

A Comparison of Methods of Facial Expression Recognition

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Abstract— Emotional recognition based on facial expressions is a very active research topic in the field of human-computer interaction. There has been a great body of work with in-depth of study in this area. In this paper, we analyze and compare the state-of-the-art facial expression recognition methods, propose some evaluation dimensions and discuss possible directions for future research.

I. INTRODUCTION

Companion robots capable of emotion recognition are arising extensive attention. Studies indicate that facial expressions play a major role in human emotional expression [1]. However, the academic community has seldom carefully analyzed the advantages and disadvantages of various methods, and seldom objectively compared various methods. We investigate the works with respect to facial expression recognition in recent years. By analyzing and comparing the related works, we aim at finding out different methods suitable for recognizing facial expressions in different scenes, and make suggestions for future work.

This paper has three main contributions: first, we carry on a detailed research to the works with respect to facial expression recognition in recent years. Secondly, after analyzing and summarizing the related works, we propose some reasonable evaluation dimensions for comparing facial expression recognition methods, and compare these methods according to the proposed evaluation dimensions. Thirdly, on the basis of analyzing the comparative results, we figure out different methods that are suitable for recognizing facial expressions in different scenes.

The rest of this paper is organized as follows. Section 2 describes the basic process of facial expression recognition and introduces some related works on facial expression recognition. Section 3 proposes some reasonable evaluation dimensions on comparing facial expression recognition methods. Section 4 compares the methods according to the proposed evaluation dimensions. In Section 5 we summarize the methods for different situations, and in section 6 we draw the conclusion and discuss the future work in this field.

II. OVERVIEW OF FACIAL EXPRESSION RECOGNITION

Most of the facial expression recognition methods generally follow the basic process as shown in Fig. 1. For methods using traditional machine learning methods, as a pattern recognition problem, facial expression recognition has

several key steps: data collection (dataset selection) and preprocessing, feature extraction, feature selection, classification and decision making. Various algorithms can be chosen in each step, especially in the selection of features and the selection of classifiers, and each choice has its own glittering [26, 27, 29]. For methods using emerging machine learning methods, such as deep learning, images are fed into neural networks and through such end-to-end methods, the input images are classified into some facial expression classes directly. In this paper, we mainly discuss the former kind of method in details.

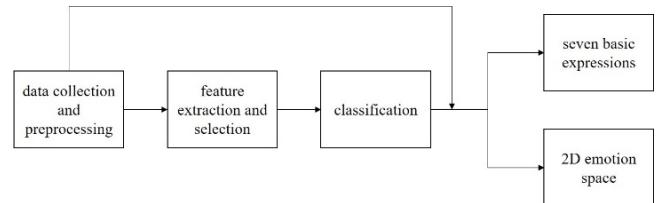


Figure 1. A flow chart of facial expression recognition.

2.1. Data collection and preprocessing

Since the facial expressions captured in the realistic scene can be affected by a variety of factors including illumination variation, head tilt angle variation, out-of-plane head rotation, partial occlusion and so on, appropriate preprocessing to facial expression image can improve the system's robustness and the recognition accuracy. Commonly used preprocessing approaches include the following steps: face detection, face segmentation, face alignment, image denoising, image enhancement and face normalization [79].

2.2. Feature extraction

Preprocessed images have less category-independent features, thus are more conducive to feature extraction. Except for several works that use the original image directly as the input of classifier, commonly used features for facial images can be divided into two types: geometric features and texture features.

Geometric feature refers to the object's position, orientation, circumference and area characteristics in an image. Geometric features of facial images mainly include the position, moving speed and mutual distance of facial feature points. Geometric features of facial images are intuitive, simple and play a very important role in facial image analysis. A 2D or 3D model is

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usually used to extract facial feature points, lines and motion information from the mouth, eyes, eyebrows and nose regions. The most frequently used approaches for extracting geometric features are: locating landmark points based on the definition of facial animation parameters (FAPs) [81]; locating feature points by the active shape model [22, 24, 26, 27]; using Candide node to extract features [13]; using deformed Candide facial grid to extract features [6] and so on.

