Geometric Modeling for Computer Graphics

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Computer Graphics

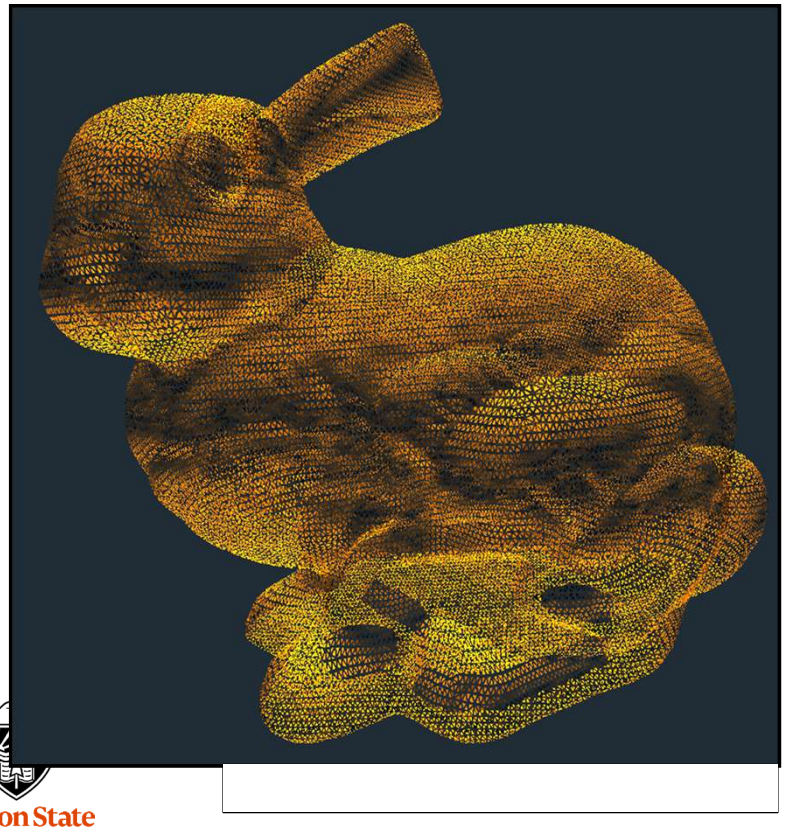
What do we mean by "Modeling"?

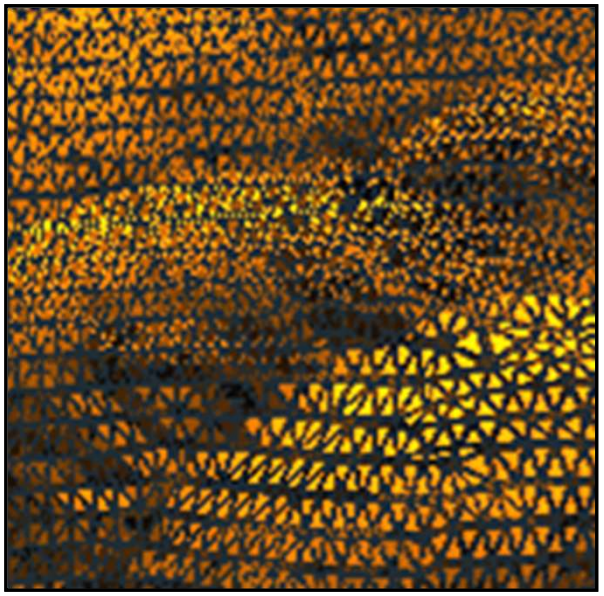
How we model geometry depends on what we would like to use the geometry for:

- Looking at its appearance
- Interacting with its shape?
- How does it interact with its environment?
- What is its surface area and volume?
- Will it need to be 3D-printed?
- Etc.

Explicitly Listing Geometry and Topology

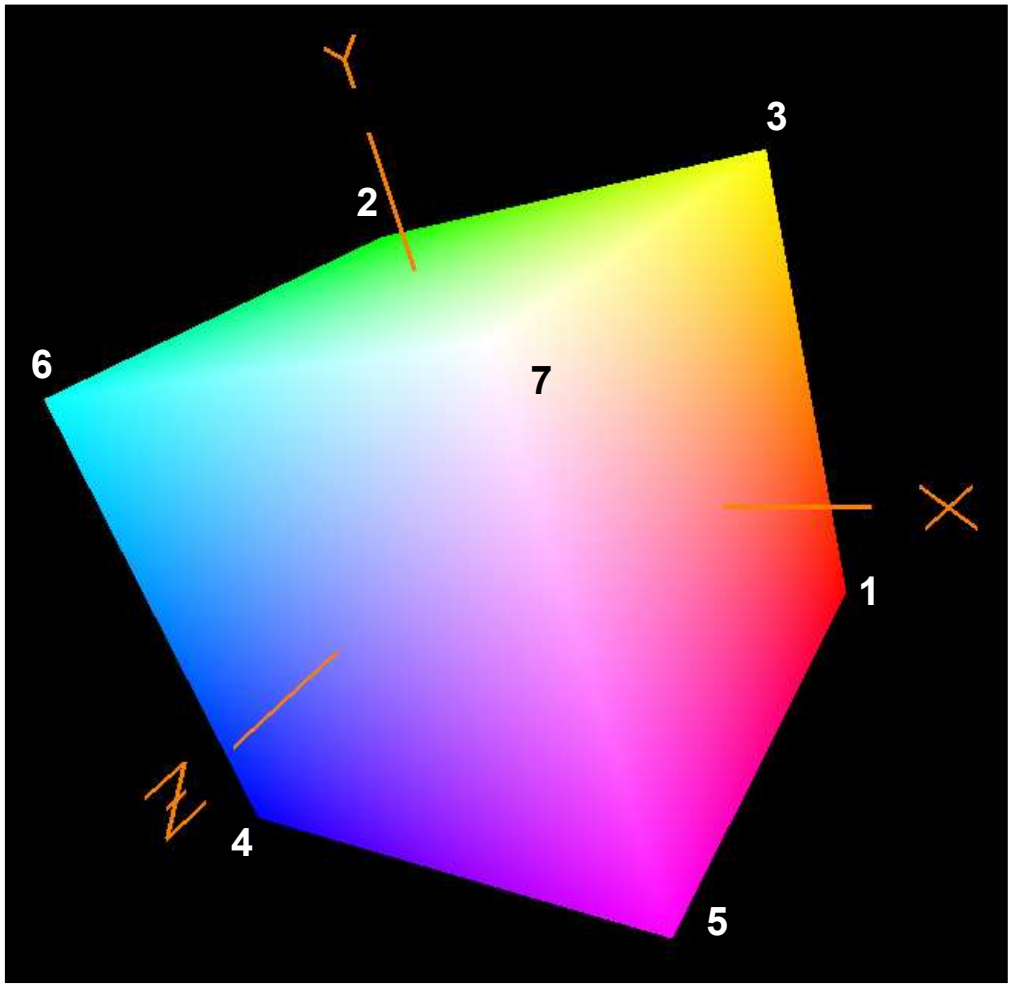
Models can consist of thousands of vertices and faces – we need some way to list them efficiently



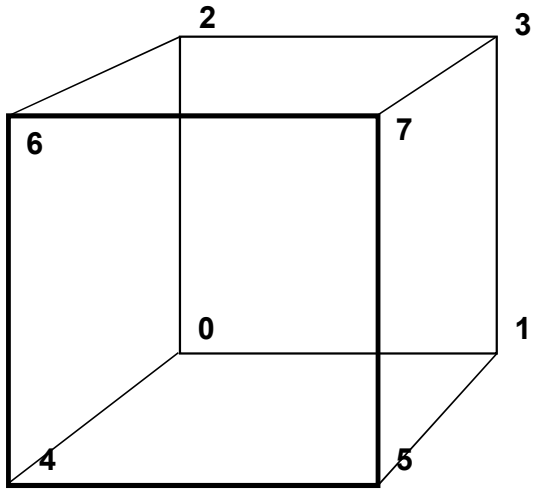


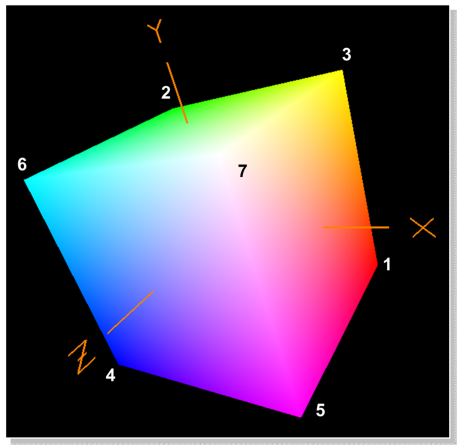
This is called a **Mesh.**
If it's in nice neat rows like this, it is called a **Regular Mesh**. If it's not, it is called an **Irregular Mesh**, or oftentimes called a **Triangular Irregular Network**, or **TIN**.

Cube Example



Explicitly Listing Geometry and Topology



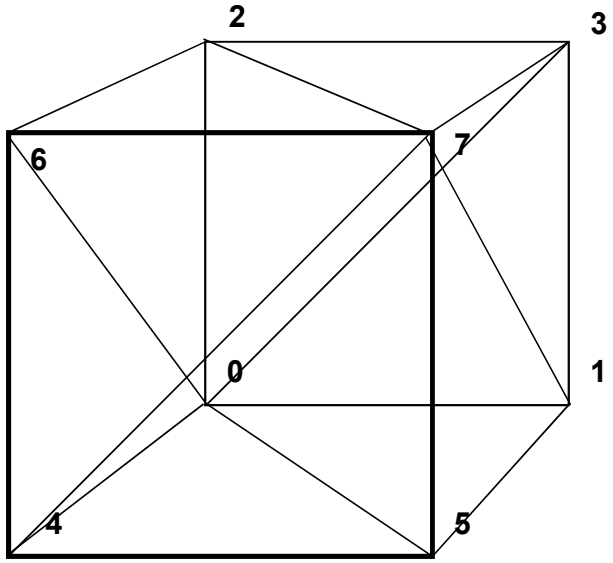


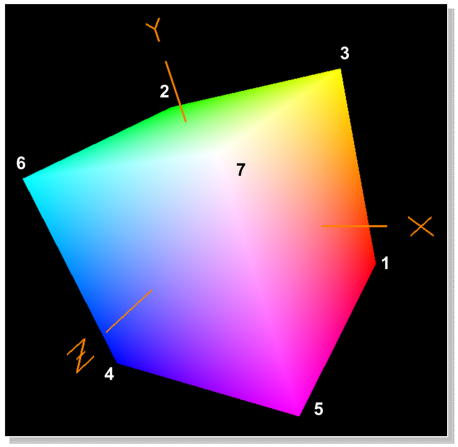

```
static GLfloat CubeVertices[ ][3] =
{
        { -1., -1., -1. },
        {  1., -1., -1. },
        { -1.,  1., -1. },
        {  1.,  1., -1. },
        { -1., -1.,  1. },
        {  1., -1.,  1. },
        { -1.,  1.,  1. },
        {  1.,  1.,  1. }
};
```

```
static GLfloat CubeColors[ ][3] =
{
        { 0., 0., 0. },
        { 1., 0., 0. },
        { 0., 1., 0. },
        { 1., 1., 0. },
        { 0., 0., 1. },
        { 1., 0., 1. },
        { 0., 1., 1. },
        { 1., 1., 1. },
};
```

```
static GLuint CubeQuadIndices[ ][4] =
{
        { 0, 2, 3, 1 },
        { 4, 5, 7, 6 },
        { 1, 3, 7, 5 },
        { 0, 4, 6, 2 },
        { 2, 6, 7, 3 },
        { 0, 1, 5, 4 }
};
```

