# Risky Jobs, Risky Assets and Risky Business Cycles*

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**Abstract**

Jobs become more precarious in recessions, while risk premia tend to rise. This paper links these two empirical regularities in a consumption-based asset pricing model with imperfectly insurable job loss risk, and shows that this offers a unified explanation for several asset pricing and macroeconomic phenomena including the equity premium, risk-free rate, and unemployment volatility puzzles. Since job loss leads to a large idiosyncratic consumption decline, agents become less willing to bear risk when jobs become more insecure. Consequently, extrinsic risk aversion increases endogenously when job insecurity rises, generating high, countercyclical risk premia. Consistent with this mechanism, the job loss rate is shown to be a robust forecaster of future excess returns. The model's equity premium Euler equation can be estimated directly, producing a value of 12 for the risk aversion parameter versus estimates for the standard CCAPM which exceed 100. The precautionary saving motive spurred by job loss risk means that the risk-free rate puzzle is avoided, and the model explains the cross-section of returns more effectively than several other consumption-based models. Embedding the mechanism into a macroeconomic model with endogenous unemployment risk allows for a resolution of the unemployment volatility puzzle due to a novel feedback loop between job loss risk, risk premia, and the hiring decision of firms.

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*"If we ask the "representative investor" in December 2008 why he or she is ignoring the high premiums offered by stocks and especially fixed income, the answer might well be "that's nice, but I' m about to lose my job, and my business might go under. I can't take any more risks right now, especially in securities that will lose value or become hard to sell if the recession gets worse.""* - John Cochrane, AFA Presidential Address: Discount Rates (2011)

# 1 Introduction

According to financial theory, an asset is risky if it performs poorly in bad states of the world for the marginal investor. But who is the marginal investor? In the consumption-based asset pricing model (CCAPM) of Lucas (1978) and Breeden (1979) the answer is a representative agent whose stochastic discount factor (SDF) only depends on aggregate consumption. However, as is well-known, this model requires the representative agent to be implausibly risk averse in order to match the risk premium on stocks because aggregate consumption does not covary strongly with equity returns.[1] Even if one accepts the notion of extreme risk aversion, the model also fails to price the cross-section of returns and generates an implausibly high risk-free rate.[2] Due to the representative agent assumption, the model has no role for undiversifiable idiosyncratic risks which vary over the business cycle, and may crowd out investors' risk-taking more broadly. Aligning with this idea, a growing body of empirical evidence suggests that investors hedge their individual labour income risk by lowering their demand for risky assets (Catherine et al., 2024; Calvet and Sodini, 2014; Betermier et al., 2012; Angerer and Lam, 2009).

In this paper, I develop a consumption-based model which deviates from the representative agent paradigm by incorporating ex-post heterogeneity in the form of a particularly salient idiosyncratic risk: job loss. Households face the uninsurable risk of losing their job, leading to a substantial decrease in their consumption — an idiosyncratic rare disaster. This background risk raises the extrinsic risk aversion of the marginal investor (Gollier and Pratt, 1996), generating countercyclical risk premia since jobs become less secure in recessions. Accordingly, assets that are riskier when the job loss rate is high (times when investors are more risk-averse) should command higher expected returns as compensation. The market portfolio

[1] This is the equity premium puzzle of Mehra and Prescott (1985). Hansen and Singleton (1982) find a high risk aversion coefficient and reject the model when estimated on US data. Parker and Julliard (2005), Jagannathan and Wang (2007), Savov (2011) and Kroencke (2017) have argued that once consumption is measured more appropriately, the model can perform better at lower levels of risk aversion, but estimates of this parameter typically remain implausibly high.

[2] See for example Breeden, Gibbons, and Litzenberger (1989), Mankiw and Shapiro (1986) and Lettau and Ludvigson (2001) on the performance of the CCAPM in the cross-section, and Weil (1989) on the risk-free rate puzzle.

and several other high-return assets exhibit this property, which is how the model can explain the equity premium and the cross-section of returns. The precautionary saving motive induced by job loss risk spurs demand for safe assets, pushing the risk-free rate down and avoiding the risk-free rate puzzle.

An objection to this theory is that high-earning households, who are more likely to be asset market participants, work in safe jobs. I provide empirical evidence that, contrary to this common assumption, richer households are still subject to substantial job loss risk. For example, many high-earning workers in the financial services industry lost their jobs during the 2008 financial crisis and numerous tech workers were laid off in 2022-23. Even in normal times, workers in the top half of the earnings distribution face this risk — in many occupations, the high salaries paid may include a compensating differential for their lack of job security. To illustrate this by means of an example, Goldman Sachs notoriously engage in a 'culling' policy where around 5-10% of their workforce, including senior employees, are laid off each year with the fraction rising substantially in downturns.[3] Mueller (2017) finds that above-average earners have a job loss rate that is 3/4 of the mean level and is around twice as countercyclical as for low-wage workers. This higher cyclicality would therefore serve to strengthen the model's key mechanism that produces countercylical risk premia. I also demonstrate that asset market participants likely experience a consumption decline upon job loss that is no less severe than non-participants.[4]

In a macroeconomic context, hiring a worker is a risky investment if the cash flows they generate for the firm fall in bad times (Kehoe et al., 2023). Thus, this decision is likely to be sensitive to the level of risk premia in the economy. By embedding the core mechanism into a DSGE model with search-and-matching frictions in the labour market, I highlight a novel negative feedback loop. An increase in job loss risk elevates risk premia, which subsequently reduces firms' willingness to hire since workers are a risky asset. This reduction in hiring exacerbates the labour market deterioration, setting off a vicious cycle of rising unemployment and risk premia. This ultimately makes unemployment significantly more volatile than in the representative agent analogue of the model where risk premia are close to constant, meaning the Shimer (2005) puzzle of insufficient volatility in the unemployment rate is resolved. The model also simultaneously matches the equity premium and several other asset pricing moments if and only if the SDF incorporates

[3] There are many other examples of this amongst high-paying firms. Meta, Google, Amazon and Microsoft all conducted substantial layoffs in 2023, while 10% of all Tesla employees lost their jobs in April 2024. See www.layoffs.fyi for more on this.

[4] Many companies typically offer a severance package to laid-off workers. It may then seem puzzling why consumption is so sensitive to job loss given that income may not decline in the period when job loss occurs. An explanation for this may be that job loss is a negative permanent income shock, while the severance pay is transitory. If consumption is proportional to permanent income, it would then decline commensurately. Supporting this hypothesis, a large literature shows that income is still 15-25% lower many years after job loss and does not fully recover. See for example Jacobson, LaLonde, and Sullivan (1993) for a classic empirical paper on this, and Lachowska, Mas, and Woodbury (2020) for a more recent study.

job loss risk.

As initial motivating evidence for the relationship between the job loss rate and risk premia, Figure 1 plots the dividend-price ratio of the S&P 500 (a common proxy for risk premia) against a measure of the US job loss rate constructed in section 3. The two are positively related to a striking degree, with a correlation coefficient of 0.81, suggesting that heightened risk of job loss is indeed associated with a rise in the risk premium on stocks.[5] Notably, the co-movement between the pair of variables holds at both high- and low-frequency, with the secular decline in the dividend-price ratio mirrored by a trend fall in the job loss rate. As direct evidence, I later demonstrate that the job loss rate is a robust forecaster of future excess returns.

I first develop a tractable version of the model, which I term the CCAPM with job loss (CCAPM-JL), where tractability is obtained via a no-trade equilibrium. Estimating the Euler equation of the CCAPM-JL using GMM reveals that a risk aversion coefficient of 12 suffices to match the observed equity premium, compared to values exceeding 100 required by the standard CCAPM. This implies that the equity premium puzzle is resolved substantially, as the model effectively explains high average risk premia on stocks in a manner linked to the business cycle without resorting to extreme risk aversion.[6] Even with the more realistic degree of risk aversion, the fit of the CCAPM-JL to the data is also much better than the CCAPM, which is rejected by Hansen's J test due to its large pricing errors. Similar results emerge when the high-minus-low (HML) and small-minus-big (SMB) portfolios of Fama and French (1993) are added to the set of test assets alongside the excess market return, meaning the CCAPM-JL is able to simultaneously explain the equity, size and value premia with reasonable risk aversion.

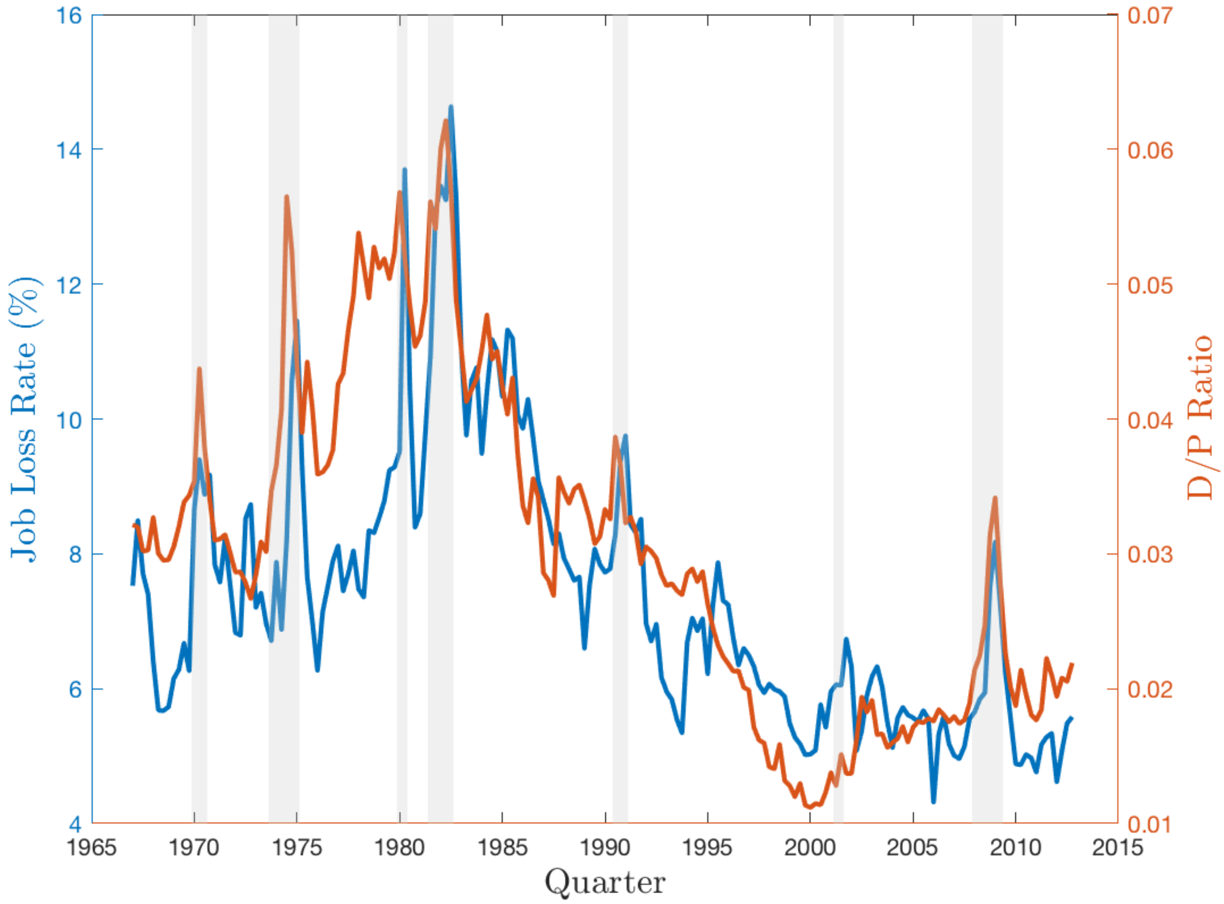
The model also avoids the risk-free rate puzzle of Weil (1989) due to the precautionary saving motive that arises from job loss risk. In the CCAPM-JL, unlike the CCAPM, increasing risk aversion lowers the risk-free rate because the precautionary saving term that arises easily overwhelms the consumption smoothing term in strength. The more risk-averse investors are, the more they fear job loss which spurs them to demand riskless assets, pushing the risk-free rate down consequently.

Linearising the model's SDF produces analytical insights into why the CCAPM-JL significantly outperforms

[5] The other possibility, of course, is that increases in the dividend-price ratio during the great moderation were primarily driven by lower expected dividend growth. A large body of empirical evidence suggests that this is not the case systematically, see for example Cochrane (2011) for a summary.

[6] Fama (1991) expressed a hope for an asset pricing model which could "relate(s) the behaviour of expected returns to the real economy in a rather detailed way". Cochrane (2017) expresses a similar sentiment.

**Figure 1:** The Dividend-Price Ratio and the Job Loss Rate




Note: The figure displays the dividend-price ratio for the S&P 500 (red line) and the job loss rate (blue line) constructed in section 3 over the period 1967:I - 2012:IV. The grey shaded areas denote NBER recessions.

the CCAPM. In the CCAPM, the single factor is consumption growth, and an asset's (approximate) expected excess return is solely determined by the unconditional covariance between the asset's returns and consumption growth multiplied by the consumption growth risk price, which is equal to the risk aversion parameter. Because this covariance is typically weak for stocks, the only way to generate a realistic excess return is with an extreme degree of risk aversion. In the CCAPM-JL however, two additional factors naturally emerge which correspond to the job loss rate and an interaction term between the job loss rate and consumption growth. As a result, assets which covary negatively with the job loss rate, or covary more positively with consumption growth when the job loss rate is high, earn a higher expected return all else equal. It is the second of these two additional factors, which I term the conditional factor, that is quantitatively important for the market return. This conditional factor captures the fact that investors dislike assets which are more correlated with consumption growth (riskier) at times when risk premia are heightened due to the increased job loss risk the marginal investor faces (Lettau and Ludvigson, 2001). This factor has a risk price which is a convex function of the risk aversion parameter, meaning that ultimately a much smaller value of this parameter is required to match the excess return on stocks.

The tractability of the model allows it to be tested in the cross-section, unlike other asset pricing models featuring heterogeneity and uninsurable labour income risk.[7] Accordingly, Fama and MacBeth (1973) regressions are used to examine whether the CCAPM-JL can better explain the cross-section of excess returns than alternative consumption-based factor models. The 25 Fama and French (1993) portfolios are considered through the lens of the CAPM, CCAPM, conditional CCAPM of Lettau and Ludvigson (2001) as well as the CCAPM-JL introduced here. The latter achieves the best performance by a considerable margin, reflected by the fact it achieves the lowest pricing errors of all four models as well as a more plausible risk price on the consumption growth factor. Once again the CCAPM-JL performs better than the CCAPM due to the fact that assets which earn high average excess returns (such as small value portfolios) weakly covary with consumption growth unconditionally, but covary more strongly with consumption growth during times when job loss risk is high.

To show that the tractability afforded by the no-trade equilibrium is not essential for the main results, I extend the model to a two-asset heterogeneous agent framework à la Krusell and Smith (1997). This version includes a non-degenerate distribution of risky capital and riskless bond holdings, and when calibrated, it still matches the equity premium and risk-free rate with a realistic level of risk aversion. The model also generates a realistic average level of wealth, marginal propensity to consume, and degree of consumption loss upon unemployment. In a simplified two-period version of the model I derive a novel analytical expression for the risky asset demand of a given household, and show that i) job loss risk depresses this, and ii) this force is stronger for more poorly insured agents with a lower wealth cushion.

I end by incorporating the mechanism into a macroeconomic model where unemployment risk is endogenous due to Diamond-Mortensen-Pissarides frictions in the labour market. Firms must post vacancies in order to hire workers, and since the cash flows workers generate depend on aggregate shocks, hiring is a form of risky investment and is naturally sensitive to fluctuations in risk premia. This mechanism generates a vicious circle where a bad aggregate shock causes firms to reduce their hiring, which increases unemployment risk for the household since they are now less likely to find a job if they separate from their current

[7] In Constantinides and Ghosh (2017) for example, the model does not yield an SDF that can be directly taken to the data, unlike the CCAPM-JL. Instead, they test the reduced form prediction of their model that consumption growth skewness should be a priced risk factor in the cross-section, which they find to be the case.

position. Since unemployment is once again associated with a large drop in idiosyncratic consumption, risk premia rise in the economy which further depresses hiring and thus catalyses the negative feedback loop. Ultimately, unemployment becomes very sensitive to aggregate shocks which means that sufficient unemployment volatility can be generated for a realistic degree of aggregate risk, resolving the Shimer (2005) puzzle. The calibrated macro model can at the same time match the mean of the equity premium and risk-free rate, while also generating 'excess volatility' in stock returns and the dividend-price ratio (Shiller, 1981). Erroneously fitting the CCAPM to the data generated by the model would lead a naive econometrician to conclude that the risk aversion parameter is four times larger than its true value.

### Related Literature

The paper shares much with others aiming to explain the equity premium puzzle and the cross-section of returns within a consumption-based framework, most notably Schmidt (2022), who develops a model with heterogeneity, incomplete markets, and idiosyncratic income risk. In his model, investors face idiosyncratic disaster risk with time-varying probability, generating a large, counter-cyclical equity premium. However, subtle differences exist between his mechanism and this paper's. The CCAPM-JL's key channel operates via fluctuations in the job loss rate, leading to variations in risk aversion and risk premia, paralleling the Campbell and Cochrane (1999) framework that generates time-varying risk premia through habit formation in utility. In Schmidt (2022), the key channel revolves around hedging demand and requires recursive preferences; negative stock returns precede increased future idiosyncratic disaster risk, making stocks a poor hedge for long-term investors, thus demanding a large equity risk premium. This approach resembles Bansal and Yaron (2004) at the individual level. Crucially, the elasticity of intertemporal substitution (EIS) must exceed one for this hedging channel, conflicting with empirical evidence suggesting EIS is significantly below one — e.g., Best et al. (2020) estimate an average value of 0.1. In contrast, the CCAPM-JL uses CRRA preferences, imposing no such EIS restriction; baseline GMM estimates imply an EIS of 0.08, close to empirical findings. A second key difference is the CCAPM-JL's greater tractability compared to Schmidt (2022), which i) removes any 'black box' element (Chen et al., 2019); ii) allows direct estimation of the model's Euler equation without specifying parametric aggregate state processes; iii) permits analytical insights into the components determining expected asset returns

The CCAPM-JL features similarities with several other existing asset pricing frameworks. It has much in common with the rare disasters framework of Barro (2006). The key asset pricing mechanism relies on *idiosyncratic* rather than *aggregate* disasters, however. The job loss rate is much more easily measurable than the probability of an aggregate consumption disaster, and the size of the consumption decline is also easier to estimate using cross-sectional data, meaning the model is far less reliant on a 'dark matter' explanation. I also draw a comparison between the CCAPM-JL and the model of Campbell and Cochrane (1999), as I show in section 3 that a rise in the job loss rate produces an increase in extrinsic risk-aversion — the same mechanism at play in the habits model. This ultimately allows both frameworks to produce time-varying risk premia. There are strong links between the model developed here and that of Constantinides and Duffie (1996), Constantinides and Ghosh (2017) and Meeuwis (2022) due to the focus on uninsurable idiosyncratic risk and asset prices. The model also has parallels to Ai and Bhandari (2021), who also build an asset pricing model that relies upon the exposure of workers to uninsurable tail risk in labour income to explain the equity premium. Relative to these papers, the model I present is significantly more tractable. This has several advantages (and disadvantages) that will become evident.

Krusell et al. (2011) also investigate the implications of asset pricing in incomplete markets, providing the foundation of the framework I develop, but neither explicitly consider job loss risk nor estimate their model. Earlier work by Mankiw (1986) found that, in a simple two-period model featuring limited consumption insurance, the equity premium could depend on the concentration of shocks so long as the precautionary motive was present. The intuition behind this result is applicable to the framework here, as ex-post job loss is concentrated on only a small fraction of the population but its incidence is ex-ante unknown to a risk-averse investor. Chang, Hong, and Karabarbounis (2018) take the equity premium as given, and instead show that the higher levels of job loss risk faced by younger workers can help to explain the empricial fact that the risky asset holding share increases with age. Brunnermeier et al. (2024) develop a model where a rise in idosyncratic risk in recessions generates an equity premium, but do not focus on job loss or labour income risk as the source of this.

This paper also shares a focus on the interaction between labour markets and asset prices that is present in a number of other papers. Kilic and Wachter (2018) develop a model featuring frictional labour market in the Diamond-Mortensen-Pissarides (DMP) tradition and show how this can explain the joint volatility of asset and labour markets. Petrosky-Nadeau, Zhang, and Kuehn (2018) are able to generate disasters endogenously in their DMP-style model, but markets are complete and there is no fluctuation in the rate of job loss. Hall (2017) and Kehoe et al. (2023) both connect variation in the SDF to the hiring decisions of firms, as I do in the macroeconomic model. The latter of these is particularly relevant as they construct a model whereby fluctuations in the price of risk allow for the simultaneous resolution of the equity premium and unemployment volatility puzzles. However, there is perfect consumption insurance in their model, meaning it lacks the feedback mechanism between job loss risk, risk premia and firm hiring decisions which is present and important in the model I present.

The paper proceeds as follows. Section 2 presents the asset pricing model and analytical results on the risk-free rate. Section 3 discusses the construction of the necessary data, discusses the model's empirical applicability, estimates the model with GMM using a small set of test assets, describes the intuition behind the results, and presents several tests of additional predictions from the model. Section 4 examines the model's ability to explain the cross-section of returns. Section 5 relaxes the no-trade equilibrium in a heterogeneous agent model. Section 6 presents the macroeconomic model and the results from it. Section 7 concludes.

## 2 Asset Pricing Model

There is a unit continuum of single-member households indexed by $i \in (0,1)$ who are ex-ante identical and derive utility from the consumption of non-durable goods and services according to the constant relative risk aversion (CRRA) utility function:

$$U_0(i) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t(i)^{1-\gamma} - 1}{1-\gamma} \right)$$

where $C_t(i)$ is consumption of household i in period t, $\beta \in (0,1)$ is the subjective discount factor, $\gamma > 0$ is the coefficient of relative risk aversion, and $\mathbb{E}_t$ is the conditional expectation function. In any given period $t$, a household is either employed or unemployed, with $\mathbb{1}_t^E$ being the respective indicator function for employment. All employed households are endowed with $Y_t^E \in [\underline{Y}^E, \overline{Y}^E]$ units of the consumption good, while all unemployed households receive an endowment of $Y_t^U \in [\underline{Y}^U, \overline{Y}^U]$. I also assume that $0 < \overline{Y}^U < \underline{Y}^E$, i.e. that the unemployed endowment is always strictly less than the employed endowment and both endowments are always strictly positive. Both $Y_t^E$ and $Y_t^U$ are thus strictly positive, bounded random variables.

At the start of each period, employed workers in the previous period either keep their job with probability $1 - \rho_t \in (0,1)$ or lose their job with the complement probability. Job losers spend at least one full period unemployed.[8] Simultaneously, households who were unemployed last period remain so with probability $1-\eta_t \in (0,1)$ and find a job with the complement probability. Upon the completion of the job loss and job finding processes, consumption and asset holding decisions are made by each household. There are N+1 assets available for each household to trade (bonds, stocks, derivatives etc.) indexed by j and each earning a real gross return of $R_t^j \in \mathbb{R}^+$ in period t. Importantly the household cannot purchase assets that are contingent upon their idiosyncratic employment status and so as a consequence, job loss constitutes an uninsurable risk. Household i's budget constraint is given by:

$$C_t(i) + \sum_{j=0}^{N} a_{t+1}^j(i) = \mathbb{1}_t^E(i)Y_t^E + (1-\mathbb{1}_t^E(i))Y_t^U + \sum_{j=0}^{N} R_t^j a_t^j(i)$$

where $a_t^j(i)$ is household i's holdings of asset j in period t. Moreover, the household faces the constraint for each asset:

$$a_{t+1}^j(i) \geq \underline{a}^j \geq 0 \qquad \forall j,t,i.$$

This assumption prevents household holdings of a given asset from falling below a threshold, and will ultimately allow for a tractable solution in the presence of heterogeneity when chosen judiciously. Using primes to indicate next-period variables, the Bellman equation for an employed household in $t$ is then given by:

$$V^E(\mathcal{A}(i),\Omega) = \max_{C(i),\mathcal{A}'(i)} \left\{U(C(i)) + \beta\mathbb{E}\left[(1-\rho')V^E(\mathcal{A}'(i),\Omega') + \rho' V^U(\mathcal{A}'(i),\Omega')\right]\right\}$$

$$s.t. \qquad C(i) + \mathcal{A}'(i) = Y^E + R\mathcal{A}(i)$$

$$\mathcal{A}A'(i) \geq \underline{\mathcal{A}}$$

$$\mathcal{A}_0(i) = \underline{\mathcal{A}}$$

where $\mathcal{A}(i)$ is the vector of i's asset holdings, $\underline{\mathcal{A}}$ is the vector of asset holding constraints and $R$ is the vector of returns, while $\Omega$ is the vector of non-asset state variables.[9] The Bellman equation for an unemployed

[8] The job loss rate should therefore be thought of as the fraction of workers who lose their job and are not able to find a new one. This is the case in the macroeconomic model presented in section 6 as job losers immediately have a chance to look for another job.

[9] For example, if the job loss rate followed an ARMA(p,q) process, it would be contained in $\Omega$. I do not specify precisely the contents of $\Omega$ since it is not necessary to do so for estimation. This stands the model in contrast with others featuring heterogeneity, which typically require the specification of parametric processes for the state variables in order to be solved and estimated/calibrated.

household is given by:

$$V^U(\mathcal{A}(i),\Omega) = \max_{C(i),\mathcal{A}'(i)} \left\{ U(C(i)) + \beta \mathbb{E}\left[(1-\eta')V^U(\mathcal{A}'(i),\Omega') + \eta' V^E(\mathcal{A}'(i),\Omega')\right]\right\}$$

$$s.t. \qquad C(i) + \mathcal{A}'(i) = Y^U + R\mathcal{A}(i)$$

$$\mathcal{A}'(i) \geq \underline{\mathcal{A}}$$

$$\mathcal{A}_0(i) = \underline{\mathcal{A}}$$

The market clearing condition for each asset is given by:

$$\int_0^1 a_t^j(i)di = \tilde{a}^j$$

meaning that $\tilde{a}^j$ is the net supply level of asset j.

**Equilibrium Definition:** An equilibrium is a set of asset prices and policies such that i) Policy functions $[C_t(i), \{a_{t+1}^j(i)\}_{j=0}^N]$ solve the household problem for all households; ii) All asset markets clear; iii) Actual and perceived laws of motion coincide.

As in Ravn and Sterk (2017) and Krusell, Mukoyama, and Smith (2011), I assume the vector of asset holding constraints is maximally tight, $\underline{a}^j = \tilde{a}^j$. This leads to the following proposition:

**Proposition 1:** The following is an equilibrium:

$$a_t^j(i) = \tilde{a}^j \qquad \forall t, i, j \tag{1}$$

$$(C_t(i)|\mathbb{1}_t^E(i) = 1) = C_t^E \qquad \forall t, i \tag{2}$$

$$(C_t(i)|\mathbb{1}_t^E(i) = 0) = C_t^U \qquad \forall t, i \tag{3}$$

$$1 = \mathbb{E}_t M_{t,t+1}^E R_{t+1}^j \qquad \forall t, j \tag{4}$$

where:

$$M_{t,t+1}^E = \beta \left[\frac{(1-\rho_{t+1})(C_{t+1}^E)^{-\gamma} + \rho_{t+1}(C_{t+1}^U)^{-\gamma}}{(C_t^E)^{-\gamma}}\right] \tag{5}$$

*Proof.* See appendix A.

