Auctions Simulations

Donny Chen, Lin Chen, April Liu, Zefeng Zhang

Feb 26, 2017

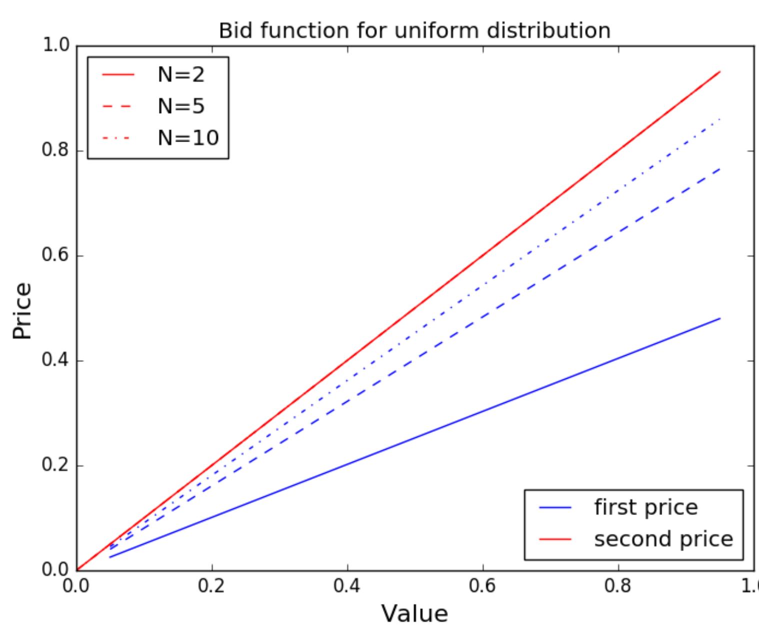
Question 1

For each of the distribution below, please:

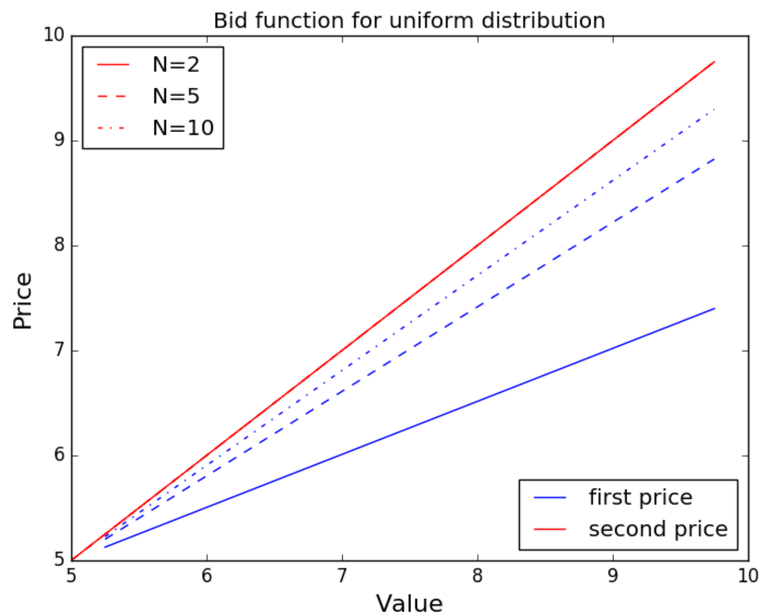
(a) Plot the bid function of a second-price auction, and a first-price auction with 2, 5, 10 bidders.

For second-price auction, the bid price will be the truthful bidder's signal. For first-price auction, the bid price will be shaded from the truthful bidder's signal according to the number of bidders present. Given a bidders' signals, We used the integral of cumulative density function to compute the amount of shade and then calculate the corresponding bidding prices. Looking at the plots, we can see that as the number of bidders increases, the bidder shade less from his signal. In addition, as the number of bidders increases, the bidding price for the second-price auction is getting closer to the bidding price for the first-price auction.