The Cube Can Also Be Defined with Triangles

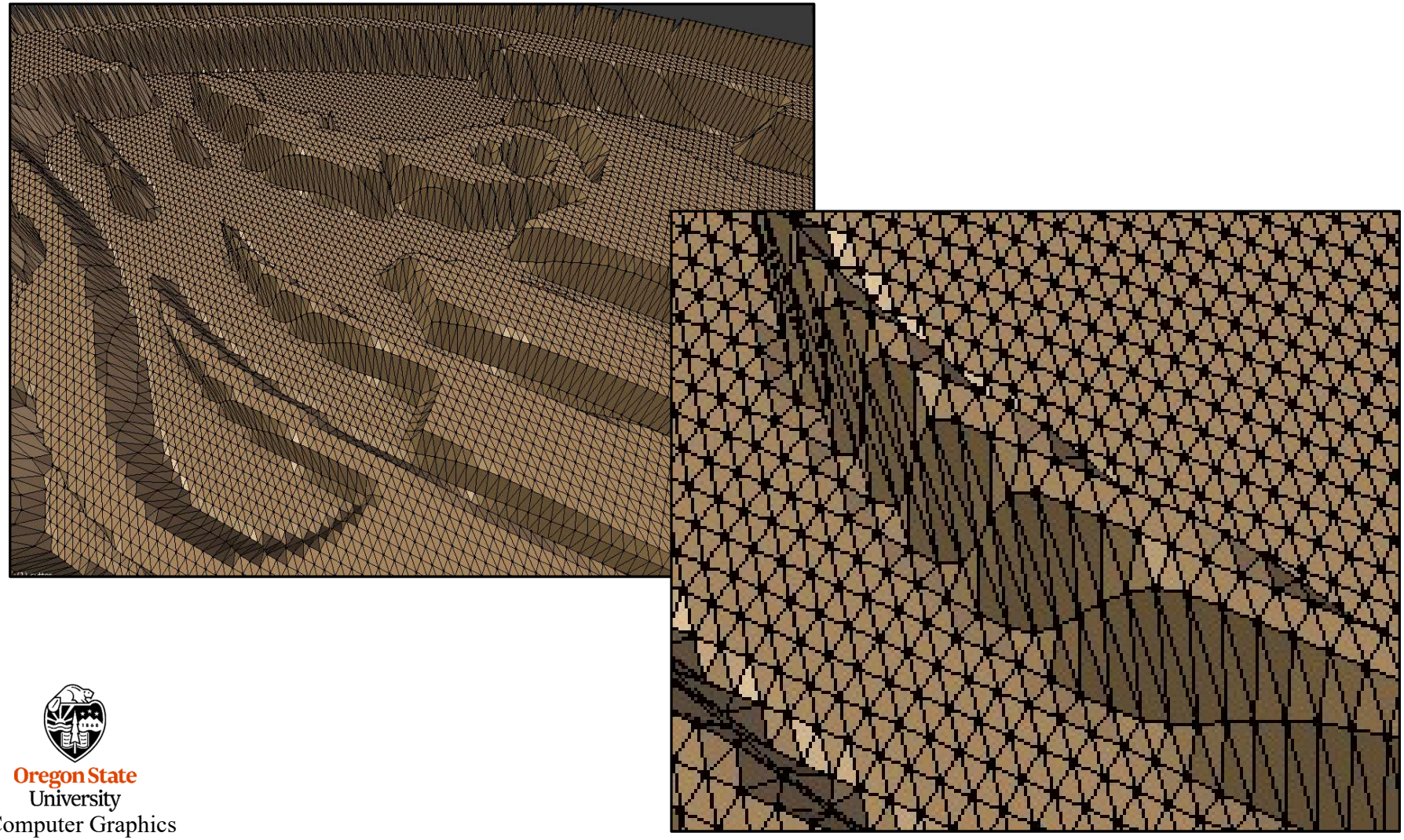





```
GLuint CubeQuadIndices[ ][4] =
{
        { 0, 2, 3, 1 },
        { 4, 5, 7, 6 },
        { 1, 3, 7, 5 },
        { 0, 4, 6, 2 },
        { 2, 6, 7, 3 },
        { 0, 1, 5, 4 }
};
```

```
GLuint CubeTriangleIndices[ ][3] =
{
        { 0, 2, 3 },
        { 0, 3, 1 },
        { 4, 5, 7 },
        { 4, 7, 6 },
        { 1, 3, 7 },
        { 1, 7, 5 },
        { 0, 4, 6 },
        { 0, 6, 2 },
        { 2, 6, 7 },
        { 2, 7, 3 },
        { 0, 1, 5 }
        { 0, 5, 4 }
};
```

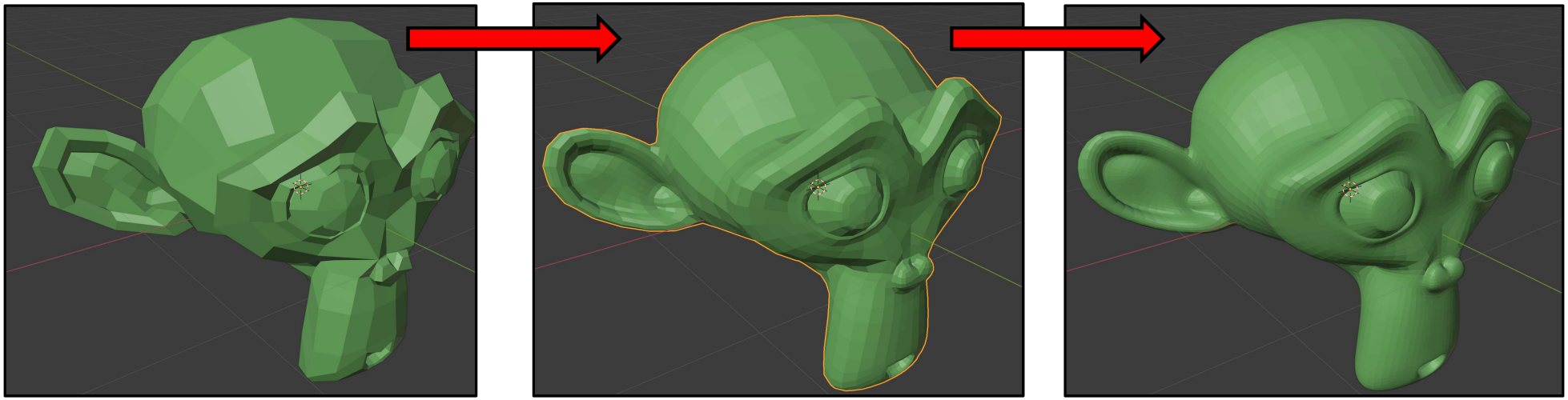
3D Printing uses an Irregular Triangular Mesh Data Format



3D Printing uses a Triangular Mesh Data Format

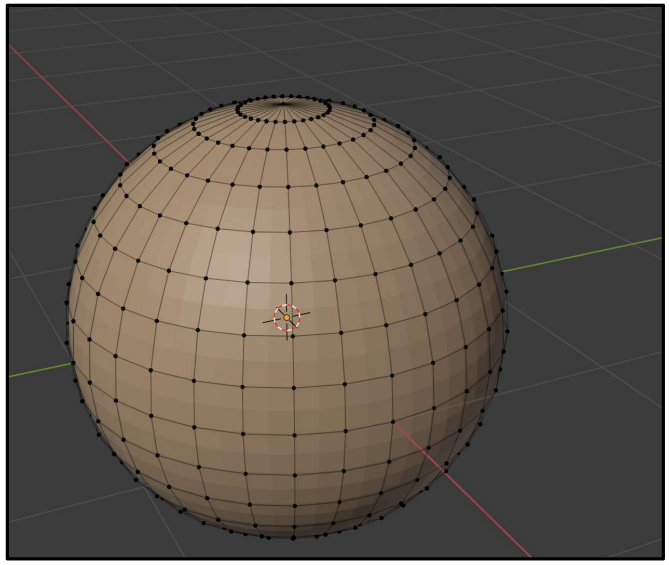
Go Beavs – mmmmmm! ☺

Meshes Can Be Smoothed

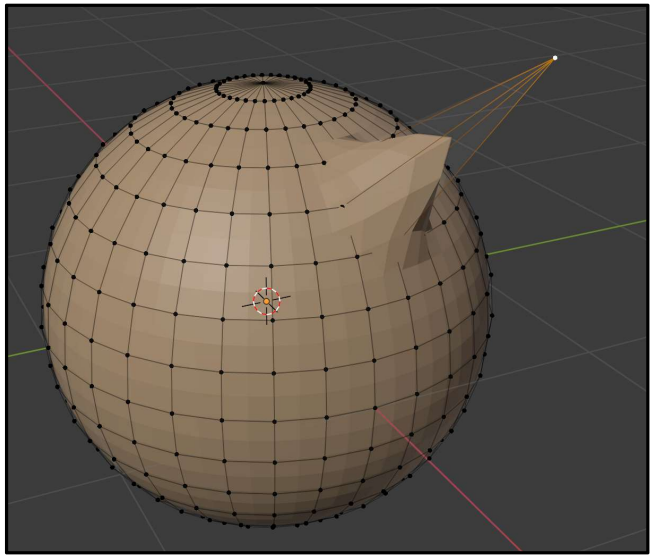


Editing the Vertices of a Mesh

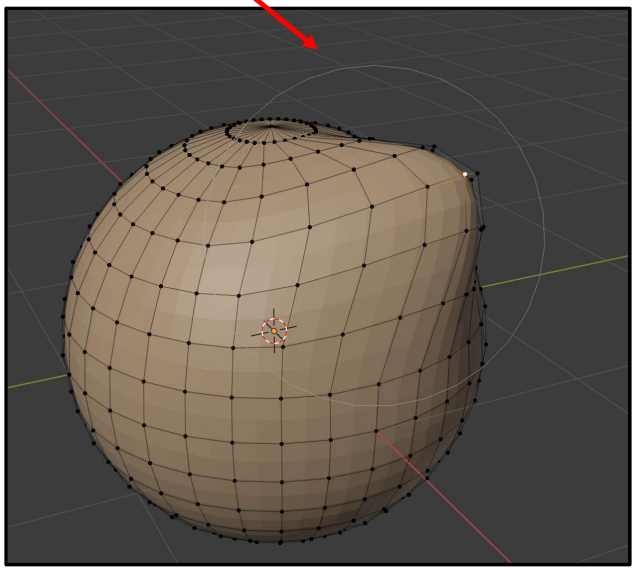
"Circle of Influence"