The asset-holding constraints are so tight that they induce autarky and a no-trade equilibrium as in Constantinides and Duffie (1996), Constantinides and Ghosh (2017) and Schmidt (2022). In equilibrium, employed households will always wish to accumulate the assets for precautionary reasons as long as they face job loss risk and are risk averse. This implies their Euler equation will hold with equality each period for each asset as they never run up against any asset-holding constraint. Unemployed households, however, wish to deplete their holdings of each asset to smooth consumption as they are currently in the worst possible idiosyncratic state, meaning the future is better than the present in expectation for them. Due to the asset-holding constraint, they are prevented from doing so, however, and consequently their Euler equation will hold with strict inequality. The employed households cannot find a counterparty, and so they do not trade either in equilibrium. Since the employed agent has the highest private valuation of each asset, they determine the asset's price and assume the role as the marginal investor in the economy.

Since households are ex-ante identical, substituting the asset holding condition into the budget constraint implies that all households of the same employment status consume the same amount in equilibrium.[10] This implies that there is effectively a representative employed agent, who faces job loss risk and prices the assets, and a representative unemployed agent who has no direct influence on asset prices. Berger et al. (2023) obtain a similar result in a more general framework. The average consumption of each group is therefore sufficient to capture the entire consumption distribution, which will consist of two mass points each period. Appendix A illustrates this in the case of an economy with a risk-free bond in zero net supply and N stocks in positive net supply. In equilibrium, the return for each asset is determined by the employed agent's Euler equation. The expression for $M^E_{t,t+1}$ clearly illustrates that the presence of uninsurable job loss risk drives a stochastic wedge into the SDF used to price assets in the economy. An increase in the probability of job loss next period will, all else equal, cause an increase in the SDF if $C^E_{t+1} > C^U_{t+1}$.

The no-trade equilibrium and the Euler equation which emerges is robust to the presence of heterogeneity

[10]The $C^E_t$ and $C^U_t$ terms will depend upon the assets which exist in the economy but will be homogenous across those of the same employment status. For example, if we have N stocks, $s^j_t$, in unit net supply with price $p^j_t$ and dividend $d^j_t \geq 0$, and a risk-free bond, $b_t$, in zero net supply, then from the budget constraints we have:

$$C^E_t(i) = C^E_t = Y^E_t + \sum_{j=1}^{N} d^j_t \qquad \forall i$$

$$C^U_t(i) = C^U_t = Y^U_t + \sum_{j=1}^{N} d^j_t \qquad \forall i$$

in permanent income, as the following proposition demonstrates. In this case, the income and consumption distributions have full support.

**Proposition 2:** Suppose that all assets are in zero net supply ($\underline{a}^j = \tilde{a}^j = 0\ \forall j$) and households vary in their permanent income, $Z_t(i)$. This scales the endowment of the household regardless of their employment status, meaning it can be thought of as their skill level in the sense that more skilled individuals always earn higher income relative to less skilled individuals of the same employment status. The budget constraint now becomes:

$$C_t(i) + \sum_{j=0}^{N} a_{t+1}^j(i) = \mathbb{1}_t^E(i) Z_t(i) Y_t^E + (1 - \mathbb{1}_t^E(i)) Z_t(i) Y_t^U + \sum_{j=0}^{N} R_t^j a_t^j(i) \tag{6}$$

where $Z_t(i)$ satisfies:

$$\frac{Z_{t+1}(i)}{Z_t(i)} = \exp\left(\epsilon_{t+1}^Z(i)\right) \tag{7}$$

$$\epsilon_{t+1}^Z(i) \sim \mathbb{N}\left(\frac{\gamma\sigma^2}{2}, \sigma^2\right) \tag{8}$$

$$\int_0^1 Z_t(i) di = 1 \qquad \forall t$$

and $\epsilon_{t+1}^Z(i)$ is independent from all other random variables. Then there is an equilibrium where the same Euler equation holds as in Proposition 1:

$$1 = \mathbb{E}_t M_{t,t+1}^E R_{t+1}^j \qquad \forall t, j \tag{9}$$

$$M_{t,t+1}^E = \beta \left[ \frac{(1 - \rho_{t+1})(C_{t+1}^E)^{-\gamma} + \rho_{t+1}(C_{t+1}^U)^{-\gamma}}{(C_t^E)^{-\gamma}} \right] \tag{10}$$

where:

$$C_t^E = \int_0^1 (C_t(i) | \mathbb{1}_t^E(i) = 1) di = Y_t^E \tag{11}$$

$$C_t^U = \int_0^1 (C_t(i) | \mathbb{1}_t^E(i) = 0) di = Y_t^U \tag{12}$$

*Proof.* See appendix A.

Introducing a risk-free asset produces the following additional condition:

$$R^f_{t,t+1} = \frac{1}{\mathbb{E}_t M^E_{t,t+1}} \tag{13}$$

The excess return on any asset j, $R^{e,j}_{t+1} = R^j_{t+1} - R^f_{t,t+1}$, is given by:

$$\mathbb{E}_t M^E_{t,t+1} R^{e,j}_{t+1} = 0 \tag{14}$$

It is this moment condition which is taken to the data in Section 3 to estimate the risk aversion parameter. No further structure is placed on the model. Crucially, there is no need to specify parametric processes for the job loss rate, consumption growth rate, the set of returns or to impose any restrictions on the relationships between the variables. As such, the estimation approach is semi-parametric in contrast to most asset pricing models with heterogeneity which are fully parametric, since they necessitate fully specifying the processes for the model's state variables. For example, Constantinides and Duffie (1996) specify a heteroskedastic process for a consumer's labour income, while Constantinides and Ghosh (2017) and Schmidt (2022) both specify processes for aggregate consumption growth, labour income, and dividend growth. The approach I take significantly reduces the degrees of freedom available, ensuring the model is strongly disciplined by the data. This, in turn, means the model avoids the 'dark matter' critique of Chen et al. (2019), who highlight the reliance of many leading asset pricing models on unobservable processes and as such are difficult to falsify.[11]

At this point, it is natural to compare the SDF in (5) to that which prevails in the standard representative agent CCAPM:

$$M^{RA}_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \tag{15}$$

This will serve as a benchmark with which to compare the performance of the model empirically. As in Ghosh et al. (2017) and Cochrane (2017), the CCAPM-JL fits into a large set of models where SDF can be expressed as the CCAPM SDF multiplied by a wedge term:

$$M^E_{t,t+1} = \beta \left( \frac{C^E_{t+1}}{C^E_t} \right)^{-\gamma} \Psi_{t+1} \tag{16}$$

[11]Dark matter is a concept in cosmology that refers to hypothetical matter which cannot be directly measured, and instead can only be observed via the effects it produces on the universe. An archetypal example of dark matter in finance is the probability of a rare disaster occurring. In the Wachter (2013) model, for example, unobservable fluctuations in this risk are responsible for observable movements in asset prices.

where:

$$\Psi_{t+1} = 1 + \rho_{t+1}\left(\chi_{U,t+1}^{-\gamma} - 1\right) \tag{17}$$

$$\chi_{U,t+1} = C_{t+1}^{U}/C_{t+1}^{E} \tag{18}$$

In contrast to almost all other models in this class (e.g. Constantinides and Duffie (1996), Campbell and Cochrane (1999), Bansal and Yaron (2004), Yogo (2006), Wachter (2013)), the wedge term is fully observable and thus does not constitute 'dark matter' since $\chi_{U,t+1}$ will ultimately be held fixed at its estimated value from microeconometric evidence and $\rho_{t+1}$ will be taken from the data directly also.[12]

The above expression spells out the conditions under which the CCAPM SDF and the CCAPM-JL SDF coincide, i.e. $\Psi_{t+1} = 1$, and gives $\Psi_{t+1}$ a natural interpretation as a stochastic job loss risk wedge. The first condition under which this occurs is with $\rho_{t+1} = 0$, when agents face no risk of losing their jobs as in a standard consumption-based asset pricing model. The second condition under which there is equivalence of the SDFs is when there is no consumption loss upon unemployment, $\chi_{U,t+1} = 1$. When this holds, the investor is perfectly insured against this source of consumption risk and so any increase in the risk of being laid off does not impact their SDF. This is not a good description of reality however, partially because the market for private unemployment insurance is missing due to the well-known issue of adverse selection in this context. As we will see, when the average level of consumption loss upon unemployment is estimated empirically, it is invariably found to be large, positive and statistically significant. The final condition is under risk neutrality, with $\gamma = 0$. This case is trivial and implies no volatility in expected returns at all and an expected excess return of zero for all assets under either of the two SDFs.

In the model, fluctuations in $\Psi_{t+1}$ — which amount from variation in either the risk of job loss or changes in the consumption loss upon unemployment — represent changes in the investor's willingness to bear risk. For the remainder of the paper (one extension aside) I focus on the case where there is no time series variation in $\chi_{U,t}$ and assume that $\chi_U < 1$, both of which we will later see are realistic empirically. When these three conditions are all not satisfied, the job loss risk wedge emerges. As a result, during recessions when there is a greater chance of the investor experiencing a consumption loss due to losing their job, the investor has less appetite to bear risk and demands higher risk premia consequently. This intuition is

[12] An alternative approach that embraces 'dark matter' in the model would be to treat $\chi_{U,t+1}$ as an unobservable stochastic process, fix risk aversion, and then reverse engineer a series for the consumption loss upon unemployment term to match various moments of the time-series and cross-section of asset returns given data for consumption growth and the job loss rate. Equally, this same procedure could be used to reverse engineer $\rho_{t+1}$ after calibrating $\gamma$ and $\chi_{U,t+1}$. The downside of both of these is that the estimated series for $\chi_{U,t+1}$ and $\rho_{t+1}$ may differ substantially from survey-based measures of the two.

formalised in the next section.

# 3 Model Estimation and Analytical Insights

## 3.1 Data

In order to estimate the model, data is needed on excess returns, consumption of the employed and unemployed, and finally on the job loss rate. All data is from the US and is quarterly, while the sample period is 1967:I - 2012:IV[13].

Starting with excess returns, I first use the return on the value-weighted CRSP portfolio in excess of the return on a three-month treasury bill as a measure of the excess market return. In a second specification I also add the excess return on the high-minus low (HML) and small-minus-big (SMB) portfolios of Fama and French (1993) to the excess market return.

In order to gather data on the average consumption of the employed and unemployed, I draw upon the work of Chodorow-Reich and Karabarbounis (2016), who first estimate the consumption loss upon unemployment relative to employment using Consumer Expenditure Survey (CEX) and Personal Survey of Income Dynamics (PSID) data. They find that there is a considerable lack of consumption insurance for job loss, estimating $\widehat{\chi}_U = 0.77$ in their preferred specification. This is repeated for two other two other statuses: out of the labour force (N) and retired (R). They then use the following accounting identity:

$$C_t^{NIPA} = \alpha_{U,t} C_t^U + \alpha_{E,t} C_t^E + \alpha_{R,t} C_t^R + \alpha_{N,t} C_t^N \tag{19}$$

where $C_t^j$ represents the consumption of those in group j, $\alpha_{j,t}$ is the share of group j at time t, and $C_t^{NIPA}$ is real NIPA non-durable goods and non-housing services consumption. Solving this equation for $C_t^E$:

$$C_t^E = \frac{C_t^{NIPA}}{\sum_j \alpha_{j,t} \chi_{j,t}} \tag{20}$$

where $\chi_{j,t}$ is the ratio of $C_t^j$ to $C_t^E$. These ratios are held fixed at time-invariant values, which implies that consumption loss upon unemployment remains constant. This is a conservative assumption that simplifies the analytical results in section 3.4 considerably. I relax this assumption in a model extension

[13] The start point of the sample period is governed by the availability of initial claims data, while the endpoint is determined by the availability of the UI take-up rate from Chodorow-Reich and Karabarbounis (2016).

following Piqueras (2024) who finds empirical evidence that $\chi_{U,t}$ is lower in recessions vs booms. It is then straightforward to solve for $C_t^U = \chi_U C_t^E$ and we have our two consumption time series which are necessary.

Finally, a measure of the job loss rate is required. To construct this, I make use of data on initial claims for unemployment insurance as in Schmidt (2022). This is an appropriate series since in the United States the criteria for unemployment insurance specifies that a claim is only eligible if the individual becomes involuntarily unemployed through no fault of his own. As such, the measure will not pick up voluntary quits as would be the case for the employment-unemployment transition rate from Current Population Survey (CPS). I do not wish to include quits in the job loss rate as their voluntary nature may suggest they do not satisfy the criterion of being an exogenous, uninsurable source of consumption risk for an investor. I construct the job loss rate by first dividing the total number of initial claims filed in a quarter by the total number of workers employed in the previous period. Finally, since job losers are more likely to file a claim when the labour market is experiencing a downturn, the take-up rate of unemployment insurance is not constant over time and is instead counter-cyclical. Chodorow-Reich and Karabarbounis (2016) estimate this take-up rate for unemployment insurance, and I use this to adjust the ratio of initial claims to employment in order to purge the series of this source of variation.

### 3.2 Discussion of Empirical Relevance

Before proceeding to the estimation of the model's Euler equation, I briefly discuss some concerns one may possibly have about the model's empirical relevance.[14]

Stock-holding is skewed towards higher income households as has been well-documented by for example Poterba and Samwick (1995) and Melcangi and Sterk (2020). A first concern may be that these households face very little job loss risk and/or that this risk is relatively constant over the business cycle ($\rho_t \approx 0$ or $\text{Var}(\rho_t) \approx 0$). Mueller (2017) finds that the job separation rate of workers with wages above the median is just over half that of below-median wage workers, meaning it will be approximately 3/4 of the average job loss rate. As such, richer working households still face a considerable risk of losing their job. Holzheu (2019) also finds, using matched employer-employee data, that there is a U-shaped relationship between prior wage growth and a worker's job separation probability.

A second key point raised by Mueller (2017) is that job loss risk for higher-income households is twice

[14] Schmidt (2022) makes similar arguments in section 3.5 of his paper.

as counter-cyclical than for lower-income households.[15] This also aligns with the findings of Guvenen et al. (2014) that the left-skewness of earnings is more counter-cyclical for households further up the earnings distribution. Pruitt and Turner (2020) find that households in the top decile of the permanent income distribution would be willing to give up around 15% of their earnings in expansions to eliminate persistent earnings risk, but this roughly doubles in recessions primarily due to the increased left-skewness of shocks at these times. This certainty equivalent is shown to be *decreasing* across the income distribution, indicative of the fact that high-income households are more exposed to large adverse earnings shocks. Given that it is covariances which matter for asset pricing, this empirical property would almost certainly strengthen the results if the model was only applied to this group of households. As shown by Malloy et al. (2009), the consumption of stockholders is also more correlated with equity returns than non-stockholders, which would also serve to strengthen the results.

To bolster this point further, Figure 2 below plots the layoff rate from JOLTS by industry for total nonfarm and the three industries with the highest average weekly wages as of 2019Q1 — financial activities, information and professional and business services. The figure illustrates that there is considerable job loss risk in these high-wage sectors, in which workers are more likely to hold stocks, and that this risk fluctuates substantially over the business cycle. The layoff rate in each of the three industries has a higher volatility than the overall series.

A second concern is that stockholders are sufficiently wealthy that they are almost completely insured against job loss in terms of their consumption. This would imply that $\chi_U \approx 1$ for these households.[16] To test this, I re-estimate the Chodorow-Reich and Karabarbounis (2016) specification for the CEX, additionally including an interaction variable between the unemployment indicator variable and an indicator that equals 1 when the individual is a stock market participant and 0 otherwise. The specification then becomes:

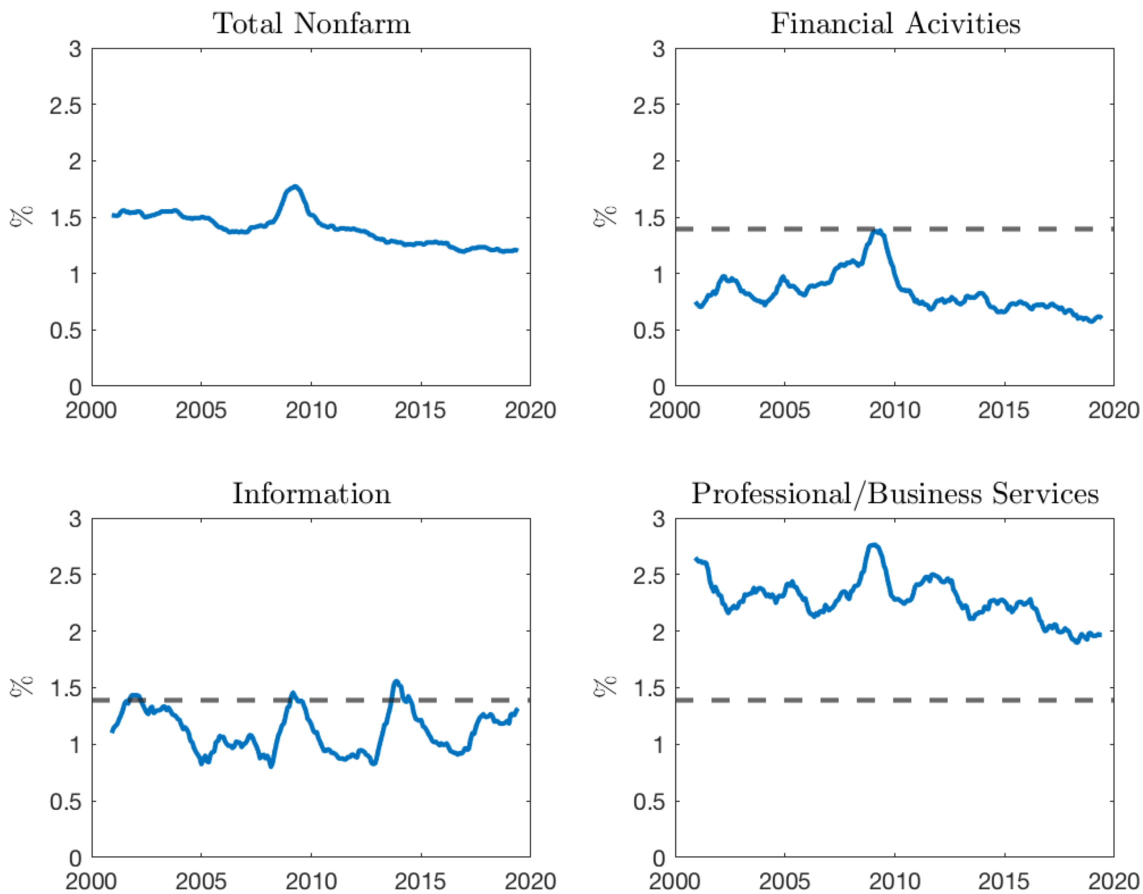
$$\ln C_{i,t} = \chi_t^0 + \phi_i \mathbf{X}_{i,t} + (\chi_U - 1) D_{i,t}^U + \Lambda D_{i,t}^U I_t^P + (\chi_N - 1) D_{i,t}^N + \epsilon_{i,t} \tag{21}$$

where $C_{i,t}$ is consumption of nondurables and services, excluding expenditures on housing, health care, and education, $\chi_t^0$ is a base level of consumption, $\mathbf{X}_{i,t}$ is a set of control variables, and $D_{i,t}^U$ and $D_{i,t}^N$ are variables that measure the fraction of the period the individual spent unemployed and not in the labour

[15] Similarly, Larkin (2019) finds that, during the Great Recession, the job loss rate of high-wage workers rose by the most.

[16] Massa and Simonov (2006) find, using rich Swedish data, that, even for high-wealth households, higher unemployment risk is associated with a lower risky asset share, suggesting that job loss is a relevant source of idiosyncratic risk for them when it comes to their portfolio choice decision.

**Figure 2:** JOLTS Layoff Rate in High-Wage Industries




Note: The figure displays the monthly layoff rate from JOLTS data for the total nonfarm sector and for financial activities, information and professional and business services. over the period 2000-2019 for which the series are available. The series are smoothed with a 12-month moving average to remove any seasonality. The red dotted line represents the mean value of the total nonfarm layoff rate.

force respectively, and $I_t^P$ is an indicator variable that takes a value of 1 if the household holds stocks and zero otherwise. The identifying assumption is that the set of controls is rich enough to capture all aspects of permanent income and any taste shocks.[17] The coefficient on the interaction term, $\Lambda$, is not found to be significant at the 5% level, and I cannot reject the null that the consumption loss upon unemployment that asset market participants experience is no different from non-participants.[18] This result aligns with

[17]The set of controls includes i) the mean age of the household head and spouse ii) the mean age squared iii) the households's marital status iv) an indicator variable for whether the household head is Caucasian or not iv) indicator variables for the following four categories of education of the household head: less than high school, high school diploma, some college, college degree, interacted with year v) indicator variables for owning a house outright, owning a house with a mortgage, or renting a house, interacted with year vi) indicator variables for quantiles of the value of the home conditional on owning, by region and year, interacted with year vii) a binary variable for having positive financial assets viii) family size ix) family size squared.

[18]At first glance this might seem to be a puzzling result, since households with financial assets should be better equipped to cushion the blow of unemployment in terms of their consumption by drawing upon their wealth. However, I find that these households tend to work in higher income professions meaning that the income loss from unemployment is higher in percentage terms. The consumption loss upon unemployment can be thought of as being roughly the product of two

Arellano et al. (2017), who find that a large negative income shock (such as job loss) leads to a sizeable decrease in consumption for high-earning households, much more so than for low-earning households in fact. They find that this consumption response is only modestly attenuated by asset-holdings.[19]

As a second way of examining this, I use a quantile regression to estimate the Chodorow-Reich and Karabarbounis (2016) specification. This offers insights into the extent of heterogeneity present in the degree of consumption loss upon unemployment. While I find some heterogeneity, the estimated effect implies a substantial consumption decline at every quantile. Even at the 90th percentile, consumption is estimated to fall by more than 15% after unemployment. The plot of these quantile regression estimates can be found in Figure A1. Later in the model of section 5 which relaxes the no-trade equilibrium and features richer heterogeneity, I find that this estimated consumption loss distribution can be matched quite well.

A third and final concern is that labour income may not constitute the primary source of income for workers at the top of the income distribution, who own a disproportionate share of stock market wealth. However, Schmidt (2022) updates the work of Piketty and Saez (2003), finding that, in 2018, 75.9% of total income for the top 10% came from wages rather than entrepreneurial sources or dividends. This implies that earnings remain the predominant source of income even for those at the upper echelons of the distribution.

### 3.3 GMM Estimation

I now proceed to estimate the risk aversion parameter from the following equation for both the SDF in the CCAPM-JL and the SDF in the CCAPM:

$$\mathbb{E}_t M^s_{t,t+1} R^{e,j}_{t+1} = 0 \tag{22}$$

A natural means of doing this is the generalised method of moments (GMM) of Hansen (1982) and Hansen and Singleton (1982). The normalisation $\beta = 1$ is used in both SDFs as is common in the literature (e.g. Savov (2011), Hansen, Heaton, and Li (2008)) since this parameter is not identified here. The

terms: the marginal propensity to consume (MPC) out of income multiplied by the derivative of income with respect to the unemployment indicator. Wealthier households tend to have a lower MPC but this is offset by the larger fall in income upon unemployment. See Patterson (2023) Figure A4 for an illustration of this.

[19]A potential explanation for this result is that many high-earning households are 'wealthy hand-to-mouth', meaning they lack liquid assets to draw upon in order to smooth the consumption impact of job loss despite having high levels of total assets.

moment conditions, where $R_{t+1}^{e,j}$ denotes a G x 1 vector of excess returns and $Z_t$ denotes a K x 1 vector of instruments, are given by:

$$\mathbb{E}\left[M_{t,t+1}^{s} R_{t+1}^{e,j} \otimes Z_t\right] = 0 \tag{23}$$

where $s \in \{E, RA\}$ denotes which SDF is being used during estimation. The test assets are either just the excess return on the market, or the excess return on the market and on the HML and SMB portfolios. The instruments are time $t$ consumption growth, the job loss rate and the price-dividend ratio. The motivation behind the choice of these instruments is that they are informative about the state of the economy and help to forecast returns. Two-step GMM is used with an identity weighting matrix in the first stage and a Newey and West (1987) weighting matrix with four lags in the second stage. I also consider one-step GMM in appendix B which yields less precise but similar estimates. Since there are GK moment equations and a single parameter is being estimated, the model has GK - 1 overidentifying restrictions. Therefore, the value of Hansen's J-statistic for the test of overidentifying restrictions is also reported.

Table 1 presents results for the GMM estimation of the CCAPM and CCAPM-JL. Replicating previous results in the literature (Mehra and Prescott (1985), Hansen and Singleton (1982), Savov (2011) and many others), the estimate of $\gamma$ in the CCAPM is implausibly high for both sets of test assets, taking a value of 92 when just the excess market return is used and 146 when the HML and SMB excess returns are added. These estimates far exceed the range of what is generally considered palatable given evidence from other economic settings. When the CCAPM-JL SDF is used in the GMM estimation, the estimates of $\gamma$ are far more reasonable at around 12 in the two cases— an order of magnitude lower. While at the high end of microeconomic estimates, this is consistent with the mean risk aversion of 12.11 estimated in Barsky et al. (1997).

Hansen's J test fails to reject the CCAPM-JL at standard significance levels in both cases and, consequently, the null hypothesis that the model successfully prices the entire array of test assets is not rejected. In contrast, this is not the case for the CCAPM as Hansen's J test rejects the model in the case with all three test assets. I conclude that the CCAPM-JL can simultaneously explain the equity, value, and size premia with a reasonable degree of risk aversion whereas the CCAPM cannot.

**Table 1:** GMM Estimation

| Panel A: Excess Market Return | | |
|---|---|---|
| Model | CCAPM-JL | CCAPM |
| $\gamma$ | 11.98 | 92.05 |
| | (4.15) | (38.33) |
| J | 1.73 | 0.88 |
| | [0.63] | [0.83] |
| **Panel B: Excess Market, HML and SMB Returns** | | |
| Model | CCAPM-JL | CCAPM |
| $\gamma$ | 12.02 | 146.11 |
| | (2.32) | (33.76) |
| J | 2.67 | 20.73 |
| | [0.99] | [0.04] |

Note: The table reports GMM estimates of the moment condition $\mathbb{E}\left[M^s_{t,t+1}R^{e,j}_{t+1} \otimes Z_t\right] = 0$,where the SDF is either given by $M^E_{t,t+1}$ or $M^{RA}_{t,t+1}$. $R^{e,j}_{t+1}$ is a vector of excess returns which consists of either just the excess return on the market portfolio (panel A) or the excess return on the market portfolio and the excess return on the HML and SMB portfolios (panel B). $Z_t$ is a vector of instruments containing consumption growth, the job loss rate and the price-dividend ratio. The market return is defined as the return on the CRSP value-weighted portfolio. The sample period is 1967:I-2012:IV. Estimation is by two-step GMM with an identity matrix in the first-stage and a Newey-West weighting matrix in the second stage. Round brackets denote standard errors for the parameter estimate, while square brackets denote p-values for a test of the null hypothesis.

