

9.5 THE SIMPLEX METHOD: MIXED CONSTRAINTS

In Sections 9.3 and 9.4, we looked at linear programming problems that occurred in *standard form*. The constraints for the maximization problems all involved \leq inequalities, and the constraints for the minimization problems all involved \geq inequalities.

Linear programming problems for which the constraints involve *both* types of inequalities are called **mixed-constraint** problems. For instance, consider the following linear programming problem.

Mixed-Constraint Problem: Find the maximum value of

$$z = x_1 + x_2 + 2x_3 \quad \text{Objective function}$$

subject to the constraints

$$\left. \begin{array}{rcl} 2x_1 + x_2 + x_3 & \leq & 50 \\ 2x_1 + x_2 & \geq & 36 \\ x_1 & + & x_3 \geq 10 \end{array} \right\} \quad \text{Constraints}$$

where $x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$. Since this is a maximization problem, we would expect each of the inequalities in the set of constraints to involve \leq . Moreover, since the first inequality does involve \leq , we can add a slack variable to form the following equation.

$$2x_1 + x_2 + x_3 + s_1 = 50$$

For the other two inequalities, we must introduce a new type of variable, called a **surplus variable**, as follows.

$$2x_1 + x_2 - s_2 = 36$$

$$x_1 + x_3 - s_3 = 10$$

Notice that surplus variables are *subtracted from* (not added to) their inequalities. We call s_2 and s_3 surplus variables because they represent the amount that the left side of the inequality exceeds the right side. Surplus variables must be nonnegative.

Now, to solve the linear programming problem, we form an initial simplex tableau as follows.

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
2	1	1	1	0	0	50	s_1
2	1	0	0	-1	0	36	s_2
1	0	(1)	0	0	-1	10	$s_3 \leftarrow \text{Departing}$
-1	-1	-2	0	0	0	0	
\uparrow Entering							

You will soon discover that solving mixed-constraint problems can be difficult. One reason for this is that we do not have a convenient feasible solution to begin the simplex method. Note that the solution represented by the initial tableau above.

$$(x_1, x_2, x_3, s_1, s_2, s_3) = (0, 0, 0, 50, -36, -10)$$

is not a feasible solution because the values of the two surplus variables are negative. In fact, the values $x_1 = x_2 = x_3 = 0$ do not even satisfy the constraint equations. In order to eliminate the surplus variables from the current solution, we basically use “trial and error.” That is, in an effort to find a feasible solution, we arbitrarily choose new entering variables. For instance, in this tableau, it seems reasonable to select x_3 as the entering variable. After pivoting, the new simplex tableau becomes

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
1	1	0	1	0	1	40	s_1
2	(1)	0	0	-1	0	36	$s_2 \leftarrow \text{Departing}$
1	0	1	0	0	-1	10	x_3
1	-1	0	0	0	-2	20	
	\uparrow						Entering

The current solution $(x_1, x_2, x_3, s_1, s_2, s_3) = (0, 0, 10, 40, -36, 0)$ is still not feasible, so we choose x_2 as the entering variable and pivot to obtain the following simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
-1	0	0	1	1	(1)	4	$s_1 \leftarrow \text{Departing}$
2	1	0	0	-1	0	36	x_2
1	0	1	0	0	-1	10	x_3
3	0	0	0	-1	-2	56	
				\uparrow			Entering

At this point, we finally obtained a feasible solution

$$(x_1, x_2, x_3, s_1, s_2, s_3) = (0, 36, 10, 4, 0, 0).$$

From here on, we apply the simplex method as usual. Note that the entering variable here is s_3 because its column has the most negative entry in the bottom row. After pivoting one more time, we obtain the following final simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
-1	0	0	1	1	1	4	s_3
2	1	0	0	-1	0	36	x_2
0	0	1	1	1	0	14	x_3
1	0	0	2	1	0	64	

Note that this tableau is final because it represents a feasible solution *and* there are no negative entries in the bottom row. Thus, we conclude that the maximum value of the objective function is

$$z = 64 \quad \text{Maximum value}$$

and this occurs when

$$x_1 = 0, \quad x_2 = 36, \quad \text{and} \quad x_3 = 14.$$

EXAMPLE 1 A Maximization Problem with Mixed Constraints

Find the maximum value of

$$z = 3x_1 + 2x_2 + 4x_3 \quad \text{Objective function}$$

subject to the constraints

$$\left. \begin{aligned} 3x_1 + 2x_2 + 5x_3 &\leq 18 \\ 4x_1 + 2x_2 + 3x_3 &\leq 16 \\ 2x_1 + x_2 + x_3 &\geq 4 \end{aligned} \right\} \quad \text{Constraints}$$

where $x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$.