Original

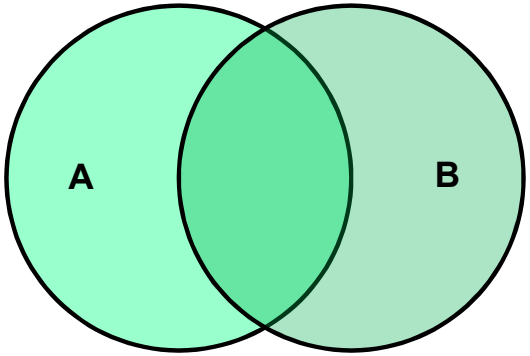


Pulling on a single Vertex

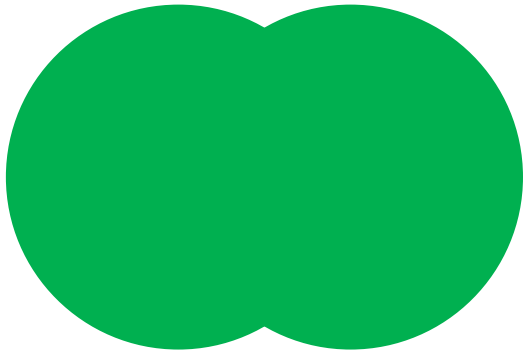


Pulling on a Vertex with Proportional Editing Turned On

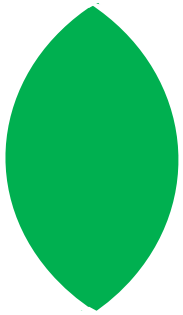
Remember Venn Diagrams (2D Boolean Operators) from High School?



Two Overlapping Shapes



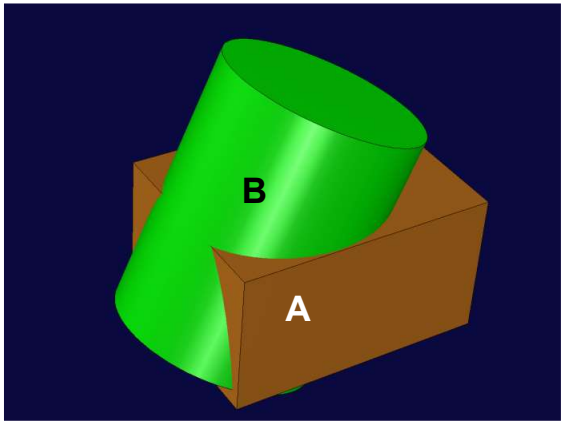
Union: A∪B



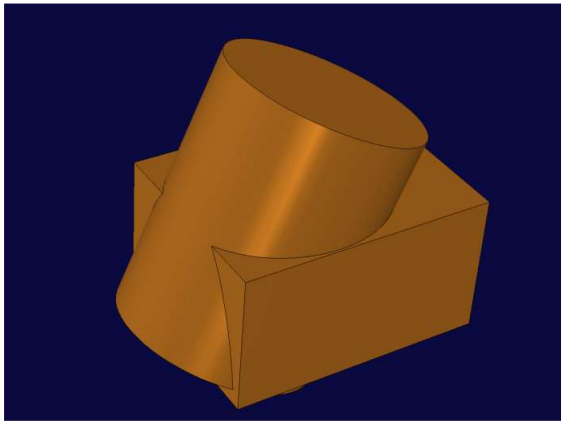
Intersection: A∩B

Difference: A-B

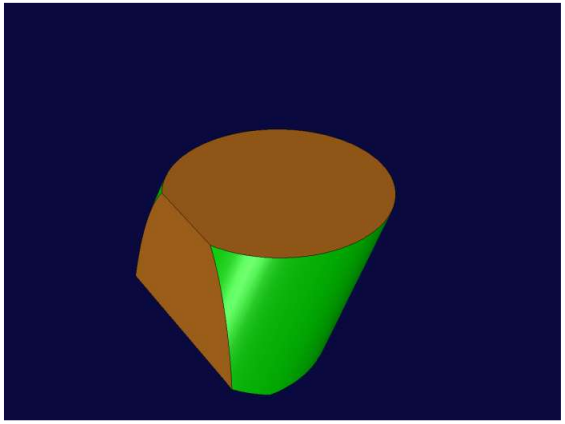
Well, Welcome to Venn Diagrams in 3D



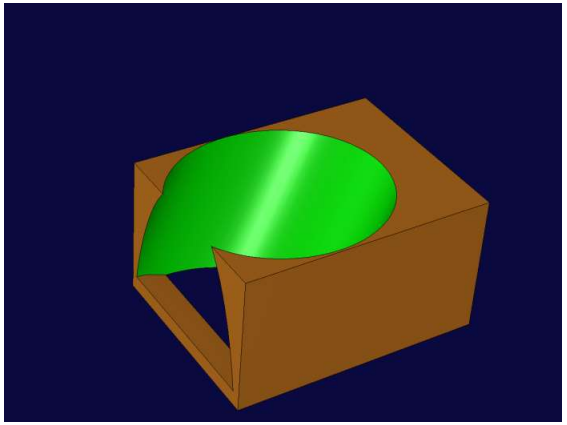
Two Overlapping Solids



Union: A∪B



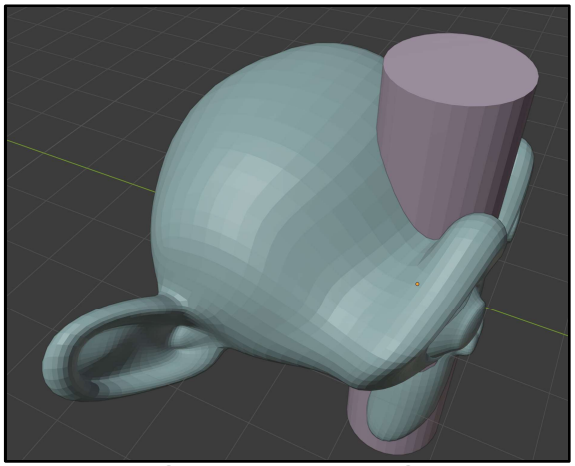
Intersection: A∩B



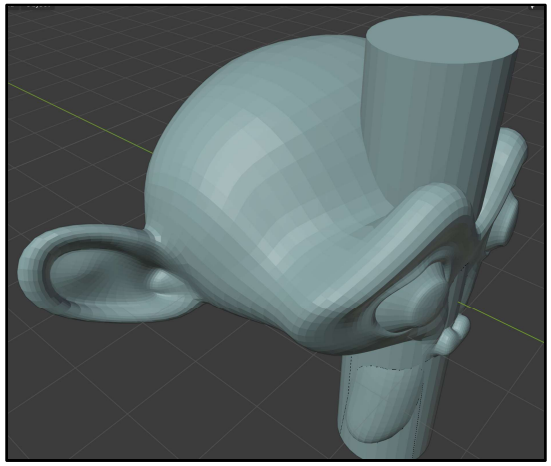
Difference: A-B

This is often called **Constructive Solid Geometry**, or CSG

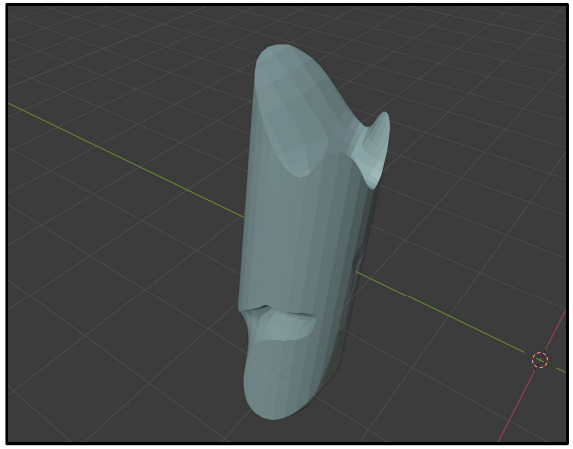
Geometric Modeling Using 3D Boolean Operators