### 3.4 Analytical Insights for the Equity Premium

To gain additional intuition for the ability of the CCAPM-JL to match the equity premium with a reasonable degree of risk aversion, consider the following approximate condition:[20]

$$\mathbb{E}(R^{e,j}_{t+1}) \approx -\operatorname{Cov}(M^s_{t,t+1}, R^{e,j}_{t+1}) \tag{24}$$

The unconditional risk premium of an asset is therefore dependent on the strength of the covariance of its returns with the SDF. As shown in appendix C, the CCAPM-JL SDF can be approximated with a first-order Taylor expansion around $\frac{C^E_{t+1}}{C^E_t} = 1, \rho_{t+1} = \rho_t$ as:[21]

$$M^E_{t,t+1} \approx \phi_0 + \phi_\rho \rho_{t+1} + \phi_c \Delta C^E_{t+1} + \phi_{\rho,c}\rho_t \Delta C^E_{t+1} \tag{25}$$

[20]This holds exactly in continous time, or can be derived in discrete time under the common assumption that returns are lognormal. We have that $1 = \mathbb{E}M^s_{t,t+1}R^j_{t+1}$ which can be manipulated to obtain an expression for the geometric risk premium $\frac{\mathbb{E}(R^j_{t+1})}{\mathbb{E}(R^f_t)} = 1 - \operatorname{Cov}(M^s_{t,t+1}, R^j_{t+1})$. Taking logs of both sides and using the $\log(1+x) \approx x$ approximation, assuming $\frac{\sigma^2_j}{2}$ is not large, gives the above expression.

[21]In simulations I find that this approximation is extremely accurate.

with:

$$\phi_0 = \beta$$
$$\phi_\rho = \beta\left(\chi_U^{-\gamma} - 1\right)$$
$$\phi_c = -\beta\gamma$$
$$\phi_{\rho,c} = -\beta\gamma\left(\chi_U^{-\gamma} - 1\right)$$

And with $\Delta C^E_{t+1} = \frac{C^E_{t+1} - C^E_t}{C^E_t} \approx \Delta C_{t+1}$ given that the shares $\alpha_{j,t}$ are not very volatile empirically. When $\chi_U < 1$ and $\gamma > 0$, we have that $\phi_0, \phi_\rho > 0$ and $\phi_c, \phi_{\rho,c} < 0$. The CCAPM is a special case of the CCAPM-JL with $\chi_U = 1$ and so $\phi_\rho = \phi_{\rho,c} = 0$, which implies:

$$M^{RA}_{t,t+1} \approx \phi_0 + \phi_c \Delta C_{t+1} \tag{26}$$

This delivers a key insight: the CCAPM-JL can be represented as a conditional version of the CCAPM, where the conditioning factor in the SDF is the job loss rate. The linearised SDF can be expressed as $M^E_{t,t+1} = a_{t+1} + b_t \Delta C^E_{t+1}$, where $a_{t+1} = \phi_0 + \phi_\rho \rho_{t+1}$ and $b_t = \phi_c + \phi_{\rho,c}\rho_t$. This is almost exactly the type of conditional version of the CCAPM considered by Lettau and Ludvigson (2001).[22]

Plugging in the approximated SDF for the CCAPM into the asset pricing condition for the excess return gives:

$$\mathbb{E}(R^{e,j}_{t+1}) \approx \gamma \operatorname{Cov}\left(\Delta C_{t+1}, R^{e,j}_{t+1}\right) \tag{27}$$

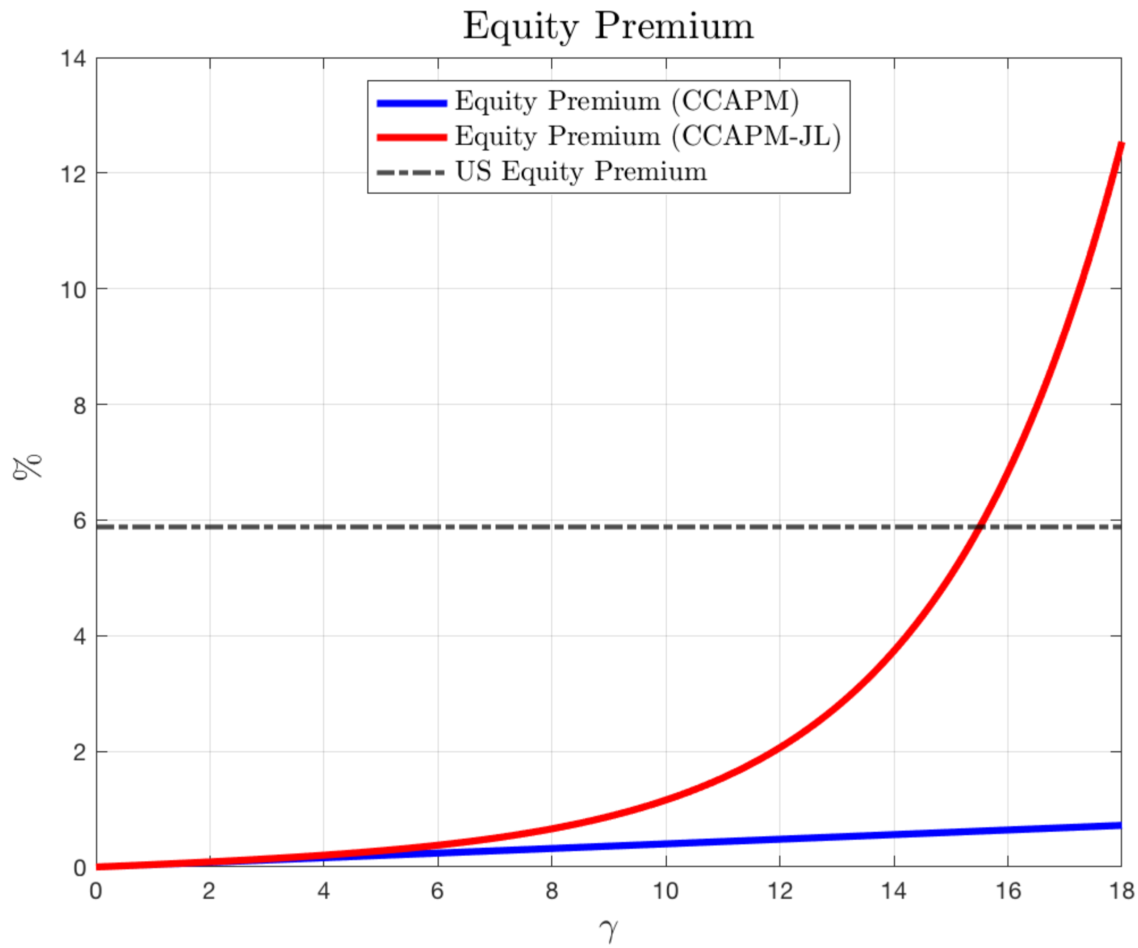
Plugging the approximated CCAPM-JL SDF gives:

$$\mathbb{E}(R^{e,j}_{t+1}) \approx \gamma \operatorname{Cov}(\Delta C_{t+1}, R^{e,j}_{t+1}) + (1 - \chi_U^{-\gamma}) \operatorname{Cov}(\rho_{t+1}, R^{e,j}_{t+1}) + \gamma\left(\chi_U^{-\gamma} - 1\right) \operatorname{Cov}\left(\rho_t \Delta C_{t+1}, R^{e,j}_{t+1}\right) \tag{28}$$

In the CCAPM, the unconditional expected excess return of asset j is (approximately) the product of two terms: the risk aversion parameter $\gamma$ and the unconditional covariance between asset j's returns and consumption growth. Since the latter term is small empirically for the market portfolio, the only way the model can match the level of the equity premium is through a very large value of $\gamma$. In the CCAPM-JL two additional covariance terms are present: the covariance between returns and the job loss rate, and the covariance between returns and the interaction of the (lagged) job loss rate and consumption growth.

[22]See Equation (7) in their paper. The only difference is that the 'intercept' term in the SDF is $a_{t+1}$ rather than $a_t$.

**Figure 3:** Equity Premium Under Approximation of SDF




The figure displays the annualised equity premium in the CCAPM-JL (red line) and the CCAPM (blue line) as a function of $\gamma$. The dotted black line represents the average annual excess market return in US data.

Therefore, expected returns are also determined by the consumption growth covariance *conditional* on the job loss rate. As shown previously, the job loss rate acts as a state variable that positively drives time-varying risk premia in the CCAPM-JL. This third term in the expression above therefore captures the fact that agents dislike assets which are riskier (in the sense that they covary more strongly with consumption growth) during times when risk premia are heightened i.e. when the risk of job loss is high. The risk prices on the two additional terms in the model are both increasing, convex functions of $\gamma$.[23]

I set all moments equal to their empirical analogues (where the CRSP value-weighted return is used as the market return), calibrate $\chi_U = 0.77$, and then plot the implied equity premium from the CCAPM and CCAPM-JL. This is presented in Figure 3.

[23] Note that in this section the risk price is the price per unit of covariance rather than per unit of $\beta$ as would be the case in a beta representation.

In order to match the large equity premium in the data, the CCAPM must feature an incredibly large value of $\gamma$ in the realm of 180, such that the weak covariance between equity returns and consumption growth is scaled by a sufficiently large risk price. The CCAPM-JL, however, is able to match the equity premium without a high risk aversion parameter due to the additional pair of covariance terms. In fact, the quantitatively important term is $\text{Cov}\left(\rho_t \Delta C_{t+1}, R^{e,M}_{t+1}\right)$, since this is positive and statistically significant in the data, while $\text{Cov}\left(\rho_{t+1}, R^{e,M}_{t+1}\right)$ is close to zero and insignificant for the market return. The risk price for this cross term is given by $\gamma\left(\chi_U^{-\gamma} - 1\right) > 0$, which is a convex function of $\gamma$, and as a result, is very large for moderate values of risk aversion as long as $\chi_U$ is significantly below 1 i.e. if consumption loss upon unemployment is substantial quantitatively. Intuitively, the lower is $\chi_U$, the more costly job loss is in terms of consumption which heightens the increase in risk premia.[24]

To consider the importance of fluctuations in the job loss rate over time, I use the fact that if the job loss rate were constant at its mean value, it would be the case that $\text{Cov}\left(\rho_t \Delta C_{t+1}, R^{e,M}_{t+1}\right) = \mathbb{E}(\rho_t)\,\text{Cov}\left(\Delta C_{t+1}, R^{e,M}_{t+1}\right)$. In this case, the presence of job loss risk, despite not varying over time, still increases the equity premium via a background risk effect à la Gollier and Pratt (1996) which effectively makes the investor more risk averse. Quantitatively, $\text{Cov}\left(\rho_t \Delta C_{t+1}, R^{e,M}_{t+1}\right)$ exceeds $\mathbb{E}(\rho_t)\,\text{Cov}\left(\Delta C_{t+1}, R^{e,M}_{t+1}\right)$ by around 50%, illustrating the importance of volatility in the job loss rate which operates via an additional coskewness channel. This mechanism captures the fact that investors dislike assets such as the market portfolio which tend to have poor returns during times when job loss risk is high and aggregate consumption growth is low simultaneously.[25]

### 3.4.1 A Parametric Model

The previous approach to finding the level of $\gamma$ which matches the equity premium in both models can be considered semi-parametric since the full distributions of $(\Delta C_{t+1}, \rho_{t+1}, R^M_{t+1})$ were not specified. I now

[24] GMM estimation of (28) for the CCAPM-JL, which accounts for estimation uncertainty around the covariance terms, yields similar results to the estimation using the full non-linear SDF. I find a point estimate of $\gamma = 20.27$ with 95% confidence interval [7.96, 32.57]. Applying this to (29) for the CCAPM yields a point estimate of $\gamma = 171.97$ with 95% confidence interval [-74.10, 418.05]. Repeating this with the HML and SMB returns added yields a point estimate of $\gamma = 12.56$ with 95% confidence interval [10.18, 14.93] for the CCAPM-JL. For the CCAPM this produces a point estimate of $\gamma = 318.99$ with 95% confidence interval [-51.73, 689.71].

[25] The two additional terms which comprise $\text{Cov}\left(\rho_t \Delta C_{t+1}, R^{e,M}_{t+1}\right)$ are $\mathbb{E}(\Delta C_{t+1})\,\text{Cov}\left(\rho_t, R^{e,M}_{t+1}\right)$ and $\mathbb{E}\left[\left(\rho_t - \mathbb{E}(\rho_t)\right)\left(\Delta C_{t+1} - \mathbb{E}(\Delta C_{t+1})\right)\left(R^{e,M}_{t+1} - \mathbb{E}(R^{e,M}_{t+1})\right)\right]$. The first term is unimportant quantitatively. The second term captures the coskewness between the job loss rate, aggregate consumption growth and returns. It is positive in the data since stock returns tend to be low when the job loss rate is high and, at the same time, consumption growth is low.

take a fully parametric approach, specifying the following stylised stochastic processes for these variables:

$$\begin{pmatrix} R^M_{t+1} \\ \rho_{t+1} \\ \Delta C_{t+1} \end{pmatrix} = \begin{pmatrix} \overline{R}^M_{t+1} \\ \overline{\rho}_{t+1} \\ 0 \end{pmatrix} + \begin{pmatrix} \sigma_R \\ -\sigma_\rho \\ \sigma_C \end{pmatrix} \epsilon_{t+1} \tag{29}$$

where $\overline{X}_{t+1} = \mathbb{E}_t X_{t+1}$, $\mathbb{E}_t \epsilon_{t+1} = 0$ and $\mathbb{E}_t \epsilon^2_{t+1} = 1$. The model specifies unexpected deviations of the market return, the job loss rate and consumption growth as a function of a common shock $\epsilon_{t+1}$. In appendix C I demonstrate that the unconditional equity premium in the CCAPM-JL can be expressed as:

$$\mathbb{E}(R^{e,M}_{t+1}) \approx \gamma \left[1 + \overline{\rho}(\chi_U^{-\gamma} - 1)\right] \sigma_C \sigma_R + (\chi_U^{-\gamma} - 1)\sigma_\rho \sigma_R \tag{30}$$

where $\overline{\rho} = \mathbb{E}\rho_{t+1}$.

The equity premium in the CCAPM is obtained as previously by setting $\overline{\rho} = 0$ or $\chi_U = 1$:

$$\mathbb{E}(R^{e,M}_{t+1}) \approx \gamma \sigma_C \sigma_R \tag{31}$$

I estimate $\sigma_R$, $\sigma_\rho$ and $\sigma_C$ as described in appendix C and then find the $\gamma$ which matches the equity premium in each model. This is depicted graphically in Figure 4. The CCAPM-JL matches the data for $\gamma \approx 9$ while the CCAPM requires less risk aversion than before, but still requires $\gamma \approx 40$.

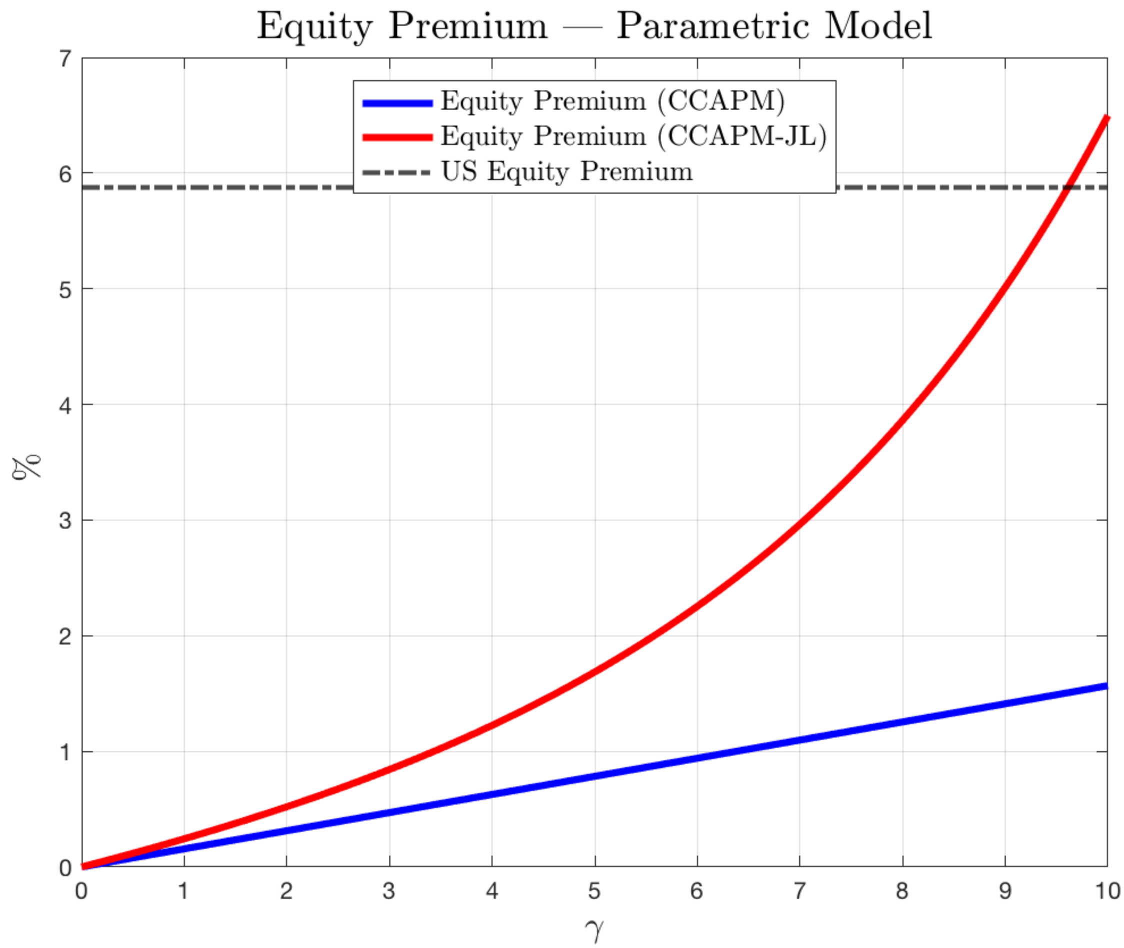
### 3.5 Hansen-Jagannathan Bounds

Another way of seeing things is through the lens of Hansen and Jagannathan (1991) (HJ) bounds, which state that the ratio of an SDF's volatility to its mean must be no less than the maximum Sharpe ratio attained by any portfolio:

$$\frac{\sigma(M^s_{t,t+1})}{\mathbb{E}(M^s_{t,t+1})} \geq \frac{\left|\mathbb{E}\left(R^e_{t+1}\right)\right|}{\sigma\left(R^e_{t+1}\right)} \tag{32}$$

Using the linearised expressions derived above for the SDF in the CCAPM and CCAPM-JL allows for the evaluation of the left-hand side of the HJ bound as a function of $\gamma$, setting the moments equal to their empirical analogues once again. We can compare this against the Sharpe ratio of the US market porfolio at the quarterly frequency, which is approximately 0.2 in the sample period. Figure 5 plots the results of this exercise. The CCAPM cannot satisfy the HJ bound for any reasonable value of $\gamma$, as a very high

**Figure 4:** Equity Premium In Parametric Model




The figure displays the annualised equity premium in the CCAPM-JL (red line) and the CCAPM (blue line) as a function of $\gamma$ in the parametric model. The dotted black line represents the average annual excess market return in US data.

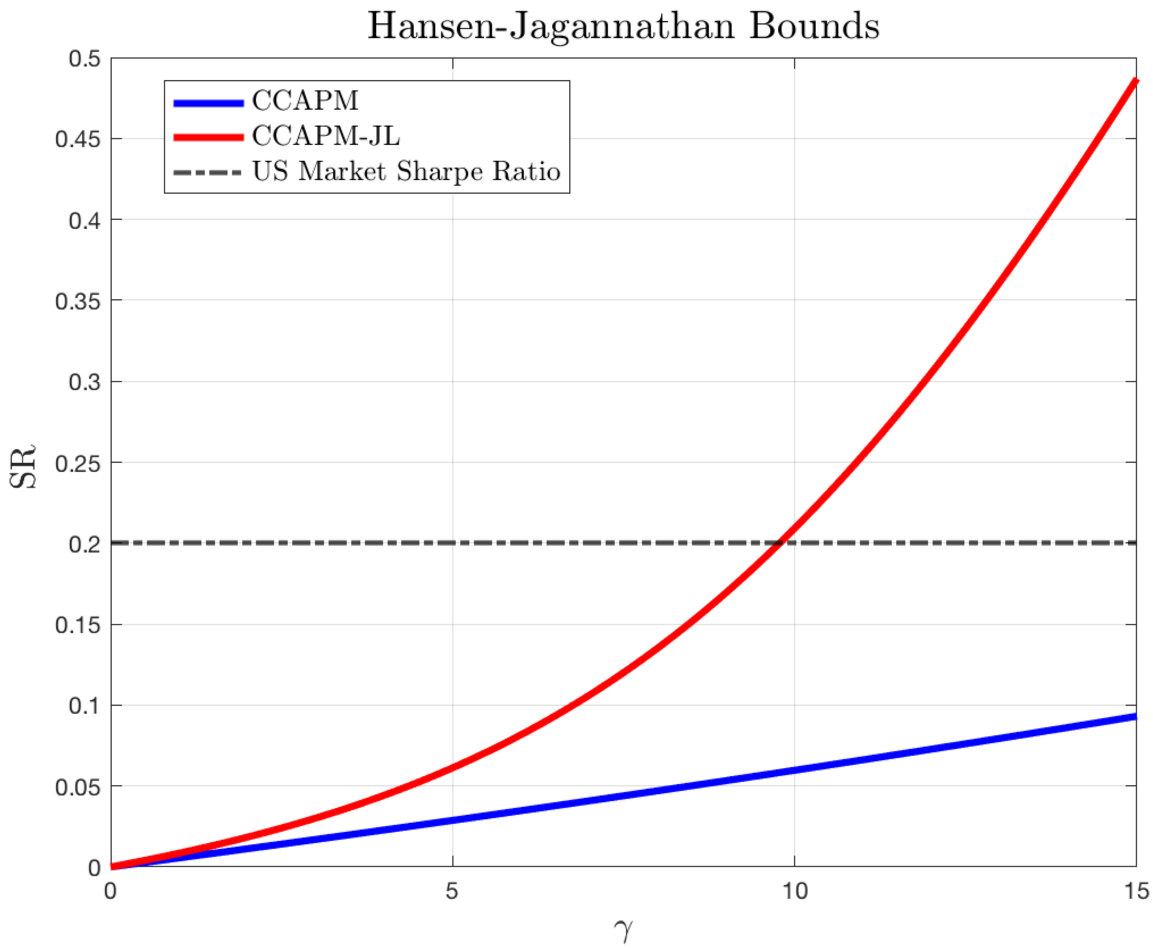
level of risk aversion is necessary to generate sufficient volatility in the SDF due to how smooth aggregate consumption is empirically.[26] The introduction of job loss risk in the CCAPM-JL introduces a natural source of additional volatility in the SDF under incomplete insurance, without having to resort to large values of $\gamma$. Instead, the bounds can be satisfied for $\gamma \approx 10$.

### 3.6 Time-Variation in the Consumption Risk Price

A fundamental issue in the CCAPM is that the model can only generate a large price of risk for consumption growth via a high risk aversion parameter, and this risk price is constant over time. This leads to large pricing errors typically seen when the model is tested empirically as was the case in Table 1. On the

[26] A $\gamma > 30$ is needed to satisfy the Hansen-Jagannathan bound in the CCAPM. As will be shown subsequently, this value would lead to an implausibly high risk-free rate.

**Figure 5:** Hansen-Jagannathan Bounds in CCAPM and CCAPM-JL




Note: The figure displays the left-hand side of the Hansen and Jagannathan (1991) bound in the CCAPM (blue line) and CCAPM-JL (red lines) as a function of $\gamma$. The black line is the emprical analogue of the right-hand side of the bound, plotting the Sharpe ratio of the US market portfolio.

contrary, the risk price for consumption growth in the conditional representation of the CCAPM-JL is given by:

$$\frac{\partial \mathbb{E}_t R^e_{t+1}}{\partial \operatorname{Cov_t}(\Delta C_{t+1}, R_{t+1})} = \beta\gamma \left(1 + \left(\chi_U^{-\gamma} - 1\right) \rho_t\right) \tag{33}$$

And so the consumption risk price is time-varying and depends on the risk aversion parameter, but also the unemployment consumption loss parameter, $\chi_U$, as well as the job loss rate $\rho_t$. The consumption risk price can be thought of as the effective level of risk aversion. To see this, consider the SDF from the CCAPM where the effective level of risk aversion is time-varying and is given by $\gamma_t$ in period t:

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma_t} \tag{34}$$

Setting $\beta = 1$ and linearising this around $\frac{C_{t+1}}{C_t} = 1$ gives:

$$M_{t,t+1} = 1 - \gamma_t \Delta C_{t+1} \tag{35}$$

The risk price for consumption growth is then given by $\gamma_t$. In this sense, an increase in the job loss rate in the CCAPM-JL can be thought of as a rise in the effective level of risk aversion with $\gamma_t = \gamma\left(1 + \left(\chi_U^{-\gamma} - 1\right)\rho_t\right)$. This produces a similar dynamic to the habits model of Campbell and Cochrane (1999). In this model $\gamma_t = \gamma(1 + \lambda(s_t))$, where $\lambda(s_t)$ is the habits sensitivity function that depends on the state variable $s_t$. A drop in consumption below its trend level in the model produces an increase in effective risk aversion, while in the CCAPM-JL this occurs when the job loss rate rises. I illustrate this in Figure 6 for $\chi_U = 0.77$ across a range of $\gamma$ values. A rise in the job loss rate causes a dramatic rise in effective risk aversion for values of $\gamma$ above 10, making the consumption risk price, and consequently the equity premium, countercyclical in line with this empirical phenomenon.

### 3.7 Return Predictability

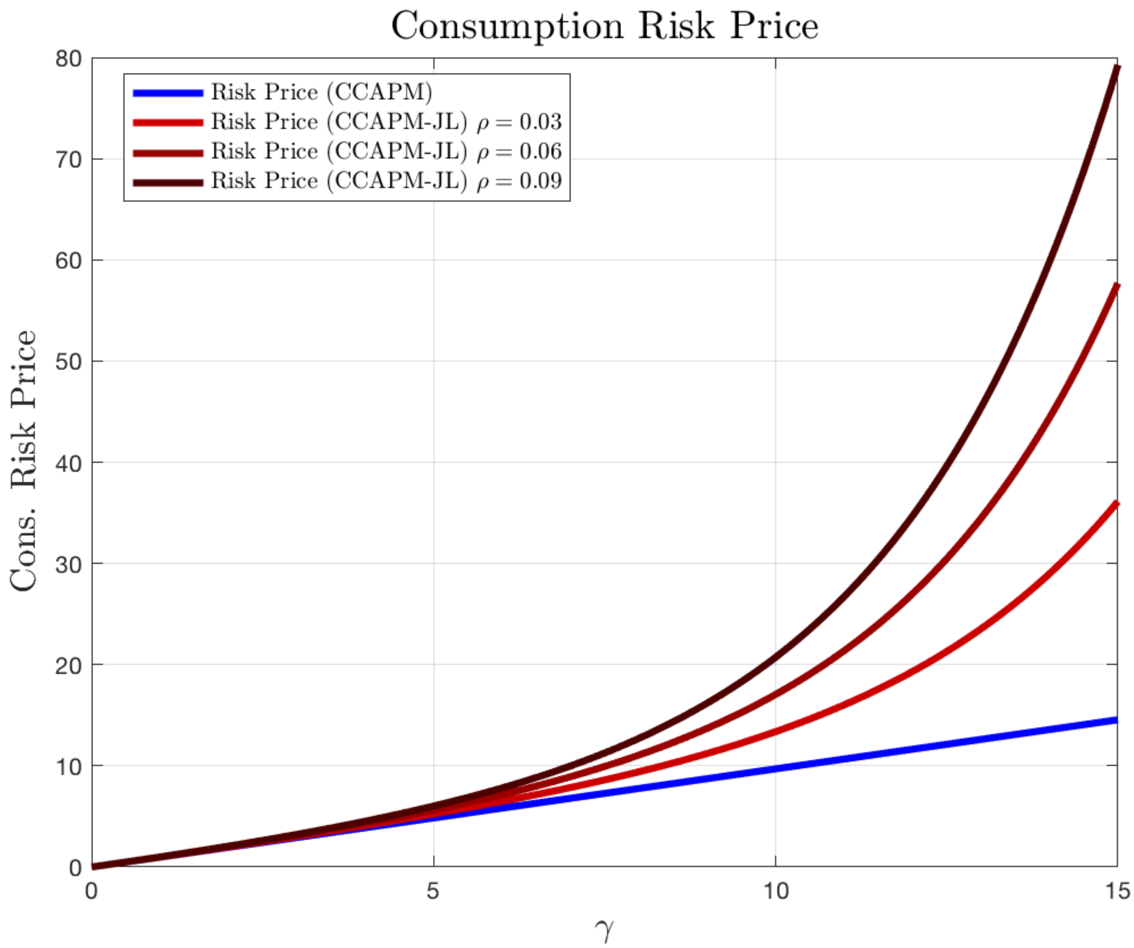
As explained in heuristic terms previously, a key part of the model's mechanism is that increases in the job loss rate should raise risk premia. To see this in formal terms, using the linearised CCAPM-JL SDF presented previously, expected excess returns can be expressed as:

$$\mathbb{E}_t R^e_{t+1} \approx -\phi_\rho \operatorname{Cov_t}(\rho_{t+1}, R_{t+1}) - (\phi_c + \phi_{\rho,c}\rho_t) \operatorname{Cov_t}(\Delta C^E_{t+1}, R_{t+1}) \tag{36}$$

Recalling that $\phi_0, \phi_\rho > 0$ and $\phi_c, \phi_{\rho,c} < 0$ under risk aversion and a positive consumption loss upon unemployment, it is evident that an increase in $\rho_t$ will increase the expected excess return (the risk premium) because it interacts with $\operatorname{Cov_t}(\Delta C^E_{t+1}, R_{t,t+1})$, which is also assumed to be positive in all t, with a positive coefficient. The sign of $\frac{d\mathbb{E}_t R^e_{t+1}}{d\rho_t}$ is thus clearly positive, meaning that a testable prediction of the model is that an increase in $\rho_t$ should be associated with an increase in future excess returns in the data. Note that this is a distinct empirical test of a prediction from the model, and does not necessarily follow from the previous results in this section.

