Year 1 Coursework Set 2

Question 1

(A) An object moves along the x-axis with velocity v_x at time t given by:

$$v_x = At + Bt^2$$

where $A = 6.0\,\mathrm{ms}^{-2}$ and $B = -1.0\,\mathrm{ms}^{-3}$.

(i) Assuming that the object is located at the origin at $t = 0\,\mathrm{s}$, deduce expressions for the position x and acceleration a_x of the object at time t. Use the expression above and the two you have deduced to calculate values of x, v_x, and a_x at $t = 0, 3,$ and $6\,\mathrm{s}$.

The acceleration a_x is the derivative of velocity with respect to time:

$$\begin{aligned} a_x &= \frac{dv}{dt} \\ &= A - 2Bt \\ &= 6.0 - 2.0t \end{aligned}$$

The position x is obtained by integrating the velocity equation:

$$x = \int (At + Bt^2)\,dt$$

$$x = \frac{1}{2}At^2 + \frac{1}{3}Bt^3 + c$$

We know when $t = 0$, $x = 0$, therefore $c = 0$:

$$x = 3.0t^2 - \frac{1}{3}t^3$$

When $t = 0$:

$$a_x = 6.0\,\mathrm{ms}^{-2}, \quad v_x = 0.0\,\mathrm{ms}^{-1}, \quad x = 0.0\,\mathrm{m}$$

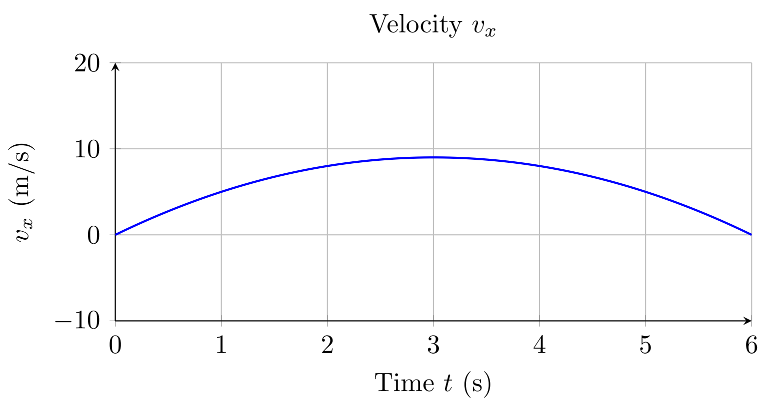
When $t = 3$:

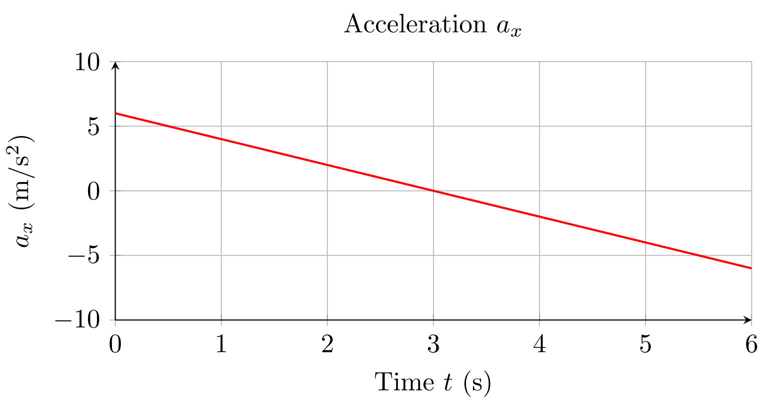
$$a_x = 0.0\,\mathrm{ms}^{-2}, \quad v_x = 9.0\,\mathrm{ms}^{-1}, \quad x = 18.0\,\mathrm{m}$$

When $t = 6$:

$$a_x = -6.0\,\mathrm{ms}^{-2}, \quad v_x = 0.0\,\mathrm{ms}^{-1}, \quad x = 36.0\,\mathrm{m}$$

(ii) Sketch stacked graphs of the velocity, v_x , and acceleration, a_x , of the object in the time interval $0 < t < 6$ s.