Texture feature is a visual feature that reflects the homogeneity in an image. It reflects the organizational and arrangement properties of objects' surface, which are slowly or periodically changing. Texture feature has the characteristics of local sequential repetition and non-random arrangement, and texture area is roughly a homogeneous unity. Texture features of facial images are mainly the changes of the image texture, such as skin wrinkles and bulges. For example, decree pattern on the sides of the nose tends to deepen while smiling. Common texture features are: Gabor filter output, pixel intensity, discrete cosine transform features, and skin color information. Most of the texture features are extracted based on: Gabor wavelet [2]; scale invariant feature transform (SIFT) [7]; local binary pattern (LBP) features [4]; Haar-like features [12]; optical flow [80]; Discriminant Non-Negative Matrix Factorization [6].

Md. Zia Uddin et al. [80] proposed a novel and effective method for feature extraction. This method uses optical flow to extract features and further processes the extracted features by principal component analysis (PCA) and generalized discriminant analysis (GDA), and then generates a codebook for the features extracted to facilitate the use of a discrete HMM next. The average recognition rate of this method on CK dataset is 99.16%. The experimental results are superior to those of PCA + linear discriminant analysis (LDA) method: 82.92%, optical flow + PCA method: 92.50% and optical flow + PCA + LDA method: 96.67%.

Other methods used for feature extraction include global spatial analysis [2], local filter [2], dense flow [3] facial feature tracking [3], high gradient detection [3], LBP [4], local phase quantization (LPQ) [4], local binary pattern from three orthogonal planes (LBP-TOP) [4] and local phase quantization from three orthogonal planes (LPQ-TOP) [4].

2.3. Feature selection

Reasonable feature selection method can not only filter out the "false features" caused by non-expression factors, but also eliminate the correlation between features, and reduce feature dimension and computational complexity. Yuqian ZHOU and Bertram E. SHI [5] have shown that feature selection plays an important role in improving the robustness of facial expression recognition system and has a great influence on the performance of the whole system.

Some researchers paid special attention to the issue of feature selection. Yong Yang and Guoyin Wang [81] used the rough set theory to study the selection of facial features. Rough set theory as a data analysis and processing theory, was proposed in 1982 by the Polish Scientist Z. Pawlak [74]. The proposed method is compared with other traditional feature selection methods for facial expression recognition, and proved to outperform other genetic methods.

Considering that feature selection is usually implemented with heuristic algorithms and requires a time-consuming search process, Guodong Guo et al. [25] proposed a method of feature selection by linear programming (FSLP), which can determine the features used for classification and the number of features based on recent optimization results. The proposed method is used to select the features for face expression recognition task, and obtains a recognition rate of 91.0%, which is higher than Bayes and Adaboost algorithm.

Yong Yang and Guoyin Wang [81] skillfully applied the rough set theory to the selection of facial features. This method combines the traditional rough set based on the equivalence relation which needs to be discretized with a self-learning attribute reduction algorithm to avoid the process of discretization. And this method doesn't need prior knowledge, which extends the scope of application of rough set theory. In their experiment, a conventional SVM is selected as classifier. The average recognition rate of this method on three datasets is 79.28%, which is better than that of genetic algorithm: 72.64%. The conclusion is that this method is superior to other traditional feature selection methods. Since this work mainly focuses on the comparison between the proposed feature selection method and the traditional feature selection methods, 33 geometric features are calculated based on 52 feature points in the feature extraction stage and SVMs with the same parameters are used in the classification stage to control the variables, thus yields a slightly lower recognition rate than other works.