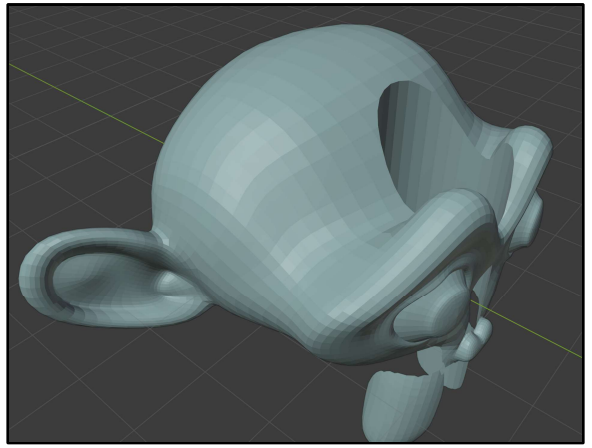
Two Overlapping Solids



Union: A∪B

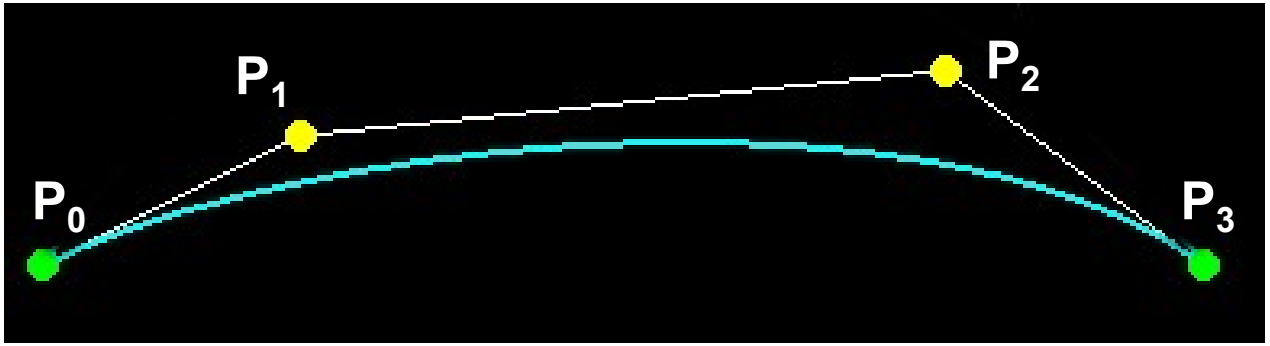


Intersection: A∩B



Difference: A-B

Another way to Model:
Curve Sculpting – Bezier Curve Sculpting



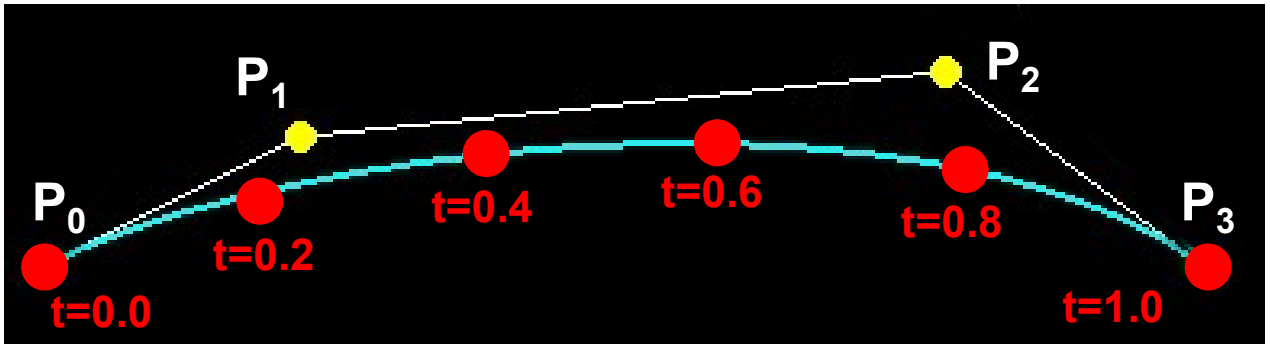
$$P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3$$

$$0. \le t \le 1.$$

where ***P*** represents $\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$

t goes from 0.0 to 1.0 in whatever increment you'd like

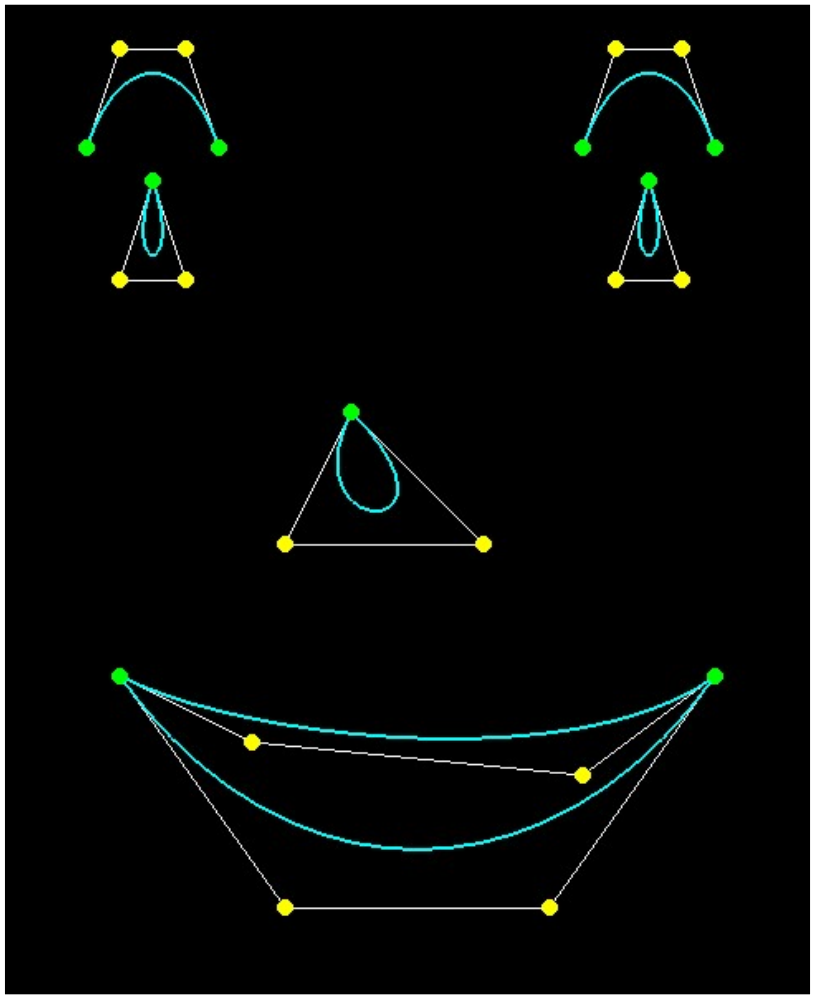
$$0. \le t \le 1.$$

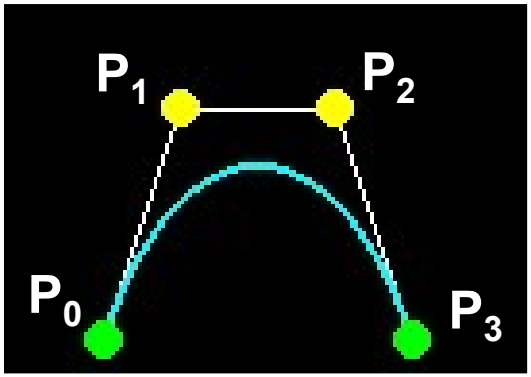


You draw the curve as a series of lines

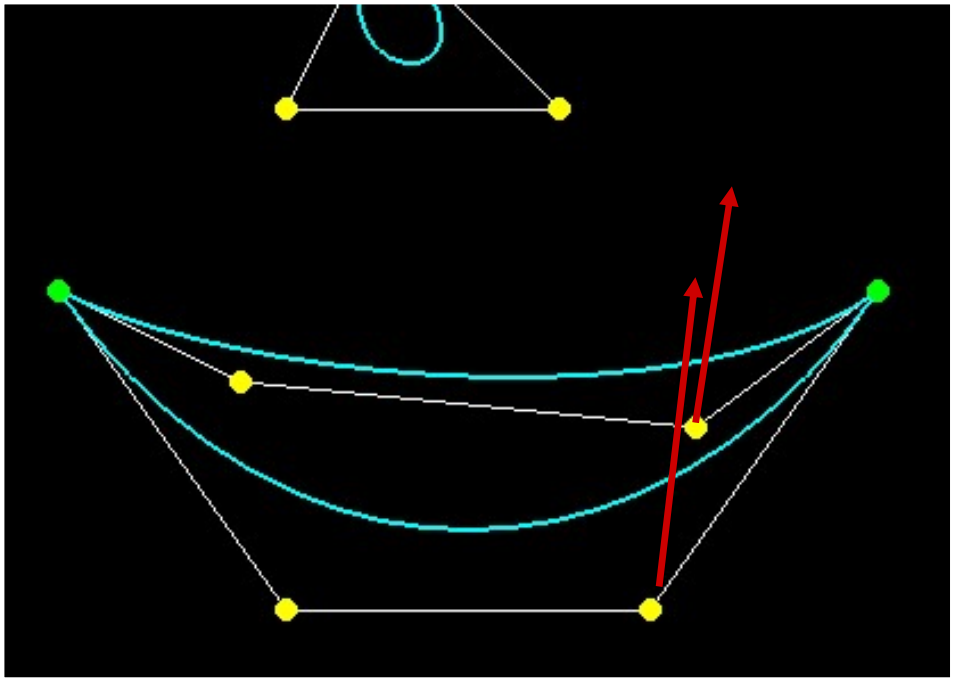
GL_LINE_STRIP is a good topology for this

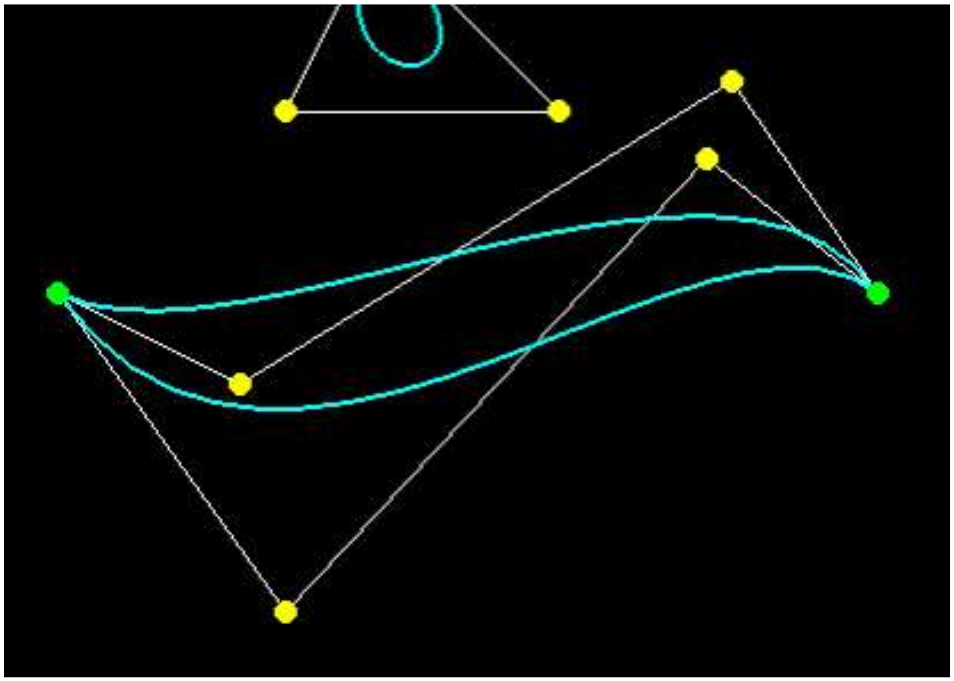
Curve Sculpting – Bezier Curve Sculpting Example





Curve Sculpting – Bezier Curve Sculpting Example





Moving a single control point moves its entire curve

A *Small* Amount of Input Change Results in a *Large* Amount of Output Change

Another way to Model:
Curve Sculpting – Catmull-Rom Curve Sculpting

The Catmull-Rom curve consists of any number of points.
The first point influences how the curve starts.
The last point influences how the curve ends.
The overall curve goes smoothly through all other points.