The predictive power of the job loss rate for returns is investigated for two different measures of the excess return on US stocks. The first is the return on the CRSP value-weighted index of U.S. stocks and the second is the return on the Standard and Poor's composite index (S&P 500). For both indexes, the excess

**Figure 6:** Consumption Risk Price in CCAPM and CCAPM-JL




Note: The figure displays the consumption growth risk price in the CCAPM (blue line) and CCAPM-JL (red lines) as a function of $\gamma$. For the CCAPM-JL, three different values of $\rho_t$ are considered: 0.03, 0.06, 0.09.

return is computed by subtracting the 30-day Treasury bill return from the respective market return. Data is quarterly, and the sample period is 1967:I - 2012:IV. The return predictive regression takes the standard form:

$$R^e_{t,t+h} = \alpha_h + \beta_h \rho_t + \epsilon_{t,t+h} \tag{37}$$

Where $R^e_{t,t+h}$ is the h-quarter ahead continuously compounded excess return on the stock market. The return predictive test involves testing for statistical significance of the $\beta_h$ coefficients. Since a strong theoretical grounding for the sign of these coefficients ($\beta_h > 0$) has been established, I follow Inoue and Kilian (2005) and perform one-sided hypothesis tests. To correct for serial correlation in the residuals, Newey and West (1987) heteroskedasticity-and-autocorrelation-robust t-statistics are computed with the lag length equal to $h$ in each regression.

**Table 2:** Baseline Excess Return Prediction Regressions

| | Forecast Horizon h | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Index | 1 | 2 | 3 | 4 | 8 | 12 | 16 | 20 |
| | 0.57 | 1.18 | 1.79 | 2.28 | 3.18 | 4.20 | 6.14 | 9.00 |
| CRSP | (1.61)* | (1.90)** | (2.24)** | (2.39)*** | (1.90)** | (2.07)** | (2.50)*** | (3.21)*** |
| | [1.13] | [2.62] | [4.29] | [5.53] | [5.12] | [5.40] | [8.03] | [11.14] |
| | 0.57 | 1.17 | 1.77 | 2.27 | 3.45 | 4.94 | 7.36 | 10.97 |
| SP500 | (1.74)** | (2.06)** | (2.39)*** | (2.49)*** | (1.99)** | (2.21)** | (2.53)*** | (3.02)*** |
| | [1.36] | [2.92] | [4.62] | [5.92] | [5.94] | [6.75] | [9.56] | [13.28] |

Note: The table reports results from return predictive regressions of the form $r_{t,t+h} = \alpha_h + \beta_h \rho_t + \epsilon_{t,t+h}$, where $r_{t,t+h}$ is the h-quarter ahead excess return and $\rho_t$ is the job loss rate. The excess return on the CRSP and S&P500 is used. For each regression, the table reports the OLS coefficient estimate, Newey-West adjusted t-statistic in parentheses and the $R^2$ value (multiplied by 100) in square brackets. *, **, and *** denotes statistical significance at the 10%, 5% and 1% levels respectively.

Table 2 presents results for these regressions for the two measures of the excess stock market return. Coefficient estimates, corresponding Newey-West t-statistics and adjusted $R^2$ values are reported for a range of horizons. The first row of the table represents the baseline case with excess returns on the CRSP index, and results show that the estimated coefficient on the job loss rate is positive and statistically significant at the 5% level at all horizons up to 5 years apart from the one-quarter horizon which is significant at the 10% level. The second row shows that results are extremely similar for the excess return on the S&P 500. Using the one-quarter ahead coefficient estimate, a one standard deviation increase in the job loss rate results in an increase in the expected excess return of around four and a half percentage points when annualised for the CRSP[27]. The effect is therefore economically significant also and expected returns rise during times of high job loss risk consistent with the mechanism previously outlined. The predictive power extends to both the medium and long horizons and increases monotonically in $h$ for both the CRSP and S&P 500 and at the 5-year horizon, around 10% and 13% of the variation in expected excess returns is explained by the job loss rate for each respective index.

The estimated $\beta_1$ coefficient can also be used to recover the implied value of $\gamma$ by re-writing the excess return condition from the CCAPM-JL as:

$$R^e_{t+1} \approx \alpha_1 + \beta_1 \rho_t + \epsilon_{t+1} \tag{38}$$

[27]For the S&P 500 the increase in expected return is very similar at around six percentage points.

where I assume conditional moments are constant. Then:

$$\alpha_1 = -\phi_\rho \operatorname{Cov}(\rho_{t+1}, R_{t+1}) - \phi_c \operatorname{Cov}(\Delta C^E_{t+1}, R_{t+1})$$

$$\beta_1 = -\phi_{\rho,c} \operatorname{Cov}(\Delta C^E_{t+1}, R_{t+1})$$

$$\epsilon_{t+1} = R^e_{t+1} - \mathbb{E}_t R^e_{t+1}$$

Setting $\chi_U = 0.77$ and $\operatorname{Cov}(\Delta C^E_{t+1}, R_{t+1})$ equal to its sample analogue, the estimate of $\beta_1$ then implies $\gamma \approx 17$, in line with previous results. The results of the return predictability regressions are therefore quantitatively as well as qualitatively consistent with the CCAPM-JL. Several robustness tests for the return predictability results can be found in the appendix, including the use of alternative sample periods and out-of-sample tests.

### 3.8 The Risk-Free Rate Puzzle

Even if one is willing to believe that the extreme risk aversion needed to match the equity premium in the CCAPM is not unreasonable, the model generates the risk-free rate puzzle of Weil (1989). The risk-free rate implied for high values of $\gamma$ ends up being implausibly high. I now show that this same issue does not occur in the CCAPM-JL.

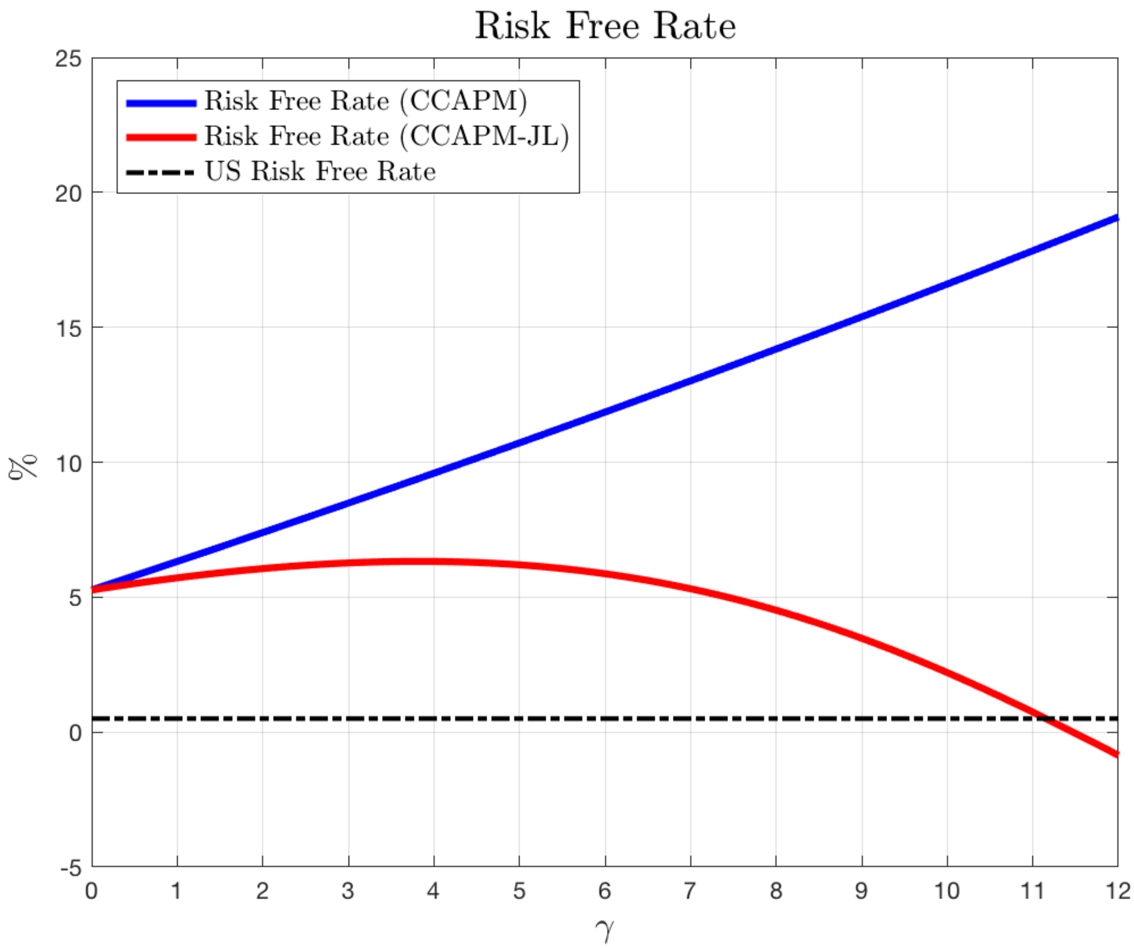
I take a second-order Taylor approximation of the SDF in the CCAPM and CCAPM-JL around the same points as previously, and then take unconditional expectations before evaluating the risk-free rate as:[28]

$$R^f = \frac{1}{\mathbb{E}(M^s_{t,t+1})} \tag{39}$$

The model frequency is quarterly and I set each of the moments that appear in the linearised expectations of the SDFs to their sample analogues, and set $\beta = 0.95$ and $\chi_U = 0.77$. Figure 7 plots the risk-free rate for each of the two models as a function of the risk aversion parameter, $\gamma$. The risk-free rate puzzle of Weil (1989) clearly emerges in the CCAPM, as the high risk aversion parameter typically necessary to generate a sufficient equity premium leads to very high levels of the expected risk-free rate. In the CCAPM-JL,

[28] I use a second-order Taylor expansion of the SDF since this is standard in the literature when linearising the SDF in the CCAPM and evaluating the implied risk-free rate. This allows for what is typically called the precautionary saving effect in this model, where greater volatility of consumption growth reduces the risk-free rate. This term is small quantitatively however due to the low volatility of consumption growth. In both the CCAPM and CCAPM-JL the results are very similar when a first-order Taylor expansion of the SDF is used to evaluate the unconditional expectation of the risk-free rate.

**Figure 7:** Risk Free Rate in Calibrated Model




Note: The figure displays the annualised risk-free rate in the CCAPM-JL (red line) and the CCAPM (blue line) as a function of $\gamma$. The dotted black line represents the annual mean risk-free rate in US data over the 1967:I - 2012:IV sample period.

however, the puzzle does not emerge as the threat of job loss spurs an additional precautionary motive which places downward pressure on the risk-free rate as the demand for this safe asset increases. This ultimately results in the risk-free rate being a concave function of $\gamma$, and with this calibration the model matches the empirical risk-free rate with $\gamma \approx 11$, which is close to the GMM estimate. The CCAPM woud only be able to match this with a negative rate of time preference for positive risk aversion, i.e. a $\beta > 1$.

### 3.9 Robustness Tests and Model Extensions

Appendix B contains a number of robustness tests and model extensions. These include i) An alternative measure of the job loss rate; ii) Allowing job finding upon job loss; iii) Allowing for the consumption loss upon unemployment ($\chi_U$) to be time-varying; iv) Incorporating a non-degenerate distribution of consumption loss upon unemployment. When taken to the data, these all lead to estimated values of $\gamma$ that are similar to the baseline case.

## 4 The Cross-Section of Returns

I now explore whether the CCAPM-JL can explain the cross-section of excess returns more effectively than the CCAPM and other consumption-based asset pricing models. The previous section asked the model to satisfyingly answer the question: why is the excess return on stocks so high? This section asks the model to answer a different question: within an asset class such as stocks, why do certain assets earn a high expected return relative to others?

As shown previously, a first-order Taylor expansion of the SDF in the CCAPM-JL demonstrates that it can be approximated as a :

$$M^E_{t,t+1} \approx \phi_0 + \phi_\rho \rho_{t+1} + \phi_c \Delta C_{t+1} + \phi_{\rho,c} \rho_t \Delta C_{t+1} \tag{40}$$

Therefore it permits a linear factor representation where the three factors are $\rho_{t+1}$, $\Delta C_{t+1}$ and $\rho_t \Delta C_{t+1}$.[29] Akin to Lettau and Ludvigson (2001) who formulate something similar in more ad hoc terms, the model essentially produces a conditional CCAPM, with the job loss rate acting as the scaling factor for consumption growth. The other linear factor models I compare against are other consumption-based models, namely the CCAPM, where the single factor is consumption growth, $\Delta C_{t+1}$, the CAPM, where the single factor is the market excess return, and finally the three factor Lettau and Ludvigson (2001) model, where the three factors are CAY, consumption growth and consumption growth scaled by CAY.

In order to test these factor models, I employ the standard method of Fama and MacBeth (1973) that is typically used in the estimation of linear factor models, and which parsimoniously accounts for cross-sectional correlation present in the errors. The same sample period is used as in the previous section, and I consider the 25 Fama-french portfolios as test assets. The first step of the Fama-Macbeth estimation involves running J time series regressions of the excess returns on the set of F factors, where J is the total

[29] In this expression, $\Delta C_{t+1}$ replaces $\Delta C^E_{t+1}$ since the two have a correlation very close to 1 empirically.

number of portfolios, in order to obtain the loadings on each factor:

$$R_t^{e,j} = a_j + \sum_{f=1}^{F} \beta_j^f f_t + \epsilon_t^j \qquad j = 1, ..., J \tag{41}$$

Following this, T cross-sectional regressions are estimated period-by-period from $t = 1, ..., T$ using the estimated factor loadings from the previous stage:

$$R_t^e = \lambda_t \hat{\beta} + \alpha_t \qquad t = 1, ..., T \tag{42}$$

Where $R_t^e$ is a $1 \times J$ vector of excess returns, $\lambda_t$ is a $1 \times F$ vector of risk prices, $\hat{\beta}$ is a $F \times J$ vector of betas, and $\alpha_t$ is a $1 \times J$ vector of pricing errors. The estimated risk factor prices and pricing errors are then given respectively by:

$$\hat{\lambda}^f = \frac{1}{T} \sum_{t=1}^{T} \lambda_t^f \qquad \hat{\alpha}_j = \frac{1}{T} \sum_{t=1}^{T} \alpha_t^j \tag{43}$$

Standard errors of the slope coefficients are then calculated from this time series. A constant is not included in the second-stage equation as this can lead to inaccurately estimated risk factor prices when the betas lack variation, while also implying that a risk-free asset would have an excess return over itself.[30] Kleibergen and Zhan (2020) highlight that when there is insufficient variation in the betas, the risk factor prices are not identified. To address this, I implement the method they propose which tests whether the matrix of betas has full rank. I find that the null hypothesis – that the beta matrix does not have full rank – can comfortably be rejected at the 1% level for every model considered forthwith, and so conclude that the risk factor prices are all likely to be identified.

Table 3 presents results of these Fama-Macbeth regressions. I report the estimates of $\lambda$ and the root mean squared pricing errors (RMSPE), which are used to assess the fit of the model. The first row corresponds to the CCAPM, where the single factor is aggregate consumption growth. The slope coefficient is positive and statistically significant, in line with the model, but is large in order to match the high expected excess return on several test assets given their weak correlation with consumption growth. The second row corresponds to the CCAPM-JL. The pricing errors are reduced by around 15% compared to the CCAPM, representing a large improvement in the model's ability to explain the cross-section of returns. The risk price on consumption growth is reduced by 2/3 also, mirroring results on the risk aversion parameter in the GMM estimation. Tellingly, the risk price on the interaction term between the job loss rate and

[30] See Savov (2011) for elaboration on this point.

**Table 3:** Fama-Macbeth Regressions

| **25 Fama-French Portfolios** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Model** | $\lambda_\rho$ | $\lambda_{\Delta C}$ | $\lambda_{\rho,\Delta C}$ | $\lambda_{CAY}$ | $\lambda_{CAY,\Delta C}$ | $\lambda_{R^M}$ | RMSPE |
| CCAPM | 0.006 | | | | | | 0.81 |
| | (2.63) | | | | | | [0.00] |
| CCAPM-JL | 0.035 | 0.002 | 0.001 | | | | 0.70 |
| | (3.13) | (0.99) | (2.39) | | | | [0.00] |
| LL (2001) | | 0.006 | | 0.007 | 0.000 | | 0.77 |
| | | (2.76) | | (2.58) | (2.45) | | [0.00] |
| CAPM | | | | | | 0.018 | 0.81 |
| | | | | | | (2.61) | [0.00] |

Note: The table reports results of Fama and Macbeth (1973) regressions for the 25 Fama-French portfolios. See text for full estimation details. RMPSE denotes the root mean squared pricing error. Round brackets denote t-statistics for the factor risk price parameter estimates, while square brackets denote p-values for a test of the null hypothesis that all pricing errors are zero. The sample period is 1967:I-2012:IV.

consumption growth, $\lambda_{\rho,\Delta C}$, is positive and statistically significant. This factor thus remains pivotal to the model's success. The risk price on the job loss rate factor, $\lambda_\rho$, is also statistically significant. The biggest improvements in terms of absolute pricing errors for the CCAPM-JL vs the CCAPM are found for the portfolios with lower size and value i.e. the small growth portfolios. The CCAPM tends to systematically undepredict the expected excess returns on the portfolios, with 18 of the 25 predicted values lying below the actual values. This is due to the weak covariances between consumption growth and returns. This deficiency is not present in the CCAPM-JL, as pricing errors are evenly split in terms of their sign.

The next row considers the CAPM, where the single factor is the excess market return, which has a positive and statistically significant slope coefficient, in line with what the model would predict for this parameter. The model has pricing errors that are higher than the CCAPM-JL but around the same as the CCAPM. The last row contains results for the Lettau and Ludvigson (2001) model. The slope coefficient on consumption growth and CAY are both not statistically significant, while the coefficient on the interaction between the two is significant. The pricing errors are once again higher than those for the CCAPM-JL. I thus conclude that the three-factor model that corresponds to the CCAPM-JL performs very well in explaining the cross-section of returns, with lower pricing errors than the three alternative models.

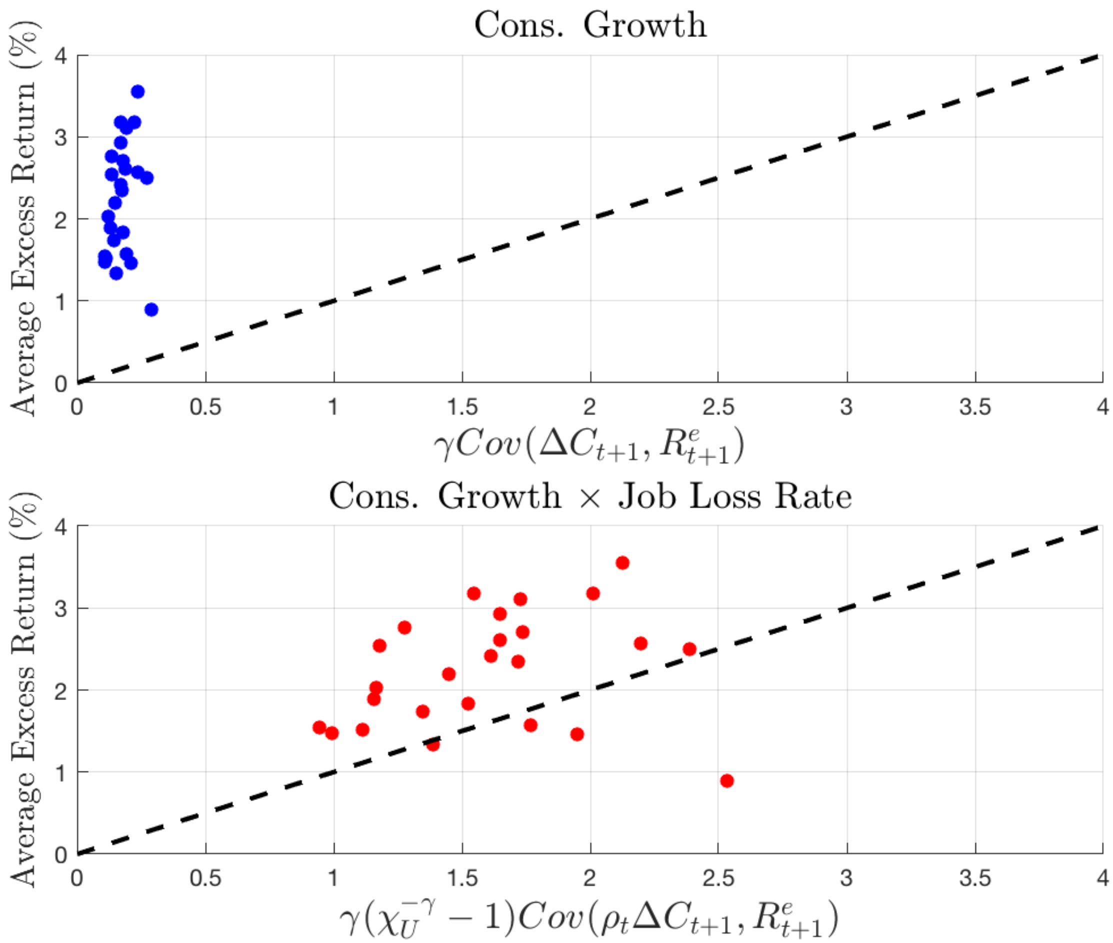
In the CCAPM, an asset's expected return is determined by its unconditional correlation with consumption growth. Since the value and small portfolios are weakly correlated with consumption growth unconditionally, the CCAPM struggles to explain their high expected returns. In the CCAPM-JL, an asset earns a high expected return if it is more correlated with consumption growth when risk premia are high (when the job loss rate is high) than when they are low (when the job loss rate is low). The value and small portfolios exhibit exactly this strong conditional correlation, explaining why the CCAPM-JL outperforms the CCAPM in this cross-section.

To visualise this further, Figure 8 plots the average excess return of each of the 25 Fama-French portfolios in the sample period against $\gamma \operatorname{Cov}\left(\Delta C_{t+1}, R^{e,j}_{t+1}\right)$ (top panel) and $\gamma(\chi_U^{-\gamma} - 1) \operatorname{Cov}\left(\rho_t \Delta C_{t+1}, R^{e,j}_{t+1}\right)$, setting $\gamma = 17$, $\chi_U = 0.77$ (bottom panel). The first plot illustrates the deficiency of the CCAPM in the cross-section of returns, as $\operatorname{Cov}\left(\Delta C_{t+1}, R^{e,j}_{t+1}\right)$ is very weak for all of the portfolios. This explains why the Fama-MacBeth regressions need to estimate such a large value of $\lambda_{\Delta C}$ to sufficiently scale up the weak covariances. Also, there is essentially no systematic relationship between the average excess return of an asset and how strongly it unconditionally covaries with consumption. For the conditional term which is added in the CCAPM-JL, $\operatorname{Cov}\left(\rho_t \Delta C_{t+1}, R^{e,j}_{t+1}\right)$, this is both quantitatively much larger for the set of assets, and also varies in a systematically positive way with the average excess return on a given asset. As a result, the CCAPM-JL produces lower pricing errors as well as a reduced estimate of the risk price on consumption growth.

## 4.1 Additional Robustness Tests

Appendix D contains several extensions and robustness tests for the cross-sectional results in this section. These include i) Looking at the 30 industry portfolios as an alternative cross-section of returns; ii) Testing the model on a cross-section of fixed income portfolios; iii) Using a different measure of the job loss rate; iv) Allowing for job finding in the model; v) Using GMM in the cross-section; iv) Using the factor-mimicking portfolio approach of Breeden et al. (1989). These all lead to similar results as the benchmark case, with the pricing errors continuing to be lower than the CCAPM and the conditional factor remaining statistically significant.

**Figure 8:** Average Excess Returns vs Scaled Covariances




Note: The figure display the plot of average excess returns for each of the 25 Fama-French portfolios vs its covariance with consumption growth (top panel) and the interaction of the lagged job loss rate and consumption growth (bottom panel). The covariances are scaled by their respective risk prices, $\gamma$ and $\gamma(\chi_U^{-\gamma} - 1)$ with $\gamma = 17$, $\chi_U = 0.77$. The black dotted line is the 45-degree line. See text for full details.

# 5 Relaxing the No-Trade Equilibrium in a Heterogeneous Agent Model

An obvious concern with the results presented so far is that they potentially hinge upon the no-trade equilibrium and the lack of a wealth distribution. To address, this I embed the mechanism within a heterogeneous agent model in the spirit of Krusell and Smith (1997). This yields more richness in the model at the expense of its direct estimability, and so there is a tradeoff between the two. I show that, if anything, this amendment strengthens previous results in the sense that the equity premium can be matched with a lower degree of risk aversion compared to that estimated in section 3.