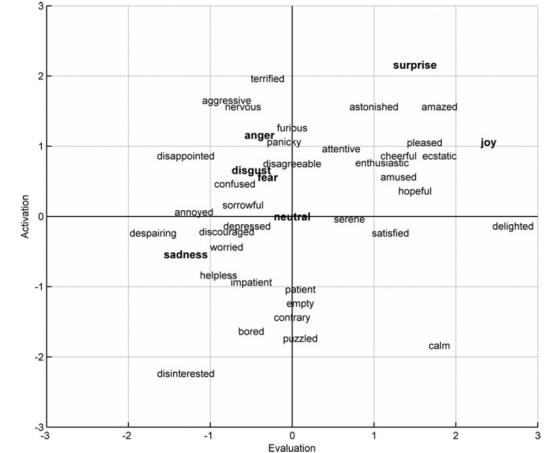


Figure 2. Several affective words positioned in evaluation-activation space [23].

2.4. Classification and decision making

2.4.1. Facial Expression classification

In the process of facial expression recognition, the core step is to classify a given facial image or video sequence, which means mapping it to a given class space based on the extracted features.

Most of the works prefer to map the given samples to seven expression classes based on the basic emotional theory proposed by Ekman et al. [22]. The theory, based on experimentation and analysis, sums up six basic emotions: happy, anger, sadness, disgust, fear and surprise. Samples are classified into six basic expression classes, plus a neutral

expression class. This expression classification method greatly simplifies the modeling of emotions and is universal to people with different cultures and ages.

Nevertheless, the expression classification method of basic expression classes is too rough for describing the emotional state, for the reason that it does not emphasize the intensity, duration, excitement and other factors of emotional expression. Therefore, some studies proposed the 2D emotional spaces, in which the evaluation-activation 2D emotional space proposed by Whissell [23] is the most common one, as shown in Fig. 2. Whissell's 2D emotional space has two dimensions: the evaluation dimension is used to measure the subjective feelings of people, from positive to negative; and the activation dimension is used to measure whether people will act in the current emotional state, from active to passive. Whissell maps around 9,000 emotional words to this 2D emotional space, and almost every emotion is corresponding to a point in the 2D space. This kind of facial expression classification method can describe more comprehensive emotional states in the form of 2D coordinates, and measure the intensity of emotion according to the division of space by two dimensions.

Because there are not many works based on 2D emotional space classification method, in this paper there is little discussion on this. We mainly compare the related works based on the facial expression classification method of seven expression classes in this paper.

2.4.2. Classification methods

The selected feature vectors can then be used as input of the selected classifier, which are divided into basic expression classes by the classifier. Commonly used classification methods for geometric features are: neural network [79], empirical classification rules [14], Hidden Markov Model (HMM) [80], Adaboost algorithm [12], support vector machine (SVM) [81] and so on. The classification methods commonly used for texture features are relatively few, mainly including neural networks [79], empirical classification rules [14], Adaboost algorithm [12], SVM [18] and so on.

Different from the popular machine learning methods, Anisha Halder et al. [82] applied the type-2 fuzzy sets to facial expression recognition task. This work introduces two kinds of type-2 fuzzy set: interval type-2 fuzzy set (IT2FS) and general type-2 fuzzy set (GT2FS). Interval Type-2 fuzzy set mainly considers the uncertainty between different subjects, namely intragroup uncertainty, but ignores intergroup uncertainty. Thus it is more prone to error. General type-2 fuzzy set takes full account of these two uncertainties by introducing the Secondary Membership Function, but it is obvious that the computational complexity is also greatly increased. These two methods have their own advantages and disadvantages in practical application, which is a trade-off between computational complexity and recognition accuracy. In the experiment, IT2FS and GT2FS obtain the average recognition rates of 90.88% and 97.328% respectively, and the conclusion is that the proposed method outperforms the traditional methods.

Other methods used for classification include Discriminant Analysis [3], HMM [3], Convolutional Neural Network [5], Median Radial Basis Function Neural Network [6], Selective

Transfer Machine [STM] [7]. TLCNN [5] achieves a recognition rate of 96.95% on CK+ dataset; the fusion of SVM and MRBFNN achieves 92.1% recognition rate on CK dataset [6].