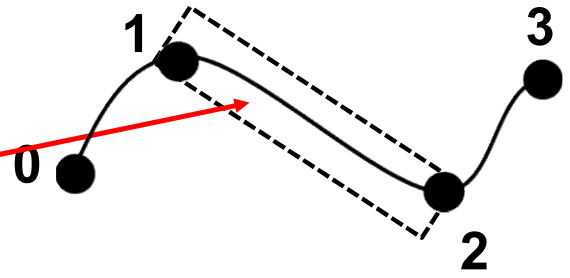
To draw the curve, grab points 0, 1, 2, and 3, call them P_0, P_1, P_2, and P_3, and loop through the following equation, varying t from 0. to 1. in an increment of your own choosing:

$$P(t) = 0.5 * \left[2 * P_1 + t * (-P_0 + P_2) + t^2(2 * P_0 - 5.* P_1 + 4P_2 - P_3) + t^3(-P_0 + 3P_1 - 3P_2 + P_3)\right]$$

$$0. \le t \le 1.$$

where ***P*** represents $\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$

For each set of 4 points, this equation just draws the line between the second and third points. That's why you keep having to use subsequent sets of 4 points



Another way to Model:
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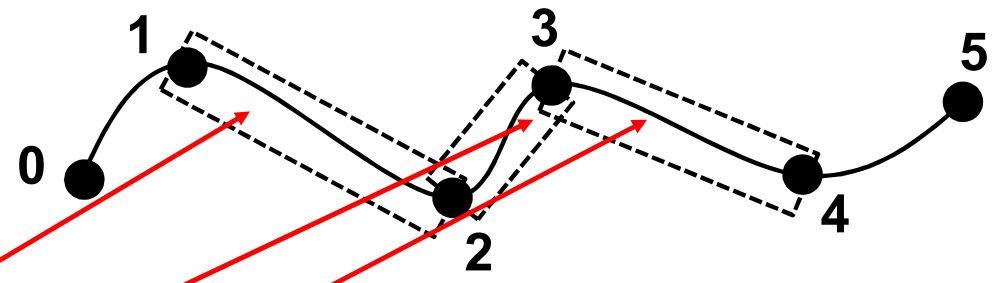


To draw the curve, grab points 0, 1, 2, and 3, call them P_0, P_1, P_2, and P_3, and loop through the equation, varying t from 0. to 1. in an increment of your own choosing.

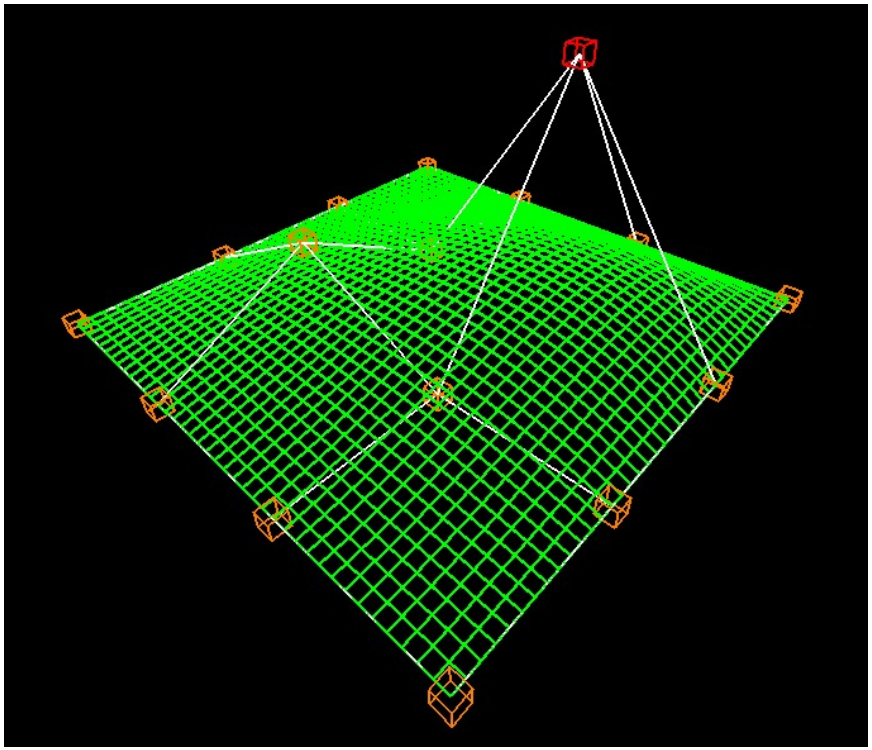
Then, grab points 1, 2, 3, and 4, call them P_0, P_1, P_2, and P_3, and loop through the same equation.

Then, grab points 2, 3, 4, and 5, call them P_0, P_1, P_2, and P_3, and loop through the same equation