Households face much the same problem as in section 2, with two assets they can hold: risky capital (a) and a risk-free one-period bond (b). The key difference is that now they are only constrained to not go short in the risky asset and to hold wealth (w) below a point $\nu$. These first of these prevents unrealistic negative positions in capital, while the second serves as a borrowing constraint. Household i's Bellman equation is given by:

$$V(b(i), a(i), \mathbb{1}^E(i), \Omega) = \max_{C(i), b'(i), a'(i)} \left\{ \frac{C(i)^{1-\gamma} - 1}{1-\gamma} + \beta \mathbb{E}\left[ V(b'(i), a'(i), \mathbb{1}^{E'}(i), \Omega') \right] \right\}$$

$$s.t. \qquad C(i) + qb'(i) + a'(i) = \mathbb{1}^E(i)W + [1 - \mathbb{1}^E(i)]\iota W + b(i) + R^K a(i)$$

$$a'(i) \geq 0$$

$$w'(i) \equiv a'(i) + qb'(i) \geq \nu$$

$\mathbb{1}^E$ is the household's idiosyncratic employment status, $\Omega$ is the set of aggregate state variables which includes the wealth distribution, W is the wage they earn if employed, $q$ is the risk-free bond price, and $R^K$ is the gross return on capital. If they are unemployed, they earn a constant fraction $\iota$ of the wage which captures an unemployment benefit or home production. The timing is the same as in the model of section 2, with job loss and job finding occurring at the start of each period. If employed today, the transition probabilities are:

$$P(\mathbb{1}^{E'}(i) = 1 | \mathbb{1}^E(i) = 1) = 1 - \rho' \tag{44}$$

$$P(\mathbb{1}^{E'}(i) = 0 | \mathbb{1}^E(i) = 1) = \rho' \tag{45}$$

If unemployed today, the transition probabilities are:

$$P(\mathbb{1}^{E'}(i) = 1 | \mathbb{1}^E(i) = 0) = \eta' \tag{46}$$

$$P(\mathbb{1}^{E'}(i) = 0 | \mathbb{1}^E(i) = 0) = 1 - \eta' \tag{47}$$

As such, $\rho$ and $\eta$ are the job loss and job finding rates. These are purely a function of TFP. Output by the representative firm is given by:

$$Y = AK^\alpha N^{1-\alpha} \tag{48}$$

$K$ is capital and $N$ is effective labour, which is normalised to 1 each period.[31] TFP, $A$, is assumed to be

[31] Labour supply follows the law of motion:

$$L = L_{-1}(1 - \rho) + \eta(1 - L_{-1})$$

Markovian. The wage and capital return are given by:

$$W = (1-\alpha)AK^{\alpha}N^{-\alpha} \tag{49}$$

$$R^K = \alpha AK^{\alpha-1}N^{1-\alpha} + 1 - \delta + \lambda(A - \overline{A}) \tag{50}$$

The additional term at the end of the capital return is ad hoc, and allows the model to match the level of return volatility seen in the data. This can be thought of as stochastic depreciation, and its presence means that capital becomes more of a risky asset since its payoffs become more procyclical. Without this, capital and bonds are too closely substitutable and essentially no agent decides to hold bonds.

Asset market clearing holds if:

$$\int_0^1 b'(i)di = 0 \tag{51}$$

$$\int_0^1 a'(i)di = K' \tag{52}$$

I solve the model using the perceived law of motion approach of Krusell and Smith (1997). Full details of this solution method can be found in the appendix. The model period is one year and I calibrate it as follows. The discount rate, $\beta$, is set to 0.90 to match the risk-free rate given other parameters. This value is lower than is typical since the presence of unemployment risk generates a precautionary saving motive which all else equal pushes the bond price up and its return down. Lower patience of households offsets this. The risk aversion parameter is set to 5 to match the equity premium. The unemployment income parameter, $\iota$, is set to generate a consumption loss of roughly 18% upon unemployment for households who hold the average level of capital; a slightly more conservative value compared to the estimate by Chodorow-Reich and Karabarbounis (2016). This is achieved with $\iota = 0.05$.[32] As such, unemployment is painful for households and job loss risk becomes particularly salient.[33] I set $\nu = -\iota\overline{W}$ which means that borrowing is fairly restricted. I set $\alpha = 0.36$ and $\delta = 0.1$ as is standard. I set $\lambda = 6$ to generate capital return volatility of 12.3%.

Labour as a factor of production is the product of labour supply and labour efficiency, $E$. The process for $E$ is assumed to be such that $N = E \cdot L = 1$ in every period. This assumption avoids the need to keep track of labour supply as a state variable.

[32] In the two-period version of the model this corresponds to the natural borrowing limit.

[33] One could alternatively treat the income earned by the unemployed in the model as unemployment insurance, and then target the replacement rate which is around 40% on average in the US. However, higher-income households who are more likely to be stock market participants will typically have a lower replacement rate due to the presence of benefit caps and perhaps their reduced propensity to claim unemployment insurance. A replacement rate of 5% is thus plausible for this relevant group.

TFP takes one of two values each period: $\Phi \in \{1-\Delta_\Phi, 1+\Delta_\Phi\}$, and I set $\Delta_\Phi = 0.02$ to roughly match the annual volatility of US output growth ($\approx 2.5\%$). Its Markov transition matrix is such that the average duration of both recessions and expansions is two years, following Krusell and Smith (1997). I discipline the Markov transition matrix for idiosyncratic productivity by imposing that the fraction of workers unemployed is on average 9% in recessions and 4% in expansions, giving an average unemployment rate of 6.5% in line with US data. I then set the job loss rate such that it is 6% on average, which corresponds to a quarterly job loss rate of around 4% that is then time aggregated to the annual frequency. I set the state-dependent values of the job loss rate such that it is symmetric around its mean, and that its volatility matches that seen in the data. This implies a job loss rate of 4% in expansions and 8% in recessions. The job finding rate is set such that all of these above conditions hold. Collectively, this implies that the unemployment rate has a volatility of 2.5%, which is consistent with the data.

In heterogeneous agent models, it is important to check that the distribution of the marginal propensity to consume (MPC) is reasonable. I find that the average MPC is 0.14, in line with most estimates from the literature. A plot of the full distribution of MPCs can be found in Figure A3. Typically Krusell and Smith (1997) models have difficulty generating a sufficiently high average MPC. The higher level of risk aversion increases the concavity of the consumption policy function, and means that this is not the case here. It is also worth noting that the mean level of financial assets held by households relative to their labour income is reasonable. In the model this ratio is around 3.6, while in the data (the 2019 SCF) this ratio is 3.5. Finally, I also find that the quantiles of the distribution of consumption loss upon unemployment produced in the model look similar to those estimated empirically in Figure A1. If anything, the model lacks some thickness in the left tail of this variable, as the worst idiosyncratic disasters experienced by households are not quite large enough. The quantile plot from the model can be found in Figure A4.

The simulated model produces an excess return on capital of 5.17%, and an average risk-free rate of 1.05%. These are in line with their empirical analogues, assuming that equity is a levered claim to capital with a debt-to-equity ratio of 0.5. The Sharpe ratio is equal to 0.42, also in line with the data. I compare these moments to those in a representative agent version of the model, which is the limiting case of the heterogeneous agent model as $\rho \to 0$.[34] The CCAPM Euler equation holds in this case. This generates a capital risk premium of just 0.24%, as well as a risk-free rate that is too high at 11%. I find that a value of $\gamma \approx 170$ is needed in this version of the model in order to match the equity premium, evidently far too

[34] I set all parameters to the same values, apart from $\lambda$ which is recalibrated to match the volatility of the capital return.

high.

## 5.1 Model Intuition

I now explore why the presence of a non-degenerate wealth distribution allows the model to produce a higher equity premium for any given $\gamma$ vs the CCAPM-JL with its no-trade equilibrium. The full infinite-horizon model is sufficiently complex as to preclude analytical results. However, in the appendix I develop a two-period version of the model and use the techniques of Devereux and Sutherland (2011) to analytically characterise the portfolio choices of each agent in the model. I show that in the limit, as aggregate risk goes to zero, the demand for the risky asset (capital) of employed household i is approximately given by:[35]

$$a^E_{t+1}(i) \approx \frac{\overline{C}^E_{t+1}(i)}{\gamma^E(i)\sigma^2_R}\bigg(\phi^E_R(i)\mathbb{E}_t R^e_{t+1} + \phi^E_{R,\rho}(i)\,\mathrm{Cov}_t(\rho_{t+1}, R^K_{t+1}) - \phi^E_{R,W}(i)\,\mathrm{Cov}_t(W_{t+1}, R^K_{t+1})\bigg) \quad (53)$$

Where $R^e_{t+1} = R^K_{t+1} - R^f_t$, $\sigma^2_R = \mathrm{Var}_t(R^K_{t+1})$, and:

$$\gamma^E(i) = \gamma(1 + \overline{\rho}_{t+1}[\overline{\chi}_U(i)^{-\gamma-1} - 1]) > 0$$

$$\phi^E_R(i) = 1 + \overline{\rho}_{t+1}[\overline{\chi}_U(i)^{-\gamma} - 1] > 0$$

$$\phi^E_{R,\rho}(i) = \overline{\chi}_U(i)^{-\gamma} - 1 \geq 0$$

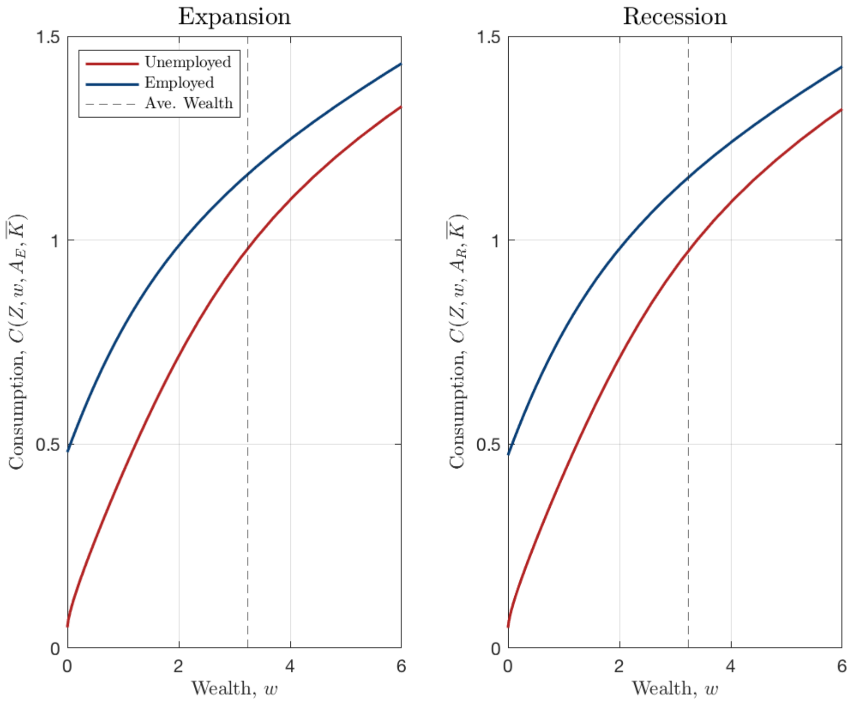
$$\phi^E_{R,W}(i) = \gamma\overline{C}^E_{t+1}(i)^{-1}(1 + \overline{\rho}_{t+1}[\iota\overline{\chi}_U(i)^{-\gamma-1} - 1]) > 0$$

With bars above variables denoting the values in the riskless limit. $C^E_{t+1}(i)$ denotes the consumption of household i if employed in the next period, while $C^U_{t+1}(i)$ denotes the consumption of household i if unemployed in the next period. $\chi_U(i) = C^U_{t+1}(i)/C^E_{t+1}(i)$ is the consumption loss that household i will experience if they lose their job at the start of next period. This will be closer to 1 for households who are better insured against unemployment risk, i.e. wealthier households, and is lower for poorer households who are not well insured. Figure 9 plots the consumption policy functions from the quantitative model, demonstrating this property. Wealth is thus the only meaningful source of heterogeneity for risky asset demand across employed households via its impact on consumption insurance.

In the model, $\mathrm{Cov}_t(\rho_{t+1}, R^K_{t+1}) < 0$ and $\mathrm{Cov}_t(W_{t+1}, R^K_{t+1}) > 0$ since wages fall in recessions and jobs

[35] I check in the infinite-horizon model that this approximation is quite accurate, with a correlation of over 0.9 with the true asset positions of households. In the appendix, I also develop the analogous approximation for unemployed households.

**Figure 9:** Consumption Decision Rules




Note: This figure plots the consumption decision rules of the employed and unemployed households in the expansion and recession state respectively. The dotted line plots the average wealth of households in each state.

become more precarious. Households who are more poorly insured against job loss risk have a lower demand for risky capital. This is because decreases in $\overline{\chi}_U(i)$ serve to increase $\gamma^E(i)$, as worse consumption insurance makes the household behave as if they are more risk-averse. This also increases $\phi^E_{R,\rho}(i)$ and reduces $\phi^E_{R,W}(i)$, making the risky asset position more sensitive to the relevant sources of risk for the household (wage and job loss risk). The fall in capital demand reduces the equilibrium capital level, which in turn increases the expected excess return on the risky asset. It is worth noting that, even if capital returns were uncorrelated with the job loss rate ($\text{Cov}_t(\rho_{t+1}, R^K_{t+1}) = 0$), the presence of job loss risk would still reduce risky asset demand and hence increase the expected return. This is because more job loss risk (higher $\overline{\rho}_{t+1}$), or an increase in the consumption effect of job loss (lower $\overline{\chi}_U(i)$), pushes up risk premia by increasing $\gamma^E(i)$ and $\phi^E_{R,W}(i)$. This aligns with previous results in the more tractable model, where it is this background risk channel that was shown to be important.

It is straightforward to show that, as job loss risk tends to zero, or equivalently as households become perfectly insured against idiosyncratic risk, (53) approaches the Merton (1969) solution, adjusted for i) the presence of labour income and ii) the covariance of capital returns with consumption growth. The equity premium is then approximately given by:

$$\mathbb{E}_t R^e_{t+1} \approx \gamma \operatorname{Cov}_t \left( \frac{C_{t+1}}{\overline{C}_{t+1}}, R^K_{t+1} \right)$$

and so the CCAPM holds, meaning that the equity premium is too low for plausible values of risk aversion. This explains why the representative agent version of the model needs such a high value of $\gamma$ to match the risk premium on capital.

An important property of risky asset demand, $a^E_{t+1}(i)$, is its strict concavity in $\chi_U(i)$. Thus, from an application of Jensen's inequality to the market clearing condition, taking the moments which appear in the demand function as given:

$$\int_0^1 a^E_{t+1}(i)di < a^E_{t+1}(i)\Big|_{\overline{\chi}_U(i)=\chi_U}.$$

i.e. risky asset demand is lower when there is a distribution of consumption insurance vs when all employed agents have $\chi_U(i) = \chi_U$ as was the case in the no-trade equilibrium. This in turn implies a higher risk premium on the asset, and this explains how the model with a non-degenerate distribution of wealth and consumption insurance can match this moment with a lower degree of risk aversion than the CCAPM-JL of section 3.

# 6 Macroeconomic Model of Risky Business Cycles

In this section, I embed the asset pricing mechanism within a fully-fledged macroeconomic model which features frictional labour markets and two assets: a risk-free bond and risky equity.

## 6.1 Environment and Equilibrium

**Environment:** Time is discrete and infinite in horizon. There is once again a unit mass of households, each of whom consumes, invests in equities, and holds a risk-free one-period nominal asset. Additionally, they supply one unit of labour inelastically. These households are ex-ante homogeneous and are indexed by i. They maximize the expected present value of their utility stream, which is discounted at a subjective discount rate denoted by $\beta$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t(i)^{1-\gamma}}{1-\gamma} \right)$$

where $C_t(i)$ is a CES aggregator of consumption varieties:

$$C_t(i) = \left( \int_j \left(C_t(i,j)\right)^{1-1/\mu} dj \right)^{1/(1-1/\mu)} \tag{54}$$

where $\mu \geq 1$ is the elasticity of substitution between the varieties. Utility is CRRA as before, with $\gamma > 0$ representing the degree of relative risk aversion.

Households have a binary employment status each period. If employed they earn a real wage of $W_t$, while if unemployed they produce $b_t$ units of the consumption good at home. All households pay lump-sum taxes, $T_t$. Employed household face the risk of losing their job and becoming unemployed, while unemployed households have the opportunity to find a job and become employed. There is a missing market for insurance that is contingent on a household's idiosyncratic employment status, and this means that unemployment is uninsurable. Household i holds $S_t(i)$ units of the market portfolio. This has a price of $Q_t$ and pays a dividend $d_t$. They hold $B_t(i)$ nominal risk-free one-period bonds which has a price of $q_t$. They face two asset constraints. The first prevents them from borrowing in the nominal bond. The second restricts their equity holdings to be above a lower bound $\underline{S}$.

The household problem in full is then:

$$V\left(I_t^E(i), S_{t-1}(i), B_{t-1}(i), \Omega_t\right) = \max_{C_t(i), S_t(i), B_t(i)} \frac{C_t(i)^{1-\gamma}}{1-\gamma} + \beta\, \mathbb{E}_t\, V\left(I_{t+1}^E(i), S_t(i), B_{t-1}(i), \Omega_{t+1}\right)$$

s.t.

$$C_t(i) + Q_t S_t(i) + q_t B_t(i) = I_t^E(i) W_t + \left(1 - I_t^E(i)\right) b_t - T_t + (Q_t + d_t) S_{t-1}(i) + \frac{B_{t-1}(i)}{\Pi_t}$$

$$S_t(i) \geq \underline{S}$$

$$B_t(i) \geq 0$$

$\Omega_t$ is a vector that contains the aggregate state variables and $\Pi_t$ is the gross inflation rate.

Home production is assumed to be constant over time and equal to a fraction of the steady-state real wage:

$$b_t = \chi_U \overline{W} \tag{55}$$

A continuum of final goods producers indexed by j each produce a differentiated good according to:

$$Y_t(j) = A_t N_t(j) \tag{56}$$

where $N_t(j)$ is the total employment of firm j and $A_t$ is TFP which follows an AR(1) in logs:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t; \qquad \rho_A \in (-1, 1) \qquad \varepsilon_t \sim N(0, \sigma_A^2) \tag{57}$$

Firms can only acquire workers by hiring in a frictional labour market. Each period begins with the resolution of a matching process and starts with a fraction $\omega \in (0, 1)$ of incumbent workers separating from the firm. After discovering the value of aggregate TFP, they then post vacancies at a cost of $\kappa > 0$ each. The probability of a given vacancy successfully leading to the formation of a match is given by $f_t \in (0, 1)$ which is the job-filling rate and is determined endogenously. Employment at firm j therefore follows the law of motion:

$$N_t(j) = (1 - \omega) N_{t-1}(j) + f_t v_t(j) \tag{58}$$

Where $v_t(j)$ is the number of vacancies posted by firm j in period t.

Following separation, unemployed workers have the chance to match with a firm that is looking to hire. This occurs with probability $\eta_t \in (0, 1)$ which we call the job finding rate. The job finding and job filling rates are determined by a Cobb-Douglas matching function which takes the number of job searchers and vacancies posted as inputs:

$$m_t = e_t^{\alpha} v_t^{1-\alpha} \tag{59}$$

$$f_t = \frac{m_t}{v_t} = \eta_t^{\frac{\alpha}{1-\alpha}} \tag{60}$$

$$\eta_t = \frac{m_t}{e_t} \tag{61}$$

where:

$$v_t = \int_j v_t(j) dj$$

and $e_t$ is the aggregate measure of job searchers pinned down by:

$$e_t = 1 - N_{t-1} + \omega N_{t-1} \tag{62}$$

Aggregate employment, $N_t = \int_j N_t(j)dj$, follows the law of motion:

$$N_t = (1-\omega)N_{t-1} + \eta_t e_t \tag{63}$$

Firms operate in a monopolistically competitive environment, and set their product prices, $P_t(j)$, taking into account adjustment frictions as in Rotemberg (1982). They maximise expected discounted dividends net of price adjustment costs, and use the prevailing equilibrium stochastic discount factor which prices assets to do so. This is in line with the notion that firms maximise shareholder value as their governance objective, and aligns the firm's valuation of future cash flows with that of the market. The firm problem is given by:

$$\max_{P_t(j),N_t(j)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t M_{t,t+1} \left[ \frac{P_t(j)}{P_t} Y_t(j) - W_t N_t(j) - \kappa v_t(j) - \frac{\psi}{2} \left( \frac{P_t(j) - P_{t-1}(j)}{P_{t-1}(j)} \right)^2 Y_t \right]$$

s.t.

$$Y_t(j) = A_t N_t(j)$$

$$N_t(j) = (1-\omega)N_{t-1}(j) + f_t v_t(j)$$

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\mu} Y_t$$

where $Y_t = \int_j Y_t(j)dj$, $P_t = (\int_j P_t(j)^{1-\mu}dj)^{\frac{1}{1-\mu}}$, and $M_{t,t+1}$ satisfies $1 = \mathbb{E}_t M_{t,t+1} R^j_{t+1}$ for any asset j. I focus on a symmetric equilibrium where all firms make the same decisions and $P_t(j)$ has a degenerate cross-sectional distribution each period.

Rather than explicitly modelling the wage determination process, for simplicity I assume that it is a function of the job finding rate:

$$\widehat{W}_t = \delta \widehat{\eta}_t \tag{64}$$

where hatted variables denote log deviations from their steady-state values. $\delta \geq 0$ is the elasticity of real wages to the job finding rate. The wage rule conveniently captures the fact that in a tight labour market where the job finding rate is high firms must pay higher wages to attract workers. Many more sophisticated wage processes capture this principle in a microfounded way. Ravn and Sterk (2021) show that this is the first-order solution in this model that emerges out of Nash bargaining. I assume that in every period $W_t > b_t$ which implies that wages do not stray too far below their steady state value and it is always preferable to work rather than not.[36] Dividends are given by output net of hiring costs and the wage bill:

$$d_t = Y_t - \kappa v_t - W_t N_t \tag{65}$$

The model is closed by specifying a Taylor rule for the monetary policy authority and a fiscal policy rule. The Taylor rule prescribes that the nominal interest rate on the one-period bond respond to inflation deviations from its target steady-state level:

$$R^n_{f,t} = \overline{R}\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\delta_\Pi} \tag{66}$$

Where $R^n_{f,t} = \frac{1}{q_t}$. The government raises tax revenue to fund spending, $G_t$, by taxing dividends:

$$T_t = d_t = G_t \tag{67}$$

Asset market clearing is satisfied if:

$$\int_0^1 B_t(i)di = 0 \tag{68}$$

$$\int_0^1 S_t(i)di = 1 \tag{69}$$

The labour market clears if labour market variables follow the laws of motion laid out above.

**Equilibrium**. Given a stochastic process for $A_t$, an equilibrium is a set of prices $(q_t, Q_t, W_t, R_t, \Pi_t)$, household policy functions $(C_t(i), S_t(i), B_t(i))$, firm policy functions $(v_t(j), N_t(j), P_t(j))$, government policy $(T_t, G_t)$, and labour market variables $(\eta_t, f_t, e_t, N_t)$ such that i) The household policy functions solve the household problem; ii) The firm policy functions solve the firm problem; iii) The goods and asset markets clear; iv) Aggregate labour market variables evolve according to their respective laws of motion; v) Monetary policy is implemented according to the monetary policy rule; vi) Fiscal policy is implemented according to the fiscal rule; vii) The equilibrium is symmetric across firms; viii) Actual and perceived laws of motion coincide.

[36] I verify this is indeed the case in the calibration.

**Proposition 3:** If $\underline{S} = 1$, there is an equilibrium where:

$$B_t(i) = 0 \qquad \forall i, t$$

$$S_t(i) = 1 \qquad \forall i, t$$

$$(C_t(i)|I_t^E(i) = 1) = W_t \qquad \forall i, t$$

$$(C_t(i)|I_t^E(i) = 0) = b_t \qquad \forall i, t$$

$$1 = \mathbb{E}_t M_{t,t+1}^E \frac{R_{f,t}^n}{\Pi_{t+1}}$$

$$1 = \mathbb{E}_t M_{t,t+1}^E R_{S,t+1}$$

$$M_{t,t+1}^E = \beta \left( \frac{C_{t+1}^E}{C_t^E} \right)^{-\gamma} [1 + \rho_{t+1}(\chi_{U,t+1}^{-\gamma} - 1)]$$

$$1 - \mu + \mu \left( \frac{1}{A_t} \right) \left( W_t + \frac{\kappa}{f_t} - \mathbb{E}_t M_{t,t+1}^E \left[ (1-\omega) \frac{\kappa}{f_{t+1}} \right] \right) = \psi(\Pi_t - 1)\Pi_t - \psi \mathbb{E}_t M_{t,t+1}^E (\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t}$$

where $\chi_{U,t+1} = \frac{C_{t+1}^U}{C_{t+1}^E} < 1$ and $\rho_{t+1} = \omega(1 - \eta_{t+1})$ is the job loss rate[37].

*Proof:* See the proof of Proposition 1.

There is once again a no-trade equilibrium, and the same Euler equation holds. The logic which underpins this remains the same as in the asset pricing model of section 2. As a result, the prevailing SDF firms use is that of the employed household, since this is the unique SDF which prices any asset in the economy.

## 6.2 Model Mechanism

As in Ravn and Sterk (2017), nominal rigidities serve to amplify the feedback loop, since a reduction in demand that emerges due to the precautionary saving motive of households causes a further cutback in hirng. However the mechanism is still present even under flexible prices. To see this, set $\psi = 0$, $\mu \to \infty$ and the condition pinning down vacancy posting becomes:

$$\frac{\kappa}{f_t} = A_t - W_t + \mathbb{E}_t M_{t,t+1}^E \left[ (1-\omega) \frac{\kappa}{f_{t+1}} \right] \tag{70}$$

Iterating this forward:

$$\frac{\kappa}{f_t} = \sum_{j=0}^{\infty} \mathbb{E}_t M_{t+j}^E \left[ (1-\omega)^j \left( A_{t+j} - W_{t+j} \right) \right] \tag{71}$$

[37] In the previous sections the job loss rate was measured as the fraction of workers who transition from employment to unemployment. This definition is therefore consistent with that notion.

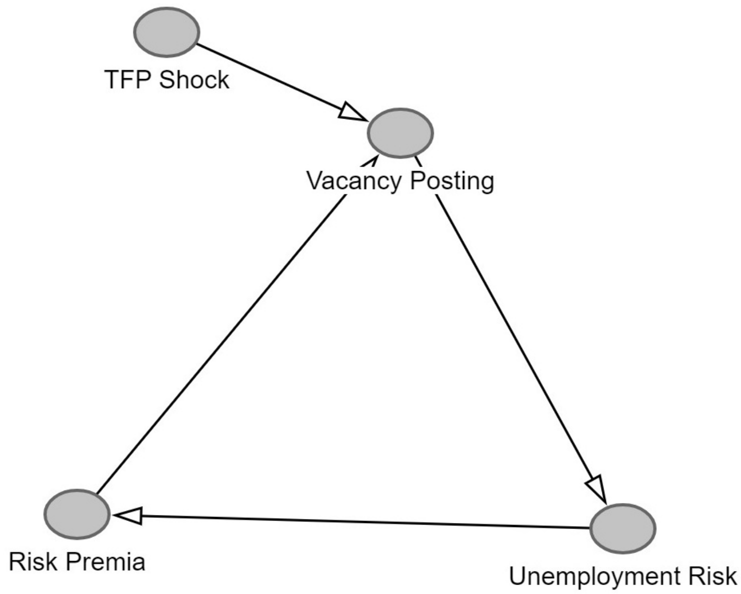



**Figure 10:** DAG of Feedback Loop in Model

where $M^E_{t+j} = \prod_{k=0}^{j} M^E_{t+k,t+k+1}$ and $M^E_t = 1$. This condition tells us that the firm optimally equalises the price of hiring (the left-hand side) with the expected risk-adjusted cash flows of hiring (the right-hand-side). The equation is an asset pricing condition akin to how the stock price equals the expected sum of risk-adjusted dividends. A fall in the expected path of productivity reduces the marginal benefits of hiring a worker, representing a cash flow shock. The price of hiring falls accordingly, translating to a rise in the job filling rate and a fall in the job finding rate.

