of gaussian. Here as well we get our solitons trapped around the central peak (3.6) of Sinc if the frequency of the rings and height of the potential are correctly balanced. However, an interesting discovery for this case was that with slight controlled imbalance, the solitons seems to oscillate towards and away from the central peak.

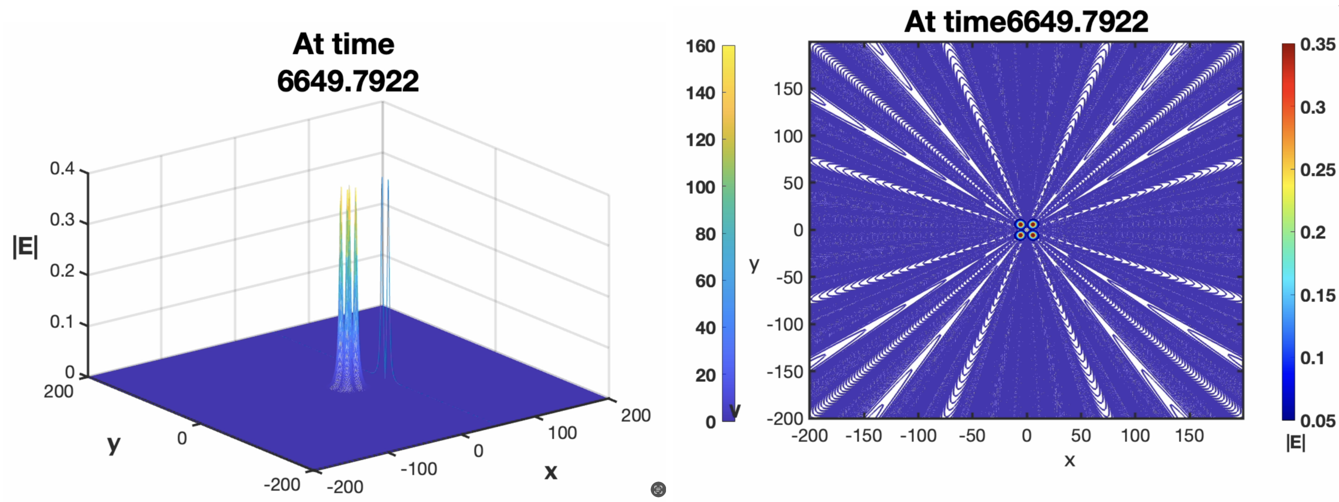


Figure 3.6: Soliton Trapping in Sinc Potential with TCMSG wave profile

3.1.4 WGM like potential with Gaussian wave profile:

Another interesting system we looked at was inspired by the famous whispering gallery phenomenon. In optics, whispering gallery modes (WGMs) specifically refer to the propagation of light waves along the curved boundary of a circular or spherical cavity. These modes are characterized by:

1. **Total Internal Reflection:** Light waves undergo multiple total internal reflections along the curved surface of the cavity, allowing them to propagate with minimal loss.

2. **Resonant Frequencies:** Only certain wavelengths of light, corresponding to specific modes, satisfy the condition for constructive interference after multiple reflections. This results in sharply defined resonances at particular frequencies.

For this system we used another compound function of Bessel and gaussian functions given by (3.2)

$$V(r) = A\exp(\frac{-r^2}{\omega})J_n(\alpha r) \tag{3.2}$$

Here, A is the amplitude, ω controls the width of the Gaussian peak, n determines the number of rings, and α adjusts the spacing of the rings.

The potential looks as shown in fig (3.7).