In order to improve practicability in real world scenarios, many methods are applied to diminish the influence of illumination variation, head pose variation, partial occlusion and other issues. Wenming Zheng et al. [83] studied non-frontal facial expressions recognition and introduced some non-frontal facial expressions recognition methods and several frequently used databases, focusing on 2D face image recognition. When the LGBP features are used, the highest recognition rate is 77.67% on the BU-3DFE database. According to their paper, non-frontal images are more practical in real world scenarios, and taking into account the symmetry of faces, non-frontal images may contain extra information than front images, thus may result in better recognition performance combined with frontal images. Future work may consider recognizing facial expressions in conjunction with face images in different views in order to get higher accuracy.

For individual differences caused by age, gender, race and etc., setting a baseline for each individual would be considered. At present, most facial expression recognition systems are subject-independent, that is to say different individuals are not distinguished. But Chuan-Yu Chang et al. [79] proposed a subject-dependent facial expression recognition system. The system performs face detection and preprocessing on the face image, and requires an additional step of face recognition before extracting facial features. After extracting the facial edge feature and the facial feature distance, the method uses the radial basis function neural network (RBFNN) to recognize facial expressions. The method achieves recognition rates of 95.87% and 88.21% on TWFE [11] and JAFFE [26] databases, respectively, and has some advantages when compared with subject-independent methods. It can be found that individual differences cannot be ignored in the presentation of facial expressions. Therefore, in the case of having enormous samples, the subject-dependent recognition method can achieve higher accuracy in most cases.

III. EVALUATION DIMENSIONS

After analyzing and summarizing the related works, we propose several reasonable dimensions of evaluating different facial expression recognition methods:

- Real-time: Due to the complexity and variability of human emotion expression, it is necessary to have a strong real-time facial expression recognition method, in order to response to user's emotions in time.
- Practicality: The generalization of a facial expression recognition method is of much significance, especially for companion robots. The good performance on dataset is far from enough. Real world scenarios should also be taken into careful consideration, such as illumination variation, face occlusion and so on.
- Accuracy: Recognition accuracy is an objective indicator while comparing different methods, and no matter what methods are compared, the ultimate goal is to design a

system which can achieve higher accuracy on facial expression recognition.

- Posed or spontaneous expressions: Facial expression datasets can be broadly divided into two categories, posed facial expression dataset and spontaneous facial expression dataset. Posed facial expressions are often obtained by instructing actors to perform a specific facial expression, and spontaneous facial expression is often captured while having the subject watch a video.

IV. COMPARISON OF RELATED WORKS

This section compares the related work in the field of facial

expression recognition in recent years based on the proposed evaluation dimensions. On the basis of analyzing and comparing the results, we evaluate various representative related work from all aspects and draw the conclusion.

4.1. Comparative results

Based on the proposed evaluation dimensions, we compare the related work. TABLE I is the comparison of related works on facial expression recognition, and for the convenience of comparison, the dimension of dataset is the main basis for the division.