In a standard Diamond-Mortensen-Pissarides type of model, where the firms are typically modelled as being risk-neutral and markets are complete, this cash flow effect comprises the entirety of the mechanism. In the model here, because households cannot insure against unemployment, the shock leads to a rise in idiosyncratic job loss risk which raises risk premia. Because hiring a worker is a risky investment with uncertain cash flows, this increase in the price of risk causes a further fall in vacancy posting via a risk premium effect. This further raises the level of unemployment risk, and a vicious cycle emerges. To see this further, (70) can be rearranged as:

$$1 = \mathbb{E}_t M^E_{t+1} R^H_{t+1}; \qquad R^H_{t+1} = \frac{(1-\omega)(\kappa/f_{t+1})}{\kappa/f_t - (A_t - W_t)} \tag{72}$$

and then using a common asset pricing decomposition:

$$\mathbb{E}_t R^H_{t+1} = R^f_t \left[1 - \text{Cov}_t \left(M^E_{t+1}, R^H_{t+1}\right)\right] \tag{73}$$

An increase in the expected return on hiring is associated with a labour market contraction i.e. a rise in $f_t$ and a fall in $\eta_t$. As with stocks, in a reasonable calibration the realised return on hiring is moves inversely with the expected return because the price of hiring is strongly mean-reverting.

The above expression highlights two competing effects which occur after an aggregate shock that raises the expected job loss rate. The first of these is a discount rate effect: via the precautionary saving channel, the increase in job loss risk pushes down the risk-free rate which stimulates hiring all else equal. Thus this effect is stabilising. The second of these is a risk premium effect: an increase in job loss risk raises extrinsic risk aversion ($\text{Cov}_t \left(M^E_{t+1}, R^H_{t+1}\right)$ decreases) which induces a desire to shift away from risky assets. This reduces hiring since, from the firm's perspective, workers are a risky asset. Thus this effect amplifies the shock, and is stronger when $\chi_U$ is further below 1 i.e. when consumption insurance is more limited.

When the second effect is strong, the productivity shock is amplified to a significant degree, which is typically not the case in Diamond-Mortensen-Pissarides models as Shimer (2005) documents. Hall (2017) was perhaps the first paper to highlight the possibility of volatility in the SDF to resolve the Shimer (2005) puzzle. Kehoe et al. (2023) develop a model where time-varying risk premia also produces a resolution of the unemployment volatility puzzle. However, because markets are complete in their framework, there is essentially no feedback between unemployment risk and risk premia. This is illustrated in the DAG below. Variation in risk premia is instead generated via habit formation in preferences as in Campbell and Cochrane (1999), meaning the utility function still requires a high degree of curvature.[38] Contrary to the empirical evidence discussed earlier, unemployment leads to no drop in consumption relative to being employed as a consequence of full insurance in the model. Borovička and Borovičková (2018) emphasise that the SDF used to discount the cash flows of the employment match should be consistent with other asset pricing moments. I show that the calibrated model avoids this critique by jointly matching several moments of the equity premium, risk-free rate and dividend-price ratio.

[38] Kehoe et al. (2023) also explore extensions with Epstein-Zin preferences and disaster risk and demonstrate this serves essentially the same role of increasing the price of risk.

### 6.3 Calibrated Model

I now calibrate the model and demonstrate its ability to fit both macroeconomic and asset price data. The model is solved via a third-order perturbation around the deterministic steady state. Each time period represents one month. The set of calibrated parameter values can be found in the table below, and I now provide a rationale for each.

**Table 4:** Calibrated Parameter Values

| Parameter | Calibrated Value | Description |
|---|---|---|
| $\beta$ | 0.97 | Subjective discount factor |
| $\alpha$ | 0.45 | Matching function elasticity |
| $\gamma$ | 7.00 | Risk aversion |
| $\mu$ | 6.00 | Elasticity of substitution between varieties |
| $\psi$ | 127.97 | Rotemberg adjustment cost |
| $\delta_{\Pi}$ | 2.00 | Taylor rule coefficient |
| $\chi_U$ | 0.83 | Home production of unemployed relative to wage |
| $\delta$ | 0.09 | Wage flexibility parameter |
| $\kappa$ | 1.73 | Vacancy cost |
| $\omega$ | 0.02 | Job separation rate |
| $\overline{W}$ | 0.82 | Steady-state wage |
| $\rho_A$ | 0.99 | Autoregressive parameter for productivity |
| $\sigma_A$ | 0.01 | Standard deviation of productivity shocks |

The subjective discount factor, $\beta$, is set to generate an annualised risk-free rate of 1% in the deterministic steady state. A low value is needed due to the presence of idiosyncratic risk. The matching function elasticity, $\alpha$, is set to 0.45 which is in the range of values highlighted by Pissarides and Petrongolo (2001). The risk aversion parameter is set to 7 — a value in the range deemed plausible by Mehra and Prescott (1985) and slightly less than the GMM estimates previously in this paper. $\mu$ is set to a standard value of 6. The Rotemberg adjustment cost parameter, $\psi$, is set to a value such that, at first-order, it is equivalent to a Calvo pricing duration of 6 months as estimated by Bils and Klenow (2004). The Taylor rule inflation coefficient is set to 2.5 which is relatively standard. The home production-wage ratio, $\chi_U$, is set to 0.83, implying a consumption loss upon job loss of around 18%. This is lower than the estimate of Chodorow-Reich and Karabarbounis (2016) and so is somewhat conservative. The wage flexibility parameter, $\delta$, is set to 0.1. This implies a high degree of wage flexibility, avoiding the critique of Kudlyak (2014) and others who argue that a reliance on very rigid wages is unrealistsic in DMP models. The choice of this parameter generates an elasticity of the real wage to productivity of 0.86, close to the estimate of 0.8 in Haefke et al. (2013) for new hires. The vacancy posting cost $\kappa$ is set such that hiring costs are

**Table 5:** Moments in the Model vs the Data

| | Data | | | Model | | |
|---|---|---|---|---|---|---|
| **Variable** | **Mean** | **Std. Dev.** | **AR(1)** | **Mean** | **Std. Dev.** | **AR(1)** |
| Nominal Risk-Free Rate (%) | 0.32 | 0.25 | 0.97 | 0.39 | 1.11 | 0.98 |
| Real Equity Return (%) | 0.70 | 4.14 | 0.03 | 0.66 | 6.50 | 0.00 |
| Log Dividend-Price Ratio | -3.50 | 0.44 | 0.99 | -8.48 | 0.34 | 0.98 |
| Log Dividends | - | 0.10 | 0.99 | - | 0.08 | 0.99 |
| Unemployment Rate (%) | 5.73 | 1.65 | 0.99 | 5.06 | 2.58 | 0.99 |
| Real Consumption Growth (Quarterly, %) | 0.72 | 0.60 | 0.34 | 0.00* | 0.71 | 0.13 |

Note: The table compares the moments of several variables in the model with their empirical analogues. A simulation of 50,000 periods is used to compute the moments in the model. See appendix F for a description of how each empirical series is obtained. The AR(1) column displays the coefficient in a regression of the variable on one lag of itself and a constant. In both the data and the model, dividends are the 12-month backward-looking moving average of aggregate dividends. An asterisk means that the moment is zero by construction given the lack of trend growth in the model. The mean of log dividends is missing due to the fact that it is detrended in the data.

around 1% of output. The job separation rate, $\omega$, is set to generate a mean quarterly job loss rate of around 4%, approximately in line with the average job loss rate in the data but slightly lower to once again err on the side of conservativism. The steady-state wage is set such that the steady-state job-finding rate is equal to 30%, in line with JOLTS data. Finally, I set the parameters governing the stochastic process for $A_t$ such that, after the series is aggregated to a quarterly frequency, it exactly matches the persistence and standard deviation of quarterly labour productivity in the US for all workers from 1947-2019.

The table below gives the mean, standard deviation and AR(1) regression coefficient for several macroeconomic and asset price variables in the data and from simulated data from the model. A description of how each data series is constructed can be found in the appendix.

Starting with the asset price variables, the model generates an annualised equity premium of 4.85%, in line with empirical estimates for the US with the well-known caveat that it is difficult to estimate this moment precisely. The model does this by generating a realistically low value for the average risk-free rate and a high value for the average return on equity. The volatility of both of these is slightly too high in the model, however. The model naturally generates persistence in the risk-free rate without resorting to ad-hoc smoothing in the monetary policy rule. The model also closely matches the persistence and volatility of the log dividend-price ratio. The numerator of this variable, dividends, is smooth and persistent in the model as is the case in the data. Returns are much more volatile than dividends, as highlighted originally by Shiller (1981), which occurs due to the discount rate variation induced by job loss risk.

Turning to macroeconomic variables, the unemployment rate displays a realistic mean, persistence and volatility. It is well known that matching the latter with a realistic labour productivity process and degree of wage flexibility in DMP models is challenging (Shimer, 2005). This is not the case here, however, as unemployment volatility is too high rather than too low in fact despite the high degree of wage flexibility and productivity process that is calibrated to its empirical analogue. Consumption growth also displays a realistic level of volatility. This is in contrast to the CCAPM, which requires consumption to be far less smooth than in the data.[39] Other asset pricing models, such as the Bansal and Yaron (2004) model, also require excessive consumption growth volatility to match the equity premium as highlighted by Beeler and Campbell (2012).

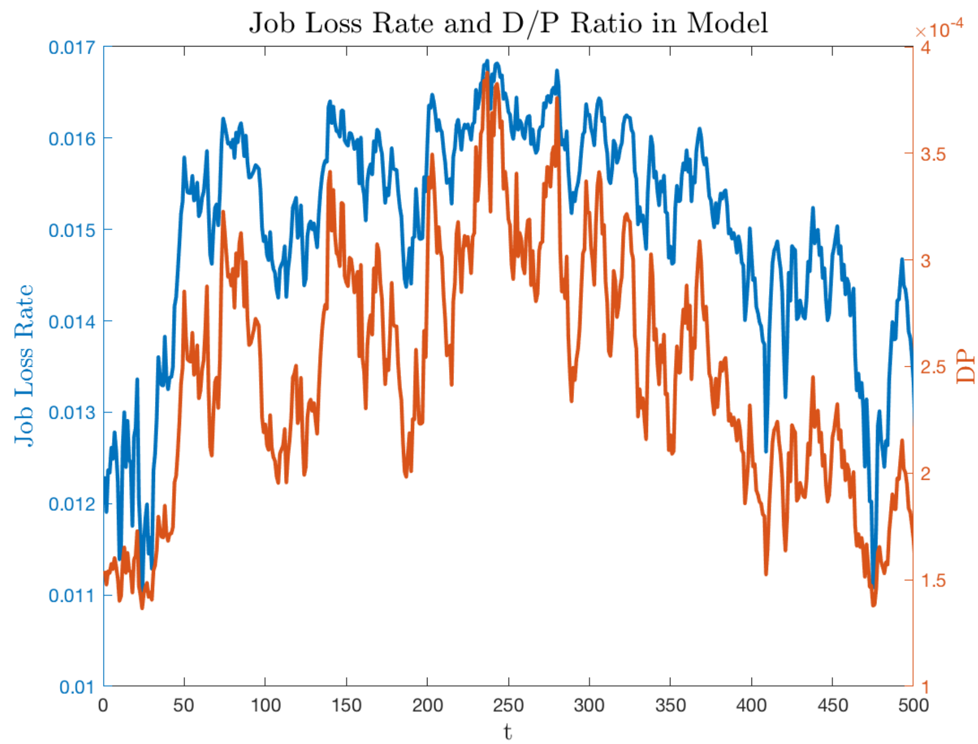
To examine this point further, suppose that there is a naive econometrician who observes the simulated data from the model and erroneously attempts to fit the Euler equation of the CCAPM to the data. They use GMM as was the case in section 3. Using the model simulated data, I find that the naive econometrician would estimate a value of $\gamma = 25$, approximately 4 times higher than the true value. As such, they would conclude that a representative agent is very risk averse whereas in reality they are simply failing to account for the stochastic job loss risk wedge term in the SDF which biases their estimate upwards.

Figure 11 plots a time series of the job loss rate and the dividend-price ratio in the model. As in the data (Figure 1), there is strong positive comovement between the two; the correlation between the two variables is 0.66 in the model vs 0.81 in the data. In the model, there is simultaneity between the level of unemployment risk and risk premia. A higher job loss rate produces increased risk premia (and in turn, an elevated dividend-price ratio), which further heightens unemployment risk, which then increases risk premia and so on.

I now perform two different experiments which are informative about the model's mechanism: i) I add perfect consumption insurance so the SDF is that of the CCAPM; ii) I model firms as being risk neutral rather than using the equilibrium SDF to discount future cash flows. In case i) with full consumption

[39] As established previously, the CCAPM requires that $\text{Cov}(\Delta C_{t+1}, R^M_{t+1})$ be much higher than in the data. This can be expressed as $\text{Cov}(\Delta C_{t+1}, R^M_{t+1}) = \text{Corr}(\Delta C_{t+1}, R^M_{t+1})\sigma(\Delta C_{t+1})\sigma(R^M_{t+1})$. Empirically, the correlation between returns and consumption growth is relatively strong, and the volatility of returns is of course high also. The reason the covariance term is low is hence ultimately due to the low volatility of consumption growth. See for example Savov (2011)) and Kroencke (2017) for elaboration on this point.

**Figure 11:** The Job Loss Rate and Dividend/Price Ratio in Model Simulated Data




insurance, the annualised equity premium generated is cut by 2/3 to only 1.48%, highlighting the importance of job loss risk in matching this moment. The volatility of the dividend-price ratio also becomes far too low, indicative of a lack of variation in risk premia. In case ii) where the firms are risk neutral, the model generates an unemployment volatility 1/3 lower than the benchmark case, as the increase in risk premia no longer affects the hiring decisions of firms directly which stymies the amplification mechanism quite substantially. The equity premium also falls to 2.41%. Therefore, the benchmark model requires an SDF that features a job loss risk wedge, and for firms to use this SDF when making hiring decisions in order to jointly match the volatility of unemployment and the equity premium.

The model purposefully contains only one shock, and its ability to better match the set of moments considered as well as others could of course be improved by adding further shocks. However, Basu et al. (2021) find that the single shock which explains the highest proportion of the equity premium also explains a large fraction of business cycle fluctuations in output and unemployment. As such, the model here replicates this property and generates risky business cycles to borrow their terminology.

## 7 Conclusion

This paper has proposed a simple and realistic amendment to the CCAPM — the incorporation of uninsurable, idiosyncratic job loss risk — that dramatically improves the model's performance along several dimensions. The model yields a plausible risk aversion coefficient when estimated, fits the cross-section of excess returns well and does not produce the risk-free rate puzzle. The central mechanism stems from the interaction of job loss risk, limited consumption insurance and risk aversion - three properties for which there is strong support empirically. Because equity returns tend to be riskier when the job loss rate is high - a time when risk premia are elevated if consumption insurance is imperfect - investors demand a substantial expected return on these assets as risk compensation. Unlike the CCAPM, an implausibly high value of the risk aversion parameters is therefore not required to match the excess return on stocks when the model is estimated. In the cross-section, the CCAPM-JL outperforms the CCAPM significantly with much lower pricing errors. Embedding the asset pricing mechanism in a macroeconomic model enables the joint resolution of the equity premium and unemployment volatility puzzles due to a novel negative feedback loop which emerges.

Unlike other asset pricing models which feature heterogeneity in labour income risk, the CCAPM-JL is far more tractable. This has three main benefits. First, the model's Euler equation can be directly estimated by GMM in the spirit of Hansen and Singleton (1982). This contrasts with other comparable models where parametric processes have to be specified for the underlying state variables, meaning they cannot be estimated semiparametrically as is the case for the CCAPM-JL. Second, the model has an approximate linear factor representation which yields concrete analytical insights into exactly why it is able to match the expected return on any given asset. Third, the linear factor representation means that the model can be directly tested in the cross-section. This is something which, to the best of my knowledge, has not been possible previously with any other asset pricing model featuring labour income heterogeneity. I also show that the key results in the model remain intact when it is enriched to feature a full wealth distribution at the cost of this tractability.

Several possible avenues of future work follow naturally. The heterogeneous agents version of the model was kept relatively stylised both to maintain comparison with the tractable benchmark version of the model and to remain computationally amenable. It could be enhanced along multiple dimensions at the cost of these two features. For example, the income process could be made more detailed by considering more than just two idiosyncratic states, and additional assets such as housing could also be incorporated. Further to this point, this paper has taken an asset pricing approach, but as the analytical results from the heterogeneous agents model illustrate, the presence of job loss risk also has clear implications for models of household finance which focus on portfolio choice. Evaluating the extent to which the incidence of job loss across different ages can explain the hump-shaped pattern of the risky asset share over the life-cycle represents an interesting open question.

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# Risky Jobs, Risky Assets and Risky Business Cycles

## Appendix

Andrew Preston

## A Asset Pricing Model Equilibrium

**Proposition 1:** The following is an equilibrium:

$$a_t^j(i) = \underline{a}^j \qquad \forall t, i, j \tag{74}$$

$$1 = \mathbb{E}_t M_{t,t+1}^E R_{t+1}^j \qquad \forall t, j \tag{75}$$

Where:

$$M_{t,t+1}^E = \beta \left[ \frac{(1-\rho_{t+1})(C_{t+1}^E)^{-\gamma} + \rho_{t+1}(C_{t+1}^U)^{-\gamma}}{(C_t^E)^{-\gamma}} \right]$$

*Proof.* An equilibrium must satisfy the following conditions, which represent, respectively, asset market clearing[40] and utility maximisation of the employed and unemployed households:

$$\int_0^1 a_t^j(i)di = \widetilde{a}^j = \underline{a}^j \qquad \forall t, j, i \tag{76}$$

$$C_t^E(i)^{-\gamma} \geq \beta \mathbb{E}_t R_{t+1}^j \left[ (1-\rho_{t+1}) C_{t+1}^E(i)^{-\gamma} + \rho_{t+1} C_{t+1}^U(i)^{-\gamma} \right] \qquad \forall t, i, j \tag{77}$$

$$C_t^U(i)^{-\gamma} \geq \beta \mathbb{E}_t R_{t+1}^j \left[ (1-\eta_{t+1}) C_{t+1}^U(i)^{-\gamma} + \eta_{t+1} C_{t+1}^E(i)^{-\gamma} \right] \qquad \forall t, i, j \tag{78}$$

Note that the asset holding constraints are maximally tight since they imply that no household can hold an amount of the asset below its fixed net supply level. The Euler equations hold with equality when the asset-holding constraint for asset j is not binding for household i and hold with strict inequality otherwise. The first condition in the proposition, combined with the budget constraint for each type of household implies that:

$$C_t^k(i) = C_t^k = Y_t^k + \sum_{j=0}^{N} (R_t^j - 1)\underline{a}^j \qquad \forall i, k \in \{E, U\} \tag{79}$$

As an illustrative example, when there is one risk free asset in zero net supply and N stocks with a fixed positive net supply of 1, which have a price of $p_t^j$ and pay a dividend of $d_t^j$, the budget constraint for a household is:

$$C_t(i) + \sum_{j=1}^{N} p_t^j [s_{t+1}^j(i) - s_t^j(i)] + b_{t+1}(i) = 1_t^E(i) Y_t^E + (1 - 1_t^E(i)) Y_t^U + \sum_{j=1}^{N} s_t^j(i) d_t^j + R_t^f b_t(i)$$

[40] Note that a corollary of Walras' law is that asset market clearing will also imply that the budget feasibility is satisfied for all households.

Where $s_t^j(i)$ denotes household i's holdings of stock j and $b_t(i)$ denotes household i's holdings of the risk-free bond. The asset-holding constraints in this example are given by:

$$s_t^j(i) \geq 1 \qquad \forall i, t, j$$

$$b_t(i) \geq 0 \qquad \forall i, t$$

We then have in the no-trade equilibrium that $s_t^j(i) = 1, b_t(i) = 0$ for all i,j,t. Plugging this into the budget constraint, we have that:

$$C_t^E(i) = C_t^E = Y_t^E + \sum_{j=1}^{N} d_t^j$$

$$C_t^U(i) = C_t^U = Y_t^U + \sum_{j=1}^{N} d_t^j$$

As such, consumption will not vary in a given period between households with the same employment status. The first condition in the proposition also trivially implies that asset market clearing, (76), is satisfied. Substituting the employed household budget constraint into the employed Euler equation in the proposition implies that utility maximisation is satisfied for all employed households, and (77) holds.

Finally, I verify that the unemployed household's optimality condition, (78), holds. Specifically, I show that it holds with strict inequality, which implies that the asset-holding constraints are binding for all unemployed households. Substituting the first condition in the proposition into the budget constraint for the unemployed households means that this will be satisfied if:

$$1 > \mathbb{E}_t M_{t,t+1}^U R_{t+1}^j \qquad \forall t, j$$

Where:

$$M_{t,t+1}^U = \beta \left[ \frac{(1-\eta_{t+1})(C_{t+1}^U)^{-\gamma} + \eta_{t+1}(C_{t+1}^E)^{-\gamma}}{(C_t^U)^{-\gamma}} \right]$$

This then holds if:

$$\mathbb{E}_t (M_{t,t+1}^E - M_{t,t+1}^U) R_{t+1}^j > 0 \qquad \forall t, j$$

Since $R_{t+1}^j$ is a strictly positive, bounded random variable, this will hold if $M_{t,t+1}^E - M_{t,t+1}^U > 0$ almost surely. Evaluating this, denoting $\chi_{U,t+1} = \frac{C_{t+1}^U}{C_{t+1}^E}$, gives:

$$M_{t,t+1}^E - M_{t,t+1}^U = \beta \left[ \left( \frac{C_{t+1}^E}{C_t^E} \right)^{-\gamma} \left[ \rho_{t+1}(\chi_{U,t+1}^{-\gamma} - 1) - \eta_{t+1}(\chi_{U,t+1}^{\gamma} - 1) \right] \right]$$

The assumptions on the endowments and support of returns mean that $\frac{C_{t+1}^E}{C_t^E}$ is also a strictly positive, bounded random variable. Therefore, $M_{t,t+1}^E - M_{t,t+1}^U > 0$ iff:

$$\rho_{t+1}(\chi_{U,t+1}^{-\gamma} - 1) > \eta_{t+1}(\chi_{U,t+1}^{\gamma} - 1)$$

This is always satisfied since $\rho_t, \eta_t \in (0,1)$ and the assumptions on the endowments imply that $\chi_{U,t+1} < 1$. The left-hand side is then strictly positive while the right-hand side is strictly negative for all t, and the inequality is satisfied. We have thus verified that $M^E_{t,t+1} - M^U_{t,t+1} > 0$ almost surely, and that the unemployed households' optimality conditions are satisfied. Since (76), (77) and (78) are all satisfied, no-trade is a valid equilibrium. □

**Proposition 2:** See text.

*Proof.* I guess and verify that the following constitutes an equilibrium:

$$(C_t(i)|1^E_t(i) = 1) = Z_t(i)Y^E_t \qquad \forall t, i \tag{80}$$

$$(C_t(i)|1^E_t(i) = 0) = Z_t(i)Y^U_t \qquad \forall t, i \tag{81}$$

$$a^j_{t+1}(i) = 0 \qquad \forall t, i, j \tag{82}$$

$$1 = \mathbb{E}_t M^E_{t,t+1} R^j_{t+1} \qquad \forall t, j \tag{83}$$

Where:

$$M^E_{t,t+1} = \beta \left[ \frac{(1-\rho_{t+1})(C^E_{t+1})^{-\gamma} + \rho_{t+1}(C^U_{t+1})^{-\gamma}}{(C^E_t)^{-\gamma}} \right] \tag{84}$$

$$C^E_t = \int_0^1 (C_t(i)|1^E_t(i) = 1)di = Y^E_t \tag{85}$$

$$C^U_t = \int_0^1 (C_t(i)|1^E_t(i) = 0)di = Y^U_t \tag{86}$$

Asset market clearing is trivially satisfied from (82). Substituting (82) into the budget constraint for both employed and unemployed households yields (80) and (81). The Euler equation for any period t employed household for asset j is given by:

$$1 = \beta \mathbb{E}_t \left[ \frac{(1-\rho_{t+1})(Z_{t+1}(i)Y^E_{t+1})^{-\gamma} + \rho_{t+1}(Z_{t+1}(i)Y^U_t)^{-\gamma}}{(Z_t(i)Y^E_t)^{-\gamma}} \right] R^j_{t+1}$$

This rearranges to:

$$1 = \mathbb{E}_t \left( \frac{Z_{t+1}(i)}{Z_t(i)} \right)^{-\gamma} M^E_{t,t+1} R^j_{t+1}$$

Where we have used (85) and (86). Using the process for $Z_{t+1}(i)$ and the properties of the log-normal distribution, it follows that this simplifies to:

$$1 = \mathbb{E}_t M^E_{t,t+1} R^j_{t+1}$$

Following the same process for the Euler equation for any unemployed household, it follows that:

$$1 > \mathbb{E}_t \left( \frac{Z_{t+1}(i)}{Z_t(i)} \right)^{-\gamma} M^U_{t,t+1} R^j_{t+1}$$

Which simplifies to:

$$1 > \mathbb{E}_t M^U_{t,t+1} R^j_{t+1}$$

Where:

$$M^U_{t,t+1} = \beta \left[ \frac{(1-\eta_{t+1})(C^U_{t+1})^{-\gamma} + \eta_{t+1}(C^E_{t+1})^{-\gamma}}{(C^U_t)^{-\gamma}} \right]$$

This holds under the same conditions as outlined in Proposition 1, and no-trade is a valid equilibrium once again.

$\square$

# B Additional Material For Section 3

## B.1 Quantile Regression Plot

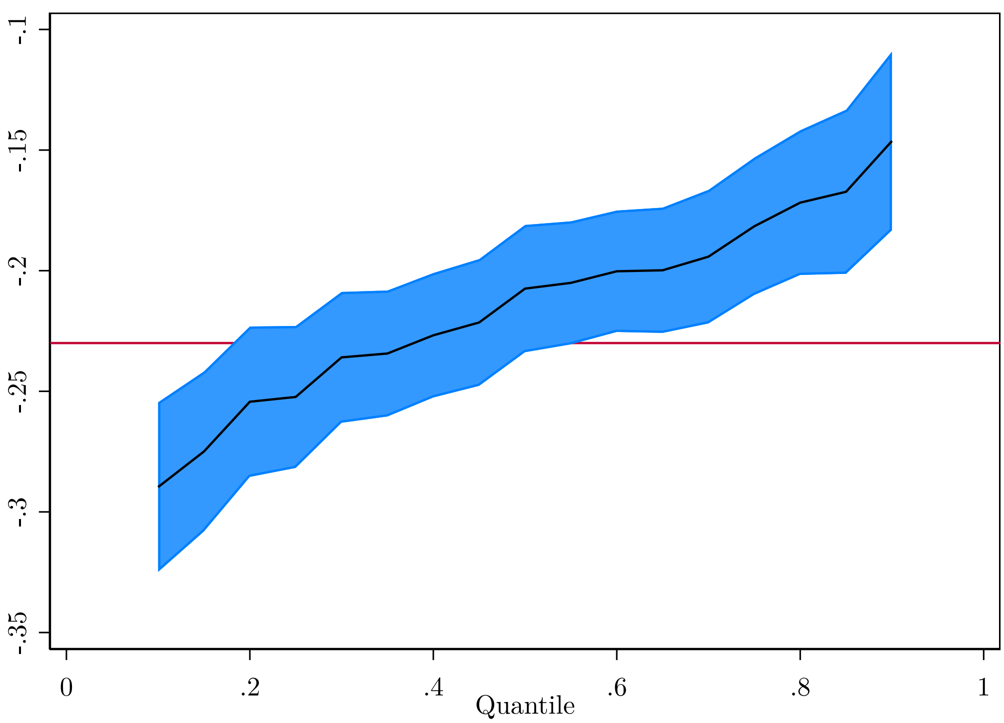



**Figure A1:** Quantile Regression of the Chodorow-Reich and Karabarbounis (2016) specification for the degree of consumption loss upon unemployment.