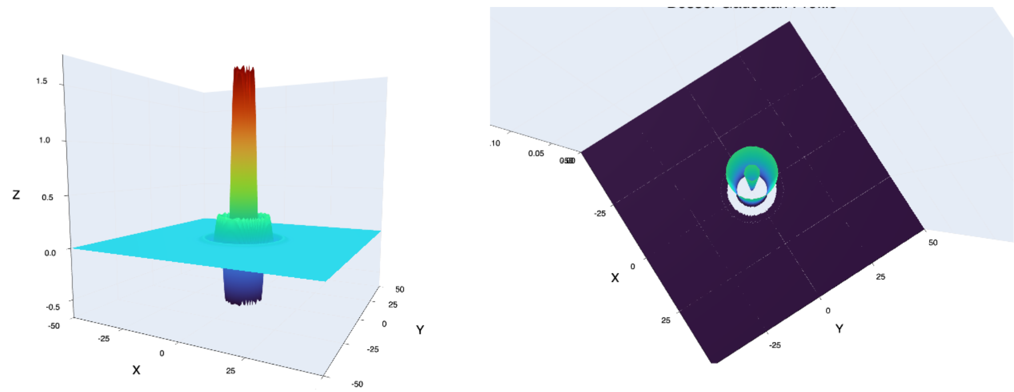


Figure 3.7: Bessel-Gaussian Profile

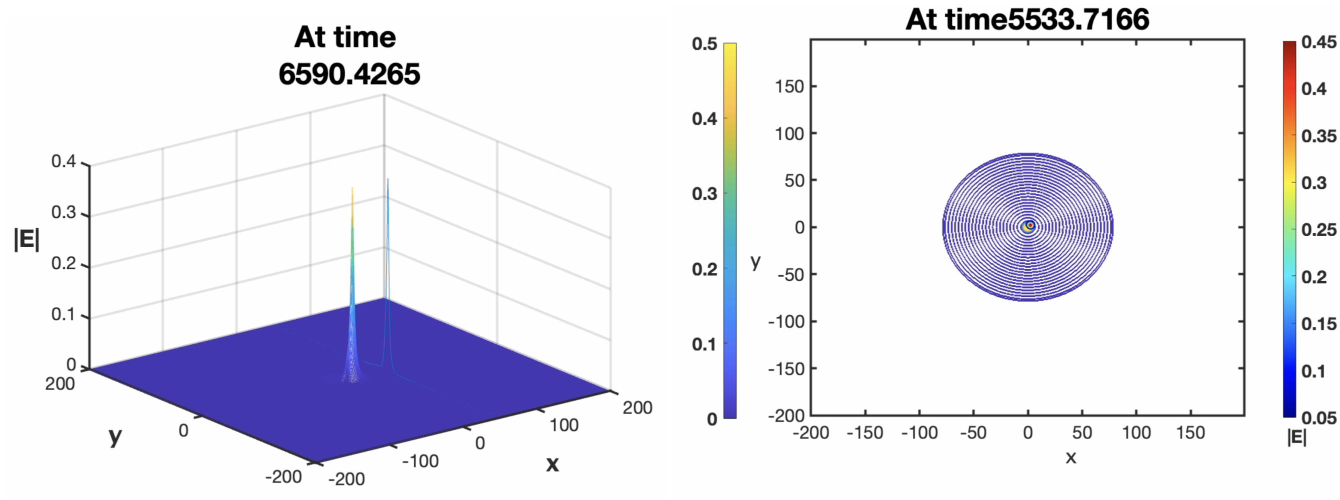


Figure 3.8: Soliton Trapping in WGM like potential with gaussian wave profile

This showed very interesting concentric rings that can be adjusted by varying n in (3.2). The cavity soliton formed here is trapped in the in inner most ring and keeps rotating on along the boundary. This particular system showed the most resilience to slight changes in parameters. This might be due to extra enforcement by surrounding concentric rings. However due to the natural uneven shape of the bessel function, no complete stationary solitons were observed. Instead they kept moving inside the innermost ring.

3.2 Stability Analysis

For any localized structure or pattern the stability (and instability) is a big issue for application as well as for fundamental concept of the dynamics of the system. Therefore, it is customary (as well as essential) to analyze the stability of the CS under the influence of different potentials.