Solution To begin, we add a slack variable to each of the first two inequalities and subtract a surplus variable from the third inequality to produce the following initial simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
3	2	5	1	0	0	18	s_1
4	2	3	0	1	0	16	s_2
2	(1)	1	0	0	-1	4	$s_3 \leftarrow \text{Departing}$
-3	-2	-4	0	0	0	0	
	\uparrow						Entering

As it stands, this tableau does not represent a feasible solution (because the value of s_3 is negative). Thus, we want s_3 to be the departing variable. We have no real guidelines as to which variable should enter the solution, but by trial and error, we discover that using x_2 as the entering variable produces the following tableau (which does represent a feasible solution).

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
-1	0	3	1	0	2	10	$s_1 \leftarrow \text{Departing}$
0	0	1	0	1	2	8	s_2
2	1	1	0	0	-1	4	x_2
1	0	-2	0	0	-2	8	

Now, because this simplex tableau does represent a feasible solution, we proceed as usual, choosing the most negative entry in the bottom row to be the entering variable. (In this case, we have a tie, so we arbitrarily choose x_3 to be the entering variable.)

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
-1	0	(3)	1	0	2	10	$s_1 \leftarrow$ Departing
0	0	1	0	1	2	8	s_2
2	1	1	0	0	-1	4	x_2
1	0	-2	0	0	-2	8	
		↑					Entering

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
$-\frac{1}{3}$	0	1	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{10}{3}$	x_3
$\frac{1}{3}$	0	0	$-\frac{1}{3}$	1	($\frac{4}{3}$)	$\frac{14}{3}$	$s_2 \leftarrow$ Departing
$\frac{7}{3}$	1	0	$-\frac{1}{3}$	0	$-\frac{5}{3}$	$\frac{2}{3}$	x_2
$\frac{1}{3}$	0	0	$\frac{2}{3}$	0	$-\frac{2}{3}$	$\frac{44}{3}$	
				↑			Entering

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
$-\frac{1}{2}$	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	x_3
$\frac{1}{4}$	0	0	$-\frac{1}{4}$	$\frac{3}{4}$	1	$\frac{7}{2}$	s_3
$\frac{11}{4}$	1	0	$-\frac{3}{4}$	$\frac{5}{4}$	0	$\frac{13}{2}$	x_2
$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	17	

Thus, the maximum value of the objective function is

$z = 17$

and this occurs when

$x_1 = 0, x_2 = \frac{13}{2},$ and $x_3 = 1.$

Mixed Constraints and Minimization

In Section 9.4, we discussed the solution of minimization problems in *standard form*. Minimization problems that are not in standard form are more difficult to solve. One technique that can be used is to change a mixed-constraint minimization problem to a mixed-constraint maximization problem by multiplying each coefficient in the objective function by -1 . We demonstrate this technique in the following example.

EXAMPLE 2 *A Minimization Problem with Mixed Constraints*

Find the minimum value of

$$w = 4x_1 + 2x_2 + x_3 \quad \text{Objective function}$$

subject to the constraints

$$\left. \begin{array}{l} 2x_1 + 3x_2 + 4x_3 \leq 14 \\ 3x_1 + x_2 + 5x_3 \geq 4 \\ x_1 + 4x_2 + 3x_3 \geq 6 \end{array} \right\} \quad \text{Constraints}$$

where $x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$.

Solution First, we rewrite the objective function by multiplying each of its coefficients by -1 , as follows.

$$z = -4x_1 - 2x_2 - x_3 \quad \text{Revised objective function}$$

Maximizing this revised objective function is equivalent to minimizing the original objective function. Next, we add a slack variable to the first inequality and subtract surplus variables from the second and third inequalities to produce the following initial simplex tableau.