## B.2 One-Step GMM Estimation of Benchmark Model

Table A1 presents estimates of the CCAPM and CCAPM-JL Euler equations using one-step rather than two-step GMM. The estimate of $\gamma$ is similar in both cases, although unsurprisingly features wider confidence intervals.

**Table A1:** One-Step GMM Estimation

| Model | CCAPM-JL | CCAPM |
|---|---|---|
| **Panel A: Excess Market Return** | | |
| $\gamma$ | 16.03 | 123.67 |
| | (3.20) | (338.88) |
| **Panel B: Excess Market, HML and SMB Returns** | | |
| Model | CCAPM-JL | CCAPM |
| $\gamma$ | 19.03 | 131.35 |
| | (3.00) | (432.37) |

The table reports GMM estimates of the moment condition $\mathbb{E}\left[M^{s}_{t,t+1}R^{e,j}_{t+1} \otimes Z_t\right] = 0$,where the SDF is either given by $M^{E}_{t,t+1}$ or $M^{RA}_{t,t+1}$. $R^{e,j}_{t+1}$ is a vector of excess returns which consists of the excess return on the market portfolio and the excess return on the HML and SMB portfolios. $Z_t$ is a vector of instruments containing consumption growth, the job loss rate and the price-dividend ratio. The market return is defined as the return on the CRSP value-weighted portfolio. The sample period is 1967:I-2012:IV. Estimation is by one-step GMM. Round brackets denote standard errors for the parameter estimate.

### B.3 Alternative Measure of the Job Loss Rate

To ensure the GMM results are not driven by the construction of the job loss rate proposed here, I consider an alternative measure suggested by Chodorow-Reich and Karabarbounis (2016). They define the job loss rate in period t as the ratio of the quantity of workers who have been unemployed for less than 15 weeks in period t +1 to the total number of employed workers in period t. The obvious drawback to this measure is that it will pick up workers who lose their job but also those who quit their job and enter unemployment, as mentioned previously in the motivation for using initial claims as the unit for the job loss rate. Conceptually it is therefore closer to a job separation rate. The correlation between this measure of the job loss rate and the main measure is 0.83, suggesting both series largely capture the same source of variation in practice, however.

I use GMM as previously to estimate the CCAPM-JL with this measure of the job loss rate utilised. Table A2 presents the estimation results, which are extremely similar to those with the main measure of the job loss rate used. The estimate of $\gamma$ is around 12 and 16 in the two specifications, and once again the J test does not reject the model by a comfortable margin as pricing errors are small.

**Table A2:** GMM Estimation II

| Panel A: Excess Market Return | |
|---|---|
| Model | CCAPM-JL |
| $\gamma$ | 16.73 |
| | (2.89) |
| J | 0.59 |
| | [0.90] |
| **Panel B: Excess Market, HML and SMB Returns** | |
| Model | CCAPM-JL |
| $\gamma$ | 11.86 |
| | (5.93) |
| J | 2.02 |
| | [1.00] |

The table reports GMM estimates of the moment condition $\mathbb{E}\left[M^E_{t,t+1}R^{e,j}_{t+1} \otimes Z_t\right] = 0$,where the alternative measure of the job loss rate is used. $R^{e,j}_{t+1}$ is a vector of excess returns which consists of either just the excess return on the market portfolio (panel A) or the excess return on the market portfolio and the excess return on the HML and SMB portfolios (panel B). $Z_t$ is a vector of instruments containing consumption growth, the job loss rate and the price-dividend ratio. The market return is defined as the return on the CRSP value-weighted portfolio. The sample period is 1967:I-2012:IV. Estimation is by two-step GMM with an identity matrix in the first-stage and a Newey-West weighting matrix in the second stage. Round brackets denote standard errors for the parameter estimate, while square brackets denote p-values for a test of the null hypothesis.

### B.4 Job Finding Upon Job Loss

A natural extension of the model is to alter the timing structure to permit the possibility of a job loser finding another job in the same period. The timing is now:

1. Job search and finding occurs amongst the pool of unemployed households.
2. Consumption and saving decisions take place.
3. A fraction, $\rho_t$, of the employed lose their job.

The SDF of the employed household, who once again prices all assets in equilibrium, is now given by:

$$\widetilde{M}^E_{t,t+1} = \beta\left[\frac{(1-\widetilde{\rho}_{t,t+1})(C^E_{t+1})^{-\gamma} + \widetilde{\rho}_{t,t+1}(C^U_{t+1})^{-\gamma}}{(C^E_t)^{-\gamma}}\right] \tag{87}$$

Where:

$$\widetilde{\rho}_{t,t+1} = \rho_t(1-\eta_{t+1})$$

In order to take this version of the model to the data, an empirical measure of the job finding rate is required. I use the unemployment-to-employment transition rate from Shimer (2012), who adjusts for time aggregation in CPS data[41]. Results using a measure of the job finding rate from Chodorow-Reich and

[41] Shimer's data ends in 2007:II and so I extend it to 2012:IV by splicing it with the job finding rate taken directly from

**Table A3:** GMM Estimation III

| **Panel A: Excess Market Return** | |
|---|---|
| Model | CCAPM-JL |
| $\gamma$ | 3.98 |
| | (32.09) |
| J | 7.88 |
| | [0.05] |
| **Panel B: Excess Market, HML and SMB Returns** | |
| Model | CCAPM-JL |
| $\gamma$ | 7.35 |
| | (11.14) |
| J | 28.82 |
| | [0.00] |

The table reports GMM estimates of the moment condition $\mathbb{E}\left[\widetilde{M}^{E}_{t,t+1} R^{e,j}_{t+1} \otimes Z_t\right] = 0$, where the SDF is for the model which features both job loss and job finding. $R^{e,j}_{t+1}$ is a vector of excess returns which consists of either just the excess return on the market portfolio (panel A) or the excess return on the market portfolio and the excess return on the HML and SMB portfolios (panel B). $Z_t$ is a vector of instruments containing consumption growth, the job loss rate and the price-dividend ratio. The market return is defined as the return on the CRSP value-weighted portfolio. The sample period is 1967:I-2012:IV. Estimation is by two-step GMM with an identity matrix in the first-stage and a Newey-West weighting matrix in the second stage. Round brackets denote standard errors for the parameter estimate, while square brackets denote p-values for a test of the null hypothesis.

Karabarbounis (2016) were extremely similar. The sample period is 1967:II - 2012:IV and the estimation follows the same procedure as outlined previously. Table A3 presents the results for the GMM estimation of this version of the model. The estimates of the risk aversion parameter are similar in both cases, albeit smaller than the baseline results. The model with job finding yields larger pricing errors, however, and is rejected by the J test in one of the two specifications. The addition of the chance to find a new job upon being laid off reduces the volatility of $\widetilde{\rho}_{t,t+1}$ relative to $\rho_{t+1}$ slightly, as the job finding rate is quite slow moving in the data.

CPS data. It is also converted to a quarterly job finding rate from an average of monthly rates.

### B.5 Time-Varying Consumption Loss Upon Unemployment

Up to this point, the assumption has been made that an employed agent who becomes unemployed experiences the same proportional change in their consumption at any point in time, i.e. that $\chi_{U,t} = \chi_U$ for all $t$. I now relax this assumption and allow consumption loss upon unemployment to vary over time. This aligns with the empirical evidence in Piqueras (2024), who finds that $\chi_{U,t}$ is higher in recessions versus booms by a factor of around 3.

Linearising the SDF with a first-order Taylor expansion around $\frac{C^E_{t+1}}{C^E_t} = 1, \rho_{t+1} = \rho_t, \chi_{U,t+1} = \chi_U$, where $\chi_U$ denotes the unconditional mean of $\chi_{U,t+1}$, yields:

$$M^E_{t,t+1} \approx \phi_0 + \phi_\rho \rho_{t+1} + \phi_c \Delta C^E_{t+1} + \phi_{\rho,c}\rho_t \Delta C^E_{t+1} + \phi_{\rho,\chi_U}\rho_t \chi_{U,t+1} + \phi_{\widetilde{\rho}}\rho_t \tag{88}$$

With $\phi_0, \phi_\rho, \phi_c$ and $\phi_{\rho,c}$ given by the same expressions as previously and:

$$\phi_{\rho,\chi_U} = -\beta\gamma\chi_U^{-\gamma-1}$$

$$\phi_{\widetilde{\rho}} = \beta\gamma\chi_U^{-\gamma}$$

The linearised SDF now contains two additional terms compared to the case where consumption loss upon unemployment is constant over time. As we will see, the more interesting of these is the interaction between the lagged job loss rate and the unemployment consumption loss term. The time $t$ expected excess return on asset j is then given by:

$$\mathbb{E}_t(R^{e,j}_{t+1}) \approx -\left[\phi_\rho \operatorname{Cov}_t(\rho_{t+1}, R^j_{t+1}) + (\phi_c + \phi_{\rho,c}\rho_t)\operatorname{Cov}_t\left(\Delta C^E_{t+1}, R^j_{t+1}\right) + \phi_{\rho,\chi_U}\rho_t \operatorname{Cov}_t\left(\chi_{U,t+1}, R^j_{t+1}\right)\right] \tag{89}$$

The risk premium on a given asset now also depends on it how its returns covary with the consumption loss variable. The risk price on $\chi_{U,t+1}$ is positive with $\rho_t > 0, \chi_U < 1$, meaning that assets whose returns covary positively with the degree of consumption loss upon unemployment must earn a higher risk premium. This effect is higher when the job loss rate is elevated, since these are periods at which the agent cares more about the degree of consumption insurance if they do lose their job as the possibility of this happening becomes larger.[42]

If the stock market return covaries positively with $\chi_{U,t+1}$, the equity premium is higher for any given value of the risk aversion parameter. This is of course a quantitative matter. To address this, I use the time series of $\chi_{U,t+1}$ produced by Chodorow-Reich and Karabarbounis (2016), where the unemployment indicator is interacted with year dummies in their specification on CEX data to construct this series at an annual frequency. I then look at the covariance of $\chi_{U,t+1}$ with the annual CRSP value-weighted return, abstracting from time-variation in this moment. This is found to be positive and statistically significant at the 5% level, meaning that, during periods of low equity returns, there is a higher average consumption

[42] A similar dynamic is present in disaster risk models where the size of the disaster is drawn from a probability distribution, as in for example Wachter (2013). In these models, larger consumption disasters are typically associated with a larger decline in the equity return, adding an additional component to the equity premium due to this positive covariance.

loss upon unemployment.[43] As a result, this extension to the model can allow it to match the equity premium for a lower degree of risk aversion.

Since the series for $\chi_{U,t+1}$ is only available on an annual basis, rather than a GMM estimation I evaluate the unconditional expectation of the approximated condition for the equity premium given above, assuming all covariances are approximately constant over time and setting them equal to their empirical analogues. I also make the assumption that $\text{Cov}\left(\chi_{U,t+1}, R^M_{t+1}\right)$ is the same at a quarterly frequency as it is at an annual frequency. As before I then vary $\gamma$ to see what value of risk aversion is necessary to match the empirical annualised equity premium of roughly 6%. This procedure yields a value of $\gamma \approx 7$, which is significantly lower than in Figure 6 for the baseline CCAPM-JL with constant consumption loss upon unemployment. While not a formal estimation, this suggests that extending the model to allow the extent of consumption insurance to vary over time means it can match the equity premium for a value of risk aversion that is substantially lower.

The model also provides a theoretical rationale as to why the consumption loss variable would covary positively with the return on the market. In the no-trade equilibrium, we have that:

$$C^E_t(i) = C^E_t = Y^E_t + \sum_{j=0}^{N} r^j_t \underline{a}^j \qquad \forall i$$

$$C^U_t(i) = C^U_t = Y^U_t + \sum_{j=0}^{N} r^j_t \underline{a}^j \qquad \forall i$$

Where $r^j_t = R^j_t - 1$. Making the simplifying assumption that there is one risky asset (the market portfolio) and one risk-free asset available in zero net supply, and that the endowments of the employed and unemployed are constant over time, we then have that:

$$\chi_{U,t+1} = \frac{Y^U + r^M_{t+1}\underline{a}^M}{Y^E + r^M_{t+1}\underline{a}^M}$$

Clearly, this will vary with the return on the market. To see this formally, a first-order Taylor approximation of $\chi_{U,t+1}$ around $r^M = \mathbb{E}(r^M_{t+1})$ yields:

$$\chi_{U,t+1} \approx \frac{Y^U + r^M\underline{a}^M}{Y^E + r^M\underline{a}^M} + \frac{\underline{a}^M(Y^E - Y^U)}{(Y^E + r^M)^2}(r^M_{t+1} - r^M)$$

And therefore:

$$\text{Cov}\left(\chi_{U,t+1}, R^M_{t+1}\right) = \frac{\underline{a}^M(Y^E - Y^U)}{(Y^E + r^M)^2}\text{Var}(r^M_{t+1}) > 0$$

This is positive since $Y^E - Y^U > 0$ by assumption. We can evaluate this covariance by calibrating the necessary parameters. First, the normalisation $Y^E = 1$ is imposed and $Y^U$ is set such that $E(\chi_{U,t+1}) = 0.77$ as in the benchmark. $r^M$ and $\text{Var}(r^M_{t+1})$ are set equal to their sample estimates. Finally, $\underline{a}^M$ is chosen to match the 2019 ratio of mean financial assets to mean income in the Survey of Consumer Finances, i.e.

[43] Kroft and Notowidigdo (2016) also find that the consumption loss upon unemployment is time-varying, as it is positively correlated with the unemployment rate.

$\underline{a}^M = 3.52$. This procedure yields $\text{Cov}\left(\chi_{U,t+1}, R^M_{t+1}\right) = 0.005$, which closely compares to the estimate of 0.006 when the Chodorow-Reich and Karabarbounis (2016) series is used.

### B.6 Non-Degenerate Distribution of Consumption Loss Upon Unemployment

In the benchmark case, there has been an implicit assumption that all job losers experience the same proportion of consumption loss upon unemployment. This assumption is now relaxed by assuming a cross-sectional distribution of consumption loss among households. Letting $\chi_U(i)$ denote the consumption loss of employed household i, their approximated to first order is then:

$$M^E_{t,t+1}(i) \approx \phi_0 + \phi_\rho(i)\rho_{t+1} + \phi_c \Delta C^E_{t+1} + \phi_{\rho,c}(i)\rho_t \Delta C^E_{t+1}$$

Where:

$$\phi_0 = \beta$$

$$\phi_\rho(i) = \beta\left(\chi_U(i)^{-\gamma} - 1\right)$$

$$\phi_c = -\beta\gamma$$

$$\phi_{\rho,c}(i) = -\beta\gamma\left(\chi_U(i)^{-\gamma} - 1\right)$$

The SDF which prices assets is assumed to be an equally-weighted average of the employed households' SDFs. This is a valid SDF, as is any other weighted average of the SDFs.

$$M^E_{t,t+1} = \int_0^1 M^E_{t,t+1}(i)di$$

Plugging this into the excess return condition then yields:

$$\mathbb{E}(R^{e,j}_{t+1}) \approx \gamma \,\text{Cov}(\Delta C_{t+1}, R^{e,j}_{t+1}) + (1 - \mathbb{E}_i(\chi_U^{-\gamma}))\,\text{Cov}(\rho_{t+1}, R^{e,j}_{t+1}) + \gamma\left(\mathbb{E}_i(\chi_U^{-\gamma}) - 1\right)\text{Cov}\left(\rho_t \Delta C_{t+1}, R^{e,j}_{t+1}\right)$$

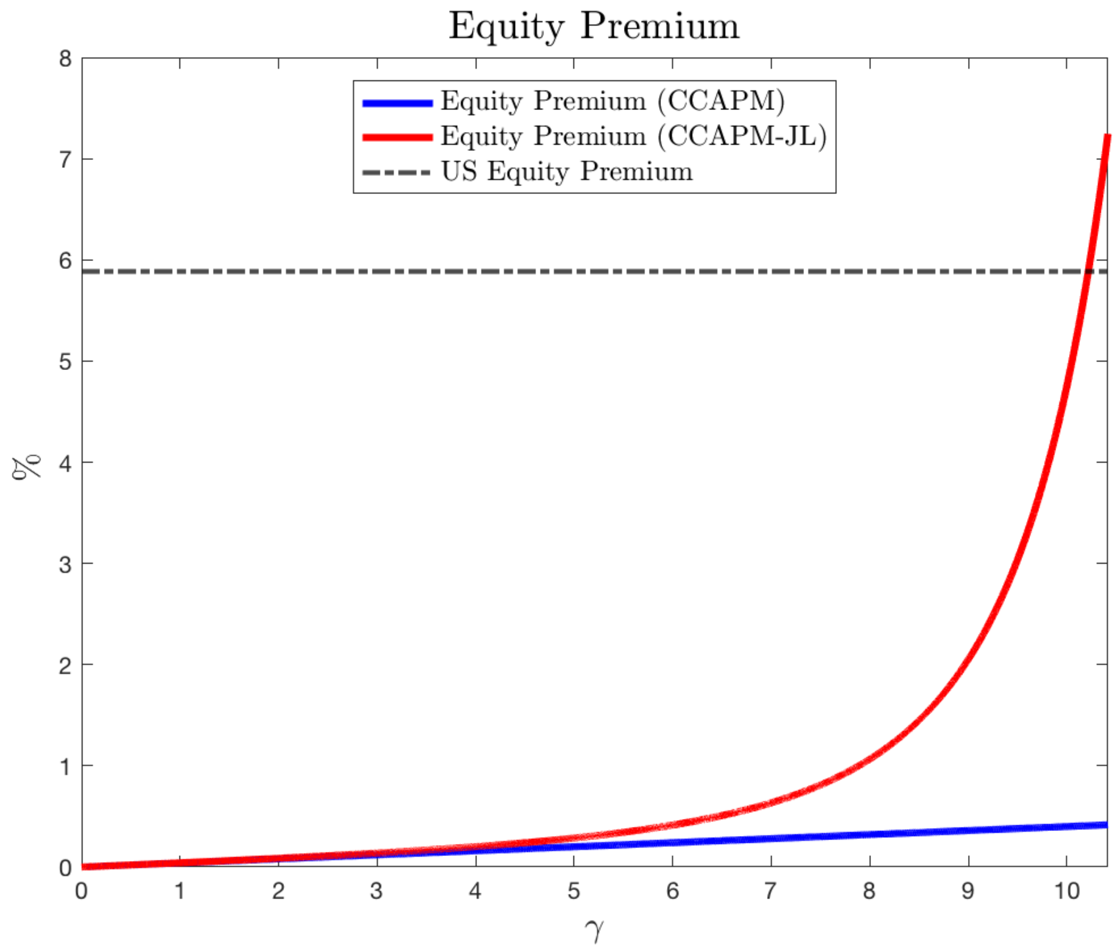
With $\mathbb{E}_i$ denoting the cross-sectional expectation.

In heterogeneous agent models with a borrowing constraint, as is the case is the model of section 5, the distribution of $\chi_U$ is typically left-skewed. This is because the consumption policy function becomes strongly non-linear in the neighbourhood of the constraint, and is close to linear away from it. Consequently, a small mass of low-wealth households experience a large drop in consumption when they lose their job. I thus assume that $-\log(\chi_U)$ follows a gamma distribution, producing left-skewness in $\chi_U$. Using the properties of the gamma distribution, it holds that:

$$\mathbb{E}_i(\chi_U^{-\gamma}) = (1 - \theta\gamma)^{-K}$$

Where $\theta$ and $K$ are the scale and shape parameters of the distribution respectively. I choose these such that $\chi_U$ has a mean of 0.77 and a standard deviation of 0.1. This generates a quantile plot for consumption loss which matches the empirical analogue in Figure A1 quite well. I then set the covariance terms equal to their empirical analogues and vary $\gamma$ to compute the implied equity premium as in Figure 3. This is

**Figure A2:** Equity Premium with Gamma Distribution for $\chi_U$.




The figure displays the annualised equity premium in the CCAPM-JL (red line) and the CCAPM (blue line) as a function of $\gamma$. The dotted black line represents the average annual excess market return in US data.

displayed below in Figure A2. The CCAPM-JL is now able to match the equity premium for a value of $\gamma$ just below 10. The mass of households with a consumption loss upon unemployment that exceeds the average serves to push up the risk price for the interaction term. Job loss is very painful for this subset and this is reflected in greater equilibrium risk compensation for holding assets which are riskier when the job loss rate is high.

### B.7 Alternative Sample Periods for the Return Predictability Regressions

A recent finding in the return predictability literature is the loss of predictive power which occurs for many common return predictor candidates when the sample is limited to the post-war period. Given the sample begins in 1967, this makes these findings of significant return predictability already rather unusual. Kostakis, Magdalinos, and Stamatogiannis (2015) find that there is little evidence of return predictability for US stocks beyond 1952, while Welch and Goyal (2008) provide evidence of particular temporal instability in the post-oil crisis period. To check the performance of the job loss rate over time

**Table A4:** Alternative Samples for Return Prediction Regressions

| Forecast Horizon h | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 8 | 12 | 16 | 20 |
| **Panel A: 1980:I - 2012:IV Sample** | | | | | | | |
| 0.45 | 0.99 | 1.52 | 1.79 | 2.60 | 4.63 | 6.49 | 9.19 |
| (1.23) | (1.51)* | (1.76)** | (1.74)** | (1.42)* | (1.90)** | (2.29)** | (2.76)*** |
| [0.59] | [2.21] | [3.70] | [3.87] | [4.26] | [8.38] | [10.80] | [14.53] |
| **Panel B: 1990:I - 2012:IV Sample** | | | | | | | |
| 1.22 | 2.43 | 4.01 | 5.44 | 13.04 | 18.93 | 22.60 | 28.81 |
| (1.74)** | (2.01)** | (2.26)** | (2.37)*** | (2.27)** | (2.01)** | (2.04)** | (3.43)*** |
| [1.17] | [3.15] | [6.55] | [9.11] | [23.48] | [25.30] | [22.29] | [24.42] |
| **Panel C: 1967:I - 2007:IV Sample** | | | | | | | |
| 0.57 | 1.13 | 1.69 | 2.16 | 2.57 | 2.98 | 4.75 | 7.73 |
| (1.54)* | (1.75)** | (2.01)** | (2.17)** | (1.41)* | (1.20) | (1.38)* | (1.83)** |
| [1.18] | [2.60] | [4.11] | [5.29] | [3.29] | [2.37] | [4.15] | [7.13] |

Note: The table reports results from return predictive regressions of the form $R^e_{t,t+h} = \alpha_h + \beta_h \rho_t + \epsilon_{t,t+h}$, where $R^e_{t,t+h}$ is the h-quarter ahead excess stock market return on the CRSP value weighted index and $\rho_t$ is the job loss rate. For each regression, the table reports the OLS coefficient estimate, Newey-West adjusted t-statistic in parentheses and the adjusted $R^2$ value in square brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively according to p-values from a one-sided hypothesis test. Each panel represents a different sample period and is labelled accordingly.

as a return predictor, I run the benchmark regressions but truncate the sample to begin in 1980:I and 1990:I to respectively exclude the oil crisis period and the monetary transition period after Paul Volcker became chairman of the Federal Reserve. A separate concern for the results here is that the large increase in layoffs during the Great Recession could be singlehandedly driving the results. To investigate this I also run the benchmark regressions for the sample 1967:I - 2007:IV. Table A4 presents the results for these four alternative samples with the excess return on the CRSP as the stock market index. The return predictability of the job loss rate is present across both alternate samples and at all horizons to varying degrees. Results from both look similar to the benchmark results, particularly for the sample which excludes the Great Recession. This would suggest that the results are not driven by the macroeconomic and monetary instability of the pre-1990 period, nor the high job loss rate regime of 2007-2009.

### B.8 Return Predictability Out-of-Sample

A well known feature of many return predictors is their lack of ability to reform well out-of-sample. Welch and Goyal (2008) show that is practically ubiquitous across the full set of candidates commonly used in the literature, with even the small number of variables performing well in-sample losing their predictive power in out-of-sample tests. It is important to note that in-sample and out-of-sample may be considered distinct tests of a return predictor, and Nagel and Xu (2019) illustrate theoretically that asset pricing models with a learning aspect can feature variables which perform adequately in-sample but fail out-of-sample. This being said, out-of sample testing remains a useful diagnostic of potential fragility in an in-sample predictor. Following much of the literature, I use the historical average excess return as the benchmark with which to

**Table A5:** Out-of-Sample Testing for Return Prediction Regressions

| | Forecast Horizon h | | | | | |
|---|---|---|---|---|---|---|
| Test | 1 | 4 | 8 | 12 | 16 | 20 |
| ENC-NEW | 1.19** | 3.14*** | 2.96*** | 3.33*** | 6.06*** | 11.69*** |
| OOS-F | 1.44** | 3.86*** | 4.47*** | 5.38*** | 8.56*** | 14.62*** |
| $R^2_{OOS}$ | 0.05* | 0.60** | 1.10*** | 1.57*** | 2.37*** | 3.94*** |

The table reports results from out-of-sample tests of return predictive regressions of the form $r_{t,t+h} = \alpha_h + \beta_h \rho_t + \epsilon_{t,t+h}$ against the benchmark constant expected return model $r_{t,t+h} = \alpha_h + \epsilon_{t,t+h}$. $r_{t,t+h}$ is the h-quarter ahead log excess CRSP value-weighted market return and $\rho_t$ is the job loss rate. The table reports the values of the test statistics from the ENC-NEW test of Clark and McCracken (2001), the OOS-F test of McCracken (2007) and the $R^2_{OOS}$ test of Clark and West (2007). The 1967:I-1999:IV period is used to estimate the model, which is then evaluated out-of-sample on the 2000:I-2012:IV period. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively according to p-values from the respective hypothesis test.

compare the predictive ability of initial claims. This involves the following specification:

$$R^e_{t,t+h} = \alpha_h + \epsilon_{t,t+h} \tag{90}$$

I.e. a model of constant expected excess returns is formulated and compared against the model with the job loss rate as an additional right-hand side variable. The latter model is nested within the former. The pre-2000 period is used to estimate each model, and then the out-of-sample performance is assessed in the post-2000 period. I use three tests to assess this for a range of horizons - the ENC-NEW test of Clark and McCracken (2001), the OOS-F test of McCracken (2007) and the Clark and West (2007) $R^2_{OOS}$ test. Table A5 presents the results of each of these. For all horizons, the model which includes the job loss rate demonstrates the ability to outperform the constant expected returns benchmark, as all test statistics exhibit significance at the 1% level at horizons over 4 quarters. Return predictability is thus retained out-of-sample. For the very short-horizon of one quarter, two out of the three tests reject the null hypothesis at the 5% level, while the other rejects at the 10% level.