(B) Give three examples of a vector quantity and three examples of a scalar quantity.

Vectors: Force, Acceleration, Jerk

Scalars: Temperature, Mass, Luminosity

In a right-handed coordinate system (x, y, z), a vector **a** has components $a_x = 2$, $a_y = 5$, $a_z = 4$, and **b** has components $b_x = 1$, $b_y = 0$, and $b_z = 2$. Calculate:

(i) The magnitude of the vectors **a** and **b**;

$$|\vec{a}| = \sqrt{2^2 + 5^2 + 4^2} = \sqrt{4 + 25 + 16} = \sqrt{45}$$

$$|\vec{b}| = \sqrt{1^2 + 0^2 + 2^2} = \sqrt{1 + 0 + 4} = \sqrt{5}$$

(ii) The dot product $\mathbf{a} \cdot \mathbf{b}$;

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= \vec{a}_x \vec{b}_x + \vec{a}_y \vec{b}_y + \vec{a}_z \vec{b}_z = 2 + 5 + 6 \\ &= 13 \end{aligned}$$

(iii) The angle θ between **a** and **b**;

$$\mathbf{a} \cdot \mathbf{b} = |a||b|cos(\vartheta)$$

$$\frac{\mathbf{a} \cdot \mathbf{b}}{|a||b|} = cos(\vartheta)$$

$$\frac{13}{\sqrt{45} \times \sqrt{5}} = cos(\vartheta)$$

$$\vartheta = cos^{-1}(\frac{13}{15}) \approx 0.522$$

(iv) The x, y, and z components of the cross product $\mathbf{c} = \mathbf{a} \times \mathbf{b}$;

$$\vec{i} = (\vec{A}_j \vec{B}_k - \vec{A}_k \vec{B}_j)$$

$$\vec{j} = (\vec{A}_k \vec{B}_i - \vec{A}_i \vec{B}_k)$$

$$\vec{k} = (\vec{A}_i \vec{B}_j - \vec{A}_j \vec{B}_i)$$

$$\vec{i} = (5 \times 2 - 4 \times 0) = 10$$

$$\vec{j} = (4 \times 1 - 2 \times 2) = 0$$

$$\vec{k} = (2 \times 0 - 5 \times 1) = -5$$

(v) The unit vectors perpendicular to both **a** and **b** (give the x, y, and z components);

$$|\vec{c}| = \sqrt{10^2 + 0^2 + 5^2} = 5\sqrt{5}$$

$$\frac{1}{5\sqrt{5}}(10i + 0j - 5k) = (\frac{2}{\sqrt{5}}i + 0j - \frac{1}{\sqrt{5}}k)$$

(iv) The dot product of $\mathbf{a}$ with the unit vector $\mathbf{i}$ that points along the x-axis is given by $\mathbf{a} \cdot \mathbf{i}$. What component of $\mathbf{a}$ does this value represent?

$$\mathbf{a} \cdot \mathbf{i} = \vec{a}_x = 2$$

(vii) In the diagram shown in Figure 1.1, the vector $\mathbf{p}$ is given by $\mathbf{p} = \mathbf{q} - \mathbf{r}$. Using the vector dot product, derive the 'cosine rule':

$$p^2 = q^2 + r^2 - 2qr\cos P$$

where p, q, and r are the magnitudes of the vectors $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{r}$ respectively, and P is the angle between $\mathbf{q}$ and $\mathbf{r}$ as shown.

Hint: Begin by writing the right-hand side of the equation $p^2 = \mathbf{p} \cdot \mathbf{p}$ in terms of $\mathbf{q}$ and $\mathbf{r}$.

$$p^2 = (q - r) \cdot (q - r)$$
$$p^2 = q \cdot q - 2r \cdot q + r \cdot r$$
$$p^2 = q^2 - 2r \cdot q + r^2$$
$$r \cdot q = qrcos(\vartheta)$$
$$p^2 = q^2 + r^2 - 2qrcos(\vartheta)$$

Question 2.