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
2	3	4	1	0	0	14	s_1
3	(1)	5	0	-1	0	4	$s_2 \leftarrow \text{Departing}$
1	4	3	0	0	-1	6	s_3
4	2	1	0	0	0	0	
	\uparrow						Entering

Note that the bottom row has the negatives of the coefficients of the revised objective function. Another way of looking at this is that for minimization problems (in nonstandard form), the bottom row of the initial simplex consists of the coefficients of the original objective function.

As with maximization problems with mixed constraints, this initial simplex tableau does not represent a feasible solution. By trial and error, we discover that we can choose x_2 as the entering variable and s_2 as the departing variable. After pivoting, we obtain the following tableau.

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
-7	0	-11	1	3	0	2	s_1
3	1	5	0	-1	0	4	x_2
-11	0	-17	0	4	-1	-10	s_3
-2	0	-9	0	2	0	-8	

From this tableau, we can see that the choice of x_2 as the entering variable was a good one. All we need to do to transform the tableau into one that represents a feasible solution is to multiply the third row by -1 , as follows.

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
-7	0	-11	1	3	0	2	s_1
3	1	5	0	-1	0	4	x_2
11	0	17	0	-4	1	10	$s_3 \leftarrow \text{Departing}$
-2	0	-9	0	2	0	-8	

\uparrow
Entering

Now that we have obtained a simplex tableau that represents a feasible solution, we continue with our standard pivoting operations as follows.

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
$\frac{2}{17}$	0	0	1	$\frac{7}{17}$	$\frac{11}{17}$	$\frac{144}{17}$	s_1
$-\frac{4}{17}$	1	0	0	$\frac{3}{17}$	$-\frac{5}{17}$	$\frac{18}{17}$	$x_2 \leftarrow \text{Departing}$
$\frac{11}{17}$	0	1	0	$-\frac{4}{17}$	$\frac{1}{17}$	$\frac{10}{17}$	x_3
$\frac{65}{17}$	0	0	0	$-\frac{2}{17}$	$\frac{9}{17}$	$-\frac{46}{17}$	

\uparrow
Entering

x_1	x_2	x_3	s_1	s_2	s_3	b	Basic Variables
$\frac{2}{3}$	$-\frac{7}{3}$	0	1	0	$\frac{4}{3}$	6	s_1
$-\frac{4}{3}$	$\frac{17}{3}$	0	0	1	$-\frac{5}{3}$	6	s_2
$\frac{1}{3}$	$\frac{4}{3}$	1	0	0	$-\frac{1}{3}$	2	x_3
$\frac{11}{3}$	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$	-2	

Finally, we conclude that the maximization value of the revised objective function is $z = -2$, and hence the minimum value of the original objective function is

$w = 2$

(the negative of the entry in the lower-right corner), and this occurs when

$x_1 = 0, x_2 = 0, \text{ and } x_3 = 2.$

Applications

EXAMPLE 3 *A Business Application: Minimum Shipment Cost*

An automobile company has two factories. One factory has 400 cars (of a certain model) in stock and the other factory has 300 cars (of the model) in stock. Two customers order this car model. The first customer needs 200 cars, and the second customer needs 300 cars. The cost of shipping cars from the two factories to the customers is shown in Table 9.3.

TABLE 9.3

	Customer 1	Customer 2
Factory 1	\$ 30	\$ 25
Factory 2	\$ 36	\$ 30

How should the company ship the cars in order to minimize the shipping cost?

Solution

To begin, we let x_1 and x_2 represent the number of cars shipped from Factory 1 to the first and second customers, respectively. (See Figure 9.20.) The total cost of shipping is then given by

$$C = 30x_1 + 25x_2 + 36(200 - x_1) + 30(300 - x_2) = 16,200 - 6x_1 - 5x_2.$$

The constraints for this minimization problem are as follows.

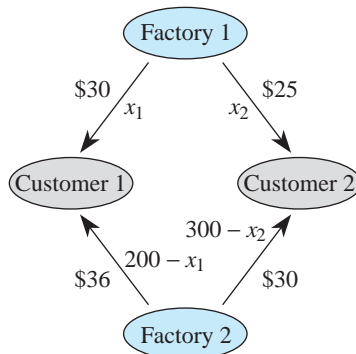
$$\begin{aligned} x_1 + x_2 &\leq 400 \\ (200 - x_1) + (300 - x_2) &\leq 300 & \rightarrow & x_1 + x_2 \geq 200 \\ x_1 &\leq 200 \\ x_2 &\leq 300 \end{aligned}$$