# C First-Order Taylor Expansion of the CCAPM-JL SDF

The CCAPM-JL SDF is given by:

$$M_{t,t+1}^E = \beta \left( \frac{C_{t+1}^E}{C_t^E} \right)^{-\gamma} \left( 1 + \rho_{t+1} \left( \chi_U^{-\gamma} - 1 \right) \right)$$

A first-order Taylor expansion of the SDF around (1,$\rho_t$) yields:

$$M_{t,t+1}^E \approx \beta \left[ 1 + \rho_t \left( \chi_U^{-\gamma} - 1 \right) \right] (1 - \gamma \Delta C_{t+1}^E) + \beta \left( \chi_U^{-\gamma} - 1 \right) (\rho_{t+1} - \rho_t)$$

So it can be approximated as:

$$M_{t,t+1}^E \approx \phi_0 + \phi_\rho \rho_{t+1} + \phi_c \Delta C_{t+1}^E + \phi_{\rho,c} \rho_t \Delta C_{t+1}^E$$

Where:

$$\phi_0 = \beta$$
$$\phi_\rho = \beta \left( \chi_U^{-\gamma} - 1 \right)$$
$$\phi_c = -\beta\gamma$$
$$\phi_{\rho,c} = -\beta\gamma \left( \chi_U^{-\gamma} - 1 \right)$$

## C.1 Parametric Model

In section 3.4.1 where I develop the parametric model, I instead linearise the SDF around $\overline{\rho}_{t+1} = \mathbb{E}_t \rho_{t+1}$. This is not substantially different if $\rho_{t+1}$ is highly persistent in the data, which it is indeed. This yields with $\beta = 1$:

$$M_{t,t+1}^E \approx \overline{M}_{t,t+1}^E + \gamma \left[ 1 + \overline{\rho}_{t+1} \left( \chi_U^{-\gamma} - 1 \right) \right] \Delta C_{t+1} + \left( \chi_U^{-\gamma} - 1 \right) (\rho_{t+1} - \overline{\rho}_{t+1})$$

where $\overline{M}_{t,t+1}^E = \mathbb{E}_t M_{t,t+1}^E = 1 + \overline{\rho}_{t+1} \left( \chi_U^{-\gamma} - 1 \right)$. Then from the asset pricing equation for the equity premium:

$$\mathbb{E}_t R_{t+1}^{e,M} \approx -\operatorname{Cov}_t(M_{t,t+1}^E, R_{t+1}^M) = -\mathbb{E}_t \left[ (M_{t,t+1}^E - \overline{M}_{t,t+1}^E)(R_{t+1}^M - \overline{R}_{t+1}^M) \right]$$

Substituting in the stochastic processes for $R_{t+1}^M$, $\rho_{t+1}$, and $\Delta C_{t+1}$ and the linearised SDf into this equation then yields:

$$\mathbb{E}_t(R_{t+1}^{e,M}) \approx \gamma \left[ 1 + \overline{\rho}_{t+1}(\chi_U^{-\gamma} - 1) \right] \sigma_C \sigma_R + (\chi_U^{-\gamma} - 1) \sigma_\rho \sigma_R$$

Using the law of iterated expectations then yields the expression in the text.

I estimate the stochastic processes for $R_{t+1}^M$, $\rho_{t+1}$, and $\Delta C_{t+1}$ by first specifying a process for $\overline{R}_{t+1}^M$ and $\overline{\rho}_{t+1}$. For $\overline{R}_{t+1}^M$ I use the regression in section 3.7 to describe this as: $\overline{R}_{t+1}^M = \alpha + \beta \rho_t$ This then allows me to compute $\sigma_R$ as $\sigma_R = \sigma(R_{t+1}^M - \alpha - \beta \rho_t)$. I set $\sigma_R = 0.07$ at the quarterly frequency. I specify an AR(1) for $\rho_{t+1} = \alpha_\rho + \beta_\rho \rho_t + \epsilon^\rho_{t+1}$. This implies that $\sigma_\rho = \sigma(\epsilon^\rho_{t+1}) = 0.01$. Finally, I set $\sigma_C = 0.006$ to match the standard deviation of quarterly real consumption growth.

# D Additional Results and Robustness Tests For Section 4

## D.1 Industry Portfolios

As noted by Lewellen et al. (2010), the 25 Fama-French portfolios exhibit a strong factor structure which can mean that models with useless factors can misleadingly obtain low pricing errors and statistically significant risk prices. As a result, I now consider the 30 industry portfolios as an alternative set of test assets, since this does not exhibit the same issue. Results are presented in Table A6 and **??**. Once again, the CCAPM-JL displays lower pricing errors than the CCAPM, although the margin is narrower. The asymmetry in pricing errors is again present in the CCAPM and corrected by the CCAPM-JL. The risk price estimate for the interaction term between consumption growth and the job loss rate in the CCAPM-JL is very similar to the benchmark results, and remains statistically significant, while the risk price for the job loss rate is no longer significant. The CAPM and Lettau and Ludvigson (2001) model both perform better in terms of pricing errors with this set of test assets.

## D.2 Bond Portfolios

To investigate the performance of the factor models in pricing fixed income securities, I consider a test asset vector consisting of ten maturity-sorted government bond portfolios with maturities in six month intervals up to five years, as well as ten corporate bond portfolios sorted on yield spread[44]. The sample period is 1975:I-2011:IV. Table A7 presents Fama-Macbeth estimation results.

The CCAPM performs woefully for this set of test assets, predicting an expected return of essentially zero for all of the portfolios. This is because the bond returns are even more weakly correlated with consumption growth unconditionally than equity returns, meaning the CCAPM predicts that they should have a very low expected return even with a large risk price for consumption growth. The CCAPM-JL performs much better, with pricing errors that are roughly a quarter of the size of those from the CCAPM. Once again, the risk price on the interaction term between consumption growth and the job loss rate is positive and statistically significant, as the bond returns are much more correlated with consumption growth conditional on the job loss rate being high. The risk price on the job loss rate factor is also significant. The CCAPM-JL outperforms the CAPM in terms of pricing errors and does marginally worse than the Lettau and Ludvigson (2001) model. Therefore, the success of the CCAPM-JL in pricing assets is not limited to equities, as it also performs remarkably well in explaining the cross-section of fixed income returns.

[44] All data comes from He, Kelly, and Manela (2017)

**Table A6:** Fama-Macbeth Regressions for 30 Industry Portfolios

| **30 Industry Portfolios** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Model** | $\lambda_\rho$ | $\lambda_{\Delta C}$ | $\lambda_{\rho,\Delta C}$ | $\lambda_{CAY}$ | $\lambda_{CAY,\Delta C}$ | $\lambda_{R^M}$ | RMSPE |
| CCAPM | | 0.005 | | | | | 1.02 |
| | | (2.26) | | | | | [0.00] |
| CCAPM-JL | 0.004 | 0.004 | 0.001 | | | | 0.98 |
| | (0.75) | (2.11) | (2.27) | | | | [0.00] |
| LL (2001) | | 0.002 | | 0.016 | 0.000 | | 0.81 |
| | | (1.22) | | (2.72) | (1.40) | | [0.00] |
| CAPM | | | | | | 0.017 | 0.67 |
| | | | | | | (2.43) | [0.00] |

Note: The table reports results of Fama and Macbeth (1973) regressions for the 30 industry portfolios. See text for full estimation details. RMPSE denotes the root mean squared pricing error. Round brackets denote t-statistics for the factor risk price parameter estimates, while square brackets denote p-values for a test of the null hypothesis that all pricing errors are zero. The sample period is 1967:I-2012:IV.

**Table A7:** Fama-Macbeth Regressions for 20 Bond Portfolios

| **20 Bond Portfolios** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Model** | $\lambda_\rho$ | $\lambda_{\Delta C}$ | $\lambda_{\rho,\Delta C}$ | $\lambda_{CAY}$ | $\lambda_{CAY,\Delta C}$ | $\lambda_{R^M}$ | RMSPE |
| CCAPM | | 0.001 | | | | | 0.80 |
| | | (0.45) | | | | | [0.00] |
| CCAPM-JL | 0.053 | 0.004 | 0.001 | | | | 0.22 |
| | (4.07) | (1.61) | (3.47) | | | | [0.00] |
| LL (2001) | | -0.004 | | -0.037 | -0.001 | | 0.13 |
| | | (1.80) | | (-5.19) | (-4.81) | | [0.00] |
| CAPM | | | | | | 0.052 | 0.33 |
| | | | | | | (3.46) | [0.00] |

Note: The table reports results of Fama and Macbeth (1973) regressions for the 20 bond portfolios. See text for full estimation details. RMPSE denotes the root mean squared pricing error. Round brackets denote t-statistics for the factor risk price parameter estimates, while square brackets denote p-values for a test of the null hypothesis that all pricing errors are zero. The sample period is 1967:I-2012:IV.

### D.3 Alternative Measures of Job Loss Rate

I once again ensure that the results are not driven by the measure of the job loss rate by testing the CCAPM-JL using the alternative constructed in section 3.3. The results can be found in Table A8, and are not meaningfully altered, with the pricing errors actually reduced relative to the CCAPM-JL with the primary measure of the job loss rate used.

I also test the variant of the model which incorporates job finding in the cross-section of returns. The job loss rate in this case is given by $\widetilde{\rho}_{t,t+1} = \rho_t(1 - \eta_{t+1})$ where $\eta_t$ is the job finding rate in t. Results can again be found in Table A8 and **??**. The incorporation of job finding does not meaningfully change the key results, and actually results in lower pricing errors.

**Table A8:** Fama-Macbeth Regressions for CCAPM-JL with Alternative Measures of Job Loss Rate

| 25 Fama-French Portfolios | | | | |
|---|---|---|---|---|
| | $\lambda_\rho$ | $\lambda_{\Delta C}$ | $\lambda_{\rho,\Delta C}$ | RMSPE |
| Alt. Job Loss Rate | 0.014 | 0.001 | 0.001 | 0.57 |
| | (3.09) | (0.41) | (2.28) | [0.00] |
| Model w/ Job Finding | 0.010 | -0.001 | 0.001 | 0.48 |
| | (4.16) | (-0.59) | (2.51) | [0.00] |

The table reports the results of Fama and Macbeth (1973) regressions for the 25 Fama-French portfolios with the alternative measures of the job loss rate used in the CCAPM-JL. The first row corresponds to the measure of the job loss rate described in section 3.3, while the second row corresponds to the job loss rate in the model that also incorporates job finding. See text for full estimation details. RMPSE denotes the root mean squared pricing error. Round brackets denote t-statistics for the factor risk price parameter estimates, while square brackets denote p-values for a test of the null hypothesis that all pricing errors are zero. The sample period is 1967:I-2012:IV.

## D.4 GMM Estimation

I also estimate the following moment condition for the 25 Fama-French portfolios as test assets:

$$\mathbb{E}\left[M^s_{t,t+1}R^{e,j}_{t+1} \otimes Z_t\right] = 0 \tag{91}$$

Where $s \in \{E, RA, CAPM\}$ and $Z_t$ again contains consumption growth, the job loss rate and the price-dividend ratio. The SDF for the CAPM is given by:

$$M^{CAPM}_{t,t+1} = 1 - \psi R^{e,m}_{t+1} \tag{92}$$

Where $R^{e,m}_{t+1}$ is the excess market return. Results from this estimation are presented in Table A9. Comparing the CCAPM and CCAPM-JL, the same patterns are present as in the previous set of GMM results — the risk aversion parameter is implausibly high in the CCAPM (taking a value of 205) but much more plausible in the CCAPM-JL (taking a value of 14), and Hansen's J test rejects the CCAPM but not the CCAPM-JL, reflecting the significantly improved fit of the latter model. The CAPM is also comfortably rejected by the J test.

## D.5 Factor-Mimicking Portfolios

An additional way to assess the risk price on a given factor is to construct a factor-mimicking portfolio as in Breeden et al. (1989), which by construction has a beta of 1 with the factor of interest and a beta of 0 with all other factors in the set. By construction, the average excess return on these factor-mimicking portfolios can be interpreted as the risk price of the respective factor. A benefit of this approach in the setting here is that data on returns is available over a much longer period than is the case for consumption growth and the job loss rate. This means we can estimate the weights for the factor-mimicking portfolios using the period in which there is data for the factors, and then use these weights to construct the portfolios for the entire 1926-2019 period over which there is data on returns. A second advantage is that it parsimoniously

**Table A9:** GMM Estimation for 25 Fama-French Portfolios

| **25 Fama-French Portfolios** | | | |
|---|---|---|---|
| **Model** | $\gamma$ | $\psi$ | J |
| CCAPM | 204.96 | | 965.03 |
| | (40.84) | | [0.00] |
| CCAPM-JL | 14.26 | | 57.20 |
| | (1.10) | | [1.00] |
| CAPM | | 7.09 | 1452.62 |
| | | (1.10) | [0.00] |

The table reports GMM estimates of the moment condition $\mathbb{E}\left[M^{s}_{t,t+1}R^{e,j}_{t+1} \otimes Z_t\right] = 0$,where the SDF is either given by $M^{E}_{t,t+1}$, $M^{RA}_{t,t+1}$ or $M^{CAPM}_{t,t+1}$. $R^{e,j}_{t+1}$ is a vector of excess returns for the 25 Fama-French portfolios. $Z_t$ is a vector of instruments containing consumption growth, the job loss rate and the price-dividend ratio. The sample period is 1967:I-2012:IV. Estimation is by two-step GMM with an identity matrix in the first stage and a Newey-West weighting matrix in the second stage. Round brackets denote standard errors for the parameter estimate, while square brackets denote p-values for a test of the null hypothesis.

deals with any classical measurement error in the factors. I follow the procedure for multi-factor models in Lehmann and Modest (1988). The first step is to estimate:

$$R^e_t = a + f_t b' + \epsilon_t \tag{93}$$

Where $f_t$ is the T x K matrix of factors, $R^e_t$ is a T x N matrix of excess returns and b is an N x K matrix of slope coefficients. Letting $\Omega$ denote the N x N variance-covariance matrix of the residuals $\epsilon_t$, which is assumed to be a diagonal matrix, the factor-mimicking portfolio weights are then given by:

$$w = (b'\Omega^{-1}b)^{-1}b'\Omega^{-1} \tag{94}$$

And the factor-mimicking portfolios are given by $w'R^e_t$.

As the set of assets with which to construct the factor-mimicking portfolio, I use the 25 Fama-French portfolios. The time series regression above is estimated for the period 1967:I - 2012:IV, but the estimated weights can be used to construct factor-mimicking portfolios over the much longer 1926:III - 2019:IV period. For the sake of brevity, I only consider the factor model corresponding to the CCAPM-JL. The implied estimate of $\lambda_\rho$ is 0.028 (t-stat = 4.02), the estimate of $\lambda_{\Delta C}$ is 0.006 (t-stat = 2.38), and the estimate of $\lambda_{\rho,\Delta C}$ is 0.001 (t-stat = 3.34). These estimates are very similar to the benchmark Fama-Macbeth regressions in Table 3, as once again the conditional factor continues to earn a positive risk premium as the model would imply. Inspecting the weights for this factor-mimicking porfolio reveals that a long position is taken in the small value portfolios, emphasising the fact that this conditional factor is strongly associated with return movements in these assets.

# E Additional Material For Section 5

## E.1 Details of Solution Method

As is well known, solving models featuring heterogeneity and aggregate shocks is challenging since the infinite-dimensional asset distribution is a state variable. I follow Krusell and Smith (1997) and assume that agents use a finite number of moments of the distribution to forecast prices next period. In practice, they only use the mean level of capital this period to forecast the bond price and the mean level of capital next period. The perceived laws of motion are thus given by:

$$\begin{aligned} \log K' &= c_0 + c_1 \log K \\ q &= d_0 + d_1 \log K \end{aligned}$$

The algorithm proceeds as follows:

1. Conjecture a perceived law of motion (PLM) for capital and the bond price:

   $$\begin{aligned} \log K' &= c_0 + c_1 \log K + c_2 1_A + c_3 1_A \log K \\ q &= d_0 + d_1 \log K + d_2 1_A + d_3 1_A \log K \end{aligned}$$

   And guess a set of coefficients. $1_A$ is an indicator function for the expansion state.

2. Given PLMs, solve HH problem for to obtain policy functions $b'(b, a, Z, K, A, q)$, $a'(b, a, Z, K, A, q)$.

3. Simulate for T periods and N households. Set initial conditions $(b_0, a_0, Z_0)$ for each household and $(A_0, K_0)$ for the economy. Draw random sequences $\{\Phi_t\}_{t=1}^T$ and $\{Z_t(i)\}_{t=1}^T$ for each i from their respective DGPs.

4. Use the policy functions to find $q$ each period such that the bond market clears, and then solve for:

   $$K' = \frac{1}{N} \sum_{i=1}^{N} a'(i)$$

5. Discard the first $T_1$ periods of the simulated data, and then run the regressions:

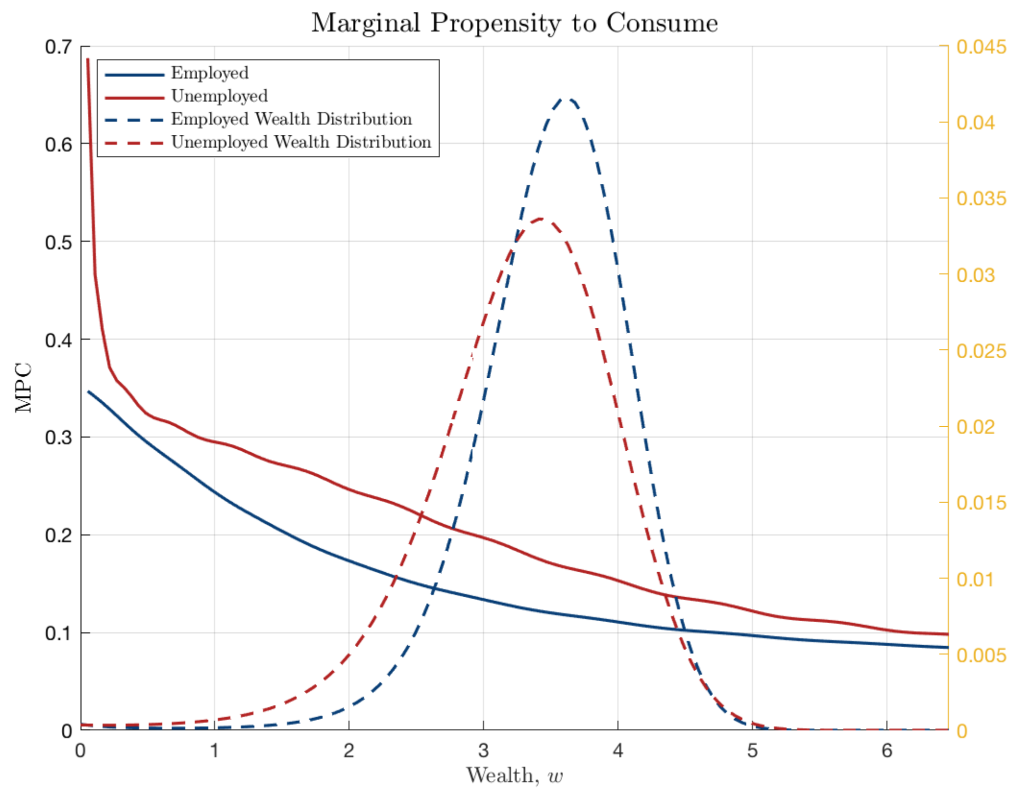
   $$\begin{aligned} \log K' &= c_0 + c_1 \log K + c_2 1_A + c_3 1_A \log K + \epsilon^K \\ q &= d_0 + d_1 \log K + d_2 1_A + d_3 1_A \log K + \epsilon^q \end{aligned}$$

   Using the simulated data.

6. If the estimated coefficients from these regressions are sufficiently close to the guesses, stop. Otherwise go back to 1, update regression coefficients and continue until convergence.

I solve the household problem using value function iteration, discretising over grids for the aggregate state variables (TFP, capital and the bond price) and the individual state variables (bond holdings, capital

**Figure A3:** MPCs in HA Model




Note: This figure plots the marginal propensity to consume of the employed (blue line) and unemployed (red line) households in the deterministic steady state which preserves idiosyncratic risk. The blue dotted line plots the wealth distribution of employed households. The red dotted line plots the wealth distribution of unemployed households

holdings and idiosyncratic productivity). The state space is large with a total of 160,000 grid points. Linear interpolation of the policy and value functions is used when necessary. I find the bond market clearing price each period using a Newton method to find the point at which excess bond demand is equal to zero. The $R^2$ of both PLM regressions exceeds 0.9999 after the coefficients have converged.

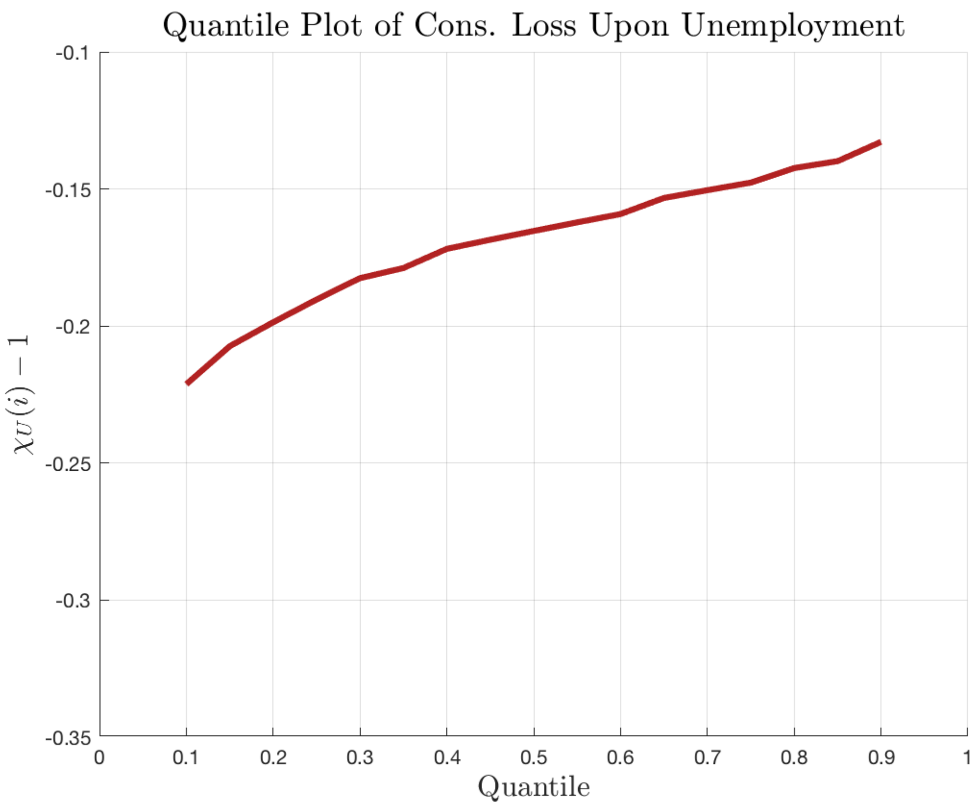
## E.2 Marginal Propensity to Consume Distribution

Figure A3 plots the marginal propensity to consume for both employed and unemployed households across the wealth distribution. Note that households at the borrowing constraint have an MPC of 1 and are not depicted.

## E.3 Quantile Plot of Consumption Loss Upon Unemployment

Figure A4 plots the quantiles of the distribution of consumption loss upon unemployment in the deterministic steady state of the HA model which preserves idiosyncratic risk. This can be compared to the empirical analogue in Figure A1.

**Figure A4:** Quantile Plot of Consumption Loss Upon Unemployment in HA Model




Note: This figure plots the quantiles of the distribution of consumption loss upon unemployment in the deterministic steady state of the HA model which preserves idiosyncratic risk.

### E.4 Two Period Version of Heterogeneous Agent Model

All aspects of the environment are the same except for the fact that the world only lasts for two periods, $t$ and $t+1$. Households only need to make decisions in period t, since in $t+1$ they simply consume their labour and asset income since it is the terminal period. Household i's problem in period $t$ is as follows:

$$V(b(i), a(i), \mathbb{1}^E(i), \Omega) = \max_{C(i), b'(i), a'(i)} \left\{ \frac{C(i)^{1-\gamma}-1}{1-\gamma} + \beta \mathbb{E}\left[\frac{C'(i)^{1-\gamma}-1}{1-\gamma}\right] \right\}$$

$$\begin{aligned} s.t. \qquad C(i) + qb'(i) + a'(i) &= W\mathbb{1}^E(i) + (1-\mathbb{1}^E(i))\iota W + b(i) + R^K a(i) \\ C'(i) &= W'\mathbb{1}^{E'}(i) + (1-\mathbb{1}^{E'}(i))\iota W' + b'(i) + R^{K\prime} a'(i) \\ a'(i) &\geq 0 \\ w'(i) &\geq -\iota \overline{W} \end{aligned}$$

For an employed household who is unconstrained in capital, the following Euler equation holds:[45]

$$0 = \mathbb{E}_t \left[ (1-\rho_{t+1})(C^E_{t+1}(i))^{-\gamma} + \rho_{t+1}(C^U_{t+1}(i))^{-\gamma} \right] R^e_{t+1}$$

Taking a second-order Taylor approximation of this equation around the riskless limit and using the period $t+1$ budget constraint (again approximated around the same point) then yields the equation for household i's demand for the risky asset.

Repeating this procedure for a household who is unemployed in period t and unconstrained in capital yields the following condition:

$$a^U_{t+1}(i) \approx \frac{\overline{C}^E_{t+1}(i)}{\gamma^U(i)\sigma^2_R} \left( \phi^U_R(i)\mathbb{E}_t R^e_{t+1} + \phi^U_{R,\eta}(i)\,\mathrm{Cov}_t(\eta_{t+1}, R^K_{t+1}) - \phi^U_{R,W}(i)\,\mathrm{Cov}_t(W_{t+1}, R^K_{t+1}) \right)$$

Where:

$$\gamma^U(i) = \gamma[\overline{\eta}_{t+1} + (1-\overline{\eta}_{t+1})\overline{\chi}_U(i)^{-\gamma-1}] > 0$$

$$\phi^U_R(i) = \overline{\eta}_{t+1} + (1-\overline{\eta}_{t+1})\overline{\chi}_U(i)^{-\gamma} > 0$$

$$\phi^U_{R,\eta}(i) = 1 - \overline{\chi}_U(i)^{-\gamma} \leq 0$$

$$\phi^U_{R,W}(i) = \gamma \overline{C}^E_{t+1}(i)^{-1}(\overline{\eta}_{t+1} + (1-\overline{\eta}_{t+1})\iota\overline{\chi}_U(i)^{-\gamma-1}) > 0$$

Unemployed agents reduce their holdings of capital if, all else equal, its returns covary more with wages or the job finding rate, or if they are wealth poor and consequently have consumption that is more sensitive to their employment outcome in $t+1$. Under reasonable calibrations, it holds that $\gamma^U(i) > \gamma^E(i)$ and unemployed households are effectively more risk averse than employed households. As such, when a greater proportion of households are unemployed, risk premia rise. This composition effect is an additional reason

[45] Note that the wealth constraint will never bind for any households, since a natural borrowing limit holds which is tighter: $w'(i) > -\iota \min(W')$.

that the heterogeneous agent model generates a countercyclical capital risk premium.

# F Data Sources and Variable Construction For Section 6

1. **Risk-Free Rate**. The return on a 1-month Treasury bill taken from Kenneth French's website.
2. **Real Equity Return**. The return on the CRSP value-weighted market portfolio adjusted by the rate of CPI inflation (FRED code: CPIAUCSL). Taken from Amit Goyal's website.
3. **Dividends**. The 12-month moving average of dividends paid on the S&P 500 price index. Taken from Amit Goyal's website.
4. **Log Price-Dividend Ratio**. The natural log of the ratio between the S&P 500 price index and the 12-month moving average of dividends paid on the S&P 500 price index. Taken from Amit Goyal's website.
5. **Unemployment Rate**. The number of unemployed as a percentage of the labour force, taken from the Current Population Survey (CPS). FRED code: UNRATE.
6. **Real Consumption Growth**. The growth rate of the sum of personal consumption expenditures: nondurable goods (FRED code: PCND) and personal consumption expenditures: services (FRED code: PCESV) from NIPA, deflated by the CPI. Available at the quarterly frequency.