TABLE I. COMPARISON OF RELATED WORKS ON FACIAL EXPRESSION RECOGNITION

No.	References	Features	Classification Method	Real-time	Practicality	Database	Posed or Spontaneous Expression	Average Accuracy (%)
1	Kaihao Zhang <i>et al.</i> [86], 2017	Original image	PHRNN ¹ -MSCNN ²	-	Basic practicality	CK+	Posed + Spontaneous	98
2	André Teixeira Lopes <i>et al.</i> [87], 2017	Original image	A combination of CNN	-	Basic practicality			97
3	Yuqian ZHOU <i>et al.</i> [5], 2017	Original image	CNN(TLCNN)	-	Basic practicality			97
4	Nianyin Zeng <i>et al.</i> [85], 2018	Geometric features: AMM ³ features Hog features	DSAE ⁴	-	Basic practicality			96
5	Patrick Lucey <i>et al.</i> [84], 2010	Geometric features: AMM features	SVM	-	Basic practicality			86
6	Irene Kotsia <i>et al.</i> [13], 2007	Geometric features: geometrical displacement of Candide node	SVM	Yes	Basic practicality	CK	Posed	99
7	Md. Zia Uddin <i>et al.</i> [80], 2013	Optical flow + PCA + GDA	HMM	No	Basic practicality			99
8	Peng Yang <i>et al.</i> [12], 2007	Texture features: dynamic haar-like features	Adaboost	-	Basic practicality			97
9	M.F. Valstar <i>et al.</i> [17], 2006	Geometric features: 20 facial feature points	SVM	-	Basic practicality			95
10	Irene Kotsia <i>et al.</i> [6], 2007	Texture features: DNMF ⁵ , Geometric features: deformed Candide facial grid	SVMs, MRBFNN ⁶	-	Basic practicality			92
11	Yong Yang <i>et al.</i> [81], 2009	33 geometric features ⁷	SVM	No	Basic practicality			79

¹ Part-based Hierarchical Bidirectional Recurrent Neural Network.

² Multi-Signal Convolutional Neural Network.

³ Active Appearance Model.

⁴ Deep Sparse Autoencoders.

⁵ Discriminant Non-Negative Matrix Factorization.

⁶ Median Radial Basis Function Neural Network.

⁷ Defined by 52 facial feature points.

12	Chuan-Yu Chang <i>et al.</i> [79], 2009	Geometric features: 16 facial feature points	RBFNN	Yes	Subject-dependent recognition.	TWFE	Posed	92
13	Anisha Halder <i>et al.</i> [82], 2013	5 facial geometric features	Custom methods (IT2FS, GT2FS)	-	Basic practicality	JAFFE	Posed	95
12*	Chuan-Yu Chang <i>et al.</i> [79], 2009	Geometric features: 16 facial feature points	RBFNN	Yes	Subject-dependent recognition.			92
11*	Yong Yang <i>et al.</i> [81], 2009	33 geometric features	SVM	No	Basic practicality			79
2*	André Teixeira Lopes <i>et al.</i> [87], 2017	Original image	A combination of CNN	-	Basic practicality			54
14	Wenming Zheng <i>et al.</i> [83], 2015	Texture features: LGBP ⁸	SVM	-	Can be used for non-frontal facial images.	BU-3DFE	Posed	78
2*	André Teixeira Lopes <i>et al.</i> [87], 2017	Original image	A combination of CNN	-	Basic practicality			73
3*	Yuqian ZHOU <i>et al.</i> [5], 2017	Original image	CNN(TLCNN)	-	Basic practicality	KDEF, RaFD	Posed	97
1*	Kaihao Zhang <i>et al.</i> [86], 2017	Original image	PHRNN-MSCNN	-	Basic practicality	MMI	Spontaneous	81

There are several commonly used datasets: (1) Cohn-Kanade (CK) Database [19]: 486 sequences from 97 subjects, each sequence containing images from the onset state to the apex state; and spontaneous facial expression databases, such as: (2) MMI Facial Expression Database [20]: 2,900 videos and images from 75 subjects, each video sequence starting from the neutral state, reaching the apex state gradually, and then returning to the neutral state; (3) RU-FACS Spontaneous Expression Database [21]: video sequences of 100 objects was collected, 2.5 minutes for each subject; (4) Extended Cohn-Kanade (CK+) Database [84]: based on the CK dataset, 107 sequences from 26 poser are added, and the peak expression in each sequence were encoded by FACS and the corresponding affective tags are modified and verified; in addition to this, 122 spontaneous smile sequences from 66 subjects are added (Most of the related works realized the recognition of posed expression, which is represented as "basic practicality" in TABLE I).