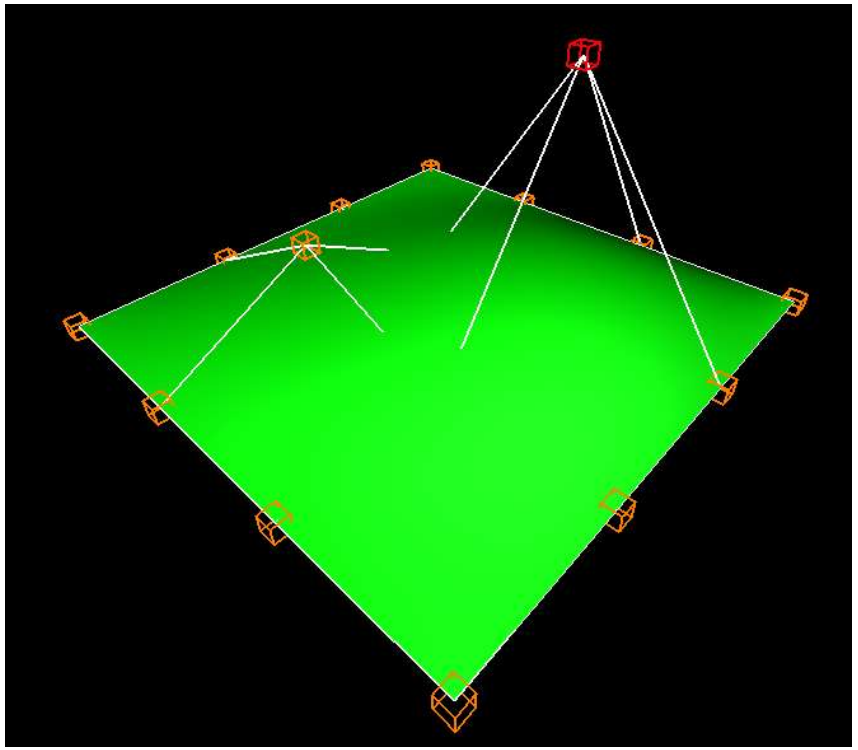
And so on…

A *Small* Amount of Input Change Results in a *Large* Amount of Output Change

Another way to Model: Surface Sculpting



Wireframe

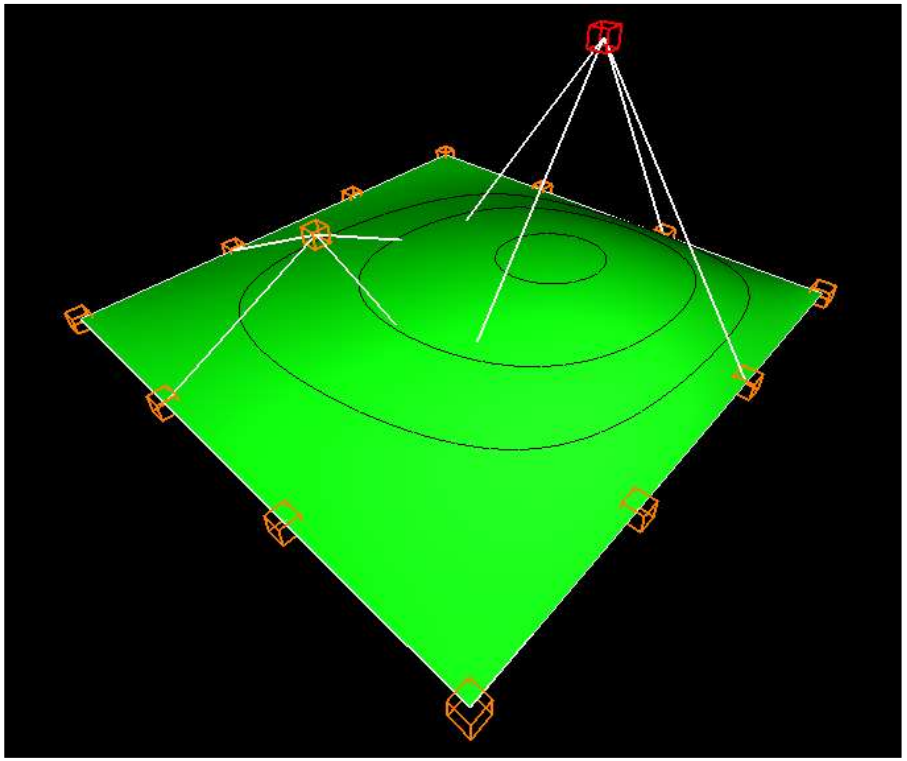


Surface

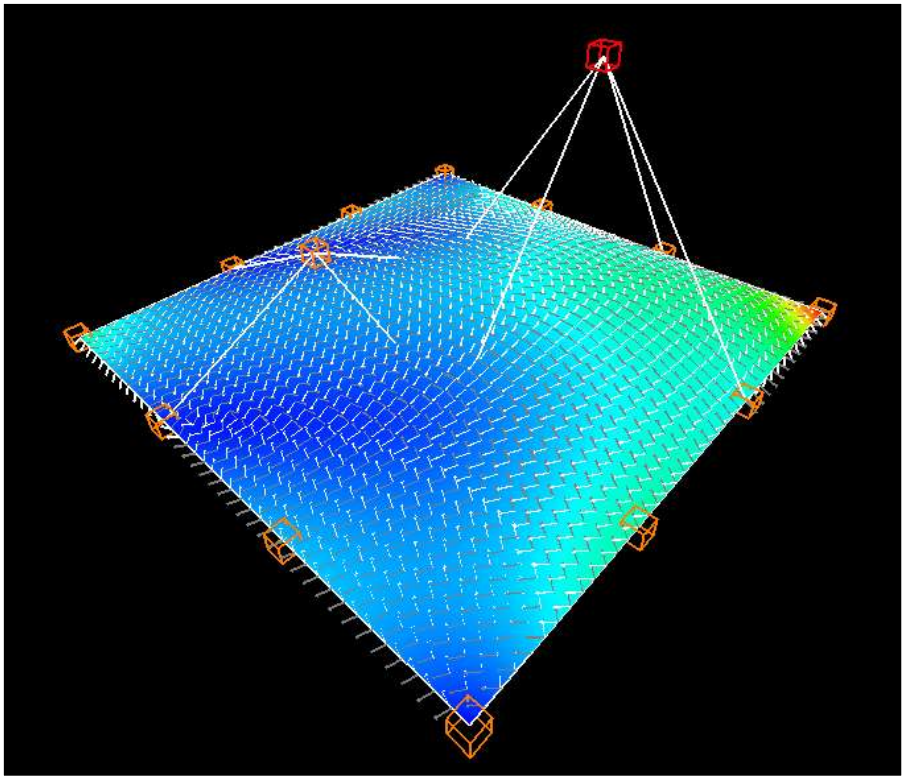
Moving a single point moves its entire surface

A *Small* Amount of Input Change Results in a *Large* Amount of Output Change

Surface Equations can also be used for Analysis

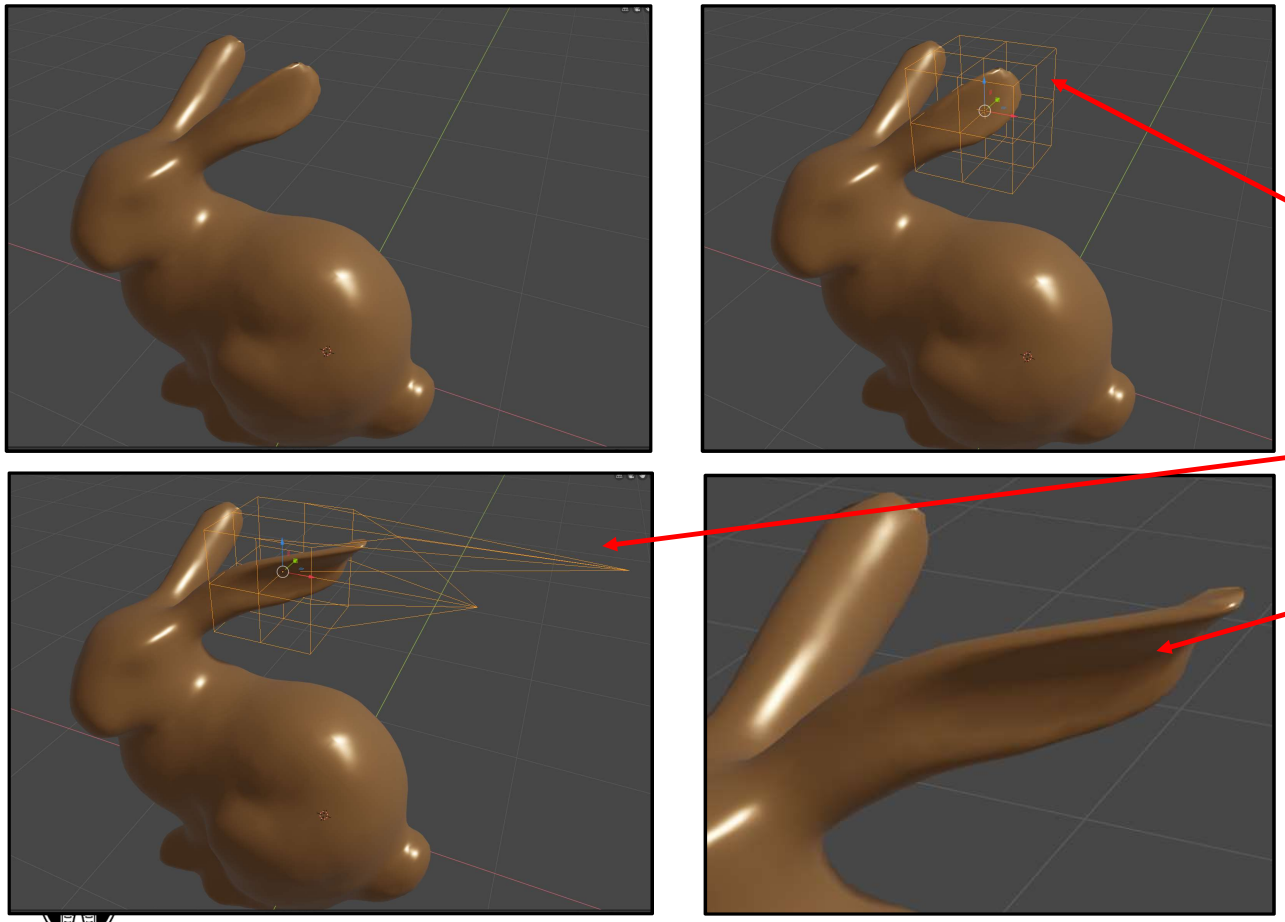


Showing Contour Lines



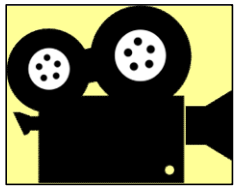
Showing Curvature

Another Way to Model: Volume Sculpting



This is often called a **"Lattice"** or a **"Cage".**

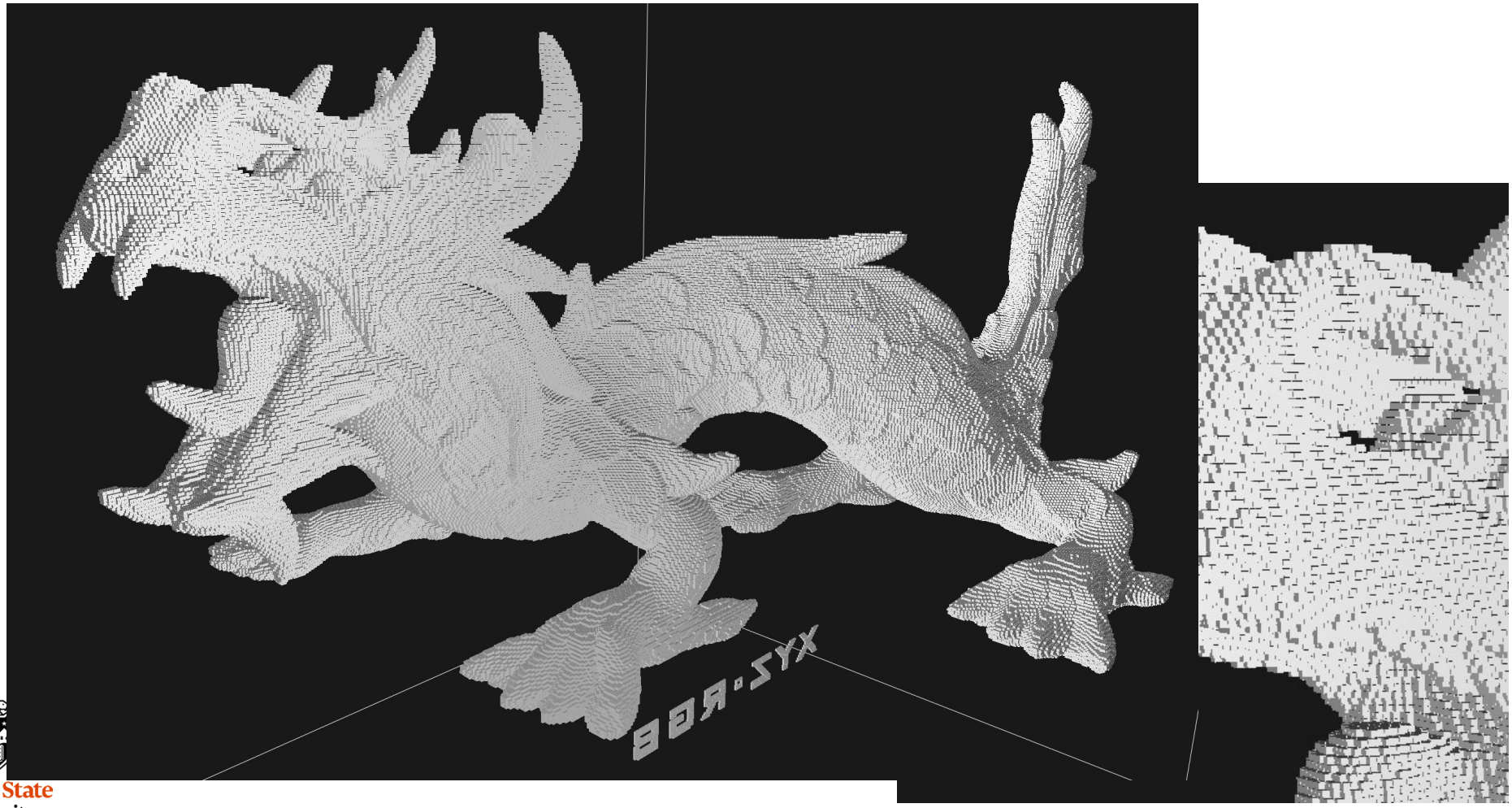
Slip a simpler object (e.g., a subdivided cube) around some of the object's vertices. As you sculpt the simpler object, all those object vertices get sculpted too