The corresponding maximization problem is to maximize $z = 6x_1 + 5x_2 - 16,200$. Thus, the initial simplex tableau is as follows.

x_1	x_2	s_1	s_2	s_3	s_4	b	Basic Variables
1	1	1	0	0	0	400	s_1
1	1	0	-1	0	0	200	s_2 ← Departing
1	0	0	0	1	0	200	s_3
0	1	0	0	0	1	300	s_4
-6	-5	0	0	0	0	-16,200	

↑
Entering

Figure 9.20



Note that the current z -value is $-16,200$ because the initial solution is

$$(x_1, x_2, s_1, s_2, s_3, s_4) = (0, 0, 400, -200, 200, 300).$$

Now, to this initial tableau, we apply the simplex method as follows.

x_1	x_2	s_1	s_2	s_3	s_4	b	Basic Variables
0	0	1	1	0	0	200	s_1
1	1	0	-1	0	0	200	x_1
0	-1	0	1	1	0	0	s_3
0	1	0	0	0	1	300	s_4
0	1	0	-6	0	0	-15,000	
			↑				
			Entering				

x_1	x_2	s_1	s_2	s_3	s_4	b	Basic Variables
0	1	1	0	-1	0	200	s_1
1	0	0	0	1	0	200	x_1
0	-1	0	1	1	0	0	s_2
0	1	0	0	0	1	300	s_4
0	-5	0	0	6	0	-15,000	
			↑				
			Entering				

x_1	x_2	s_1	s_2	s_3	s_4	b	Basic Variables
0	1	1	0	-1	0	200	x_2
1	0	0	0	1	0	200	x_1
0	0	1	1	0	0	200	s_2
0	0	-1	0	1	1	100	s_4
0	0	5	0	1	0	-14,000	

From this tableau, we see that the minimum shipping cost is \$14,000. Since $x_1 = 200$ and $x_2 = 200$, we conclude that the number of cars that should be shipped from each factory is as shown in Table 9.4.

TABLE 9.4

	Customer 1	Customer 2
Factory 1	200 cars	200 cars
Factory 2	0	100 cars

SECTION 9.5 EXERCISES

In Exercises 1–6, add the appropriate slack and surplus variables to the system and form the initial simplex tableau.

1. (Maximize)

Objective function:

$$w = 10x_1 + 4x_2$$

Constraints:

$$2x_1 + x_2 \geq 4$$

$$x_1 + x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

3. (Minimize)

Objective function:

$$w = x_1 + x_2$$

Constraints:

$$2x_1 + x_2 \leq 4$$

$$x_1 + 3x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

5. (Maximize)

Objective function:

$$w = x_1 + x_3$$

Constraints:

$$4x_1 + x_2 \geq 10$$

$$x_1 + x_2 + 3x_3 \leq 30$$

$$2x_1 + x_2 + 4x_3 \geq 16$$

$$x_1, x_2, x_3 \geq 0$$

2. (Maximize)

Objective function:

$$w = 3x_1 + x_2 + x_3$$

Constraints:

$$x_1 + 2x_2 + x_3 \leq 10$$

$$x_2 + 5x_3 \geq 6$$

$$4x_1 - x_2 + x_3 \geq 16$$

$$x_1, x_2, x_3 \geq 0$$

4. (Minimize)

Objective function:

$$w = 2x_1 + 3x_2$$

Constraints:

$$3x_1 + x_2 \geq 4$$

$$4x_1 + 2x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

6. (Maximize)

Objective function:

$$w = 4x_1 + x_2 + x_3$$

Constraints:

$$2x_1 + x_2 + 4x_3 \leq 60$$

$$x_2 + x_3 \geq 40$$

$$x_1, x_2, x_3 \geq 0$$

In Exercises 7–12, use the given entering and departing variables to solve the given mixed constraint problem.

7. (Maximize)

Objective function:

$$w = -x_1 + 2x_2$$

Constraints:

$$x_1 + x_2 \geq 3$$

$$x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Entering x_2 , departing s_1 .

8. (Maximize)

Objective function:

$$w = 2x_1 + x_2$$

Constraints:

$$x_1 + x_2 \geq 4$$

$$x_1 + x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

Entering x_1 , departing s_1 .