We performed the stability analysis with respect to the significant system parameters like pumping parameter for active medium μ and feedback strength σ for CS in the above discussed systems. Here we employed, largest Lypunov Exponent method to numerically analyse the stability of the above mentioned systems.[1] The Lypunov exponent, as given by (3.3), measures the average rate of convergence or divergence of nearby trajectories, thereby acting as a description for chaotic systems [37]. They also describe how senstive the system is to the initial conditions.

$$\lambda = \lim_{t\to\infty} \lim_{\delta\to 0} \frac{1}{t} \ln \frac{d(t)}{\delta} \tag{3.3}$$

Where d is the separation between the trajectories at a given time t, and δ is the initial purturbation.

The steady state solution of the system are found by directly solving the differential equation for steady state condition. The Maximal Lyapunov Exponent is numerically calculated as follows:

1. First find the steady state solution by directly solving the differential equation.
2. Create a perturbed state $E_{perturbed} = E + \delta$, where δ is a small perturbation
3. Evolve both E and $E_{perturbed}$
4. Calculate the new seperation $d = ||E_{new} - E_{newperturbed}||$
5. Compute the local Lypunov exponent : $\lambda_{local} = \ln(d/\sigma)/\Delta t$
6. Repeat the process many times (1000 iterations are taken for our code)
7. Calculate the final Lypunov exponent by averaging all the local exponents $\lambda = (1/N)\sum \lambda_{local}$

The Largest or Maximal Lyapunov Exponent, gives us a good idea about the stability of the system. Negative or smaller values correspond to the stable system, while

[1]The code is available on the following link: ...

positive or larger value corresponds to unstability.

The following figures Fig: 3.9 & Fig: 3.10 shows the stability analysis of systems with initial profile to be gaussian and TCMSG respectively.

1. The first figure is a 3D landscape of Lyapunov Exponent on the z axis as the μ and σ are changing.

2. The second figure shows how the exponent is changing with respect to σ while μ is constant at 0.5.

3. The third figure show similar to previous, how the exponent is changing with respect to μ while σ is kept constant at 0.5.