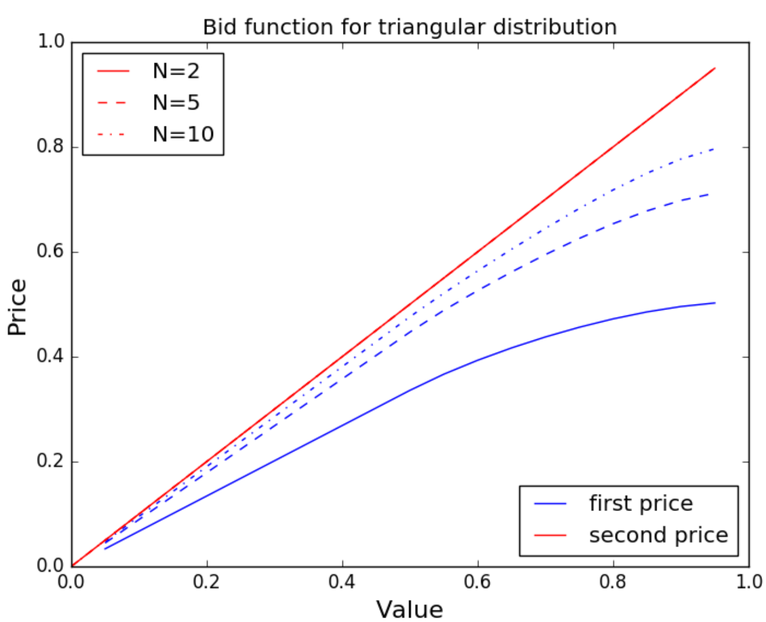
Distribution 1. Uniform [0, 1]



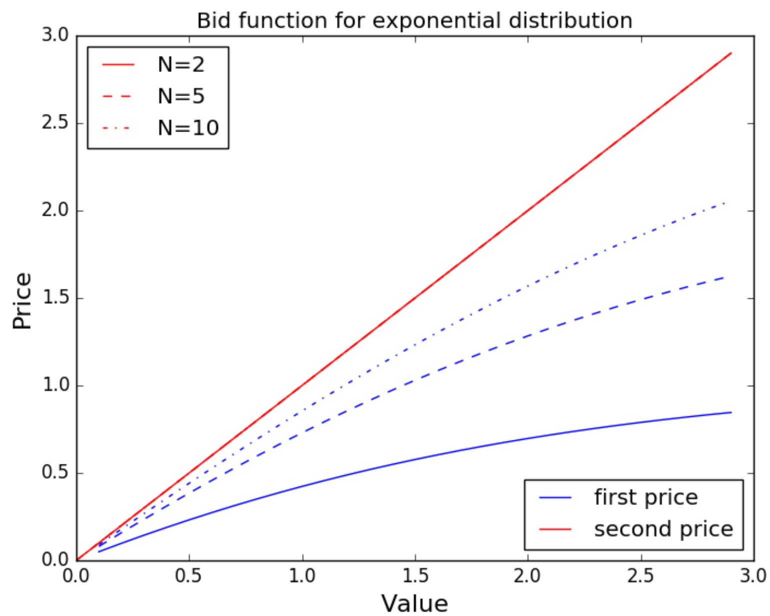
Distribution 2. Uniform [5, 10]



Distribution 3. Triangle [0, 1, 0.5]



Distribution 4. Exponential (λ = 1)



(b) Compute the expected revenue (how much money the seller receives) in both the first-price auction and the second-price auction with 2, 5, 10 bidders. Also compute the standard deviation of the seller's revenue.

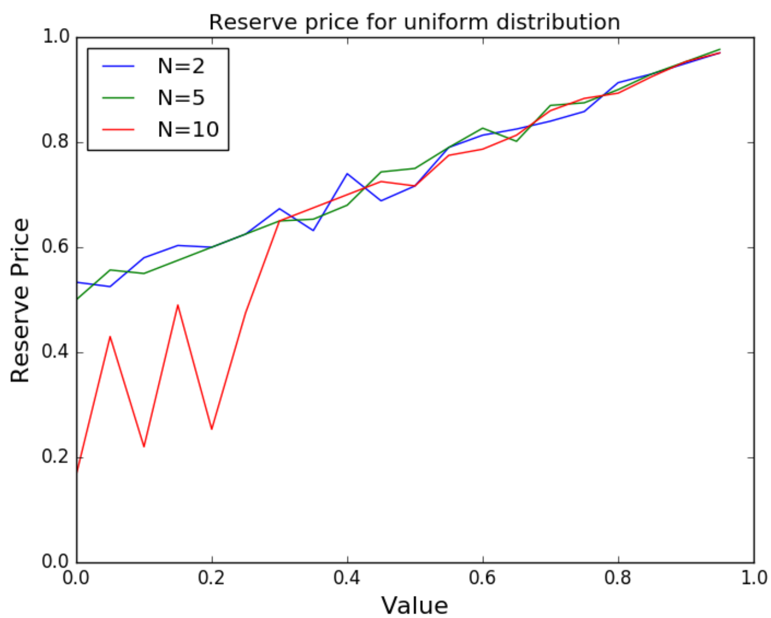
We simulated 5000 auctions and computed the expected revenue in both the first-price and the second-price auction with 2, 5, 10 bidders for the above 4 distributions. Our results demonstrate that the expected revenue for each distribution in first and second price auctions is approximately the same, while the expected revenue of the first-price auction shows less variance than the expected revenue of the second-price auction.