9. (Minimize)

Objective function:

$$w = x_1 + 2x_2$$

Constraints:

$$2x_1 + 3x_2 \leq 25$$

$$x_1 + 2x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Entering x_2 , departing s_2 .

11. (Maximize)

Objective function:

$$w = x_1 + x_2$$

Constraints:

$$-4x_1 + 3x_2 + x_3 \leq 40$$

$$-2x_1 + x_2 + x_3 \geq 10$$

$$x_2 + x_3 \leq 20$$

$$x_1, x_2, x_3 \geq 0$$

Entering x_2 , departing s_2 .

10. (Minimize)

Objective function:

$$w = 3x_1 + 2x_2$$

Constraints:

$$x_1 + x_2 \geq 20$$

$$3x_1 + 4x_2 \leq 70$$

$$x_1, x_2 \geq 0$$

Entering x_1 , departing s_1 .

12. (Maximize)

Objective function:

$$w = x_1 + 2x_2 + 2x_3$$

Constraints:

$$x_1 + x_2 \geq 50$$

$$2x_1 + x_2 + x_3 \leq 70$$

$$x_2 + 3x_3 \geq 40$$

$$x_1, x_2, x_3 \geq 0$$

Entering x_2 , departing s_1 .

In Exercises 13–20, use the simplex method to solve the given problem.

13. (Maximize)

Objective function:

$$w = 2x_1 + 5x_2$$

Constraints:

$$x_1 + 2x_2 \geq 4$$

$$x_1 + x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

14. (Maximize)

Objective function:

$$w = -x_1 + 3x_2$$

Constraints:

$$2x_1 + x_2 \leq 4$$

$$x_1 + 5x_2 \geq 5$$

$$x_1, x_2 \geq 0$$

15. (Maximize)

Objective function:

$$w = 2x_1 + x_2 + 3x_3$$

Constraints:

$$x_1 + 4x_2 + 2x_3 \leq 85$$

$$x_2 - 5x_3 \geq 20$$

$$3x_1 + 2x_2 + 11x_3 \geq 49$$

$$x_1, x_2, x_3 \geq 0$$

16. (Maximize)

Objective function:

$$w = 3x_1 + 5x_2 + 2x_3$$

Constraints:

$$9x_1 + 4x_2 + x_3 \leq 70$$

$$5x_1 + 2x_2 + x_3 \leq 40$$

$$4x_1 + x_2 \geq 16$$

$$x_1, x_2, x_3 \geq 0$$

17. (Minimize)

Objective function:

$$w = x_1 + x_2$$

Constraints:

$$x_1 + 2x_2 \geq 25$$

$$2x_1 + 5x_2 \leq 60$$

$$x_1, x_2 \geq 0$$

19. (Minimize)

Objective function:

$$w = -2x_1 + 4x_2 - x_3$$

Constraints:

$$3x_1 - 6x_2 + 4x_3 \leq 30$$

$$2x_1 - 8x_2 + 10x_3 \geq 18$$

$$x_1, x_2, x_3 \geq 0$$

18. (Minimize)

Objective function:

$$w = 2x_1 + 3x_2$$

Constraints:

$$3x_1 + 2x_2 \leq 22$$

$$x_1 + x_2 \geq 10$$

$$x_1, x_2 \geq 0$$

20. (Minimize)

Objective function:

$$w = x_1 + x_2 + x_3$$

Constraints:

$$x_1 + 2x_2 + x_3 \geq 30$$

$$6x_2 + x_3 \leq 54$$

$$x_1 + x_2 + 3x_3 \geq 20$$

$$x_1, x_2, x_3 \geq 0$$

In Exercises 21–24, maximize the given objective function subject to the following constraints.

$$x_1 + x_2 \leq 5$$

$$-x_1 + x_2 \leq 3$$

$$x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

21. $w = 2x_1 + x_2$ **22.** $w = x_1 + 2x_2$ **23.** $w = x_2$ **24.** $w = -x_1 - x_2$

In Exercises 25–28, maximize the given objective function subject to the following constraints.

$$3x_1 + 2x_2 \geq 6$$

$$x_1 - x_2 \leq 2$$

$$-x_1 + 2x_2 \leq 6$$

$$x_1 \leq 4$$

$$x_1, x_2 \geq 0$$

25. $w = x_1 + x_2$ **26.** $w = x_1 - 2x_2$ **27.** $w = -4x_1 + x_2$ **28.** $w = 4x_1 - x_2$

In Exercises 29–32, a tire company has two suppliers, S_1 and S_2 . S_1 has 900 tires on hand and S_2 has 800 tires on hand. Customer C_1 needs 500 tires and customer C_2 needs 600 tires. Minimize the cost of filling the orders subject to the given table (showing the shipping cost per tire).

