Exam 3
MATH 141 - Calculus II
Wednesday, April 8

Instructions: You may use your textbook and class notes. You MAY NOT use any online resource or mathematical software or calculator. You MAY NOT be in communication with anyone else, whether in person or via phone, text, email, chat, etc., for the duration of the examination.

You will be asked to write and sign the honor pledge on the last page of your exam:

"*I pledge on my honor that I have not given or received any **unauthorized assistance** on this examination.*

There are 5 problems. Work must be shown to receive credit, and partial credit will be given for progress towards a solution.

Please upload your completed exam to elms for grading by 11:59 p.m. *Points will be deducted for each minute late your exam is uploaded.*

(1) (a) (10 points) Consider the integral

$$\int \sec^6 x \tan^5 x \, dx$$

Evaluate this integral in TWO ways.

(b) (10 points) Find the volume of the solid obtained by rotating the region bounded by $y = \tan^2 x$, $y = 0$, $x = 0$, and $x = \pi/4$ about the x-axis.

(2) (a) (10 points) Using a trigonometric substitution, evaluate the indefinite integral

$$\int x\sqrt{x^2-9}\,dx$$

(b) (10 points) Using a trigonometric substitution, evaluate the definite integral

$$\int_0^4 x^3\sqrt{16-x^2}\,dx$$

(3) (a) (10 points) Find the area of the region under the curve

$$y = \frac{1}{x^2 - 6x + 8}$$

from $a = 5$ to $b = 10$.

(b) (10 points) Evaluate the integral

$$\int \frac{x^3 + x^2 + 4x + 5}{x^2 + 4}\, dx$$

(4) (a) (7 points) Write down the approximation using Simpson's rule for the integral

$$\int_0^2 e^{x^2}\,dx$$

using $n = 4$ subintervals. You do not need to find the final numerical answer.

(b) (3 points) Show that the fourth derivative of $f(x) = e^{x^2}$ is

$$f^{(4)}(x) = (12 + 48x^2 + 16x^4)e^{x^2}$$

(c) (8 points) Now find an upper bound

$$E_n^S \le \frac{K_S}{180n^4}(b-a)^5$$

for the error E_4^S in the approximation from part (a).

(d) (2 points) Using the error estimates E_n^T and E_n^S, in one sentence tell why Simpson's Rule generally gives a better estimate of a given integral than the trapezoidal rule if n is large enough.

(5) Determine whether each integral is convergent or divergent. Evaluate those which are convergent.

(a) (10 points)

$$\int_0^\infty x^2 e^{-x}\, dx$$

(b) (10 points)

$$\int_2^\infty \frac{1}{x^2 - 4x + 4}\, dx$$

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Please don't forget to upload your work to elms.