Distribution	n	1st Price Exp Rev	std	2nd Price Exp Rev	std
unif [0, 1]	2	0.334569	0.119732	0.331712	0.235551
unif [0, 1]	5	0.671176	0.110750	0.667474	0.177250
unif [0, 1]	10	0.821772	0.075131	0.819331	0.111726
unif [5, 10]	2	6.673708	0.600184	6.658559	1.177754
unif [5, 10]	5	8.358377	0.556268	8.337370	0.886250
unif [5, 10]	10	9.112812	0.377477	9.096655	0.558632
triangle [0, 1, 0.5]	2	0.384939	0.084454	0.382373	0.168065
triangle [0, 1, 0.5]	5	0.610059	0.073668	0.606992	0.118836
triangle [0, 1, 0.5]	10	0.716734	0.062457	0.714515	0.094161
exp (λ=1)	2	0.504090	0.247491	0.499459	0.511610
exp (λ=1)	5	1.296432	0.414278	1.290422	0.696830
exp (λ=1)	10	1.943824	0.480527	1.939917	0.748877

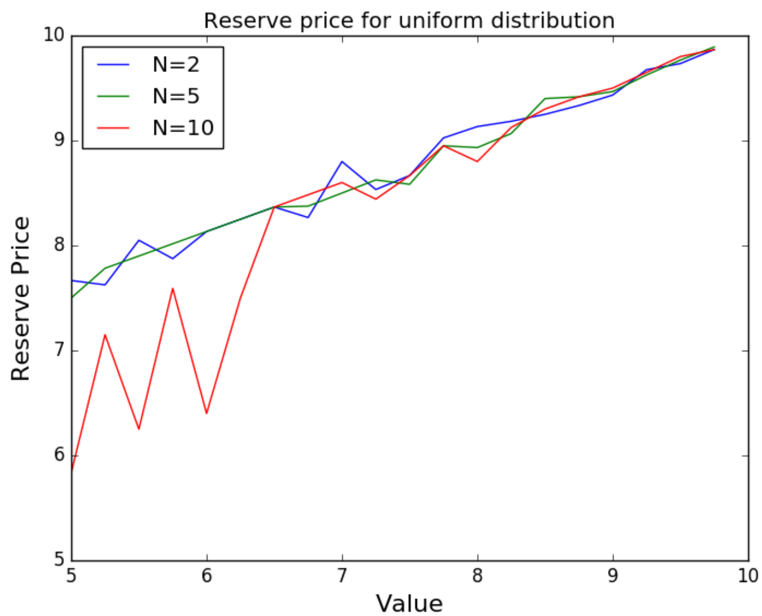
(c) Assume that the seller can choose a reserve price and has their own value for the item. Plot the optimal reserve price as a function of the value of the item to the seller. Assume that a second-price auction is held. Plot this in the case of 2, 5, and 10 bidders.

We simulated 3000 second-price auctions for the above four distributions to get the optimal reserve price given the seller's own value of the item. This is assuming that the optimal reserve price maximizes the seller's profit (profit = second price - seller's value). Please see more detailed discussion in Question 3.