29.

	C_1	C_2
S_1	0.60	1.20
S_2	1.00	1.80

30.

	C_1	C_2
S_1	0.80	1.00
S_2	1.00	1.20

31.

	C_1	C_2
S_1	1.20	1.00
S_2	1.00	1.20

32.

	C_1	C_2
S_1	0.80	1.00
S_2	1.00	0.80

33. An automobile company has two factories. One factory has 400 cars (of a certain model) in stock and the other factory has 300 cars (of the model) in stock. Two customers order this car model. The first customer needs 200 cars, and the second customer needs 300 cars. The cost of shipping cars from the two factories to the two customers is as follows.

	<i>Customer 1</i>	<i>Customer 2</i>
<i>Factory 1</i>	\$36	\$30
<i>Factory 2</i>	\$30	\$25

How should the company ship the cars in order to minimize the shipping cost?

34. Suppose in Exercise 33 that the shipping costs for each of the two factories are as follows.

	<i>Customer 1</i>	<i>Customer 2</i>
<i>Factory 1</i>	\$25	\$30
<i>Factory 2</i>	\$35	\$30

How should the company ship the cars in order to minimize the shipping cost?

35. A company has budgeted a maximum of \$600,000 for advertising a certain product nationally. Each minute of television time costs \$60,000 and each one-page newspaper ad costs \$15,000. Each television ad is expected to be viewed by 15 million viewers, and each newspaper ad is expected to be seen by 3 million readers. The company's market research department advises the company to use at least 6 television ads and at least 4 newspaper ads. How should the advertising budget be allocated to maximize the total audience?

36. Rework Exercise 35 assuming that each one-page newspaper ad costs \$30,000.

In Exercises 37 and 38, use the following information. A computer company has two assembly plants, Plant A and Plant B, and two distribution outlets, Outlet I and Outlet II. Plant A can assemble 5000 computers in a year and Plant B can assemble 4000 computers in a year. Outlet I must have 3000 computers per year and Outlet II must have 5000 computers per year. The transportation costs from each plant to each outlet are indicated in the given table. Find the shipping schedule that will produce the minimum cost. What is the minimum cost?

37.

	Outlet I	Outlet II
Plant A	\$4	\$5
Plant B	\$5	\$6

38.

	Outlet I	Outlet II
Plant A	\$4	\$5
Plant B	\$6	\$4

CHAPTER 9 REVIEW EXERCISES

In Exercises 1–6, sketch a graph of the solution of the system of inequalities.

- $x + 2y \leq 160$
 $3x + y \leq 180$
 $x \geq 0$
 $y \geq 0$
- $2x + 3y \leq 24$
 $2x + y \leq 16$
 $x \geq 0$
 $y \geq 0$
- $3x + 2y \geq 24$
 $x + 2y \geq 12$
 $2 \leq x \leq 15$
 $y \leq 15$
- $2x + y \geq 16$
 $x + 3y \geq 18$
 $0 \leq x \leq 25$
 $0 \leq y \leq 15$
- $2x - 3y \geq 0$
 $2x - y \leq 8$
 $y \geq 0$
- $x - y \leq 10$
 $x \geq 0$
 $y \geq 0$

In Exercises 7 and 8, determine a system of inequalities that models the given description, and sketch a graph of the solution of the system.

7. A Pennsylvania fruit grower has 1500 bushels of apples that are to be divided between markets in Harrisburg and Philadelphia. These two markets need at least 400 bushels and 600 bushels, respectively.

8. A warehouse operator has 24,000 square meters of floor space in which to store two products. Each unit of product I requires 20 square meters of floor space and costs \$12 per day to store. Each unit of product II requires 30 square meters of floor space and costs \$8 per day to store. The total storage cost per day cannot exceed \$12,400.

In Exercises 9–14, find the minimum and/or maximum values of the given objective function by the graphical method.

9. Maximize: $z = 3x + 4y$