4.2. Analysis and summary of comparative results

Mostly related works choose to classify the facial expression images or video sequences into seven expression classes, and some others choose to classify expressions into six basic expression classes without neutral expression. Only a small part of the studies classifies expressions into less or more expressions classes based on their experimental needs. For example, [82] divided expression into four out of six basic

facial expression classes: happy, anger, disgust and fear. While conversely [27] divided expression into 22 different facial expressions including basic expressions and combined expressions.

Among methods recognizing seven expressions from posed video sequence that only contains one single expression [6, 12, 13, 17, 23], different features and classifiers have obvious influence on these methods' performance. The most prominent recognition performance is gained by the work which uses grid tracking and deformation system to extract the maximum geometric deformation of facial grid and chooses multi-class SVM to classify [13]. Its recognition rate on CK database is 99.7%.

In terms of practicality, it can be seen that methods taking real world scenarios into account, such as: [9, 26], usually have lower accuracy. Methods using spontaneous facial expression database, such as: [84], have lower recognition accuracy than methods only using posed facial expression image, such as: [13]. In real world scenarios, facial expressions may appear continuously. For example, it is likely that before an expression completely disappeared, another expression has already occurred. An application for facial expression recognition need to be able to capture the user's spontaneous emotions without too much restrictions to shooting environment, so higher requirements to facial expression recognition system have been put forward. A good

⁸ Local binary pattern feature extracted from Gabor images.

facial expression recognition system should roundly take into account the variation of illumination, head rotation, occlusion and other issues, and can deal with spontaneous emotions occurred continuously with strong robustness. With respect to how to solve these problems, there is still a lot of room for research.

V. METHODS SUITABLE FOR DIFFERENT SITUATIONS

In the related works involved in the comparison, most of them choose the expression classification method of seven expressions classes, which is relatively simple to implement, but can only recognize a limited number of expressions, and cannot describe the intensity of expression. Methods are more practical while taking into account the issues existing in real world scenarios, such as head post changes, partial occlusion but sacrificing a little accuracy.

On posed facial expression databases, the method employing grid tracking and deformation system to extract the maximum geometric deformation of facial grid and using SVM to classify [13] obtains the best recognition performance, which is up to 99.7% and maybe the best on CK database.

In the aspect of the recognition on non-frontal facial images, [9] and [83] are all opted to use 3D databases to obtain facial images from different views. [9] uses VGG-Face for classification, with a recognition rate of 78% on BP4D datasets and [83] uses SVM for classification, with a 78% recognition rate on BU-3DFE datasets, and both of them have large improving space.

VI. CONCLUSION & FUTURE WORK

In recent years, many researches have made in-depth progress in the field of facial expression recognition. Nevertheless, the advantages and disadvantages of various methods have seldom been carefully analyzed, and various methods have seldom been compared. On the basis of analyzing and summarizing the related works, some reasonable evaluation dimensions for the comparison of facial expression recognition methods are proposed. In accordance with that, various works are compared under the proposed dimensions, and the advantages and disadvantages are analyzed according to the comparative results.

Future work should pay more attention to real-time spontaneous expression recognition tasks in real world scenarios, taking full account of the impact of problems such as variation of illumination, head motion, partial occlusion to enhance the robustness of facial expressions recognition system. Current works on facial expression recognition all focus on one dataset or several datasets. Only few works consider recognizing facial expressions from images or video sequences obtained in real world scenarios. However, a practical facial expression recognition method needs to be employed in complex external environment. Therefore, stable facial expression recognition should be implemented in unconstrained real world scenarios.

At the same time, individual differences caused by different factors such as age, gender, race and so on should also be

considered. And in practical applications, subject-dependent or subject-independent method can be chosen depending on the specific problems. In addition, there are many important expressions in real life that are not covered in seven expressions, such as shame, shyness, and embarrassment. Future work can selectively recognize more expressions according to different needs.

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