Question 1

An experiment makes a series of voltage measurements that are distributed as in the table:

Voltage / V	5.0-5.2	5.2-5.4	5.4-5.6	5.6-5.8	5.8-6.0	6.0-6.2	6.2-6.4
Frequency	4	17	49	65	41	21	3

(a) Estimate the mean voltage measurement.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}(x_i)$$

$$\bar{x} = \frac{1139.4}{200} = 5.697$$

(b) Estimate the variance of the measurements.

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$S^2 = 0.0613$$

(c) How many standard deviations away from the mean would a measurement of 6.45V be?

$$S = 0.247$$

$$\frac{6.45 - 5.697}{0.247} = 3.048 \approx 3$$

Question 2

An atomic force microscope (AFM) measures the distance between two atoms to be 2.6 nm with an error of 0.3 nm. A second AFM measures the distance to be 3.1 nm, with an error of 0.4 nm.

(a) Assuming that the errors are Gaussian, what is the probability that the difference between measurements is as large as observed, or larger, purely due to random errors?

$$3.1 - 2.6 = 0.5$$

$$\mu_{ab} = \sqrt{\mu_a^2 + \mu_b^2} = \sqrt{0.4^2 + 0.3^2} = 0.5$$

$$Z = \frac{0.5}{0.5} = 1 = 68\%$$

The measurements are repeated 9 times with each AFM. The means of the two sets of measurements are 2.7 nm and 3.2 nm, respectively.

(b) What is the error on each mean value?

$$\bar{\mu_1} = \frac{\mu_1}{\sqrt{n}} = \frac{0.3}{\sqrt{9}} = 0.1$$

$$\bar{\mu_2} = \frac{\mu_2}{\sqrt{n}} = \frac{0.4}{\sqrt{9}} = \frac{2}{15} \approx 0.1333$$

(c) What is the probability that the difference in the means is due to random error?

$$3.2 - 2.7 = 0.5$$

$$\mu = \sqrt{\mu_a^2 + \mu_b^2} = \sqrt{0.1^2 + (\frac{2}{15})^2} = \frac{1}{6}$$

$$Z = \frac{0.5}{1/6} = 3 = 99.7\%$$

$$P = 100 - 99.7 = 0.3\%$$

(d) Are the two sets of measurements consistent with each other?

The measurements are consistent as the probability of the difference in the means being random is very low

Question 3

A student is taking four modules. The probability that they pass any individual module depends on the fraction of lectures attended, f, such that the probability of passing a module is $0.97f$.

(a) If the student attends 90% of lectures, ($f = 0.9$), calculate the probability that they will

(i) pass all four modules,

$$P = (0.97 \times 0.9)^4 = 0.58 = 58\%$$

(ii) fail one module and pass the other three

$$P = 4 \times (0.97 \times 0.9)^3(1 - (0.97 \times 0.9)) = 0.338 = 33.8\%$$

(iii) pass no modules.

$$P = (1 - (0.97 \times 0.9))^4 = 0.00026 = 0.026\%$$

(b) What is the probability that the student passes at least three modules?

$$P = 0.338 + 0.58 = 0.92 = 92\%$$

(c) What fraction of lectures must the student attend to have a 40% chance of passing all four modules?

$$(0.97 \times x)^4 = 0.4$$

$$x = 0.82$$

(d) Why is this not a realistic way to model the probability that the student will pass the modules?

Lecture attendance is not the only factor affecting module passes, as well as this the correlation is unlikely to be linear