4. The fourth is the contour plot showing the region for which the system shows stability.

5. The fifth figure is the heatmap of Lyapunov exponent for the values of μ and σ.

6. The figures sixth to ninth, shows the phase plot of the system for different values of μ and σ.

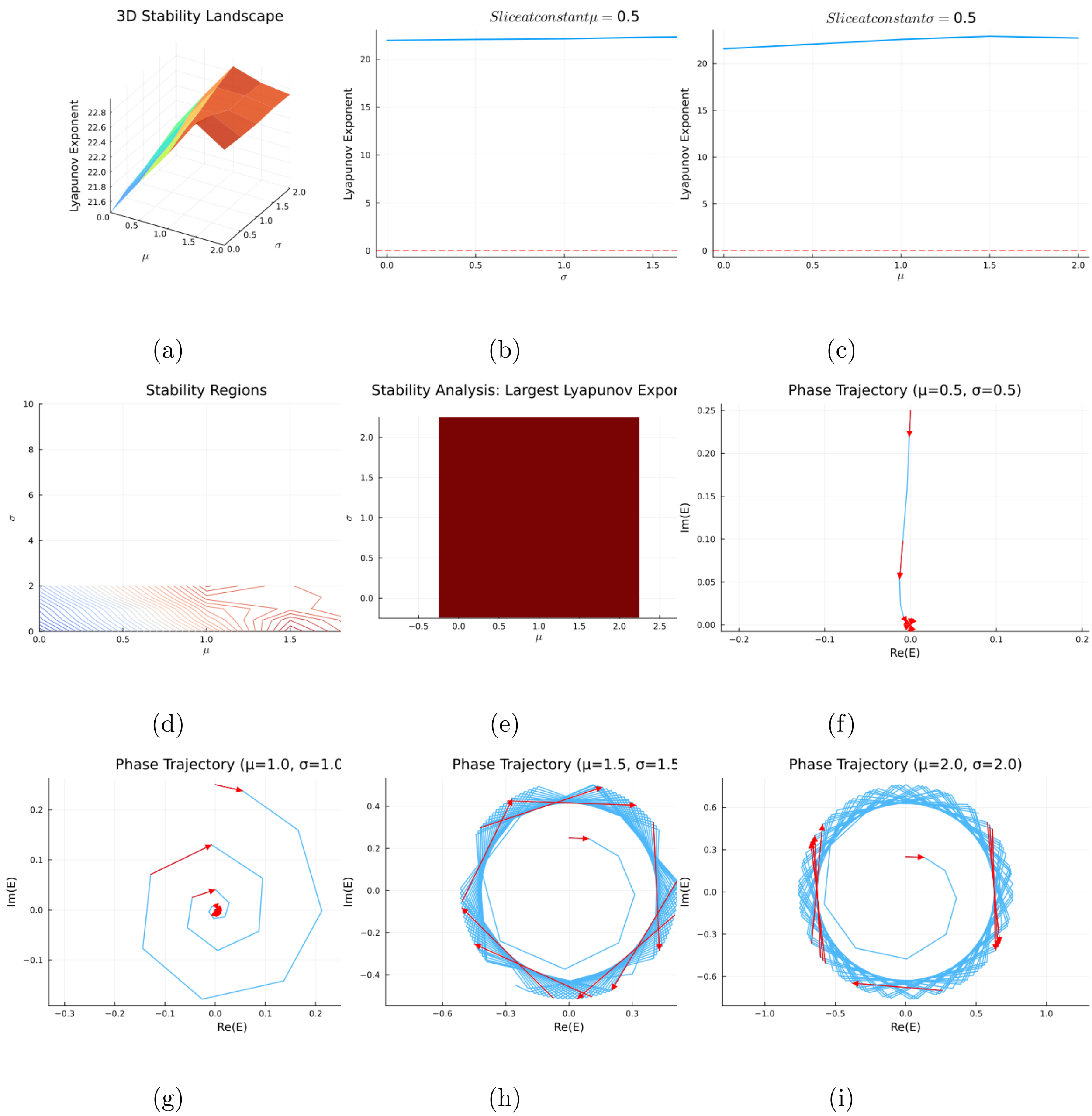
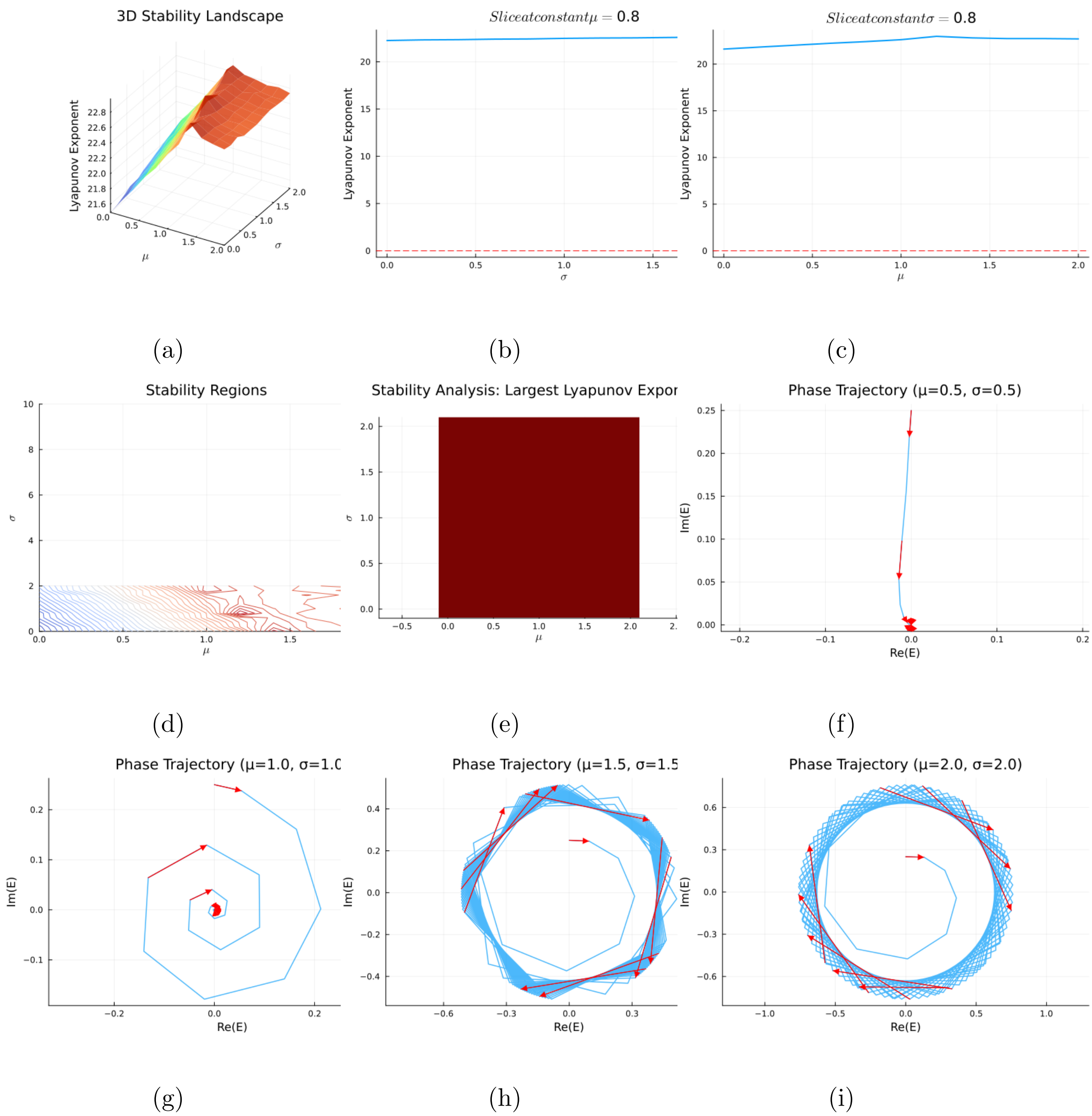