lattice.mp4

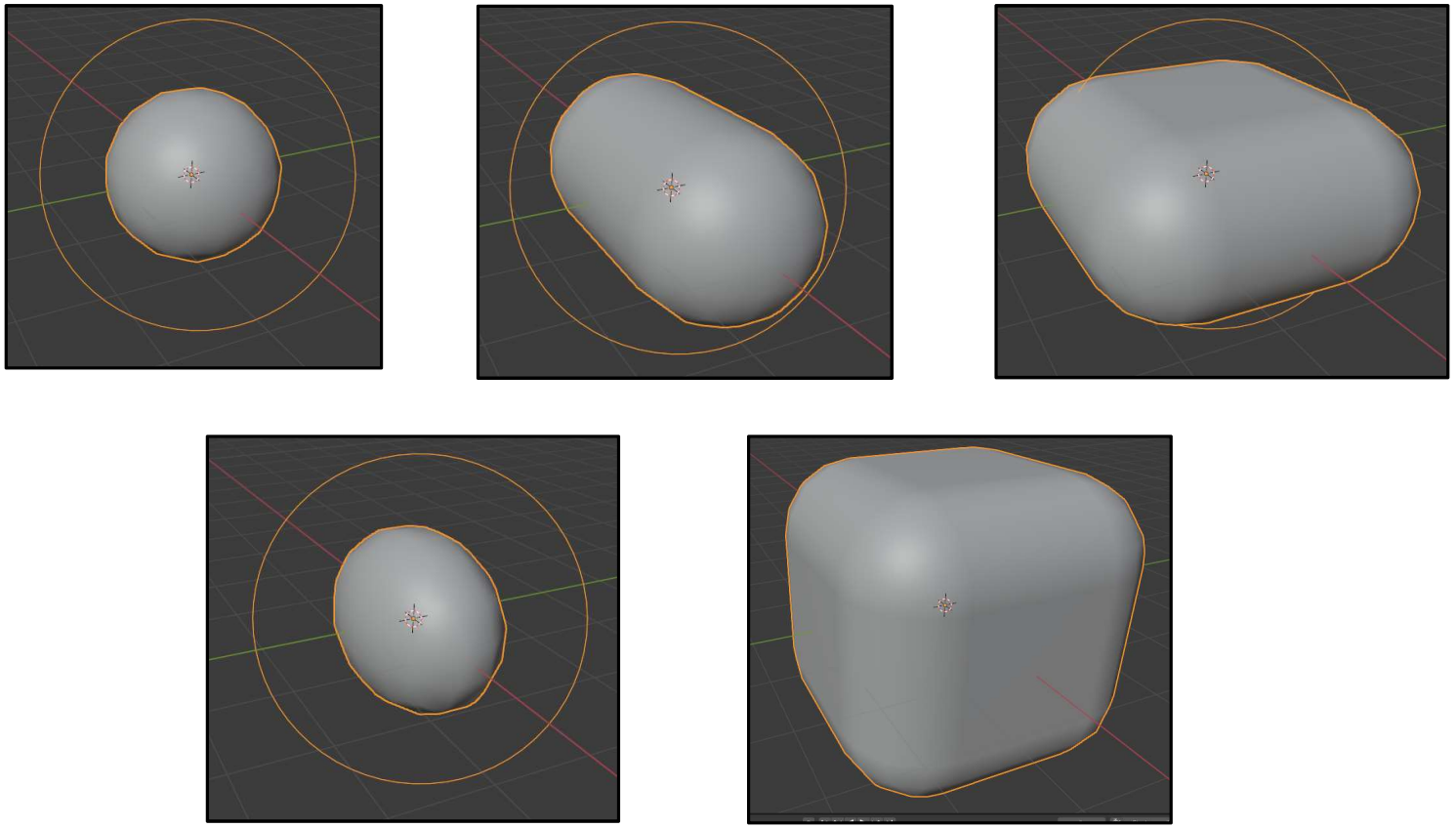
A *Small* Amount of Input Change Results in a *Large* Amount of Output Change