A. A rocket takes off carrying an astronaut. The mass of the astronaut, including the space suit, is 100 kg. The rocket accelerates at $a = 4 \times 9.8\,\mathrm{ms}^{-2}$ during the first stage of the launch.

(i) What is the weight of the astronaut (including suit) before launch?

$$980N$$

(ii) What is the weight of the astronaut (including suit) during the first stage of the launch, when the rocket is accelerating vertically?

$$w = mg$$
$$g = 4 \times 9.8 - 9.8 = 3 \times 9.8$$
$$w = 3 \times 9.8 \times 100 = 2940N$$

The rocket accelerates to a speed sufficient to put the rocket into a stable circular orbit around the Earth and then the engines shut down.

(iii) Considering the radial acceleration of the rocket, explain why the astronaut is weightless in orbit.

As both the astronaut and the rocket have the same radial acceleration the rocket and the astronaut are experiencing the same acceleration towards the earth, the rocket cannot provide a normal force on the astronaut which is how we experience weight.

(iv) The astronaut's rocket docks with a fictional orbiting spacecraft. The craft is shaped like a hollow cylinder with inside diameter 100 m, and it rotates around its axis. Once inside the craft, the astronaut can stand up on the inner curved surface of the cylinder. How fast would the spacecraft need to rotate for the astronaut's weight on the spacecraft to be half that on the Earth? Express your answer in terms of the number of revolutions per minute required.

$$F = m\omega^2 r$$

$$4.9 = 200\omega^2$$

$$\omega = 0.1565$$

$$\omega = 2\pi f$$

$$f = 0.0249$$

$$rpm = 0.0249 \times 60 \approx 1.5rpm$$

B. Figure 2.1 shows a child's toy in which a ball is launched by a rotating arm. The ball is held at the end of the arm while it rotates, at a distance $r = 0.5\,\text{m}$ from the centre of rotation, as shown in the figure. The arm starts in the vertical position and rotates anti-clockwise, with constant angular acceleration, α, before releasing the ball. The arm moves through an angle of $\frac{5\pi}{4}$ radians before release. The ball then lands a horizontal distance X from the release point after time T. [$g = 9.8\,\text{ms}^{-2}$]

(i) Deduce an expression for the speed of the ball, v_0, on its release, in terms of α and r.

$$\omega^2 = 2\alpha\vartheta$$

$$v_0 = \omega^2 r$$

$$v_0 = r\sqrt{2\alpha\vartheta}$$

(ii) Show that the flight time from release is given by the expression

$$T = \sqrt{2}\frac{v_0}{g}$$

$$0 = v_0 t + \frac{1}{2} g T^2$$

$$v_{0y} = sin(\frac{\pi}{4}) = \frac{v_0}{\sqrt{2}}$$

$$\frac{v_0}{\sqrt{2}} = \frac{1}{2} g T$$

$$T = \frac{2v_0}{g\sqrt{2}} = \sqrt{2}\frac{v_0}{g}$$

(iii) Show that a flight time of 2 s implies that the arm has an angular acceleration of approximately $100\,\mathrm{rad\,s^{-2}}$.

$$2 = \sqrt{2}\frac{v_0}{9.8}$$

$$v_0 \approx 13.85929$$

$$13.85929 = 0.5\sqrt{2\alpha\frac{5\pi}{4}}$$

$$\alpha = 97.8255 \approx 100$$

(iv) Derive an expression for distance travelled, X, in terms of v_0 and g.

$$X = v_0 t + \frac{1}{2} g T^2$$

$$T = \sqrt{2}\frac{v_0}{g}$$

$$X = v_0 t + \frac{1}{2} g (\sqrt{2}\frac{v_0}{g})^2$$

$$X = v_0 t + \frac{v_0^2}{g}$$