1.

Express the fraction

$$\frac{1}{2x^3 + 7x^2 + 4x - 4}$$

As partial fractions

$$(2x - 1)(x^2 + 4x + 4)$$

$$(2x - 1)(x + 2)^2$$

$$\frac{a}{(x + 2)} \frac{b}{(x + 2)^2} \frac{c}{(2x - 1)}$$

$$a(x + 2)(2x - 1) + b(2x - 1) + c(x + 2)^2 = 1$$

Let $x = -2$

$$1 = b((2 \times -2) - 1) \qquad b = -\frac{1}{5}$$

Let $x = \frac{1}{2}$

$$1 = c(\frac{1}{2} + 2)^2 \qquad c = \frac{4}{25}$$

Let $x = 1$

$$1 = a(3) - \frac{1}{5} + \frac{4}{25}(3)^2 \qquad c = -\frac{2}{25}$$

$$\frac{4}{25(2x - 1)} - \frac{1}{5(x + 2)^2} - \frac{2}{25(x + 2)}$$

2.

Find all the roots of the equation

$$z^5 + 4z^3 + 3z = 0$$

$$(z)(z^4 + 4z^2 + 3) = 0$$

$$z^2 = x$$

$$x^2 + 4x + 3 = 0$$

$$x = -1, 3$$

$$z^2 = -1$$

$$z = i$$
$$re^{i\vartheta} = e^{i(\pi/2+2\pi n)}$$
$$r = 1$$
$$\vartheta = \pi/2, 3\pi/2$$
$$e^{i\pi/2}, e^{i3\pi/2}$$

$$z^2 = -3$$
$$z = 3i$$
$$re^{i\vartheta} = 3e^{i(2\pi n)}$$
$$r = \sqrt{3}$$
$$\vartheta = \pi/2, 3\pi/2$$
$$\sqrt{3}e^{i\pi}/2, \sqrt{3}e^{3i\pi/2}$$

Roots

$$0, \pm i, \pm i\sqrt{3}$$
$$0, e^{i\pi/2}, e^{i3\pi/2}, \sqrt{3}e^{i\pi/2}, \sqrt{3}e^{3i\pi/2}$$

3.

Find the first three non-zero terms in the Taylor series of the following functions about the points given:

a) $x\exp(x^2)$ about $x = 1$

$$f(1) = e$$

$$f'(x) = uv' + u'v$$
$$\exp(x^2) + 2x^2\exp(x^2)$$
$$f'(1) = 3e$$

$$f''(x) = 2x^2\exp(x^2) + uv' + u'v$$
$$uv' + u'v = 2x^2(2x^2\exp(x^2)) + \exp(x^2)(4x)$$
$$2x^2\exp(x^2) + 2x^2(2x^2\exp(x^2)) + \exp(x^2)(4x)$$
$$f''(1) = 2e + 4e + 4e = 10e$$

$$e + 3e(x-1) + 5e(x-1)^2$$

b) $(1+\tan x)^{-1}$ about $x = \pi/3$

$$tan(\pi/3) = \sqrt{3}$$

$$f(pi/3) = \frac{1}{1+\sqrt{3}}$$

First differentiation

$$(1+\tan x)^{-1}$$

$$u = 1 + \tan x$$

$$y = u^{-1}$$

$$\frac{du}{dx} = (1+\tan x^2)$$

$$\frac{dy}{du} = -1u^{-2}$$

$$\frac{dy}{dx} = -(1+\tan x)^{-2}(1+\tan x^2)$$

$$f'(\pi/3) = -\frac{4}{(1+\sqrt{3})^2}$$

$$(1+\sqrt{3})^2 = 4+2\sqrt{3}$$

$$-\frac{4}{(1+\sqrt{3})^2} \times \frac{4-2\sqrt{3}}{4-2\sqrt{3}}$$

$$-\frac{16-8\sqrt{3}}{4} = -4+2\sqrt{3}$$

Second differentiation

$$-(1+\tan x)^{-2}(1+\tan x^2)$$

$$u = -(1+\tan x)^{-2}$$

$$v = (1+\tan x^2)$$

$$z = (1+\tan x)$$

$$u = -z^{-2}$$

$$\frac{du}{dz} = 2z^{-3}$$

$$\frac{dz}{dx} = (1 + \tan x^2)$$
$$u' = 2(1 + \tan x)^{-3}(1 + \tan x^2)$$

$$z = \tan x$$
$$v = u^2$$
$$\frac{du}{dz} = 2u$$
$$\frac{dz}{dx} = 1 + \tan x^2$$
$$v' = 2\tan x(1 + \tan x^2)$$

$$uv' + u'v$$
$$-(1 + \tan x)^{-2} 2\tan x(1 + \tan x^2) + (1 + \tan x^2)2(1 + \tan x)^{-3}(1 + \tan x^2)$$
$$-2\sqrt{3}(4)(1 + \sqrt{3})^{-2} + 2(1 + \sqrt{3})^{-3}4^2$$
$$\frac{32}{(1 + \sqrt{3})^3} - \frac{8\sqrt{3}}{(1 + \sqrt{3})^2}$$
$$(1 + \sqrt{3})^3 = 10 + 6\sqrt{3}$$
$$(1 + \sqrt{3})^2 = 4 + 2\sqrt{3}$$

$$\frac{32}{10 + 6\sqrt{3}} \times \frac{10 - 6\sqrt{3}}{10 - 6\sqrt{3}} = \frac{32(10 - 6\sqrt{3})}{-8} = -40 + 24\sqrt{3}$$
$$\frac{8\sqrt{3}}{4 + 2\sqrt{3}} \times \frac{4 - 2\sqrt{3}}{4 - 2\sqrt{3}} = \frac{32\sqrt{3} - 48}{4} = 8\sqrt{3} - 12$$
$$-40 + 24\sqrt{3} - 8\sqrt{3} - 12 = 16\sqrt{3} - 28$$

$$\frac{1}{1 + \sqrt{3}} - \frac{4}{(1 + \sqrt{3})^2}(x - \pi/3) + 8\sqrt{3} - 14(x - \pi/3)^2$$
$$\frac{1}{1 + \sqrt{3}} - 4 + 2\sqrt{3}(x - \pi/3) + 8\sqrt{3} - 14(x - \pi/3)^2$$