Understanding “V functions”

- Here we take simple static general equilibrium models and derive the “V function”
- We show that although there is an “externality”, competitive equilibrium is efficient.

- Consider a simple consumption labor supply problem, where agent i has preferences $U(c_i, e_i)$, and has the budget constraint

$$c_i = we_i + \frac{\Pi}{N}$$

  where $\frac{\Pi}{N}$ is the share of profits
- There are N agents and they are all the same
- We will focus on symmetric allocations $e_j = e \; \forall j$
- There is a competitive firm with technology $F(Ne)$
- The consumption good is the numéraire

Understanding "V functions"

To do

1. From firms behaviour, derive equilibrium wage as a function of Ne.
2. Use it to write household i utility as a function $V(e_i, e)$, when the wage function is the competitive one.
3. Find household i FOC and the equation that defined Walrasian GE
4. Now take function V and wage function and look for a planner symmetric solution, that would internalise the wage effect.
5. Show that this solution is the same than the competitive one.

Understanding "V functions"

Competitive Equilibrium

- Firm profit :

$$\Pi = F(Ne) - wNe$$

- Firm profit maximisation leads to

$$F'(Ne) = w \text{ or equivalently } w = w(Ne)$$

- so that

$$\Pi(Ne) = F(Ne) - w(Ne)Ne$$

- In this setup, one can write household i decision problem using the following V function :
$$V(e_i, e) = U\left(w(Ne)e_i + \frac{\Pi(Ne)}{N}, e_i\right)$$
- The individual FOC is
$$wU_1 + U_2 = 0$$

- Let us now consider the social optimum
- We have $V(e,e) = U\left(w(Ne)e + \frac{\Pi(Ne)}{N}, e\right)$
- Planner FOC is

$$V_1 + V_2 = 0$$

- Note that

$$\frac{\Pi(Ne)}{N} = \frac{F(Ne) - w(Ne)Ne}{N}$$

and

$$\frac{\partial \frac{\Pi(Ne)}{N}}{\partial e} = \frac{1}{N}\left[NF'(Ne) - w\frac{\partial Ne}{\partial e} - \frac{\partial w}{\partial e}Ne\right]$$
$$= \left(F'(Ne) - w\right) - \frac{\partial w}{\partial e}e$$

- For the Planner :

$$V_1 = wU_1 + U_2$$

and

$$V_2 = U_1 \left[\frac{\partial w}{\partial e} e + \left(F'(Ne) - w \right) - \frac{\partial w}{\partial e} e \right]$$

- Because $F'(Ne) = w$, this implies

$$V_2 = 0$$

- Planner FOC is therefore

$$V_1 = wU_1 + U_2 = 0$$

- So that we are back to the competitive equilibrium condition.
- As we see, the actions of the others have an impact on one's payoff V, but nevertheless there are no inefficiencies at the Walrasian equilibrium.