Voxelization as a Way to Model 3D Geometry



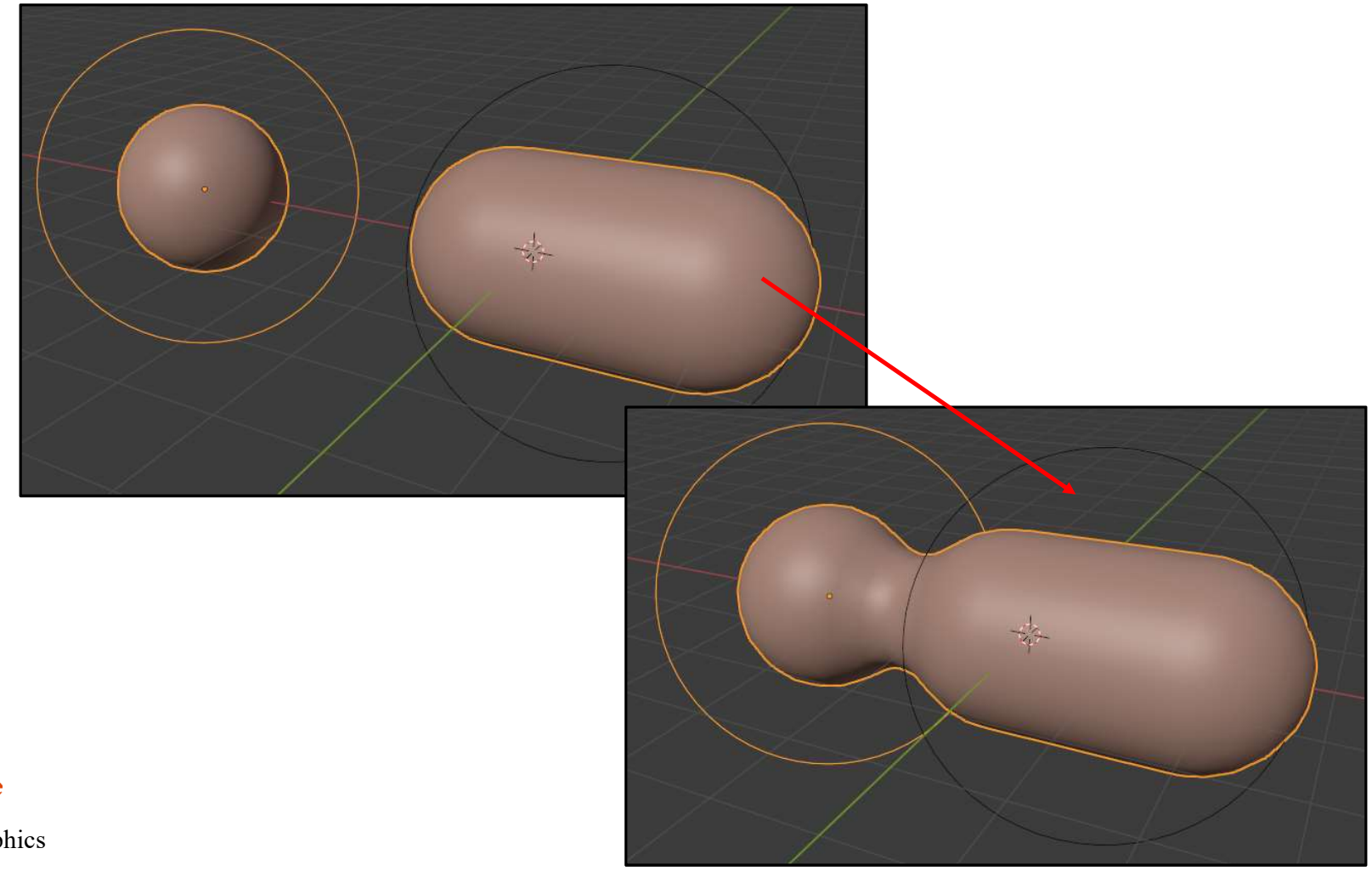
Randy Rauwendaal

Metaball Objects

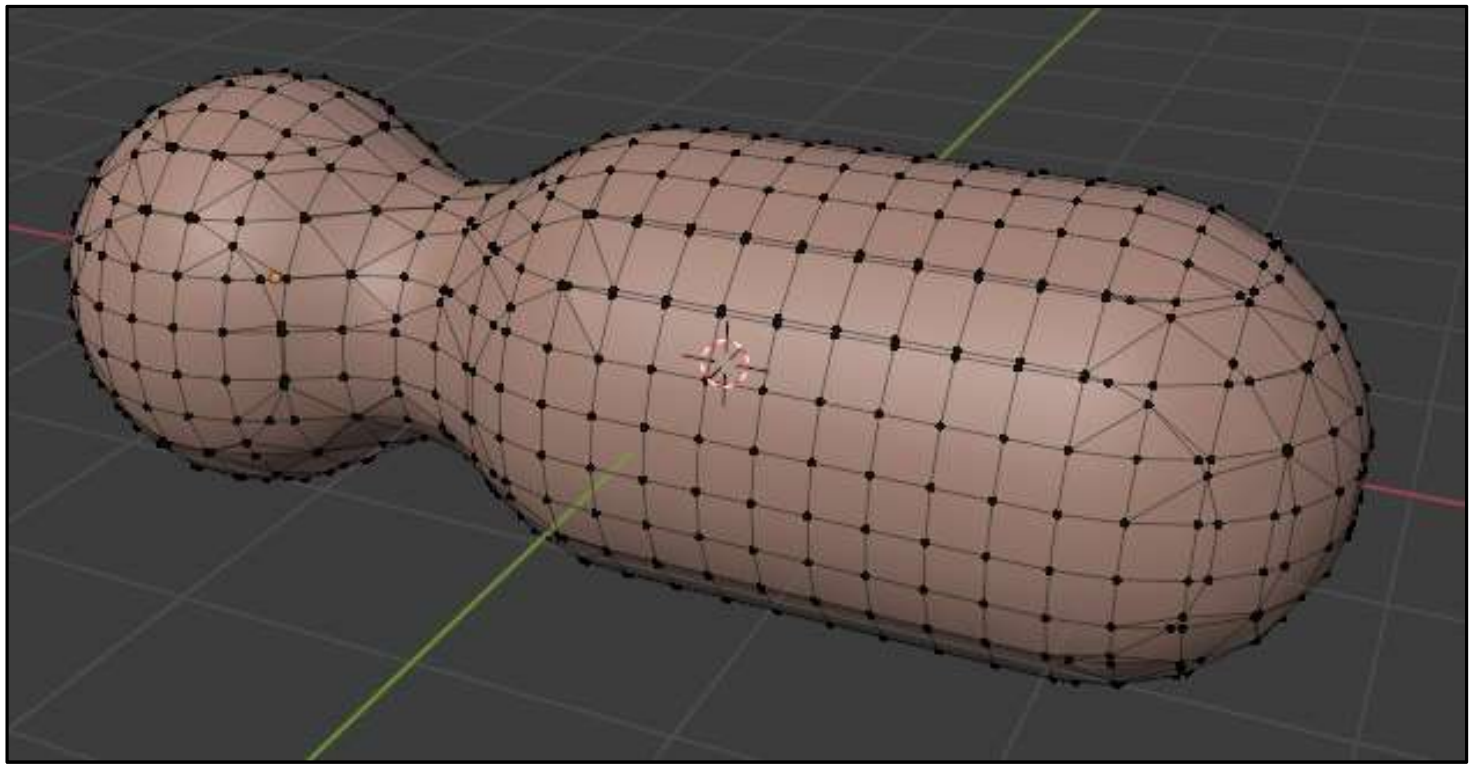


Metaball Objects

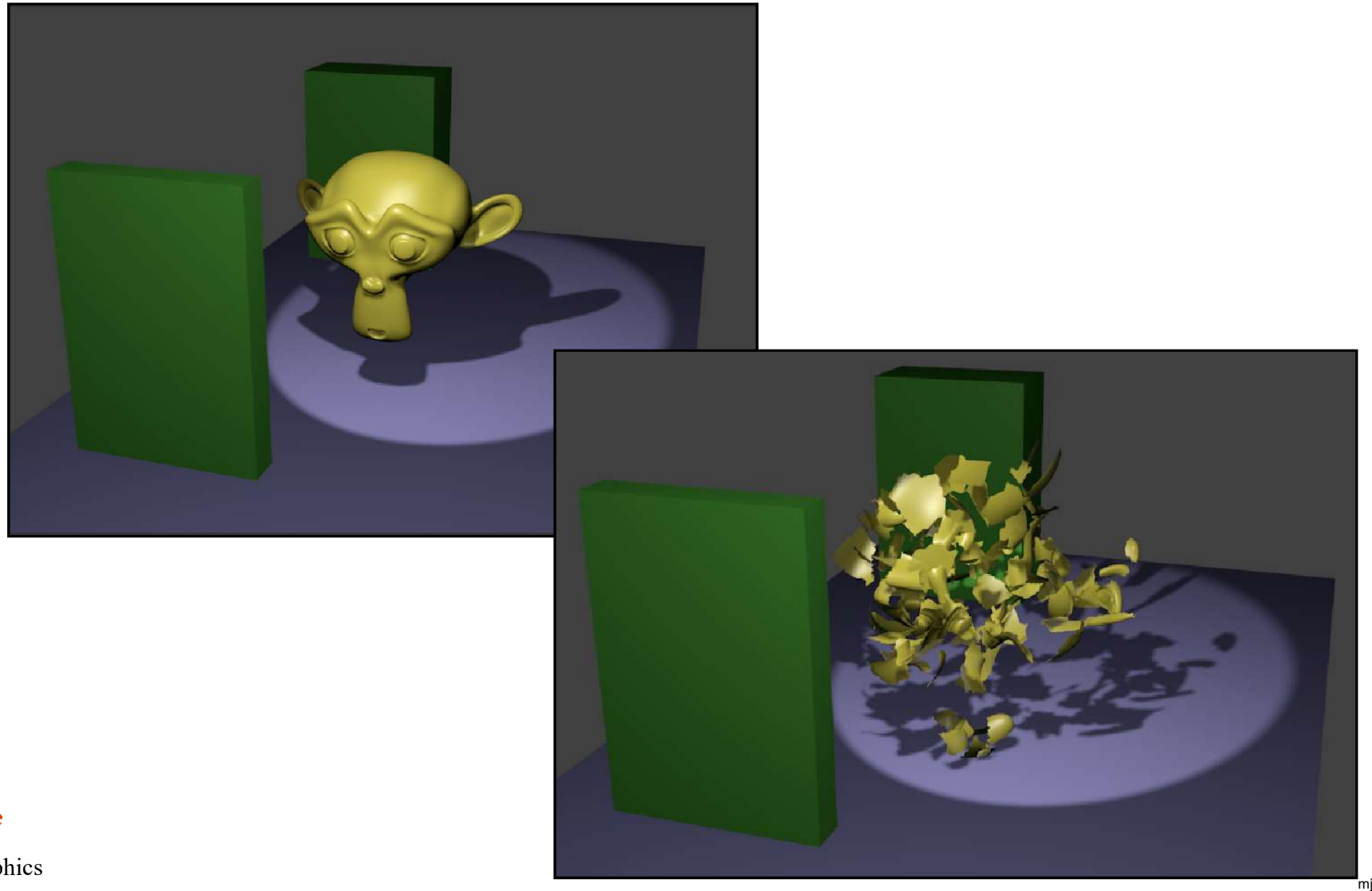
The cool thing is that, if you move them close enough together, they will “glom” into a single object



Metaball Objects Can Be Turned into Meshes for Later Editing

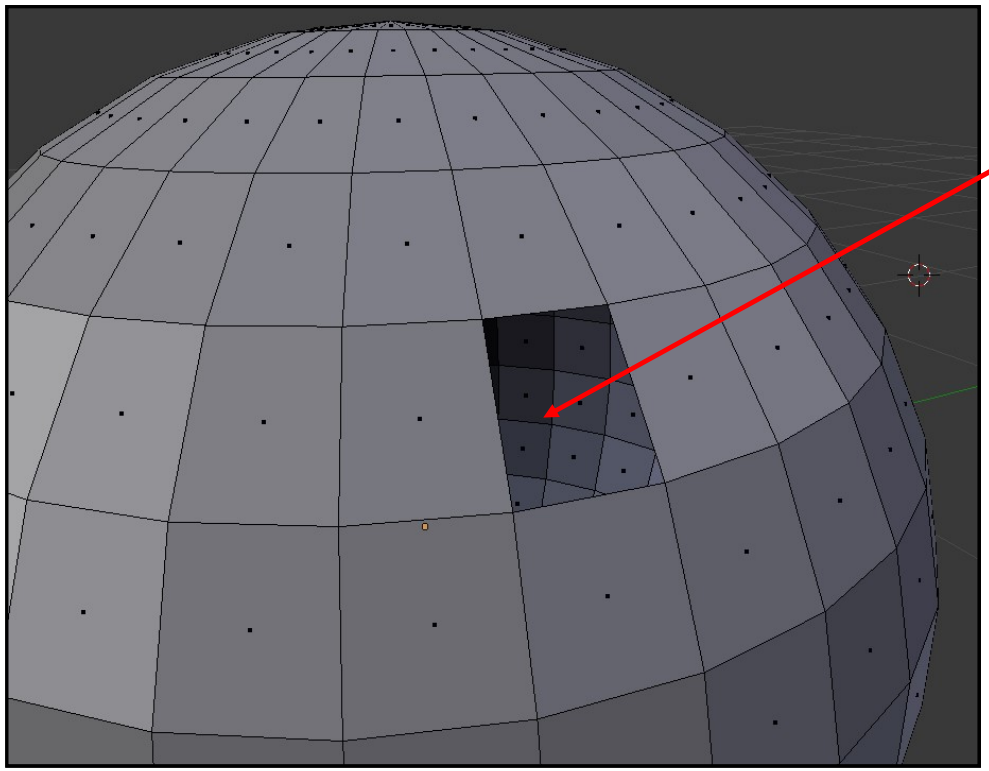


Modeling → Simulation (Explosion)



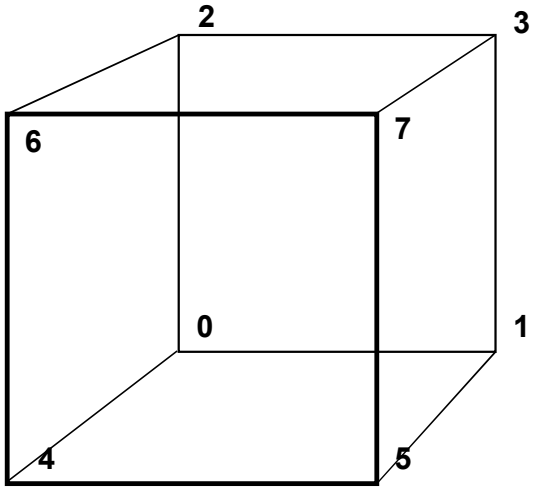
Object Modeling Rules for 3D Printing

The object must be a legal solid. It must have a definite inside and a definite outside. It can't have any missing face pieces.



"Definite inside and outside" is sometimes called **"Two-manifold"** or **"Watertight"**

The Simplified Euler's Formula* for Legal Solids



For a cube, 6 – 12 + 8 = 2

*sometimes called the Euler-Poincaré formula

F – E + V = 2

F	**Faces**
E	**Edges**
V	**Vertices**

The full formula is:

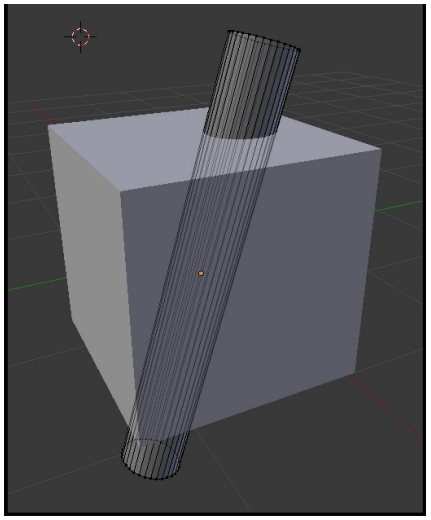
F – E + V – L = 2(B – G)

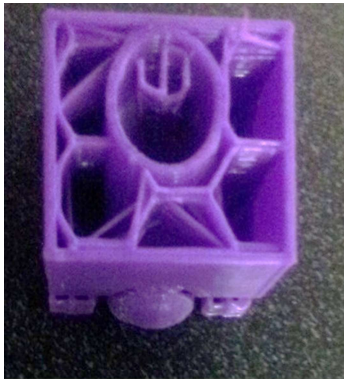
F	**Faces**
E	**Edges**
V	**Vertices**
L	**Inner Loops (within faces)**
B	**Bodies**
G	**Genus (number of through-holes)**

Object Modeling Rules for 3D Printing

Objects cannot pass through other objects. If you want two shapes together, do a Boolean union on them so that they become one complete object.

Overlapped in 3D -- bad





Boolean union -- good