Figure 3.9: Stability Analysis for the Gaussian input profiles



(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

Figure 3.10: Stability Analysis for the TCMSG input profiles

Results: The stability region for all the above system did not show any significant variation. We can see from the plots above, the system where TCMSG was the input profile (3.10) the region of stability is slightly more for bigger values of μ than for all the other systems that had gaussian input profile (3.9) stopped at $\mu = 1$. While the upper value of

σ for which stability can be seen, is the same in both the cases. Nonetheless, the overall nature of dynamics seems to be consistent. Looking at the phase plots we see that around the values of $\mu = 1$ and $\sigma = 1$ to be equal we get a stable system, and above that the system becomes unstable and exhibits Hopf bifurcation typical of CGLE.

Conclusion

In the work, we have studied the formation and entrapment of cavity soliton (CS) in vertical cavity surface emitting laser (VCSELs) coupled with saturable absorber (SA) and frequency selective feedback mechanism (FSF) under central peak shaped potential functions. The governing equation for the system mentioned, complex Ginzburg-Landau equation (CGLE), is used to numerically simulate the dynamics using split-step Fourier method (SSFM) for four different potentials. It was found that the central peak in the potentials can successfully trap the CS. This is important because of various potential applications in optical communication and optical computing.

Further the stability analysis using Lyapunov Exponent with respect to the significant system parameters: pump parameter for active medium μ and feedback strength parameter σ, showed us the region where the system can be stable.

The future direction should be to find out more potential that can effectively trap the cavity soliton and figure out how effectively we can manipulate the dynamics of CS.

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