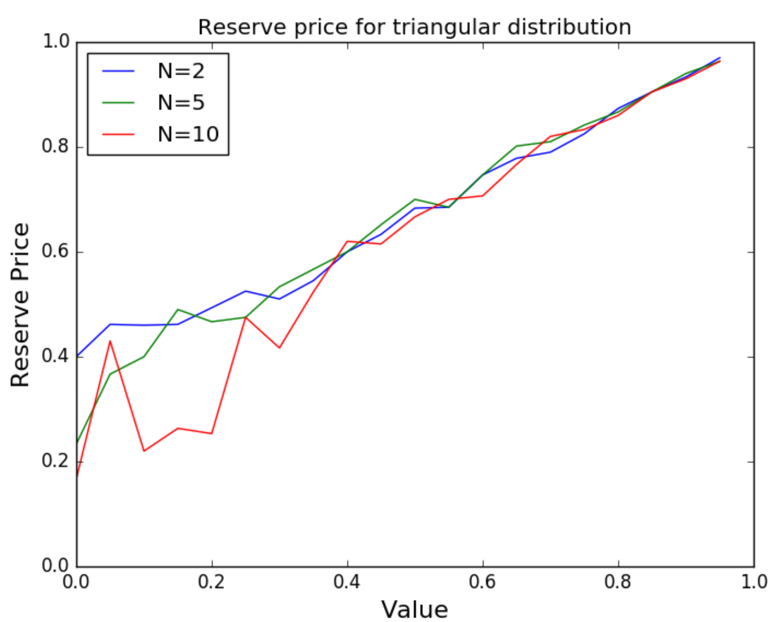
Distribution 1. Uniform [0,1]



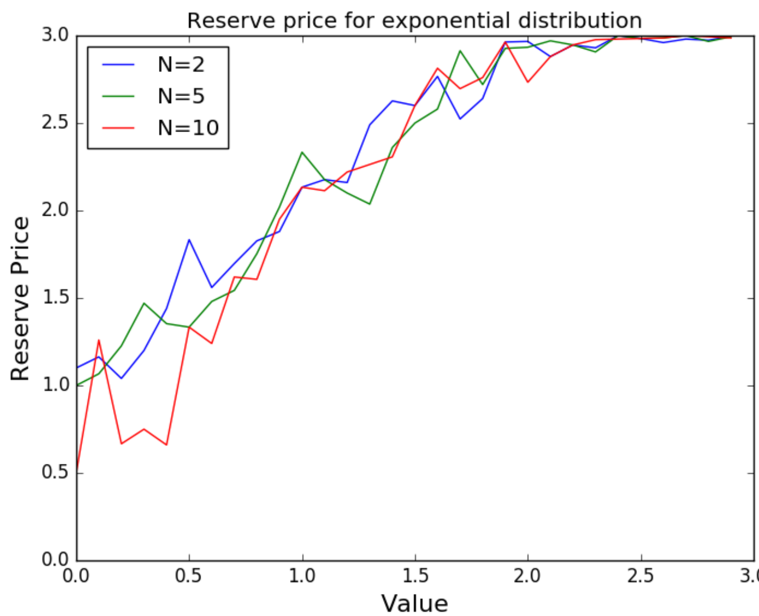
Distribution 2. Uniform [5,10]



Distribution 3. Triangle [0,1]



Distribution 4. Exponential (λ = 1)



Question 2

To intuitively explain the relationship between the expected revenue in the first and second-price auctions, we can transform the question into: *how should a seller choose between first and second price auctions to maximize his/her revenue?* This seems to be a dilemma for the seller. Suppose that all bidders bid truthfully, on one hand, in a first-price auction, the bidder pays the highest winning price in the first-price auction while the bidder pays the second highest price in the second-price auction, on the other hand, bidders tend to shade down their bids in the first-price auction.

We can examine the above opposing factors trade-off by our results. According to our results, with independent private values and truthful bidders, the two types of auctions provide the same expected revenue to the seller. This supports the revenue equivalence theory which asserts that "a seller's revenue will be the same across a broad class of auctions and arbitrary independent distributions of bidder values, when bidders follow equilibrium strategies". As far as the variance's concern (variance in revenue of the first-price auction is less than the second-price auction), a risk-averse seller facing risk neutral bidders with independent private values would fare better with a first-price auction than a second-price auction.

Question 3

Based on the plots above, the optimal reserve price indicates an approximately linear growth as the seller's value increases for uniform and triangular distributions. It is intuitive that as the seller's value goes up, the reserve price should also increase accordingly to avoid potential loss and maximize the seller's profit. Despite a large variance in the reserve price for the exponential distribution, a similar increasing trend is also observed as the seller's value grows.

Additionally, the plots above also confirm the marginal dependence of the optimal reserve price upon the number of bidders for all 4 distributions. Although the optimal reserve price indicates a high variance at small values, especially for a large number of bidders, the reserve prices for different numbers of bidders gradually converge as the seller's value increases. It makes sense that given a small seller's value but a large number of bidders, the optimal reserve price is expected to be arbitrary and variant due to very small chances of losing money and thereby small impact of reserve prices on the total profit. Nevertheless, the estimated reserve prices gradually stablize as the likehihood of losing money increases.